

This is an Open Access document downloaded from ORCA, Cardiff University's institutional repository: <https://orca.cardiff.ac.uk/id/eprint/106149/>

This is the author's version of a work that was submitted to / accepted for publication.

Citation for final published version:

Chen, Guan-Nian, Cleall, Peter , Li, Yu-Chao, Yu, Ze-Xi, Ke, Han and Chen, Yun-Min
2018. Decoupled advection-dispersion method for determining wall thickness of slurry trench cutoff walls. *International Journal of Geomechanics* 18 (5) , 06018007.
10.1061/(ASCE)GM.1943-5622.0001130

Publishers page: [http://dx.doi.org/10.1061/\(ASCE\)GM.1943-5622.00011...](http://dx.doi.org/10.1061/(ASCE)GM.1943-5622.00011...)

Please note:

Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher's version if you wish to cite this paper.

This version is being made available in accordance with publisher policies. See <http://orca.cf.ac.uk/policies.html> for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.



1 **Title:** A Decoupled Advection-Dispersion Method for Determining Wall Thickness of
2 Slurry Trench Cut-off Walls

3

4 **Authors:**

5 Guan-Nian Chen¹; Peter John Cleall²; Yu-Chao Li³; Ze-Xi Yu⁴; Han Ke⁵; Yun-Min
6 Chen⁶

7 ¹ PhD Candidate, MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering,
8 Department of Civil Engineering, Zhejiang University, Hangzhou 310058, China. E-
9 mail: 1354128629@qq.com

10 ² Reader, Cardiff School of Engineering, Cardiff University, Cardiff, CF24 3AA, Wales,
11 UK. E-mail: cleall@cf.ac.uk

12 ³ Professor, MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering,
13 Department of Civil Engineering, Zhejiang University, Hangzhou 310058, China. E-
14 mail: yuchao_li@hotmail.com (corresponding author)

15 ⁴ MSc student, MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering,
16 Department of Civil Engineering, Zhejiang University, Hangzhou 310058, China. E-
17 mail: 1979073949@qq.com

18 ⁵ Professor, MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering,
19 Department of Civil Engineering, Zhejiang University, Hangzhou 310058, China. E-
20 mail: boske@126.com

21 ⁶ Professor, MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering,
22 Department of Civil Engineering, Zhejiang University, Hangzhou 310058, China. E-
23 mail: chenyunmin@zju.edu.cn

24

25 **Abstract:** Low-permeability slurry trench cut-off walls are commonly constructed as
26 barriers for containment of subsurface point source pollution or as part of seepage control
27 systems on contaminated sites. A method to estimate wall thickness in slurry wall design
28 is proposed based on decoupling the advective and dispersive components of contaminant
29 fluxes through the wall. The relative error of the result obtained by the proposed method
30 to that by an analytical solution increases as the ratio of the specified breakthrough exit
31 concentration (c^*) to the source concentration (c_0) increases. For c^*/c_0 of less than 0.1,
32 which covers common practical situations, the relative error is not greater than 4% and is
33 always conservative indicating that the proposed method provides sufficient accuracy for
34 design. For a given breakthrough criterion (that is, c^*/c_0), the relative error is low for the
35 scenarios having either a low or high column Peclet number, where either dispersion and
36 advection dominate the contaminant migration, respectively; and the relative error is high
37 for the scenario having an intermediate column Peclet number, in which case the
38 coupling effect of advective and dispersive migrations is relatively high.

39

40

41 **Keywords:** advection; breakthrough time; dispersion; slurry wall; wall thickness

42

43 **Introduction**

44

45 Slurry trench cut-off walls (termed as slurry walls hereafter) utilizing low-permeability
46 backfill materials are commonly constructed as barriers for containment of subsurface
47 point source pollution or as part of seepage control systems on contaminated sites. In
48 slurry wall design, one key requirement is that the breakthrough time t_b should be not less
49 than the designed service life. Transient contaminant transport through slurry walls can
50 be regarded as a one-dimensional advective-dispersive process (see Fig. 1). Analytical
51 solutions (Lapidus and Amundson 1952; Ogata and Banks 1961; Brenner 1962;
52 Lindstrom et al. 1967) are available for varied boundary conditions to calculate the
53 contaminant transport in slurry walls. However, these analytical solutions contain non-
54 elementary functions (such as complementary error function) or require the solution of
55 eigen equations. The evaluation of these analytical solutions is nontrivial and generally
56 requires the use of a computer (Rowe et al. 2004). Accordingly the wall thickness
57 corresponding to a designed service life has to be searched in a trial and error manner.
58 This often leads to determination of the wall thickness by practical experience instead of
59 contaminant transport analysis in design. An alternative simplification of the analytical
60 solution of Ogata and Banks (1961) was presented by Cavalcante and de Farias (2013)
61 however a numerical computation was required to iteratively obtain a solution.

62

63 A new method for determining the wall thickness of slurry walls is proposed in this paper.
64 Representation of contaminant migration through the slurry wall is simplified by
65 superposition of decoupled solutions for advective and dispersive fluxes. The error of the

66 proposed method is investigated by comparing the results with those obtained by an
67 analytical solution commonly used for slurry wall design. Finally, an example is
68 presented to illustrate the procedure of implementing the proposed method for slurry wall
69 design.

70

71

72 **Method**

73

74 As illustrated in Fig. 1, the slurry wall keys into the impervious soil layer. The backfill is
75 assumed to be homogenous, fully saturated and non-deformable. The pore water flow in
76 the backfill is assumed to be in a steady state condition. Contaminant migration in the
77 slurry wall can be regarded as a one-dimensional advective-dispersive process (Freeze
78 and Cherry 1979), that is,

$$79 \quad R \frac{\partial c}{\partial t} = D_h \frac{\partial^2 c}{\partial x^2} - v_s \frac{\partial c}{\partial x} \quad (1)$$

80 where c is the volume-average concentration of contaminant in the pore water of backfill;
81 t is time; R is the retardation factor of contaminant for the backfill; and D_h is the
82 hydrodynamic dispersion coefficient of contaminant in the backfill. v_s is the seepage
83 velocity of the pore water flow and can be written as follows,

$$84 \quad v_s = \frac{v_d}{n} = \frac{kh}{nL} \quad (2)$$

85 where v_d is the discharge velocity of pore water flow given by Darcy's law (see Eq. (2));
86 n and k are the porosity and hydraulic conductivity of the backfill, respectively; L is the

87 thickness of the slurry wall; and h is the hydraulic head difference between the entrance
88 and exit boundaries of the slurry wall and is assumed to be independent of L .

89

90 The backfill is assumed to be initially free of contaminant. A constant concentration (that
91 is, $c=c_0$, where c_0 is the source concentration of contaminant at the upstream) at the
92 entrance boundary, as typically recommended for vertical barrier design and performance
93 assessment (Rabideau and Khandelwal 1998), is assumed in this paper. The
94 breakthrough time is commonly defined to be the time when the exit concentration
95 reaches a specified value (c^*) which is often based on groundwater quality or other
96 standards.

97

98 As shown in Eq. (1), advective and dispersive migrations of the contaminant are coupled,
99 which leads to relatively complex analytical solutions. In this paper, advection and
100 dispersion are assumed to be decoupled to allow development of a simplified method for
101 performing a suitable design of the wall thickness. The error caused by this assumption
102 is investigated in the next section. The concentration of contaminant for the pure
103 advection segment is equal to the source concentration (that is, $c=c_0$) due to the effect of
104 dispersion being ignored. At the breakthrough time the distance between the advection
105 front and the entrance boundary due to pure advection, x_a , with consideration of
106 adsorption is

$$107 \quad x_a = v_s \frac{t_b}{R} \quad (3)$$

108 The pure advection segment provides a moving constant concentration boundary, whose
109 velocity is equal to v_s/R , making the subsequent segment one of pure dispersion in a

110 semi-infinite medium, as illustrated in Fig. 2. Concentration continuity is assumed at the
 111 interface of the two segments, in other words, the concentration at the inlet boundary of
 112 the pure dispersion segment is equal to that of the advection front (which is c_0). The
 113 analytical solution presented by Carslaw and Jaeger (1959) gives the following equation
 114 for the pure dispersion segment at the breakthrough time,

$$115 \quad \operatorname{erfc}\left(\frac{x_d}{2\sqrt{D_h t_b / R}}\right) = \frac{c^*}{c_0} \quad (4)$$

116 where x_d is the dispersion distance between the advection front and the exit boundary, as
 117 illustrated in Fig. 2. The ratio of the specified breakthrough exit concentration to the
 118 source concentration, that is, c^*/c_0 , represents the breakthrough criterion.

119

120 The complementary error function in Eq. (4) is a non-elementary function and a variable
 121 m can be defined as the solution of the following equivalent equation,

$$122 \quad \operatorname{erfc}(m) = \frac{c^*}{c_0} \quad (5)$$

123 For c^*/c_0 in the range of 0.001 to 0.1, which covers the problems commonly considered,
 124 the following approximating formula is proposed for the relationship between m and c^*/c_0
 125 by fitting with least-square method

$$126 \quad m = 3.56 - 3.33(c^*/c_0)^{0.142} \quad (6)$$

127 The relative error of Eq. (6) to Eq. (5) is less than 0.7% for the range of c^*/c_0 of 0.001 to
 128 0.1. x_d can then be written as follows by substitution of Eq. (5) into Eq. (4),

$$129 \quad x_d = 2m\sqrt{D_h \frac{t_b}{R}} \quad (7)$$

130 At the breakthrough time the wall thickness L is equal to the sum of x_a and x_d , and so can
131 be expressed as

$$132 \quad L = v_s \frac{t_b}{R} + 2m \sqrt{D_h \frac{t_b}{R}} \quad (8)$$

133 The wall thickness corresponding to the designed service life of t_b for a breakthrough
134 criterion of c^*/c_0 can be obtained explicitly from Eq. (9) using Eq. (2), that is,

$$135 \quad L = \left(m + \sqrt{m^2 + P_L} \right) \sqrt{D_h \frac{t_b}{R}} \quad (9)$$

136 where P_L is the column Peclet number (van Genuchten and Parker 1984; Shackelford
137 1994; Shackelford 1995; Rabideau and Khandelwal 1998; Prince et al. 2000), which
138 represents the relative importance of advective migration to the dispersive migration and
139 is defined by

$$140 \quad P_L = \frac{v_s L}{D_h} = \frac{kh}{nD_h} \quad (10)$$

141 For many cases of slurry walls, the value of hydraulic conductivity of backfills, the range
142 of typical values of P_L is 0.01~1000. If the wall thickness L is given, the breakthrough
143 time for a breakthrough criterion of c^*/c_0 can be estimated as follows based on Eq. (9),

$$144 \quad t_b = \frac{L^2 R}{\left(m + \sqrt{m^2 + P_L} \right)^2 D_h} \quad (11)$$

145

146

147 **Error analysis**

148

149 The error associated with the assumption of the advective and dispersive fluxes being
 150 decoupled is investigated in this section. The results found by the proposed method are
 151 compared to those obtained from the analytical solution commonly used in slurry wall
 152 design (Lapidus and Amundson 1952; Ogata and Banks 1961) that gives the following
 153 equation at the breakthrough time t_b , when the exit concentration rises to c^* at $x=L$,

$$154 \quad \frac{c^*}{c_0} = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2} A - \frac{1}{2} \frac{P_L}{A}\right) + \frac{1}{2} \exp(P_L) \operatorname{erfc}\left(\frac{1}{2} A + \frac{1}{2} \frac{P_L}{A}\right) \quad (12)$$

155 where A is a dimensionless parameter defined by

$$156 \quad A = \sqrt{\frac{R}{D_h t_b}} L \quad (13)$$

157

158 The relative error, e_r , of the value of A obtained by the proposed method with respect to
 159 that by the analytical solution (Eq. 12) is calculated as follows,

$$160 \quad e_r = \frac{A_d - A_c}{A_c} \times 100\% \quad (14)$$

161 where A_d is the value of A obtained by the proposed method; and A_c is that found solving
 162 the analytical solution with a Newton-Raphson method based search. The relationship
 163 between e_r and P_L for varied breakthrough criteria (that is, c^*/c_0) is shown in Fig. 3. The
 164 value of e_r is always positive, which indicates that the proposed method gives a
 165 conservative result in terms of wall thickness for the designed service life (see Eq. (13)).

166

167 The relative error increases as c^*/c_0 increases and is not greater than 4% for $c^*/c_0 \leq 0.1$,
 168 which is commonly used as a breakthrough criterion in slurry wall design. For $c^*/c_0 = 0.2$
 169 and 0.3, the peak relative errors are 5.9% and 8.1%, respectively. So it is noted that the

170 wall thickness evaluated by the proposed method may be over 8.1 % higher than that
171 required according to contaminant transport analysis if $c^*/c_0 > 0.2$. Fig. 3 also shows that
172 the value of P_L corresponding to the peak value of e_r increases with decreasing c^*/c_0 due
173 to the higher impact of dispersion/diffusion on contaminant breakthrough.

174

175 For any given ratio of c^*/c_0 the relative error is low for relatively low or high values of P_L ,
176 as in these scenarios dispersion and advection dominate contaminant migration,
177 respectively. In such cases, the coupling effects between advection and dispersion are
178 relatively low and subsequently the impact of assuming the two processes to be
179 decoupled becomes less significant. The relative error has a peak value for an
180 intermediate value of P_L , where both dispersion and advection are significant with a
181 relatively high degree of coupling occurring between the two migration processes.

182

183

184 **Example**

185

186 The procedure of implementing the proposed method to determine the wall thickness of
187 slurry walls is illustrated in this section. In the example considered, the porosity and
188 hydraulic conductivity of the backfill are taken as 0.4 and 1×10^{-9} m/s, respectively. The
189 contaminant is phenol, and its retardation factor is 30 based on Malusis et al. (2010) and
190 hydrodynamic dispersion coefficient is taken as 5×10^{-10} m²/s (Rowe, et al. 2004). The
191 hydraulic head difference between the entrance and exit boundaries is assumed to be 0.8
192 m, and the entrance reservoir concentration of phenol is 1.0 mg/L according to the data of

193 typical landfill leachate (Rowe, et al. 2004). A concentration of 0.002 mg/L at the exit
 194 boundary is used as the breakthrough criterion as per class III of Chinese Quality
 195 Standard of Ground Water (GB/T 14848-1993). The designed service life of the slurry
 196 wall is taken as 50 years.

197

198 The wall thickness of the slurry wall can be determined by the following steps using the
 199 proposed method:

200 Step 1: Calculate the ratio of the specified breakthrough exit concentration to the source
 201 concentration,

$$202 \quad \frac{c^*}{c_0} = \frac{0.002}{1.0} = 0.002 \quad (15)$$

203 Step 2: Approximate the value of m via Eq. (6),

$$204 \quad m = 3.56 - 3.33 \left(\frac{c^*}{c_0} \right)^{0.142} = 3.56 - 3.33 \times 0.002^{0.142} = 2.18 \quad (16)$$

205 Step 3: Calculate the value of P_L using Eq. (10),

$$206 \quad P_L = \frac{kh}{nD_h} = \frac{1 \times 10^{-9} \times 0.8}{0.4 \times 5 \times 10^{-10}} = 4.0 \quad (17)$$

207 Step 4: Calculate the wall thickness L from Eq. (9),

$$208 \quad L = \left(m + \sqrt{m^2 + P_L} \right) \sqrt{D_h \frac{t_b}{R}} = \left(2.18 + \sqrt{2.18^2 + 4.0} \right) \sqrt{5 \times 10^{-10} \frac{50 \times 3.1536 \times 10^7}{30}} = 0.83 \text{ m} \quad (18)$$

209

210 The calculated wall thickness of 0.83 m corresponds to a designed service life of 50 years
 211 for the specified breakthrough criterion. As a result, $L=0.9$ m can be used as the designed
 212 wall thickness of the slurry wall.

213

214 The concentration profiles in the slurry wall at the calculated breakthrough time are
215 shown in Fig. 4. For the proposed method, the advection front x_a is 0.14 m and the
216 dispersion distance x_d is 0.76 m, which indicates that dispersion/diffusion dominates the
217 contaminant migration for the scenario considered (that is, $P_L=4.0$). At the breakthrough
218 time the contaminant concentration profile obtained by the proposed method is close to
219 that produced by the analytical solution in which the advective and dispersive migrations
220 are coupled.

221

222 Concentration profiles, at the calculated breakthrough time, for the scenarios with $P_L=0.4$,
223 40 and 400, are also shown in Fig. 4. It can be observed that for cases having low and
224 high P_L the proposed method does not result in a significant error in the determination of
225 wall thickness or breakthrough time despite the assumption of a decoupled advection-
226 dispersion problem. For $P_L=0.4$, the concentration profiles obtained by these two
227 methods are close with a relative error of 0.2% in the calculated t_b due to this scenario
228 being dispersion dominated. For $P_L=400$, advection dominates contaminant migration,
229 and the concentration profiles obtained by the two methods are also close to each other.

230

231 For the scenario with $P_L=40$ the relative error of the predicted t_b is 2.5%, which is fully
232 acceptable in design, though the difference between the concentration profiles is relative
233 large compared to the other scenarios. The advection front x_a is 0.45 m and the
234 dispersion distance x_d is also 0.45 m (see Fig. 4). This indicates that for this scenario the
235 contaminant migration is controlled by both advection and dispersion.

236

237

238 **Conclusions**

239

240 A simplified method has been proposed to determine the thickness of slurry walls via an
241 assumption of decoupled advection-dispersion in the analysis of contaminant migration.

242 The relative error for the column Peclet number P_L in the range of 0.01 and 1000 is not
243 greater than 4% when the breakthrough criterion of c^*/c_0 is less than 0.1, which covers

244 common practical situations in slurry wall design. For a given breakthrough criterion, the
245 relative error is relatively low for a low or high P_L , when dispersion or advection

246 dominate the contaminant migration, respectively; but for intermediate values of P_L ,
247 when the coupling effects between dispersion and advection migrations are more

248 significant the relative errors are higher. Finally, it should be fully recognised that such a
249 decoupling approach may not be extended to other contaminant transport problems

250 without careful calculation and comparison of the results to those obtained by suitable
251 analytical solutions.

252

253

254 **Acknowledgements**

255

256 The financial supports received from the National Natural Science Foundation of China
257 (NSFC) by grant No. 51378465 and 41672284 and the Science Technology Department

258 of Zhejiang Province by grant No. 2016C31G2010015 are gratefully acknowledged.

259

260

261 **References**

262

263 Brenner, H. (1962). "The diffusion model of longitudinal mixing in beds of finite
264 length - numerical values." *Chem Eng Sci*, 17(4), 229-243.

265 Carslaw, H. S., and Jaeger, J. C. (1959). *Conduction of Heat in Solids*, Oxford
266 University Press.

267 Cavalcante, A. L. B., and de Farias, M. M. (2013). "Alternative solution for
268 advective-dispersive flow of reagent solutes in clay liners." *Int J Geomech*, 13(1), 49-56.

269 Freeze, R. A., and Cherry, J. A. (1979). *Groundwater*, Prentice-Hall, Inc.,
270 Englewood Cliffs, New Jersey.

271 Lapidus, L., and Amundson, N. R. (1952). "Mathematics of adsorption in beds. 6.
272 The effect of longitudinal diffusion in ion exchange and chromatographic columns." *J*
273 *Phys Chem-U.S.*, 56(8), 984-988.

274 Lindstrom, F. T., Haque, R., Freed, V. H., and Boersma, L. (1967). "Theory on the
275 movement of some herbicides in soils: Linear diffusion and convection of chemicals in
276 soils." *Environmental Science and Technology*, 1(7), 561-565.

277 Malusis, M. A., Maneval, J. E., Barben, E. J., Shackelford, C. D., and Daniels, E. R.
278 (2010). "Influence of adsorption on phenol transport through soil-bentonite vertical
279 barriers amended with activated carbon." *J Contam Hydrol*, 116(1-4), 58-72.

280 Ogata, A., and Banks, R. B. (1961). "A solution of the differential equation of
281 longitudinal dispersion in porous media." *U. S. Geological Survey Professional Paper*
282 *411-A*, A1-A9.

283 Prince, M. J., Maneval, J. E., and Evans, J. C. (Year). "Analysis of boundary
284 conditions for contaminant transport through adsorptive, low-permeability slurry trench
285 cutoff walls." *Proc., Environmental Geotechnics*, ASCE, 58-72.

286 Rabideau, A., and Khandelwal, A. (1998). "Boundary conditions for modeling
287 transport in vertical barriers." *J Environ Eng-Asce*, 124(11), 1135-1139.

288 Rowe, R. K., Quigley, R. M., Brachman, R. W. I., and Booker, J. R. (2004). *Barrier*
289 *Systems for Waste Disposal Facilities*, E & FN Spon, Abingdon, Oxon.

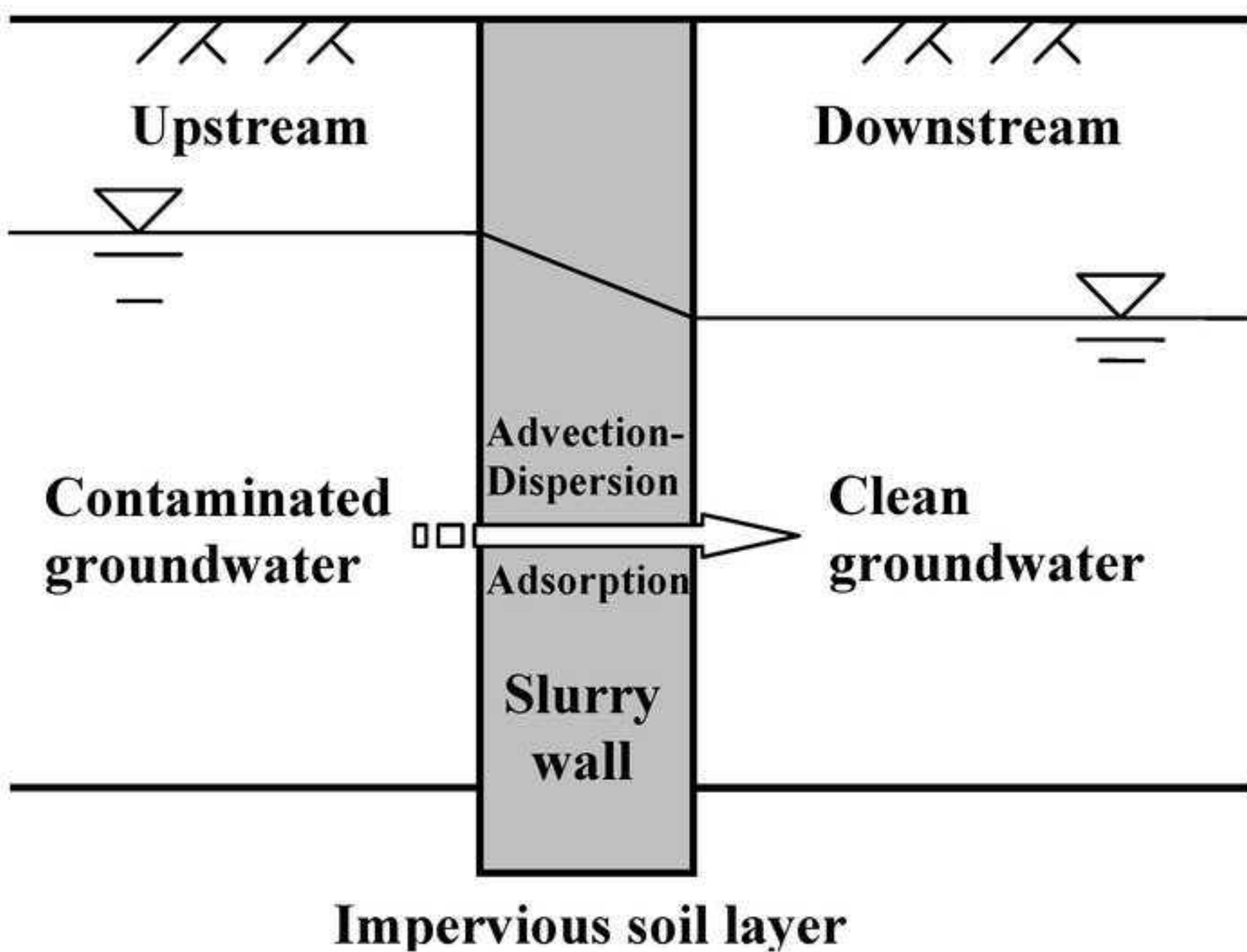
290 Shackelford, C. D. (1994). "Critical concepts for column testing." *Journal of*
291 *Geotechnical Engineering-ASCE*, 120(10), 1804-1828.

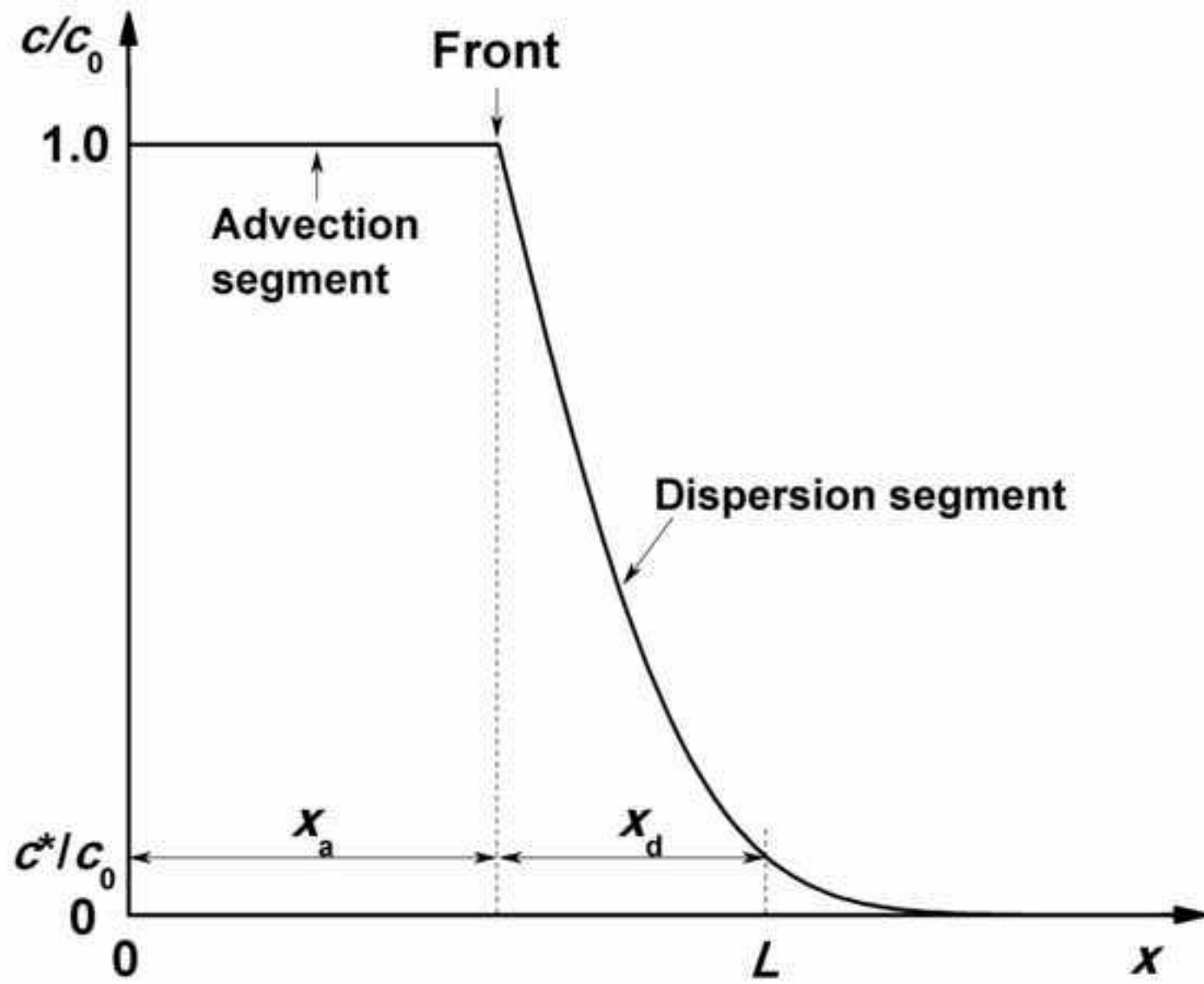
292 Shackelford, C. D. (1995). "Analytical models for cumulative mass column
293 testing." *Proceedings of ASCE Specialty Conference. Geoenvironment 2000*, ASCE, New
294 York, 355-372.

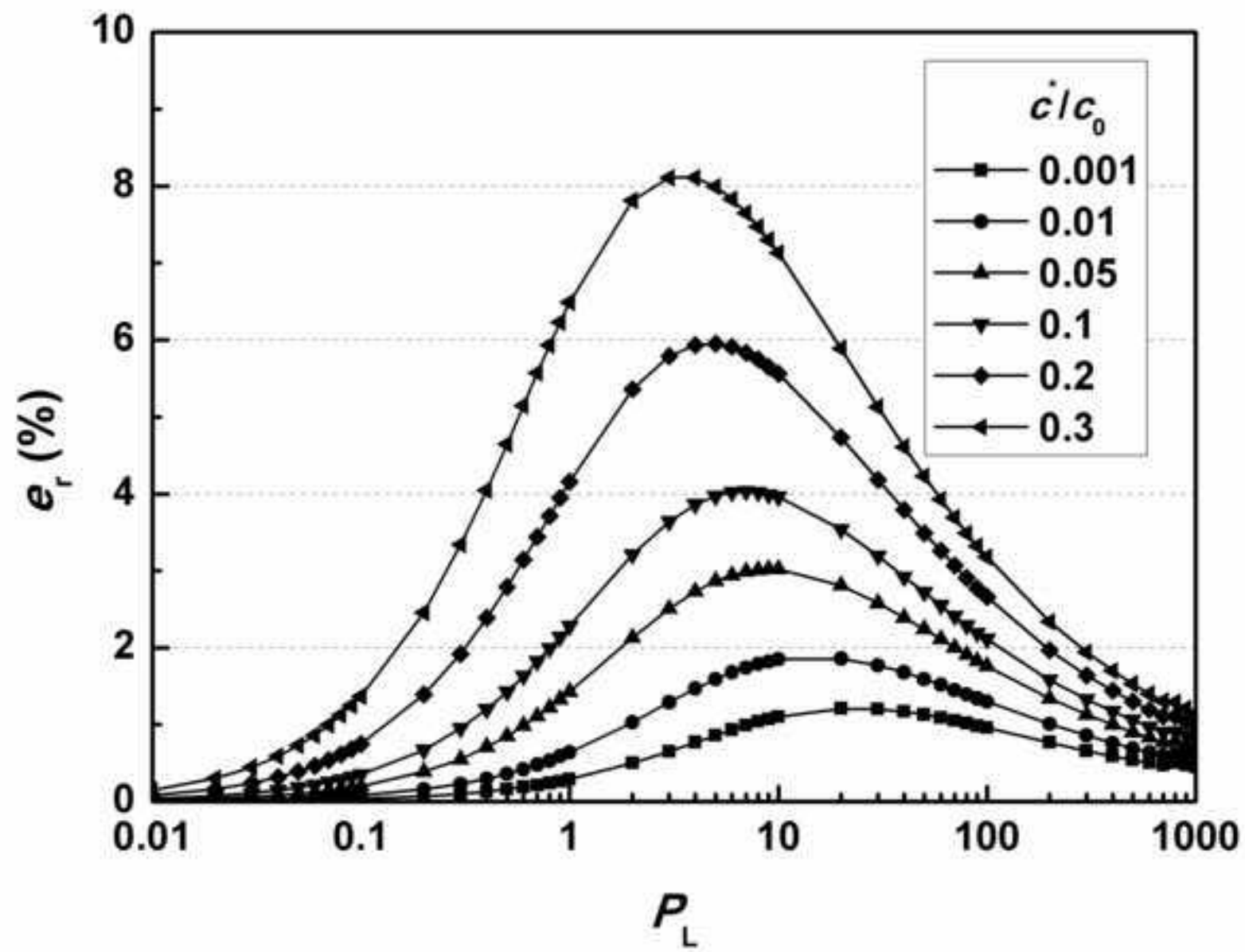
295 van Genuchten, M. T., and Parker, J. C. (1984). "Boundary conditions for
296 displacement experiments through short laboratory soil columns." *Soil Science Society of*
297 *America Journal*, 48(4), 703-708.

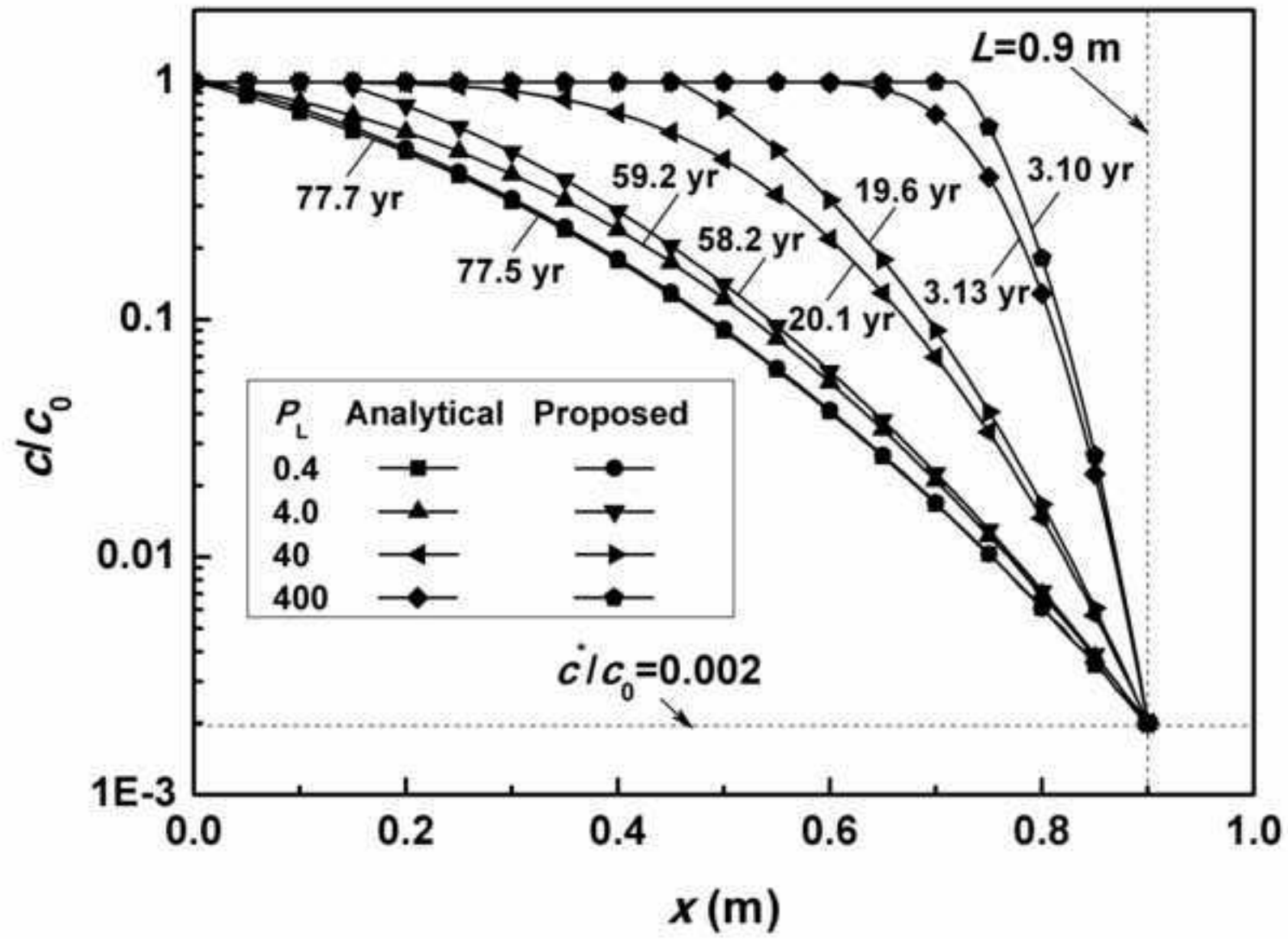
298

299









List of figure captions

Fig. 1. Configuration of contaminant transport through a slurry trench cut-off wall

Fig. 2. Configuration of contaminant concentration profile in slurry wall at
breakthrough time for the proposed method

Fig. 3. Relationship between relative error of A and P_L for the proposed method

Fig. 4. Concentration profiles of analytical solution and proposed method for varied
assumed P_L for the example