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Multi-objective operation optimization of an electrical distribution network with soft open point

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HIGHLIGHTS

- The SOP's capability of bringing benefits on multiple objectives simultaneously was investigated.
- A multi-objective framework was developed to improve distribution network operation with SOP.
- An optimization method integrating both global and local search techniques was proposed.
- The optimization method is capable of obtaining diverse Pareto optimal solutions.

ARTICLE INFO

Keywords:
Soft open point (SOP)
Multi-objective optimization
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ABSTRACT

With the increasing amount of distributed generation (DG) integrated into electrical distribution networks, various operational problems, such as excessive power losses, over-voltage and thermal overloading issues become gradually remarkable. Innovative approaches for power flow and voltage controls are required to ensure the power quality, as well as to accommodate large DG penetrations. Using power electronic devices is one of the approaches. In this paper, a multi-objective optimization framework was proposed to improve the operation of a distribution network with distributed generation and a soft open point (SOP). An SOP is a distribution-level power electronic device with the capability of real-time and accurate active and reactive power flow control. A novel optimization method that integrates a Multi-Objective Particle Swarm Optimization (MOPSO) algorithm and a local search technique – the Taxi-cab method, was proposed to determine the optimal set-points of the SOP, where power loss reduction, feeder load balancing and voltage profile improvement were taken as objectives. The local search technique is integrated to fine tune the non-dominated solutions obtained by the global search technique, overcoming the drawback of MOPSO in local optima trapping. Therefore, the search capability of the integrated method is enhanced compared to the conventional MOPSO algorithm. The proposed methodology was applied to a 69-bus distribution network. Results demonstrated that the integrated method effectively solves the multi-objective optimization problem, and obtains better and more diverse solutions than the conventional MOPSO method. With the DG penetration increasing from 0 to 200%, on average, an SOP reduces power losses by 58.4%, reduces the load balance index by 68.3% and reduces the voltage profile index by 62.1%, all compared to the case without an SOP. Comparisons between SOP and network reconfiguration showed the outperformance of SOP in operation optimization.

1. Introduction

Growing awareness of energy and environment, and the demand for a reliable, secure, and sustainable power grid lead to the continuously expanding deployments of Distributed Generators (DG). However, high penetrations of DG significantly affect the operation of electrical distribution networks, where the technical challenges are mainly power loss increments, line and transformer overloads, and voltage violations [1–5].

The use of power electronic devices provides alternative solutions to overcome these challenges. The application of power electronics to High Voltage Direct Current (HVDC) transmission systems has gained increasingly importance in the bulk power transfer. The extensive growing demand for power electronic devices and their continuous developments offer significant reductions in converter costs [6], which provide a chance for their further applications in medium voltage (MV) and low voltage (LV) distribution networks.

Power electronic devices were applied in distribution networks for

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Nomenclature

Network parameters

I_k current flow through branch k
$I_{k,rated}$ rated current of branch k
I_k^{max} maximum current limit of branch k
LBI load balance index
N_{bus} total number of buses
N_{branch} total number of branches
P_i, Q_i active and reactive power flowing from bus i to bus $i+1$
$P_{load\,(i)}$, $Q_{load\,(i)}$ active and reactive power demand at bus i
$P_{loss\,(i,i+1)}$, $Q_{loss\,(i,i+1)}$ active and reactive power loss of the branch connecting buses i and $i+1$
P_{C1}, Q_{C1} active and reactive power provided by VSC1
P_{C2}, Q_{C2} active and reactive power provided by VSC2
P_{loss} total active power loss of a network
$P_{SOP\,loss}$ power loss within an SOP
S_{C1} rated capacity of VSC1
S_{C2} rated capacity of VSC2
V_i voltage at bus i
V_k voltage drop through branch k
V_{C1} voltage at VSC1
V_{C2} voltage at VSC2
$V_{i,ref}$ nominal voltage of bus i
V^{min}, V^{max} minimum and maximum bus voltage limits
VPI voltage profile index
x_k, r_k reactance and resistance of branch k

Optimization parameters

A archive that stores Pareto optimal solutions in each iteration
c_1, c_2 cognitive learning and social learning factors
e_m set of standard base vectors
F_n value of the n^{th} objective function
g_{best} global best position
g_m optimal step size generated by the golden section search technique
M total number of decision variables
m the m^{th} decision variable
N_{obj} total number of objective functions
OBJ set of objective functions
obj_n the n^{th} objective function
p_{best} individual/personal best position
r_1, r_2 random numbers $\in [0,1]$
v_i velocity vector of the i^{th} particle
ω inertia weight
x_i position vector of the i^{th} particle
X position of a particle selected from the archive and used in the local search process

different purposes [7–16]. A power electronic grid interconnector was introduced to decouple the frequency and voltage from the upstream grid [7]. A loop power flow controller and a loop balance controller implemented with back-to-back converters to form an active meshed distribution system were proposed in [8] and [9]. Voltage source converter-based smart power router was proposed for minimizing load shedding [10]. Specifically, the use of soft open point (SOP) in distribution networks was investigated in [11–16]. SOP is a power electronic device that can be installed in place of a normally open/closed point in distribution networks, with the capability to accurately control active and reactive power flows between the feeders that it is connected to. Moreover, it has the advantages of fast response, frequent actions and continuous control. In [11], the capability of SOP was quantified for voltage regulation in order to increase DG penetration, and the network with SOP showed better performance compared with other voltage control options. A combination of SOP with energy storage was investigated to mitigate the transient effects caused by photovoltaic systems [12]. Two control modes for the SOP operation were developed in [13] for the power flow control and supply restoration. The operational benefits of SOPs installed in a distribution network were quantified in [14], in which an improved Powell's Direct Set method was used to determine the optimal SOP operation. In [15], a Jacobian matrix-based sensitivity method was proposed to define the operating region of SOP considering different objectives separately. The results illustrated a confliction between the objectives of voltage profile improvement and energy loss minimization. Optimal siting and sizing of SOPs was investigated in [16] to minimize the annual expense of the overall distribution system under study.

Previous research on SOP has mainly focused on the following attributes: (1) development of control strategies for SOP; (2) minimization of network energy losses considering the influence and increase of DG; (3) analysis and quantification of benefits of SOP considering different objectives separately. However, in order to achieve the potential benefits and wider applications of SOP, it is important to investigate the device's capability of bringing advantages on multiple objectives simultaneously, which makes the problem of distribution network operation with SOP a multi-objective optimization problem.

Over the past few decades, multi-objective optimization problems have attracted considerable interests from researchers motivated by the real-world engineering problems [17]. Many multi-objective optimization problems are solved under the concept of *a priori* method, in which the decision maker defines the importance amongst objectives before the search performs, and the multi-objective optimization problem is transformed into a single objective one. Afterwards, a classical single objective optimization algorithm is used to find the optimum. The key drawback of the *a priori* method is the arbitrarily imposed limitation of the search space, which does not allow findings of all solutions in a feasible set [18]. In addition, since it is common that real-world objectives are incommensurable in nature and can be conflicting with each other, aggregating multiple objectives into one may result in losing significance. Pareto optimality, on the contrary, is based on a simultaneous optimization of multiple objective functions. It provides a set of non-dominated solutions named Pareto optimal solutions, illustrating the nature of trade-offs among conflicting objectives. Evolutionary algorithms, e.g., the widely recognized SPEA2 [19] and NSGA-II [20], are suitable to solve multi-objective optimization problems using the concept of Pareto optimality, since these techniques deal with a set of possible solutions simultaneously, which allow obtaining an entire set of Pareto optimal solutions in a single run. Particle Swarm Optimization is one of the most recently used evolutionary algorithms. It is a global search technique with the most attractive features of simple concept, easy implementation, fast computation and robust search ability. Compared with other evolutionary algorithms, Particle Swarm Optimization shows incomparable advantages in searching speed and precision [21]. There are different Pareto-based multi-objective Particle Swarm Optimization variants, and a state-of-the-art review was given in [22].

Despite the global exploration capability, evolutionary algorithms are comparatively inefficient in fine tuning solutions (the exploitation) [23]. To overcome this deficiency and to enhance the search capability of evolutionary algorithms, appropriate integrations of global and local search techniques to maintain the balance between exploration and exploitation have been proposed. In [24] an adaptive local search method for hybrid evolutionary multi-objective algorithms was developed. In [25] a biogeography-based optimization technique that combines the periodic re-initialization with local search operators to search for global optimal solution was proposed. A memetic algorithm with a local search operator to improve accuracy and convergence speed simultaneously was proposed in [26]. The impact of various local search methodologies on the multi-objective memetic algorithm was investigated in [27].

In this paper, a multi-objective optimization framework was proposed to find a compromise among incommensurable objectives of distribution network operation. In order to determine the optimal SOP set-points, a novel optimization method that integrates both global and local search techniques was proposed. In this method, a Multi-Objective Particle Swarm Optimization (MOPSO) method is used as a global search technique to ensure the search capability of the entire solution space, while the Taxi-cab method is used as a local search technique to improve the solution quality and avoid stagnation. The framework was applied to a 69-bus network. Results showed great benefits of SOP in power loss reduction, load balancing and voltage profile improvement, as well as its capability to accommodate high DG penetrations. Results also showed the effectiveness of the proposed integrated method in solving the multi-objective optimization problem.

The new contributions of this work include: (1) investigating the SOP's capability of bringing benefits to the distribution networks on multiple objectives simultaneously; (2) providing a multi-objective optimization framework to improve the distribution network operation with an SOP; and (3) proposing a novel optimization method integrating both global and local search techniques, which has the capability of obtaining better and more diverse Pareto optimal solutions than the conventional MOPSO method.

2. Mathematical model of SOP in distribution networks

An SOP can be implemented in different topologies. In this study a Back-To-Back Voltage Source Converter (B2B VSC) topology was used. The schematic diagram of a simple distribution network installed with an SOP is shown in Fig. 1.

The B2B VSCs can operate in four quadrants. The reactive power at both AC terminals of the SOP is independent and can be assigned as required. This makes the device able to provide flexible reactive power to the network. In addition, the active power flow of the SOP can be controlled rapidly and accurately.

To fully evaluate the potential effects of the SOP on steady-state network operations, a mathematic power injection model of the device was developed. Using this model, active and reactive power injections at the SOP terminals are integrated into the load flow algorithm without considering the detailed design of converter controllers. The backward forward sweep method was used for load flow calculations. Taking Feeder 1 in Fig. 1 as an example, the load flow was calculated by the following recursive equations [28]:

$$P_{i+1} = P_i - P_{loss(i,i+1)} - P_{load(i+1)} = P_i - \frac{r_i}{|V_i|^2}\cdot(P_i^2 + Q_i^2) - P_{load(i+1)} \tag{1}$$

$$Q_{i+1} = Q_i - Q_{loss(i,i+1)} - Q_{load(i+1)} = Q_i - \frac{x_i}{|V_i|^2}\cdot(P_i^2 + Q_i^2) - Q_{load(i+1)} \tag{2}$$

$$|V_{i+1}|^2 = |V_i|^2 + \frac{r_i^2 + x_i^2}{|V_i|^2}\cdot(P_i^2 + Q_i^2) - 2\cdot(r_i P_i + x_i Q_i) \quad i \in \{1,2,\dots,N_{bus}\} \tag{3}$$

where P_i and Q_i are the active and reactive power flowing from bus i to bus $i + 1$. $P_{load(i)}$ and $Q_{load(i)}$ are the active and reactive power demand at bus i. $P_{loss(i,i+1)}$ and $Q_{loss(i,i+1)}$ are the power losses within the branch connecting busses i and $i + 1$, and r_i and x_i are the resistance and reactance of that branch. V_i is the voltage at bus i. N_{bus} is the total number of busses in a network.

The operational boundaries of the SOP are:

$$P_{C1} = P_p - P_{loss(p,C1)} \tag{4}$$

$$P_{C2} = P_q - P_{loss(q,C2)} \tag{5}$$

where P_{C1} and P_{C2} are the active power flows of each VSC of the SOP. $P_{loss(p,C1)}$ is the power loss between bus p and VSC 1, and $P_{loss(q,C2)}$ is the power loss between bus q and VSC 2.

The active power exchange of the two VSCs is constrained by:

$$P_{C1} + P_{C2} + P_{SOP\,loss} = 0 \tag{6}$$

where $P_{SOP\,loss}$ is the power losses at the SOP. These power losses are relatively low compared to the power losses of the entire network and thus can be neglected. Therefore, Eq. (6) is simplified as:

$$P_{C1} = -P_{C2} \tag{7}$$

The constraints on the SOP capacity and terminal voltages are:

$$\sqrt{P_{C1}^2 + Q_{C1}^2} \leqslant S_{C1} \tag{8}$$

$$\sqrt{P_{C2}^2 + Q_{C2}^2} \leqslant S_{C2} \tag{9}$$

$$V^{min} \leqslant |V_{C1}| \leqslant V^{max} \tag{10.1}$$

$$V^{min} \leqslant |V_{C2}| \leqslant V^{max} \tag{10.2}$$

where Q_{C1} and Q_{C2} are the reactive power injections of each VSC of the SOP. S_{C1} and S_{C2} are the rated capacity of each VSC. V^{min} and V^{max} are the minimum and maximum allowed voltages of the network. V_{C1} and V_{C2} are voltages at each of the SOP terminals.

Generally, the ac side of the VSC can be controlled in either PV mode or PQ mode. In this study the latter was considered. By choosing optimal SOP set-points, power flows within a network can be controlled actively. Therefore, specific operational objectives can be achieved.

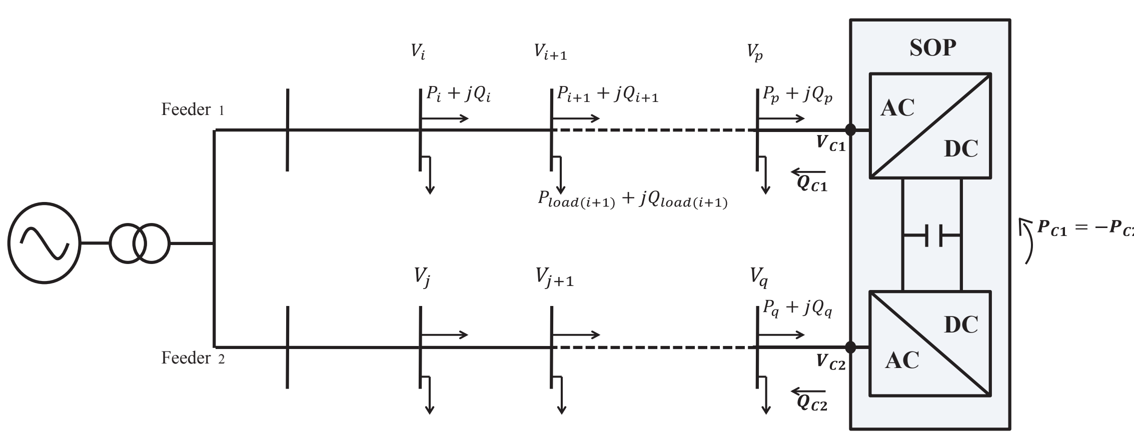

Fig. 1. A distribution network installed with an SOP.

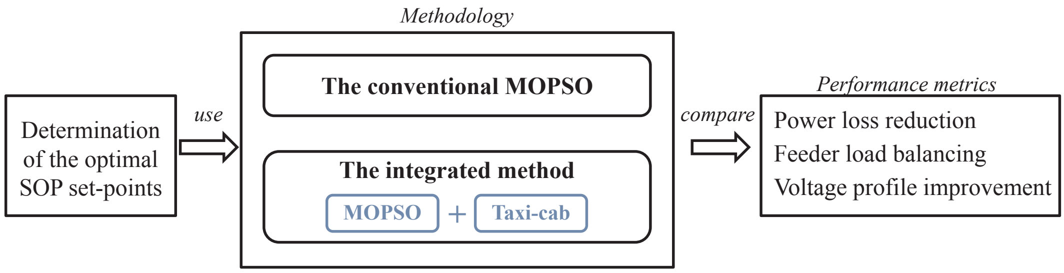

Fig. 2. Schematic overview of the proposed approach in this paper.

3. Problem formulation

A schematic overview of this work is presented in Fig. 2. To determine the optimal set-points of an SOP in a distribution network, a novel optimization method that integrates both global (the MOPSO method) and local (the Taxi-cab method) search techniques was developed. The performance metrics were compared with those of the conventional MOPSO method from the aspects of network power losses, the feeder load balancing, and the voltage profile improvement.

3.1. Objective functions

Objectives considered in the proposed multi-objective optimization framework were power loss reduction, load balancing and voltage profile improvement. They are described in the mathematical expressions as follows:

$$OBJ = min\,[obj_1, obj_2, obj_3] \tag{11}$$

3.1.1. Power loss reduction

$$obj_1 = P_{loss} = \sum_{k=1}^{N_{branch}} I_k^2 \times r_k = \sum_{k=1}^{N_{branch}} \frac{P_k^2 + Q_k^2}{|V_k|^2} \times r_k \tag{12}$$

where I_k, V_k, P_k and Q_k are the current flow, voltage drop, active and reactive power flow through branch k. r_k is the resistance of branch k. N_{branch} is the total number of branches.

3.1.2. Load balancing

Load balancing is achieved by minimizing the Load Balance Index (LBI), which is defined as

$$obj_2 = LBI = \sum_{k=1}^{N_{branch}} \left(\frac{I_k}{I_{k-rated}}\right)^2 \tag{13}$$

where $I_{k-rated}$ is the rated current of branch k

3.1.3. Voltage profile improvement

The improvement in voltage profiles is achieved by minimizing the Voltage Profile Index (VPI), which is defined as:

$$obj_3 = VPI = \sum_{i=1}^{N_{bus}} |V_i - V_{i,ref}| \tag{14}$$

where $V_{i,ref}$ is the nominal voltage of bus i, i.e. 1 p.u. was taken as $V_{i,ref}$ for all busses.

3.2. Constraints

In addition to the constraints as illustrated in Eqs. (1)–(10), the following limits were also considered:

3.2.1. Bus voltage limits

$$V^{min} \leqslant |V_i| \leqslant V^{max} \quad i \in \{1,2,...,N_{bus}\} \tag{15}$$

3.2.2. Branch capacity limits

$$|I_k| \leqslant I_k^{max} \quad k \in \{1,2,...,N_{branch}\} \tag{16}$$

where I_k^{max} is the maximum allowed current of branch k.

3.3. DG penetration

In order to evaluate the impact of DG penetrations on distribution network operation, a range of DG penetrations was considered. DG penetration is defined as the ratio of the active power injection from DG to the minimum active power demand of the network [29]. This case is recognized as the worst-case scenario [30,31] and provides vulnerable network operation conditions.

4. Optimization framework

4.1. Pareto optimality and dominance

Under the concept of Pareto optimality, candidate solutions which satisfy the imposed constraints are compared according to their dominances, and thereby a set of non-dominated solutions that are of equal interests amongst different objectives can be obtained. Assuming a collection of objective functions are to be minimized, solution 'A' dominates solution 'B' if:

$$\forall\ n \in [1,2,...,N_{obj}]\text{: } F_n(A) \leqslant F_n(B) \cap \exists\ n \in [1,2,...,N_{obj}]\text{: } F_n(A) < F_n(B) \tag{17}$$

where F_n is the n_{th} objective function value of a solution, and N_{obj} is the total number of the objective functions. The set of all Pareto optimal solutions is called the Pareto set, and the mapping of the Pareto set in the solution space is called the Pareto frontier, on which alternative solutions bringing out the flexible operation of the network are presented. For instance, some solutions may lead to lower power losses while some others may cause branch loadings more balanced. The availability of the Pareto frontier provides a set of feasible solutions for the distribution network operators (DNOs), and allows them to choose based on their priority or the network condition.

4.2. Overall optimization framework

The schematic of the proposed method, which integrates the global search technique (i.e. the MOPSO method) and the local search technique (i.e. the Taxi-cab method [32]), is shown in Fig. 3. The MOPSO is used for global search so as to obtain a set of Pareto optimal solutions. Then in order to avoid local optima trapping, the Taxi-cab method is used to fine tune the obtained Pareto solutions. The search space around each Pareto optimal solution is exploited further by the Taxi-cab method.

To illustrate the process of applying the Taxi-cab method to the Pareto solutions, a two-objective minimization problem is taken as an example. In Fig. 3(a), the point at the center of the dashed lines represents one Pareto solution. The Taxi-cab method can only optimize one objective function at a time. Therefore, the local search is carried

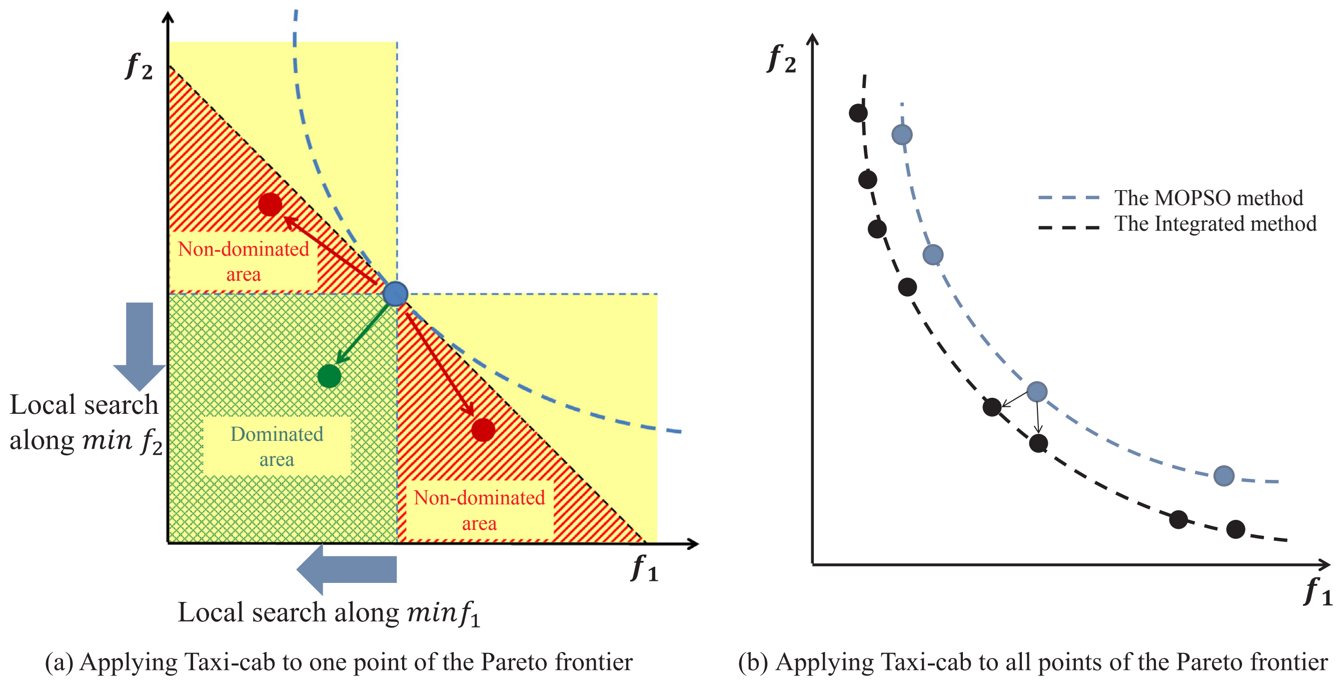

Fig. 3. Overall schematic of the integrated method (MOPSO + Taxi-cab).

out along the two objective functions separately, i.e. for one Pareto solution, a number of N_{obj} local searches are carried out and N_{obj} new solutions are generated. The possible area where new solutions might be located is shown in yellow. The possible area can be further divided into dominated area and non-dominated area. The dominated area refers to that both objective function values of the new solution are better than those values of the initial solution. The non-dominated area refers to that the new solution has a smaller value along one objective while sacrificing the value of another objective. The Taxi-cab method is then applied to all the Pareto solution points and an improved Pareto frontier is therefore obtained, as shown in Fig. 3(b). The Pareto solution points obtained by the MOPSO method are shown in blue, and the new Pareto frontier is shown in black. It can be seen clearly that with the integrated method (i.e. MOPSO + Taxi-cab), the Pareto frontier is pushed to lower values for both objectives, and better and more diverse Pareto solutions are generated.

4.3. Multi-Objective Particle Swarm Optimization (MOPSO)

4.3.1. Particle Swarm optimization (PSO)

PSO algorithm is a population-based multi-point search technique developed by Eberhart and Kennedy in 1995 [33]. The search starts with a population of random search points named particles. Each particle is encoded by a position vector (x) containing M-dimensional information (i.e. M is the number of decision variables). In this study, the active and reactive power injections of the SOP: [P_{C1},Q_{C1},Q_{C2}] (P_{C2} is not included since it is determined by P_{C1}) are considered as the decision variables. The position vector (x) is updated using the particle's velocity in successive iterations. In each iteration, the velocity vector (v) of a particle is updated using two best values. The first one is the individual/personal best position (p_{best}) achieved by each particle itself. The other one is the global best position (g_{best}) obtained by any particle in the population, which is used as a guide leading the population toward optimum. The velocity and position update equations for the i_{th} particle are:

$$v_i^{iter+1} = \omega v_i^{iter} + c_1 r_1 (p_{best,i}^{iter} - x_i^{iter}) + c_2 r_2 (g_{best,i}^{iter} - x_i^{iter}) \tag{18}$$

$$x_i^{iter+1} = x_i^{iter} + v_i^{iter+1} \tag{19}$$

where ω is the inertia weight controlling the effect of the particle's previous velocity on the current one, i.e. the tendency of a particle to continue in the same direction it has been traveling. c_1 is the cognitive learning factor representing the attraction that a particle has toward its own best. c_2 is the social learning factor representing the attraction that a particle has toward the best among its neighbors. c_1 and c_2 are usually defined as positive constants [34]. r_1 and r_2 are two random numbers $\in [0,1]$, which are used to keep away from entrapment on local optimum as well as to permit the diversity of particles in the search space. The updating process of Particle Swarm Optimization algorithm is illustrated in Fig. 4.

4.3.2. Multi-Objective Particle Swarm Optimization (MOPSO)

4.3.2.1. Selection of p_{best} and g_{best}. In Particle Swarm Optimization algorithm, the selection of p_{best} and g_{best} relies on the fitness value of

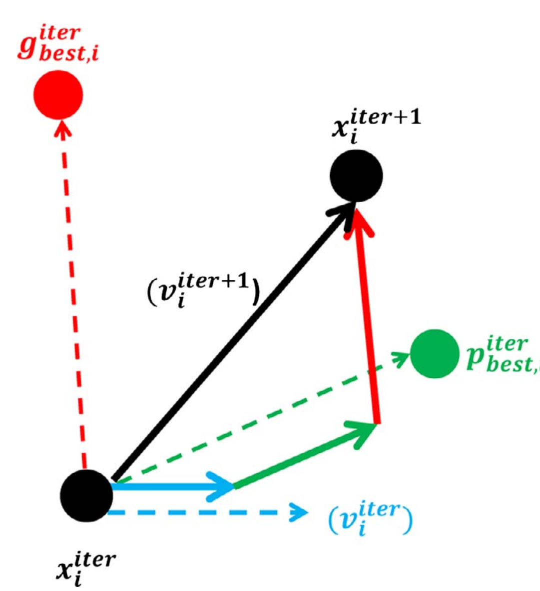

Fig. 4. The updating process of a particle's position and velocity in Particle Swarm Optimization.

particles, which is determined by the objective function. However, in a multi-objective problem, the concept of the best position is substituted with a set of non-dominated solutions and each of the non-dominated solutions is a potential guide for particles.

The selection of p_{best} is straightforward: if the current position of the i_{th} particle dominates its personal best position, p_{best} is replaced by the current position. If the current and personal best positions non-dominate each other, p_{best} is replaced by either of them with equal probability. Otherwise, p_{best} remains unchanged.

Regarding the selection of g_{best}, research work has been carried out to avoid defining a new concept of the guide by adopting approaches that aggregate all the objectives into a single function [35,36], or approaches that assign the objectives in order of importance. The solution is then obtained from optimizing the most important one and proceeding according to the assigned order of importance [37,38]. However, it is important to indicate that the majority of the currently proposed MOPSO approaches select guides based on the Pareto dominance. Several variations of the guide selection scheme are feasible and the MOPSO algorithm in this study is similar to the ones in [39–41], in which an archive is used to store the non-dominated solutions.

In the proposed MOPSO, the archive is updated iteratively using Pareto optimality. A random method is then used to select a guide for a particle from the archived solutions. Let a be the solutions in the archive A and x_i a particle's position. According to this method, if $A_{x_i} = \{a \in A | a \prec x_i\}$ (where the sign $\prec$ means 'dominate') is the set of archived solutions that dominate x_i, then the g_{best} for x_i is randomly chosen from A_{x_i} with equal probability. If $x_i \in A$, clearly A_{x_i} is empty. In this case the g_{best} for x_i is selected from the entire archive A [41]. Thus:

$$g_{best,i} = \begin{cases} a \in A \text{ with probability } |A|^{-1} & \text{if } x_i \in A \\ a \in A_{x_i} \text{ with probability } |A_{x_i}|^{-1} & \text{otherwise} \end{cases} \tag{20}$$

4.3.2.2. Retaining and spreading solutions in the archive. It is important to restrict the size of the archive. Since the archive has to be updated in each iteration, the update may become computationally expensive if the size of the archive grows too large.

In the research of evolutionary multi-objective optimization problems, different techniques have been adopted by researchers to bind the archive size while maintaining the diversity of solutions in the archive, e.g., niche technique [42], crowding distance sorting [20] and clustering [43]. More recently, the use of relaxed forms of dominance has been proposed. The main one adopted in Particle Swarm Optimization is the ε-dominance method [44], which is used to filter solutions in the archive [22]. Moreover, it is found in [45] that when comparing it against the existing clustering techniques for fixing the archive size, the ε-dominance method obtains solutions with much faster speed. Therefore this method is adopted in the proposed MOPSO for retaining and spreading the non-dominated solutions in the archive. An example of using the ε-dominance method to filter solutions in an archive is illustrated in Fig. 5.

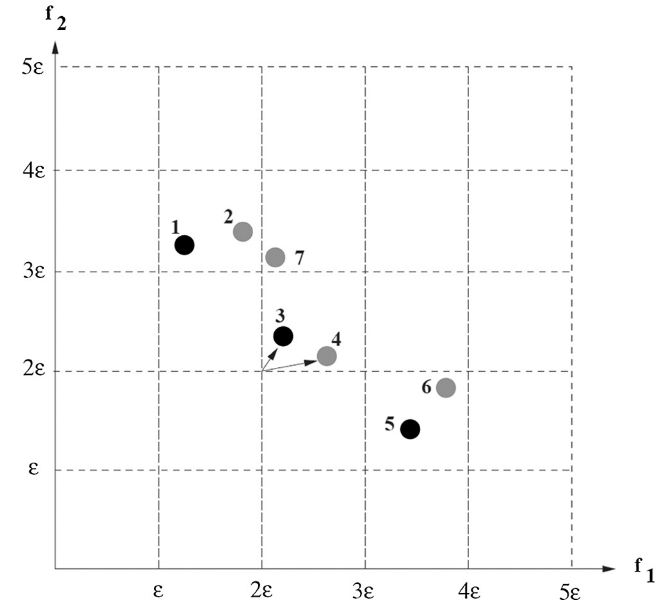

Fig. 5. An illustration of using the ε-dominance method (assuming minimizing f_1 and f_2 simultaneously): when selecting between solutions 1 and 2, solution 1 is preferred since it dominates solution 2 (same with selecting between solutions 5 and 6). Solutions 3 and 4 are incomparable. However, solution 3 is preferred since it is closer to the lower left corner represented by point (2ε, 2ε). Solution 7 is not accepted since its box, represented by point (2ε, 3ε) is dominated by the box represented by point (2ε, 2ε) [22].

4.3.2.3. Mutation operation. The appropriate promotion of diversity in Particle Swarm Optimization is an important issue in order to control its normally fast convergence [22]. Cauchy and Gaussian are two popular methods of mutation operations. An important difference between them is that the Cauchy distribution is heavier-tailed. This means that it is more prone to produce values that are far from its mean, thus making the Cauchy method have a higher chance of escaping premature convergences than the Gaussian method [46]. Therefore, in the proposed MOPSO algorithm, the Cauchy mutation operation is used. After each particle has completed its search, the Cauchy mutation is applied: if the previous position is dominated by the new one after mutation, the particle's position is then replaced by the new one. If neither of the previous position nor the new position dominates the other, one of them is selected to be that particle's position with equal probability. Otherwise the position remains unchanged.

4.4. Taxi-cab method

By applying the Taxi-cab method, a nonlinear function can be optimized in finite steps with fast convergence. The Taxi-cab method does not need any information of the derivative of the objective function, in which the search is performed by moving the decision variables along standard base vectors.

The process of the Taxi-cab method is as follows:

(1) Select a particle from the archive and set its position as X;
(2) Initialize the standard base vectors $e_m = [0,...,1_m,...,0]^T$, where $m = 1,2,...,M$, and M is the number of decision variables;
(3) Select the objective function obj_n that to be optimized, where $n = 1,2,...,N_{obj}$. Set $X_0 = X$
(4) Let X_0 be the starting point of the search. Along each base vector, obj_n is treated as the function of one decision variable only, where a one-dimensional search technique, the golden section search [47] is applied to generate an optimal step size g_m. The search is performed by proceeding along each of the base vectors successively and generating a sequence of improved values along the objective function:

$$X_m = X_{m-1} + g_m e_m \tag{21}$$

(5) The process is continued until X_M is obtained. Stop the search and set $n = n + 1$ then go back to step 3) if:

$$|obj_n(X_M) - obj_n(X_{M-1})| \leqslant \text{convergence criteria} \tag{22}$$

Otherwise, set $X_0 = X_M$ and go back to step 4);

The Taxi-cab method is applied to all particles in the archive after each iteration. Any new solutions obtained by the Taxi-cab method that are not dominated by any members in the archive are added into the archive, and any members in the archive which are dominated by the new solutions are deleted from the archive. This ensures that the

1: archive $A = \emptyset$
2: for $i = 1{:}N_{particle}$
3: Initialize: $x_i \quad v_i$
4: $EnforceConstraint(x_i)$
5: $p_{best,i} = x_i$
6: end
7: $A = Dominance(p_{best})$
8: $iter = 0$
9: while $iter \leq MAXiter$
10: for $i = 1{:}N_{particle}$
11: $v_i^{iter+1} = \omega v_i^{iter} + c_1 r_1 (p_{best,i}^{iter} - x_i^{iter}) + c_2 r_2 (g_{best,i}^{iter} - x_i^{iter})$
12: $x_i^{iter+1} = x_i^{iter} + v_i^{iter+1}$
13: $EnforceConstraint(x_i)$
14: $x_i = Mutate(x_i)$
15: if x_i $dominates$ $p_{best,i}$
16: $p_{best,i} = x_i$
17: end
18: end
19: $A = Dominance(p_{best})$
20: Taxi-cab (A)
21: Update A
22: $iter = iter + 1$
23: end

Fig. 6. Pseudo code of the integrated method.

archive always contains a set of non-dominated solutions.

4.5. Integrated method (MOPSO and Taxi-cab)

In the integrated method, the MOPSO is used to explore the solution space globally, and the Taxi-cab method is used for fine-tuning the non-dominated solutions in the archive of the MOPSO. The pseudo code of the integrated method is shown in Fig. 6.

At the beginning of the optimization process, the archive A is empty (Line 1) and the positions and velocities of all particles are initialized randomly (Line 3). Since it is possible that the particle positions lie outside the feasible region, it must be ensured that the decision variables, which are the active and reactive power injections of SOP, are constrained within the SOP rated capacity. This is indicated in Line 4 and 13 by using the function *EnforceConstraint*, in which an upper and a lower limits are used to bind the particle positions. The initial personal best positions of all particles are set to their starting positions (Line 5). The function *Dominance* (Line 7 and 19) is used to select the non-dominated solutions from all particles according to the concept of Pareto optimality, and store them in the archive. In each iteration, the particle positions and velocities are updated using Eqs. (18) and (19) (Line 11 and 12). Then the mutation operation (Line 14) as explained in Section 4.3.2.3 is applied to improve the diversity of solutions. Line 20 illustrates the local search procedure, where the Taxi-cab method is applied to the non-dominated solutions in the archive. When updating the archive (Line 21), the technique described in Section 4.3.2.2 is adopted for retaining and spreading the solutions in the archive.

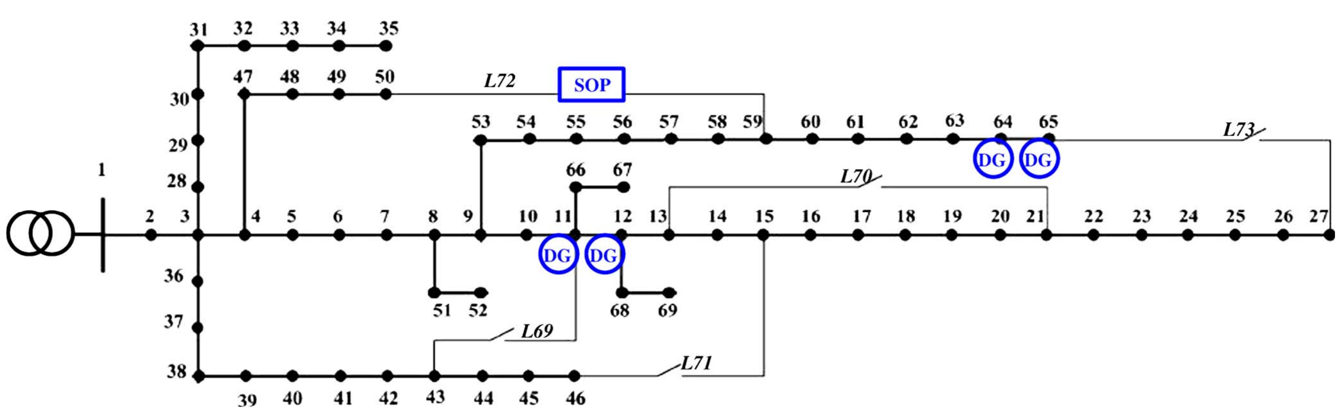

Fig. 7. A modified 69-bus distribution network.

5. Case study and results

5.1. Description of the test network

The proposed multi-objective optimization framework was developed in MATLAB and applied to a modified 69-bus distribution network [48]. The network is rated at 12.66 kV, with a total demand of 3802.19 kW and 2694.6 kVar, which was considered as the minimum demand. Four DGs were installed at bus 11, bus 12, bus 64 and bus 65, and an SOP was installed at *L72*, as show in Fig. 7. P_{loss}, *LBI* and *VPI* of the initial network are 225.01 kW, 10.82 and 23.26 respectively.

Several assumptions were made:

- The system is three-phase balanced.
- The rated capacity of the SOP is 5 MVA.
- Busses with DG installations are PQ busses with unity power factor.
- The range of DG penetrations is from 0 to 200%, with a 5% increment.

5.2. Multi-objective operation optimization results

Fig. 8 depicts the Pareto frontiers obtained by the MOPSO method and the integrated method, which gives a set of Pareto optimal solutions for the network operation under 50% DG penetration along multiple objectives with an SOP. The integrated method refers to the one integrating the MOPSO and the Taxi-cab methods.

The corresponding 2-dimentional, i.e. two-objective plots of the Pareto frontiers in Fig. 8 are presented in Fig. 9. It can be seen clearly that, the integrated method resulted in more numbers of Pareto optimal solutions than the MOPSO method. Moreover, better solutions were obtained by the integrated method, since these solutions resulted in smaller values for all objectives compared to those obtained by the MOPSO method.

The extreme points along each axis of the Pareto frontier illustrate the optimal values that can be obtained along each objective function. These extreme points searched by both MOPSO and integrated methods are listed in Table 1. Values of power loss, load balance index, and voltage profile index are presented in a pair of brackets, i.e. (P_{loss}, *LBI*, *VPI*). The improvements in percentage were calculated in comparison to those values of the network under the same DG penetration but without the SOP.

In Table 1, the improvements in each column illustrate that, the network operation along all the objectives were improved by using SOP for all DG penetrations varying from 0 to 200%.

Table 1 reveals the correlations between different pairs of objective functions. The rows of 'min P_{loss} points' show that minimizing the power loss resulted in decreases in *LBI* and *VPI* as well. The same situation occurred when minimizing the *LBI*. As shown by the rows of 'min *LBI* points', P_{loss} and *VPI* were also decreased for all DG penetrations. The minimization of *VPI*, however, caused increases in P_{loss} for all DG penetrations except for the case of zero DG penetration. It also caused increases in *LBI* when DG penetrations were high. Such conflictions are marked in red in Table 1.

It can be observed that improvements obtained by the integrated method were higher than those of the MOPSO method along all objectives and for all DG penetrations. This was due to the exploitation capability of the local search technique to fine tune the Pareto non-dominated solutions. The number of solutions obtained by the integrated method was 100, which reached the restricted archive size, whist the average number of solutions found by the MOPSO method was 42. This proved that the integrated method is capable to find more diverse Pareto solutions, and hence providing enhanced feasibility for decision making.

5.3. The impact of DG penetrations on SOP performance

In order to evaluate the impact of DG penetrations on the network operation, a range of DG penetrations were set as input during the optimization process. For each penetration, a set of non-dominated solutions were obtained by the integrated method and plotted against that penetration value. Hence, variations of the objective functions with increased DG penetrations were obtained as shown in Fig. 10, where each 'o' denotes one non-dominated solution.

It is observed in Fig. 10 that, all the variations present U-shape trajectories. The network power loss, load balance index and voltage profile index started to decrease when DG penetration increased from 0. Once these minimum values were reached, if the DG penetration continued to increase, the objective function values started to increase. With further increase of the DG penetration, the objective function values were even higher than those of the network without DG. Fig. 10 also provides an optimal range of DG penetrations that leads to improved network operations. For the test network under study, the optimal range of DG penetrations was around 20–80%.

5.4. Performance assessment of the integrated method

The performance assessment of multi-objective optimization methods is different from that of single-objective optimization methods,

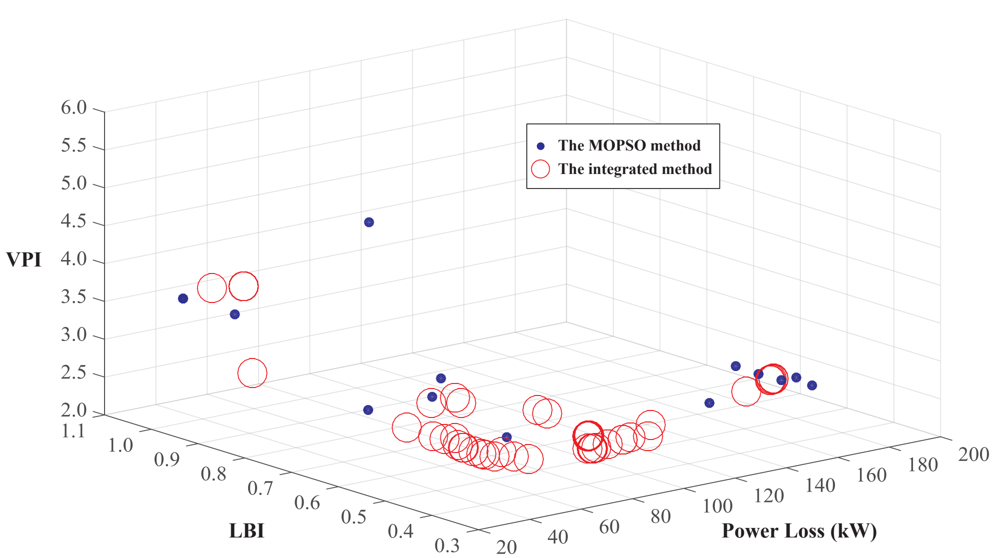

Fig. 8. Pareto frontiers obtained by the MOPSO method and the integrated method (50% DG Penetration).

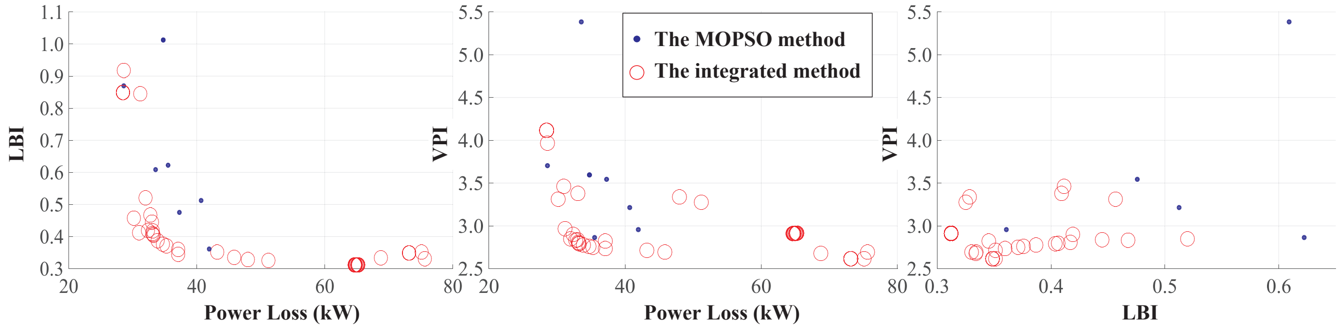

Fig. 9. 2-Dimentional plots of the Pareto frontiers shown in Fig. 8.

Table 1
Extreme points obtained by MOPSO and integrated methods & corresponding improvements to the case without SOP.

DG Penetration		0	50%	100%	150%	200%
Without SOP		(225.0, 10.8, 23.3)	(97.7, 4.3, 11.5)	(107.9, 2.2, 2.2)	(227.5, 3.7, 10.3)	(436.6, 7.3, 19.4)
MOPSO	min$\boldsymbol{P_{loss}}$ point	(60.0, 2.6, 9.3)	(29.2, 1.0, 4.9)	(50.2, 0.8, 1.9)	(121.2, 2.3, 7.6)	(234.8, 4.6, 11.9)
	Improvement	73.3%	70.1%	53.5%	46.7%	46.2%
Integrated (MOPSO+Taxi-cab)	min$\boldsymbol{P_{loss}}$ point	(60.0, 2.4, 9.2)	(28.5, 0.9, 4.1)	(49.7, 0.8, 1.8)	(119.3, 2.1, 6.5)	(233.5, 4.8, 11.6)
	Improvement	73.3%	70.8%	53.9%	47.6%	46.5%
MOPSO	min***LBI*** point	(190.2, 1.0, 6.9)	(95.4, 0.3, 3.2)	(70.7, 0.7, 1.9)	(128.5, 2.1, 5.2)	(255.4, 4.3, 9.0)
	Improvement	90.7%	93.0%	68.2%	43.2%	41.1%
Integrated (MOPSO+Taxi-cab)	min***LBI*** point	(137.9, 1.0, 6.7)	(67.4, 0.3, 3.0)	(61.2, 0.7, 1.9)	(122.5, 2.0, 5.1)	(246.4, 4.1, 8.9)
	Improvement	90.7%	93.0%	68.2%	45.9%	43.8%
MOPSO	min***VPI*** point	(197.2, 1.2, 5.9)	(174.0, 0.6, 2.6)	(187.9, 1.9, 1.8)	(977.5, 5.4, 3.5)	(1052.8, 7.1, 6.9)
	Improvement	74.7%	77.4%	18.2%	66.0%	64.4%
Integrated (MOPSO+Taxi-cab)	min***VPI*** point	(196.5, 1.2, 5.9)	(176.2, 0.5, 2.6)	(113.3, 1.2, 1.7)	(970.4, 5.5, 3.4)	(1509.2, 11.2, 6.1)
	Improvement	74.7%	77.4%	22.7%	67.0%	68.6%

The numbers in a pair of brackets represent the optimization results of power loss, load balance index and voltage profile index, i.e. ($P_{loss(kW)}$,LBI,VPI). The numbers marked in red represent the results that are worse than the case without SOP.

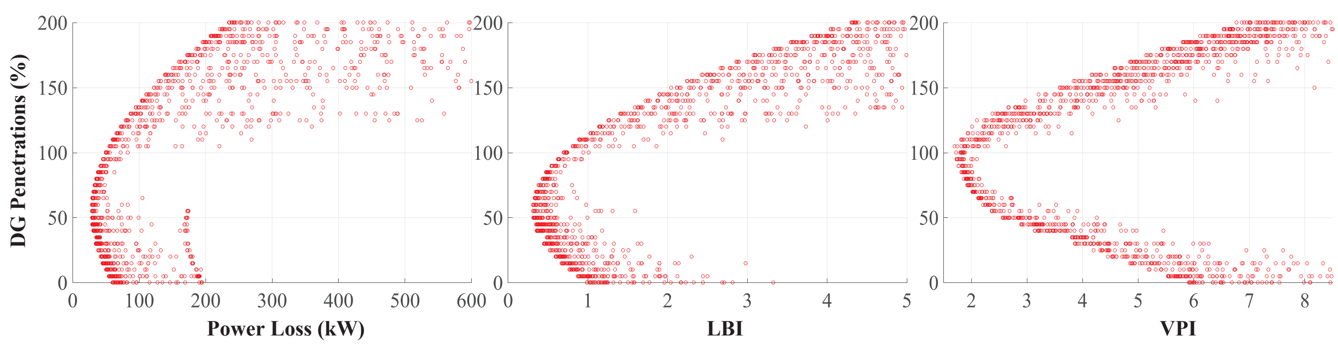

Fig. 10. Variations of the objective functions with increased DG penetrations.

Table 2
Performance metrics of Pareto solutions obtained by the MOPSO and integrated methods.

Methods	Metrics	
	diversity Δ	mean ideal distance MID
The MOPSO method	8.00	5.02
The integrated method	10.02	4.52
Improvement (%)	25.3%	10.0%

since a set of solutions rather than a single one are obtained. The diversity metric (Δ) and the mean ideal distance metric (MID) were used in this study to evaluate the quality of Pareto solutions. These two metrics give visions of how the Pareto solutions are dispersed and how they are close to the ideal values, which are formulated as follows:

$$\Delta = \sqrt{\sum_{n=1}^{N_{obj}} \left(max_{j=1,...,N_{PS}}\left\{of_n^j\right\} - min_{j=1,...,N_{PS}}\left\{of_n^j\right\}\right)^2} \tag{23}$$

$$MID = \sum_{j=1}^{N_{PS}} C_j/N_{PS} \tag{24}$$

Table 3
Comparisons of SOP and network reconfiguration in operation optimization on the 69-bus distribution network.

DG Penetration Without reconfiguration/SOP		0 (225.0, 10.8, 23.3)	50% (97.7, 4.3, 11.5)	100% (107.9, 2.2, 2.2)	150% (227.5, 3.7, 10.3)	200% (436.6, 7.3, 19.4)
Network Reconfiguration	min $\boldsymbol{P_{loss}}$ point Improvement	(105.7, 6.3, 12.2) 53.0%	(61.4, 3.8, 7.3) 37.2%	(75.7, 2.6, 2.9) 29.8%	(155.4, 2.7, 6.1) 31.7%	(296.8, 3.8, 12.3) 32.0%
SOP	min $\boldsymbol{P_{loss}}$ point Improvement	(60.0, 2.4, 9.2) 73.3%	(28.5, 0.9, 4.1) 70.8%	(49.7, 0.8, 1.8) 53.9%	(119.3, 2.1, 6.5) 47.6%	(233.5, 4.8, 11.6) 46.5%
Network Reconfiguration	min $\boldsymbol{LBI}$ point Improvement	(106.2, 6.1, 13.7) 43.2%	(61.6, 3.7, 6.6) 12.1%	(105.4, 2.1, 2.2) 2.0%	(279.4, 2.3, 4.3) 38.7%	(501.7, 4.4, 19.8) 40.0%
SOP	min $\boldsymbol{LBI}$ point Improvement	(137.9, 1.0, 6.7) 90.7%	(67.4, 0.3, 3.0) 93.0%	(61.2, 0.7, 1.9) 68.2%	(122.5, 2.0, 5.1) 45.9%	(246.4, 4.1, 8.9) 43.8%
Network Reconfiguration	min $\boldsymbol{VPI}$ point Improvement	(114.9, 6.3, 9.9) 57.3%	(100.2, 4.2, 3.6) 68.4%	(106.0, 2.1, 1.8) 20.7%	(170.6, 3.1, 4.0) 61.4%	(320.3, 4.6, 7.9) 59.1%
SOP	min $\boldsymbol{VPI}$ point Improvement	(196.5, 1.2, 5.9) 74.7%	(176.2, 0.5, 2.6) 77.4%	(113.3, 1.2, 1.7) 22.7%	(970.4, 5.5, 3.4) 67.0%	(1509.2, 11.2, 6.1) 68.6%

The numbers in a pair of brackets represent the optimization results of power loss, load balance index and voltage profile index, i.e. ($P_{loss(kW)}$,LBI,VPI).

where n denotes the n_{th} objective function and N_{obj} is the number of objective functions. j is the j_{th} Pareto solution in the Pareto set and N_{PS} is the number of solutions in the Pareto set. of_n^j is the value of the n_{th} objective function corresponding to the j_{th} solution. $max_{j=1,\ldots,N_{PS}}\{of_n^j\}$ and $min_{j=1,\ldots,N_{PS}}\{of_n^j\}$ represent the maximum and the minimum values of the n_{th} objective function. C_j is the Euclidean distance of the j_{th} solution from the ideal point (0,0,0).

The larger the diversity metric (Δ), the more diverse the Pareto solutions are. The smaller the mean ideal distance metric (MID), the closer to the ideal point the Pareto solutions are. These two metrics were calculated based on the solutions obtained by the integrated method as well as the MOPSO method and are listed in Table 2.

As shown in Table 2, the integrated method improves Δ by 25.3% compared to that obtained by the MOPSO method, which illustrates that the integrated method results in more diverse solutions. The improvement of MID by 10% shows that, the integrated method is capable of obtaining Pareto solutions with higher quality than the MOPSO.

5.5. Comparisons of SOP with network reconfiguration

Comparisons of SOP and network reconfiguration in operation optimization on the 69-bus distribution network with different DG penetrations are shown in Tables 3. Results listed are the optimal points along each objective function, which were selected from the Pareto frontiers obtained by using SOP and reconfiguration respectively. Again, the values of power losses, load balance index, and voltage profile index are presented in a pair of brackets, i.e. (P_{loss}, LBI, VPI), and improvements in percentage were calculated in comparison to those values of the network without reconfiguration or SOP. The results of Table 3 are visualized in Fig. 11.

It can be seen from Table 3 that, with different DG penetrations, SOP outperformed network reconfiguration along all objective functions. For example, when the DG penetration is 50%, the power loss reduction obtained by reconfiguration is 37.2% while the reduction obtained by SOP is 70.8%. Under the same condition, improvements of load balance and voltage profile obtained by reconfiguration are 12.1% and 68.4%, while the corresponding improvements obtained by SOP are 93.0% and 77.4%.

6. Conclusion

In this study, a multi-objective optimization framework was proposed to improve the operation of a distribution network with an SOP. Power loss reduction, load balancing and voltage profile improvement were taken as objectives, and the various penetrations of DG were taken into consideration. Firstly, a load flow algorithm incorporating the model of SOP was developed. Then a novel method that integrates both global and local search techniques was proposed to determine the optimal SOP set-points. In the integrated method, a MOPSO algorithm is used to explore the solution space globally, which contains an archive to store the non-dominated solutions, as well as a mutation operator to search for a wider space and avoid pre-convergences. A local search technique - the Taxi-cab method is used for solution space exploitation, which refines the quality of non-dominated solutions in the archive of

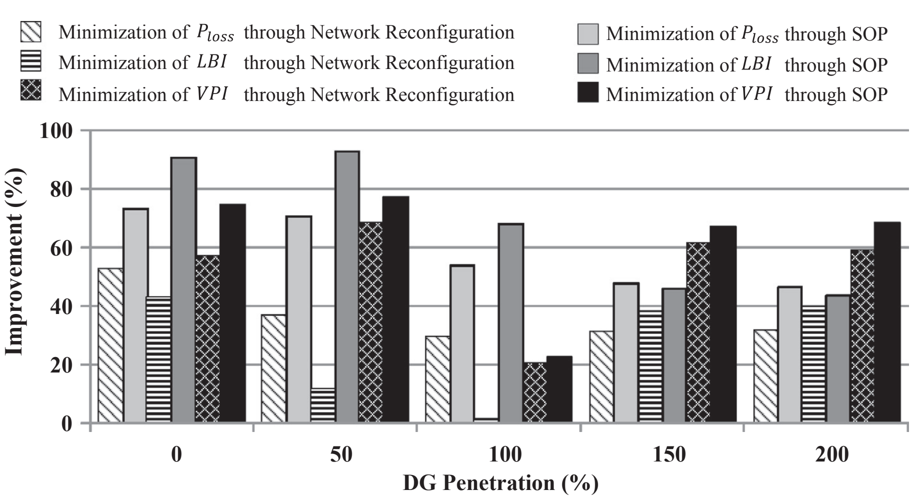

Fig. 11. Comparisons of the improvements along different objectives obtained through network reconfiguration and SOP.

MOPSO and enhances the search capability.

The multi-objective optimization framework was applied to a 69-bus test network. The results elaborated that SOP is an effective tool to improve the network operation in power loss reduction, load balancing and voltage profile improvement. With the DG penetration increasing from 0 to 200%, on average, an SOP reduces power losses by 58.4%, reduces the load balance index by 68.3% and reduces the voltage profile index by 62.1%, all compared to the case without an SOP. The analysis of the impact of DG penetrations on SOP performance showed that, the use of SOP facilitates a large increase in DG penetration and provides a significant increase in the flexibility of distribution network operation. When compared with the conventional MOPSO method, the proposed integrated method increases the diversity metric by 25% and reduces the mean ideal distance metric by 10%. It is also found that the network with an SOP outperformed the one using network reconfiguration in operation optimization under various DG penetrations. With the DG penetration increasing from 0 to 200%, on average, an SOP outperforms network reconfiguration on power loss reduction, feeder load balancing and voltage profile improvement by 21.7%, 41.1% and 8.7% respectively.

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