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Using Bradley-Terry Models to Analyse Test Match Cricket

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In this paper we investigate the use of Bradley-Terry models to analyse test match cricket. Specifically, we develop a new and alternative team ranking and compare our rankings with those produced by the International Cricket Council, we forecast the outcomes of a selected number of test cricket matches and show that our predictions perform well compared to bookmaker predictions. We offer ratings of individual players and use these ratings to predict the results of some recent matches. The general purpose of the paper is to illustrate the potential of Bradley-Terry models, which are effectively models of $P(i \text{ is preferred to } j)$, and thus can be applied in a number of settings where there are paired comparisons. Popular applications include analysing taste test experiments and modelling sports competitions. More creative examples of applications include statistical modelling of citation exchange among statistics journals, predicting the fighting ability of lizards and estimating driver crash risks.

Keywords: Bradley-Terry models; test match cricket; forecasting.

1. Introduction

Suppose that there are a set of entities that we wish to consider according to some common attribute. Pairs of these entities can be compared with respect to some quantifiable aspect of this attribute. Such comparisons are generally referred to as 'paired comparisons'. An example of such a comparison is a taste test, where a judge tastes two specimens before declaring a preference for a particular specimen.

The Bradley-Terry model is a useful model that was developed with a view to analysing the results from a set of paired comparisons. Such models can produce scores for the entities based on a set of results, and can also be used to make predictions for the results of future comparisons. Furthermore, if we instead consider a comparison as a contest between two competitors, a variety of possible applications becomes apparent, most notably in sport.

In this paper we investigate the use of Bradley-Terry models to analyse test match cricket. There are a number of other papers which have analysed at least one version of cricket. Some recent papers include Scarf & Shi (2005), Scarf et al. (2011), Scarf & Akhtar (2011),Akhtar & Scarf (2012), Perera & Swartz (2012), Davis et al. (2015) and Akhtar et al. (2015). Each of these papers also has numerous references of interest to those in analysing cricket. Specifically, we do the following:

- We develop a new and alternative team ranking and compare our rankings with those produced by the International Cricket Council (ICC).
- We forecast the outcomes of a selected number of matches and compare our predictions with bookmaker odds.
- We offer a method to obtain ratings of individual players.

The purpose of the paper is to illustrate the potential of Bradley-Terry models, which are effectively models of $P(i \text{ is preferred to } j)$. This is an interesting construct in itself, with many potential applications. The Bradley-Terry model (Bradley & Terry (1952)) is named after Ralph A. Bradley and Milton

E. Terry, who devised the method in 1952. However, as noted in Simons et al. (1999), the method was discovered independently in Zermelo (1929), Ford (1957) and Jech (1983). Agresti (2014) discusses the Bradley-Terry model and presents a simple example of how it can be used to rank baseball teams.

Applications of the Bradley-Terry model are both plentiful and diverse. Popular applications include analysing taste test experiments (Hopkins (1954), Bliss et al. (1956)) and modelling sports competitions, with racquetball (Strauss & Arnold (1987)), soccer (Hallinan (2005)) and basketball (Cattelan et al. (2013)) all receiving attention in this regard. More creative examples of applications include statistical modelling of citation exchange among statistics journals (Varin et al. (2013)), predicting the fighting ability of lizards (Whiting et al. (2006)) and estimating driver crash risks (Li & Kim (2000)).

The structure of the paper is as follows. In Section 2 we offer a brief description of Bradley-Terry models and some of the areas in which they have been successfully applied thus far. In Section 3 we describe the three sources of data we have used to model the aspects of test match cricket, as described in the previous paragraph. Our analyses are described in Section 4 and we conclude our findings in Section 5. We deliberately leave technical content to a minimum in an aim to improve the readability of the paper.

Cricket is a notoriously difficult game to model, as it contains many subtle nuances and variables. Accordingly we believe it is appropriate as a test-bed for the potential of Bradley-Terry models. More generally, we show the capability of these models for predicting contests and ranking individuals. These are useful applications with wide-bearing implications.

2. Bradley-Terry Models

2.1 *Description*

For any pair of entities, say i and j, the Bradley-Terry model takes an input as some measure of quality for the respective entities and computes the probability that i is preferred to j. For a system of n entities, the model introduces parameters $\pi_1, \pi_2, \dots, \pi_n$, which can be interpreted as some measure of quality of the respective entries, such that

$$p_{ij} = P(i \text{ is preferred to } j) = \frac{\pi_i}{\pi_i + \pi_j}$$

where $\sum_{i=1}^{n} \pi_i = 1$. It follows that

$$\log \frac{p_{ij}}{p_{ji}} = \lambda_i - \lambda_j$$

where $\lambda_i = \log \pi_i$. For n parameters $\lambda_1, \lambda_2, \dots, \lambda_n$ then

$$p_{ij} = \frac{\exp(\lambda_i - \lambda_j)}{1 + \exp(\lambda_i - \lambda_j)}\,.$$

In many paired comparison experiments there is often a factor that, independent of the attributes of the respective entities, influences the outcome of the experiment. In a taste test, this could refer to some advantage gained by the first sample tasted, or could refer to a perceived home advantage in a sports contest. It is possible to include such factors into a Bradley-Terry model. Let $\delta \geqslant 0$ be the advantage gained by entity i from the external effect in question. Then we may write

$$\log \frac{p_{ij}}{p_{ji}} = \lambda_i - \lambda_j + \delta\,. \tag{2.1}$$

In order to fit our Bradley-Terry models we use the BradleyTerry2 package in R, see Turner & Firth (2012).

3. Data

In this paper we consider three different sets of data, each of which are described below. The data to be considered are (i) results of test matches since 2004, (ii) bookmakers odds for selected test matches and (iii) ball-by-ball data for selected test matches. The data are for a relatively long period of time. In such a period the relative abilities of teams may change. In this paper we do not pursue the fitting of a 'dynamic' Bradley-Terry model but note that there has been recent work in this area, see Cattelan et al. (2013). We also note that the ICC rankings have remained fairly consistent throughout the time period considered, perhaps making this less of an issue.

3.1 *Test Match Results*

We consider data collected from ESPN Cricinfo which comprises of the results of 442 test matches between 8th March 2004 and 30th June 2014. For each match, the data recorded contains the home team, the away team, the outcome (indicated as 1 for a home win, 0 for an away win and 0.5 for a draw), which ground the match was played at, which team batted first (1 if the home team batted first, 0 for the away team) and the start date of the match. A one-off test match between an ICC World XI and Australia in October 2005 has been excluded from the data since this is the only test that an ICC World XI has played and it is inappropriate to compare them with regular test playing nations. Table 1 contains an example extract of the data taken from ESPN Cricinfo.

Home	Away	Result	Ground	Bat First	Start Date
Sri Lanka	Australia	0	Galle	0	2004-03-08
New Zealand	South Africa	0.5	Hamilton	0	2004-03-10
West Indies	England	0	Kingston	1	2004-03-11
Sri Lanka	Australia	0	Kandy	0	2004-03-16
New Zealand	South Africa	1	Auckland	0	2004-03-18
West Indies	England	0	Port of Spain	1	2004-03-19
Sri Lanka	Australia	0	Colombo (SSC)	0	2004-03-24
New Zealand	South Africa	0	Wellington	1	2004-03-26
Pakistan	India	0	Multan	0	2004-03-28
West Indies	England	0	Bridgetown	1	2004-04-01
Pakistan	India	1	Lahore	0	2004-04-05
West Indies	England	0.5	St John's	1	2004-04-10
Pakistan	India	0	Rawalpindi	1	2004-04-13

Table 1. Extract of data of test match results taken from ESPN Cricinfo

Due to security concerns, Pakistan has not hosted a test match since February 2009, with their 'home' matches instead being played at neutral venues. The majority of these matches have taken place in the United Arab Emirates and, given the regularity with which they have played there and the relative similarity to Pakistani conditions, we have decided to treat Pakistan as the home side in such matches. There was, however, a test series played between Australia and Pakistan in England in 2009, but Pakistan were officially stated to be the 'home' team.

Bangladesh and Zimbabwe are often perceived to be far weaker than the eight other test playing nations. As shown in Table 2, they have played fewer games than the other sides, and have worse records than the other teams, between them winning just seven of their 75 test matches included in the data, four of which have come in matches between the two sides. As a result of this gulf in quality, the stronger teams often field weakened line-ups when playing against either of these sides and so results in these matches are often not a reflection of the true ability of the teams.

The most notable example of this is regarding Bangladesh's tour of West Indies in 2009. A contract dispute led to several leading West Indies players boycotting the match and so their side featured numerous uncapped players. Bangladesh won both test matches in the series and these remain their only victories over a major international team. After consideration, matches involving Bangladesh and Zimbabwe have been excluded from the analysis, leaving a dataset of 372 test matches played between eight major international teams. Initial analysis showed that parameter estimates concerning these teams suffered from huge standard errors, rendering them unusable. Table 3 gives a summary of team performances excluding matches involving Bangladesh and Zimbabwe.

Team	Matches	Wins	Losses	Draws	Win %	Loss %	Draw %
Australia	118	69	28	21	58.47%	23.73%	17.80%
Bangladesh	55	4	42	9	7.27%	76.36%	16.36%
England	131	59	35	37	45.04%	26.72%	28.24%
India	107	46	28	33	42.99%	26.17%	30.84%
New Zealand	87	24	39	24	27.59%	44.83%	27.59%
Pakistan	81	26	33	22	32.10%	40.74%	27.16%
South Africa	99	47	28	24	47.47%	28.28%	24.24%
Sri Lanka	93	36	29	28	38.71%	31.18%	30.11%
West Indies	93	14	50	29	15.05%	53.76%	31.18%
Zimbabwe	20	3	16	1	15.00%	80.00%	5.00%

Table 2. Summary of team performances over 442 test matches between 2004 and 2014

Team	Matches	Wins	Losses	Draws	Win%	Loss%	Draw%
Australia	116	67	28	21	57.76%	24.14%	18.10%
England	125	53	35	37	42.40%	28.00%	29.60%
India	99	39	28	32	39.39%	28.28%	32.32%
New Zealand	74	14	39	21	18.92%	52.70%	28.38%
Pakistan	76	22	32	22	28.95%	42.11%	28.95%
South Africa	93	41	28	24	44.09%	30.11%	25.81%
Sri Lanka	78	23	29	26	29.49%	37.18%	33.33%
West Indies	83	8	48	27	9.64%	57.83%	32.53%

Table 3. Summary of team performances over 372 test matches excluding matches involving Bangladesh and Zimbabwe

3.2 *Bookmakers' Odds*

Data regarding the average pre-match odds of all test matches between March 2012 and June 2014 were taken from Oddsportal. The odds that were taken were those of a home win, away win and a draw. Average odds were gathered for a total of 64 matches.

3.3 *Ball-by-ball Data*

Further data regarding test matches was collected from Cricsheet. The data consists of detailed information on all test matches played since 2009. Basic information about each match, such as the competing teams, the location, the dates and the result were provided along with information about each delivery (a delivery or ball in cricket is a single action of bowling a cricket ball toward the batsman) in the match, including the bowler, the batsman, how many runs were scored and whether or not a wicket was taken.

Match	Innings	Batting Team	Bowler	Batsman	Runs	Wicket
SA Eng 2009-12-16	1	South Africa	JM Anderson	GC Smith	0	0
SA Eng 2009-12-16	1	South Africa	JM Anderson	GC Smith	0	0
SA Eng 2009-12-16	1	South Africa	JM Anderson	GC Smith	0	0
SA Eng 2009-12-16	1	South Africa	JM Anderson	GC Smith	0	0
SA Eng 2009-12-16	1	South Africa	JM Anderson	GC Smith	0	0
SA Eng 2009-12-16	1	South Africa	JM Anderson	GC Smith	0	0
SA Eng 2009-12-16	1	South Africa	SCJ Broad	AG Prince	0	0
SA Eng 2009-12-16	1	South Africa	SCJ Broad	AG Prince	1	0
SA Eng 2009-12-16	1	South Africa	SCJ Broad	GC Smith	0	1
SA Eng 2009-12-16	1	South Africa	SCJ Broad	HM Amla	3	0
SA Eng 2009-12-16	1	South Africa	SCJ Broad	AG Prince	0	0
SA Eng 2009-12-16	1	South Africa	SCJ Broad	AG Prince	0	0

Table 4. Extract of ball-by-ball data

The information extracted from the data comprised of ball by ball details of every match played between Australia, England, India and South Africa between December 2009 and August 2013. An extract of the data is given in Table 4. Using this data, it was possible to compute traditionally referenced statistics for cricket players such as batting and bowling averages for each player. Tables 5 and 6 give these statistics for the top 15 run scorers and wicket takers respectively, from the matches considered.

4. Analysis

4.1 *An alternative team ranking system*

The International Cricket Council (ICC), the governing body of international cricket, currently uses a method devised by David Kendix to rank its teams in each form of the game. Although the precise methodology that is used to compute these rankings is not freely available, several aspects of the method are known. The rankings are based on a rating points value assigned to each team depending on their results. These take account of the results of matches in the past three to four years, with greater weighting given to more recent matches. Points are assigned after each match, with the amount of points available dependent on the respective ratings of the two teams. Furthermore, a team earns bonus points for winning a series of games (such as the Ashes).

A simple Bradley-Terry model, of the form (2.1) was fitted to estimate the 'ability' of each team based on the data described in Section 3.1. We set West Indies as our reference category and thus this team is nominally given an ability rating of 0. Note that any team may be set to be the reference category, the results will remain the same. Matches that resulted in draws were considered as half a win for each side.

Table 7 contains the ability ratings for the teams considered based on a simple Bradley-Terry model

Batsman	Runs	Balls Faced	Innings	Times Out	Batting Av.	Strike Rate
AN Cook	2435	5333	44	41	59.39	45.66
MJ Clarke	2263	3668	44	41	55.20	61.70
IR Bell	2024	4200	41	36	56.22	48.19
KP Pietersen	2015	3354	40	40	50.38	60.08
HM Amla	1899	3634	25	21	90.43	52.26
IJL Trott	1537	3391	41	40	38.43	45.33
SR Tendulkar	1480	2713	36	33	44.85	54.55
SR Watson	1353	2553	36	37	36.57	53.00
MJ Prior	1347	2216	38	33	40.82	60.79
MEK Hussey	1317	2418	28	27	48.78	54.47
JH Kallis	1225	2303	25	23	53.26	53.19
GC Smith	1169	2175	25	24	48.71	53.75
MS Dhoni	1048	1902	32	27	38.81	55.10
RT Ponting	983	1789	27	24	40.96	54.95
CA Pujara	933	1701	16	11	84.82	54.85

Table 5. Batting summary statistics for the top 15 run-scorers for data considered

Bowler	Runs Conc.	Balls Bowled	Wickets	Bowling Av.	Economy	Strike Rate
JM Anderson	3072	6156	109	28.18	2.99	56.48
GP Swann	3338	6779	103	32.41	2.95	65.82
PM Siddle	2255	4522	79	28.54	2.99	57.24
SCJ Broad	2105	4401	74	28.45	2.87	59.47
DW Steyn	1618	2951	67	24.15	3.29	44.04
M Morkel	1545	3110	56	27.59	2.98	55.54
R Ashwin	1884	3878	55	34.25	2.91	70.51
NM Lyon	1760	3293	48	36.67	3.21	68.6
BW Hilfenhaus	1326	3079	46	28.83	2.58	66.93
RJ Harris	1019	2102	45	22.64	2.91	46.71
Z Khan	1164	2322	41	28.39	3.01	56.63
TT Bresnan	1171	2315	39	30.03	3.03	59.36
PP Ojha	1198	2863	36	33.28	2.51	79.53
I Sharma	1806	3414	34	53.12	3.17	100.41
MG Johnson	1265	2049	33	38.33	3.70	62.09

Table 6. Bowling summary statistics for the top 15 run-scorers for data considered

(2.1) where parameters for batting first, and playing at home have been included. More details as to the fitted model are given in the next section. Table 8 contains the specific Bradley-Terry fit, with standard errors. Also reported is the ICC ratings and rankings. We see that the systems produce similar results. South Africa enjoy a sizeable rating advantage at the top of each ranking system, with Australia, England and India occupying the next three spots. Note that the ICC rankings consider data from 36-48 months prior to the date given.

Team	ICC Rating	ICC Ranking	Bradley-Terry Ranking
South Africa	127	1	1
Australia	115	2	2
India	112	3	4
England	107	4	3
Pakistan	100	5	5
Sri Lanka	89	6	7
New Zealand	87	7	6
West Indies	87	8	8

Table 7. Comparison of rankings from ICC and the Bradley-Terry model. Model fitted using data from 8th March 2004 and 30th June 2014. ICC rankings as at 30th June 2014.

Variable	Estimate	Std. Error	p
Australia	1.76447	0.32832	< 0.001
England	1.28945	0.30601	< 0.001
India	1.27814	0.32324	< 0.001
New Zealand	0.22980	0.34062	0.4999
Pakistan	0.87838	0.34854	< 0.05
South Africa	1.33587	0.32945	< 0.001
Sri Lanka	0.89390	0.34861	< 0.05
Home	0.55484	0.11635	< 0.001
Batting first	0.04885	0.11655	0.6751

Table 8. Initial Bradley-Terry model fitted to data described in Section 3.1. Model fitted using data from 8th March 2004 and 30th June 2014. West Indies set as reference category. Estimates given as log-probability ratios as given in (2.1).

4.2 *Forecasting match outcomes*

In this section we consider how Bradley-Terry models can be used to forecast the outcomes of test cricket matches. We again use the data given in Section 3.1 to form our models.

4.2.1 *Initial model* Table 8 contains an initial Bradley-Terry model fitted to the test match data where we have included home advantage and batting first as order effects, described algebraically by a simple extension of (2.1). The West Indies have been used as our reference category. Surprisingly, and perhaps against common perception, the model has provided little evidence to suggest that batting first has an effect on the outcome of test cricket matches.

Excluding the order effect of batting first (consequently this will also be removed from subsequent models considered in this paper), we now consider home advantage in more detail. Table 9 shows the win probabilities for matches by fitting a Bradley-Terry model excluding home advantage. It can be seen that Australia are favoured against all opposition, since they have been computed as the strongest team by the Bradley-Terry model. Similarly, West Indies, are considered unfavourable in matches against any opponent by the model. Table 10 gives the updated win probabilities after fitting a Bradley-Terry model including home advantage. Here we can see that Australia are no longer favourites in all possible matches. When enjoying home advantage, England, India and South Africa are all slightly favoured in matches against Australia. Similarly, West Indies are no longer considered outsiders for all matches, being favoured when playing at home to New Zealand. Omitted from study is the effect of winning the coin-toss. This is naturally another factor which could be given due consideration.

	A	E	I	NZ	P	SA	SL	WI
A		0.608	0.611	0.804	0.719	0.600	0.701	0.845
E	0.392		0.503	0.726	0.622	0.492	0.602	0.778
I	0.389	0.497		0.723	0.619	0.489	0.599	0.776
NZ	0.196	0.275	0.277		0.384	0.268	0.364	0.571
P	0.282	0.378	0.381	0.616		0.370	0.478	0.681
SA	0.400	0.508	0.512	0.732	0.630		0.610	0.784
SL	0.299	0.398	0.401	0.636	0.522	0.390		0.000
WI	0.155	0.222	0.224	0.429	0.319	0.216	0.301	

Table 9. Probabilities of victory for the teams in the left hand column against teams on top row. A: Australia, E: England, I: India, NZ: New Zealand, P: Pakistan, SA: South Africa, SL: Sri Lanka, WI: West Indies.

4.2.2 *Predicting draws* So far, the probabilities computed assume that there is a winner, i.e. that there is no chance of a draw. In order to derive a method for computing the probability of a draw in any given match, the unique nature of draws in test cricket need to be considered. In many sports, such as soccer, rugby and hockey, a draw is often synonymous with a tie, an outcome whereby both teams end up with the same score. As a result, the probability of a draw may be considered as a function of the abilities of the respective teams, with similarly able teams more likely to draw with each other than teams with greatly differing abilities. In cricket, however a draw refers to an outcome whereby the allocated time for the match has elapsed before either team could win. Therefore, it may be argued that probability of a draw does not depend as heavily on the teams' respective abilities. Work in Allsopp & Clarke (2002) is alignment with this claim.

For a team to win a match, they necessarily must take twenty wickets over the course of the match, and so any aspects of a match which influence the chance of taking wickets are likely to contribute to the probability of a draw. One such aspect is the weather, where lengthy rain interruptions are clearly conducive to the probability of a draw. Furthermore, it is often considered that cloudier conditions can increase the chances of wickets being taken, and thus reducing the chances of a draw. Weather is not considered in this paper, but would be an interesting topic for future research. Another such factor is the state of the pitch, where flatter pitches are considered to be more difficult to take wickets on. In a similar manner day-night games might also affect the number of wickets taken. Also, if the quality of bowling

	A	E	I	NZ	P	SA	SL	WI
A		0.738	0.742	0.890	0.811	0.729	0.806	0.912
E	0.518		0.639	0.833	0.726	0.624	0.720	0.864
I	0.513	0.630		0.831	0.722	0.620	0.716	0.862
NZ	0.272	0.376	0.381		0.479	0.366	0.472	0.689
P	0.413	0.532	0.537	0.766		0.521	0.628	0.807
SA	0.529	0.645	0.649	0.839	0.735		0.729	0.869
SL	0.420	0.540	0.545	0.772	0.642	0.529		0.811
WI	0.227	0.322	0.326	0.577	0.420	0.312	0.413	

Table 10. Probabilities of victory for the home teams in the left hand column against away teams on top row. A: Australia, E: England, I: India, NZ: New Zealand, P: Pakistan, SA: South Africa, SL: Sri Lanka, WI: West Indies.

Variable	Estimate	Std. Error	p
Australia	-2.39260	0.53607	< 0.001
England	-1.15563	0.39512	< 0.01
India	-1.03145	0.40315	< 0.05
New Zealand	-0.61504	0.44865	0.17042
Pakistan	-0.99483	0.51802	0.05480
South Africa	-1.95886	0.50228	< 0.001
Sri Lanka	-0.86322	0.45881	0.05991
West Indies	-0.81985	0.45231	0.06990

Table 11. Fit of logistic regression model to predict draw

is weak compared to that of the batting, wickets are less likely to be taken, and thus the probability of a draw will increase.

Extensions to the Bradley-Terry model have been offered to accommodate the possibility of ties. These extensions consider the probability of a tie as a function of the difference in ability between the two teams. Since in cricket there may be other confounding variables which more heavily dominate the probability of a draw, such extensions are inappropriate for computing the probabilities for draws in test match cricket, and a different method needs to be devised.

We fit a logistic regression model with the home team and away team as explanatory variables, with the probability of a draw occurring as the dependent variable. The parameters estimated from this model can be interpreted as 'draw abilities', with each team having two separate draw abilities for whether they are playing at home or away. The probability of a draw in a match with team i at home to team j, θ_{ij}, given the home draw ability of team i, $\lambda_i^{(h)}$, and the away draw ability of team j, $\lambda_j^{(a)}$ is computed as follows:

$$\theta_{ij} = \frac{\exp(\lambda_i^{(h)} + \lambda_j^{(a)})}{1 + \exp(\lambda_i^{(h)} + \lambda_j^{(a)})}.$$

Table 11 contains the the parameters of the fitted logistic regression model.

A Bradley-Terry model is then fitted, using all matches that did not end in a draw. The abilities from this model are then used to compute win probabilities as in previous sections. These probabilities are then scaled using the previous calculated draw probabilities so that for any combination of home team and away team, we now have probabilities for any possible outcome. Thus, we have, if the Bradley-Terry model gives abilities $\lambda_1, \lambda_2, \ldots, \lambda_8$ for the eight respective teams considered and δ is the order effect of playing at home, the probability of team i winning at home to team j as

$$p_{ij}^{(h)} = (1 - \theta_{ij}) \left[\frac{\exp(\lambda_i - \lambda_j + \delta)}{1 + \exp(\lambda_i - \lambda_j + \delta)} \right].$$

Combining the methodology of this section with Section 4.2.1, Table 12 contains the match outcome probabilities as computed on 30th June 2014 for each possible match for the data as described at the outset of the paper. These probabilities reflect historical form, and on current form the home win probabilities for India appear to be quite low.

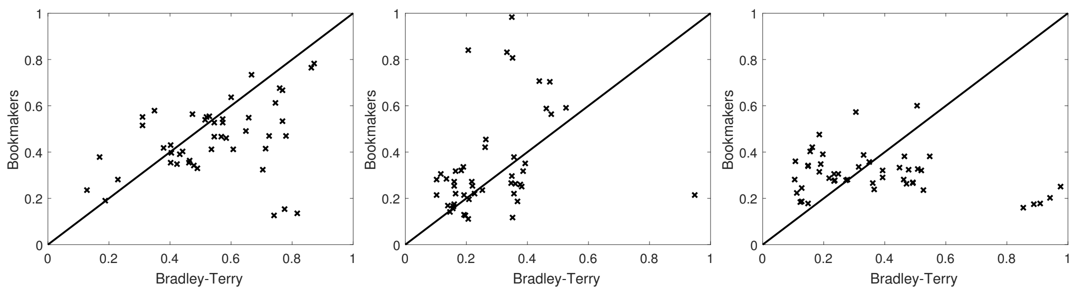
The gambling industry makes heavy use of statistical modelling to compute their odds, although the precise methodology is confidential. Figure 1 displays a comparison of the outcome probabilities computed by the model with the implied probabilities based on average bookmakers odds, taken from

Home	Away	Home	Away	Draw
A	E	0.616	0.255	0.129
A	I	0.697	0.183	0.119
A	NZ	0.865	0.102	0.330
A	P	0.727	0.199	0.741
A	SA	0.607	0.280	0.113
A	SL	0.748	0.139	0.113
A	WI	0.841	0.386	0.120
E	A	0.545	0.188	0.267
E	I	0.513	0.132	0.354
E	NZ	0.769	0.110	0.122
E	P	0.636	0.119	0.245
E	SA	0.426	0.233	0.341
E	SL	0.553	0.107	0.340
E	WI	0.611	0.335	0.356
I	A	0.507	0.220	0.273
I	E	0.410	0.206	0.383
I	NZ	0.756	0.119	0.125
I	P	0.569	0.180	0.251
I	SA	0.406	0.246	0.348
I	SL	0.527	0.126	0.347
I	WI	0.600	0.365	0.363
NZ	A	0.309	0.300	0.391
NZ	E	0.243	0.243	0.515
NZ	I	0.289	0.219	0.492
NZ	P	0.390	0.247	0.364
NZ	SA	0.234	0.290	0.477
NZ	SL	0.339	0.186	0.476
NZ	WI	0.447	0.591	0.494

Home	Away	Home	Away	Draw
P	A	0.389	0.253	0.358
P	E	0.321	0.199	0.480
P	I	0.363	0.180	0.457
P	NZ	0.640	0.185	0.175
P	SA	0.288	0.270	0.442
P	SL	0.405	0.154	0.441
P	WI	0.485	0.560	0.459
SA	A	0.698	0.205	0.971
SA	E	0.602	0.247	0.151
SA	I	0.689	0.171	0.140
SA	NZ	0.876	0.843	0.393
SA	P	0.694	0.219	0.876
SA	SL	0.739	0.129	0.132
SA	WI	0.827	0.327	0.141
SL	A	0.404	0.244	0.352
SL	E	0.312	0.214	0.474
SL	I	0.374	0.176	0.451
SL	NZ	0.683	0.146	0.171
SL	P	0.471	0.202	0.327
SL	SA	0.313	0.252	0.436
SL	WI	0.506	0.421	0.452
WI	A	0.194	0.475	0.332
WI	E	0.170	0.379	0.451
WI	I	0.199	0.373	0.428
WI	NZ	0.454	0.387	0.159
WI	P	0.310	0.383	0.307
WI	SA	0.145	0.441	0.414
WI	SL	0.249	0.339	0.412

Table 12. Match outcome probabilities evaluated on 30th June 2014 for each possible match.

Oddsportal, for a selection of matches that have taken place over the past few years (the data is described in Section 3.2). This selection covers matches from 2012 to 2014 where pre-match odds are available. We can see that whilst the probabilities for home wins and away wins are strongly correlated between bookmakers odds and the Bradley-Terry model, there is a much weaker correlation in the predicted probabilities for draws. This could be because people are less inclined to bet on draws. Further work could be conducted to investigate the potential on betting on draws as a winning strategy.



(a) Home win (b) Away win (c) Draw

FIG. 1. Plot of average bookmaker odds against Bradley-Terry predicted odds

4.3 *Rating individual players*

The following section presents a method by which bowlers and batsmen can be rated based on their performance. The data used in this section is the ball-by-ball test match data described in Section 3.3.

4.3.1 *Wickets* The quality of any given delivery in cricket will initially be judged on whether or not a wicket is taken. Thus, an appropriate method of determining the ability of a bowler is by considering the likelihood that they will take a wicket on any given delivery. Similarly, a batsman can be judged on the likelihood that they will not be dismissed on a delivery.

A logistic regression model was fitted for the data set described in Section 3.3 to compute parameters for each player on the likelihood of a wicket. The model takes the bowler and the batsman as independent variables, with the probability of a wicket occurring as the dependent variable. Thus the estimated parameters from the fitted model are used to inform 'wicket-taking ability', for bowlers, and 'wicket-preservation ability', for batsman. These abilities are computed by the inverse logit function. Of course it can be argued that there is more to batting than this, but it is difficult to account for and model all nuances of cricket.

Thus the 'wicket-taking ability' for bowler i is given by

$$\phi_i^{(w)} = \frac{\exp(\omega_i)}{1+\exp(\omega_i)}$$

where $\omega_1, \omega_2, \ldots, \omega_n$ are obtained from a logistic regression. Similarly the 'wicket-preservation ability' for batsman i is given by

$$\psi_i^{(w)} = \frac{\exp(\mu_i)}{1+\exp(\mu_i)}$$

where $\mu_1, \mu_2, \ldots, \mu_m$ are obtained from a logistic regression.

4.3.2 *Runs* If a wicket is not taken, then a delivery is judged by whether the batsman scored any runs, and if so, how many. Therefore, another way of assessing a players quality is by analysing the amount of runs scored, if the player is a batsman, or the amount of runs conceded, if the player is a bowler.

Table 13 contains the distribution of the number of runs scored per ball as observed in the dataset described in Section 3.1. The mean number of runs per ball is 0.5101, and the variance is 1.2213.

Runs	No. of balls
0	66196
1	12410
2	3152
3	905
4	5529
5	11
6	254

Table 13. Distribution of number of runs scored per ball.

We may model the number of runs scored per delivery using a negative binomial regression, with independent variables corresponding to the bowler and the batsman. The model produces parameters representing each player's contribution to the amount of runs scored on any given delivery. These parameters can be used to inform a 'run-scoring ability' for batsmen and a 'run-prevention ability' for bowlers.

For example suppose the negative binomial regression gives parameters $\gamma_1, \gamma_2, \ldots, \gamma_n$ for each of our n bowlers. The 'run-prevention ability' for bowler i is given by $\phi_i^{(r)} = \exp(\gamma_i)$. Similarly if we have parameters $\mu_1, \mu_2, \ldots, \mu_m$ for the batsmen then the 'run-scoring ability' for batsman i is given by $\psi_i^{(r)} = \exp(\mu_i)$.

4.3.3 *Computing rating values* Combining the results of Sections 4.3.1 and 4.3.2, a rating can now be assigned to each player as a function of these two abilities. We can compute a rating for a bowler's overall ability as

$$\phi_i = \phi_i^{(r)}\left(1 - \phi_i^{(w)}\right). \tag{4.1}$$

Similarly we can compute a rating for batsmen as

$$\psi_i = \psi_i^{(r)}\left(1 - \psi_i^{(w)}\right). \tag{4.2}$$

Thus a low rating value indicates quality for a bowler, whereas a higher rating value indicates a better batsman.

In test match cricket, a batsman's ability to preserve his wicket is usually valued greater than his ability to score quickly, although there are certain situations in which aggressive batting is preferred. Similarly, a bowler's ability to take wickets is treated with greater significance compared to how economical they are. There are, however, very few situations in test cricket in which more economical bowling is valued greater than wicket taking ability.

Proposed here is the modification of the ratings formulae (4.1) and (4.2) to include a parameter that allows for one to place more emphasis on one of the two ability parameters used to compute the ratings. This is analogous to the proposal of Barr and Kantor Barr & Kantor (2004) who devised a criterion by which to measure the performance of batsmen in one-day cricket, allowing one to alter the emphasis placed on aggressive batting.

First we consider the ratings of batsmen. By introducing a parameter $0 < \alpha < 1$, that represents the emphasis placed between the run-scoring and the wicket-preservation abilities, we gain flexibility in our rating system that allows us to consider the quality of a batsman in different scenarios. The updated final rating formula for batsman is thus given by

$$\psi_{i,\alpha} = \left(\frac{1}{2}\psi_i^{(r)}\right)^{\alpha} (1 - \psi_i^{(w)})^{1-\alpha}.$$

The run-scoring ability has been multiplied by $\frac{1}{2}$ to ensure that the reference point for the player rating for all values of α remains at $\frac{1}{2}$.

A similar parameter $0 < \beta < 1$ is introduced for bowlers, to allow for emphasis to be placed on either wicket-taking ability or run-prevention ability yielding the rating formula:

$$\phi_{i,\beta} = \left(\frac{1}{2}\phi_i^{(r)}\right)^{\beta} (1 - \phi_i^{(w)})^{1-\beta}.$$

We have imposed a qualifying criterion for a player to receive a rating, with batsman needing to have played a minimum of 10 innings to qualify for a rating, and bowlers requiring to have delivered 720 balls to qualify.

Table 14 contains rankings of a selection of batsmen for different values of α and Table 15 contains rankings of bowlers for different values of β evaluated from the data described in Section 3.3. If we accept $\alpha = 0.3$ and $\beta = 0.3$ to be suitable parameter values then we may argue that it appears that RJ Harris is the best bowler and CA Pujara is the best batsman. In Tables 16 and 17 we compare the player rankings we have computed with the batting and bowling averages obtained from the matches considered. Of the 15 batsman with the highest batting averages, all of them are within the top 15 for the player ratings, with the exception of AB de Villiers, who is in 16th position.

Similarly, we can see that TT Bresnan is the only bowler who ranks in the top 15 for bowling average but not for player rating, where he ranks 17th. Interestingly, the player who is ranked 3rd, MA Starc, does not rank within the top 15 for bowling average. We can use these rankings to select a hypothetical ‘World XI’, comprising of the best players from each of the teams considered. Typically, a team will select 7 batsmen and 4 bowlers. Furthermore, one of the batsmen must also be able to play as a wicketkeeper. Since no attempt has been made in this paper to assess the wicket-keeping ability of players, we will select the wicketkeeper with the strongest batting rating, which is AB De Villiers. Note that this is not wholly inappropriate since it is a common strategy for teams to select their wicketkeepers with great consideration for their batting ability. Adding to our selection the top 6 ranked batsmen and the top 4 ranked bowlers, we have a World XI as CA Pujara, HM Amla, AN Cook, IR Bell, M Vijay, MJ Clarke, AB De Villiers, VD Philander, MA Starc, RA Jadeja and RJ Harris.

4.4 *Using player ratings to predict outcomes*

We have seen how player ratings can be used either to rank players for general interest, or to inform selection decisions for teams. It will now be shown how these ratings could be used to predict the outcome of a match, given the players which have been selected.

4.4.1 *Deriving team ratings from player ratings* First, we will look at how the player ratings given in Section 4.3.3 can be used to form overall ratings for each team. We propose that each team have a

Batsman	Runs	Faced	Inn.	Out	Bat. Av.	Str. Rate	Ranking for value of α				
							0.1	0.3	0.5	0.7	0.9
AB de Villiers	926	1823	23	22	42.09	50.8	17	16	19	20	24
AJ Strauss	813	1772	26	26	31.27	45.88	27	31	35	36	42
AN Cook	2435	5333	44	41	59.39	45.66	3	3	9	22	34
BJ Haddin	779	1503	30	28	27.82	51.83	36	34	34	24	23
DA Warner	805	1173	25	25	32.2	68.63	38	36	24	9	6
DW Steyn	213	536	20	17	12.53	39.74	49	50	52	53	53
GC Smith	1169	2175	25	24	48.71	53.75	9	7	7	13	19
GP Swann	638	780	31	23	27.74	81.79	43	39	23	3	2
Harbhajan Singh	149	210	13	13	11.46	70.95	57	55	49	31	5
HM Amla	1899	3634	25	21	90.43	52.26	2	2	2	8	18
IJL Trott	1537	3391	41	40	38.43	45.33	13	17	26	28	33
IR Bell	2024	4200	41	36	56.22	48.19	6	4	11	21	31
JE Root	432	1120	12	10	43.2	38.57	4	9	28	42	51
JM Anderson	181	518	32	23	7.87	34.94	56	56	56	56	56
JM Bairstow	361	774	10	10	36.1	46.64	19	21	30	33	39
JP Duminy	264	656	14	12	22	40.24	37	42	46	50	50
KP Pietersen	2015	3354	40	40	50.38	60.08	14	10	4	6	8
M Morkel	197	398	19	15	13.13	49.5	53	52	53	51	38
M Vijay	649	1351	12	12	54.08	48.04	5	5	17	30	44
MA Starc	332	442	13	8	41.5	75.11	35	27	5	2	3
MEK Hussey	1317	2418	28	27	48.78	54.47	15	13	10	14	20
MG Johnson	297	509	19	17	17.47	58.35	50	46	44	32	13
MJ Clarke	2263	3668	44	41	55.2	61.7	12	6	3	4	7
MS Dhoni	1048	1902	32	27	38.81	55.1	23	18	16	15	17
MV Boucher	400	611	11	10	40	65.47	34	25	8	5	4
PD Collingwood	427	962	13	12	35.58	44.39	21	28	33	38	45
PJ Hughes	444	925	22	21	21.14	48	41	43	42	37	29
PM Siddle	497	1125	35	33	15.06	44.18	46	47	47	47	43
R Dravid	767	1742	20	17	45.12	44.03	7	11	20	29	40
RJ Harris	165	272	16	12	13.75	60.66	55	51	48	39	12
RT Ponting	983	1789	27	24	40.96	54.95	24	23	21	16	15
SCJ Broad	541	840	27	26	20.81	64.4	47	45	41	23	10
SK Raina	223	427	11	11	20.27	52.22	44	44	45	41	28
SPD Smith	665	1409	20	18	36.94	47.2	22	22	27	25	26
SR Tendulkar	1480	2713	36	33	44.85	54.55	16	15	14	17	22
SR Watson	1353	2553	36	37	36.57	53	26	26	25	18	21
V Kohli	772	1674	21	19	40.63	46.12	8	12	22	26	36
V Sehwag	922	1118	29	30	30.73	82.47	42	38	12	1	1
VD Philander	198	389	11	9	22	50.9	39	40	39	40	32

Table 14. Rankings of batsmen for different values of α. Also given, in order of left to right, runs, number of balls faced, number of times out, batting average and strike rate.

Bowler	Con.	Bowled	Wkts	Bowl Av.	Econ.	Str. Rate	Ranking for value of β				
							0.1	0.3	0.5	0.7	0.9
A Mishra	570	1202	7	81.43	2.85	171.71	32	32	29	25	20
BW Hilfenhaus	1326	3079	46	28.83	2.58	66.93	16	12	8	6	5
CT Tremlett	493	1029	21	23.48	2.87	49	12	10	9	9	11
DW Steyn	1618	2951	67	24.15	3.29	44.04	3	6	11	20	24
GP Swann	3338	6779	103	32.41	2.95	65.82	22	21	19	16	13
Harbhajan Singh	1229	2586	31	39.65	2.85	83.42	26	25	22	17	9
JL Pattinson	961	1885	32	30.03	3.06	58.91	5	8	10	18	22
JM Anderson	3072	6156	109	28.18	2.99	56.48	15	14	17	14	16
M Morkel	1545	3110	56	27.59	2.98	55.54	11	11	13	12	17
MA Starc	818	1500	26	31.46	3.27	57.69	2	3	7	21	25
MG Johnson	1265	2049	33	38.33	3.7	62.09	18	24	27	29	30
MS Panesar	456	1098	17	26.82	2.49	64.59	20	13	5	3	2
NM Lyon	1760	3293	48	36.67	3.21	68.6	14	20	21	24	26
PL Harris	726	1543	16	45.38	2.82	96.44	30	26	25	22	12
PM Siddle	2255	4522	79	28.54	2.99	57.24	4	5	6	13	21
PP Ojha	1198	2863	36	33.28	2.51	79.53	24	22	15	5	4
RA Jadeja	535	1581	27	19.81	2.03	58.56	13	2	1	1	1
RJ Harris	1019	2102	45	22.64	2.91	46.71	1	1	2	7	19
S Sreesanth	600	896	11	54.55	4.02	81.45	25	29	32	32	32
SCJ Broad	2105	4401	74	28.45	2.87	59.47	17	15	14	10	8
SR Watson	589	1353	15	39.27	2.61	90.2	21	18	16	8	6
ST Finn	1010	1583	31	32.58	3.83	51.06	7	16	23	27	28
TT Bresnan	1171	2315	39	30.03	3.03	59.36	19	17	18	15	14
UT Yadav	625	893	18	34.72	4.2	49.61	9	19	24	28	31
VD Philander	647	1536	30	21.57	2.53	51.2	6	4	3	2	3

Table 15. Rankings of bowlers for different values of β. Also given, from left to right, number of runs conceded, number of balls bowled, number of wickets taken, bowling average, economy and strike rate.

Batsman	Bat Av.	Rank
HM Amla	90.43	2
CA Pujara	84.82	1
AN Cook	59.39	3
IR Bell	56.22	4
MJ Clarke	55.20	6
M Vijay	54.08	5
JH Kallis	53.26	8
KP Pietersen	50.38	10
MEK Hussey	48.78	13
GC Smith	48.71	7
R Dravid	45.12	11
SR Tendulkar	44.85	15
AN Petersen	43.46	14
JE Root	43.20	9
AB de Villiers	42.09	16

Table 16. Comparison of the batting average and Bradley-Terry player ranking for players in the ‘Top 15’ for batting average

Bowler	Bowl Ave.	Rank
RA Jadeja	19.81	2
VD Philander	21.57	4
RJ Harris	22.64	1
CT Tremlett	23.48	10
DW Steyn	24.15	6
MS Panesar	26.82	13
P Kumar	27.38	7
M Morkel	27.59	11
JM Anderson	28.18	14
Z Khan	28.39	9
SCJ Broad	28.45	15
PM Siddle	28.54	5
BW Hilfenhaus	28.83	12
JL Pattinson	30.03	8
TT Bresnan	30.03	17

Table 17. Comparison of the bowling average and Bradley-Terry player ranking for players in the ‘Top 15’ for bowling average

separate rating for bowling and batting. From this point onwards, any reference to a batsman's rating refers to the rating produced from taking $\alpha = 0.3$ in equation (4.1), and, similarly all reference to bowling ratings refer to the ratings produced where $\beta = 0.3$ in equation (4.2). These are taken as example values. One could examine approaches to estimate these parameters, or to update them during over time. This is a possible area for future investigation.

Given that any player in a team may be expected to bat, we have considered a team's batting rating, BAT_{team} to simply be the sum of each player's batting rating. If a player has not reached the qualification criterion of having played 10 innings, they are not assigned a batting rating and therefore do not contribute to the team rating. It is however, inappropriate to compute the bowling rating of a team in such a manner. Typically four players in a team will be expected to bowl the majority of overs in a match, with other 'part-time' bowlers occasionally alleviating the workload on the main bowlers. Therefore, we postulate that a team's bowling rating should be based only on the top four individual bowler ratings within their team.

We define a team's bowling rating as follows:

$$BOWL_{team} = \sum_{i=1}^{4} \frac{1}{4\phi_{i,0.3}},$$

where $i = 1,2,3,4$ are indices representing the four players with the best bowling rating in a selected team. Since a low rating indicates greater quality for bowlers, it is necessary to compute the team bowling rating as a sum of reciprocal values of the individual bowler ratings, so that a higher rating indicates a better bowling attack. Multiplying by $1/4$ simply ensures that the reference value for the ratings of individual bowlers remains at $1/2$.

4.4.2 *Predicting outcomes based on player selection* We have fitted a Bradley-Terry model to the results of the matches in the dataset described in Section 3.1, with home advantage, batting quality and bowling quality each being considered as order effects, with a parameter for random effects also included. The model is described below.

For teams A and B with respective team batting abilities BAT_A and BAT_B, and respective team bowling abilities $BOWL_A$ and $BOWL_B$, the probability of team A winning at home to team B is given by

$$p_{AB}^{(h)} = \frac{\exp\left(\delta_{bat}(BAT_A - BAT_B) + \delta_{bowl}(BOWL_A - BOWL_B) + \delta_{home}\right)}{1 + \exp\left(\delta_{bat}(BAT_A - BAT_B) + \delta_{bowl}(BOWL_A - BOWL_B) + \delta_{home}\right)} \tag{4.3}$$

where δ_{home} is the order effect representing the advantage from playing at home, and δ_{bat} and δ_{bowl} are order effects representing the influence that team batting and bowling abilities have on the expected outcome of a match respectively.

Table 18 contains the Bradley-Terry predicted match outcomes and bookmakers' predictions for matches between 2013 and 2014 where pre-match bookmakers' odds were available. We make the following remarks. The Bradley-Terry model predicts the correct outcome more often than the outcome suggested from the bookmakers' average odds, although it must be recognised that the bookmakers also perform very well. There are examples when both the bookmakers and the model get it wrong: see games on 1/8/13, 21/8/13, 14/10/13 for example. An interesting difference can be observed for the game held on 21/11/13. The bookmakers have a more even spread across all possible outcomes, whilst the Bradley-Terry model gives a more pronounced (correct) prediction for a home win. A big discrepancy occurs for the game held on 3/12/13, where the Bradley-Terry model correctly predicts a draw whilst the bookmakers placed a lot of confidence in a win for the home side. Indeed on inspection, for the games where the bookmakers have struggled to offer a definite outcome (by putting equal probabilities on a win, lose and a draw), the Bradley-Terry model seems more able to pick out an outcome, and this is often the correct outcome.

5. Conclusion

The objective of this paper was to use Bradley-Terry models to analyse various aspects of test cricket. The main areas of investigation were ranking teams, predicting match outcomes and rating individual players.

The Bradley-Terry model was used to devise an alternative ranking system for test cricket. Aside from the obvious computational differences, the consideration of home advantage distinguished this system from that employed by the ICC. The strong correlation between the results of this system and the official ICC rankings indicates that it provides an accurate reflection of the abilities of the respective teams at any given point.

The Bradley-Terry model was also used to produce a system that predicts the outcome of test matches based on previous results. The predicted outcomes were compared to bookmakers odds, showing a strong correlation between the predicted probabilities of home wins and away wins, but only a moderate correlation with the predicted probabilities of draws.

We also produced an individual player rating system for batsmen and bowlers. The player ratings derived were then used to inform batting and bowling ratings for a team, given the players that were selected. A Bradley-Terry model was then used to investigate whether these team ratings could be used to predict matches, successfully predicting the result of eight out of ten test matches.

More generally, this paper has shown the potential for Bradley-Terry models in wider settings. Problems of rating, ranking and evaluating can be tackled by fitting such models and these applications have wide-bearing use.

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			Bookmakers			Bradley-Terry			
Date	Home	Away	HW	AW	D	HW	AW	D	Result
02/01/2013	SA	NZ	0.765	0.703	0.174	0.864	0.474	0.888	HW
03/01/2013	A	SL	0.735	0.841	0.186	0.666	0.207	0.127	HW
11/01/2013	SA	NZ	0.784	0.706	0.161	0.871	0.439	0.854	HW
01/02/2013	SA	P	0.613	0.157	0.250	0.746	0.156	0.976	HW
14/02/2013	SA	P	0.676	0.142	0.201	0.759	0.147	0.941	HW
22/02/2013	I	A	0.430	0.235	0.358	0.401	0.252	0.348	HW
22/02/2013	SA	P	0.666	0.168	0.179	0.769	0.140	0.908	HW
02/03/2013	I	A	0.404	0.221	0.388	0.442	0.227	0.331	HW
06/03/2013	NZ	E	0.134	0.592	0.289	0.816	0.527	0.392	D
14/03/2013	I	A	0.329	0.111	0.572	0.489	0.206	0.305	HW
14/03/2013	NZ	E	0.127	0.564	0.334	0.741	0.478	0.448	D
22/03/2013	I	A	0.538	0.196	0.282	0.514	0.209	0.278	HW
22/03/2013	NZ	E	0.152	0.588	0.281	0.776	0.461	0.462	D
16/05/2013	E	NZ	0.532	0.832	0.391	0.769	0.333	0.197	HW
24/05/2013	E	NZ	0.470	0.806	0.477	0.780	0.351	0.185	HW
10/07/2013	E	A	0.411	0.271	0.304	0.536	0.217	0.247	HW
18/07/2013	E	A	0.467	0.255	0.277	0.544	0.221	0.236	HW
01/08/2013	E	A	0.526	0.214	0.274	0.573	0.193	0.234	D
09/08/2013	E	A	0.467	0.255	0.277	0.568	0.160	0.273	HW
21/08/2013	E	A	0.411	0.271	0.304	0.607	0.159	0.234	D
14/10/2013	P	SA	0.189	0.454	0.382	0.187	0.264	0.548	HW
23/10/2013	P	SA	0.280	0.420	0.328	0.230	0.261	0.509	AW
06/11/2013	I	WI	0.544	0.117	0.322	0.573	0.351	0.392	HW
14/11/2013	I	WI	0.638	0.984	0.239	0.600	0.350	0.365	HW
21/11/2013	A	E	0.355	0.318	0.343	0.463	0.387	0.149	HW
03/12/2013	NZ	WI	0.514	0.126	0.267	0.311	0.196	0.493	D
04/12/2013	A	E	0.364	0.252	0.404	0.464	0.381	0.155	HW
11/12/2013	NZ	WI	0.551	0.219	0.236	0.309	0.165	0.526	HW
13/12/2013	A	E	0.564	0.261	0.179	0.474	0.377	0.149	HW
18/12/2013	SA	I	0.548	0.129	0.338	0.658	0.193	0.149	D
19/12/2013	NZ	WI	0.578	0.167	0.270	0.350	0.158	0.492	HW
26/12/2013	A	E	0.551	0.220	0.246	0.519	0.355	0.127	HW
26/12/2013	SA	I	0.492	0.176	0.348	0.648	0.161	0.191	HW
31/12/2013	P	SL	0.397	0.305	0.323	0.405	0.116	0.479	D
03/01/2014	A	E	0.554	0.267	0.183	0.529	0.347	0.124	HW
07/01/2014	P	SL	0.419	0.281	0.322	0.379	0.103	0.519	AW
16/01/2014	P	SL	0.355	0.285	0.380	0.402	0.134	0.464	HW
06/02/2014	NZ	I	0.236	0.188	0.601	0.128	0.367	0.505	HW
12/02/2014	SA	A	0.470	0.319	0.223	0.724	0.165	0.111	AW
14/02/2014	NZ	I	0.379	0.379	0.264	0.170	0.357	0.472	D
20/02/2014	SA	A	0.325	0.337	0.359	0.704	0.191	0.106	HW
01/03/2014	SA	A	0.415	0.322	0.281	0.712	0.184	0.104	AW
08/06/2014	WI	NZ	0.392	0.296	0.287	0.433	0.350	0.217	AW
12/06/2014	E	SL	0.460	0.215	0.337	0.583	0.103	0.314	D
16/06/2014	WI	NZ	0.347	0.350	0.316	0.423	0.392	0.185	HW
20/06/2014	E	SL	0.526	0.213	0.267	0.544	0.949	0.361	AW
26/06/2014	WI	NZ	0.342	0.263	0.421	0.477	0.361	0.162	AW

Table 18. Comparison of Bradley-Terry model predictions with actual results. Bookkeeper odds also provided. HW = Home Win, AW = Away Win, D = Draw.

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