Reliability Analysis of Subsea Pipelines under Spatially Varying Ground Motions by Using Subset Simulation

Yuyin Li[a], Yahui Zhang[a*], David Kennedy[b]

[a] *State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, International Center for Computational Mechanics, Dalian University of Technology, Dalian 116023, PR China;*

[b] *School of Engineering, Cardiff University, Cardiff CF24 3AA, Wales, UK*

Corresponding author:

Dr. Y. H. Zhang

State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology, Dalian 116023, PR China

Email: zhangyh@dlut.edu.cn

Tel: +86 411 84706337

Fax: +86 411 84708393

Abstract

A computational framework is presented to calculate the reliability of subsea pipelines subjected to a random earthquake. This framework takes full account of the physical features of pipelines and the earthquake, and also retains high computing precision and efficiency. The pipeline and the seabed are modelled as a Timoshenko beam and a Winkler foundation, respectively, while the unilateral contact effect between them is considered. The random earthquake is described by its power spectrum density function and its spatial variation is considered. After suitable discretizations in the spatial domain by the finite element method and the time domain by the Newmark integration method, the dynamic unilateral contact problem is derived as a linear complementarity problem (LCP). Subset Simulation (SS), which is an advanced Monte Carlo simulation approach, is used to estimate the reliability of pipelines. By means of numerical examples, the accuracy and robustness of SS are demonstrated by comparing with the direct Monte Carlo simulation (DMCS). Then a sensitivity analysis of the reliability and a failure analysis are performed to identify the influential system parameters. Finally, failure probabilities of subsea pipelines are assessed for three typical cases, namely, with and without the unilateral contact effect, with different grades of spatial variations and with different free spans. The influences of these effects or parameters on the reliability are discussed qualitatively.

Key words: subsea pipeline; random earthquake; spatially varying ground motion; unilateral contact; reliability; subset simulation

1 Introduction

Subsea pipelines always rest freely on the seabed, rather than being buried or anchored. Due to the scouring or unevenness of the seabed, pipelines will not touch down uniformly along the length of the pipe, and free spanning inevitably occurs. Since subsea pipelines are generally important and costly facilities, their reliability has been a fundamental problem of interest throughout the world. In recent years, attention has mainly been focused on corrosion failure [1], vortex-induced vibration fatigue damage [2], on-bottom lateral instability [3] and so on. As an occasional random excitation, a strong earthquake poses a tremendous threat to the safety of pipelines, and hence the dynamic response and reliability of pipelines under an earthquake have also received great attention. However, the emphasis has been on buried pipelines, with much less research on unburied pipelines. The relevant standards, such as DNV-OS-F101 [4], do not provide design methods or guidelines for the earthquake reliability of unburied subsea pipelines. The failure of structures under an earthquake is a typical first-excursion problem. To assess the first-excursion reliability, the main difficulties arise from (1) the solution of random responses of structures under the earthquake and (2) the evaluation of the reliability by using the random responses obtained in (1).

In the solution of random responses, one of the most important problems is how to consider the relationship between pipelines and seabed as exactly as possible. In the literature on the dynamic analysis of unburied pipelines, pipelines are widely simplified

as multi-supported beams or beams on an elastic foundation [5-8]. However, in reality unburied pipelines are constrained unilaterally by the seabed, which means that the reaction of the seabed can only be compressive, but not tensile. Hence, during the vibration of pipelines, particularly when the deformation takes place predominantly in the vertical plane, a separation of pipelines and the seabed will occur. Clearly, both the multi-supported beam model and the elastic foundation beam model will overestimate the constraint between pipelines and the seabed, and neither of these two models can take the separation of pipelines and the seabed into consideration. Therefore, a unilateral contact model is more appropriate to simulate the relationship between unburied pipelines and the seabed, and such models have been applied to various kinds of static and dynamic analysis of the subsea pipelines, such as the elastic and elasto-plastic analysis of subsea pipelines subjected to vertical static loads [9], stress analysis problems involving subsea pipelines freely resting upon irregular seabed profiles [10], optimal control of the dynamic response of subsea cables constrained by a frictionless rigid seabed [11] and so on. Nevertheless, due to the contact nonlinearity, obtaining the dynamic response of a unilateral contact system is a challenging task, and some classical methods of structural analysis, such as the analytical method used in [6] or the frequency-domain method used in [8], are no longer feasible. As a consequence, the unilateral model is not used frequently for the dynamic analysis of subsea pipelines under an earthquake, despite its good approximation to the relationship between subsea pipelines and the seabed. In

general, the unilateral contact problem is dealt with by numerical methods, e.g., the combination of the finite element method and step-by-step integration method. In each time step, the nonlinear problem is solved by the Newton-Raphson method or a similar iterative method [10]. The unilateral contact problem can also be treated by deriving it as a linear complementary problem (LCP). There are many well-developed methods to solve the LCP and most of them have been included in commercial software, making it much more convenient to solve the unilateral contact method by the LCP method than Newton-Raphson methods.

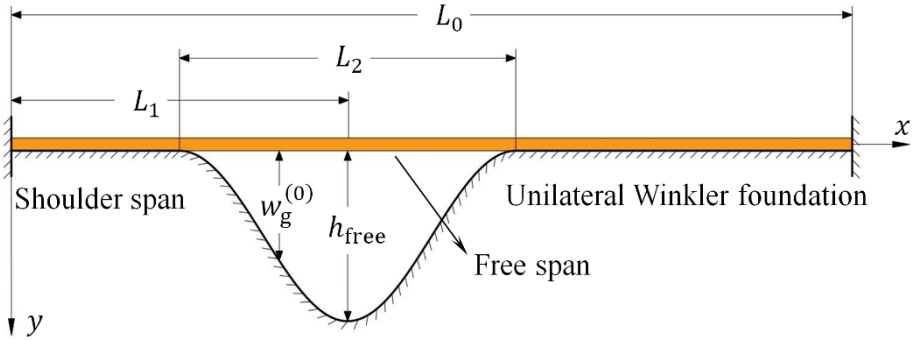
The earthquake excitation model is another key point in the process of the solution of random responses of subsea pipelines. Due to the natural random factors of the soil, the motion of the seabed is more likely to exhibit strong randomness during an earthquake, as are the responses of structures. Hence it is more rational to study the responses of structures subjected to an earthquake from the point of view of the random vibration. On the other hand, variations can be found during earthquake wave propagation along the length of long-span structures, such as subsea pipelines, which result in differences in the amplitude and phase of ground motions at the supports of the structures. This phenomenon is termed as spatially varying ground motions [13]. Many random vibration methods have been developed for the analysis of multi-span structures subjected to spatially varying ground motions [14-16]. However, these methods are no longer feasible after taking the contact of pipelines and the seabed into consideration, for two reasons.

Firstly, these methods are based on the power spectrum density or response spectrum, which are essentially frequency-domain methods, and thus cannot be used to treat the contact problem because of the nonlinearity. Secondly, the response of a nonlinear system is always non-Gaussian even if the input is a Gaussian random process, and these methods can only provide the first- and second-order statistical moments of the response, which are insufficient to describe totally the statistical properties of the non-Gaussian response. In the circumstances, the Monte Carlo simulation (MCS) method, which is suitable for both linear and nonlinear random vibration analysis, seems to be the best and only choice, despite its relatively large computational requirements [17].

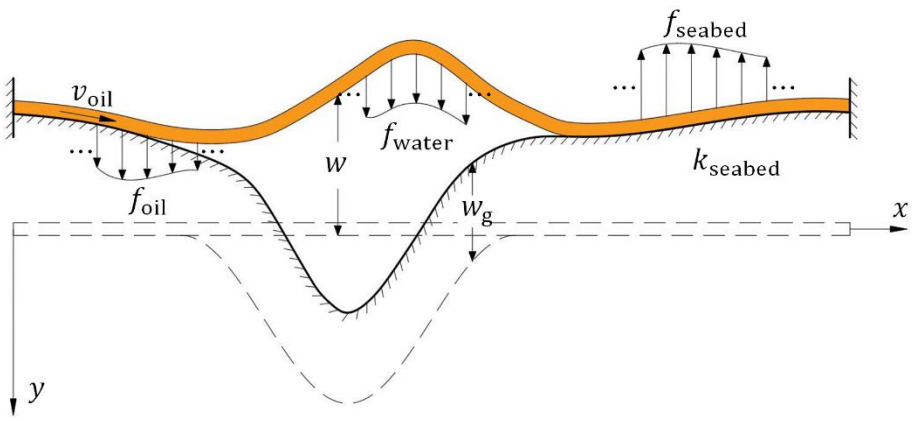
After obtaining the random response of subsea pipelines, the problem which follows is how to estimate the reliability of subsea pipelines through this random response. Due to the complex nature of the first-excursion failure and the unilateral contact problem, the limit state function of subsea pipelines is extremely complicated and has no explicit expression, while extreme values of the random response are not Gaussian distributed. Therefore, popular methods of reliability analysis such as the first order reliability method (FORM) [18], second order reliability method (SORM) [19], point estimate method (PEM) [20], etc. are unable to predict accurately the reliability of subsea pipelines under an earthquake. The MCS is one of most well-known methods for reliability analysis because it is independent of the complexity and dimension of the problem. However, the number of samples required by the MCS is proportional to the reciprocal of the failure

probability. Hence, when the failure probability is very small, e.g., of order 10^{-3}, this method will suffer from inefficiency due to its demand for a large number of samples. In order to reduce the computational cost of the MCS, an advanced MCS called Importance Sampling (IS) was developed [21,22]. The IS requires prior knowledge of the system in the failure region, so it works well when applied to a linear and low-dimensional problem, whose failure region is quite simple. However, the failure region geometry of subsea pipelines under an earthquake is complicated and prior knowledge of the random responses is unavailable, hence the IS is not suitable for the problem considered in this paper. In order to carry out reliability analysis with small failure probabilities, Au et al. [23,24] developed another advanced MCS named Subset Simulation (SS). The basic idea of SS is to express a small failure probability event as a product of a series of intermediate events with larger conditional probabilities. Through setting these intermediate events properly, the conditional probabilities can be large enough to be estimated with a small number of samples. SS is a robust and efficient method and has been used for predicting small failure probabilities in engineering fields, such as the time-dependent reliability of underground flexible pipelines [25], the probabilistic dynamic behaviour of mistuned bladed disc systems [26], radioactive waste repository performance assessment [27], the stochastic dynamic stiffness of foundations for large offshore wind turbines [28] and so on. A general form of SS is presented in [29] is mainly, with application to a seismic risk problem involving dynamic analysis.

As mentioned above, reliability analysis of subsea pipelines subjected to random earthquakes faces two difficulties: the solution of random responses and estimation of reliability, and these are the focus of this paper. Regarding random response solutions, mathematical models with reasonable simplifications and assumptions are firstly estimated for pipelines, seabed and ground motions, and then a corresponding solution strategy is given based on LCP. Regarding reliability estimation, SS is introduced to increase the computational efficiency for the predictions of first-excursion failure probabilities of pipelines. This paper therefore provides a practical computational framework for the reliability analysis of subsea pipelines subjected to random earthquakes. The work is structured as follows. In section 2, the mathematical formulation of a subsea pipeline under a random earthquake is given. In section 3, by combining the finite element method and Newmark integration method, a solution strategy is obtained by deriving the unilateral contact problem as a LCP. In section 4 the fundamental concept and implementation procedures of SS are briefly presented. Section 5 gives some numerical examples. The feasibility of SS is verified with respect to direct MCS, and the contribution of some random parameters to the failure of pipelines is addressed through a sensitivity analysis. Then, influences of the unilateral contact effect, the spatial variation and the free span on the reliability of pipelines are investigated. Finally, conclusions are given in section 6.



(a) Without deformation



(b) With deformation

Fig. 1. Schematic of a subsea pipeline

2 Problem formulations

2.1 Governing equations of the pipeline

A schematic of a subsea pipeline and the seabed is shown in Fig. 1(a). There is a free span in the middle of the pipeline due to the scouring or unevenness of the seabed. The length of the pipeline is denoted by L_0, while the location and length of the free span are denoted by L_1 and L_2, respectively. Because of the complex formation mechanism and

the lack of practical measured data of the free span, the seabed profile $w_{\mathrm{g}}^{(0)}$ is approximated with the following function

$$w_{\mathrm{g}}^{(0)} = \begin{cases} 0 & 0 \le x < L_1 - L_2/2 \\ \dfrac{h_{\mathrm{free}}}{2}\left[1 - \cos\dfrac{2\pi(x - L_1 + L_2/2)}{L_2}\right] & L_1 - L_2/2 \le x < L_1 + L_2/2 \\ 0 & L_1 + L_2/2 \le x \le L_0 \end{cases} \tag{1}$$

where h_{free} is the maximum depth of the free span.

The pipeline is modelled based on the Timoshenko beam theory and hydrodynamic forces of the internal oil and the surrounding seawater are considered. The seabed is simplified as a Winkler foundation. In the coordinates shown in Fig. 1(b), the governing equations for the vibration of the pipeline in the vertical plane can be written as

$$\begin{aligned} &\rho I \frac{\partial^2 \theta}{\partial t^2} - EI\frac{\partial^2 \theta}{\partial x^2} - \kappa GA\left(\frac{\partial w}{\partial x} - \theta\right) + d_1 \rho I \frac{\partial \theta}{\partial t} \\ &\qquad - d_2\left[EI\frac{\partial^3 \theta}{\partial x^2 \partial t} + \kappa GA\left(\frac{\partial^2 w}{\partial x \partial t} - \frac{\partial \theta}{\partial t}\right)\right] = M_{\mathrm{oil}} \\ &m_{\mathrm{pipe}}\frac{\partial^2 w}{\partial t^2} - \kappa GA\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \theta}{\partial x}\right) + d_1 m_{\mathrm{pipe}}\frac{\partial w}{\partial t} - d_2 \kappa GA\left(\frac{\partial^3 w}{\partial x^2 \partial t} - \frac{\partial^2 \theta}{\partial x \partial t}\right) \\ &\qquad + N_0 \frac{\partial^2 w}{\partial x^2} = f_{\mathrm{oil}} + f_{\mathrm{water}} - f_{\mathrm{seabed}} \end{aligned} \tag{2}$$

where t is time, θ is the cross-section rotation, w is the vertical displacement of the pipeline, ρI is the moment of inertia, EI is the flexural rigidity, κGA is the effective shear rigidity, m_{pipe} is the mass per unit length, d_1 and d_2 are Rayleigh damping factors corresponding to the mass and stiffness, respectively, N_0 is the axial compression, M_{oil} and f_{oil} are respectively the hydrodynamic forces per unit length in the rotational

and vertical directions, f_{water} is the hydrodynamic force per unit length due to the surrounding seawater, and f_{seabed} is the reaction force per unit length of the seabed.

It is assumed that the internal oil is an incompressible and inviscid fluid with a constant flow velocity v_{oil}, and the effects of the oil are considered as external forces on the pipeline. Thus, as a fluid-conveying beam [30], M_{oil} and f_{oil} can be expressed as

$$\begin{gathered} M_{\mathrm{oil}} = -\rho_{\mathrm{oil}} I_{\mathrm{oil}} \frac{\partial^2 \theta}{\partial x^2} \\ f_{\mathrm{oil}} = -m_{\mathrm{oil}} v_{\mathrm{oil}}^2 \frac{\partial^2 w}{\partial x^2} - 2 m_{\mathrm{oil}} v_{\mathrm{oil}} \frac{\partial^2 w}{\partial x \partial t} - m_{\mathrm{oil}} \frac{\partial^2 w}{\partial t^2} \end{gathered} \tag{3}$$

in which $\rho_{\mathrm{oil}} I_{\mathrm{oil}}$ is the moment of inertia of the oil and m_{oil} is its mass per unit length.

For slender cylindrical structures such as pipelines, Morison's equation [31] is widely used to evaluate the resulting hydrodynamic force of the surrounding water, defined as the summation of the inertia and drag forces. It is assumed that the surrounding water is static, while the drag force is small and hence can be neglected, so that f_{water} is determined by

$$f_{\mathrm{water}} = -C_{\mathrm{m}} \rho_{\mathrm{water}} \pi R_{\mathrm{out}}^2 \frac{\partial^2 w}{\partial t^2} \tag{4}$$

where C_{m} is the added mass coefficient and is generally equal to 1.0, ρ_{water} is the density of the seawater and R_{out} is the outer radius of the pipe.

The friction of the seabed is ignored. Unilateral contact in the vertical plane is considered, and thus the reaction force of the seabed f_{seabed} is

$$f_{\text{seabed}} = \begin{cases} 0 & \xi > 0 \\ \lambda k_{\text{seabed}} & \xi = 0 \end{cases} \tag{5}$$

where k_{seabed} is the stiffness of the seabed, and

$$\xi = \lambda + w_{\text{g}}^{(0)} + w_{\text{g}} - w \tag{6}$$

is the relative displacement between the pipe and the seabed, λ is the compressional deformation of the seabed, $w_{\text{g}}^{(0)}$ is the initial profile of the seabed and w_{g} is the motion of the seabed. Since the pipeline is constrained unilaterally by the seabed, the reaction of the seabed can only be compressive, but not tensile. On the other hand, the pipe can be either above or in contact with the seabed, but never under it. Hence, f_{seabed} and ξ satisfy the following linear complementarity conditions,

$$\xi \geq 0, \qquad f_{\text{seabed}} \geq 0, \qquad \xi f_{\text{seabed}} = 0 \tag{7}$$

Obtaining the solution of Eq. (2) is a quite challenging task because of two difficulties. Firstly, the earthquake is a random process and the spatial variation is considered, so the motion of the seabed is actually a random field. Secondly, contact regions of the pipeline and seabed are not known a priori and will change with the pipeline motion. The contact nonlinearity further increases the difficulty of solution.

2.2 Random earthquake with spatial variation

The acceleration of the ground motion due to the earthquake is assumed to be a nonstationary random process which can be expressed as

$$\ddot{w}_{\mathrm{g}} = g(t)\ddot{d}(t) \tag{8}$$

where $\ddot{d}(t)$ is a stationary Gaussian random process with zero mean value and $g(t)$ is a slowly varying deterministic envelope function which is given as

$$g(t) = 2.5974(\mathrm{e}^{-0.2t} - \mathrm{e}^{-0.6t}) \tag{9}$$

Assuming that $\ddot{d}(t)$ is homogeneous in the spatial domain, then the cross power spectral density (PSD) of the acceleration at two arbitrary points can be expressed as

$$S\,(\Delta x, \omega) = \gamma(\Delta x, \omega) S_0(\omega) \tag{10}$$

where ω is the circular frequency, $\Delta x = |x_i - x_j|$ is the distance between the two points x_i and x_j on the ground, $S_0(\omega)$ is the auto-PSD of the acceleration of the ground motion and $\gamma(\Delta x, \omega)$ is the coherency function which can be written as

$$\gamma(\Delta x, \omega) = \gamma^{(\mathrm{w})}(\Delta x, \omega)\gamma^{(\mathrm{c})}(\Delta x, \omega) \tag{11}$$

in which

$$\gamma^{(\mathrm{w})}(\Delta x, \omega) = \exp\left(-\frac{\mathrm{i}\omega\Delta x}{v_{\mathrm{app}}}\right) \tag{12}$$

indicates the complex-valued wave passage effect, $\mathrm{i} = \sqrt{-1}$, v_{app} is the apparent velocity of the earthquake waves, and

$$\gamma^{(\mathrm{c})}(\Delta x, \omega) = \exp\left(-\alpha \frac{\omega \Delta x}{2\pi v_{\mathrm{app}}}\right) \tag{13}$$

characterizes the real-valued incoherence effect. The L-Y model [32] is used in this paper and $\alpha = 0.125$. The modified Kanai-Tajimi PSD of the acceleration is used and S_0 is given by [33]

$$S_0(\omega) = \frac{1 + 4\xi_{\mathrm{g}}^2\left(\omega/\omega_{\mathrm{g}}\right)^2}{\left[1 - \left(\omega/\omega_{\mathrm{g}}\right)^2\right]^2 + 4\xi_{\mathrm{g}}^2\left(\omega/\omega_{\mathrm{g}}\right)^2} \times \frac{(\omega/\omega_{\mathrm{f}})^4}{[1 - (\omega/\omega_{\mathrm{f}})^2]^2 + 4\xi_{\mathrm{f}}^2(\omega/\omega_{\mathrm{f}})^2} S_{\mathrm{g}} \tag{14}$$

where ω_{g} and ξ_{g} are the resonant frequency and damping ratio of the first filter, and ω_{f} and ξ_{f} are those of the second filter. S_{g} is the amplitude of the white-noise bedrock acceleration which depends on the soil condition.

Since the SS used in this paper is based on the MCS method, the time history samples of the ground motion should be generated from the PSD of the acceleration as shown in Eq. (10). Here the Auto-Regressive Moving-Average (ARMA) method is used to generate the time history samples of the ground acceleration. For brevity in this paper, details of the ARMA are not presented and interested readers are referred to [34]. In addition, since it is required to estimate the force acting on the pipeline, and to detect the contact between the pipeline and the seabed in each time step, the time histories of the velocity and displacement of the seabed also need to be evaluated. A simple and direct approach to

obtaining the velocity and displacement is to make use of the inherent integral relations between the displacement, velocity and acceleration by assuming zero initial conditions. However, due to the accumulation of random noise in accelerations, direct integration of the acceleration data often causes unrealistic drifts, namely baseline offsets in the velocity and displacement. In order to eliminate the baseline offsets, a commonly used correction scheme suggested by Berg and Housner [35] is used, in which the zero-acceleration baseline is assumed to be of polynomial form, the constants of which are determined by minimizing the mean square computed velocity.

3 Linear complementarity method for the dynamic contact of pipeline and seabed

Because of the contact nonlinearity, it is impossible to obtain an analytical solution of Eq. (2). Numerical solution seems to be the only choice, and thus a suitable discretization is needed in the spatial and time domains. The pipeline is discretized into Timoshenko beam elements with two nodes, considering the effects of both the oil conveyed through the pipeline and the surrounding seawater, while the seabed is modelled as spring elements. Considering the spatial variation of the ground motion, the governing equation of the pipeline can be represented in the following discrete form,

$$\begin{bmatrix} \boldsymbol{M}_{\mathrm{s}} & \boldsymbol{M}_{\mathrm{sb}} \\ \boldsymbol{M}_{\mathrm{sb}}^{\mathrm{T}} & \boldsymbol{M}_{\mathrm{b}} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{X}}_{\mathrm{s}} \\ \ddot{\boldsymbol{X}}_{\mathrm{b}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_{\mathrm{s}} & \boldsymbol{C}_{\mathrm{sb}} \\ \boldsymbol{C}_{\mathrm{sb}}^{T} & \boldsymbol{C}_{\mathrm{b}} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{X}}_{\mathrm{s}} \\ \dot{\boldsymbol{X}}_{\mathrm{b}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{\mathrm{s}} & \boldsymbol{K}_{\mathrm{sb}} \\ \boldsymbol{K}_{\mathrm{sb}}^{\mathrm{T}} & \boldsymbol{K}_{\mathrm{b}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{X}_{\mathrm{s}} \\ \boldsymbol{X}_{\mathrm{b}} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{R}_{\mathrm{s}} \\ \boldsymbol{R}_{\mathrm{b}} \end{Bmatrix} \tag{15}$$

in which the subscripts “b” and “s” indicate the support and non-support degrees of freedom (DOF), respectively, so that $\boldsymbol{X}_{\mathrm{b}}$ are the enforced displacements of the supports on both sides, $\boldsymbol{X}_{\mathrm{s}}$ are all nodal displacements except those at the supports, $\boldsymbol{R}_{\mathrm{b}}$ are the enforced forces at the supports and $\boldsymbol{R}_{\mathrm{s}}$ are the reaction forces of the seabed. $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices, respectively. Expanding the first row of Eq. (15) gives

$$\boldsymbol{M}_{\mathrm{s}}\ddot{\boldsymbol{X}}_{\mathrm{s}} + \boldsymbol{C}_{\mathrm{s}}\dot{\boldsymbol{X}}_{\mathrm{s}} + \boldsymbol{K}_{\mathrm{s}}\boldsymbol{X}_{\mathrm{s}} = \boldsymbol{R}_{\mathrm{s}} + \boldsymbol{P} \tag{16}$$

where $\boldsymbol{P} = -\boldsymbol{M}_{\mathrm{sb}}\ddot{\boldsymbol{X}}_{\mathrm{b}} - \boldsymbol{C}_{\mathrm{sb}}\dot{\boldsymbol{X}}_{\mathrm{b}} - \boldsymbol{K}_{\mathrm{sb}}\boldsymbol{X}_{\mathrm{b}}$ is the effective earthquake force acting on the non-support DOF.

Each node of the beam element used in this paper has two DOF, namely translation and rotation in the vertical plane. However, it is assumed that the reaction force of the seabed acts only on the translation DOF. For the convenience of the following derivation procedures, rearranging the DOF in Eq. (16) gives

$$\begin{bmatrix} \boldsymbol{M}_{\mathrm{w}} & \boldsymbol{M}_{\mathrm{wq}} \\ \boldsymbol{M}_{\mathrm{qw}} & \boldsymbol{M}_{\mathrm{q}} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{w}} \\ \ddot{\boldsymbol{q}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_{\mathrm{w}} & \boldsymbol{C}_{\mathrm{wq}} \\ \boldsymbol{C}_{\mathrm{qw}} & \boldsymbol{C}_{\mathrm{q}} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{w}} \\ \dot{\boldsymbol{q}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_{\mathrm{w}} & \boldsymbol{K}_{\mathrm{wq}} \\ \boldsymbol{K}_{\mathrm{qw}} & \boldsymbol{K}_{\mathrm{q}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{w} \\ \boldsymbol{q} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{r}_{\mathrm{w}} \\ \boldsymbol{0} \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{P}_{\mathrm{w}} \\ \boldsymbol{P}_{\mathrm{q}} \end{Bmatrix} \tag{17}$$

where $\boldsymbol{w}$ and $\boldsymbol{q}$ are the translation and rotation DOF, respectively; $\boldsymbol{P}_{\mathrm{w}}$ and $\boldsymbol{P}_{\mathrm{q}}$ are the effective forces acting on the translation and rotation DOF, respectively, and $\boldsymbol{r}_{\mathrm{w}}$ is the reaction force of the seabed.

Based on the Newmark integration method, Eq. (17) is discretized in the time domain. With appropriate gathering of terms, the governing equation at time t_{k+1} can be written as

$$\begin{bmatrix} \widehat{\boldsymbol{K}}_{\mathrm{w}} & \widehat{\boldsymbol{K}}_{\mathrm{wq}} \\ \widehat{\boldsymbol{K}}_{\mathrm{qw}} & \widehat{\boldsymbol{K}}_{\mathrm{q}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{w} \\ \boldsymbol{q} \end{Bmatrix}_{k+1} = \begin{Bmatrix} \boldsymbol{r}_{\mathrm{w}} \\ \boldsymbol{0} \end{Bmatrix}_{k+1} + \begin{Bmatrix} \widehat{\boldsymbol{P}}_{\mathrm{w}} \\ \widehat{\boldsymbol{P}}_{\mathrm{q}} \end{Bmatrix}_{k+1} \tag{18}$$

where

$$\begin{bmatrix} \widehat{\boldsymbol{K}}_{\mathrm{w}} & \widehat{\boldsymbol{K}}_{\mathrm{wq}} \\ \widehat{\boldsymbol{K}}_{\mathrm{qw}} & \widehat{\boldsymbol{K}}_{\mathrm{q}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{K}_{\mathrm{w}} & \boldsymbol{K}_{\mathrm{wq}} \\ \boldsymbol{K}_{\mathrm{qw}} & \boldsymbol{K}_{\mathrm{q}} \end{bmatrix} + a_0 \begin{bmatrix} \boldsymbol{M}_{\mathrm{w}} & \boldsymbol{M}_{\mathrm{wq}} \\ \boldsymbol{M}_{\mathrm{qw}} & \boldsymbol{M}_{\mathrm{q}} \end{bmatrix} + a_1 \begin{bmatrix} \boldsymbol{C}_{\mathrm{w}} & \boldsymbol{C}_{\mathrm{wq}} \\ \boldsymbol{C}_{\mathrm{qw}} & \boldsymbol{C}_{\mathrm{q}} \end{bmatrix} \tag{19}$$

$$\begin{aligned} \begin{Bmatrix} \widehat{\boldsymbol{P}}_{\mathrm{w}} \\ \widehat{\boldsymbol{P}}_{\mathrm{q}} \end{Bmatrix}_{k+1} = \begin{Bmatrix} \boldsymbol{P}_{\mathrm{w}} \\ \boldsymbol{P}_{\mathrm{q}} \end{Bmatrix}_{k+1} &+ \begin{bmatrix} \boldsymbol{M}_{\mathrm{w}} & \boldsymbol{M}_{\mathrm{wq}} \\ \boldsymbol{M}_{\mathrm{qw}} & \boldsymbol{M}_{\mathrm{q}} \end{bmatrix} \left(a_0 \begin{Bmatrix} \boldsymbol{w} \\ \boldsymbol{q} \end{Bmatrix}_k + a_2 \begin{Bmatrix} \dot{\boldsymbol{w}} \\ \dot{\boldsymbol{q}} \end{Bmatrix}_k + a_3 \begin{Bmatrix} \ddot{\boldsymbol{w}} \\ \ddot{\boldsymbol{q}} \end{Bmatrix}_k \right) \\ &+ \begin{bmatrix} \boldsymbol{C}_{\mathrm{w}} & \boldsymbol{C}_{\mathrm{wq}} \\ \boldsymbol{C}_{\mathrm{qw}} & \boldsymbol{C}_{\mathrm{q}} \end{bmatrix} \left(a_1 \begin{Bmatrix} \boldsymbol{w} \\ \boldsymbol{q} \end{Bmatrix}_k + a_4 \begin{Bmatrix} \dot{\boldsymbol{w}} \\ \dot{\boldsymbol{q}} \end{Bmatrix}_k + a_5 \begin{Bmatrix} \ddot{\boldsymbol{w}} \\ \ddot{\boldsymbol{q}} \end{Bmatrix}_k \right) \end{aligned} \tag{20}$$

$$\begin{gathered} a_0 = \frac{1}{\alpha(\Delta t)^2} \quad a_1 = \frac{\delta}{\alpha \Delta t} \quad a_2 = \frac{1}{\alpha \Delta t} \quad a_3 = \frac{1}{2\alpha} - 1 \\ a_4 = \frac{\delta}{\alpha} - 1 \quad a_5 = \frac{\Delta t}{2}\left(\frac{\delta}{\alpha} - 2\right) \end{gathered} \tag{21}$$

in which Δt is the time step, δ and α are parameters of the Newmark integration method, which satisfy $\delta \geq 0.5$ and $\alpha \geq 0.25(0.5+\delta)^2$ to ensure the unconditional stability of the integration scheme. The acceleration and the velocity at time t_{k+1} are

$$\begin{gathered} \begin{Bmatrix} \ddot{\boldsymbol{w}} \\ \ddot{\boldsymbol{q}} \end{Bmatrix}_{k+1} = a_0 \left(\begin{Bmatrix} \boldsymbol{w} \\ \boldsymbol{q} \end{Bmatrix}_{k+1} - \begin{Bmatrix} \boldsymbol{w} \\ \boldsymbol{q} \end{Bmatrix}_k \right) - a_2 \begin{Bmatrix} \dot{\boldsymbol{w}} \\ \dot{\boldsymbol{q}} \end{Bmatrix}_k - a_3 \begin{Bmatrix} \ddot{\boldsymbol{w}} \\ \ddot{\boldsymbol{q}} \end{Bmatrix}_k \\ \begin{Bmatrix} \dot{\boldsymbol{w}} \\ \dot{\boldsymbol{q}} \end{Bmatrix}_{k+1} = \begin{Bmatrix} \dot{\boldsymbol{w}} \\ \dot{\boldsymbol{q}} \end{Bmatrix}_k + a_6 \begin{Bmatrix} \ddot{\boldsymbol{w}} \\ \ddot{\boldsymbol{q}} \end{Bmatrix}_k + a_7 \begin{Bmatrix} \ddot{\boldsymbol{w}} \\ \ddot{\boldsymbol{q}} \end{Bmatrix}_{k+1} \end{gathered} \tag{22}$$

in which $a_6 = \Delta t(1-\delta)$ and $a_7 = \delta\Delta t$.

Expanding Eq. (18) and eliminating terms related to $\boldsymbol{q}$ gives

$$\bar{\boldsymbol{K}}_{\mathrm{w}}\boldsymbol{w}_{k+1} = \boldsymbol{r}_{\mathrm{w},k+1} + \bar{\boldsymbol{P}}_{\mathrm{w},k+1} \tag{23}$$

where $\bar{\boldsymbol{K}}_{\mathrm{w}} = \hat{\boldsymbol{K}}_{\mathrm{w}} - \hat{\boldsymbol{K}}_{\mathrm{wq}}\hat{\boldsymbol{K}}_{\mathrm{q}}^{-1}\hat{\boldsymbol{K}}_{\mathrm{qw}}$ and $\bar{\boldsymbol{P}}_{\mathrm{w},k+1} = \hat{\boldsymbol{P}}_{\mathrm{w},k+1} - \hat{\boldsymbol{K}}_{\mathrm{wq}}\hat{\boldsymbol{K}}_{\mathrm{q}}^{-1}\hat{\boldsymbol{P}}_{\mathrm{q},k+1}$.

According to Eq. (6), the relative vertical distance between the seabed and pipeline at time t_{k+1} is

$$\boldsymbol{\xi}_{k+1} = \boldsymbol{\lambda}_{k+1} + \boldsymbol{w}_{\mathrm{g}}^{(0)} + \boldsymbol{w}_{\mathrm{g},k+1} - \boldsymbol{w}_{k+1} \tag{24}$$

where

$$\boldsymbol{\lambda}_{k+1} = \boldsymbol{K}_{\mathrm{g}}^{-1}\boldsymbol{r}_{\mathrm{w},k+1} \tag{25}$$

is the compressional deformation of the seabed at time t_{k+1}, $\boldsymbol{K}_{\mathrm{g}}$ is the stiffness matrix of the seabed and $\boldsymbol{w}_{\mathrm{g}}^{(0)}$ is the initial profile of the seabed.

Combining Eqs. (23) to (25) gives

$$\tilde{\boldsymbol{K}}\boldsymbol{\xi}_{k+1} = \boldsymbol{r}_{\mathrm{w},k+1} + \tilde{\boldsymbol{P}}_{\mathrm{w},k+1} \tag{26}$$

in which $\tilde{\boldsymbol{K}} = \left(\bar{\boldsymbol{K}}_{\mathrm{w}}\boldsymbol{K}_{\mathrm{g}}^{-1} - \boldsymbol{I}\right)^{-1}\bar{\boldsymbol{K}}_{\mathrm{w}}$, $\tilde{\boldsymbol{P}}_{\mathrm{w},k+1} = \left(\bar{\boldsymbol{K}}_{\mathrm{w}}\boldsymbol{K}_{\mathrm{g}}^{-1} - \boldsymbol{I}\right)^{-1}\left[\bar{\boldsymbol{K}}_{\mathrm{w}}\left(\boldsymbol{w}_{\mathrm{g}}^{(0)} + \boldsymbol{w}_{\mathrm{g},k+1}\right) - \bar{\boldsymbol{P}}_{\mathrm{w},k+1}\right]$ and $\boldsymbol{I}$ is an identity matrix.

Eq. (7) can be expressed in the following discretized form

$$\boldsymbol{\xi}_{k+1} \geq \mathbf{0}, \qquad \boldsymbol{r}_{\mathrm{w},k+1} \geq \mathbf{0}, \qquad \boldsymbol{\xi}_{k+1}^{\mathrm{T}} \boldsymbol{r}_{\mathrm{w},k+1} = 0 \tag{27}$$

Eqs. (26) and (27) together form a mathematical structure known as a LCP, which is equivalent to the following quadratic programming problem

$$\begin{gathered} \min f(\boldsymbol{\xi}_{k+1}) = \frac{1}{2} \boldsymbol{\xi}_{k+1}^{\mathrm{T}} \widetilde{\boldsymbol{K}} \boldsymbol{\xi}_{k+1} - \widetilde{\boldsymbol{P}}_{\mathrm{w},k+1}^{\mathrm{T}} \boldsymbol{\xi}_{k+1} \\ \mathrm{s.t.}\ \boldsymbol{\xi}_{k+1} \geq \mathbf{0} \end{gathered} \tag{28}$$

Because of the symmetry and positive definiteness of $\widetilde{\boldsymbol{K}}$, Eq. (28) is a convex optimization problem and the common solution to it is guaranteed to exist and be unique [12]. There are many well-developed methods to solve the LCP and most of them have been included in some commercial software. For simplicity, the solution procedures of the LCP are not given in this paper.

By solving the LCP problem of Eq. (28), $\boldsymbol{\xi}_{k+1}$ can be determined. Then, by substituting $\boldsymbol{\xi}_{k+1}$ into Eq. (26), the reaction force of the seabed $\boldsymbol{r}_{\mathrm{w},k+1}$ is obtained. The displacement of the pipeline $\boldsymbol{w}_{k+1}$ is further determined by substituting $\boldsymbol{r}_{\mathrm{w},k+1}$ into Eq. (23). It is worthwhile to point out that an iterative procedure is needed in most solution methods for the LCP, and thus the solution at the current time step can be used as the initial trial solution for the next time step in order to accelerate convergence.

4 Subset simulation for reliability estimation

The first-excursion probabilities of a subsea pipeline subjected to a spatially varying

ground motion are considered in this paper. The failure event of the pipelines can be represented as the exceedance of an arbitrary response $\boldsymbol{s}(t,\boldsymbol{\theta})$, which can be the displacement, internal force, stress or any other response, above the threshold value b within a specified time interval, i.e.,

$$F = \left\{ \max_{l=1,\cdots,n_r} \left(\max_{t\in[0,T]} |\boldsymbol{s}(t,\boldsymbol{\theta})| \right) > b \right\} \tag{29}$$

where n_r denotes the dimension of the response $\boldsymbol{s}(t,\boldsymbol{\theta})$, T is the duration of the earthquake, $\boldsymbol{\theta}$ is a random variable vector which characterizes the randomness in the system, and whose probability density function (PDF) is $q(\boldsymbol{\theta})$. It is noted that bending stresses are used to identify the failure of pipelines in this paper. The probability of the occurrence of the failure event F, namely, the failure probability, can be expressed in terms of the following probability integral

$$P(F) = \int_{\boldsymbol{\theta}\in\boldsymbol{\Omega}} I_F(\boldsymbol{\theta}) q(\boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\theta} \tag{30}$$

where I_F is the indicator function, which is equal to 1 when the pipeline has failed and 0 otherwise. $\boldsymbol{\Omega}$ denotes the value space of $\boldsymbol{\theta}$.

Generally, the integral in Eq. (30) cannot be calculated efficiently by means of direct numerical integration due to the high dimension of $\boldsymbol{\theta}$ and the complicated geometry of the failure region. MCS is commonly used in problems with high dimension and a complicated failure region, by virtue of its computational robustness. However, the main

drawback of MCS is that it is not suitable for evaluating small failure probabilities due to its demand for a large number of samples. In order to reduce the computational cost of MCS, SS [23, 24] is used, of which the main procedures are as follows.

By introducing a sequence of ascending threshold values $0 < b_1 < b_2 < \cdots < b_n = b$, one can obtain the corresponding intermediate failure events $F_1 \supset F_2 \supset \cdots \supset F_n = F$. By the definition of conditional probability, the failure probability of the pipeline can be expressed as a product of conditional probabilities,

$$P(F) = P(F_1)\prod_{i=1}^{n-1} P(F_{i+1}|F_i) = \prod_{i=1}^{n} P_i \tag{31}$$

where P_1 denotes $P(F_1)$ and $P_i(i = 2,3,\cdots,n)$ denotes $P(F_i|F_{i-1})$. Eq. (32) expresses a small failure probability as a product of relatively large conditional probabilities. For example, assume $P_i \sim 0.1, i = 1,2,\cdots 4$, then the failure probability $P(F) \sim 10^{-4}$ which is too small for efficient estimation by MCS. However, the conditional probabilities $P_i(i = 1,2,\cdots 4)$, are of order 0.1, and so can be evaluated efficiently by simulation.

The probability P_1 can be evaluated readily by the application of direct MCS simulation as

$$P_1 = \frac{1}{N_1}\sum_{h=1}^{N_1} I_{F_1}\left(\boldsymbol{\theta}_h^{(1)}\right) \tag{32}$$

in which $\boldsymbol{\theta}_h^{(1)}(h = 1,2,\cdots,N_1)$ are independently distributed samples simulated from the PDF $q(\boldsymbol{\theta})$, and N_1 is the number of samples $\boldsymbol{\theta}_h^{(1)}$.

To estimate the conditional probabilities $P_i(i = 2,3,\cdots,n)$ samples should be generated according to the conditional PDF $q(\boldsymbol{\theta}|F_{i-1}) = q(\boldsymbol{\theta})I_{F_i}(\boldsymbol{\theta})/P(F_{i-1})$. However, efficient sampling from a conditional PDF is usually not a trivial task. Fortunately, this can be achieved by the Markov chain Monte Carlo (MCMC) simulation based on the Metropolis-Hastings (M-H) algorithm which provides a powerful method for generating samples that satisfy the prescribed conditional probability. Readers are referred to [23] for more details regarding the MCMC and the modified M-H algorithm.

After generating the conditional samples, the conditional failure probability P_i can be determined as

$$P_i = \frac{1}{N_i}\sum_{h=1}^{N_i} I_{F_i}\left(\boldsymbol{\theta}_h^{(i)}\right) \tag{33}$$

where $\boldsymbol{\theta}_h^{(i)}(h = 1,2,\cdots,N_i)$ are independent distributed conditional samples according to the conditional density probability $q(\boldsymbol{\theta}|F_{i-1})$. Through choosing the intermediate threshold values b_i adaptively, the conditional probabilities $P_i(i = 1,2,\cdots,n-1)$ can be ensured to be equal to a certain value p_0 ($p_0 = 0.1$ is used here). Substituting Eqs. (32) and (33) into Eq. (31), the failure probability can be expressed as

$$P_F = p_0^{n-1}\frac{1}{N_n}\sum_{h=1}^{N_n} I_{F_n}\left(\boldsymbol{\theta}_h^{(n)}\right) \tag{34}$$

The main procedures of SS can be summarized as follows.

1. Generate N samples $\boldsymbol{\theta}_h^{(0)}(h = 1,2,\cdots,N)$ by direct MCS from the original PDF $q(\boldsymbol{\theta})$. The superscript "0" denotes these samples correspond to conditional level 0.

2. Set $i = 0$.

3. Calculate the corresponding response variable $\tilde{\boldsymbol{s}}\left(\boldsymbol{\theta}_h^{(i)}\right) = \max\left(\left|\boldsymbol{s}\left(t, \boldsymbol{\theta}_h^{(i)}\right)\right|\right)$

4. Choose the intermediate threshold value b_{i+1} as the $(1 - p_0)N$th value in the ascending order of $\tilde{\boldsymbol{s}}\left(\boldsymbol{\theta}_h^{(i)}\right)$ (calculated at step 3). Hence the sample estimate of P_{i+1} is always equal to p_0. Note that it has been assumed that $p_0 N$ is an integer value.

5. If $b_{i+1} > b_n$, proceed to step 10 below.

6. Otherwise, if $b_{i+1} < b$, with the choice of b_{i+1} performed at step 4, identify the $p_0 N$ samples $\boldsymbol{\theta}_H^{(i)}(H = 1,2,\cdots,p_0 N)$ among $\boldsymbol{\theta}_h^{(i)}(h = 1,2,\cdots,N)$ whose response $\tilde{\boldsymbol{s}}\left(\boldsymbol{\theta}_H^{(i)}\right)$ lies in the region $F_{i+1} = \left\{\tilde{\boldsymbol{s}}\left(\boldsymbol{\theta}_H^{(i)}\right) > b_{i+1}\right\}$. These samples are at conditional level $i + 1$ and distributed as the conditional probability $q(\quad|F_{i+1})$.

7. The samples $\boldsymbol{\theta}_H^{(i)}(H = 1,2,\cdots,p_0 N)$ (identified at step 6) provide seeds for applying the MCMC simulation to generate $(1 - p_0)N$ additional conditional samples distributed as the conditional probability $q(\quad|F_{i+1})$, so that it obtains a total of N conditional samples $\boldsymbol{\theta}_h^{(i+1)}(h = 1,2,\cdots,N) \in F_{i+1}$ at conditional level $i + 1$.

8. Set $i \leftarrow i + 1$.

9. Return to step 3 above.

10. Stop the algorithm.

It is noted that the total number of samples employed is $N_T = N + (n-1)(1-p_0)N$.

The sensitivity of the reliability with respect to variations in system parameters reflects the contributions of these parameters to the failure of structures, and hence it is useful to perform a reliability sensitivity analysis. The reliability sensitivity is defined as the partial derivative of the failure probability with respect to distributional parameters of the system parameter. In the framework of SS, the reliability sensitivity can be expressed as [36]

$$\left.\frac{\partial P_F}{\partial \eta}\right|_{\bar{\eta}} = p_0^{n-1}\frac{1}{N_n}\sum_{h=1}^{N_n} I_{F_n}\left(\boldsymbol{\theta}_h^{(n)}\right)\frac{\frac{\partial q\left(\boldsymbol{\theta}_h^{(n)}\right)}{\partial \eta}}{q\left(\boldsymbol{\theta}_h^{(n)}\right)} \tag{35}$$

where η denotes the distribution parameters, for example, the mean value or the standard deviation, of the PDF of the uncertain system parameters $\boldsymbol{\theta}$. $\bar{\eta}$ is the value of the distribution parameter where the sensitivity is evaluated. For a better comparison of the contribution of different system parameters, the reliability sensitivity can be normalized as follows,

$$e_\eta = \frac{\partial P_F}{\partial \eta}\frac{\bar{\eta}}{P_F} \tag{36}$$

5 Numerical examples

In the following numerical examples, information about the system parameters is

given first. Then, SS is applied for estimating the failure probability of a subsea pipeline subjected to a random earthquake with spatial variation, and a comparison is made with the direct MCS (DMCS) to verify the SS. Then a sensitivity and failure analysis is performed to identify the influential parameters on the pipeline failure. Finally, the reliabilities of subsea pipelines in three typical cases are investigated.

5.1 Description of system parameters

The subsea pipeline in Fig. 1 is adopted as an example structure. Unless otherwise specified, the physical and geometric parameters of the pipeline are as follows: material grade X60 with specified minimum yield strength (SMYS) $\sigma_y = 414 \times 10^6$Pa, Young's modulus $E = 207 \times 10^9$Pa, mass density $\rho_{\text{pipe}} = 7850\,\text{kg/m}^3$, Poisson's ratio $\nu = 0.3$, Rayleigh damping factors corresponding to the mass $d_1 = 0.01$ and the stiffness $d_2 = 0.05$, total length of pipeline $L_0 = 100\text{m}$, shear correction factor $\kappa = 2(1+\nu)/(4+3\nu)$, outer radius $R_{\text{out}} = 0.6\text{m}$, wall thickness $h = 0.02\text{m}$. The mass densities of the oil in the pipeline and surrounding water are $\rho_{\text{oil}} = 800\,\text{kg/m}^3$ and $\rho_{\text{water}} = 1025\,\text{kg/m}$, respectively, and the velocity of the oil is $3\,\text{m/s}$. According to the design standard [4], the effective axial compression N_0 should not exceed $0.5N_{\text{cr}}$, where N_{cr} is the critical buckling load, and hence $N_0 = 0.3N_{\text{cr}}$ is used in this paper. The pipeline is discretized into 100 elements and both ends are simply supported. The failure criterion for the subsea pipeline is defined as when the bending stress exceeds 80% of SMYS [37].

The location and length of the free span, L_1 and L_2, are assumed to be Gaussian distributed variables with mean values 50m and 30m, respectively, and coefficient of variation (COV) 0.3. As physical parameters these must be strictly positive, and in order to guarantee that one free span always exists, L_1 and L_2 are required to satisfy the following constraints,

$$\begin{cases} L_0 \geq L_2 \geq 0 \\ L_1 - L_2/2 \geq 0 \\ L_0 - L_1 - L_2/2 \geq 0 \end{cases} \tag{37}$$

Strictly speaking, these constraints rule out the use of Gaussian models for the random variables L_1 and L_2. Hence, an acceptance-rejection method is used to generate samples of L_1 and L_2 from Gaussian distributions with constraints as expressed in Eq. (37).

The maximum depth of the free span $h_{\text{free}} = 0.3\text{m}$ and the stiffness of the seabed $k_{\text{seabed}} = 2.293 \times 10^6\ \text{N/m}^2$ are used here. Parameters of the ground motion PSD and the spatial variation are respectively $S_{\text{g}} = 0.018\,\text{m}^2/\text{s}^3$, $\omega_{\text{g}} = 15\,\text{rad/s}$, $\omega_{\text{f}} = 0.1\omega_{\text{g}}$, $v_{\text{app}} = 1000\,\text{m/s}$, $\xi_{\text{g}} = \xi_{\text{f}} = 0.6$ [38]. The duration of the earthquake is $T = 10.92\text{s}$, and the time step for the numerical integration is $\Delta t = 0.01\text{s}$. Hence the number of time steps is $N_t = 1093$. In order to generate the ground motion time histories from the above ground PSD, the following procedures are implemented. A $N_{\text{node}} \times N_t$ discrete-time white noise matrix $\boldsymbol{W}$ is first generated, where the elements of $\boldsymbol{W}$ have a mean value of 0 and standard deviation of $\sqrt{2\pi/\Delta t}$, N_{node} is the number of nodes of the discrete pipeline. Then the ARMA method is used to modulate $\boldsymbol{W}$ to generate the required

ground motion samples.

During the SS procedures, the choice of the proposal PDF and the grouping of uncertain parameters affect the distribution and the acceptance rate of the samples and consequently the efficiency of the SS. It is suggested in [29] that deciding what type of proposal PDF to use for a group depends on the contribution of the uncertain parameters to the failure and on the information available for constructing the proposal PDF. In this paper, there are two types of uncertain parameters. The first type is the discrete-time white noise matrix $\boldsymbol{W}$, whose parameters play a similar role in affecting failure. These parameters affect failure significantly as a whole, but not individually. Hence, each of these parameters is grouped individually and their proposal PDFs can be chosen to be uniform with width 2. The second type is the structural parameters L_1 and L_2, whose contribution to the failure cannot be known a priori. Hence, L_1 and L_2 are collected into one group and the proposal PDF is chosen to be Gaussian with mean and covariance estimated from the current seed samples.

5.2 Validation of the subset simulation

SS and DMCS are used to estimate the failure probability of the subsea pipeline under the earthquake and the results are shown in Fig. 2. SS is applied with a conditional failure probability $p_0 = 0.1$ and the number of samples at each level is $N = 1000$. Four levels of conditional simulations are used in one simulation run, so the total number of samples is $N_T = 3700$. For comparison, the failure probabilities from the DMCS with

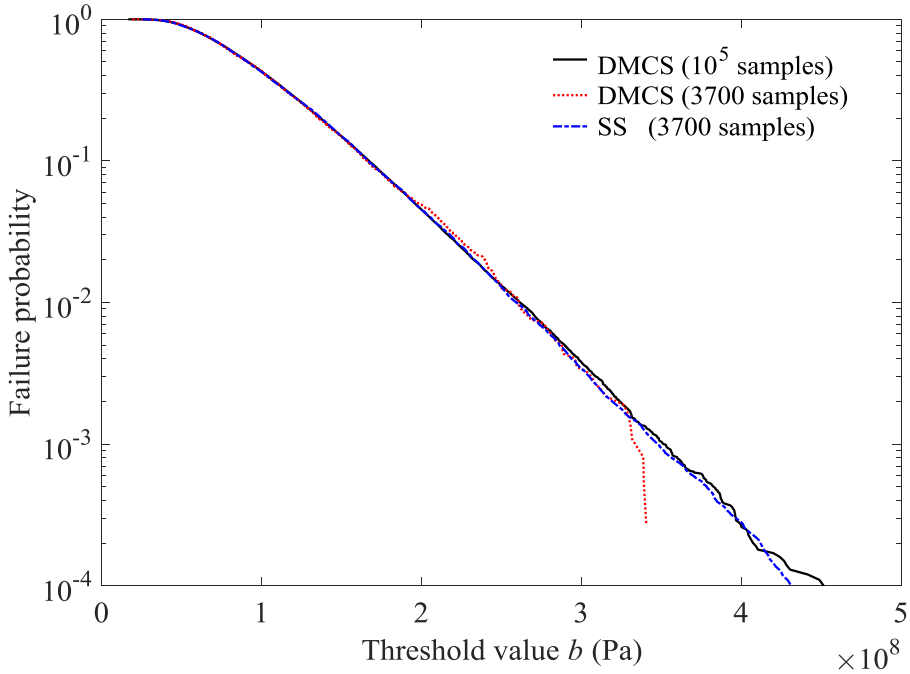


Fig. 2. Comparison of failure probability from SS and DMCS

10^5 and 3700 samples are also given in Fig. 2. It is seen that the results of these three simulation cases agree very well in the region with relative large failure probability (above 10^{-2}). However, in the region with small failure probability (below 10^{-3}), results of SS and DMCS with 10^5 samples still agree quite well, while those of the DMCS with 3700 samples show a significant discrepancy.

To investigate the variability of the SS results, the COV of the failure probability from 10 independent SS runs is shown in Fig. 3. Since each conditional level contains 1000 samples, the total numbers of samples, N_T, used for obtaining estimates of failure probability level at 10^{-1}, 10^{-2}, 10^{-3} and 10^{-4} are 1000, 1900, 2800 and 3700, respectively. For comparison, the COV of the results of the DMCS are given at particular

failure probability level by using the same total numbers of samples. It should be noted that DMCS is unable to estimate the failure probability accurately with a relatively small number of samples, e.g., 3700, so the COV of DMCS is obtained from an simple approximate formula [23], i.e., $\mathrm{COV} = \sqrt{\left(1 - P(F)\right)/(P(F)N)}$, rather than from many independent DMCS runs. It can be seen from Fig. 3 that the COV of SS increases gradually with decreasing failure probability, while the COV of DMCS increases much more rapidly. The COV of SS and DMCS are quite close in the region with relatively large failure probability (10^{-2}~10^{-1}). However, when the failure probability is below 10^{-3}, it can be observed that the COV of DMCS is much larger than that of SS.

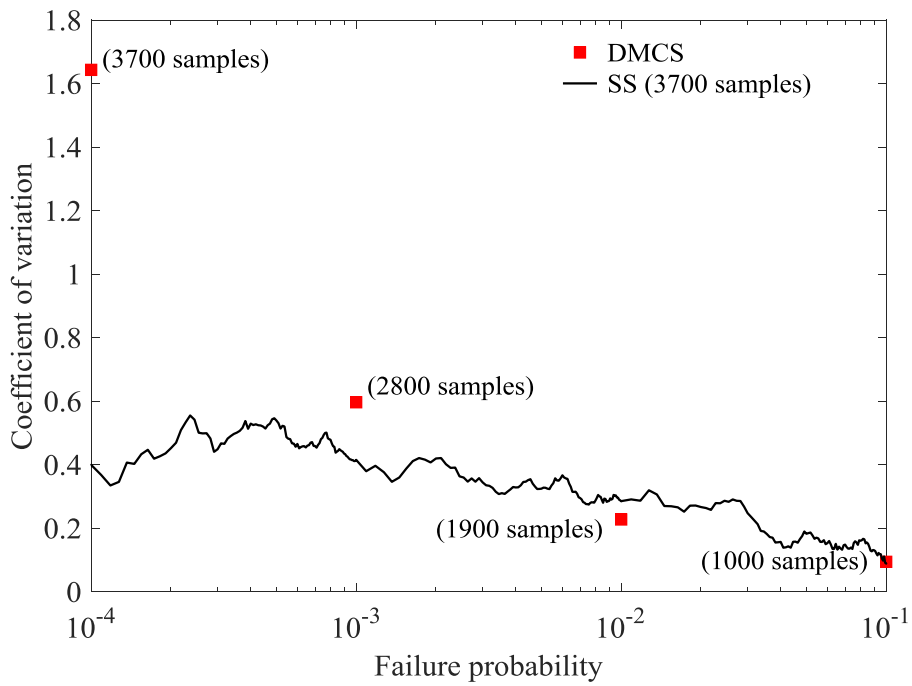


Fig. 3. Comparison of the COV of the failure probabilities from SS and DMCS

The results in Fig. 2 and 3 show that, compared to DMCS, SS can estimate the failure probability with much fewer samples, and its results have a smaller COV, especially in the low failure probability region. Hence, SS is proved to be a highly accurate and robust method to estimate the small failure probability of subsea pipelines under a random earthquake.

5.3 Sensitivity and failure analysis

To identify the contribution of the uncertain structural parameters to the failure of the subsea pipeline, a sensitivity analysis of the failure probability with respect to the mean values and variations of L_1 and L_2 is performed using SS, and the results are compared to those of DMCS, as shown in Table 1. It can be observed that the results of SS and DMCS agree quite well from the perspective of the sensitivity which is in fact a higher order quantity. It is also observed that the sensitivity with respect to L_2 is larger than that with respect to L_1, implying that the length of the free span L_2 is more influential on the failure of subsea pipelines than its location L_1.

The Markov chain samples generated during SS can be used not only for estimating the conditional probabilities, but also for the failure probability [29]. From Bayes' theorem,

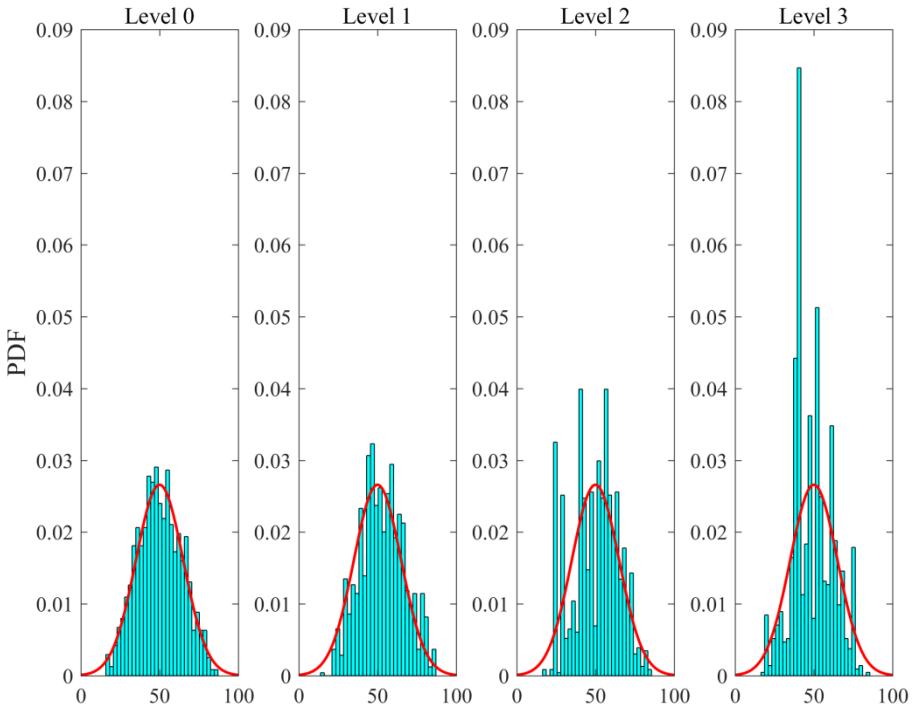
$$P(F|\boldsymbol{\theta}_l) = \frac{q(\boldsymbol{\theta}_l|F)}{q(\boldsymbol{\theta}_l)} P(F), \qquad l = 1,2 \cdots N_{\boldsymbol{\theta}} \tag{39}$$

Table 1 Normalized sensitivities of the failure probability

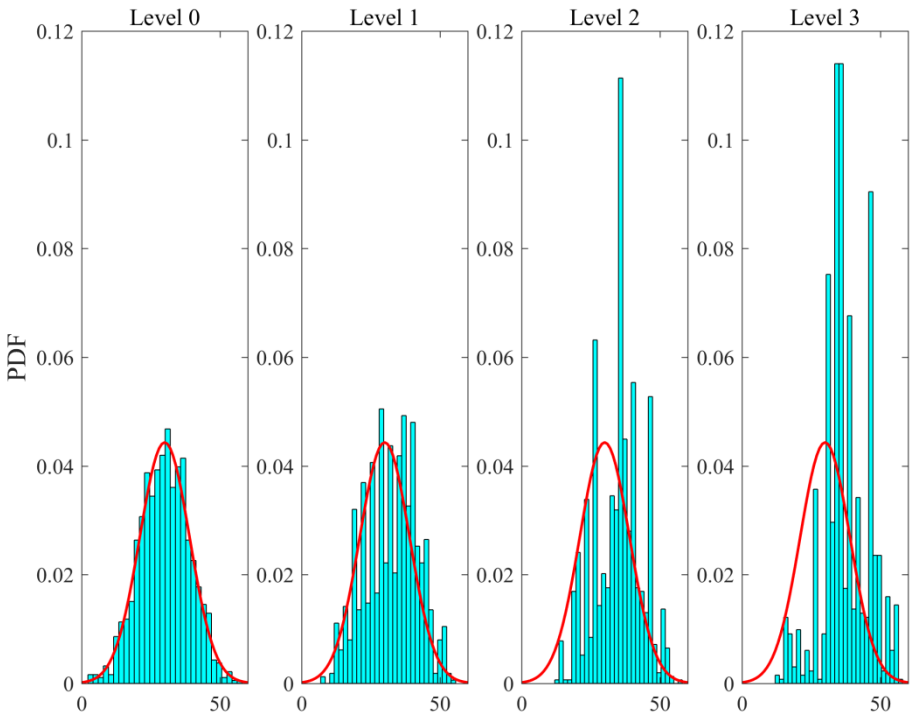
	SS	DMCS
$e_{\mu_{L_1}}$	0.147	0.125
$e_{\mu_{L_2}}$	1.035	0.893
$e_{\sigma_{L_1}}$	0.0371	0.0439
$e_{\sigma_{L_1}}$	0.348	0.298

where $N_{\boldsymbol{\theta}}$ is the dimension of $\boldsymbol{\theta}$. Thus when $q(\boldsymbol{\theta}_l|F)$ is similar to $q(\boldsymbol{\theta}_l)$, it is deduced that $P(F|\boldsymbol{\theta}_l) \approx P(F)$, so that the failure probability is insensitive to $\boldsymbol{\theta}_l$. Hence, by comparing the conditional PDF $q(\boldsymbol{\theta}_l|F)$ with the unconditional PDF $q(\boldsymbol{\theta}_l)$, one can obtain an indication of how much the uncertain parameter $\boldsymbol{\theta}_l$ influences the system failure.

Fig. 4 shows histograms of the conditional samples of the uncertain parameters L_1 and L_2 at different levels for a single SS run. It is noticed that the conditional PDFs of the uncertain parameters are obviously too large for certain values. This is because there are inevitably some repeated samples during the MCMC procedure. Despite some peaks, it is seen that the conditional PDFs of L_1 are almost symmetric about the mean value of L_1 at different levels, while those of L_2 are not symmetric and have a significant rightward shift, especially at the final level. From the comparison of the shapes of the conditional PDFs, it is concluded qualitatively that L_2 contributes more to the failure



(a) L_1



(b) L_2

Fig. 4. Empirical conditional PDFs of L_1 and L_2 at different conditional levels (histograms) compared to their unconditional PDFs (solid lines)

of subsea pipelines than L_1. This is in agreement with the conclusion of the parametric sensitivity analysis above.

5.4 Study of three typical cases

To study the influences of more parameters or effects on the failure of subsea pipelines, three typical cases are considered.

Case 1: *Unilateral contact model and permanent contact model*

As pointed out in the introduction, the separation of pipelines and the seabed is not considered in the dynamic analysis in some literature [5, 6]. This pipeline-seabed model is called a “permanent contact model”, while the model used in this paper is called a “unilateral contact model”. In order to investigate the influence of the unilateral contact effect on the reliability of subsea pipelines, failure probabilities of the unilateral and permanent models are calculated by SS and results are shown and compared in Fig. 5. It is seen that the failure probability of the permanent contact model is smaller than that of the unilateral contact model at the same threshold value, so that the permanent contact model is a more dangerous model in the earthquake design of subsea pipelines. The comparison also shows the necessity of the consideration of the unilateral contact effect in the earthquake reliability analysis of subsea pipelines.

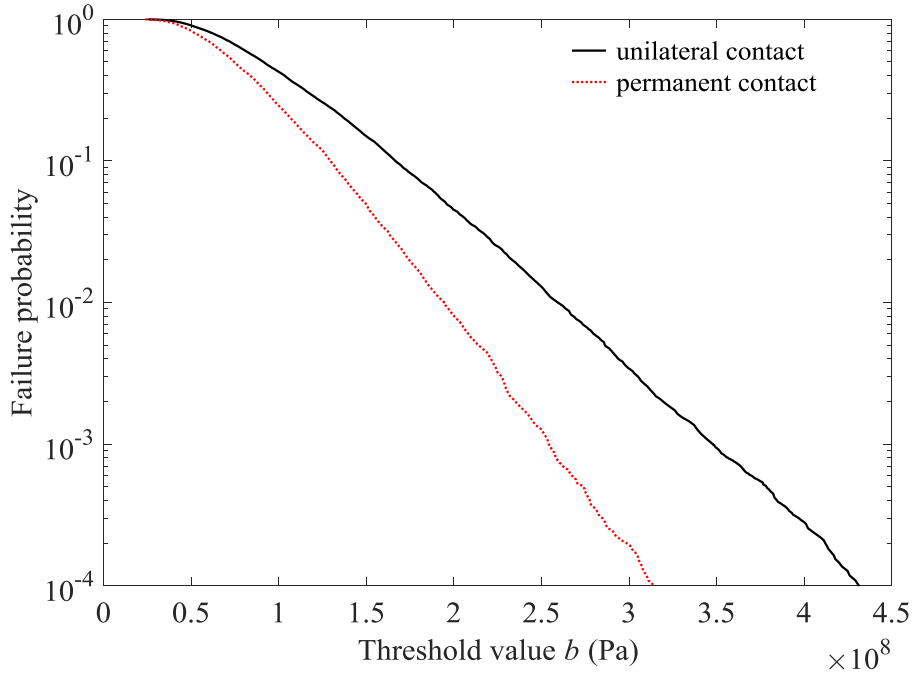


Fig. 5. Failure probabilities of subsea pipelines using unilateral and permanent contact models

Case 2: *Different apparent velocity*

As expressed in Eqs. (12) and (13), the apparent velocity of the earthquake waves v_{app} is one of the most important parameters affecting the spatial variation of the ground motion. The spatial variation decreases with increasing v_{app}. When v_{app} approaches infinity, the spatial variation of the ground motion vanishes and the earthquake is reduced to a uniform excitation. To study the influences of the spatial variation of the ground motion on the reliability, four cases with different v_{app} are considered, namely $v_{\mathrm{app}} = 500\,\mathrm{m/s}$, $1000\,\mathrm{m/s}$, $2000\,\mathrm{m/s}$ and uniform excitation, as shown in Fig. 6. It is observed that the influence of v_{app} on failure is not significant in the region with relative large failure probability. But when the failure probability is at a small level, it increases

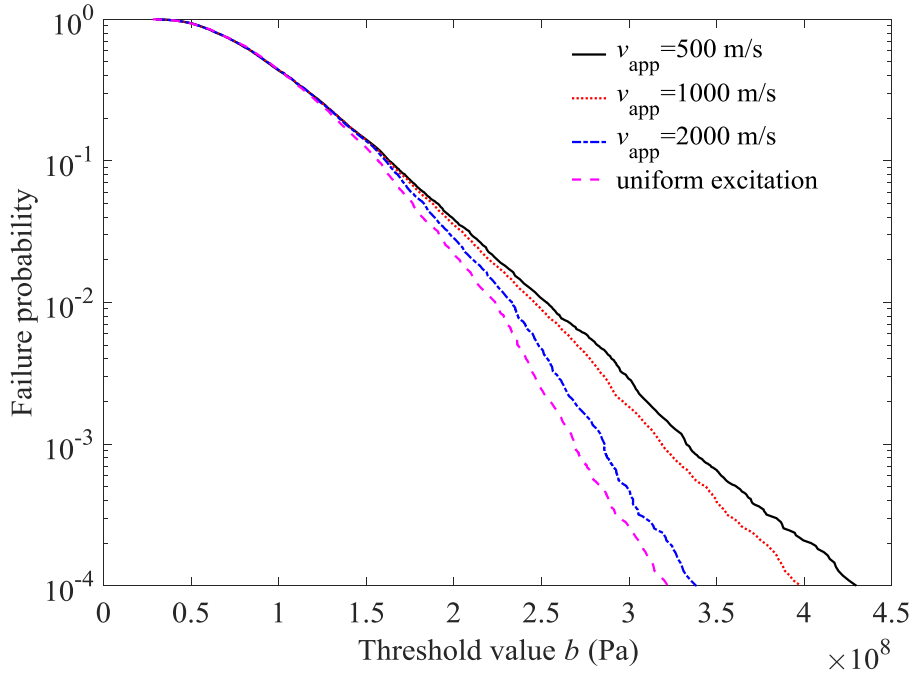


Fig. 6. Failure probabilities of subsea pipeline with different apparent velocity v_{app}

with decreasing apparent velocity v_{app}.

Case 3: *With and without free span*

In subsection 5.3, sensitivities of the failure probability of pipelines with respect to the location and length of the free span were studied. To study further the influence of the free span itself on the failure of subsea pipelines, three different cases are considered, i.e., (1) without free span, (2) with a deterministic free span at location $L_1 = 50\text{m}$ with length $L_2 = 30\text{m}$ and (3) with the uncertain free span used in subsection 5.3. The results are given in Fig. 7. It is shown that the free span has significant influence on the reliability of the subsea pipeline. Neglect of the free span or its uncertainty in the earthquake reliability analysis will lead to an underestimate of the failure probability.

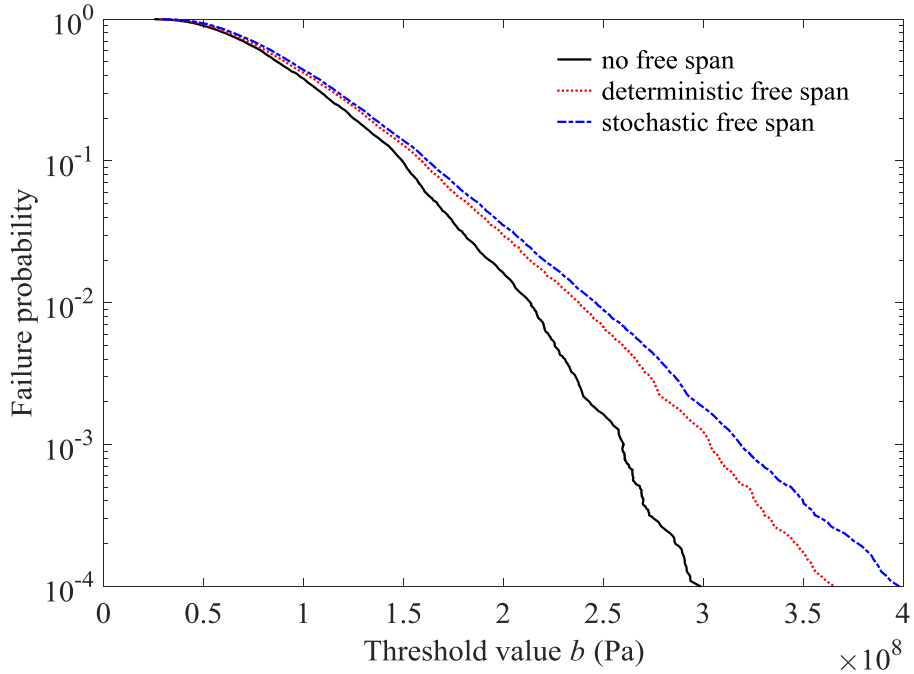


Fig. 7. Failure probabilities of subsea pipeline with different types of free span

6 Conclusions

In this paper, a computational framework based on subset simulation (SS) is proposed for the reliability analysis of subsea pipelines subjected to a random earthquake with spatial variation. Firstly, a mathematical model of subsea pipelines under random earthquake is established with consideration of the unilateral contact effect between the pipeline and the seabed. Then, by using the finite element method and Newmark integration method, the governing equation is discretized and the dynamic contact problem is derived as a linear complementarity problem. Finally, SS is applied for estimating the failure probability of subsea pipelines. SS expresses a small failure probability as a product of a sequence of large conditional probabilities, and provides the

potential to reduce the number of samples required in estimating small failure probability.

In numerical examples, direct Monte Carlo simulation (DMCS) is used to validate the feasibility of SS in the earthquake reliability problem. It is found that the failure probabilities calculated by SS agree well with those from DMCS, while the efficiency of SS, as indicated by the smaller number of samples, is much greater than that of DMCS. A coefficient of variation analysis shows that SS is more robust in small failure probability estimation than DMCS. Results from a sensitivity analysis indicate that the free span length is more influential on the failure of subsea pipelines than the free span location. Three typical cases with different parameters or effects are studied. It is shown that the unilateral contact effect between the seabed and pipelines, the spatial variation of the ground motion and the uncertainty of the free span have great influence on the failure of seabed pipelines and should be considered in earthquake reliability analysis.

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