Title: Design Charts for Contaminant Transport through Slurry Trench Cut-off Walls

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Abstract: Slurry trench cut-off walls with low-permeability backfill material, such as soil-bentonite and slag-cement-bentonite, are used widely for containment of subsurface pollution. In the design of slurry walls the potential service life for a given thickness or the wall thickness for a target service life are typically determined via analyses of one-dimensional contaminant transport. The difficulty of selecting appropriate inlet and outlet boundary conditions and the mathematical complexity of analytical solutions hinder engineers from undertaking a contaminant transport analysis based design. Design charts for non-dimensionalized effluent flux are presented by developing and utilizing an analytical solution. The methodologies of using these charts in design are demonstrated.

Keywords: contaminant transport; slurry wall; design chart; subsurface contamination

Introduction

Slurry trench cut-off walls (termed as slurry walls hereafter) with low-permeability backfills, such as soil-bentonite and slag-cement-bentonite, are widely used for containment of subsurface pollution (D'Appolonia, 1980; LaGrega, et al., 2001; Opdyke and Evans, 2005; Jefferis, 2012). Many laboratory studies have been conducted to evaluate properties of the backfills (Evans, 1994; Filz, et al., 2001; Yeo, et al., 2005; Joshi, et al., 2010; Soga, et al., 2013) with a focus on hydraulic conductivity (*k*). Typically $k \leq 10^{-9}$ m/s is specified for backfills in slurry wall designs as in such a condition diffusion of contamination can be reasonably assumed to be the significant transport process (Devlin and Parker, 1996).

In the design of slurry walls the determination of either the potential service life (which is usually indicated by the breakthrough time of the target contaminant) for a given wall thickness or the wall thickness required for a target service life is typically required. In such problems contaminant transport through the slurry wall can be considered as a one-dimensional advective-dispersive process, as illustrated in Fig. 1. The appropriate choice of boundary conditions is critical in analyzing contaminant transport through slurry walls (van Genuchten and Parker, 1984; Rabideau and Khandelwal, 1998; Prince, et al., 2000). Use of first-type (Dirichlet) boundary conditions at the inlet (up-stream) boundary fails to satisfy conservation of mass and the impact of this discrepancy is not always negligible (van Genuchten and Parker, 1984). Due to this limitation it has been suggested that solutions for a semi-infinite system with a first-type boundary at the inlet boundary

(Lapidus and Amundson, 1952; Ogata and Banks, 1961) should not be used in the design of slurry walls (Prince, et al., 2000). Use of a third-type (Robin) boundary condition, which is a more accurate representation of mass balance between the total flux into the backfill and the mass of contaminant in the backfill, is recommended for the inlet boundary in the analysis of contaminant transport through a slurry wall (van Genuchten and Parker, 1984; Prince, et al., 2000). However, analytical solutions with such a boundary condition typically utilize complementary error functions (Lindstrom, et al., 1967) or require solution of eigen equations (Brenner, 1962). This thereby restricts their usefulness to practicing engineers and limits their implementation in the slurry wall design process.

In this paper, design charts for contaminant transport through slurry walls are presented. They are established in terms of non-dimensionalized effluent flux and concentration by developing and utilizing an appropriate analytical solution. Methods for using these design charts to determine the effluent flux of contaminant or to estimate the thickness of slurry walls are demonstrated.

Method and Charts

A slurry wall keying into impermeable layer (see Fig. 1) is considered. The backfill is assumed to be homogenous, fully saturated and non-deformable. The pore water flow in the backfill is assumed to be in a steady state condition. A coordinate system (x), whose direction is coincident with that of the pore-water flow, is adopted, and the inlet boundary is chosen as the origin. Contaminant transport through the slurry wall can be described by the one-dimensional advection-dispersion equation for soils (Bear and Cheng, 2010), that is,

$$nR\frac{\partial c}{\partial t} = nD_{\mathrm{e}}\frac{\partial^2 c}{\partial x^2} - v\frac{\partial c}{\partial x} \tag{1}$$

where c is the volume-average concentration of contaminant in the pore water of backfill; t is time; n is the porosity of the backfill; R and D_{e} are the retardation factor and effective diffusion coefficient of contaminant in the backfill, respectively. v is the discharge (superficial) velocity and is assumed to be determined by Darcy's law, so can be expressed as:

$$v = k\frac{h}{L} \tag{2}$$

where h is the hydraulic head difference between the inlet boundary and outlet (down-stream) boundary of the slurry wall; and L is thickness of the slurry wall. Chemical equilibrium between the pore water and the soil particles of backfills is assumed to be instantaneously reached. For linear, instantaneous and reversible equilibrium adsorption of reactive contaminants, the linear adsorption, R, is given by

$$R = 1 + \frac{\rho K_{\mathrm{d}}}{n} \tag{3}$$

where ρ is bulk (dry) density of the backfill; and K_{d} is the linear partition coefficient of the contaminant. The first term on the right side of Eq. (1) represents dispersive and diffusive transport of contaminant in soils and the second term represents advective transport. Initially, the backfill is assumed to be free of contaminant.

$$c\left(x,0\right)=0 \qquad 0\le x\le L \tag{4}$$

A third-type boundary condition is used in this paper at the inlet boundary of the slurry wall following the discussions of van Genuchten and Parker (1984) and Prince et al. (2000), that is,

$$-nD_{\mathrm{e}}\frac{\partial c}{\partial x}+vc=vc_0 \qquad x=0 \tag{5}$$

where c_0 is the inlet concentration.

The choice of outlet boundary condition is less straightforward (Rabideau and Khandelwal, 1998; Prince, et al., 2000). The suitability of the semi-infinite assumption for finite columns or barriers is itself questionable, which is particular true when the Peclet number is low, as in the case of adsorptive, low-permeability slurry wall barriers (Prince, et al., 2000). For the scenario that the regional ground-water flow is parallel to the slurry wall, advection can remove contaminant from the barrier exit much faster than the rate of diffusion from within the barrier. Therefore, the zero concentration boundary condition, which implies a "flushing" effect, is recommended as a conservative starting point of design (Rabideau and Khandelwal, 1998), that is,

$$c=0 \qquad x=L \tag{6}$$

Eqs. (1), (4)~(6) can be non-dimensionalized as follows,

$$\frac{\partial C}{\partial T}=\frac{1}{P_{\mathrm{L}}}\frac{\partial^2 C}{\partial X^2}-\frac{\partial C}{\partial X} \tag{7}$$

$$C\left(X,0\right)=0 \qquad 0\le X\le 1 \tag{8}$$

$$-\frac{1}{P_{\mathrm{L}}}\frac{\partial C}{\partial X}+C=1 \qquad X=0 \tag{9}$$

$$C = 0 \qquad X = 1 \tag{10}$$

where

$$P_{\mathrm{L}} = \frac{vL}{nD_{\mathrm{e}}} \tag{11}$$

$$T = \frac{vt}{nRL} \tag{12}$$

$$C(X,T) = \frac{c(x,t)}{c_0} \tag{13}$$

$$X = \frac{x}{L} \tag{14}$$

Substitution of Eq. (2) into Eqs. (11) and (12) yields

$$P_{\mathrm{L}} = \frac{kh}{nD_{\mathrm{e}}} \tag{15}$$

$$T = \frac{kht}{nRL^2} \tag{16}$$

The column Peclet number P_{L} (van Genuchten and Parker, 1984; Shackelford, 1994; Shackelford, 1995; Rabideau and Khandelwal, 1998) represents the relative importance of advection to dispersion in the soil matrix. Eq. (15) indicates P_{L} is independent of L if h is assumed to be not changed by the thickness of the slurry wall.

The following analytical solution to Eq. (7), with the initial and boundary conditions of Eqs. (8), (9) and (10), can be developed following Li and Cleall (2011),

$$C(X,T) = 1 - \exp\left(P_{\mathrm{L}}X - P_{\mathrm{L}}\right) + \sum_{m=1}^{\infty}\left[A_m \sin\left(\beta_m X\right) + B_m \cos\left(\beta_m X\right)\right]\exp\left(\frac{P_{\mathrm{L}}X}{2} - \frac{P_{\mathrm{L}}T}{4} - \frac{\beta_m{}^2 T}{P_{\mathrm{L}}}\right) \tag{17}$$

where

$$A_m = -\frac{4\dfrac{\beta_m}{\alpha^2+{\beta_m}^2}}{\dfrac{\alpha^2+{\beta_m}^2}{\alpha^2}+\dfrac{\sin\beta_m}{\beta_m\alpha}\left(\beta_m\sin\beta_m-\alpha\cos\beta_m\right)} \tag{18}$$

$$B_m = \frac{\beta_m}{\alpha}A_m \tag{19}$$

$$\alpha = \frac{P_\mathrm{L}}{2} \tag{20}$$

β_m are the positive roots of the following eigen equation

$$\beta_m\cot\beta_m + \frac{P_\mathrm{L}}{2} = 0 \tag{21}$$

For the scenario described above the effluent flux of contaminant at the outlet boundary can be used as the breakthrough criterion. The normalized effluent flux can be written as

$$F\left(1,T\right) = 1-\sum_{m=1}^{\infty}\left(\frac{A_m\beta_m}{P_\mathrm{L}}\cos\beta_m-\frac{B_m\beta_m}{P_\mathrm{L}}\sin\beta_m+\frac{A_m}{2}\sin\beta_m+\frac{B_m}{2}\cos\beta_m\right)\exp\left(\frac{P_\mathrm{L}}{2}-\frac{P_\mathrm{L}T}{4}-\frac{{\beta_m}^2T}{P_\mathrm{L}}\right) \tag{22}$$

where

$$F\left(1,T\right) = \frac{f\left(L,t\right)}{vc_0} \tag{23}$$

$$f\left(x,t\right) = -nD_\mathrm{e}\frac{\partial c}{\partial x}+vc \tag{24}$$

and $f(x,t)$ is the flux of contaminant. For use of Eq. (22) in design calculations engineers have to solve the eigen equation thereby reducing its usefulness. In this paper, the eigen values (β_m) in Eq. (21) are generated numerically, using the Newton-Raphson method (Chapra and Canale, 2006). The obtained design charts for the relationships between the normalized effluent flux and time are plotted in Fig. 2, following the format used by

Rowe et al. (2004). Similarly, those for the relationships between the normalized effluent concentration and time for the scenario with second-type (Neumann) boundary condition at the barrier exit are given in the Appendix using the analytical solution of Brenner (1962).

Examples

The methodology of using the presented charts to design slurry walls is outlined via a series of examples and follows the work of Acar and Haider (1990) and Rowe et al. (2004). The material parameters of backfill used in the examples considered are listed in Table 1.

The effluent concentration at an, arbitrarily, selected time of interest of 30 years, for a 0.9 m-thick slurry wall is first estimated. The values of coefficients P_L=10.0 and T=0.47 can be obtained by Eqs. (15) and (16), respectively. Based on these values, F=0.085 (that is, f=9.4×10^{-12} kg/(m^{2}s)) can be found from Fig. 2(c). If the breakthrough criterion that the effluent concentration F=0.01 (i.e., f=1.1×10^{-12} kg/(m^{2}s)) at 30 years is used the thickness of 0.9 m is not sufficient and a greater thickness is required.

The thickness satisfying the breakthrough criterion above can also be determined using the design charts. The value of P_L is unchanged as it is independent of L (see Eq. (15)). To satisfy the breakthrough criterion of F=0.01 for a service life of 30 years, T is required to be less than 0.33 for the curve of P_L=10.0 according to Fig. 2(c). Consequently, the thickness of the slurry wall should be greater than 1.07 m using Eq. (16).

Conclusions

Design charts for non-dimensional contaminant transport through slurry walls, based on a newly developed analytical solution, have been presented. They can be used to estimate the effluent flux of contaminant or to determine the thickness of slurry walls. Calculation of complex functions or search of eigen values in the alternative solutions are no longer required. These charts can help engineers design slurry walls based on contaminant transport.

Acknowledgements

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Appendix

The normalized effluent concentration of contaminant at the outlet boundary can be written as follows using the analytical solution of Brenner (1962) for the scenario with second-type boundary condition at the barrier exit,

$$C\left(1,T\right)=1-\sum_{m=1}^{\infty}\frac{2P_{\mathrm{L}}\beta_m\left(\beta_m\cos\beta_m+\frac{P_{\mathrm{L}}}{2}\sin\beta_m\right)\exp\left(\frac{P_{\mathrm{L}}}{2}-\frac{P_{\mathrm{L}}T}{4}-\frac{\beta_m{}^2T}{P_{\mathrm{L}}}\right)}{\left(\beta_m{}^2+\frac{P_{\mathrm{L}}{}^2}{4}+P_{\mathrm{L}}\right)\left(\beta_m{}^2+\frac{P_{\mathrm{L}}{}^2}{4}\right)} \tag{A1}$$

where β_m are the positive roots of the eigen equation

$$\beta_m\cot\beta_m-\frac{1}{P_{\mathrm{L}}}\beta_m{}^2+\frac{P_{\mathrm{L}}}{4}=0 \tag{A2}$$

The design charts for the relationships between the normalized effluent concentration of contaminant and time are plotted in Fig. A1 using Eq. (A1).

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List of Table and Figure Captions

Table 1. Parameters used in example.

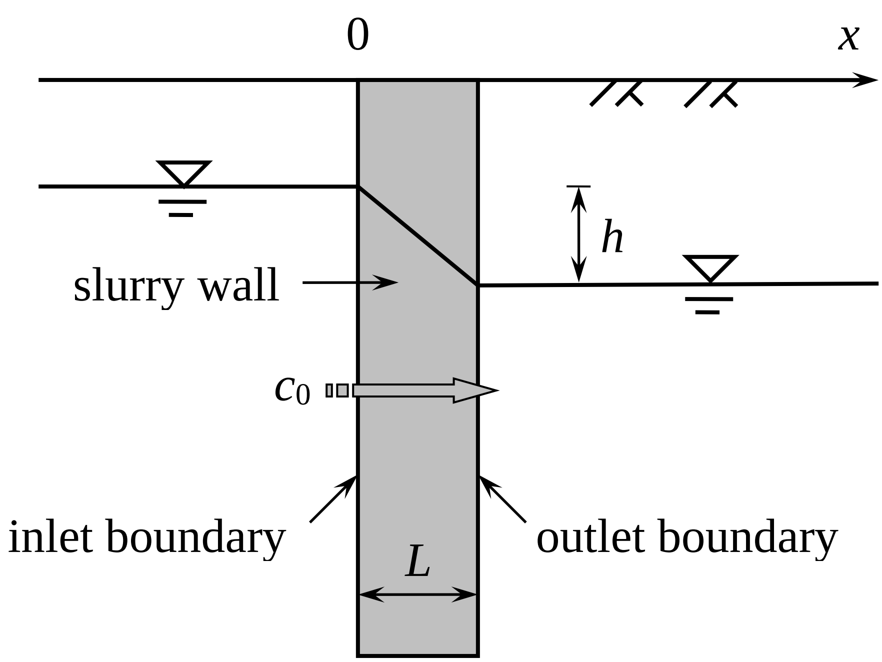
Fig. 1 Configuration of contaminant transport through a slurry wall.

Fig. 2 Design charts for normalized effluent contaminant flux of slurry walls.

Fig. A1 Design charts for normalized effluent contaminant concentration of slurry walls.

Table 1. Parameters used in example.

Parameter	Values	Unit
n	0.25	/
k	1×10^{-9}	m/s
R	10.0	/
D_e	4×10^{-10}	m^2/s
h	1.0	m
c_0	100	mg/L



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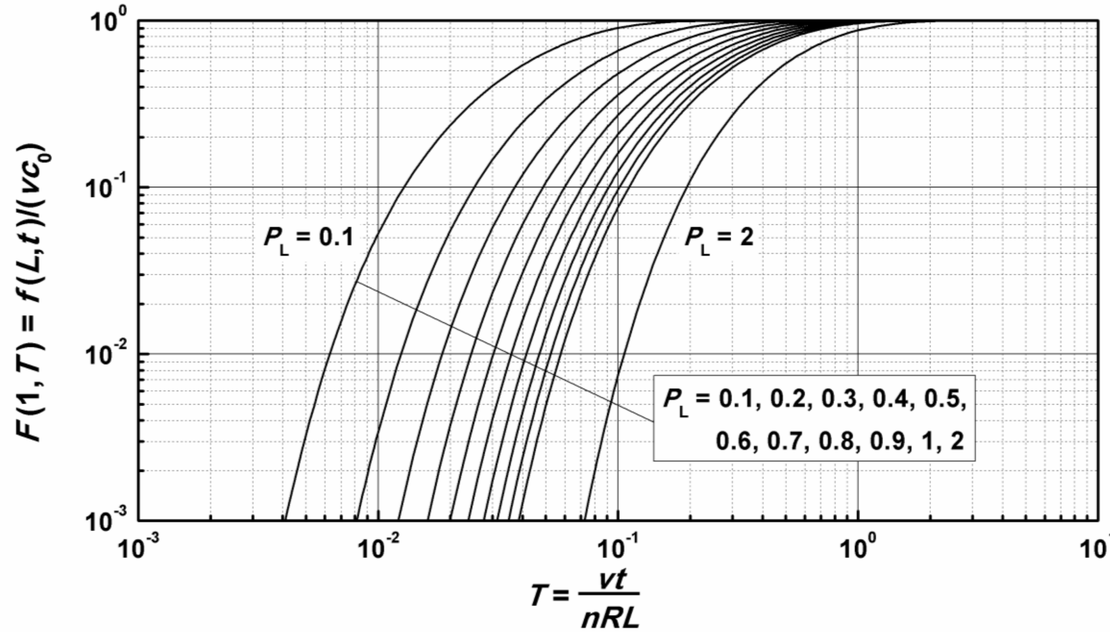
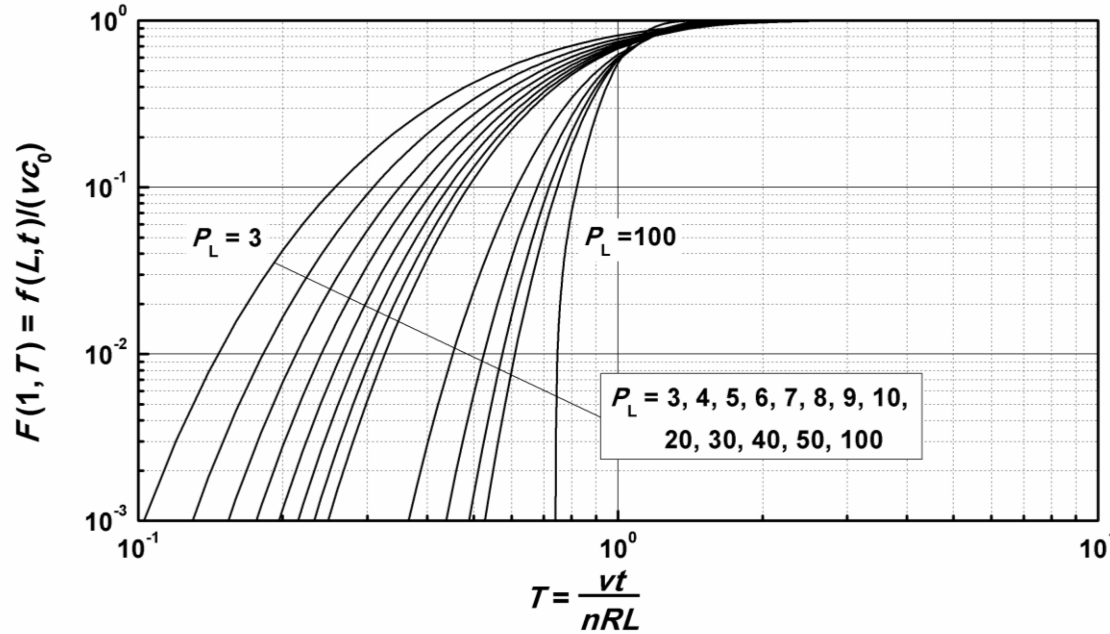
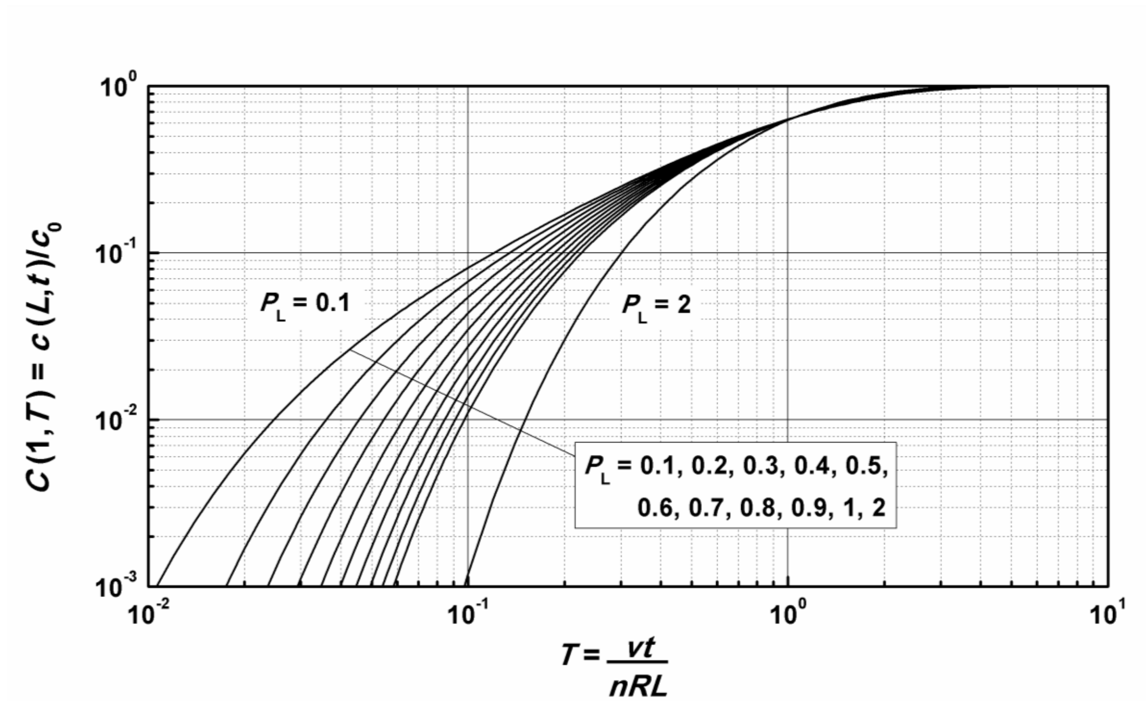


(a) P_L=0.1~2

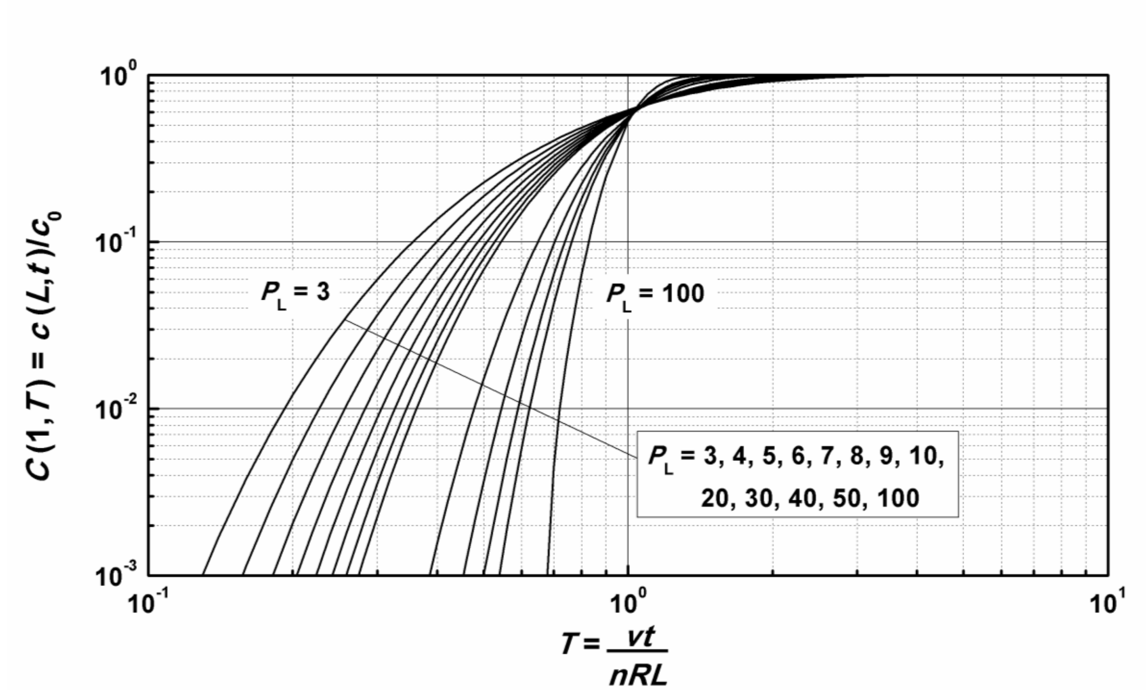


(b) P_L=3~100

Fig. 2 Design charts for normalized effluent contaminant flux of slurry walls.



(a) P_L=0.1~2



(b) P_L=3~100

Fig. A1 Design charts for normalized effluent contaminant concentration of slurry walls.