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1 **Title:** Design Charts for Contaminant Transport through Slurry Trench Cut-off Walls

2

3

4 **Authors:** Yu-Chao Li¹; Guan-Nian Chen²; Yun-Min Chen³; Peter John Cleall⁴

5

6 ¹ Associate Professor, MOE Key Laboratory of Soft Soils and Geoenvironmental
7 Engineering, Department of Civil Engineering, Zhejiang University, Hangzhou 310058,
8 China.

9 ² MSc student, MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering,
10 Department of Civil Engineering, Zhejiang University, Hangzhou 310058, China.

11 ³ Professor, MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering,
12 Department of Civil Engineering, Zhejiang University, Hangzhou 310058, China.

13 ⁴ Senior Lecturer, Cardiff School of Engineering, Cardiff University, Cardiff, CF24 3AA,
14 Wales, UK. E-mail: cleall@cf.ac.uk. Tel: +44 (0)29 2087 5795. Fax: +44 (0)29 2087
15 4716 (corresponding author)

16

17 **Abstract:** Slurry trench cut-off walls with low-permeability backfill material, such as
18 soil-bentonite and slag-cement-bentonite, are used widely for containment of subsurface
19 pollution. In the design of slurry walls the potential service life for a given thickness or
20 the wall thickness for a target service life are typically determined via analyses of one-
21 dimensional contaminant transport. The difficulty of selecting appropriate inlet and
22 outlet boundary conditions and the mathematical complexity of analytical solutions
23 hinder engineers from undertaking a contaminant transport analysis based design. Design
24 charts for non-dimensionalized effluent flux are presented by developing and utilizing an
25 analytical solution. The methodologies of using these charts in design are demonstrated.

26

27

28 **Keywords:** contaminant transport; slurry wall; design chart; subsurface contamination

29

30 **Introduction**

31

32 Slurry trench cut-off walls (termed as slurry walls hereafter) with low-permeability
33 backfills, such as soil-bentonite and slag-cement-bentonite, are widely used for
34 containment of subsurface pollution (D'Appolonia, 1980; LaGrega, et al., 2001; Opdyke
35 and Evans, 2005; Jefferis, 2012). Many laboratory studies have been conducted to
36 evaluate properties of the backfills (Evans, 1994; Filz, et al., 2001; Yeo, et al., 2005;
37 Joshi, et al., 2010; Soga, et al., 2013) with a focus on hydraulic conductivity (k).
38 Typically $k \leq 10^{-9}$ m/s is specified for backfills in slurry wall designs as in such a condition
39 diffusion of contamination can be reasonably assumed to be the significant transport
40 process (Devlin and Parker, 1996).

41

42 In the design of slurry walls the determination of either the potential service life (which is
43 usually indicated by the breakthrough time of the target contaminant) for a given wall
44 thickness or the wall thickness required for a target service life is typically required. In
45 such problems contaminant transport through the slurry wall can be considered as a one-
46 dimensional advective-dispersive process, as illustrated in Fig. 1. The appropriate choice
47 of boundary conditions is critical in analyzing contaminant transport through slurry walls
48 (van Genuchten and Parker, 1984; Rabideau and Khandelwal, 1998; Prince, et al., 2000).
49 Use of first-type (Dirichlet) boundary conditions at the inlet (up-stream) boundary fails to
50 satisfy conservation of mass and the impact of this discrepancy is not always negligible
51 (van Genuchten and Parker, 1984). Due to this limitation it has been suggested that
52 solutions for a semi-infinite system with a first-type boundary at the inlet boundary

53 (Lapidus and Amundson, 1952; Ogata and Banks, 1961) should not be used in the design
54 of slurry walls (Prince, et al., 2000). Use of a third-type (Robin) boundary condition,
55 which is a more accurate representation of mass balance between the total flux into the
56 backfill and the mass of contaminant in the backfill, is recommended for the inlet
57 boundary in the analysis of contaminant transport through a slurry wall (van Genuchten
58 and Parker, 1984; Prince, et al., 2000). However, analytical solutions with such a
59 boundary condition typically utilize complementary error functions (Lindstrom, et al.,
60 1967) or require solution of eigen equations (Brenner, 1962). This thereby restricts their
61 usefulness to practicing engineers and limits their implementation in the slurry wall
62 design process.

63

64 In this paper, design charts for contaminant transport through slurry walls are presented.
65 They are established in terms of non-dimensionalized effluent flux and concentration by
66 developing and utilizing an appropriate analytical solution. Methods for using these
67 design charts to determine the effluent flux of contaminant or to estimate the thickness of
68 slurry walls are demonstrated.

69

70

71 **Method and Charts**

72

73 A slurry wall keying into impermeable layer (see Fig. 1) is considered. The backfill is
74 assumed to be homogenous, fully saturated and non-deformable. The pore water flow in
75 the backfill is assumed to be in a steady state condition. A coordinate system (x), whose

76 direction is coincident with that of the pore-water flow, is adopted, and the inlet boundary
 77 is chosen as the origin. Contaminant transport through the slurry wall can be described
 78 by the one-dimensional advection-dispersion equation for soils (Bear and Cheng, 2010),
 79 that is,

$$80 \quad nR \frac{\partial c}{\partial t} = nD_e \frac{\partial^2 c}{\partial x^2} - v \frac{\partial c}{\partial x} \quad (1)$$

81 where c is the volume-average concentration of contaminant in the pore water of backfill;
 82 t is time; n is the porosity of the backfill; R and D_e are the retardation factor and effective
 83 diffusion coefficient of contaminant in the backfill, respectively. v is the discharge
 84 (superficial) velocity and is assumed to be determined by Darcy's law, so can be
 85 expressed as:

$$86 \quad v = k \frac{h}{L} \quad (2)$$

87 where h is the hydraulic head difference between the inlet boundary and outlet (down-
 88 stream) boundary of the slurry wall; and L is thickness of the slurry wall. Chemical
 89 equilibrium between the pore water and the soil particles of backfills is assumed to be
 90 instantaneously reached. For linear, instantaneous and reversible equilibrium adsorption
 91 of reactive contaminants, the linear adsorption, R , is given by

$$92 \quad R = 1 + \frac{\rho K_d}{n} \quad (3)$$

93 where ρ is bulk (dry) density of the backfill; and K_d is the linear partition coefficient of
 94 the contaminant. The first term on the right side of Eq. (1) represents dispersive and
 95 diffusive transport of contaminant in soils and the second term represents advective
 96 transport. Initially, the backfill is assumed to be free of contaminant.

97 $c(x,0) = 0 \quad 0 \leq x \leq L$ (4)

98 A third-type boundary condition is used in this paper at the inlet boundary of the slurry
 99 wall following the discussions of van Genuchten and Parker (1984) and Prince et al.
 100 (2000), that is,

101 $-nD_e \frac{\partial c}{\partial x} + vc = vc_0 \quad x = 0$ (5)

102 where c_0 is the inlet concentration.

103 The choice of outlet boundary condition is less straightforward (Rabideau and
 104 Khandelwal, 1998; Prince, et al., 2000). The suitability of the semi-infinite assumption
 105 for finite columns or barriers is itself questionable, which is particular true when the
 106 Peclet number is low, as in the case of adsorptive, low-permeability slurry wall barriers
 107 (Prince, et al., 2000). For the scenario that the regional ground-water flow is parallel to
 108 the slurry wall, advection can remove contaminant from the barrier exit much faster than
 109 the rate of diffusion from within the barrier. Therefore, the zero concentration boundary
 110 condition, which implies a “flushing” effect, is recommended as a conservative starting
 111 point of design (Rabideau and Khandelwal, 1998), that is,

112 $c = 0 \quad x = L$ (6)

113

114 Eqs. (1), (4)~(6) can be non-dimensionalized as follows,

115 $\frac{\partial C}{\partial T} = \frac{1}{P_L} \frac{\partial^2 C}{\partial X^2} - \frac{\partial C}{\partial X}$ (7)

116 $C(X,0) = 0 \quad 0 \leq X \leq 1$ (8)

117 $-\frac{1}{P_L} \frac{\partial C}{\partial X} + C = 1 \quad X = 0$ (9)

118 $C = 0 \quad X = 1$ (10)

119 where

120 $P_L = \frac{vL}{nD_e}$ (11)

121 $T = \frac{vt}{nRL}$ (12)

122 $C(X, T) = \frac{c(x, t)}{c_0}$ (13)

123 $X = \frac{x}{L}$ (14)

124 Substitution of Eq. (2) into Eqs. (11) and (12) yields

125 $P_L = \frac{kh}{nD_e}$ (15)

126 $T = \frac{kht}{nRL^2}$ (16)

127 The column Peclet number P_L (van Genuchten and Parker, 1984; Shackelford, 1994;
 128 Shackelford, 1995; Rabideau and Khandelwal, 1998) represents the relative importance
 129 of advection to dispersion in the soil matrix. Eq. (15) indicates P_L is independent of L if
 130 h is assumed to be not changed by the thickness of the slurry wall.

131

132 The following analytical solution to Eq. (7), with the initial and boundary conditions of
 133 Eqs. (8), (9) and (10), can be developed following Li and Cleall (2011),

134 $C(X, T) = 1 - \exp(P_L X - P_L) + \sum_{m=1}^{\infty} [A_m \sin(\beta_m X) + B_m \cos(\beta_m X)] \exp\left(\frac{P_L X}{2} - \frac{P_L T}{4} - \frac{\beta_m^2 T}{P_L}\right)$
 135 (17)

136 where

137
$$A_m = -\frac{4\frac{\beta_m}{\alpha^2 + \beta_m^2}}{\frac{\alpha^2 + \beta_m^2}{\alpha^2} + \frac{\sin \beta_m}{\beta_m \alpha} (\beta_m \sin \beta_m - \alpha \cos \beta_m)}$$
 (18)

138
$$B_m = \frac{\beta_m}{\alpha} A_m$$
 (19)

139
$$\alpha = \frac{P_L}{2}$$
 (20)

140 β_m are the positive roots of the following eigen equation

141
$$\beta_m \cot \beta_m + \frac{P_L}{2} = 0$$
 (21)

142

143 For the scenario described above the effluent flux of contaminant at the outlet boundary

144 can be used as the breakthrough criterion. The normalized effluent flux can be written as

145
$$F(1, T) = 1 - \sum_{m=1}^{\infty} \left(\frac{A_m \beta_m}{P_L} \cos \beta_m - \frac{B_m \beta_m}{P_L} \sin \beta_m + \frac{A_m}{2} \sin \beta_m + \frac{B_m}{2} \cos \beta_m \right) \exp \left(\frac{P_L}{2} - \frac{P_L T}{4} - \frac{\beta_m^2 T}{P_L} \right)$$

 146 (22)

147 where

148
$$F(1, T) = \frac{f(L, t)}{vc_0}$$
 (23)

149
$$f(x, t) = -nD_e \frac{\partial c}{\partial x} + vc$$
 (24)

150 and $f(x, t)$ is the flux of contaminant. For use of Eq. (22) in design calculations engineers

151 have to solve the eigen equation thereby reducing its usefulness. In this paper, the eigen

152 values (β_m) in Eq. (21) are generated numerically, using the Newton-Raphson method

153 (Chapra and Canale, 2006). The obtained design charts for the relationships between the

154 normalized effluent flux and time are plotted in Fig. 2, following the format used by

155 Rowe et al. (2004). Similarly, those for the relationships between the normalized effluent
156 concentration and time for the scenario with second-type (Neumann) boundary condition
157 at the barrier exit are given in the Appendix using the analytical solution of Brenner
158 (1962).

159

160

161 **Examples**

162

163 The methodology of using the presented charts to design slurry walls is outlined via a
164 series of examples and follows the work of Acar and Haider (1990) and Rowe et al.
165 (2004). The material parameters of backfill used in the examples considered are listed in
166 Table 1.

167

168 The effluent concentration at an, arbitrarily, selected time of interest of 30 years, for a 0.9
169 m-thick slurry wall is first estimated. The values of coefficients $P_L=10.0$ and $T=0.47$ can
170 be obtained by Eqs. (15) and (16), respectively. Based on these values, $F=0.085$ (that is,
171 $f=9.4\times 10^{-12}$ kg/(m²s)) can be found from Fig. 2(c). If the breakthrough criterion that the
172 effluent concentration $F=0.01$ (i.e., $f=1.1\times 10^{-12}$ kg/(m²s)) at 30 years is used the thickness
173 of 0.9 m is not sufficient and a greater thickness is required.

174

175 The thickness satisfying the breakthrough criterion above can also be determined using
176 the design charts. The value of P_L is unchanged as it is independent of L (see Eq. (15)).
177 To satisfy the breakthrough criterion of $F=0.01$ for a service life of 30 years, T is required

178 to be less than 0.33 for the curve of $P_L=10.0$ according to Fig. 2(c). Consequently, the
179 thickness of the slurry wall should be greater than 1.07 m using Eq. (16).

180

181

182 **Conclusions**

183

184 Design charts for non-dimensional contaminant transport through slurry walls, based on a
185 newly developed analytical solution, have been presented. They can be used to estimate
186 the effluent flux of contaminant or to determine the thickness of slurry walls. Calculation
187 of complex functions or search of eigen values in the alternative solutions are no longer
188 required. These charts can help engineers design slurry walls based on contaminant
189 transport.

190

191

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193

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198

199

200 **Appendix**

201

202 The normalized effluent concentration of contaminant at the outlet boundary can be
203 written as follows using the analytical solution of Brenner (1962) for the scenario with
204 second-type boundary condition at the barrier exit,

$$205 \quad C(1,T) = 1 - \sum_{m=1}^{\infty} \frac{2P_L \beta_m \left(\beta_m \cos \beta_m + \frac{P_L}{2} \sin \beta_m \right) \exp \left(\frac{P_L}{2} - \frac{P_L T}{4} - \frac{\beta_m^2 T}{P_L} \right)}{\left(\beta_m^2 + \frac{P_L^2}{4} + P_L \right) \left(\beta_m^2 + \frac{P_L^2}{4} \right)} \quad (A1)$$

206 where β_m are the positive roots of the eigen equation

$$207 \quad \beta_m \cot \beta_m - \frac{1}{P_L} \beta_m^2 + \frac{P_L}{4} = 0 \quad (A2)$$

208 The design charts for the relationships between the normalized effluent concentration of
209 contaminant and time are plotted in Fig. A1 using Eq. (A1).

210

211

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275

276 **List of Table and Figure Captions**

277

278

279 Table 1. Parameters used in example.

280

281

282 Fig. 1 Configuration of contaminant transport through a slurry wall.

283

284 Fig. 2 Design charts for normalized effluent contaminant flux of slurry walls.

285

286 Fig. A1 Design charts for normalized effluent contaminant concentration of slurry walls.

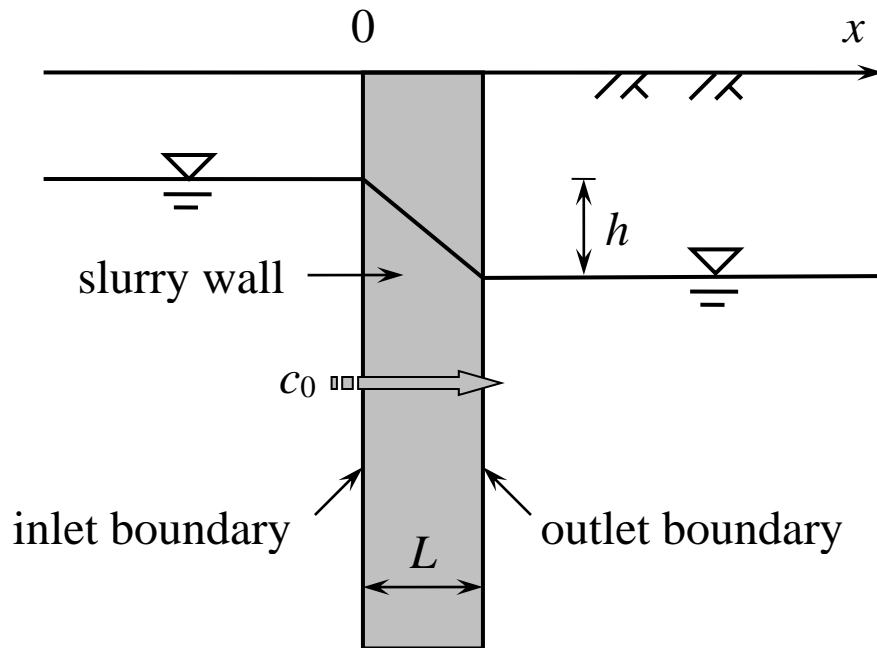
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Table 1. Parameters used in example.

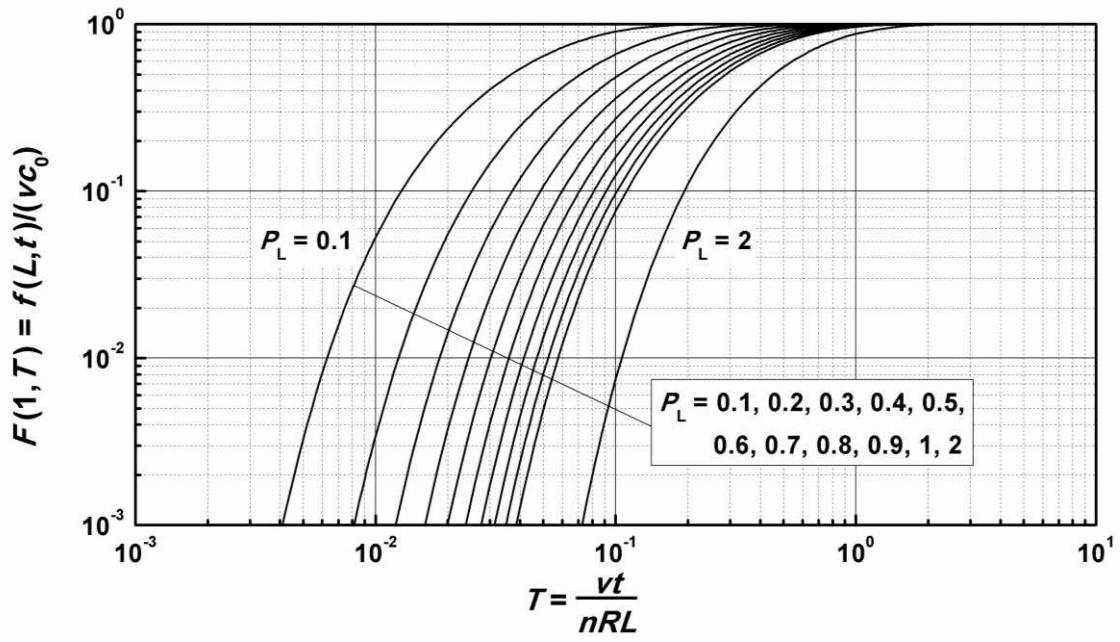
Parameter	Values	Unit
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k	1×10^{-9}	m/s
R	10.0	/
D_e	4×10^{-10}	m^2/s
h	1.0	m
c_0	100	mg/L

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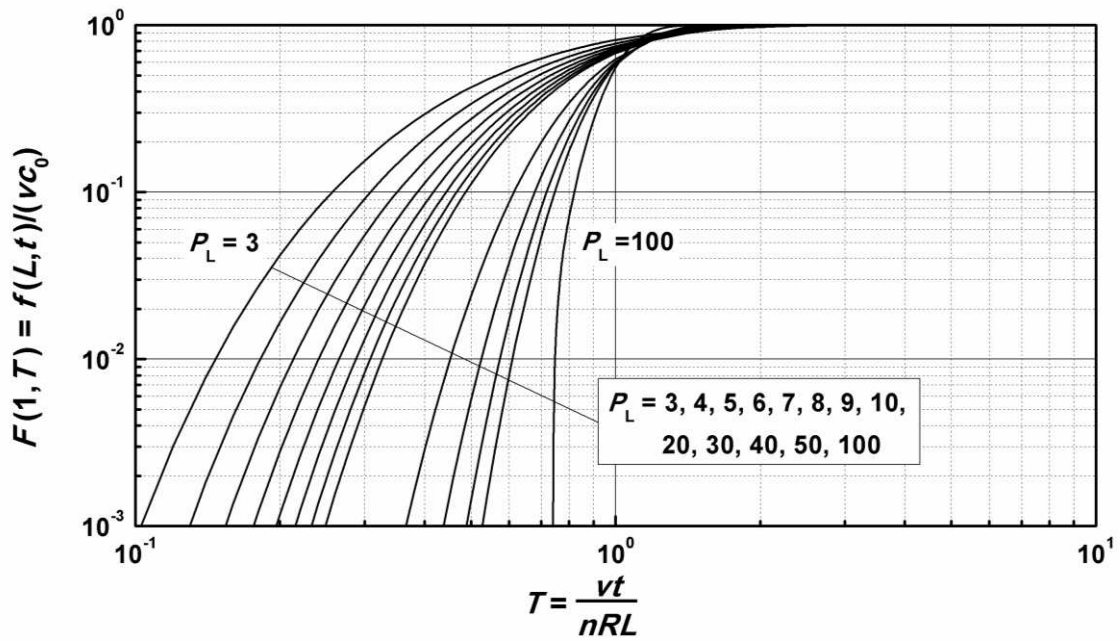
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Fig. 1 Configuration of contaminant transport through a slurry wall.



(a) $P_L = 0.1 \sim 2$

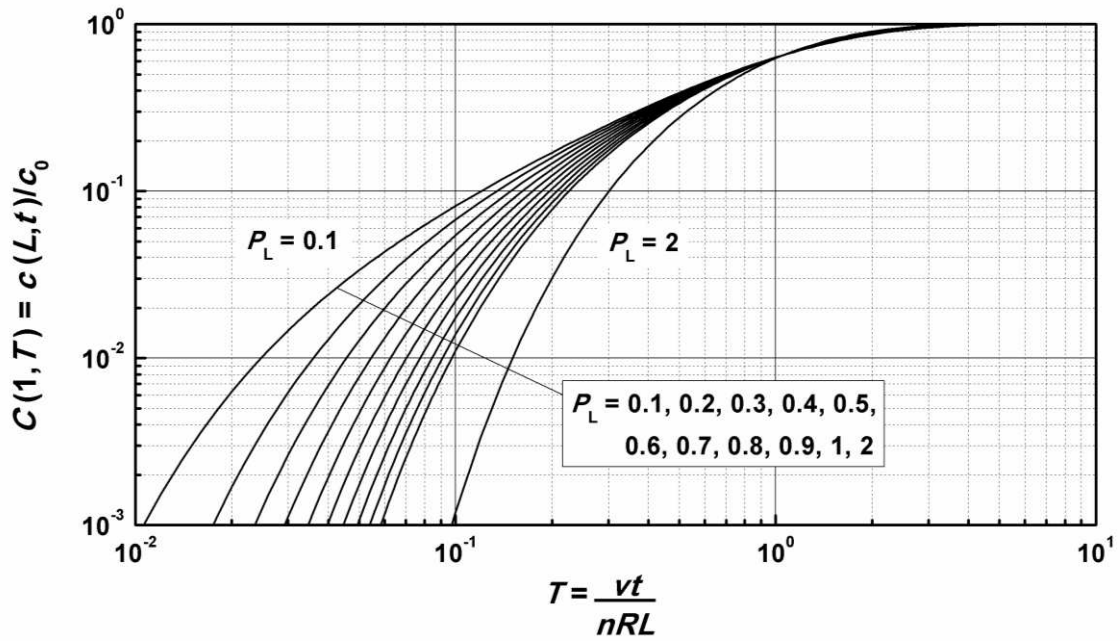
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(b) $P_L = 3 \sim 100$

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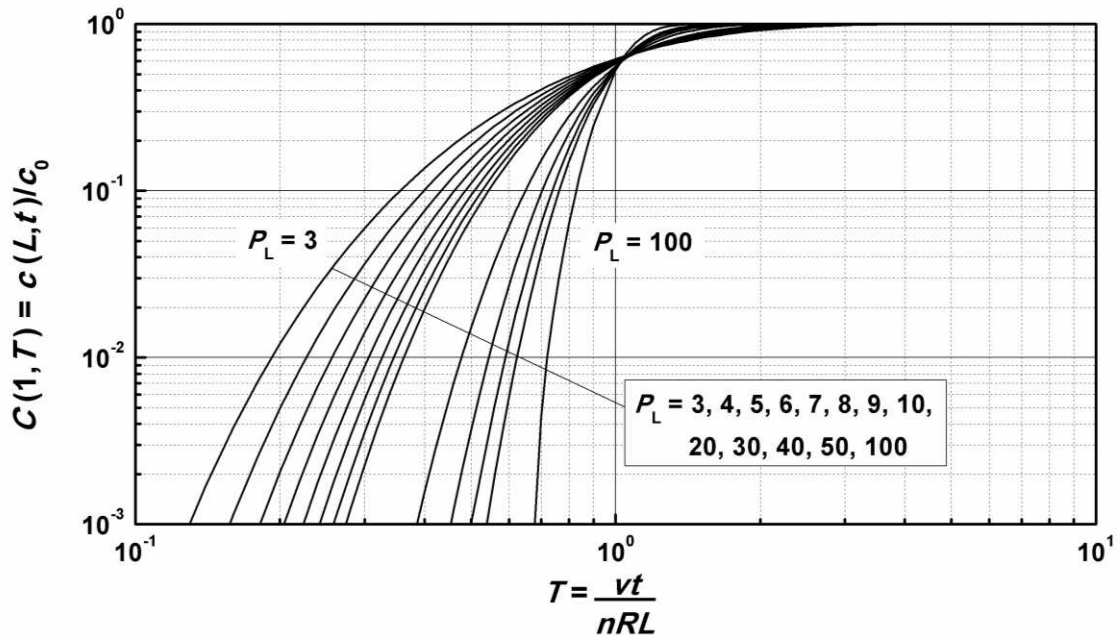
Fig. 2 Design charts for normalized effluent contaminant flux of slurry walls.



305

306

(a) $P_L=0.1\sim 2$



307

308

309

310

(b) $P_L=3\sim 100$

Fig. A1 Design charts for normalized effluent contaminant concentration of slurry walls.