The accuracy of AVA approximations in isotropic media assessed via synthetic numerical experiments: Implications for the determination of porosity

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Abstract

Analyses of seismic amplitude vs. angle are widely used to estimate hydrocarbon reservoir properties. In this paper we have investigated the accuracy of existing approximations based on the Zoeppritz equation, using synthetic numerical experiments that correlate P-wave reflectivity in isotropic media with reservoir porosity. An effective medium non-interacting approach (NIA) in rock physics modelling was used to compute the properties of fluid-saturated (water + gas) reservoir, which were then used in seismic modelling. In parallel, a Bayesian approach was used to estimate reservoir porosities from angle-dependent reflection coefficients and seismic amplitudes. A *Maximum a posteriori* solution of the Bayesian approach was also utilised to obtain an inverted porosity distribution in the reservoir model. The results of our forward models are important as they suggest that most of the approximations deviate from the exact Zoeppritz solutions with increasing angles of incidence of seismic waves. The results from the Bayesian inversion show that the Rüger and Bortfeld approximations agree with the exact Zoeppritz solutions to accurately estimate reservoir porosity. All the other approximations, except for Smith's, underestimate reservoir porosity and should be used in pre-stack inversion with caution. Smith’s and Fatti’s approximations failed to estimate reservoir porosity because of associated uncertainties.

Keywords: Seismic amplitude vs. angle; Rock physics modelling, Non-interacting approach, Bayesian approach, Metropolis algorithm, Pre-stack seismic inversion.

1. Introduction

The analysis of seismic amplitude variation with angle of incidence (AVA) is commonly used in the evaluation of porosity, lithology and fluids in hydrocarbon reservoirs. This analysis can become further useful when integrated with appropriate rock physics models. The roots of AVA analyses derive from the classical writings of Green (1839) and Knott (1899), who have studied the effect of interfaces on the reflection and transmission of seismic waves. In addition, Zoeppritz (1919) published a series of equations, among which the most important is the Zoeppritz equation regarding the partition of energy across isotropic media. This equation expressed the partition of energy of a plane wave when it hits the interface between two isotropic layers with different properties. As a result, several approximations to the exact Zoeppritz solution have been used in AVA analyses of isotropic media around the globe due to their relative ease of applicability (Bortfeld 1961; Aki and Richards 1980; Shuey 1985; Smith and Gidlow 1987; Hilterman 1989; Fatti et al. 1994; Rüger 2002).

The validity of the approximations described above depends on key assumptions used in their formulation. Each approximation describes the P-wave reflection coefficient as a function of a wave's angle of incidence and local rock properties such as the compressional and shear wave velocities, density, Poisson's ratio and other elastic parameters. These approximations are widely used because they are empirical and able to explain the AVA phenomenon on seismic, to then return meaningful physical properties of sub-surface units. For instance, the Shuey's approximation (Shuey 1985) tells us about the intercept (reflection strength at zero offset), gradient (the rate of change of reflection strength with incident angle) and curvature (the rate of change of reflectivity gradient) of a seismic wave. These AVA attributes are particularly helpful in the identification of low impedance gas sands (Castagna and Swan 1997).

Rock Physics Modelling acts as a bridge between seismic and rock properties, having a crucial role in seismic reservoir characterization (Avseth et al. 2005). Rock Physics Modelling plays a significant role in linking elastic parameters, such as impedance and velocity, to reservoir properties of interest (lithology, porosity and pore fluids; Wang 2001; Bosch et al. 2010). Rock Physics Modelling is widely combined with geostatistical techniques in seismic inversion. Combining rock physics data with geo-statistics during seismic inversion can be helpful to reduce uncertainties (Mukerji et al. 2001). By comparing this modelled (calculated) seismic data with raw (observed) seismic data, desired rock parameters can be calculated by iteratively using stochastic approaches (Grossman, 2003; Shahraini et al. 2011; Ali and Jakobsen 2011a; Ali and Jakobsen 2011b; Ali and Jakobsen 2014; Ali et al. 2015).

Pre-stack seismic inversion is widely used to estimate reservoir properties in the petroleum industry. This is a complicated process because it is an ill-posed problem with a non-unique solution. Therefore, it is important to overcome these challenges in order to estimate reservoir properties up to a satisfactory level. In order to overcome the problem of model instability, Du and Yan (2013) proposed a method for the estimation of fluid factors by utilising offset-limited data. Liang et al., (2017) addressed the same problem by utilising edge-preserving regularization and a Markov random field. Chiappa and Mazzotti (2009) formulated a linear Bayesian inversion method to estimate petrophysical properties. Sun et al. (2015) introduced pre-stack elastic integration techniques by considering the impact of rock physics and amplitude-preserving processing algorithms on pre-stack inversion. Finally, Anwer et al. (2017) utilised an anisotropic T-matrix approach in a Bayesian inversion scheme to characterise anisotropic sand-shale medium.

The aim of this study is to investigate the accuracy of existing approaches used in AVA modelling to determine porosity in isotropic media. In order to accomplish our aim, we have followed the workflow presented in Figure 1. A rock physics model based on an effective medium approach was used to compute the effective properties for fluid saturated isotropic reservoir rocks. These properties were then utilised to compute P- and S-wave velocities (V_P and V_S respectively) and density (ρ), and these two latter parameters were applied in a forward model to compute angle dependent P-wave reflection coefficients using the exact Zoeppritz solution, or approximations to the exact solution. AVA synthetic gathers were computed by convolving these reflection coefficients with a source wavelet. The exact Zoeppritz solution or approximations to the exact solution were used to invert the data to estimate porosity using a Bayesian approach and the Metropolis Algorithm of the Monte Carlo method (Tarantola 2005). Porosity distribution throughout the reservoir was also estimated using the *maximum a posteriori* solution of the Bayesian approach for each approximation, so we could investigate any implications of our methods to the determination of reservoir porosity.

2. Forward Modelling

The forward model used in this study can be written as:

$$\boldsymbol{d} = \boldsymbol{G}(\boldsymbol{m}). \quad (1)$$

Here d is the vector of observed seismic AVA data, $\boldsymbol{m}$ is a vector of unknown parameters (porosity in our case) and $\boldsymbol{G}$ is a forward modelling operator, which is a combination of rock physics modelling and seismic attribute generation (AVA data). In the following section we present a brief description of rock physics and seismic modelling.

2.1 Rock Physics Modelling

The main purpose of rock physics is to understand the influence of rock properties, e.g. lithology, porosity, saturation, etc., on seismic velocities. There are several theories to estimate the elastic properties of dry and saturated rocks containing pores and cracks of different aspect ratios. Ali et al. (2015) showed a comparison between rock physics models based on effective stiffness and compliance methods for fractured reservoir characterization. A good rock physics model can efficiently estimate reservoir properties, which can then be correlated with seismic data to allow the modelling of an entire reservoir. Hence, a realistic model was assumed in this study containing a quartz matrix, interconnected spherical pores, randomly oriented micro-cracks that do not contribute to porosity, and a mixture of water and gas as pore saturating fluids (Figure 2). The input to rock physics model, in the form of elastic properties of quartz matrix and fluid, is given in Table 1.

We used a non-interacting approach (NIA) based on effective medium modelling to compute effective properties of isotropic reservoirs. Hudson and Knopoff (1989) proposed a relationship to obtain effective compliance $\boldsymbol{S}^{*}$ for an isotropic medium, based on a NIA, as follows:

$$\boldsymbol{S}^{*} = \boldsymbol{S}^{(0)} - \sum_{r=1}^{N}\left(v^{(r)}\left(\boldsymbol{S}^{(0)}:\boldsymbol{C}^{(r)} - \boldsymbol{I}_4\right):\boldsymbol{K}^{(r)}\right), \quad (2)$$

in which $\boldsymbol{S}^{(0)}$ represents the compliance tensor of background matrix, $v^{(r)}$ is the volume concentration of pores and randomly oriented micro-cracks, the stiffness tensor $C^{(r)}$ is associated with the inclusions (pores and randomly oriented micro-cracks), $\boldsymbol{I}_4$ is the identity for fourth-rank tensors, and $K^{(r)}$ represents the K-tensor of Eshelby (1957) which can be given as (Jakobsen and Johansen 2005):

$$\boldsymbol{K}^{(r)} = \boldsymbol{A}^{(r)} : \boldsymbol{S}^{(0)}, \tag{3}$$

where,

$$\boldsymbol{A}^{(r)} = \left[\boldsymbol{I}_4 - \boldsymbol{G}^{(r)} : \left(\boldsymbol{C}^{(r)} - \boldsymbol{C}^{(0)}\right)\right]^{-1}. \tag{4}$$

Here $\boldsymbol{G}^{(r)}$ is a fourth-rank tensor given by the Green's function integrated over a characteristic spheroid with the same shape as inclusions (pores and randomly oriented micro-cracks) of type r (Jakobsen et al. 2003; Ali and Jakobsen 2011a; Ali and Jakobsen 2011b; Ali and Jakobsen 2014). In order to incorporate the case of empty inclusions, the stiffness parameter is taken out of the equation so that Equation (2), for a dry rock, can be rewritten as (Hu and McMechan 2009):

$$\boldsymbol{S}^{*} = \boldsymbol{S}^{(0)} + \sum_{r=1}^{N} v^{(r)} \boldsymbol{K}^{(r)} \tag{5}$$

For the effect of fluid saturation we used the isotopic Gassmann's equation (Gassmann 1951), which is given by:

$$K_{sat} = K_{dry} + \left(\frac{\left(1 - \left(\frac{K_{dry}}{K_{frame}}\right)\right)^2}{\left(\frac{\varphi}{K_f}\right) + \left(\frac{1 - \varphi}{K_{frame}}\right) - \left(\frac{K_{dry}}{\left(K_{frame}\right)^2}\right)} \right). \tag{6}$$

In Equation (6) K_{sat}, K_{dry}, K_{frame} and K_f represent the bulk modulus of fluid-saturated rock, dry rock, dry rock frame and pore-saturating fluid, respectively, and φ represents porosity. K_f is computed using Wood's relationship (Wood 1955), which is given by:

$$\frac{1}{K_f} = \frac{S_g}{K_g} + \frac{S_O}{K_O} + \frac{S_W}{K_W}. \qquad (7)$$

In Equation (7), S_g and K_g, S_O and K_O, S_W and K_W are the saturation and bulk modulus of gas, oil and pore saturating water, respectively.

The effective moduli and density from this rock physics model were estimated for different porosities, and then used to compute P- and S-wave velocities. As the Lamé's parameter (λ) and shear modulus (μ) are sufficient to characterise an isotropic medium, these moduli can also be expressed in terms of the stiffness parameters C11 and C44. The effect of porosity on P- and S-wave velocities, and stiffness parameters (C11 and C44), is shown in Figure 3. An increase in porosity weakens a volume of rock by decreasing the bulk and shear moduli. As a result of this reduction, P- and S-wave velocities, together with the stiffness parameters above, decrease with increasing porosity. Such trends are more or less expected, but they would have been difficult to quantify without a suitable rock physics model.

2.2 Seismic Modelling

The solution of seismic forward modelling started with the development of numerical solutions for the wave equation (Krebes 2004). Two-dimensional (2D) seismic forward modelling can be undertaken using ray tracing, matrix method, finite difference and finite-element methods. One of the key parts in seismic forward modelling is the computation of reflection coefficients by utilising P- and S-wave velocities and density in the exact Zoeppritz solution, or approximations to the exact solution, as previously discussed. When an incident P-wave strikes an interface between two layers of different properties, at a non-zero incident angle, it is converted into four rays as shown in Figure 4. The energy partition at the interface can be calculated using the Zoeppritz energy equation re-written as follows (Pujol 2003):

$$\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = \begin{bmatrix} -\sin e & \cos f & \sin e' & \cos f' \\ \cos e & \sin f & \cos e' & -\sin f' \\ \sin 2e & -\left(\frac{Vp}{Vs}\right)\cos 2f & \left(\frac{\rho'}{\rho}\right)\left(\frac{Vp}{Vp'}\right)\left(\left(\frac{Vs'}{Vs}\right)^2\right)\sin 2e' & \left(\frac{\rho'}{\rho}\right)\left(\frac{Vp}{Vs'}\right)\left(\left(\frac{Vs'}{Vs}\right)^2\right)\cos 2f' \\ -\cos 2f & -\left(\frac{Vs}{Vp}\right)\sin 2f & \left(\frac{\rho'}{\rho}\right)\left(\frac{Vp'}{Vp}\right)\cos 2f' & \left(\frac{\rho'}{\rho}\right)\left(\frac{Vs'}{Vp}\right)\sin 2f' \end{bmatrix}^{-1} \begin{bmatrix} \sin e \\ \cos e \\ \sin 2e \\ \cos 2f \end{bmatrix}. \quad (8)$$

with:

R_{PP} - P-wave reflection coefficient

R_{PS} - S-wave reflection coefficient

T_{PP} - P-wave transmission coefficient

T_{PS} - S-wave transmission coefficients.

In Equation (8), Vp, Vs and ρ are the P- and S-wave velocities and density of upper medium, respectively. Vp', Vs' and ρ' are P- and, S-wave velocities and density of the gas reservoir respectively. In addition, e and e', and f and f' are the reflection and transmission angles of P-and converted S-wave respectively.

From the original Zoeppritz equations, different seismic amplitude vs. offset (AVO) approximations can be classified into linear and nonlinear AVO approximations (Rüger 2002). The key assumptions leading to linear AVO approximations are justified by the fact that certain sedimentary rocks show weak to moderate contrasts in elastic parameters (Thomsen 1986; Thomsen 1995). The exact Zoeppritz solution and approximations to exact solution are dependent upon the angle of incidence, at which the seismic wave strikes an interface, but generally the seismic data is a function of offset. This same offset should be converted in an angle during the application of processing algorithms. This type of analysis is called as AVA instead of AVO.

In order to perform AVA modelling, P and S-wave velocities and density obtained from rock physics modelling are placed in Equation (8), or approximations to Equation (8), to compute reflection coefficients for different angles of incidence of a seismic wave. These angle dependent reflectivities can be convolved with source wavelet to obtain an AVA seismic response.

In this study, a rock physics model was iteratively used to compute effective moduli and density for different porosities at reservoir level. The sensitivity of P-wave reflection coefficient for porosity, using the exact Zoeppritz solution and approximations to the exact solution, is shown in Figure 5. The values of P-wave reflection coefficient are greater for smaller porosity and decrease with increase in porosity. In turn, velocity and density of the medium are responsible for this behaviour because they increase with decreasing porosity, and vice-versa. P-wave reflection coefficient for the exact Zoeppritz solution along with its approximations decreases with a relative increase in the angle of incidence, and form Class-1 AVO anomalies according to Rutherford and Williams (1989).

3. Inverse modelling

Seismic inversion is an important tool to estimate rock properties from seismic data using a combination of rock physics and statistical techniques. There are different approaches for the quantitative estimation of reservoir properties using seismic inversion. The scientific study of a physical system can be conducted in three steps: a) parameterisation of the system, b) forward modelling, and c) inverse modelling (Tarantola 2005). A non-linear inverse problem is considered in this study as:

$$\boldsymbol{G}(\boldsymbol{m}) \approx \boldsymbol{d}. \quad (9)$$

Here $\boldsymbol{m}$ represents a vector of physical parameters related to the porosity of the Earth model, and $\boldsymbol{d}$ is data vector of observed values, i.e. in this work, the angle dependent reflectivity/seismic AVA gathers. $\boldsymbol{G}$ is a combination of the rock physics and seismic attributes, i.e. angle-dependent reflectivity/seismic gathers as a function of porosity.

A real physical system is best modelled by incorporating the effect of noise in forward model. Therefore, by including the noise term in Equation (9) (Aster et al. 2005), we have:

$$\boldsymbol{G}(\boldsymbol{m}) \approx \boldsymbol{d} + \eta. \tag{10}$$

In Equation (10), η represents the noise vector and generally it is assumed to be Gaussian (Tarantola 2005; Aster et al. 2005). The noise in seismic data is mainly introduced during its acquisition, and can be coherent (originated due to seismic source) or incoherent (noise introduced due to some other sources like traffic, wind, river, high tension wires above geophones etc.). Incoherent noise is also called random noise because its behaviour varies for each shot gather in a data volume. This noise can be minimised by increasing the fold of seismic data. Coherent noise includes diffractions, refractions, multiples, etc., and should be removed by the application of sophisticated processing algorithms before performing AVA inversion (Grossman 2003; Zhang et al. 2014; Marfurt and Alves 2015).

We used a Bayesian approach to get the probability distribution of porosities from our forward modelling. The Bayes' theorem (Aster et al. 2005) provides a framework in which the posterior probability of the variables of interest, derived from uncertain data, is obtained using *a priori* information. This *a priori* information is used to obtain unique maxima of Probability Density Functions (PDF) and makes solutions stable when using uncertain data (Duijndam 1988a, 1988b). The probabilistic prediction provides a natural way of understanding the uncertainty of the problem. Uncertainties in a Bayesian framework for AVO inversion were discussed by Houck (2002).

An inverse problem is solved using a Bayesian approach that combines the prior distribution $P(\boldsymbol{m})$ for the model parameters with the likelihood function $P(\boldsymbol{d}|\boldsymbol{m})$. This way, one can obtain a posterior probability distribution $P(\boldsymbol{m}|\boldsymbol{d})$ over a model space such as (Aster et al. 2005):

$$P(\boldsymbol{m}|\boldsymbol{d}) = \frac{P(\boldsymbol{m})P(\boldsymbol{d}|\boldsymbol{m})}{P(\boldsymbol{d})}. \tag{11}$$

In this equation, the posterior probability distribution $P(\boldsymbol{m}|\boldsymbol{d})$ represents the solution of the inverse problem, and $P(\boldsymbol{d})$ is considered as normalisation constant. The solution for a posterior probability density function (Aster et al. 2005) using a Gaussian approach is given by:

$$P(\boldsymbol{m}|\boldsymbol{d}) = N \,.\, e^{-\boldsymbol{J}(\boldsymbol{m})}. \tag{12}$$

In Equation (12), the constant N is called the normalization constant, and $\boldsymbol{J}(\boldsymbol{m})$ represents the objective function to be minimised. The objective function by assuming Gaussian statistics can be given as (Aster et al. 2005):

$$\boldsymbol{J}(\boldsymbol{m}) = ½\left[(\boldsymbol{G}(\boldsymbol{m}) - \boldsymbol{d})^T \boldsymbol{C}_{\boldsymbol{D}}^{-\boldsymbol{1}}(\boldsymbol{G}(\boldsymbol{m}) - \boldsymbol{d}) + (\boldsymbol{m} - \boldsymbol{m_0})^T \boldsymbol{C}_{\boldsymbol{M}}^{-\boldsymbol{1}}(\boldsymbol{m} - \boldsymbol{m_0})\right]. \tag{13}$$

Here, $\boldsymbol{m_0}$ represents the mean value of the *a priori* distribution, $\boldsymbol{C_D}$ is the covariance matrix for the data, and $\boldsymbol{C_M}$ is the covariance matrix representing the model space. In case of uninformative prior information, Equation (13) can be represented by the likelihood function. The posterior distribution represents the full solution to an inverse problem. The evaluation of posterior distribution depends on the number of unknown parameters. As, in this study, we have only one unknown parameter (porosity), the posterior distribution given by Equations 12-13 represents the solution of the inverse problem. In case of higher number of unknown parameters, the exploration of posterior distribution can be performed using the methods presented by Ali and Jakobsen (2011a), Ali and Jakobsen (2011b), Shahraini et al. (2011) and Ali and Jakobsen (2015).

We also used the Metropolis algorithm of the Monte Carlo method to estimate reservoir porosity using the exact Zoeppritz solution, and approximations to the exact solution. This method was developed by Metropolis and Ulam (1949), Metropolis et al. (1953) and Hastings (1970), consisting of a Markov Chain Monte Carlo (MCMC) technique used to estimate a solution by sampling through a posterior (arbitrary) distribution. The basic idea of this method is to sample the target distribution by performing a random walk, from sample to sample, and modify the walk according to some pre-defined conditions (Tarantola 2005).

4 Results and Discussion

4.1. Accuracy in forward modelling

Rock Physics Modelling was used to compute the effective moduli and density of a model presented in Figure 2. The aspect ratio of randomly oriented micro-cracks used in Rock Physics Modelling was set to 1/1000. The properties of the quartz matrix and fluids (water + gas) are given in Table 1.

The seismic velocity for the model was computed from these moduli, and from density, by iterating the rock physics model for different porosities discussed in section 2.1. These velocities and density were utilised in Equation (8), and approximations to Equation (8), in order to obtain angle-dependent P-wave reflectivity for different porosity values discussed in section 2.2. The properties of overburden strata required to compute reflection coefficients are shown in Table 1. The comparison of the exact Zoeppritz solution (and approximations to the exact solution) for different porosities, with respect to the angle of incidence of seismic waves, is shown in Figure 6.

In Figure 6 one can observe that almost all the approximations, except Fatti and Smith's, are in good agreement with the exact Zoeppritz's solution at relatively small incidence angles. However, they start to deviate from the Zoeppritz's solution as the angle of incidence increases (Figure 6). The Fatti’s approximation (Fatti et al. 1994) has a comparatively higher gradient and deviates from the exact Zoeppritz solution even at near-incidence angles. Importantly, Smith’s reflectivity values (Smith and Gidlow 1987) start to increase and move away from the exact Zoeppritz solution with increasing porosity (Figure 6). All other approximations do not change their behaviour significantly with increasing porosity values (Figure 6).

The P-wave reflection coefficients obtained from a combination of rock physics and seismic modelling were convolved with source wavelet to obtain synthetic seismic AVA gathers for different porosity levels. These P-wave reflection coefficients are displayed in Figure 7 to 10 for porosity values of 0.1, 0.2, 0.3 and 0.4, respectively. All the approximations reveal a polarity reversal, with the Fatti’s approximation having the largest negative amplitude - hence disagreeing with the exact Zoeppritz solution (Figures 7-10). The amplitude of synthetic AVA gathers shows a decreasing trend with respect to angle of incidence (Figures 7-10). Synthetic amplitude is higher at low porosities for all approximations, and decreases with increasing porosity.

4.2 Accuracy in inverse modelling

In order to check the accuracy of P-wave reflection coefficient approximations for AVO inversions, we tried to retrieve true reservoir porosity (0.15) from synthetic reflection coefficient and amplitude data (with 25% noise/uncertainty/standard deviation of observed seismic data) using the Bayesian approach and the Monte Carlo method discussed in Section 3. Normally, the uncertainty (standard deviation/noise level) in seismic AVA data is within the range of 10-30% (Ren et al. 2017). Noise represents the uncertainty left in the observed data after the application of sophisticated seismic AVA-processing algorithms such as amplitude preserving migration (Grossman 2003; Zhang et al. 2014). The amplitude of seismic data is the most important factor in seismic AVA analyses, and preserving the true amplitude via sophisticated algorithms is crucial.

We have considered an uninformative prior in our inverse problem, so the objective function is only represented by the likelihood function. The choice of uninformative prior gives an equal likelihood for all unknown parameters to be estimated during the inversion process. The source of prior information comes from independent measurements (e.g. well logs and laboratory measurements of porosity). The results of this inversion are shown in Figure 11 to 14. Figures 11-12 represent the inversion result in the form of a posterior distribution, whereas Figures 13-14 represent the inversion result obtained via the sampling of the posterior distribution, i.e. a Monte Carlo method.

Rüger and Bortfeld's (Bortfeld 1961; Rüger 2002) approximations show a good agreement with the exact Zoeppritz solution, and associated uncertainties are comparatively smaller. Aki and Richards, Hilterman and Shuey's approximation (Aki and Richards 1980; Hilterman 1989; Shuey 1985) underestimates reservoir porosity. In addition, the uncertainty for Smith's approximation is quite high and cannot be used in AVA inversion for reservoir porosity. It is also interesting to note that the inversion results from angle-dependent reflection coefficients and seismic amplitudes are the same (Figures 11-12). This stems out from the fact that seismic amplitudes are basically the result of convolving reflection coefficients with a source wavelet.

Finally the *Maximum a posteriori* solution of the Bayesian approach was used to recover the porosity distribution in the reservoir. Initially, a Gaussian random porosity field (a smoothly varying field; Buland and Omre 2003) representing reservoir porosity was generated (100×100 grid blocks), and this field was then compared with the porosity fields recovered by the exact Zoeppritz solution, and approximations to the exact solution, by minimising the objective function as:

$$\mathrm{J}(\mathbf{m}) = \sum_{i=1}^{40} \left[\frac{R_i^c(\boldsymbol{m}) - R_i^o}{\Delta R_i}\right]^2. \tag{14}$$

In this equation, R_i^C and R_i^O are respectively the calculated and observed reflectivities. The term 'ΔR_i' in the denominator ΔR_irepresents the standard deviation (noise/uncertainty) present in the synthetic AVA data.

Results obtained from this latter procedure are shown in Figure 15. Porosities recovered by Rüger and Bortfeld's approximations are in good agreement with the exact Zoeppritz solution, and recover reservoir porosities to a satisfactory level. In comparison, Aki and Richards, Hilterman and Shuey's approximations underestimate porosity, with its effect being more prominent for smaller porosities. Fatti and Smith's approximations completely failed to recover reservoir porosity due to their high associated uncertainty. Uncertainty associated with each approximation is shown in Table 2.

It is important to mention that the inversion results in Figures 11 to 14 represent the results in the form of a single grid block. The inverse numerical experiment presented in Figure 15, in the form of Gaussian field on 100×100 grid blocks for porosity, is very important in the context of its application on raw data. More specifically, the inversion procedure presented in Figures 11 to 14 is repeated for 100×100 grid blocks and the optimum (true) value of porosity is recovered utilising the *Maximum a posteriori* solution of the Bayesian approach. One can visually inspect the performance of approximations to the exact solution by comparing them with the result of exact Zoeppritz solution. This approach is, in this work, suggested as the most practical way of estimating the distribution of porosity in reservoir intervals using raw seismic data.

4.3 Applications on raw seismic data

For applications on raw seismic data, pre-stack seismic data processed typically for AVA/AVO analyses using the workflows given by Ostrander (1984), Chiburis (1984), Castagna and Backus, 1993, Grossman (2003) and Zhang et al. (2014), are required along with well-log and any ancillary laboratory information. The petrophysical analysis of well-log data and laboratory measurements will provide the basic input parameters required to perform the Rock Physics Modelling necessary to obtain the effective elastic properties of the gas sand reservoir. Using these effective elastic properties, seismic modelling can be performed by exact Zoeppritz solution, or approximations to the exact solution (calculated data). From pre-stack seismic data (near, mid and far angle gathers), the amplitudes/angle–dependent reflection coefficients can be obtained at reservoir level (observed data). For angle-dependent reflection coefficients, the amplitudes obtained at reservoir level should be convolved with the inverse source wavelet extracted from seismic and well-log data.

The calculated and observed data are used in the Bayesian approach and their misfit is minimised in the form of porosity or desired parameter describing the reservoir character via the objective function. The sensitivity of the desired reservoir parameter with seismic AVA amplitudes, or angle-dependent reflection coefficient, is crucial for the inversion procedure, i.e. if porosity changes, the seismic AVA data must also change.

In a nutshell, the results presented in this study using synthetic numerical experiments are important to everyone working with AVA data. The analysis of seismic amplitude variation with angle of incidence (AVA) is commonly used in the evaluation of reservoir character. It can be very useful to know which seismic AVA model is suitable to provide reliable results during AVA analyses and seismic-data inversion.

5 Conclusions

AVA analysis and inversion in isotropic media require computation of P-wave reflection coefficients between two layers with different properties. There are several approximations to the exact Zoeppritz solution for this purpose and these are often used in practice. It may be an interesting idea to investigate the accuracy of these existing approximations within the context of seismic reservoir characterization via AVA analysis or inversion. In this study, we have investigated the accuracy of AVA approximations and their implications to the determination of reservoir porosity both in synthetic forward and inverse numerical experiments.

Forward modelling results show that all the approximations to the exact solution, except for Fatti's and Smith's, are in good agreement with the exact Zoeppritz solutions at smaller angles of incidence. However, they start to deviate from it as incidence angle increases from 20^{o}.

In synthetic AVA gathers, all the approximations to the exact solution show a decrease in seismic amplitude with increasing porosity, and polarity reversals at relatively small porosity values.

Fatti’s approximation shows the largest negative amplitude, whereas Smith’s approximation returns large positive amplitudes. They are both in disagreement with the exact Zoeppritz solution.

In AVA inversion tests using Bayesian and Monte Carlo methods, Rüger and Bortfeld approximations show a good agreement with the exact Zoeppritz solution, while the Aki and Richard, Hilterman and Shuey’s approximations underestimate the reservoir porosity and should be used in AVA inversion with caution. The uncertainty for Smith’s approximation is significantly high and it cannot be used in AVA inversion for reservoir porosity.

The *Maximum a posteriori* solution for porosity inversion shows that porosities recovered by Rüger and Bortfeld’s approximations are in good agreement with the exact Zoeppritz solution and recover reservoir porosities to a satisfactory level. Aki and Richard, Hilterman and Shuey’s approximations underestimate porosity and the effect is more prominent for smaller porosities. Fatti's and Smith’s approximations completely failed to recover reservoir porosity due to their associated high uncertainty. We hope that this study will provide the readers an insight on choosing a suitable approximation for AVA analyses and inversion as methods in reservoir characterisation.

Table 1: Elastic properties of solid mineral, fluid and overburden used in the computation of reflection coefficients in this work ($GPA\ (Gigapascal) = 10^9 Pa = 10^9 Kg\,.m^{-1}.s^{-2}$).

Material	Bulk Modulus (GPA)	Shear Modulus (GPA)	Density (g/cm^3)
Quartz matrix	37.6	44	2.65
Fluid (water/brine)	2.2	0	1

Fluid (gas)	0.02	0	0.065
Overburden (Shale)	18	7	2.35

Table 2: Uncertainty percentage of different approximations for the estimation of porosity using a *Maximum a posteriori* solution.

Approximation	Uncertainty (%)	Status	Remarks
Shuey	53	Underestimates porosity	Should be used with caution
Hilterman	67	Considerably underestimates porosity	Should not be used in inverse modelling
Fatti	100	Fails to recover porosity	Should not be used in inverse modelling
Aki and Richards	42	Underestimates porosity	Should be used with caution
Smith	639	High uncertainty, overestimates porosity	Should not be used in inverse modelling
Bortfeld	04	Closer to exact Zoeppritz	Satisfactory
Ruger	04	Closer to exact Zoeppritz	Satisfactory

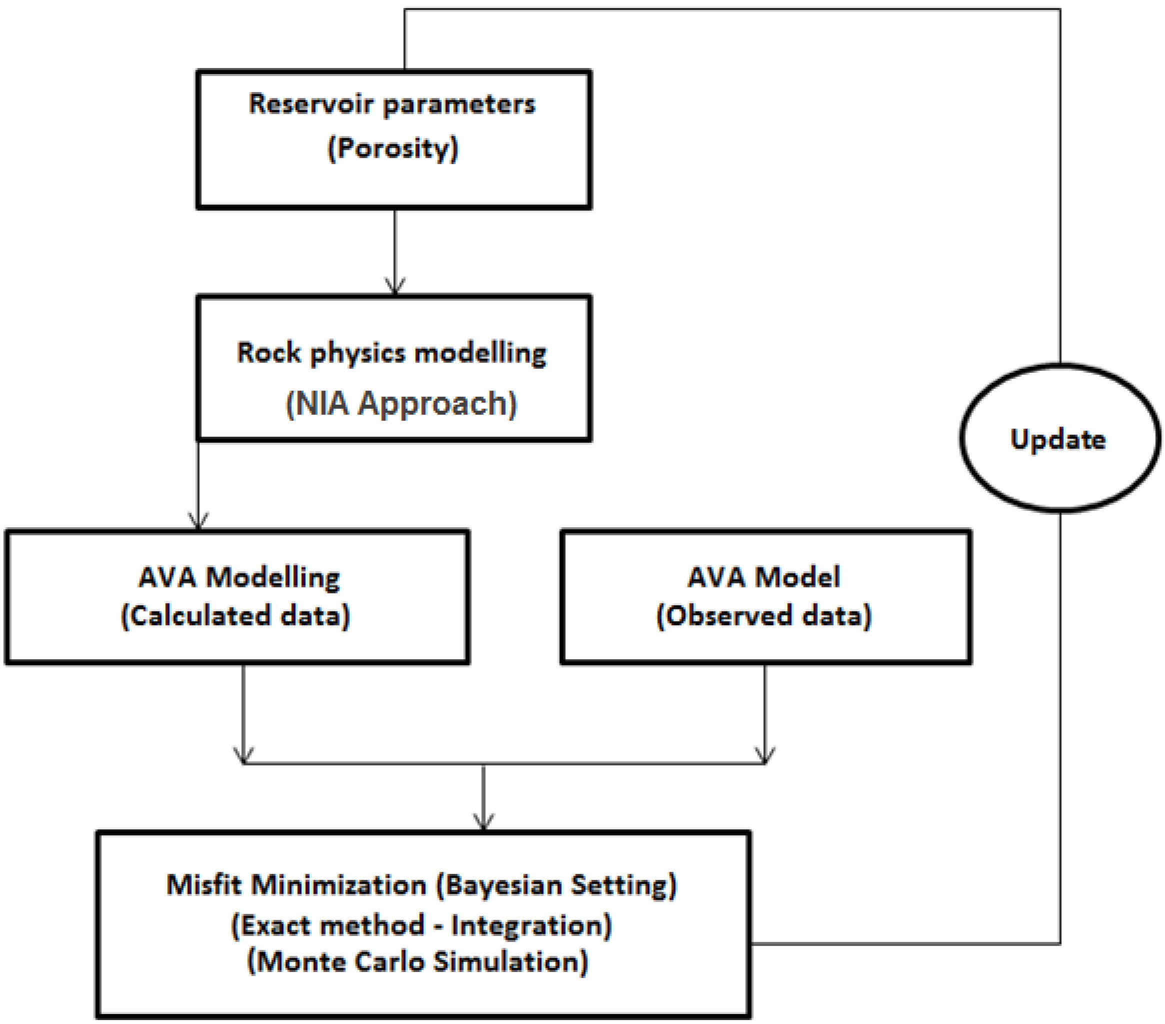

Figure 1: Schematic workflow used to estimate porosity from seismic AVA analyses.

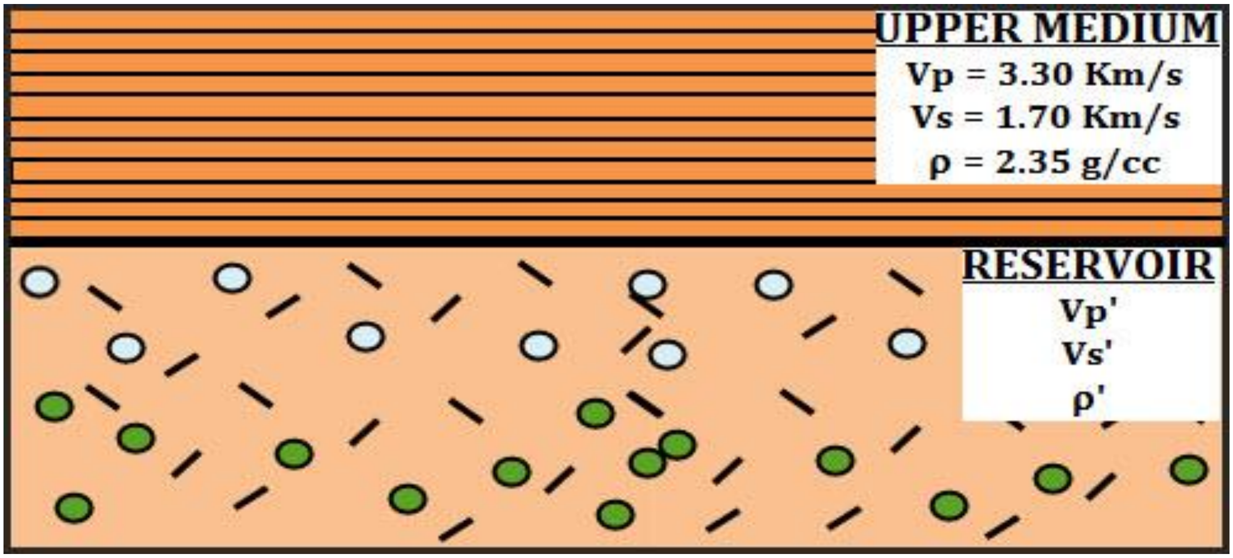

Figure 2: Rock physics model includes quartz matrix, interconnected spherical pores, and randomly oriented micro-cracks that do not contribute to the porosity of the rock, with water and gas as pore-saturating fluids. The aspect ratio of randomly oriented micro-cracks was set to 1/1000.

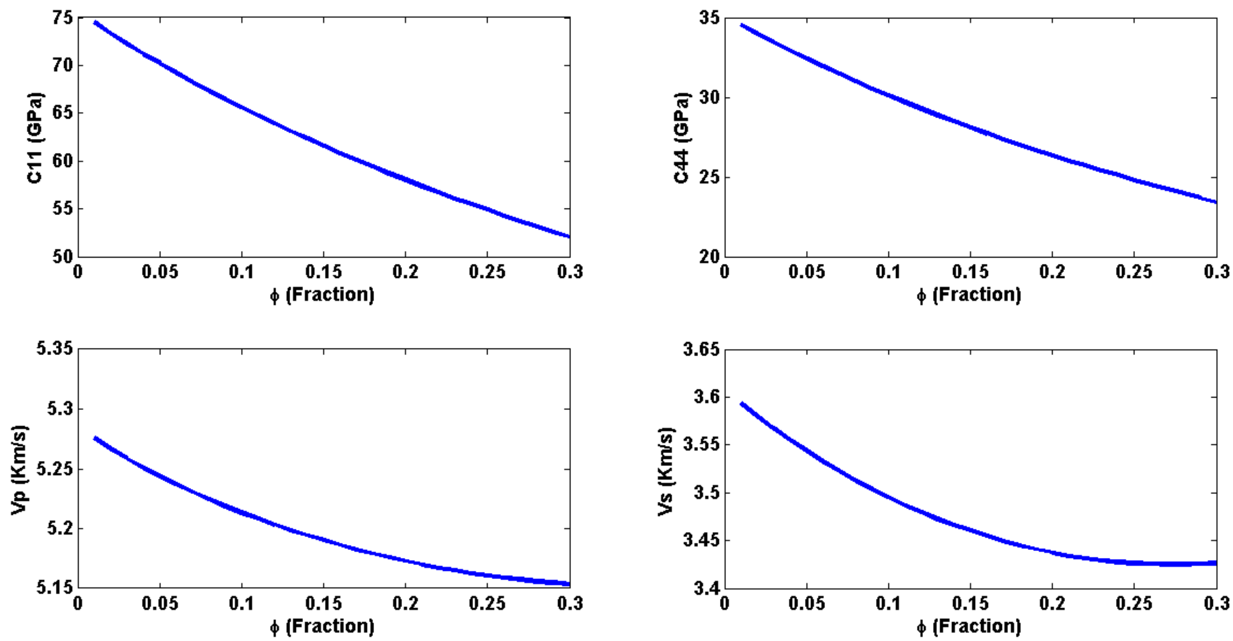

Figure 3: C11, C44 and Vp, Vs plotted against porosity show a relative decrease with increasing porosity.

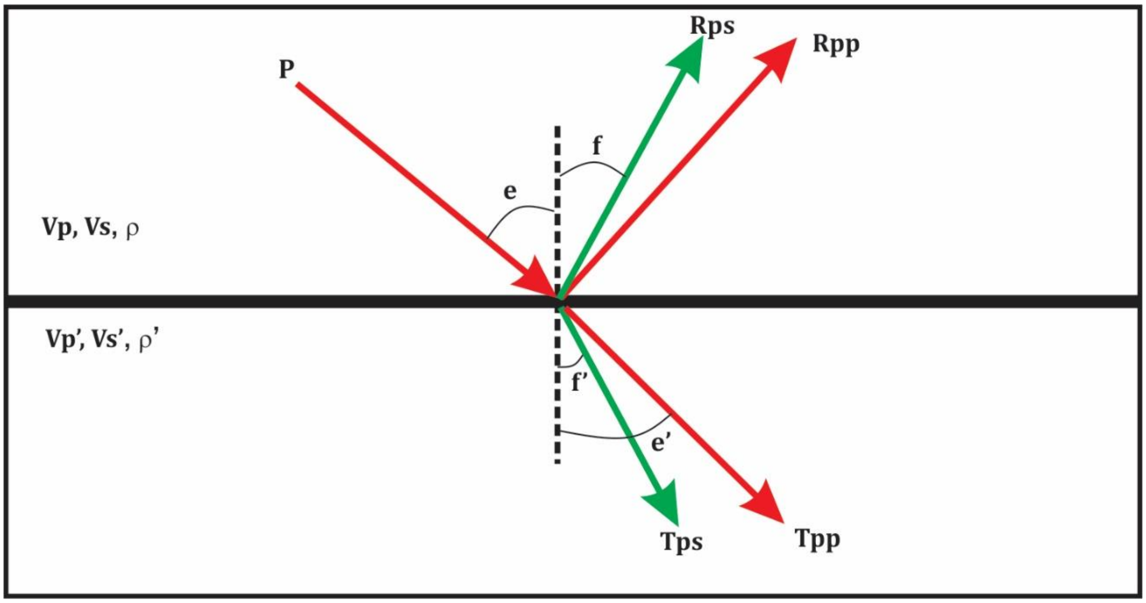

Figure 4: Partitioning of energy at an interface. Modified from Castagna and Backus (1993).

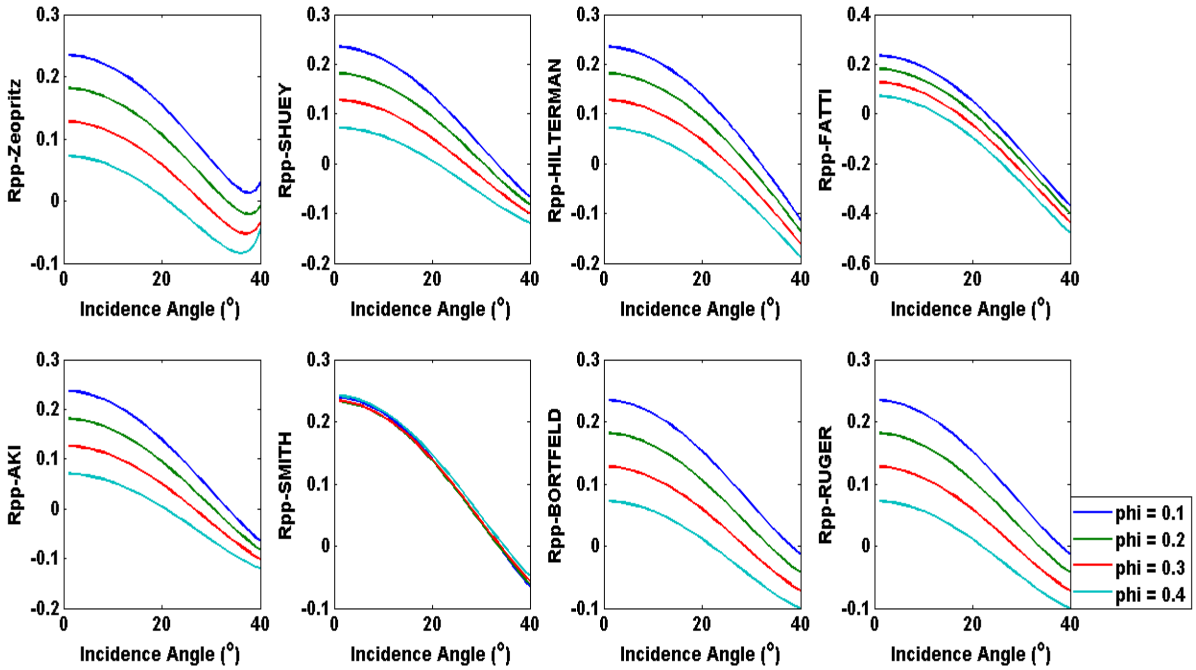

Figure 5: Reflection Coefficient (RC) sensitivity to porosity. The exact Zoeppritz equation, and all its approximations show a general decrease in reflection coefficient with increasing angles of incidence and porosity. The properties of overburden rocks are given in Table 1.

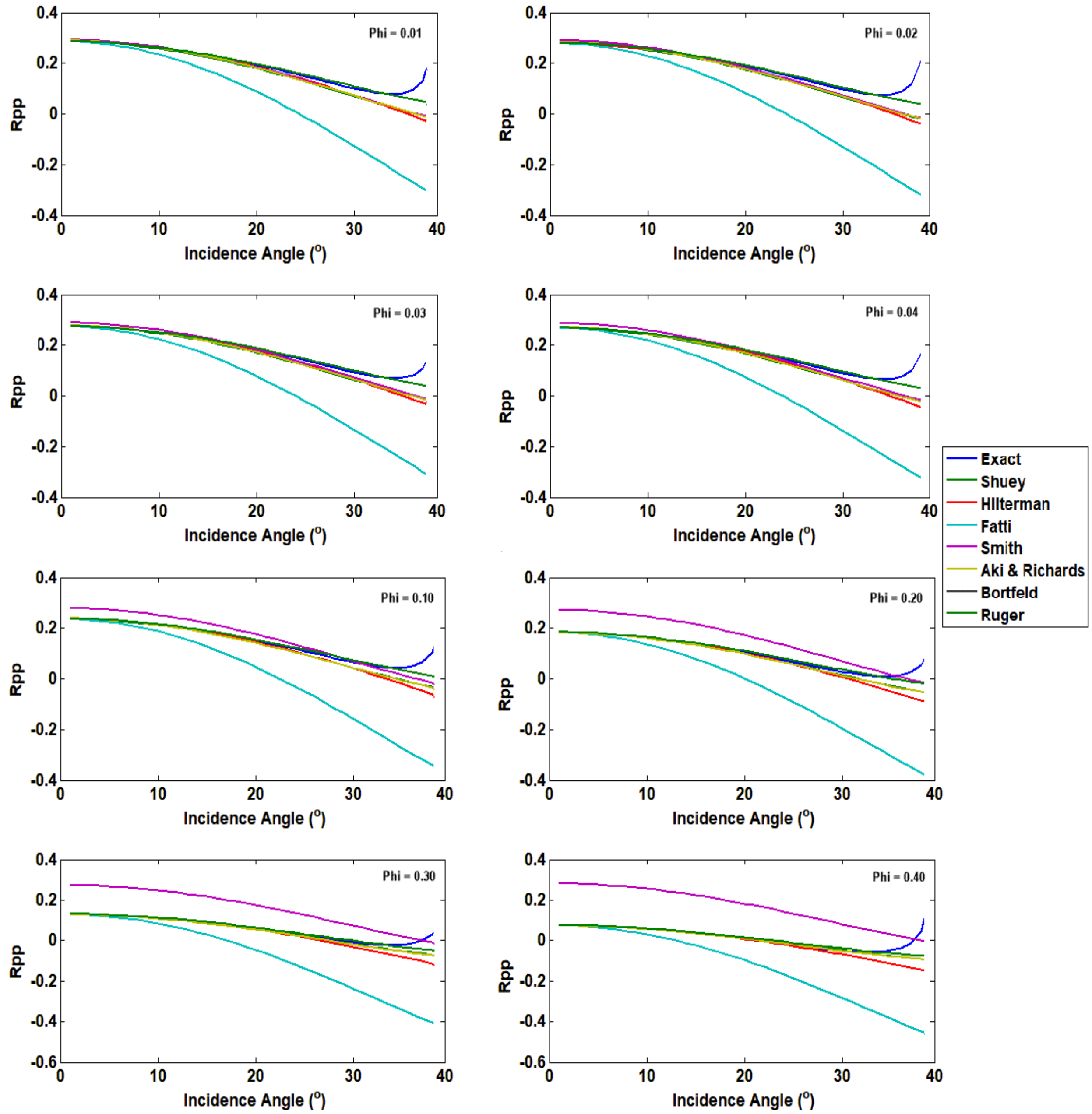

Figure 6: Comparison between the exact Zoeppritz solution, and approximations to the exact solution, for small and high porosity values. All the approximations, except for Fatti and Smith's, are in agreement with Zoeppritz's at small incidence angles (between 0^0 and 20^0), and deviate from it at large incidence angles. The properties of overburden rocks are given in Table 1.

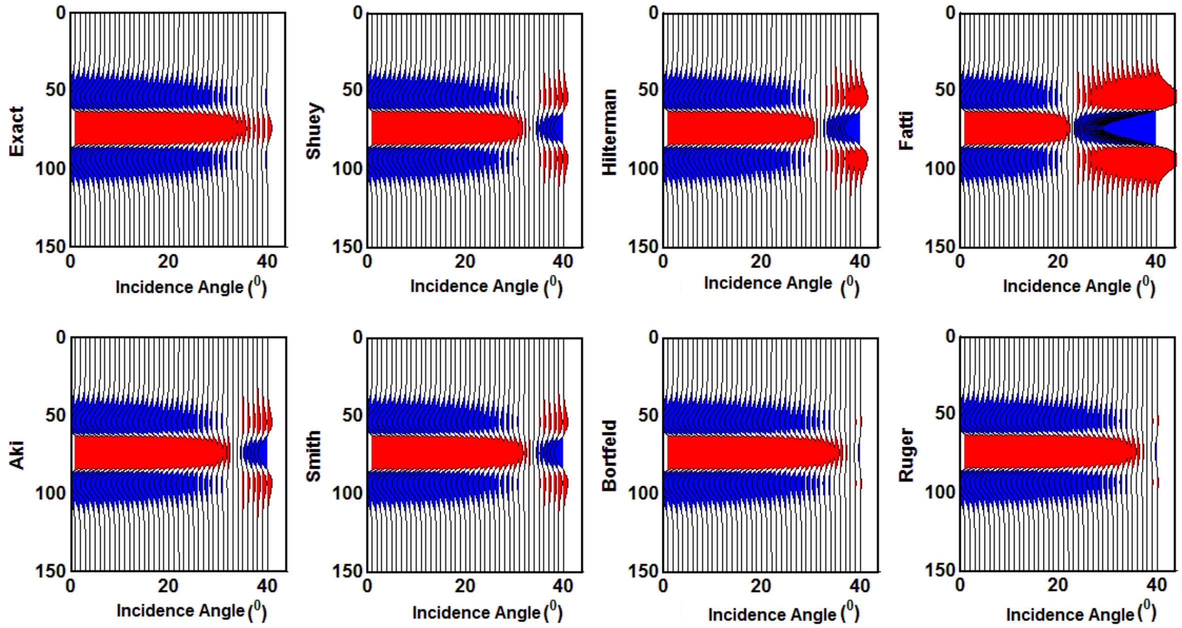

Figure 7: AVA response of the exact Zoeppritz and approximations to the exact solution (Phi-Fraction = 0.10).

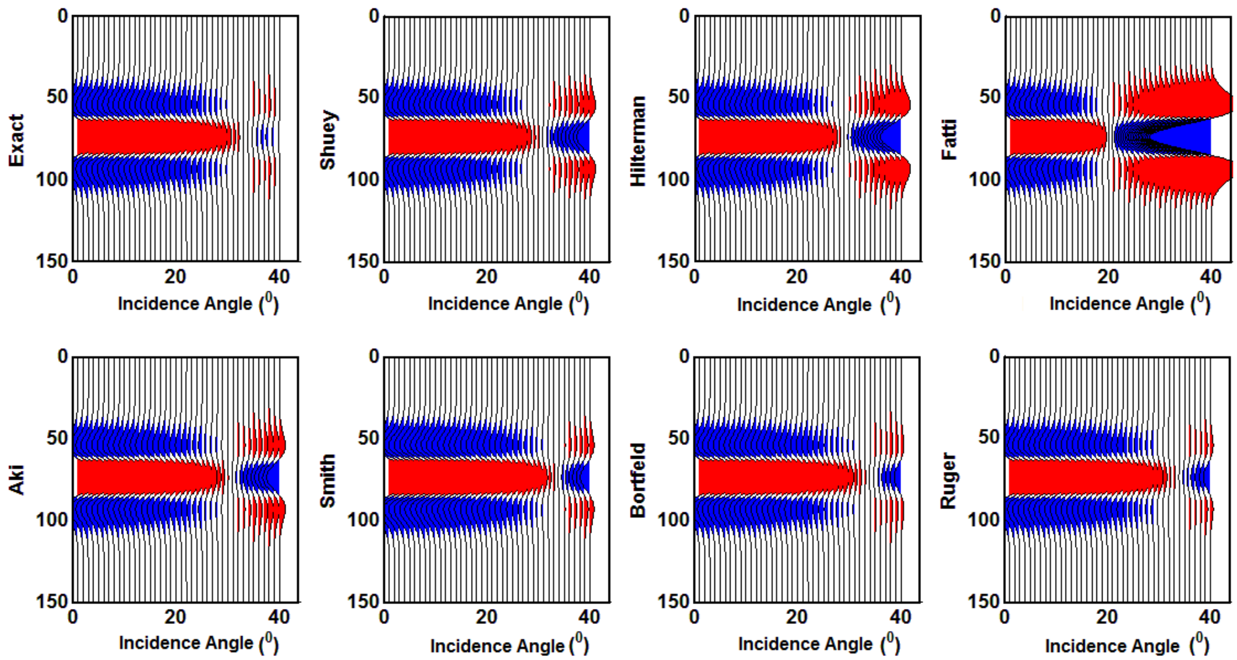

Figure 8: AVA response of the exact Zoeppritz equation and its approximations to the exact solution (Phi-Fraction = 0.20). Every approximation shows polarity reversal at relatively large incident angles.

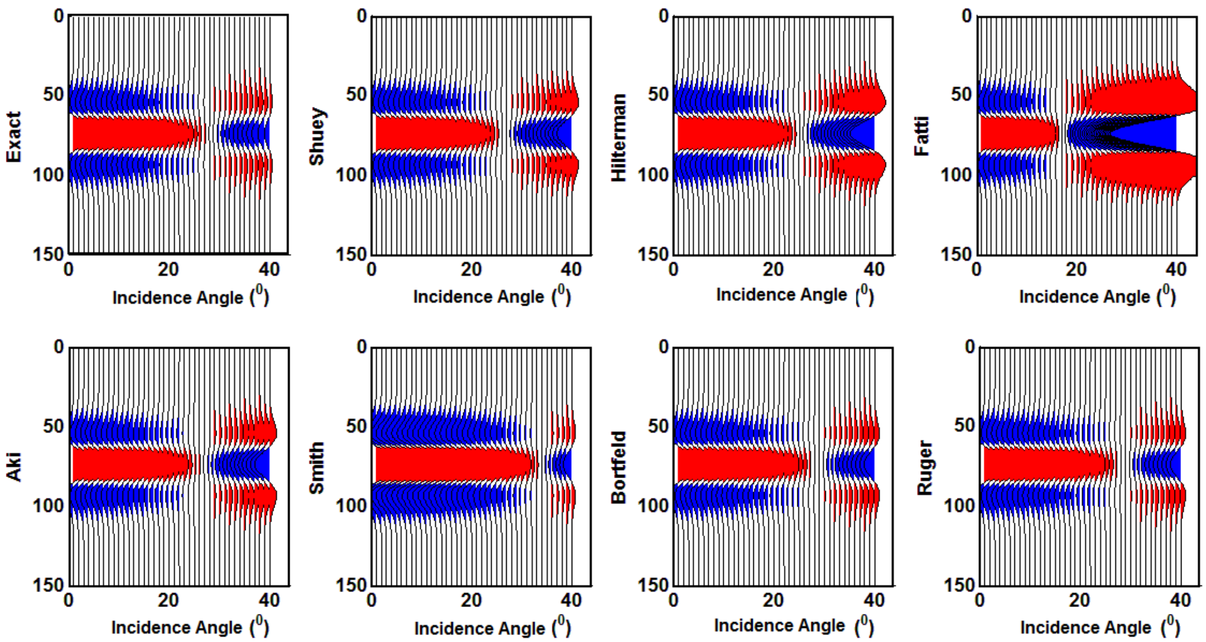

Figure 9: AVA response of the exact Zoeppritz equation and its approximations to the exact solution (Phi-Fraction = 0.30).

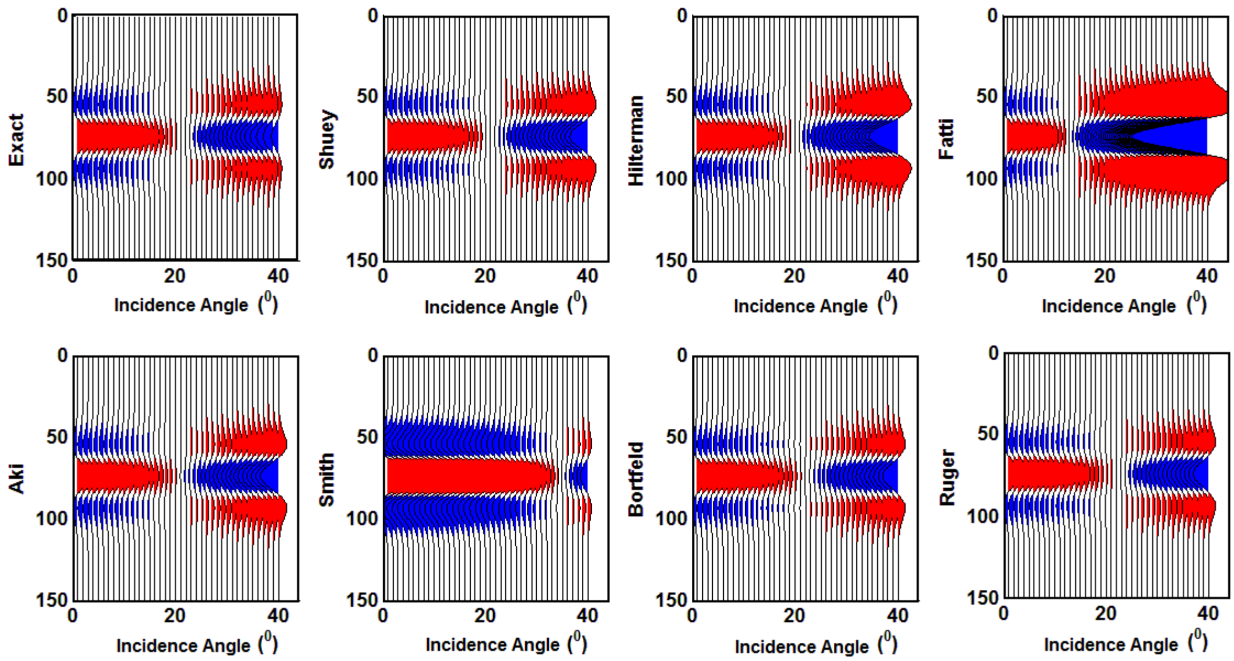

Figure 10: AVA response of the exact Zoeppritz and approximations to the exact solution (Phi-Fraction = 0.40).

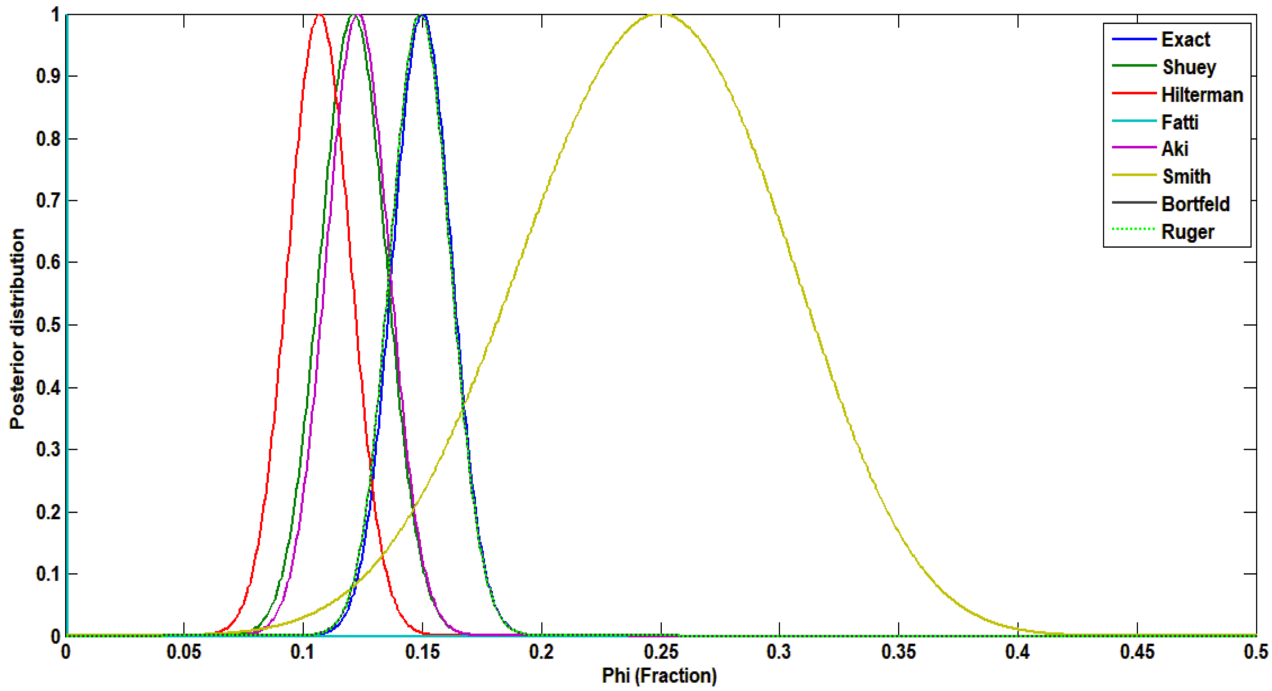

Figure 11: Results of the Bayesian inversion for porosity from angle-dependent reflectivity data (for incident angle ranging between 0^0 and 40^0 degrees) using the exact Zoeppritz equations and its approximations. True porosity is set at 0.15. The Rüger and Bortfeld approximations are in good agreement with the Zoeppritz solution. Aki and Richards, Shuey and Hilterman's approximations underestimate porosity. The uncertainty for Smith's approximation is very large, and Fatti's approximation failed to recover a meaningful value for porosity.

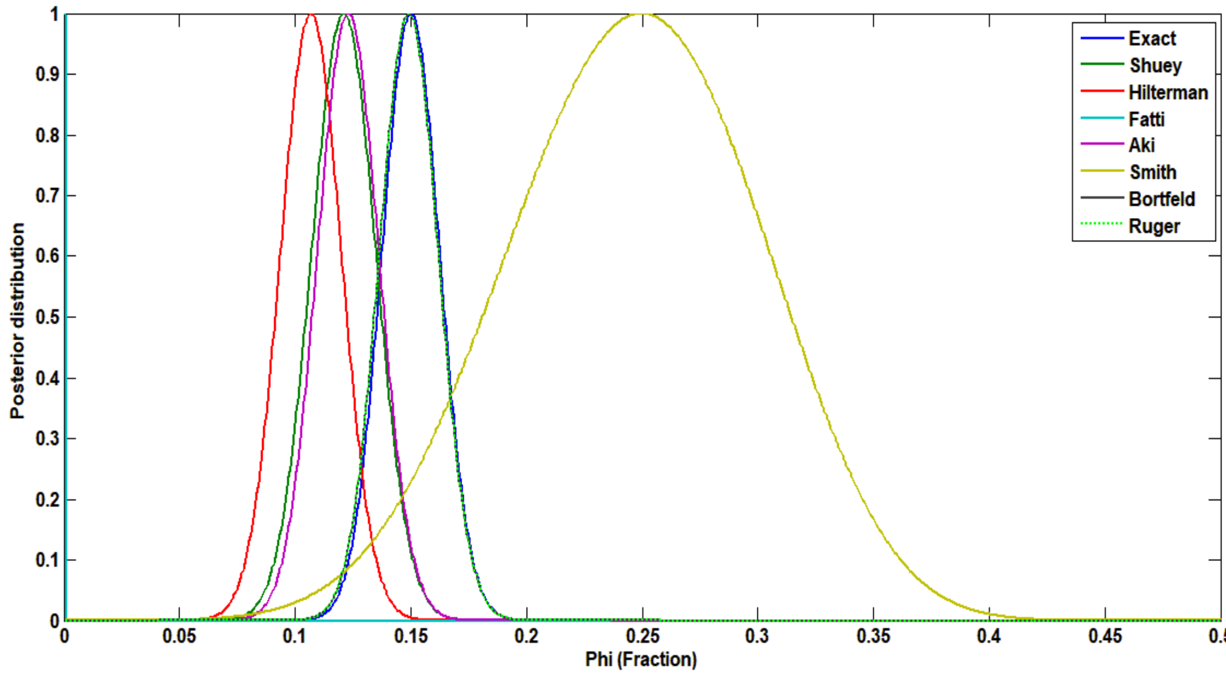

Figure 12: Results of the Bayesian inversion for porosity from synthetic AVA gathers (for incident angle ranging between 0^0 and 40^0 degrees) using the exact Zoeppritz equation and its approximations. True porosity is set at 0.15. The Rüger and Bortfeld approximations are in good agreement with the Zoeppritz solution. Aki and Richards, Shuey and Hilterman's approximations underestimate porosity. The uncertainty for Smith's approximation is very large and Fatti's approximation failed to recover a meaningful value for porosity.

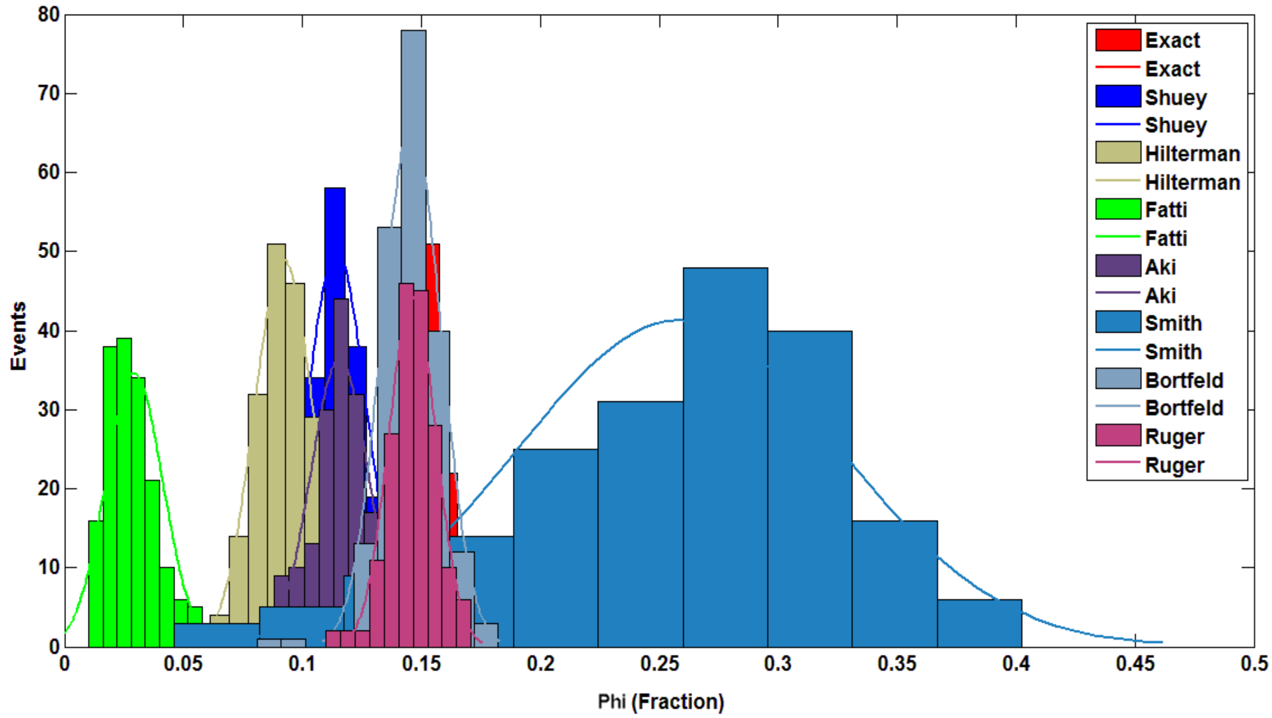

Figure 13: Monte Carlo method for sampling the *a posteriori* distribution for porosity from angle-dependent reflectivity data (incidence angles between 0^0 and 40^0degrees). True porosity is set to 0.15.

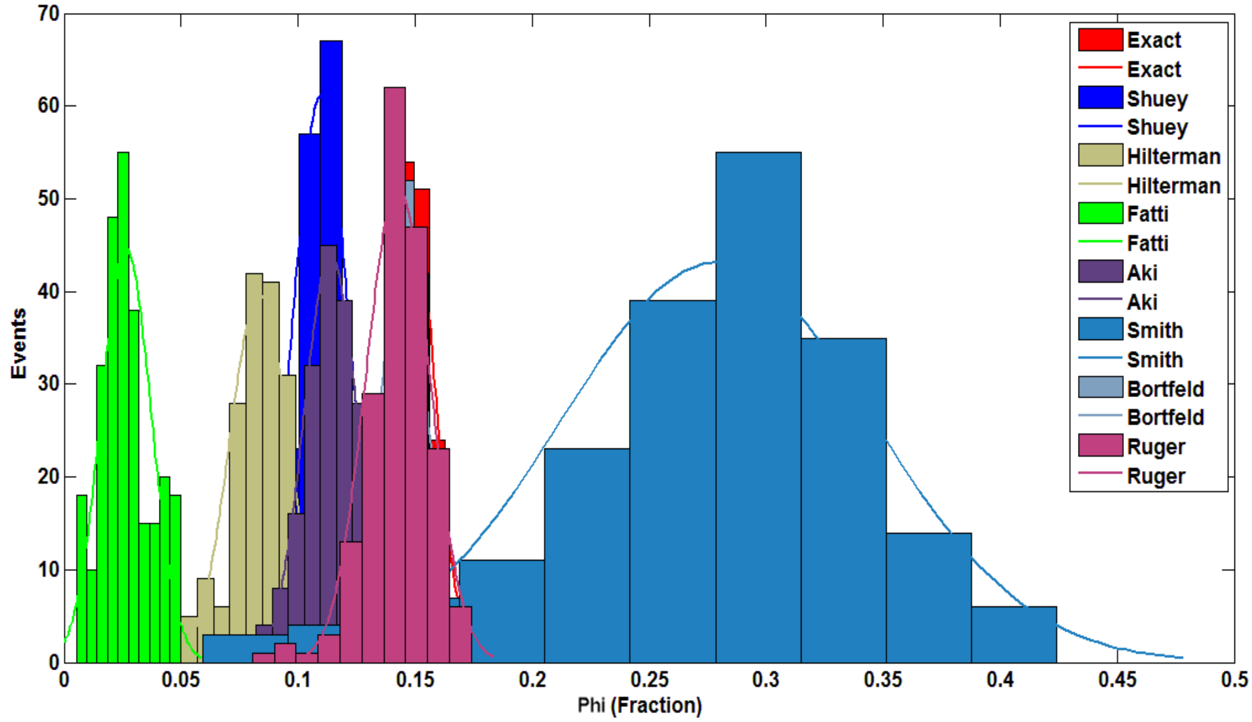

Figure 14: Monte Carlo method for sampling the *a posteriori* distribution for porosity from synthetic AVA gathers (incidence angles between 0^0 and 40^0 degrees). True porosity is set to 0.15.

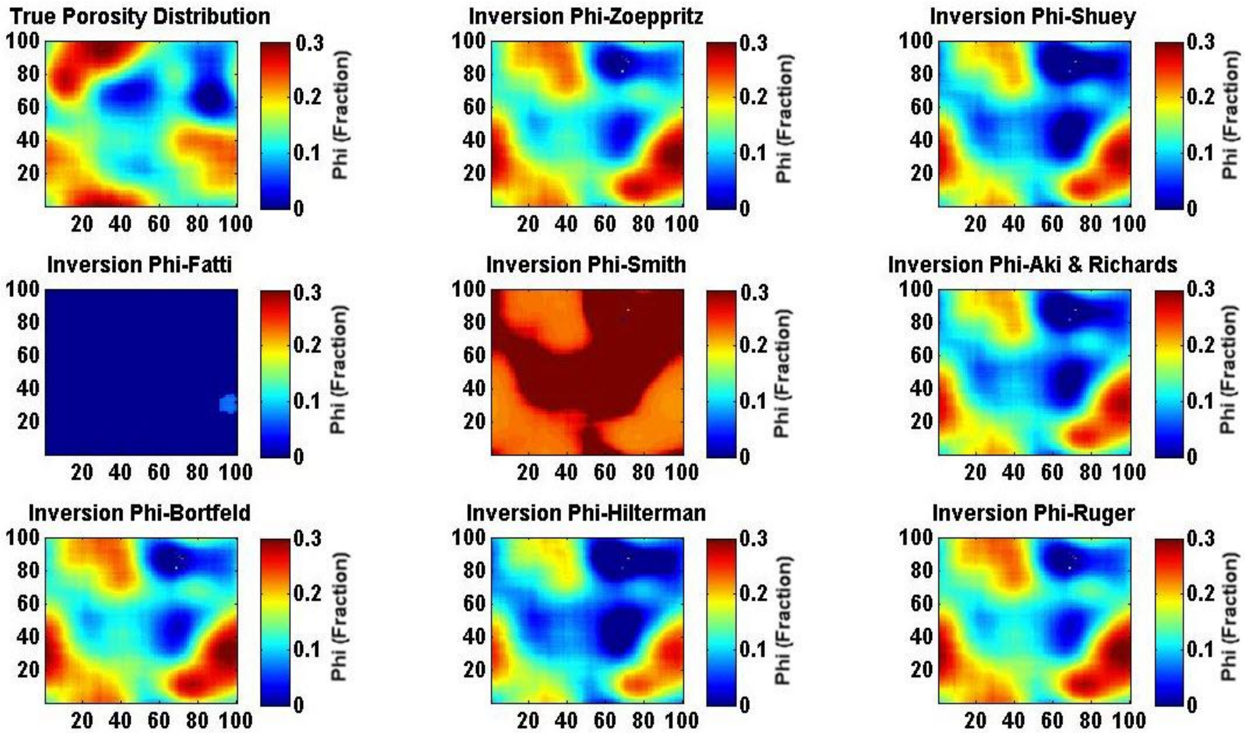

Figure 15: *Maximum a posteriori* solution used to recover the reservoir porosity distribution, for the exact Zoeppritz equation and its approximations under 25% noise settings. Aki and Richards, Bortfeld, Shuey and Rüger's approximations recover reservoir porosity distribution up to some extent, but Fatti and Smith's approximations failed to recover porosity to a satisfactory level.

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