Imperfect Quality Items in Inventory and

Supply Chain Management

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Acknowledgments

ii

Preface

Structured abstract

Motivation: To relax some assumptions embedded in the Economic Order Quantity (EOQ) model in order to enhance inventory control of items with imperfect quality.

Aim and objectives: The aim of this research is to advance the current state of knowledge in the field of inventory mathematical modelling and management by means of providing theoretically valid and empirically viable generalised inventory frameworks to assist inventory managers towards the determination of optimum order/production quantities that minimise the total system cost. The aim is reflected on six main objectives:

- 1) To explore the implications of the inspection process on inventory decision-making and link such process with the management of perishable inventories;
- 2) To derive a general, step-by-step solution procedure for continuous intra-cycle periodic review applications;
- 3) To demonstrate how the terms "deterioration", "perishability" and "obsolescence" may collectively apply to an item;
- 4) To develop a new dispatching policy that is associated with simultaneous consumption fractions from an owned warehouse (OW) and a rented warehouse (RW). The policy developed is entitled "Allocation-In-Fraction-Out (AIFO)";
- 5) To relax the inherent determinism related to the maximum fulfilment of the capacity of OW to maximising net revenue; and
- 6) To assess the impact of learning on the operational and financial performance of an inventory system with a single-level storage and a two-level storage.

Method: A deductive approach is employed to utilise non-linear programming techniques in order to derive the solution procedures for the proposed models.

Contributions: Four general EOQ models for items with imperfect quality are presented. The first model underlies an inventory system with a single-level storage (OW) and the other three models relate to an inventory system with a two-level storage (OW and RW). The three models with a two-level storage underlie the following three dispatching policies, respectively: Last-In-First-Out (LIFO), First-In-First-Out (FIFO) and AIFO.

Implications: The versatile nature of each model allows the consideration of the appropriate demand, screening, defectiveness and deterioration function suitable to a particular case. The inspection process is linked with the management of perishable and non-perishable inventories in order to take into account several practical concerns with regards to product quality related issues. Each model manages and controls the flow of perishable and non-Structured abstract

perishable products so as to reduce cost and/or waste for the benefit of economy, environment and society. General solution procedures to determine the optimal policy for continuous intra-cycle periodic review applications are derived for each model. A detailed method that illustrates how deterioration, perishability and obsolescence may collectively affect inventories is explored. The value of the temperature history and flow time through the supply chain is also used to model the shelf lifetime of an item.

Findings and managerial insights: Relaxing the inherent determinism of the maximum capacity associated with OW, not only produces better results and implies comprehensive learning, but may also suggest outsourcing the inventory holding through vendor managed inventory. In the case of managing perishable products, LIFO and FIFO may not be the right dispatching policies since the total sum of inventory that perishes in each cycle is likely greater than that experienced under the AIFO policy. Under an AIFO policy, a discounted holding cost can be gained if a continuous and long-term rental contract is used and hence further reduction in the total minimum cost can be achieved. Special cases that demonstrate application of the theoretical models in different settings lead to the generation of further interesting managerial insights. The behaviour of time-varying demand, screening and deterioration rates, defectiveness and value of information (VOI) are tested. We find that time-varying rates and VOI significantly impact on the optimal order quantity. The resulting insights offered to inventory managers are thought to be of great value since many of these issues have neither been recognised nor analytically examined before.

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Structured abstract iii

Abstract

The assumption that all items are of good quality is technologically unattainable in most supply chain applications. Moreover, inventory theories are often built upon the assumption that the rates of demand, screening, deterioration and defectiveness are constant and known, even though this is rarely the case in practice. In addition, the classical formulation of a two-warehouse inventory model is often based on the Last-In-First-Out (LIFO) or First-In-First-Out (FIFO) dispatching policy. The LIFO policy relies upon inventory stored in a rented warehouse (RW), with an ample capacity, being consumed first, before depleting inventory of an owned warehouse (OW) that has a limited capacity. Consumption works the other way around for the FIFO policy.

This PhD research aims to advance the current state of knowledge in the field of inventory mathematical modelling and management by means of providing theoretically valid and empirically viable generalised inventory frameworks to assist inventory managers towards the determination of optimum order/production quantities that minimise the total system cost. The aim is reflected on the following six objectives: 1) to explore the implications of the inspection process in inventory decision-making and link such process with the management of perishable inventories; 2) to derive a general, step-by-step solution procedure for continuous intra-cycle periodic review applications; 3) to demonstrate how the terms "deterioration", "perishability" and "obsolescence" may collectively apply to an item; 4) to develop a new dispatching policy that is associated with simultaneous consumption fractions from an owned warehouse (OW) and a rented warehouse (RW). The policy developed is entitled "Allocation-In-Fraction-Out (AIFO)"; 5) to relax the inherent determinism related to the maximum fulfilment of the capacity of OW to maximising net revenue; and 6) to assess the impact of learning on the operational and financial performance of an inventory system with a two-level storage. Four general Economic Order Quantity (EOQ) models for items with imperfect quality are presented. The first model underlies an inventory system with a singlelevel storage (OW) and the other three models relate to an inventory system with a two-level storage (OW and RW). The three models with a two-level storage underlie, respectively, the LIFO, FIFO and AIFO dispatching policies. Unlike LIFO and FIFO, AIFO implies simultaneous consumption fractions associated with RW and OW. That said, the goods at both warehouses are depleted by the end of the same cycle. This necessitates the introduction of a key performance indicator to trade-off the costs associated with AIFO, LIFO and FIFO. Each lot that is delivered to the sorting facility undergoes a 100 per cent screening and the percentage of defective items per lot reduces according to a learning curve. The mathematical formulation reflects a diverse range of time-varying forms.

The behaviour of time-varying demand, screening and deterioration rates, defectiveness, and value of information (VOI) are tested. Special cases that demonstrate application of the theoretical models in different settings lead to the generation of interesting managerial insights. For perishable products, we demonstrate that LIFO and FIFO may not be the right dispatching policies. Further, relaxing the inherent determinism of the maximum capacity associated with OW, not only produces better results and implies comprehensive learning, but may also suggest outsourcing the inventory holding through vendor managed inventory.

Abstract iV

List of acronyms

AIFO	Allocation-In-Fraction-Out
EOQ	Economic order quantity
EPQ	Economic production quantity
FEFO	First-Expired-First-Out
FIFO	First-In-First-Out
KPI	KEY performance indicator
LHS	Left-hand side
LIFO	Last-In-First-Out
OR	Operational Research
OW	Owned warehouse
RFID	Radio-frequency identification
RHS	Right-hand side
RW	Rented warehouse
TTH	Time and temperature history
VMI	Vendor managed inventory
VOI	Value of Information
w.r.t	With respect to

List of acronyms

Notations and symbols¹

Chapter 4	
j	Cycle index
D(t)	Demand rate per unit time
x(t)	Screening rate per unit time
$\delta(t)$	Deterioration rate per unit time
	$g(t) = \int \delta(t) dt \& G(t) = \int e^{-g(t)} dt$
p_j	Percentage of defective items per lot
С	Unit purchasing cost
d	Unit screening cost
h_g	Holding cost of good items per unit per unit time
h_d	Holding cost of defective items per unit per unit time
k	Ordering cost per cycle
Q_j	Lot size delivered for cycle <i>j</i>
$T_{1j} = f_{1j}(Q_j)$	Time to screen Q_j units
$T_{2j} = f_{2j}(Q_j)$	Cycle length
$I_{gj}(t)$	Inventory level of good items at time t
$I_{dj}(t)$	Inventory level of defective items at time t
W	Total cost per unit time
w	Total cost per cycle
L	

Notations and symbols Vi

$w_{Q_j}^{'}$	Derivative of \boldsymbol{w} with respect to \boldsymbol{Q}_j
$f_{2j,Q_j}^{'}$	Derivative of \boldsymbol{f}_{2j} with respect to \boldsymbol{Q}_j
ω_j	Number of deteriorated items for cycle <i>j</i>
	Chapter 5 – Perishable items
$\underline{Q_j} = \left(q_{mj}, q_{m-1j}, \dots, q_{0j}\right)$	Lot size delivered for cycle <i>j</i>
q_{ij}	Number of units with $i(i=0,1,\ldots,m)$ useful periods of shelf lifetime
$q_{0j} = p_j Q_j$	Newly replenished items that have arrived already perished or items not satisfying certain quality standards (defective items)
ω_{ij}	Quantity of the on-hand inventory of shelf lifetime i that perishes by the end of period i
D_{ij}	Actual demand observed up to the periodic review \emph{i}
d_{ij}	Number of items of shelf lifetime $\it i$ that deteriorate while on storage
q_{0sj}	Number of defective items isolated up to the periodic review $\it i$
q_{0rj}	Number of defective items remaining after the review
Δ	Lead-time
$^{\circ}C_{y}$	Temperature of an item in a supply chain entity y
t_y	Time elapsed of an item in a supply chain entity y
M	Remaining shelf lifetime in a supply chain
L	Remaining shelf lifetime in a supply chain with VOI

Notations and symbols Vii

$I_{rgj}(t) \hspace{1cm} \text{Inventory level of good items at time t in RW}$ $I_{rdj}(t) \hspace{1cm} \text{Inventory level of defective items at time t in RW}$ $I_{ogj}(t) \hspace{1cm} \text{Inventory level of good items at time t in OW}$ $I_{odj}(t) \hspace{1cm} \text{Inventory level of defective items at time t in OW}$ $\delta_y(t) \hspace{1cm} \text{Deterioration rate per unit time}$ $g_y(t) = \int \delta_y(t) dt \& G_y(t) = \int e^{-g_y(t)} dt , y = o, r$	
$I_{rdj}(t)$ Inventory level of defective items at time t in RW $I_{ogj}(t)$ Inventory level of good items at time t in OW $I_{odj}(t)$ Inventory level of defective items at time t in OW $\delta_y(t)$ Deterioration rate per unit time	
$I_{ogj}(t)$ Inventory level of good items at time t in OW $I_{odj}(t)$ Inventory level of defective items at time t in OW $\delta_y(t)$ Deterioration rate per unit time	
$I_{odj}(t)$ Inventory level of defective items at time t in OW $\delta_y(t)$ Deterioration rate per unit time	
$\delta_y(t)$ Deterioration rate per unit time	
$g_{y}(t) = \int \delta_{y}(t) dt \& G_{y}(t) = \int e^{-g_{y}(t)} dt, y = o, r$	
$Q_{ij} = q_{rij} + q_{oij}$ Lot size delivered for cycle j for $i = L, F, A$	
L = LIFO, F = FIFO and $A = AIFO$	
$q_{\it oij}$ and $q_{\it rij}$ Sub-replenishment delivered to OW and RW	
$T_{rj} = f_{rj}(q_{rj})$ Screening time of items stored in RW	
$T_{oj} = f_{oj}(q_{rj})$ Screening time of items stored in OW	
$T_{Rj} = f_{Rj}(q_{rj})$ Depleting time of items stored in RW	
This time also represents the cycle length for FIFO	
$T_j = f_j(q_{rj})$ Depleting time of items stored in OW	
This time also represents the cycle length for LIFO a	nd AIFO
h_{rg} Holding cost of good items per unit per unit time for	r RW
h_{rd} Holding cost of defective items per unit per unit tim	e for RW
h_{og} Holding cost of good items per unit per unit time for	r OW
h_{od} Holding cost of defective items per unit per unit time for	OW

Notations and symbols
Viii

W_i	Total cost per unit time for $i = L, F, A$
w_i	Total cost per cycle for $i = L, F, A$
S_0	Unit transportation cost for OW
S_r	Unit transportation cost for RW
$\emptyset_{oj} = \emptyset_j(q_{rj})$	Fraction of the demand satisfied from OW for AIFO
$\emptyset_{rj} = 1 - \emptyset_{oj}$	Fraction of the demand satisfied from RW for AIFO
c_L	Charge payable per unit time if RW remains idle for LIFO
c_F	Cost incurred per unit time if OW remains idle for FIFO
Δ_{ij}	KPI, i.e. an upper-bound (cost applied if OW (RW) is idle) for $i=L,F$ that renders LIFO or FIFO the optimal dispatching policy
ω_{rkj}	Quantity of the on-hand inventory of shelf lifetime k that perishes by the end of period k in RW
ω_{okj}	Quantity of the on-hand inventory of shelf lifetime k that perishes by the end of period k in OW
Т	Remaining shelf lifetime in a supply chain with VOI

Notations and symbols jX

¹All other notations and symbols (that are not included in the list) are solely used (but not elsewhere) for the propose of some cases (implications) as they are presented and identified in the thesis.

Definition of key terms used in the thesis

We provide below a summary of definitions of some key terms used in this PhD thesis. This is to ensure clarity and avoid any potential ambiguities as to the meaning of those terms.

Quality refers to the degree to which an item satisfies the expected standards or value characteristics. That is, the quality of an item is determined by: 1) the degree to which explicit characteristics related to its physical status are satisfied; 2) changes in its value as perceived by the customer; or 3) a risk of reduction of its future functionality/desirability. In this PhD thesis, quality is associated with, and reflected on defectiveness, deterioration, perishability and obsolescence of an item.

Perishability refers to the state of an item with a fixed lifetime (expiration date) exceeding its maximum shelf lifetime and thus it must be discarded. This refers to the degree to which explicit characteristics related to an item's physical status are not satisfied.

Defectiveness refers to the state of newly replenished items that are found by inspection to be either already perished or not satisfying certain quality standards. This refers to the degree to which explicit characteristics related to an item's physical status are not satisfied.

Deterioration indicates the process of decay, damage or spoilage of a product, i.e. the product loses its value characteristics and can no longer be sold/used for its original purpose. This refers to the degree to which explicit characteristics related to an item's physical status are not satisfied and/or there are changes in its value as perceived by the customer and/or there is a risk of reduction of its future functionality/desirability.

Obsolescence refers to items incurring a partial or a total loss of value in such a way that the value for a product continuously decreases with its perceived utility/desirability. This refers to the changes in its value as perceived by the customer and/or a risk of reduction of its future functionality/desirability.

Last-In-First-Out (LIFO²) is a dispatching policy according to which inventory stored in a Rented Warehouse (RW), with ample capacity, is consumed first, before depleting inventory of an Owned Warehouse (OW) that has limited capacity.

First-In-First-Out (FIFO²) is a dispatching policy according to which inventory stored in an OW is consumed first, before depleting inventory of a RW.

Allocation-In-Fraction-Out (AIFO) is a dispatching policy that implies simultaneous consumption fractions associated with RW and OW. That said, the goods at both warehouses are depleted by the end of the same cycle.

First-Expired-First-Out (FEFO) is a dispatching policy according to which an item with fewer useful periods of shelf lifetime is depleted first, before consuming the one with longer useful periods of shelf lifetime.

²The terms LIFO and FIFO are often associated with cost accounting, and indeed there is a considerable amount of research that has been conducted in this area. However, for the purposes of this PhD thesis, these terms relate only to the two-warehouse inventory problem and are solely used to indicate which warehouse is being utilised first.

Table of Contents

Acknowle	edgmentsi
Preface	ii
Structu	red abstractii
Abstrac	tiv
List of a	cronymsv
Notatio	ns and symbols vi
Definiti	on of key terms used in the thesis x
Part A: EC	OQ model for imperfect quality Items 1 -
1. Intro	oduction 2 -
1.1.	Research background 2 -
1.2.	Aim and objectives 5 -
1.3.	Research motivation and contribution 6 -
1.4.	Methodology 8 -
1.5.	Thesis structure 10 -
2. Liter	ature review 13 -
2.1.	Inventory quality issues 13 -
2.1.1	. Perishability and lifetime constraints 14 -
2.1.2	. Deterioration 15 -
2.2.	Information sharing and inspection process 18 -
2.2.1	. Value of information (VOI) 18 -
2.2.2	. Learning effects 19 -
2.2.3	. Inspection process 21 -
2.3.	Inventory models with imperfect quality items 22 -

xiii

2.3.	1. Single-warehouse model	23 -
2.3.	2. Single-warehouse model with learning	24 -
2.3.	3. Vendor–buyer supply chain modelling	25 -
2.4.	Two-warehouse model	27 -
2.5.	Summary	30 -
3. Res	earch methodology	33 -
3.1.	Introduction	33 -
3.2.	Epistemological and ontological orthodoxies	
3.3.	Research methods	
3.4.	Validity and reliability of the research	
3.5.	Summary	
Part B: L	ot size inventory model with one level of storage	41 -
4. A g	eneral EOQ model for imperfect quality items	42 -
4.1.	Introduction	42 -
4.2.	Need for the research	43 -
4.3.	Formulation of the general EOQ model	44 -
4.3.	1. Assumptions and notation	44 -
4.3.	2. The model	45 -
4.4.	Solution procedures	49 -
4.5.	Illustrative examples for different settings	51 -
4.5.	Varying demand, screening, defectiveness and deterioration rates	51 -
4.5.	2. Sensitivity analysis	55 -
4.5.	3. Findings	58 -
4.6.	Conclusion and further research	59 -
5. Spe	cial cases of the general EOQ model	62
Table of Con	-	- 02 - viii

	5.1.	Introduction	- 62 -
	5.2.	Intra-cycle periodic review	- 63 -
	5.2.1	1. Solution procedure	- 63 -
	5.2.2	2. Numerical verification	- 66 -
	5.3.	Perishable products	- 67 -
	5.3.1	1. The model	- 68 -
	5.3.2	2. Numerical verification	- 71 -
	5.3.3	3. Time and temperature history (TTH)	- 72 -
	5.4.	Renewal theory	- 74 -
	5.5.	Coordination mechanisms	- 76 -
	5.6.	Stochastic parameters	- 78 -
	5.7.	A 100 per cent inspection and sampling test	- 79 -
	5.8.	Further implications	- 80 -
	5.9.	Summary of implications and managerial insights	- 80 -
	5.10.	Conclusion and further research	- 82 -
Ρ	art C: Lo	ot size inventory model with two levels of storage	- 85 -
_			
6		eneral EOQ model for imperfect quality items under LIFO dispatching policy	
	6.1.	Introduction	
	6.2.	Need for the research	- 88 -
	6.3.	Formulation of the general model under LIFO dispatching policy	- 89 -
	6.3.1	1. Assumptions and notation	- 89 -
	6.3.2	2. The model	- 91 -
	6.4.	Solution procedures	- 97 -
	6.5.	Numerical analysis	- 99 -
	6.5.1	1. Varying rates	100 -

	6.5.2	2.	Sensitivity analysis	101 -
	6.5.3	3.	Findings	103 -
	6.6.	Con	clusion and further research	104 -
7.	A ge	enera	al EOQ model for imperfect quality items under FIFO dispatching p	olicy 107 -
	7.1.	Intro	oduction	107 -
	7.2.		d for the research	
	7.3.	Forr	nulation of the general model under FIFO dispatching policy	109 -
	7.3.1	1.	Assumptions and notation	109 -
	7.3.2	2.	The model	109 -
	7.4.	Solu	tion procedures	114 -
	7.5.	Illus	trative examples for different settings	115 -
	7.5.1	1.	Varying demand, screening, defectiveness, and deterioration rates	115 -
	7.5.2	2.	Sensitivity analysis	117 -
	7.5.3	3.	Findings	119 -
	7.6.	Con	clusion and further research	120 -
8.	Gen	eral	EOQ models for imperfect quality items under LIFO, FIFO and AIFO	כ
di	spatchi	ing p	olicies	123 -
	8.1.	Intro	oduction	123 -
	8.2.	Nee	d for the research	124 -
	8.3.	Forr	nulation of the general models	129 -
	8.3.1	1.	Assumptions and notation	129 -
	8.4.	AIFO	O dispatching policy	131 -
	8.5.	Solu	ition procedures	134 -
	8.6.	LIFO	dispatching policy	136 -
	8.7.	FIFO	dispatching policy	137 -

8.8.	Nun	nerical analysis and special cases 1	138 -
8.8	.1.	Formulation of the upper-bound 1	138 -
8.8	.2.	Varying rates 1	139 -
8.8	.3.	AIFO vs. LIFO/FIFO 1	142 -
8.8	.4.	Sensitivity analysis 1	144 -
8.8	.5.	Findings 1	151 -
8.9.	Spe	cial cases of the general EOQ models 1	153 -
8.9.	.1.	Perishable products and lifetime constraints - 1	154 -
8.9.	.2.	Stochastic parameters - 1	156 -
8.10.	Sum	nmary of implications and managerial insights 1	157 -
8.11.	Con	clusion and further research 1	159 -
Part D: (Concl	usion1	L 62 -
9. Sur	mmar	y of contributions and further research1	163 -
9.1.		oduction1	
9.1.		dels overview 1	
9.3.		size inventory model with one level of storage 1	
9.3		Research contribution 1	
9.3		Key Findings - 1	
9.3.		Implications and managerial insights - 1	
9.4.	Lot	size inventory model with two levels of storage 1	
9.4.	.1.	Research contribution 1	174 -
9.4	.2.	Key Findings - 1	176 -
9.4	.3.	Implications and managerial insights 1	178 -
9.5.	Furt	her research 1	179 -
Referen	ces	1	81 -

Appendix A. EOQ model with one level of storage2	.11 -
Appendix B. EOQ model with two levels of storage (LIFO)2	:15 -
Appendix C. EOQ model with two levels of storage (FIFO)2	:17 -
Appendix D. EOQ model with two levels of storage (AIFO)2	19 -

Table of Contents

List of Figures

Fig.1.1 Thesis structure 12 -
Fig. 4.1. Inventory variation of an Economic Order Quantity (EOQ) model for one cycle 47
-
Fig. 4.2. The effect of each additional model parameter on the Economic Order Quantity
(EOQ) 56 -
Fig. 4.3. EOQ with same and differing holding costs 57 -
Fig. 4.4. A comparison of the optimal lot sizes 57 -
Fig. 6.1. Inventory variation of the two-warehouse model during one cycle when $T_{oj} \leq$
<i>T_{Rj}</i> (LIFO) 92 -
Fig. 6.2. Inventory variation of the two-warehouse model during one cycle when $T_{Rj} < T_{oj}$
(LIFO) 95 -
Fig. 7.1. Inventory variation of the two-warehouse model during one cycle when $T_{rj} \leq T_j$
(FIFO) 110 -
Fig. 7.2. Inventory variation of the two-warehouse model during one cycle when $T_j < T_{rj}$
(FIFO) 111 -
Fig. 8.1. Inventory variation of the two-warehouse model during one cycle (LIFO) 126 -
Fig. 8.2. Inventory variation of the two-warehouse model during one cycle (FIFO) 126 -
Fig. 8.3. Inventory variation of the two-warehouse model during one cycle (AIFO) 127 -
Fig. 8.4. Inventory variation of the two-warehouse model during one cycle (AIFO) 131 -

Fig. 8.5. A comparison of the optimal lot sizes of AIFO and LIFO for S-shaped and Power
earning curves 150
Fig. 8.6. A comparison of the maximum rental cost per year for S-shaped and Power
earning curves150

List of Tables

Table of Contentsxii
Table 4.1. Input parameters for Example 4.1 53 -
Table 4.2. Optimal results for varying demand, screening and deterioration rates 54 -
Table 4.3. Sensitivity analysis for the general model 56 -
Table 5.1. Input parameters for Example 5.2 71 -
Table 5.2. Input parameters for comparison examples for renewal theory 75 -
Table 6.1. Input parameters for example 6.1 100 -
Table 6.2. Sensitivity analysis for the general model 102 -
Table 6.3. The effect of Wright's learning curve on the optimal values of the general
model 102 -
Table 7.1. Input parameters for example 7.1 116 -
Table 7.2. Sensitivity analysis for the general model 118 -
Table 7.3. The effect of Wright's learning curve on the optimal values of the general
model 118 -
Table 8.1. Input parameters for example 8.1 140 -
Table 8.2. Optimal results for varying demand, screening, defectiveness and deterioration
rates 141 -
Table 8.3. Sensitivity analysis for transportation costs 145 -
Table 8.4. Sensitivity analysis for holding costs 146 -

Table 8.5. Sensitivity analysis for deterioration rates	147 -
Table 8.6. Sensitivity analysis for special cases of the general models	148 -
Table 8.7. The effect of Wright's learning curve on variable capacity of the OW	149 -

List of Tables

Part A: EOQ model for imperfect quality Items

This part contains three chapters. The first chapter presents a general introduction, followed by a literature review, where we³ provide a summary of research gaps that are found in the literature to shape the objectives of this PhD thesis. The third chapter advocates our epistemological position.

The introductory chapter discusses the research background, sets the aim and objectives of the research, states the motivation for conducting this research and summarises its contribution and briefly introduces the research methodology embraced in the study. It closes with escribing the structure of this PhD thesis.

The literature review chapter is organised around four main streams of research: 1) inventory quality related issues; 2) information sharing and inspection processes; 3) model formulations and related solution techniques that consider imperfect quality items; and 4) lot size inventory modelling with two levels of storage.

The third chapter is dedicated to discussing our epistemological position and stating, in detail, the research methodology embraced in the study.

Part A - 1 -

³The use of word "we" throughout the thesis is purely conventional. The work discussed in this thesis has been developed by the author only, albeit with support from his University (supervisory team).

1. Introduction

This introductory chapter aims to outline the research background, set the aim and objectives of the research, state the motivation for undertaking this research, discuss the research methodology embraced in the study and introduce the structure of the thesis.

1.1. Research background

The goal of supply chain management is best described as obtaining the right commodity in the right quantities to the right place at the right time, the first time. This necessitates coordination mechanisms that integrate supply chain entities, such as suppliers, manufacturers, wholesalers/distributors and retailers, in order to satisfy service level requirements, while minimising system-wide costs (Chopra and Meindl, 2007; Simchi-Levi et al., 1999).

In today's competitive markets, supply chains cannot tolerate process failures and, therefore, the dimensions of risk among inter-related business entities must be recognised. One of the elements related to that risk is the amount of inventories that companies must hold in order to be responsive to market needs. Ordering excessive inventory reduces ordering cost and may reduce purchase cost, but it may also tie up capital, which may lead to unnecessary holding cost and products that may deteriorate. On the other hand, ordering too little inventory reduces the holding cost, but can result in lost sales and, consequently affect the reliability of the operation of an inventory system. Therefore, one fundamental problem frequently encountered in this field is the determination of when products are ordered and

Introduction - 2 -

how many products will be ordered per order cycle. This constitutes the core of inventory control problems.

According to the 22nd Annual State of Logistics Report published in 2012, the world is sitting on approximately eight trillion dollars' worth of goods held for sale. The amount of money tied up in inventories has implications, not only for the financial state of organisations and supply chains, but also for national and international economies. However, the broad spectrum of supply chain management makes it impossible for a single existing theory to adequately capture all aspects of the relevant processes and the inventory problems associated with them. Accordingly, the extent of these problems is dependent upon the type of inventory system that each entity adopts. In particular, from an Operational Research (OR) perspective, solving the inventory problem entails building mathematical models which explain inventory fluctuations over planning horizons.

Since the introduction of the Economic Order Quantity (EOQ) model by Harris (1913), frequent contributions have been made in the literature towards the development of alternative models that overcome the unrealistic assumptions embedded in the EOQ formulation (Glock et al., 2014). One of the unrealistic assumptions underlying the EOQ model is that all items are of good quality. In practice, this assumption is technologically unattainable in most supply chain applications, as defective items may affect the operational and financial performance of an inventory system (Chan et al., 2003; Cheng, 1991; Khan et al., 2011; Pal et al., 2013; Salameh and Jaber, 2000).

The complexity and drivers associated with product waste and loss have been increasingly discussed in the academic literature and include such issues as imperfect quality items (that necessitate an inspection to take place at various supply chain stages to ensure the quality of

Introduction - 3 -

the product is adequate) (Gunders, 2012). For example, in the food and drink industry, different proportions of food waste are attributed to different stages in the supply chain, from production to handling and storage, processing and packaging, distribution and retail, and finally at the household consumption stage. In particular, the fresh meat sector has been identified as the largest producer of waste and accounts overall for 25 per cent of the waste, ahead of fruit and vegetables at 13 per cent (WRAP, 2012a). The waste and spoilage related to inventory decisions represent a large proportion, and it is estimated that around 10 per cent of all perishable goods are spoiled before they reach consumers (Roberti, 2005; Tortola, 2005; Boyer, 2006). WRAP (2012b) published that "5-25 per cent of fruit and vegetable crop might not get through the supply chain to retail customers". For example, in the onion supply chain, losses related to grading account for 9-20 per cent; storage 3-10 per cent and in the packing process they equate to 2-3 per cent loss (WRAP (2012b). The main causes of waste in these examples relate to product specification, product deterioration and reliance on (excessive) storage to cope with fluctuations in actual and/or forecasted demand.

EOQ models are associated with another implicit assumption that stored items may retain the same utility indefinitely, i.e. they do not lose their value as time goes on. This assumption may be valid for certain items. However, real-life systems analysis suggests that goods are subject to "obsolescence", "perishability" and "deterioration" that have a direct impact on the flow of an item as it moves through the supply chain (Goyal and Giri, 2001b; Bakker et al., 2012; Pahl and Voß, 2014). Common examples are packaged foods, seafood, fruit, cheese, processed meat, pharmaceutical, agricultural or chemical products that are transported over long distances in refrigerated containers, where temperature variability has a significant impact on product shelf lifetime (Doyle, 1995; Koutsoumanis et al., 2005; Taoukis et al., 1999).

Introduction - 4 -

The product shelf lifetime also depends on various environmental factors, such as the product's temperature history, humidity, transportation and handling (Ketzenberg et al., 2015). Further, increases in the time products are being stored, as well as changes in the environment of the storage facilities (e.g. temperature storage and controlled atmosphere storage), may result in an increase (or decrease) of the deterioration rate of certain commodities. The sole and/or collective impact of defectiveness, deterioration, perishability and obsolescence on goods is an important factor in any inventory and production system. This means that the identification of an appropriate ordering policy is an essential but challenging task.

Finally, an important issue involved in decision making in this area is whether we refer to a single storage facility (often termed as 'Owned Warehouse', OW) or dual storage facility (that in addition to the OW also involved a 'Rented Warehouse', RW). As will be discussed later in the thesis, this is an important factor both for modelling and real-world decision-making purposes.

1.2. Aim and objectives

On the one hand, inventory management is a field which has been relatively mature for several decades, but on the other hand, there is no single existing theory that can adequately capture all aspects of the relevant processes and the inventory problems associated with them. Therefore, making a contribution that scholars would deem "significant" is not an easy task.

This study aims to advance the current state of knowledge in the field of inventory

Introduction - 5 -

mathematical modelling and management by means of providing theoretically valid and empirically viable generalised inventory frameworks to assist inventory managers towards the determination of optimum order/production quantities that minimise the total system cost. The aim is reflected on six main objectives:

- 1) To explore the implications of the inspection process on inventory decision-making and link such process with the management of perishable inventories;
- 2) To derive a general, step-by-step solution procedure for continuous intra-cycle periodic review applications;
- 3) To demonstrate how the terms "deterioration", "perishability" and "obsolescence" may collectively apply to an item;
- 4) To develop a new dispatching policy that is associated with simultaneous consumption fractions from an owned warehouse (OW) and a rented warehouse (RW). The policy developed is entitled "Allocation-In-Fraction-Out (AIFO)";
- 5) To relax the inherent determinism related to the maximum fulfilment of the capacity of OW to maximising net revenue; and
- 6) To assess the impact of learning on the operational and financial performance of an inventory system with a single-level storage and a two-level storage.

1.3. Research motivation and contribution

Although, the literature related to the formulation of EOQ models is quite mature, the

Introduction - 6 -

inventory formulation may still have space for further contributions. For example, inventory theories are often built upon the assumption that the rates of demand, screening, deterioration and defectiveness. are constant and known, even though this is rarely the case in practice. Moreover, even if those rates are stochastic, the key parameters (moments) of the relevant distribution(s), typically the mean and variance, are assumed to be known and stable.

A survey of the inventory literature reveals that there is no published work that investigates the EOQ model for items with imperfect quality under time-varying demand and product deterioration. Product life cycle analysis suggests that a constant demand rate assumption is usually valid in the mature stage of the life cycle of the product. In the growth and/or declining stages, the demand rate can be well approximated by a linear demand function (e.g. Alamri, 2011). Also, one implicit assumption is that the stored items that are screened may retain the same utility indefinitely, i.e. they do not lose their value as time goes on. In fact, the variation of demand and/or product deterioration with time (or due to any other factors) is a quite natural phenomenon. In order to enhance this line of research, we present four general EOQ models for items with imperfect quality. The first model underlies an inventory system with a single-level storage (OW) and the other three models relate to an inventory system with a two-level storage (OW and RW). The three models with a two-level storage underlie the following three dispatching policies, respectively: Last-In-First-Out (LIFO), First-In-First-Out (FIFO) and AIFO.

The versatile nature of each model allows the consideration of the appropriate demand, screening, defectiveness and deterioration function suitable to a particular case. The inspection process is linked with the management of perishable and non-perishable

Introduction - 7 -

inventories in order to take into account several practical concerns with regards to product quality related issues. Each model manages and controls the flow of perishable and non-perishable products so as to reduce cost and/or waste for the benefit of economy, environment and society. General solution procedures to determine the optimal policy for continuous intra-cycle periodic review applications are derived for each model. A detailed method that illustrates how deterioration, perishability and obsolescence may collectively affect inventories is explored. The value of the temperature history and flow time through the supply chain is also used to model the shelf lifetime of an item.

The proposed models may be viewed as realistic in today's competitive markets and reflective of several practical concerns with regard to product quality related issues. These issues relate to imperfect items received from suppliers, deterioration of goods during storage, potential dis-location of good and defective items, tracking the quality of perishable products in a supply chain, and transfer of knowledge from one inventory cycle to another. We show that the solution to each underlying inventory model, if it exists, is unique and global optimal. Practical examples that are published in the literature for generalised models in this area are shown to be special cases of our proposed models.

1.4. Methodology

Research paradigms are linked to specific underlying assumptions about the reality, knowledge, values and logic of the subject being investigated. Consequently, they may often be perceived as ambiguous, implicit, or be taken for granted, often resulting in the terms "paradigm", "methodology" and "method" being used interchangeably in the literature

Introduction - 8 -

(McGregor and Murnane, 2010). Recognising each paradigm by its philosophical underpinnings in "methodologies" constitutes a means by which scientists perceive the world or reality under investigation. Methodology involves apprising the methods and techniques embraced to shape research in this paradigm (McGregor, 2007; 2008).

The preference of each paradigm is based largely on its ability to answer the two fundamental questions constituting the ontological (the nature of reality) and epistemological (the nature of knowledge) assumptions. In the domain of epistemology, scientists explore the nature of how the world is perceived. In the domain of ontology, scholars investigate the form and nature of reality (Guba and Lincoln, 1994, p. 108). The nature of the problem under investigation and the intention to come up with generalisable solutions implies following the positivist paradigm for the purposes of this work.

Closely associated with the positivist paradigm is deductive mathematical modelling and its associated techniques, which constitute the most common methods adopted in supply chain research (Sachan and Datta, 2005; Burgess et al., 2006; Spens and Kovács, 2006; Aastrup and Halldórsson, 2008). Mathematical optimisation is used widely in this environment as an effective aid to solve problems involving decision making. In this thesis, and once an appropriate mathematical formulation is assumed and built for the total cost (objective) function, non-linear optimisation techniques are adopted to derive the solution procedure needed to obtain the optimal order/manufactured quantity that minimises the total system cost.

By its very nature such research does not involve any ethical concerns other than the obvious ones associated with accurately reporting both the methods and the results to allow interested readers to derive, verify and compare research findings with currently available

Introduction - 9 -

models. More details about the research methodology are discussed in Chapter 3.

1.5. Thesis structure

This thesis is organised into four parts:

Part A discusses the context of this research and is organised around three chapters:

Chapter 1 (already conducted) presented the research background of this work, its aim and objectives, its research motivation and contribution and the methodology undertaken to conduct the research.

Chapter 2 discusses the literature on product quality related issues, models that consider imperfect quality items for single-level and two-level storage scenarios. This chapter closes with a summary of some existing gaps in the literature that are adopted to shape the research conducted in this PhD.

Chapter 3 discusses the research methodology adopted in this thesis.

Part B discusses the development of an EOQ model for items with imperfect quality for a single-level storage, along with special cases; it comprises two chapters:

Chapter 4 outlines our single-level storage EOQ model for items with imperfect quality. The chapter presents the context, need for the research, model formulation, solution procedures, illustrative examples that demonstrate the application of the theoretical results in practice and concluding remarks as well as opportunities for further research in this area.

Introduction - 10 -

Chapter 5 emphasises the versatile nature of the proposed model. In particular, we derive a general step-by-step solution procedure for continuous intra-cycle periodic review applications, account for an appropriate management of perishable inventories, explore coordination mechanisms, link the model to some practical situations for inventory management and provide a summary of implications and managerial insights.

Part C presents EOQ models for a two-level storage and special cases. It consists of three chapters:

Chapter 6 introduces our first two-level storage EOQ model for items with imperfect quality that underlies a Last-In-First-Out (LIFO) dispatching policy. We present the context, need for the research, model formulation, solution procedures, illustrative examples that demonstrate the application of the theoretical results in practice and concluding remarks, as well as opportunities for further research in this area.

Chapter 7 proposes our second two-level storage EOQ model for items with imperfect quality that considers a First-In-First-Out (FIFO) dispatching policy. The chapter is organised as above.

Chapter 8 suggests a new framework for a two-level storage EOQ model for items with imperfect quality. A new dispatching policy entitled Allocation-In-Fraction-Out (AIFO) is presented. This chapter also follows the structure adopted in Chapter 6.

Part D comprises one chapter that summarises the overall contributions presented in this PhD and provides a discussion of avenues for further research.

Introduction - 11 -

Chapter 9 highlights the overall contribution and findings of this work and the next steps of research.

A pictorial overview of the structure of the thesis is offered in Fig. 1.1.

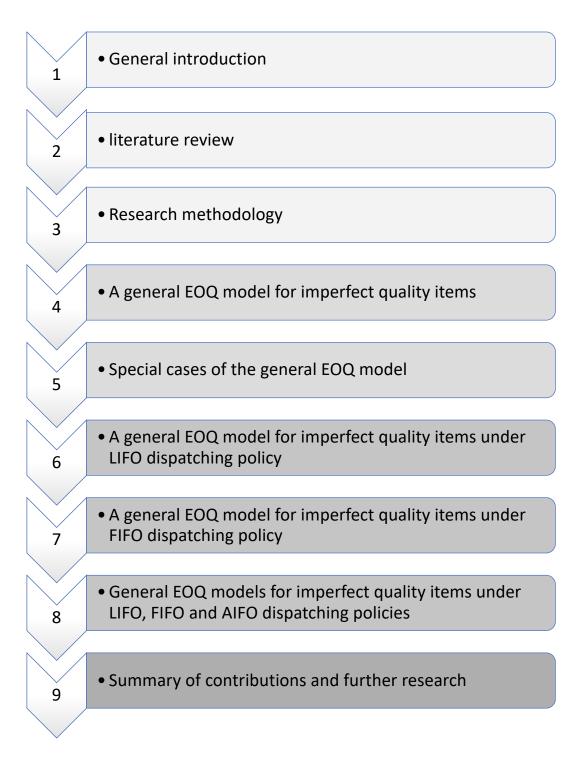


Fig.1.1 Thesis structure.

Introduction

2. Literature review

The academic literature related to inventory control for imperfect quality items is multidisciplinary in nature and, for presentation purposes in this PhD thesis, is thematically organised around four main streams of research: 1) inventory quality related issues; 2) information sharing and inspection processes; 3) model formulations and related solution techniques that consider imperfect quality items; and 4) lot size inventory models with two levels of storage. The academic literature related to the first, third and fourth themes are reviewed. For the second theme, some discussion on the Value of Information (VOI) and learning effect is conducted to enable linkage with the inspection process. This provides the necessary background to position our study in the current body of literature and elaborate on its research contributions. This review will also summarise and highlight the research gaps identified in the literature that led to the formulation of the general EOQ models stated in the previous chapter.

2.1. Inventory quality issues

One implicit assumption embedded in the EOQ model is that stored items preserve their physical characteristics indefinitely. This assumption may hold true for certain commodities. However, in real-life settings, items are subject to "perishability", "deterioration" and "obsolescence" that affect the physical state/fitness and behaviour of an item while in storage or as it moves through the supply chain (Goyal and Giri, 2001b; Bakker et al., 2012; Pahl and Voß, 2014). Next, we present an overview of previous studies related to the quality issues considered in this thesis.

Literature review - 13 -

2.1.1. Perishability and lifetime constraints

Nahmias (1975, 1977) introduced the fixed lifetime case and analysed the problem of a random lifetime product managed under periodic review with stationary stochastic demand. He assumed no fixed order cost and backlogged demand and orders perishing in the same sequence that they enter stock (i.e. a FIFO dispatching policy). Padmanabhan and Vrat (1995) investigated an inventory model for perishable items with stock dependent selling rate. Abad (1996) presented pricing and lot-sizing models under conditions of perishability and partial backordering. Giri and Chaudhuri (1998) studied deterministic inventory models of perishable product with stock dependent demand rate.

Skouri and Papachristos (2002) investigated a continuous review inventory model for deteriorating items and time-varying demand rate. They assumed linear replenishment cost and partial time-varying backlogging. Abad (2003) studied an optimal pricing and lot sizing problem considering perishability, finite production, partial backordering and lost sales.

Ketzenberg et al. (2012) extended the work of Nahmias (1977) and addressed the random lifetime as a function of the product's time and temperature history (TTH) in the supply chain. They allowed for orders to perish out of sequence, to discard inventory that remains good for sale and to sell inventory that may have already perished.

Amorim et al. (2013) presented a classification of models for perishable items that have explicit characteristics related to their physical status (e.g. by spoilage, decay or depletion) and/or changes in their value as perceived by the customer and/or a risk of future reduced functionality according to specialist opinion. Pahl and Voß (2014) provided a comprehensive literature review that addresses deterioration and lifetime constraints of items.

Literature review - 14 -

Ketzenberg et al. (2015) considered a case in which, unsatisfied demand having been lost, products may arrive already perished and orders may not perish in sequence.

2.1.2. Deterioration

Ghare and Schrader (1963) were among the first authors to address inventory problems considering deteriorating products. Covert and Philip (1973) formulated an EOQ model in which the deterioration rate follows a two parameter Weibull distribution. Shah and Jaiswal (1977) and Aggarwal (1978) developed inventory models with a constant rate of deterioration. Dave and Patel (1981) formulated an inventory model for deteriorating items with time proportional demand. Hollier and Mak (1983) developed an inventory replenishment policy for deteriorating items. They assumed constant rate of deterioration and exponentially negative decreasing demand rate. Roychowdhury and Chaudhuri (1983) presented an order level inventory model for deteriorating items with finite rate of replenishment. Sachan (1984) extended the model of Dave and Patel (1981) to allow for shortages. Dave (1986a, 1986b) proposed an order level inventory model for constant rate of deterioration. Baker and Urban (1988) investigated a deterministic inventory system allowing for stock dependent demand rate.

Datta and Pal (1988) developed an order level inventory model with power demand pattern for items with variable rate of deterioration. Bahari-Kashani (1989) extended the model of Dave and Patel (1981) to allow for variable replenishment periods. Datta and Pal (1990) presented a note on an inventory model with stock dependent demand. Pal et al. (1993) and Giri et al. (1996) developed deterministic inventory models for deteriorating items with stock dependent demand rate. Wee (1993) proposed an economic production lot size model for

Literature review - 15 -

deteriorating items with partial back ordering. Hill (1995) studied EOQ models for deteriorating items with time varying demand. Chang and Dye (1999) studied an order level inventory model for deteriorating items with time-varying demand rate and partial backlogging. Mandal and Maiti (1999) developed an inventory model of damageable items with variable replenishment and stock dependent demand rate.

Gupta and Aggarwal (2000) presented an EOQ model for deteriorating items allowing for production rate to be dependent on a linear trend in demand. Abad (2001) presented pricing and lot-sizing models allowing for a variable rate of deterioration and partial backlogging. Chang and Dye (2001) proposed an inventory model for deteriorating items with partial backlogging and permissible delay in payments. Goyal and Giri (2001a) conducted an extensive review of papers addressing deteriorating items since the early 1990s. Goyal and Giri (2001b) extended the model of Chang and Dey (1999) for deteriorating items with time varying demand and partial backlogging. Wee and Law (2001) proposed a deterministic inventory model for deteriorating items under time-value of money and price-dependent demand. Wang (2002) investigated an inventory replenishment policy for deteriorating items with shortages and partial backlogging. Wu (2002) developed an EOQ inventory model for items considering Weibull distribution deterioration rate, time-varying demand and partial backlogging.

Khanra and Chaudhuri (2003) re-established an order-level inventory model for deteriorating items with time-dependent quadratic demand. Zhou et al. (2003) formulated a new variable production scheduling strategy for deteriorating items with time-varying demand and partial lost sale. Chu and Chung (2004) discussed the sensitivity of the inventory model with partial backorders. Chu et al. (2004) presented a note on inventory model with a mixture of back

Literature review - 16 -

orders and lost sales. Sana et al. (2004) formulated a production inventory model for deteriorating items with trended demand and shortages. Zhou et al. (2004) studied a finite horizon lot-sizing inventory model with time-varying demand and waiting time dependent partial backlogging. Giri et al. (2005) proposed an economic production lot size inventory model with increasing demand and partial backlogging. Teng and Chang (2005) studied an EPQ model for deteriorating items with price and stock dependent demand.

Manna and Chaudhuri (2006) developed an EOQ model with ramp type demand and time dependent deterioration rate. In that model, the unit production cost is inversely proportional to the demand rate. Ouyang et al. (2006) studied an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Pal et al. (2006) suggested an inventory model for deteriorating items with demand rate being dependent on the displayed stock level. Wu et al. (2006) studied an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Teng et al. (2007) extended the work of Abad (2003) considering shortage and lost sales costs into the objective function. Liao (2008) developed an EOQ model with non-instantaneous receipt and exponentially deteriorating items under two-level trade credit. Chung (2009) presented a complete proof on the solution procedure for non-instantaneous deteriorating items with permissible delay in payment.

Skouri et al. (2009) studied inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. Ahmed et al. (2013) formulated inventory models with ramptype demand rate, partial backlogging and general deterioration rate. Sarkar and Sarkar (2013) presented an inventory model with stock-dependent demand, partial backlogging and time-varying deterioration rate. Sicilia et al. (2014) developed a deterministic inventory

Literature review - 17 -

model for deteriorating items with shortages and time-varying demand.

2.2. Information sharing and inspection process

There is a unanimous agreement among researchers and practitioners on the benefits of information sharing that allows more timely material flow in a supply chain (Costantino et al., 2013). In many situations, products entail inspection to ensure an appropriate service to the customers (White and Cheong, 2012). In this section, we first address the importance of VOI in supply chains, followed by some discussion that links the VOI and learning effect with the inspection process associated with the formulation of EOQ inventory models.

2.2.1. Value of information (VOI)

Value of information (VOI) in supply chains has become increasingly important and may relate to sharing data over and above demand and inventory information (Dong et al., 2014; Kahn 1987; Metters 1997). For example, modern technologies, such as radio-frequency identification (RFID) systems, data loggers and time—temperature integrators and sensors, are capable of recording, tracking and transmitting information regarding an item as it moves through the supply chain (Jedermann et al., 2008). The deployment of such technologies increases supply chain visibility, which in turn lowers safety stocks and improves customer service level (Gaukler et al., 2007; Kim and Glock 2014).

Ketzenberg et al. (2007) conducted an extensive literature review of papers that: (1) address VOI in the context of inventory control, (2) provide a numerical study to explore VOI over a

Literature review - 18 -

set of varying operating characteristics and (3) compare two or more scenarios. In addition, they developed and tested a VOI framework to help identify the determinants of VOI. The researchers pointed out that the dominant research stream in this area focuses on the value of demand information to enhance supply chain performance.

Accurate shelf lifetime monitoring is a goal of technologies that have been developed to collect and transmit data about the state of a product. For certain items, if temperature departs from a pre-defined range, the items are spoiled and must be discarded (Zacharewicz et al., 2011). Ketzenberg and Ferguson (2008) examined the VOI for a product with fixed lifetime in the context of a serial supply chain. They evaluated the case in which a supplier shares retailer demand and inventory information, as well as the case where a centralised decision maker collects full information at both echelons. Recently, Ketzenberg et al. (2015) addressed the VOI for inventory replenishment decisions to demonstrate the wide fluctuations in a supply chain's TTH, the applicability and accuracy of using RFID temperature tags to capture the TTH, and the use of TTH to model shelf lifetime.

The focus of this PhD thesis is on the value and use of technologies, such as RFID, to capture the TTH to model shelf lifetime and not the technologies themselves. For further details related to this technology, see Jedermann et al. (2008) and Wessel (2007).

2.2.2. Learning effects

The 'Learning Phenomenon' introduced by Wright (1936) implies that the performance of a system engaged in a repetitive task improves with time. The learning phenomenon is reflected by the "learning curve" theory, which links the performance of a specific task with

Literature review - 19 -

the number of times that the task is repeated. Wright's power function formulation (which was based on empirical data) suggests that as production accumulates, the unit production time decreases by a constant percentage (e.g. 80 per cent, 70 per cent, etc.) each time the quantity doubles.

Wright's simple mathematical formulation is a commonly used model because it is easier to implement and understand by practitioners than those complex ones (e.g. Hackett, 1983; Towill, 1982, 1985). However, it implies that production time can be neglected as the cumulative quantity produced takes on relatively larger values. This is an unreasonable conclusion, since in real-world problems, after a certain time of cumulative learning in a production system, the system plateaus, in which case the production time will attain an almost certain value.

Jordan (1965) and Carlson and Rowe (1976) argued that in practice, the learning function is an "S"-shaped curve. The task "life cycle" depicted by an S-shaped logistic learning curve comprises three phases. In the first phase (incipient), the worker is becoming acquainted with the techniques, tools, procedures, work place, facilities, etc. In this phase improvement is slow, which is consistent with the behaviour observed in many industrial situations. The second phase (learning) is where most improvement occurs. The third phase (maturity) denotes the levelling of the curve.

There is almost unanimous agreement among researchers and practitioners that the preponderant form of a learning curve is either an S-shaped (Jordan, 1958; Carlson, 1973) or a power one, as suggested by Wright (1936) (Alamri and Balkhi, 2007; Dar-El, 2000; Jaber, 2006). For more discussion on learning curves, interested reader may refer to Grosee et al. (2015).

Literature review - 20 -

2.2.3. Inspection process

The above discussion raised in Sections 2.2.1 and 2.2.2 relates very much to knowledge acquired from an inspection process conducted at the retailer site. This means that the quality issues that render an item defective can be communicated to the supplier in order to reduce the potential risks affecting such defectiveness. These risks can be attributed to production, handling or transportation errors.

Although the buyer is often credited, so that no costs apply for defective items, the potential interest remains to eliminate the presence of defects in subsequent replenishments. Therefore, coordination may be pursued between supply chain members implying that any information gained through previous replenishments can be used to enhance subsequent deliveries.

White and Cheong (2012) considered the benefit of observing the quality of a perishable product in a food supply chain that is processed in multiple stages from origin to destination. At each stage, it is presumed essential to decide whether to inspect the quality of the product at a certain cost. Moussawi-Haidar et al. (2013) investigated an instantaneous replenishment model under the effect of a sampling policy for defective items.

Inspection may also be presumed essential for updating the Information System records with good items that are actually available in stock, so as to avoid shortages. Moreover, inspection may eliminate the return service cost associated with product recalls (Klassen and Vereecke 2012). It can be used in real-life settings where the impact of allowing through defective items could be severe. Different types of inspection can occur, including seal inspection, outer case label inspection or damaged carton inspection. The service cost may include goodwill cost, transportation cost, and re-processing cost, and that may affect all supply chain members.

Literature review - 21 -

Inspection may also reduce holding costs due to the deployment of less preserving environmental conditions, i.e. the defective items are not usually stored in the same warehouse where the good items are stored (e.g. Wahab and Jaber, 2010).

2.3. Inventory models with imperfect quality items

The classical EOQ has been a widely-accepted model for inventory control purposes due to its simple and intuitively appealing mathematical formulation. However, it is true to say that the operation of the model is based on a number of explicitly or implicitly made, unrealistic, mathematical assumptions that are never actually met in practice (Jaber et al., 2004; Liao et al., 2013). For example, the assumption of perfect quality items is technologically unattainable in most supply chain applications and it is an important restriction in the classical formulation of the EOQ model (Cheng, 1991).

In a centralised scenario, there is a single decision-maker who is concerned with maximising the entire chain's profit. The objective is to find a more profitable joint production and inventory strategy, as compared to the strategy resulting from independent decision making. In a decentralised, coordinated scenario, the supplier and the retailer cooperate in order to render the total minimum (maximum) cost (revenue) closer to that associated with a centralised one.

In this section, we present a review of previous studies related to the formulation of the EOQ model with imperfect quality items. Section 2.3.1 will focus on papers that considered the formulation of a single warehouse inventory model, followed by a review of studies investigating the effect of learning on a single warehouse inventory model with imperfect

Literature review - 22 -

quality items in Section 2.3.2. Previous studies related to the formulation of a joint-vendorbuyer inventory model with imperfect quality items are presented in Section 2.3.3.

2.3.1. Single-warehouse model

Porteus (1986) studied the impact of imperfect products when the production process may shift randomly from an in-control state to an out-of-control state during a production run. He assumed that all imperfect items could be reworked. Similarly, Rosenblatt and Lee (1986) considered the effects of an imperfect manufacturing process on the determination of an optimal manufacturing cycle time.

Salameh and Jaber (2000) developed a mathematical model that permits some of the items to drop below the quality requirements, i.e. a random proportion of defective items is assumed for each lot size shipment, with a known probability distribution. The researchers assumed that each lot is subject to a 100 per cent screening, where defective items are kept in the same warehouse until the end of the screening process and then can be sold at a price lower than that of perfect quality items. Cárdenas-Barrón (2000) corrected a minor error appearing on Salameh and Jaber's model but did not diminish the main idea and the contribution. Goyal and Cárdenas-Barrón (2002) developed a practical approach for determining the economic production quantity. Chan et al. (2003) presented a similar model, in which products are categorised as good quality, good quality after reworking, imperfect quality, and scrap.

Chung and Huang, (2006) presented an inventory model which accounts for imperfect quality items under the condition of permissible delay in payments. Papachristos and Konstantaras

Literature review - 23 -

(2006) investigated the sufficient condition proposed by Salameh and Jaber (2000) and Chan et al. (2003), which is related to the issue of non-shortages. They pointed out that this condition is not a sufficient condition to ensure that shortages will not occur. They also extended the work of Salameh and Jaber in which imperfect items are removed by the end of the cycle. Rezaei (2005), Eroglu and Ozdemir (2007) and Wee et al. (2007) extended the model of Salameh and Jaber (2000) by allowing for shortages. Konstantaras et al. (2007) investigated the case where imperfect quality items can be sold as a single batch at a lower price as well as the case in which these items are reworked at a cost. Maddah and Jaber (2008) suggested a new model that rectifies a flaw in the one presented by Salameh and Jaber (2000) using renewal theory.

Jaggi and Mittal (2011) investigated the effect of deterioration on a retailer's EOQ when the items are of imperfect quality. In that paper, defective items were assumed to be kept in the same warehouse until the end of the screening process. Jaggi et al. (2011) and Sana (2012) presented inventory models, which account for imperfect quality items under the condition of permissible delay in payments. Moussawi-Haidar et al. (2014) extended the work of Jaggi and Mittal (2011) to allow for shortages.

2.3.2. Single-warehouse model with learning

Jaber et al. (2008) extended the work of Salameh and Jaber (2000) by considering that the percentage of defective items per lot reduces according to a learning curve. They examined empirical data from the automotive industry for several learning curve models and the S-shaped logistic learning curve (Jordan, 1958; Carlson, 1973) was found to fit well.

Literature review - 24 -

Wahab and Jaber (2010) presented the case where different holding costs for good and defective items are assumed. They showed that if the system is subject to learning, then the lot size with the same assumed holding costs for the good and defective items is less than the one with differing holding costs. When there is no learning in the system, the lot size with differing holding costs increases with the percentage of defective items.

Konstantaras et al. (2012) pointed out that as learning in quality increases, then the lot size, number of defective items and shortages decrease. Hlioui et al. (2015) investigated replenishment, production and quality control strategies in a three-stage supply chain with imperfect quality items. They pointed out that the integration of 100 per cent screening process or discarding decisions is more beneficial, and assures better coordination at a lower cost.

2.3.3. Vendor-buyer supply chain modelling

Zhang and Gerchak (1990) developed a joint lot sizing and inspection policy under an EOQ model, in which a random proportion of the lot was considered defective. Wee (1995a) presented a joint pricing and replenishment policy for deteriorating items with a declining market. In a follow-up, Wee (1995b) developed a deterministic inventory model for deteriorating items with shortages and a declining market. Yang and Wee (2002) investigated the effect of deterioration and constant production and demand rates on a production inventory policy with a single vendor and multiple buyers. Rau et al. (2003) proposed an integrated inventory model for deteriorating items and derived an optimal joint total cost for a multi-echelon supply chain environment.

Literature review - 25 -

Huang (2004) developed a model to determine an optimal integrated vendor—buyer inventory policy for flawed items in a just-in-time manufacturing environment. Huang and Yao (2005) studied an optimally coordinating inventory model for deteriorating items in a supply chain system with a single vendor and multiple buyers. Lo et al. (2007) derived an integrated vendor—buyer supply chain model with imperfect production processes. They assumed a varying rate of deterioration, partial backordering and inflation. Sung and Gong (2008) proposed a production inventory model for deteriorating production with defective items being reworked and joint material replenishment policy being considered.

Singh and Diksha (2009) formulated an integrated vendor-buyer cooperative inventory model for deteriorating items, allowing for multivariate demand and progressive credit period. Wahab et al. (2011) developed an EOQ inventory model for a coordinated two-level supply chain by allowing for shortages and environmental effects. Moussawi-Heidar and Jaber (2013) proposed a joint model for cash and inventory management for a retailer under delay in payments. Khan et al. (2014) presented an integrated vendor—buyer inventory policy by accounting for learning in production at the vendors' end and quality inspection errors at the buyers' end.

Paul et al. (2014) presented a joint replenishment policy with imperfect quality items for multiple products. Rad et al. (2014) derived an integrated vendor—buyer for a demand-driven pricing model with imperfect production and shortages. Ongkunaruk et al. (2016) determined the optimal reordering policy for multiple products in a joint replenishment problem. They integrated shipment constraint, budget constraint and transportation capacity constraint. Yu and Hsu (2017) derived a single-vendor, single-retailer, production-inventory model when a 100 per cent inspection is assumed with imperfect items being returned to vendor

Literature review - 26 -

immediately under an unequally sized shipment.

2.4. Two-warehouse model

The classical EOQ model is often based on the assumption that the OW has unlimited capacity. However, there are many factors that may lead to purchasing a number of units that may exceed the limited capacity of OW, resulting in the excessive units being stored in another RW, which is assumed to be of ample capacity. Such factors may include a discounted price of goods offered by the supplier, revenue (acquisition price) being higher than the holding cost in RW, and evading high inflation rates.

The earliest approach to address the basic two-warehouse inventory model was recognised by Hartley (1976). Sarma (1983) presented a deterministic two-warehouse inventory model under an optimum release rule, in which the cost of transporting an item from RW to OW is assumed. Murdeshwar and Sathe (1985) discussed some aspects of lot size model with two levels of storage and derived the solution for optimum lot size under finite production rates.

Sarma (1987) considered a deterministic order level inventory model for deteriorating items. He assumed a lower rate of deterioration in RW due to better preserving facilities. Goswami and Chaudhuri (1992) developed a two-warehouse inventory model allowing for linear demand and deterioration rates. Pakkala and Achary (1992) explored a deterministic two-warehouse inventory model for deteriorating items with a finite replenishment rate assuming uniform demand and shortages.

Bhunia and Maiti (1994) studied a two-warehouse inventory model for a single item with an

Literature review - 27 -

infinite rate of replenishment. They assumed a linear trend in demand and shortages were allowed and fully backlogged. Ishii and Nose (1996) proposed a two-warehouse inventory model for a perishable product with two types of customers, high priority and low priority (the former buys only the newest commodity, while the latter buys either one). They assumed that stock is depleted by high priority demand first and then low priority demand using FIFO issuing policy; different selling prices reflecting the remaining lifetime of commodity are assumed.

Roy and Maiti (1997) established a fuzzy EOQ model with two-level storage allowing for price dependent demand. In that model, the setup cost varies with the quantity produced/purchased. Bhunia and Maiti (1998a, 1998b) formulated a two-warehouse inventory model for deteriorating items with linear trend in demand. Zhou (1998) and Lee and Ma (2000) investigated a two-warehouse model for deteriorating items and time varying demand. Kar et al. (2001) established a deterministic inventory model with two levels of storage. They assumed linear time-dependent demand and finite time horizon. Zhou (2003) considered a multi-warehouse inventory model for items with time-varying demand and shortages.

Yang (2004) proposed a two-warehouse inventory problem for deteriorating items under inflation. In that model, the inventory deteriorates at a constant rate and shortages were allowed. Wee et al. (2005) investigated a two-warehouse inventory model with Weibull distribution deterioration under inflation. They assumed partial backordering and applied the discounted cash flow in problem analysis. Zhou and Yang (2005) presented a two-warehouse model allowing for stock-level dependent demand rate.

Mandal et al. (2006) considered a two-warehouse inventory model for deteriorating items

Literature review

with constant demand and deterioration rates. Yang (2006) extended his earlier model, Yang (2004), accounting for partial backlogging and inflation. Dye et al. (2007) suggested a deterministic two-warehouse inventory model for deteriorating items. They allowed for shortages to occur in the OW with the backlogging demand rate being dependent on the duration of the stock out.

Hsieh et al. (2008) formulated a deterministic two warehouses inventory model for deteriorating items by minimising the net present value of the total cost. Rong et al. (2008) investigated a two-warehouse inventory model for deteriorating items and shortages. Thangam and Uthayakumar (2008) investigated a two-level supply chain model with partial backordering and approximated Poisson demand. Lee and Hsu (2009) proposed a two-warehouse production inventory model for deteriorating items and time-dependent demands. Ghosh and Chakrabarty (2009) studied an order-level inventory model under two levels of storage for deteriorating items and time-dependent demand. Jaggi et al. (2010) proposed a two-warehouse inventory model for deteriorating items when demand is price sensitive. Dem and Singh (2012) investigated a two-warehouse production model for deteriorating items under quality consideration. Sett et al. (2012) developed a two-warehouse production model assuming quadratically increasing demand and time-varying deterioration. Agrawal et al. (2013) presented a two-warehouse inventory model with constant deterioration rates, ramp-type demand and partial backlogging.

Literature review - 29 -

2.5. Summary

The effect of the presence of defective items on lot size has received the attention of many researchers in the field. However, there has been little research that links the inspection process with the management of perishable inventories. In parallel, it would be interesting to explore the implications of the inspection process into inventory decision-making. Moreover, a review of the literature reveals that there is no published work that derives solution procedures for periodic review applications for the EOQ model with imperfect quality items.

The terms "deterioration", "perishability" and "obsolescence" are used interchangeably in the literature and may often be perceived as ambiguous because they are linked to specific underlying assumptions regarding the physical state/fitness and behaviour of items over time. In this thesis, we provide clearer definitions that distinguish the role of each term. For example, deterioration indicates the process of decay, damage or spoilage of a product, i.e. the product loses its value characteristics and can no longer be sold/used for its original purpose. In contrast, an item with a fixed lifetime (expiration date) perishes once exceeding its maximum shelf lifetime and thus it must be discarded. Obsolescence refers to items incurring a partial or a total loss of value in such a way that the value for a product continuously decreases with its perceived utility/desirability. Therefore, it would be value adding to the literature of supply chain management if a detailed method is provided which underpins and demonstrates how these terms may collectively apply to an item. It appears also that the accuracy of continuous automated inventory control systems to model the shelf lifetime of an item with imperfect quality is another research gap in this area.

The above identified research gaps are addressed in our first model that underlies an

Literature review - 30 -

inventory system with a single-level storage (OW). In particular, we present a general EOQ model for items with imperfect quality under varying demand, defective items, a screening process and deterioration rates for an infinite planning horizon. Consequently, the generality of the model goes beyond academic interests to enable inventory managers to establish the optimum order quantities that minimise the total system cost in different realistic situations.

It seems plausible to address the same identified research gaps when formulating an inventory system with a two-level storage (OW and RW). However, this necessitates formulation of LIFO and FIFO EOQ models for investigation and numerical comparison purposes. As such, we propose and compare three general EOQ models with a two-level storage considering different transportation costs associated with OW and RW. The first model underlies LIFO, the second model underlies FIFO and the third model relates to our new suggested dispatching policy, which implies simultaneous consumption fractions associated with OW and RW and is entitled AIFO. In order to enhance this line of research, we formulate a key performance indicator (KPI), i.e. an upper-bound (cost associated with OW (RW) being idle) that renders AIFO the optimal dispatching policy.

The mathematical formulations consider arbitrary functions of time that allow the decision maker to assess the consequences of a diverse range of strategies by employing a single inventory formulation for each model. The proposed models may be viewed as realistic in today's competitive markets and reflective of several practical concerns with regard to product quality related issues. These issues relate to imperfect items received from suppliers, deterioration of goods during storage, potential dis-location of good and defective items, tracking the quality of perishable products in a supply chain and transfer of knowledge from one inventory cycle to another. We show that the solution to each underlying inventory

Literature review - 31 -

model, if it exists, is unique and global optimal.

In summary, the aim of this PhD thesis is to advance the current state of knowledge in the field of inventory mathematical modelling and management by means of providing theoretically valid and empirically viable generalised inventory frameworks to assist inventory managers towards the determination of optimum order/production quantities that minimise the total system cost. The obvious implications that can be derived from the general formulations, along with the fact that many practical examples published in the literature for generalised models in this area constitute special cases of our proposed models, make, we feel, an important contribution to the supply chain literature.

3. Research methodology

3.1.Introduction

Research paradigms may often be perceived as ambiguous, misleading or assumed, frequently resulting in the terms "paradigm", "methodology" and "method" being considerably tangled and used interchangeably in the literature (McGregor and Murnane, 2010). Consequently, it is often difficult to recognise each paradigm that is linked to particular, underlying assumptions about the reality, knowledge, values and logic of the subject being investigated. Fortunately, recognition of the feasibility of each paradigm by its philosophical underpinning in "methodologies" is key to reconciling the differences in individual perceptions of the same social phenomenon, which in turn involves apprising the methods and techniques adopted to shape research in this paradigm (McGregor, 2007; 2008).

Despite acknowledgment of these traditions as a means of shaping research, preference for each tradition is based largely on its ability to answer the two fundamental questions constituting epistemological and ontological orthodoxies. In the domain of ontology (the nature of reality), scientists explore the form and nature of reality and, therefore, what can be known about it. In the domain of epistemology (the nature of knowledge), scholars investigate the nature of the relationship between the knower and what can be known (Guba and Lincoln, 1994, p. 108). Hence, long-standing epistemological and ontological debate has been centred on whether social reality and natural sciences can be conducted by using the same ideologies (Bryman, 2015). In this context, positivism has emerged as a broad position seeking to integrate deductive logic, precise empirical observations and individual behaviour while aiming to explain numerous patterns of human activity.

Research methodology - 33 -

3.2. Epistemological and ontological orthodoxies

A survey of the literature reveals that "the rumours of positivism's death within sociology have been greatly exaggerated. Though it is not dead, positivism has become increasingly encapsulated within the USA and disavowed in the UK, particularly in its 'instrumental' form" (Gartrell and Gartrell, 2002, p.653). Positivism is generally known as an approach to the methodology of science in which patterns and causal relations can be investigated and disclosed through research. Moreover, and among a variety of definitions found in the literature about positivism, one can deduce that a positivistic philosophical basis provides an approach to investigate social reality by assuming that theory and observation are dependent on each other and involve employing natural science methods and theories to explore current observations, seeking regularities for the purpose of generalisation. With researchers considering themselves isolated from the research setting and in an unbiased position of observation, positivism can also be regarded as aiming to understand and predict reality, where deterministic and reactive stances among people are assumed. Reality is assumed to be objective, tangible, and fragmentable, with research findings being context independent in order to provide a clear interpretation of separate objective and subjective data (Mackenzie, 2011).

Given that positivism is the dominant epistemological position, researchers need to become more conscious and seek appropriate materials to avoid the possible risk of misunderstanding the nature of their own research. The presence of a few fundamentally misleading passages found in many books and articles on positivism and the absence of unanimous agreement among social scientists on a clear definition of the term "positivism", could lead to rejection for the wrong reasons (Persson, 2010).

Research methodology - 34 -

There is almost unanimous agreement among social scientists that a paradigm is a set of values, concepts, assumptions and practices that collectively constitute a means by which scientists perceive the world or reality under investigation and the effects of this knowledge on research findings (McGregor and Murnane, 2010). It is worth noting that having an appropriate paradigm is the basic platform a scientist uses for analysing research, which is the true orientation of his or her control to infer unbiased perception of reality. This unbiased perception, whether it relies on a recognised methodological approach, is appreciated as a combination of multiple methods, or is a claim towards advancing the current state of knowledge in either approaches, will remain insignificant unless it presents originality and/or new insights that illuminate a specific line of research. Although positivism has been described by many authors in pejorative terms (Mackenzie, 2011; Nairn, 2012), supply chain research is known to be marginalised outside the positivist research paradigm.

Answering the above ontological question is a core characteristic that creates and tests management theories. In other words, knowing what constitutes reality, and what the consideration of a philosophy of science offers to supply chains, plays a significant role in how to shape and form the subsequent knowledge generated (Burgess et al., 2006; Miller and Tsang, 2010). Therefore, philosophies, worldviews and attitudes form the basis that enables researchers to pose a research question, embrace a research strategy and employ appropriate methods and reasoning approaches (Solem, 2003). In this regard, positivist research paradigm comprises the empirical methodology where social reality is perceived as being independent and data is derived from experimentation and observation, which in turn can be measured and evaluated by rigorous scientific approaches (generating supportive evidence) (Rohmann, 1999). Thus, scientific examinations are produced quantitatively in

Research methodology - 35 -

which the research findings must be repeatable to assure reliability as measures of research quality; i.e. interested researchers can generate any future results employing the same research methods. Moreover, these findings are governed by the cause-effect doctrines aimed at defining stable patterns of social reality (Crotty, 1998; Marczyk et al., 2005; Neuman, 2002).

3.3. Research methods

Closely associated with the positivist paradigm is deductive mathematical modelling and its associated scientific methods and techniques which are the most common quantitative ones adopted in supply chain research (Sachan and Datta, 2005; Burgess et al., 2006; Spens and Kovács, 2006; Aastrup and Halldórsson, 2008). Moreover, most research published in the top Operational Research (OR) international journals primarily comprises theoretical mathematical modelling, where research results are produced almost entirely within the quantitative research domain (Lewis and Suchan, 2003; Sachan and Datta, 2005; Stock et al., 2010).

Although researchers accept that supply chain management is more likely to be approached quantitatively, this method has been subjected to criticism for not considering a more inclusive, supply chain-wide system perspective and undermining the influence of social structures and their relations to activity (Sachan and Datta, 2005). In this regard, recent calls in top OR journals have invited scholars to consider more fully the importance of human factors in supply chain management and to conduct behavioural studies.

Other examples of criticisms of the dominant supply chain management research paradigm

Research methodology - 36 -

from different philosophical domains includes, but not limited to, those raised by Faria and Wensley (2002), Näslund (2002), Lewis and Suchan (2003) and New (2004). Within the long-standing epistemological debate about the most appropriate philosophical position, other researchers have claimed that the designability of supply chains adheres to a positivist research paradigm with an inherent assumption that managers responsible for supply chain-related activities and other related parties, such as being isolated from the context within which they work and live, may act in predictable ways and accept designs as proposed without questioning them (Aastrup and Halldórsson, 2008). This can be a valid criticism if, for example, the reactions being experienced rely heavily on the managerial life experience of the subjects. Instead, in well-established organisations, reactions are presumed to be dependent on the implementation of either optimal and near optimal values derived or statistically projected parameters.

Similarly, other works have focused primarily on the exclusion of elements such as context and distribution of power and, consequently, call for the adoption of a new philosophy of science perspectives that can discover reasons for the existence of phenomena, rather than a perspective that can describe them and present solutions for reducing their effects (New and Payne, 1995; Mangan et al., 2004; Spens and Kovács, 2005; Burgess et al., 2006; Boyer and Swink, 2008; Stock et al., 2010; Tokar, 2010).

It is the very nature (not people related) of the majority of problems involved in OR that has resulted in the dominance of quantitative methods. This is particularly true in statistical inventory optimisation, whereby the scope of the problem (tremendous number of stock keeping units involved) and the need for statistical optimisation calls for fully quantitative approaches.

Research methodology - 37 -

3.4. Validity and reliability of the research

As discussed above, supply chain research has been characterised as being significantly governed by a positivistic epistemological position that justifies the dominant ontological and epistemological orthodoxies. In the domain of ontology, scholars agree that supply chains are more likely to be perceived as inter-organisational entities that have ontological identities generated independently of social bodies, relations and practices.

Despite the frequency and veracity of claims made regarding the ability of quantitative methods to generate innovation, there is a natural dominance of quantitative, fully positivistic approaches (Arlbjørn and Halldórsson, 2002; Näslund, 2002; Sachan and Datta, 2005). This is because the intention is always to generalise and propose solutions that are sufficiently robust to successfully apply in many different contexts. For example, in supply chain mathematical modelling, the adoption of the positivistic paradigm is presumed essential for facilitating the production of guidelines needed for optimised configurations and improvement purposes, which, in most cases, are easy to implement among business entities. Furthermore, the nature of this research is to formulate, solve and analyse mathematical inventory models. Hence, the only ethical concerns to be considered when conducting research is to accurately report both the methods and the results to allow interested readers to derive, verify and compare research findings with currently available models.

Adamides et al. (2012) pointed out that the designability embedded in supply chains assumes a flat ontology constructed on regularities at the activity level. In such a case, the constituting activities of supply chains are being shaped according to the desired values of certain performance levels. The potential question that arises here is how to attain validity and/or

Research methodology – 38 –

reliability without the presence of designability. For example, this designability is the cornerstone for determining optimisation procedure required for the formulation, creation and operationalisation of specific variables. These are collectively, under rigorous mathematical assumptions, valid and reliable, particularly mathematical modelling formulations and their associated science methods and techniques. Moreover, this designability allows scholars to rectify any shortcomings that may appear in any mathematical model and, thus, be able to refine the line of research. This is because researchers can generate any future results produced by using the same research methods, whereby the entire research process can be made value free to eliminate biased interpretations of the results (Jantzen and Østergaard, 2001). Given that all research methodologies have their weaknesses as well as their strengths, the methods and analytical tools employed by a scholarly community may influence the very problems that do and do not receive the attention of researchers (Martin, 1980).

3.5. Summary

So far, the motivation associated with relaxing some of the assumptions embedded in the EOQ model for items with imperfect quality has been addressed. As the intended outcome is presumed to be a theory-based research leading to widely applicable solutions, the positivist epistemological framework will be embraced. It is often the case that a variety of practical and complex inventory problems can be perceived as fully quantitative research studies. Thus, a deductive approach is employed to incorporate non-linear programming techniques so as to derive the solution procedure for the proposed models. It is, therefore, the aim of this research to advance the current state of knowledge in the field of inventory mathematical

Research methodology - 39 -

modelling and management by means of providing theoretically valid and empirically viable generalised inventory frameworks to assist inventory managers towards the determination of optimum order/production quantities that minimise the total system cost.

To ascertain the validity of the research, and also for comparison purposes, all papers contributing in the generalised work will be considered and the relevant models will be solved to show that they constitute special cases of our proposed models.

Research methodology - 40 -

Part B: Lot size inventory model with one level of storage

This part consists of two chapters. Chapter 4 underlies a general EOQ model for imperfect quality items and Chapter 5 covers special cases generated from the general model. In Chapter 4, we present a general EOQ model for items that are subject to inspection for imperfect quality. Each lot that is delivered to the sorting facility undergoes a 100 per cent screening and the percentage of defective items per lot reduces according to a learning curve. The generality of the model is viewed as important both from an academic and practitioner perspective. The mathematical formulation considers arbitrary functions of time that allow the decision maker to assess the consequences of a diverse range of strategies by employing a single inventory model. A rigorous method is utilised to show that the solution is unique and global optimal, and a general step-by-step solution procedure is presented for continuous intra-cycle periodic review applications. The value of the temperature history and flow time through the supply chain is also used to determine an appropriate policy. Furthermore, coordination mechanisms that may affect the supplier and the retailer are explored to improve inventory control at both echelons. In Chapter 5, we provide illustrative examples and special cases that demonstrate application of the theoretical model in different settings and lead to the generation of interesting managerial insights.

Part B is based on the paper entitled "Efficient inventory control for imperfect quality items" by Adel Alamri, Irina Harris and Aris Syntetos, which has been published in the *European Journal of Operational Research* (Alamri, et al. (2016)).

Part B

4. A general EOQ model for imperfect quality items

4.1.Introduction

In this chapter, we present a general EOQ model for items with imperfect quality under varying demand, defective items, a screening process and deterioration rates for an infinite planning horizon. Consequently, the generality of the model extends beyond academic interests to enable inventory managers to establish optimum order quantities that minimise the total system cost. In the model, each lot is subject to a 100 per cent screening where items that do not conform to certain quality standards are stored in a different warehouse. Therefore, different holding costs for the good and defective items are considered in the mathematical model.

Items deteriorate while they are in storage, with demand, screening and deterioration rates being arbitrary functions of time. Thus, the decision maker can assess the consequences of a diverse range of strategies by employing a single inventory model. The percentage of defective items per lot reduces according to a learning curve. After a 100 per cent screening, imperfect quality items may be sold at a discounted price as a single batch at the end of the screening process or incur a disposal penalty charge.

A rigorous method is utilised to show that the solution, if it exists, is unique and global optimal. Previously published models in this area are shown to be special cases of our model. The behaviour of different conditions (such as using functions for varying demand, screening, defectiveness and deterioration rates) is studied using illustrative examples, and interesting insights are offered to practitioners.

The remainder of the chapter is organised as follows: in Section 4.2 we emphasise the need for the research. Our general EOQ model for items with imperfect quality, the assumptions and notation of the inventory system are presented in Section 4.3. The solution procedures are presented in Section 4.4, followed, in Section 4.5, by illustrative examples that demonstrate the application of the theoretical results in practice. Managerial insights and concluding remarks are provided in Section 4.6. The proof of the optimality and uniqueness of our solution is presented in Appendix A.

4.2. Need for the research

The classical EOQ has been a widely accepted model for inventory control purposes due to its simple and intuitively appealing mathematical formulation. However, the model is based on a number of explicitly or implicitly made, unrealistic, mathematical assumptions that are never actually met in practice (Jaber et al., 2004; Liao et al., 2013). Salameh and Jaber (2000) presented a mathematical model in which a random proportion of defective items is assumed for each lot size shipment. Maddah and Jaber (2008) developed a new model that rectifies a flaw in the model presented by Salameh and Jaber (2000) using renewal theory. Jaber et al. (2008) extended this by assuming the percentage of defective items per lot reduces according to a learning curve. They examined empirical data from the automotive industry for several learning curve models and the S-shaped logistic learning curve (Jordan, 1958; Carlson, 1973) was found to fit well. Jaggi and Mittal (2011) investigated the effect of deterioration on a retailer's EOQ when the items are of imperfect quality.

In a real manufacturing environment, the defective items are not usually stored in the same

warehouse as the good items. As a result, the holding cost must be different for the good items and the defective ones (e.g. Paknejad et al., 2005). With this consideration in mind, Wahab and Jaber (2010) presented the case where different holding costs for the good and defective items are assumed. In this section, we have cited only references that are directly relevant to this chapter. For more detail about the extension of a modified EOQ model for imperfect quality items, see Khan et al. (2011).

One basic assumption of the above cited contributions is that the demand rate is assumed to be constant and known. A survey of the inventory literature reveals that there is no published work that investigates the model of Wahab and Jaber (2010) for time-varying demand and product deterioration. Product life cycle analysis suggests that a constant demand rate assumption is usually valid in the mature stage of the life cycle of the product. In the growth and/or declining stages, the demand rate can be well approximated by a linear demand function (e.g. Alamri and Balkhi (2007)). Also, one implicit assumption is that the stored items that are screened may retain the same utility indefinitely, i.e. they do not lose their value as time goes on. In fact, the variation of demand and/or product deterioration with time (or due to any other factors) is a quite natural phenomenon.

4.3. Formulation of the general EOQ model

4.3.1. Assumptions and notation

The mathematical model is developed under the following assumptions and notation:

- 1. A single item is held in stock.
- 2. The lead-time is negligible, and no capacity restrictions are assumed, i.e. any

replenishment ordered at the beginning of a cycle arrives just prior to the end of that same cycle.

- 3. The demand, screening and deterioration rates are arbitrary functions of time denoted by D(t), x(t) and $\delta(t)$ respectively.
- 4. The percentage of defective items per lot reduces according to a learning curve denoted by p_j , where j is the cycle index.
- 5. Shortages are not allowed, i.e. we require that $(1-p_j)x(t) \ge D(t) \ \forall \ t \ge 0$.
- 6. The following notations are used for the cost parameters:

c is the unit purchasing cost.

d is the unit screening cost.

 h_{q} denotes the holding cost of good items per unit per unit time.

 h_d denotes the holding cost of defective items per unit per unit time.

k is the ordering cost per cycle.

4.3.2. The model

At the beginning of each cycle j (j=1,2,...), a lot of size Q_j is delivered, which covers the actual demand and deterioration during both the first phase (screening) and the second phase (non-screening). Each lot is subjected to a 100 per cent screening process at a rate of x(t) that starts at the beginning of the cycle and ceases by time T_{1j} , by which point in time Q_j units have been screened and y_j units have been depleted, which is the summation of demand and deterioration. During this phase, items not conforming to certain quality

standards are stored in a different warehouse.

The variation in the inventory level during the first and second phases (Fig. 4.1) and the variation in the inventory level for the defective items (shaded area) are given by (4.1), (4.3) and (4.4) respectively.

$$\frac{dI_{gj}(t)}{dt} = -D(t) - p_j x(t) - \delta(t) I_{gj}(t), \qquad 0 \le t < T_{1j} \quad (4.1)$$

with the boundary condition $I_{gj}(0) = Q_j$,

where

$$Q_{j} = \int_{0}^{T_{1j}} x(u) du. \tag{4.2}$$

$$\frac{dI_{gj}(t)}{dt} = -D(t) - \delta(t)I_{gj}(t), T_{1j} \le t \le T_{2j} {(4.3)}$$

with the boundary condition $I_{gj}(T_{2j}) = 0$.

$$\frac{dI_{dj}(t)}{dt} = p_j x(t), \qquad 0 \le t \le T_{1j} \quad (4.4)$$

with the boundary condition $I_{dj}(0) = 0$.

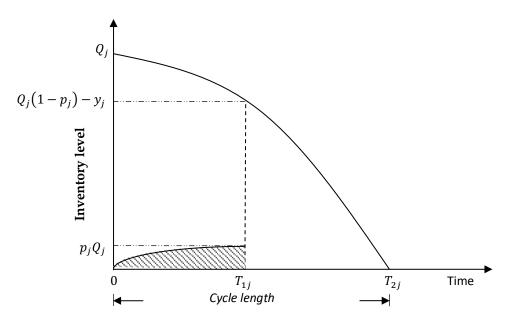


Fig. 4.1. Inventory variation of an Economic Order Quantity (EOQ) model for one cycle.

The solutions of the above differential equations are:

$$I_{gj}(t) = e^{-\left(g(t) - g(0)\right)} \int_0^{T_{1j}} x(u) du - e^{-g(t)} \int_0^t \left[D(u) + p_j x(u)\right] e^{g(u)} du, \ 0 \le t < T_{1j} \quad (4.5)$$

$$I_{gj}(t) = e^{-g(t)} \int_{t}^{T_{2j}} D(u)e^{g(u)} du, \qquad T_{1j} \le t \le T_{2j} \quad (4.6)$$

$$I_{dj}(t) = \int_0^t p_j x(u) du,$$
 $0 \le t \le T_{1j}$ (4.7)

respectively, where

$$g(t) = \int \delta(t) dt. \tag{4.8}$$

The per cycle cost components for the given inventory system are as follows:

Purchasing cost $=c\int_0^{T_{1j}}x(u)du$. Note that this cost includes the defective and deteriorated items.

Holding cost =
$$h_g[I_{gj}(0, T_{1j}) + I_{gj}(T_{1j}, T_{2j})] + h_d I_{dj}(0, T_{1j}).$$

Thus, the total cost per unit time of the underlying inventory system during the cycle $[0, T_{2j}]$, as a function of T_{1j} and T_{2j} , say $Z(T_{1j}, T_{2j})$ is given by:

$$Z(T_{1j}, T_{2j}) = \frac{1}{T_{2j}} \Big\{ (c+d) \int_0^{T_{1j}} x(u) du + h_g \left[-G(0)e^{g(0)} \int_0^{T_{1j}} x(u) du + \int_0^{T_{1j}} D(u)G(u)e^{g(u)} du + \int_0^{T_{1j}} p_j x(u)G(u)e^{g(u)} du + \int_{T_{1j}}^{T_{2j}} D(u)G(u)e^{g(u)} du \right] + h_d \left[\int_0^{T_{1j}} [T_{1j} - u]p_j x(u) du + k \right],$$

$$(4.9)$$

where

$$G(t) = \int e^{-g(t)} dt. \tag{4.10}$$

Our objective is to find T_{1j} and T_{2j} that minimise $Z(T_{1j},T_{2j})$. However, the variables T_{1j} and T_{2j} are related to each other as follows:

$$0 < T_{1j} < T_{2j}, (4.11)$$

$$e^{g(0)} \int_0^{T_{1j}} x(u) du = \int_0^{T_{2j}} D(u) e^{g(u)} du + \int_0^{T_{1j}} p_j x(u) e^{g(u)} du.$$
 (4.12)

Thus, our goal is to solve the following optimisation problem, which we shall call problem (m)

$$(m) = \begin{cases} \text{minimise } Z(T_{1j}, T_{2j}) \text{ given by (4.9)} \\ \text{subject to (4.11) and } h_j = 0 \end{cases}$$

where

$$h_j = e^{g(0)} \int_0^{T_{1j}} x(u) du - \int_0^{T_{1j}} p_j x(u) e^{g(u)} du - \int_0^{T_{2j}} D(u) e^{g(u)} du.$$

It can be noted from Eq. (4.12), that $T_{1j}=0 \Longrightarrow T_{2j}=0$ and $T_{1j}>0 \Longrightarrow T_{1j}< T_{2j}$. Thus Eq. (4.12) implies constraint (4.11). Consequently, if we temporarily ignore the monotony

constraint (4.11) and call the resulting problem (m_1) then (4.11) does satisfy any solution of (m_1) . Hence (m) and (m_1) are equivalent. Moreover, $T_{1j}>0 \Longrightarrow$ right-hand side (RHS) of (4.6) >0, i.e. Eq. (4.12) guarantees that the number of good items is at least equal to the demand during the first phase.

4.4. Solution procedures

First, we note from (4.2) that T_{1j} can be determined as a function of Q_j , say

$$T_{1j} = f_{1j}(Q_j). (4.13)$$

Taking also into account Eq. (4.12) we find that T_{2j} can be determined as a function of T_{1j} , and thus of Q_j , say

$$T_{2j} = f_{2j}(Q_j). (4.14)$$

Thus, if we substitute (4.12)-(4.14) in (4.9) then problem (m) will be converted to the following unconstrained problem with the variable Q_j (which we shall call problem (m_2)).

$$W(Q_{j}) = \frac{1}{f_{2j}} \Big\{ (c+d) \int_{0}^{f_{1j}} x(u) du + h_{g} \left[-G(0)e^{g(0)} \int_{0}^{f_{1j}} x(u) du + \int_{0}^{f_{1j}} p_{j} x(u) G(u) e^{g(u)} du + \int_{0}^{f_{2j}} D(u) G(u) e^{g(u)} du \right] + h_{d} \left[\int_{0}^{f_{1j}} \left[f_{1j} - u \right] p_{j} x(u) du \right] + k \Big\}.$$

$$(4.15)$$

Now, the necessary condition for having a minimum for problem (m_2) is

$$\frac{dW}{dQ_j} = 0. ag{4.16}$$

To find the solution of (4.16), let $W = \frac{w}{f_{2j}}$ then

$$\frac{dW}{dQ_j} = \frac{w'_{Q_j} f_{2j} - f'_{2j,Q_j} w}{f_{2j}^2},\tag{4.17}$$

where w_{Q_j}' and f_{2j,Q_j}' are the derivatives of w and f_{2j} with respect to (w.r.t) Q_j , respectively. Hence, (4.16) is equivalent to

$$w'_{Q_j} f_{2j} = f'_{2j,Q_j} w. (4.18)$$

Also, taking the first derivative of both sides of (4.12) (w.r.t) Q_i we obtain

$$e^{g(0)} - p_j e^{g(f_{1j})} = f'_{2j,Q_j} D(f_{2j}) e^{g(f_{2j})}.$$
(4.19)

From which and (4.13)-(4.15) we have

$$w'_{Q_j} = (c+d) + h_g \left[\left(G(f_{2j}) - G(0) \right) e^{g(0)} + \left(G(f_{1j}) - G(f_{2j}) \right) p_j e^{g(f_{1j})} \right] + \frac{h_d}{x(f_{1j})} \int_0^{f_{1j}} p_j x(u) du.$$
(4.20)

Also, (4.18)
$$\Leftrightarrow W = \frac{w}{f_{2j}} = \frac{w'_{Q_j}}{f'_{2j,Q_j}},$$
 (4.21)

where W is given by (4.15) and w_{Q_j}' is given by (4.20). Eq. (4.21) can be used to determine the optimal value of Q_j and its corresponding total minimum cost. Then the optimal values of T_{1j} and T_{2j} can be found from (4.13) and (4.14), respectively.

4.5. Illustrative examples for different settings

In this section, we present examples to illustrate the theoretical application of our mathematical model and solution procedure, whereby we consider scenarios with varying demand, screening, defectiveness and deterioration rates.

4.5.1. Varying demand, screening, defectiveness and deterioration rates

In practice, the demand for products relies heavily on price (when price elasticity holds), time and quality (Karmarkar and Pitbladdo, 1997). In addition, increasing (decreasing) demand functions over time with quadratic, linear, exponential and stock-dependent trends is a natural phenomenon (Murdeshwar, 1988; Hariga and Benkherouf, 1994; Datta et al. 1998; Alamri, 2011; Benkherouf et al., 2014). For example, essential commodities and seasonal products may follow steadily increasing quadratic or linear demand functions over time (Mandal and Maiti, 2000). On the other hand, exponentially increasing demand applies to products such as new spare parts, new electronic chips and seasonal goods in which the demand rate is likely to increase very rapidly with time (Sana, 2010). Moreover, in some countries (e.g. Saudi Arabia), the entire national (private and government) economy is influenced by such phenomenon. For example, during the month of Ramadan, demand rate for certain commodities and services is likely to increase very fast with time. This can be justified by the fact that the number of visitors increases with time and the fact that customers consumption behaviour during this month differs form that during other months. However, before the end of this month, demand rate for such commodities is likely to decrease very fast with time. Such phenomenon appears also in Saudi Arabia during the

period of Hajj, i.e. the period in which the world's biggest annual gathering of people takes place. In such period, demand rate for certain commodities and services is likely to increase very fast with time until a specific point in time and then demand rate is likely to decrease very fast with time. As such, the mathematical formulation presented in this PhD thesis considers arbitrary functions of time, i.e. the mathematical formulation has no restriction on all functions modelled. This implies that many functions can be incorporated to allow the decision maker to assess and compare the consequences of a diverse range of strategies by employing a single inventory model.

It is worth noting that the dominant form of a learning curve implemented by researchers and practitioners alike is either an S-shaped (Jordan, 1958; Carlson, 1973), or a power one as suggested by Wright (1936); please refer to Jaber (2006) for discussion on this issue.

In this example (Example 4.1), we consider the following functions for varying demand, screening, defectiveness, and deterioration rates:

$$x(t) = at + b,$$
 $D(t) = \alpha t + r,$

$$p_j = \frac{\tau}{\pi + e^{\gamma j}},$$
 $\delta(t) = \frac{l}{z - \beta t}$

where $b, r, \pi, z > 0$; $a, \alpha, \gamma, l, \tau, \beta, t \ge 0$, and $\beta t < z$.

The parameter " α " represents the rate of change in the demand. The case of $\alpha=0$ reflects a constant demand rate, when then $D(t)=r\ \forall\ t\geq 0$. A similar behaviour is observed for the effect of " α ", the rate of change in the screening rate. Note that $\delta(t)$ is an increasing function of time. The case of $\beta=0$ reflects a constant deterioration rate and l=0 corresponds to the case associated with no deterioration. The percentage of defective items per lot reduces

according to an S-shaped logistic learning curve (Jordan, 1958; Carlson, 1973), where τ and π are model parameters, γ is the learning exponent and j is the cycle index. The case $\gamma=0$ applies to a constant percentage of defective items per lot.

The problem (m_2) has been coded in *MATLAB* for the above demand, screening, defectiveness and deterioration rates and solutions were obtained using Eq. (4.21) for a wide range of the control parameter values. Here, we adopt the values considered in the study by Wahab and Jaber (2010), that are presented in Table 4.1 below.

Table 4.1. Input parameters for Example 4.1.

h	h	С	d	а	b
h_g	h_d	C	и	и	D
20	5	100	0.50	1000	100200
Dollars/unit/year	Dollars/unit/year	Dollars/unit	Dollars/unit	Units/year	Units/year
α	r	l	Z	β	k
500	50000	1	20	25	3000
Units/year	Units/year	Units/year	Units/year	Units/year	Dollars/cycle
τ	π	γ			
70.067	819.76	0.7932			
Units/year	Units/year	Units/year			

The optimal values of $Q_j^*, T_{1j}^*, T_{2j}^*, \omega_j^*$, and the corresponding total minimum cost for 10 successive cycles are obtained and the results are shown in Table 4.2. In the first cycle, we have taken $p_1=0.08524$ resulting in a total number of $Q_1^*=3550$ units, which is screened by time $T_{11}^*=0.0354\cong 13$ days and consumed by time $T_{21}^*=0.0648\cong 24$ days. The total minimum cost per year is $W_1^*=5585464$ dollars and the total minimum cost per cycle is $w_1^*=362030$ dollars. The number of defective items is $p_1Q_1^*=303$ units and the number of deteriorated items is $\omega_1^*=5.4$ units, which is the difference between the actual demand and the amount held in stock at the beginning of the cycle, excluding the number of defective

items. The amount $p_1Q_1^*$ may be sold at a salvage price at time T_{11}^* or incur a disposal penalty charge.

In the next section, we analyse the behaviour of the theoretical models in different settings. Table 4.3 depicts the effect of each model parameter on the optimal values. Fig. 4.2 shows the impact of defects and varying demand and deterioration rates on the optimal order quantity. Fig. 4.3 compares the lot size with the same assumed holding costs for the good and defective items with that of differing holding costs. Fig. 4.4 indicates the effect of different learning curves on the optimal order quantities.

Table 4.2. Optimal results for varying demand, screening and deterioration rates with $p_j = \frac{70.067}{819.76 + e^{0.7932 imes j^*}}$

j	p_{j}	f_{1j}^*	f_{2j}^*	Q_j^*	$p_j Q_j^*$	ω_j^*	W_j^*	w_j^*
1	0.08524	0.035424	0.06482	3550	303	5.4	5585464	362030
2	0.08497	0.035419	0.06483	3550	302	5.4	5583830	361980
3	0.08436	0.035407	0.06485	3548	299	5.4	5580142	361850
4	0.08305	0.035380	0.06489	3546	294	5.4	5572240	361580
5	0.08030	0.035324	0.06498	3540	284	5.4	5555724	361020
6	0.07482	0.035212	0.06516	3529	264	5.5	5523107	359900
7	0.06502	0.035013	0.06548	3509	228	5.5	5465734	357890
8	0.05042	0.034715	0.06594	3479	175	5.6	5382467	354900
9	0.03369	0.034376	0.06644	3445	116	5.7	5290159	351490
10	0.01944	0.034088	0.06686	3416	66	5.7	5214030	348600

4.5.2. Sensitivity analysis

The results presented in Table 4.3 summarise the sensitivity analysis of the optimal order quantity, total minimum cost per unit time and total minimum cost per cycle with respect to all model parameters. The first row represents the original values of the proposed model and the last one yields the values of the EOQ model. Fig. 4.2 depicts the effect of each additional model parameter on the EOQ, i.e. the first three values represent the lot sizes of the last three rows (EOQ, EOQ with defect and EOQ with the dis-location of good and defective items, respectively). Whereas the following values reflect the effect of each additional model parameter on the EOQ (Fig. 4.2). Fig. 4.3 replicates the first two rows of Table 4.3 for 10 consecutive cycles to compare the case of having the same holding costs for the good and defective items with that of differing holding costs. Example 4.1 is replicated for 20 consecutive cycles to compare $p_j = \frac{\tau}{\pi + e^{\gamma j}}$ (Jordan, 1958; Carlson, 1973) with $p_j = \frac{\tau}{\pi + 1} j^{-\gamma}$ (Wright, 1936) and the result is shown in Fig. 4.4 for $\tau = 40, \pi = 999, \gamma = 0.75$.

Table 4.3. Sensitivity analysis for the general model.

а	b	α	r	p_{j}	h_g	h_d	l	Z	β	f_{2j}^*	Q_j^*	ω_j^*	W_j^*	w_j^*
1000	100200	500	50000	0.08524	20	5	1	20	25	0.06482	3550	5.4	5585464	362030
1000	100200	500	50000	0.08524	20	20	1	20	25	0.06397	3504	5.2	5586696	357370
1000	100200	500	50000	0.08524	20	5	1	40	25	0.06874	3762	3	5580407	383600
1000	100200	500	50000	0.08524	20	5	1	10	25	0.05827	3196	9	5595130	326040
1000	100200	500	50000	0.08524	20	5	1	20	50	0.06440	3528	5.5	5585745	359730
1000	100200	500	50000	0.08524	20	5	1	20	10	0.06505	3563	5.4	5585305	363310
1000	100200	500	50000	0.08524	20	5	1	20	0	0.06519	3571	5.6	5585202	364110
0	100200	500	50000	0.08524	20	5	1	20	25	0.06482	3550	5.4	5585464	362030
-1000	100200	500	50000	0.08524	20	5	1	20	25	0.06482	3550	5.4	5585464	362030
1000	100200	0	50000	0.08524	20	5	1	20	25	0.06612	3621	5.6	5583646	369190
1000	100200	-500	50000	0.08524	20	5	1	20	25	0.06751	3695	5.9	5581790	376800
0	100200	0	50000	0.08524	20	5	1	20	25	0.06612	3621	5.6	5583646	369190
0	100200	500	50000	0.08524	20	5	0	20	25	0.07321	4003	0	5575183	408140
0	100200	0	50000	0.08524	20	5	0	20	25	0.07511	4105*	0	5573127	418590
0	100200	0	50000	0.08524	20	20	0	20	25	0.07380	4034*	0	5574546	411390
0	100200	0	50000	0	20	20	0	20	25	0.07746	3873	0	5102460	395230

^{*} The order quantity as in Wahab and Jaber (2010).

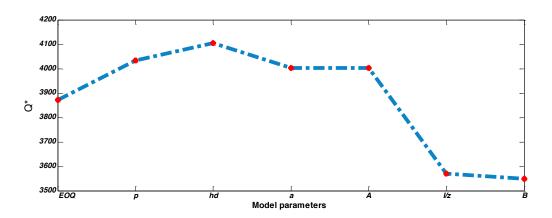


Fig. 4.2. The effect of each additional model parameter on the Economic Order Quantity (EOQ).

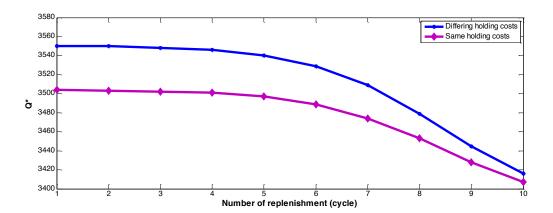


Fig. 4.3. EOQ with same and differing holding costs when $p_j = \frac{70.067}{819.76 + e^{0.7932 imes j}}$.

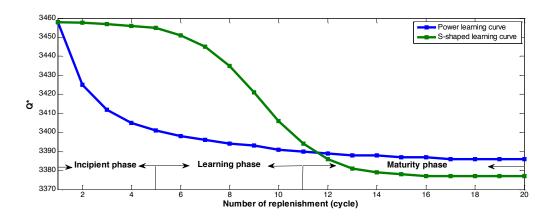


Fig. 4.4. A comparison of the optimal lot sizes for $p_j=rac{40}{999+e^{0.75} imes j}$ and $p_j=rac{40}{999+1}j^{-0.75}$.

In the next section, we list some key findings that depict and emphasise the behaviour of the theoretical model in different settings and relate the results of the study to the general body of knowledge in the discipline.

4.5.3. Findings

- Table 4.2 indicates that the total minimum cost per year and the total minimum cost per cycle decrease as learning increases, which supports the findings presented by Jaber et al. (2008) and Wahab and Jaber (2010).
- The tabulated results indicate that all optimal quantities decrease as learning increases, except for the amount of deteriorated items that incur a minor increase that can be justified by the slight increase in the cycle length (Table 4.2).
- The presence of defects and varying demand and deterioration rates significantly impact on the optimal order quantity (Table 4.2 and Fig. 4.2).
- The lot size with the same assumed holding costs for the good and defective items is less than the one with differing holding costs. However, the difference between the two quantities vanishes as p_j takes on relatively small values (Fig. 4.3). Such finding is consistence with that presented by Jaber et al. (2008).
- The results in Table 4.2 show a slight decrease in the total minimum cost per year due to a slight decrease in p_j . This is true in the incipient phase when an S-shaped logistic learning curve is assumed, which is consistent with the behaviour of slow improvement observed in this short phase, making the S-shaped learning curve an appropriate model to use (Dar-El, 2000). On the other hand, this is not the case when Wright's learning curve is considered, which then leads to smaller quantities in the incipient phase and hence the total minimum cost per year behaves similarly (Fig. 4.4). Fig. 4.4 indicates that the reduction in the total minimum cost per year and the optimal order quantities follow the same fashion as that of p_j .
- The effect of α , the rate of change in the demand significantly influences the optimal order quantity and the total minimum cost per year (Table 4.3). Moreover, this effect

holds true for the case in which the deterioration rate is assumed to be of a fixed value as well as for the case associated with no deterioration (Table 4.3).

- The presence of deterioration has a significant impact on the optimal order quantity and the total minimum cost per year (Table 4.3). Such finding is consistence with that presented by Moussawi-Haidar et al. (2014).
- A comparison between the results obtained in Tables 4.2 and 4.3 reveals that the
 reduction of the optimal order quantity does not imply that the total minimum cost
 per year decreases; in fact, it may increase.
- Previously published models in this area are shown to be special cases of our model (Table 4.3).

4.6. Conclusion and further research

In this section, we summarise and emphasise the financial implications and managerial aspects of our work to illustrate the theoretical application of our mathematical model.

In this chapter, a general EOQ model for items with imperfect quality was presented. The general model developed in this chapter reflects a number of practical concerns with regard to product quality related issues and may assist operations managers to respond to many real-world challenges/opportunities for inventory improvements. Those opportunities include poor supplier service levels (imperfect items received from suppliers), potential dislocation of good and defective items (different warehouses for the good and defective items), and transfer of knowledge from one inventory cycle to another.

Each lot is subjected to a 100 per cent screening and the percentage of defective items per

lot reduces according to a learning curve. Items not conforming to certain quality standards are stored in a separate facility with different holding costs for the good and defective items being considered.

We presented illustrative examples to support application of the model and solution procedure in different realistic situations. The obtained numerical results reflect the learning effects incorporated in the proposed model. The presence of product deterioration and varying demand rate significantly impact on the optimal order quantity. We observed the effect of changing all model parameters and found that a reduction in the optimal order size does not necessarily lead to a lower total minimum cost per unit time.

The generality of our model stems from the fact that the demand, screening, and product deterioration rates are arbitrary functions of time. The proposed model unifies and extends the academic literature related to imperfect quality items, which is quite diverse in nature. Practical examples that are published in the literature for generalised models are used to demonstrate that the solution is the same as in published sources, i.e. the validity of the general model is ascertained. The versatile nature of our model and the fact that it may accommodate many real-world concerns has been emphasised, whereby the results obtained are compatible with the behaviour observed in many real-life settings. A mathematical proof was presented (Appendix A), which shows that the solution to the underlying inventory model, if it exists, is unique and global optimal. To the best of our knowledge, this appears to be the first time that such a general EOQ model is formulated, investigated, and numerically verified.

Based on the findings of this chapter, several interesting lines of further inquiry can be addressed for finite or infinite planning horizons; for example:

- To consider the screening rate follows learning and forgetting curves with allowed shortages.
- To allow for the risk of failure during screening (Type I and Type II errors).
- To consider different supplier trade credit practices, such as a permissible delay in payment.
- To formulate an EPQ model in which product quality levels depend on an instantaneous cost of investing in product innovation.
- To assess the formulation of a two-warehouse system (due to the capacity limitations
 of the OW), where a comparison between LIFO and FIFO dispatching policies governed
 by a fixed shelf life time may be implemented. (This constitutes Part C of this thesis.)

As illustrated in Section 4.5, the dis-location of good and defective items together with other forms of varying demand, screening, defectiveness and deterioration rates may be incorporated to allow managers to assess the consequences of a diverse range of strategies.

The proposed model is not limited to the above contributions; its formulation may trigger other applications as shown in Chapter 5 below.

5. Special cases of the general EOQ model

5.1.Introduction

In this chapter, we further demonstrate the versatile nature of our model and the fact that it may accommodate many real-world concerns. Specifically, we offer several special cases to illustrate the theoretical application of our mathematical model. We aim to address the quality related issues discussed in Chapter 2 when modelling inventories for items that require 100 per cent screening; therefore, underpinning and demonstrating how the terms deterioration, perishability and obsolescence may collectively apply to an item.

The number of special cases covered, and the resulting insights are considered to be of great value to practitioners, since many of these issues have neither been recognised nor analytically examined before. Consequently, inventory managers can establish the optimum order quantities that minimise total system cost.

The behaviour of different conditions, including functions for varying demand, screening, defectiveness and deterioration rates, VOI and perishable items that are subject to deterioration while in storage, is studied using illustrative examples, and interesting insights are offered to practitioners. In this chapter, we consider perishable and non-perishable (infinite shelf lifetime) items, which are subject to deterioration while they are in storage.

The remainder of the chapter is organised as follows: A general solution procedure for intracycle periodic review is presented in Section 5.2. In Section 5.3, we present a detailed example that explains how to manage perishable products. The renewal theory is treated in Section 5.4, followed, in Section 5.5, by coordination mechanism. Stochastic processes, sampling test

and further implications are introduced in Sections 5.6, 5.7 and 5.8, respectively. A summary of implications and managerial insights is given in Section 5.9. Concluding remarks are provided in Section 5.10.

5.2. Intra-cycle periodic review

It is often desirable to adjust input parameters to be responsive to a new policy due to acquired new knowledge. Such adjustment may occur due to the dynamic nature of demand, screening and deterioration rates or as a result of price fluctuations. Therefore, the periodic review is also beneficial to illustrate what happens if the decision maker deviates from the optimal solution to assess the consequences of such a deviation. In this section, we present a step-by-step solution procedure to determine the optimal policy for intra-cycle periodic review applications.

5.2.1. Solution procedure

For each periodic review:

- 1. Reset the new input parameters and obtain the optimal values using Eq. (4.21).
- 2. The optimal quantity that needs to be added to the on-hand inventory for the next replenishment is given by

$$Q_{rj} = Q_j - I_{gj-1}(t_{rj}), (5.1)$$

where t_{ri} is the time up to the periodic review.

From Eq. (5.1) we distinguish two cases.

Case 1: $0 \le t_{ri} < T_{1i-1}$.

Considering Eqs. (4.5)-(4.7) and (5.1) we have

$$Q_{rj} = \int_0^{T_{1j}} x(u) du - e^{-\left(g(t_{rj}) - g(0)\right)} \int_0^{T_{1j-1}} x(u) du + e^{-g(t_{rj})} \int_0^{t_{rj}} \left[D(u) + p_j x(u)\right] e^{g(u)} du,$$

$$0 \le t_{rj} < T_{1j-1}$$
(5.2)

from which the number of units to be screened is given by

$$q_{rj} = Q_{rj} + \int_{t_{rj}}^{T_{1j-1}} x(u) du. \qquad 0 \le t_{rj} < T_{1j-1}$$
 (5.3)

Note that the time T_{qj} , by which q_{rj} units are screened can be readily determined by

$$q_{rj} = \int_0^{T_{qj}} x(u) du$$
, where $q_{rj} \geq Q_{rj}$ and $s_{rj} = \int_0^{t_{rj}} x(u) du$.

Thus, the total cost per unit time of the underlying inventory system during the periodic review is adjusted as:

$$W(Q_{j}) = \frac{1}{f_{2j}} \left\{ cQ_{rj} + dq_{rj} + h_{g} \left[-G(0)e^{g(0)} \int_{0}^{f_{qj}} x(u)du + \int_{0}^{f_{qj}} p_{j}x(u)G(u)e^{g(u)}du + \int_{0}^{f_{2j}} D(u)G(u)e^{g(u)}du \right] + h_{d} \left[f_{qj} \left(\int_{0}^{f_{rj}} px(u)du + \int_{0}^{f_{qj}} px(u)du \right) - \int_{0}^{f_{qj}} up_{j}x(u)du \right] + k \right\}.$$

$$(5.4)$$

It is worth noting here that the number of defective items held up to the periodic review may be sold at a salvage price at time t_{rj} . In this case, we can set $t_{rj}=0$ (without loss of generality) in Eq. (5.4) or it can be kept as is in Eq. (5.4) up to time T_{qj} by which the screening process ceases. Moreover, in the extreme case $t_{rj}=0 \implies T_{qj}=T_{1j-1}$, then the left-hand side (LHS) of (5.2) is equal to zero (recall (5.3)), then the optimal values resulting from solving

Eq. (4.21) constitute the optimal policy for the decision maker. Alternatively, Q_{rj} is to be substituted by q_{rj} in Eq. (5.4).

Case 2: $T_{1j-1} \le t_{rj} \le T_{2j-1}$.

$$Q_{rj} = \int_0^{T_{1j}} x(u) du - e^{-g(t_{rj})} \int_{t_{rj}}^{T_{2j-1}} D(u) e^{g(u)} du, \qquad T_{1j-1} \le t_{rj} \le T_{2j-1}$$
 (5.5)

Note that $Q_{rj}=q_{rj}$, i.e. the items ordered to fulfil the demand, defects and deterioration during the planning horizon are the only ones that need to be screened (recall that the onhand inventory has already been screened).

Thus, the total cost per unit time of the underlying inventory system during the periodic review is adjusted to:

$$W(Q_{j}) = \frac{1}{f_{2j}} \Big\{ (c+d)Q_{rj} + h_{g} \Big[-G(0)e^{g(0)} \int_{0}^{f_{qj}} x(u)du + \int_{0}^{f_{qj}} p_{j}x(u)G(u)e^{g(u)}du + \int_{0}^{f_{2j}} D(u)G(u)e^{g(u)}du \Big] + h_{d} \Big[f_{qj} \Big(\int_{0}^{f_{rj}} px(u)du + \int_{0}^{f_{qj}} px(u)du \Big) - \int_{0}^{f_{qj}} up_{j}x(u)du \Big] + k \Big\}.$$

$$(5.6)$$

From Eq. (5.6), the extreme case $t_{rj}=T_{2j-1}\Longrightarrow T_{qj}=T_{1j}$ (recall (5.5)).

Remark 5.1

The above suggested procedure is valid for $t_{rj} \in [0, T_{2j-1}]$ as well as for the generalised models and the proposed idea can be further extended to be implemented in inventory

mathematical modelling. Note that the structure of the model allows for both continuous and discrete periodic review.

5.2.2. Numerical verification

Let us now assume that the decision maker would like to change the current status within the fifth cycle. Here, we consider the same set of values as in the previous example (Example 4.1) except that a different demand rate is assumed, r = 45000, and the coordination regarding the on-hand inventory for the fifth batch of defective items has been made, i.e. $p_6=0.07482$ is to be implemented. In this example (Example 5.1), the optimal values of Q_6^* , Q_{r6}^* , Q_{r6}^* , Q_{r6}^* , Z_{16}^* , Z_{26}^* , Z_{26}^* , and the total minimum cost are obtained for a given periodic review time, say $t_{r6} = 0.0137 = 5$ days. In this periodic cycle, we have taken $p_6 = 0.07482$ resulting in a total number of $Q_6^*\cong 3348$ units, which is consumed by time $T_{26}^*=0.0687\cong$ 25 days. The optimal quantity that needs to be added to the on-hand inventory is $Q_{r6}^* = 617$ units. The number of units that need to be screened is $q_{r6}^* \cong 2784$ units, which is being done by time $T_{q6}^* = 0.0278 \cong 10$ days, by which point in time the total number of defective items is accumulated. The total minimum cost per year is $W_{r6}^{*}=877640~\mathrm{dollars}$ and the total minimum cost in this periodic cycle is $w_{r6}^* = 60270$ dollars. The number of defective items is $p_6q_{r6}^*=208$ units. Note that this amount is to be added to the previous defective items that have been accumulated during time $t_{r6}=0.0137$, i.e. $p_5s_{r6}=103$ units, where both quantities constitute the total number of defective items.

5.3. Perishable products

In real life settings, a large number of perishable items encounter deterioration that occurs out of sequence. This can be attributed to random lifetimes that are associated with the time elapsing for the items to flow through the supply chain. Packaged foods, seafood, fruit, baked goods, milk, cheese, processed meat, pharmaceutical and blood products, etc. would be examples of such items (Lashgari et al., 2016).

Appropriate management of perishable inventories, in conjunction with modern technologies, play an important role in monitoring the condition of those goods in different stages of the supply chain. Ketzenberg et al. (2015) emphasised the importance of the value of information generated from using different systems in the decision-making process in the grocery industry (which is associated with low net margins). For example, continuous automated inventory control systems are capable of tracking, recording and transmitting relevant information regarding an item as it moves through the network.

The deployment of RFID systems, data loggers and time—temperature integrators and sensors lead to a reduction in product spoilage and economic benefits (Ketzenberg et al., 2015). The potential benefits of RFID for logistics, transportation and warehousing relate to increased supply chain visibility, which in turn lowers safety stocks and provides the same or even better customer service level (Gaukler et al., 2007; Kim and Glock 2014).

This study intersects the areas of fixed and random items' lifetimes, since the assumption that each lot is subjected to a 100 per cent screening will render the (potential) random lifetime of a product deterministic. In this regard, a 100 per cent screening assumption not only guarantees the isolation of defective and/or already perished items, but also classifies the

order quantity based on a FEFO policy, rather than a FIFO one. The focus of this chapter is on the value and use of technologies such as RFID to capture the TTH to model shelf lifetime and not the technologies themselves.

As discussed in Chapter 2, the terms "deterioration", "perishability" and "obsolescence" are linked to specific underlying assumptions regarding the physical state/fitness and behaviour of items over time. As we attempt to provide a general special case model that considers many possible practical scenarios, the behaviour of these conditions will be discussed through illustrative examples given in Section 5.3.2 below. In this study, we consider perishable and non-perishable (infinite shelf lifetime) items, which are subject to deterioration while they are in storage.

5.3.1. The model

To show that our model can be easily responsive to manage such perishable items, consider the amount ordered $\underline{Q_j} = \left(q_{mj}, q_{m-1j}, \dots, q_{0j}\right)$ where q_{ij} is the number of units with $i(i=0,1,\dots,m)$ useful periods of shelf lifetime. The special case of shelf lifetime equal to zero refers to newly replenished items that have arrived already perished or items not satisfying certain quality standards (defective items). It is worth noting here that the assumption that each lot is subject to a 100 per cent screening underpins such classification, where $p_jQ_j=q_{0j}$. It is often the case that the system is credited so that no outdating costs apply for this quantity. However, the potential interest exists so as to reduce the presence of both defective and already perished items in subsequent replenishments. Therefore, coordination can be made between inter-related entities from which we can set $p_{j+1}=\xi\left(\frac{q_{0j}}{Q_i}\right)$ without loss of

generality. This is so, since such an assumption seems realistic, given that any information gained from previous replenishments can be incorporated to enhance the subsequent delivery.

Now, let ω_{ij} denote the quantity of the on-hand inventory of shelf lifetime i that perishes by the end of period i. Thus, we have

$$\omega_{ij} =$$

$$\begin{cases} q_{ij} - \left[D_{ij} - \left(\sum_{x=1}^{i-1} q_{xj} - \sum_{x=1}^{i-1} \omega_{xj} - \sum_{x=1}^{i} d_{xj} \right) \right], & D_{ij} < \left(\sum_{x=1}^{i} q_{xj} - \sum_{x=1}^{i-1} \omega_{xj} - \sum_{x=1}^{i} d_{xj} \right) \\ 0 & \text{otherwise,} \end{cases}$$

where D_{ij} is the actual demand observed up to the periodic review i, and d_{ij} is the number of items of shelf lifetime i that deteriorate while on storage. Hence, $\sum_{i=1}^m \omega_{ij}$ denotes the total sum of inventory that perishes in cycle j, excluding any replenished items that have arrived already perished, and $\sum_{y=i}^m d_{yj}$ refers to the total sum of deteriorated items in period i, i.e. an item may not retain the same utility throughout its shelf lifetime. Therefore, the two amounts that need to be discarded in each periodic review i are ω_{ij} and $\sum_{y=i}^m d_{yj}$. Assuming an automated inventory control system, the observation of D_{ij} seems realistic since all items are tracked. Thus, the information gained so far, collectively, constitutes a means by which the input parameters can be known and then may or may not be adjusted.

Note that $\underline{Q_j} = \left(q_{mj}, q_{m-1j}, \dots, q_{0j}\right)$ and the amounts ω_{ij} and $\sum_{y=i}^m d_{yj}$ are known and that D_{ij} is fulfilled based on a FEFO policy. Then we have

$$I_{gj}(t_{ij}) = \begin{cases} Q_j - q_{0sj} - D_{ij} - \sum_{x=1}^{i} \omega_{xj} - \sum_{x=1}^{i} \sum_{y=x}^{m} d_{yj}, & 0 \le t_{ij} < T_{1j}, \\ (1 - p_j)Q_j - D_{ij} - \sum_{x=1}^{i} \omega_{xj} - \sum_{x=1}^{i} \sum_{y=x}^{m} d_{yj}, & T_{1j} \le t_{ij} \le T_{2j}, \end{cases}$$
(5.7)

$$\Leftrightarrow I_{gj}(t_{ij}) = \begin{cases} (q_{m-ij}, q_{m-i-1j}, \dots, q_{1j}, q_{0rj}), & 0 \le t_{ij} < T_{1j}, \\ (q_{m-ij}, q_{m-i-1j}, \dots, q_{1j}), & T_{1j} \le t_{ij} \le T_{2j}, \end{cases}$$

where $q_{0rj} = \int_{t_{ij}}^{T_{1j}} p_j x(u) du$, $q_{0sj} + q_{0rj} = p_j Q_j$. Thus, the optimal quantity that needs to be added to the on-hand inventory for the next replenishment is given by

$$\underline{Q_{ij}} = \underline{Q_{j+1}} - I_{gj}(t_{ij} + \Delta). \tag{5.8}$$

The necessary condition to place an order is given by

$$I_{gj}(t_{ij}) \le (D_{ij} + \omega_{ij} + \sum_{y=i}^{m} d_{yj} - D_{i-1j})\Delta,$$
 (5.9)

with a lead-time $\Delta(\Delta \leq T_{2j}-t_{ij})$, the initial amount $D_{0j}=0$ and t_{ij} being the time up to the periodic review. If condition (5.9) holds true for periodic review i, then Eq. (5.8) calculates the next optimal replenishment quantity that needs to be added to the on-hand inventory (given by (5.7)). In Eq. (5.9), the quantity $(D_{ij}+\omega_{ij}+\sum_{y=i}^m d_{yj}-D_{i-1j})$ is taken as an approximation for the behaviour of inventory fluctuation during the lead-time Δ .

Note that if $\underline{Q_{ij}} = \underline{Q_{j+1}}$, then we may assume that unsatisfied demand is lost. On the other hand, if demand is fulfilled based on a record of known quantity, then the unsatisfied demand $D_{lj} = \left|I_{gj}(t_{ij} + \Delta)\right|$ is known and consequently any relevant cost may apply. In this case, $(1-\varphi)\left|I_{gj}(t_{ij} + \Delta)\right|$ forms the lost sales quantity with a fraction $\varphi(0 \le \varphi \le 1)$ being backordered, i.e. $I_{gj}(t_{ij} + \Delta) < 0$.

5.3.2. Numerical verification

We now introduce another example (Example 5.2) where we consider the values summarised in Table 5.1 below for an item with a maximum shelf life-time m = 5, i.e. $i \in [0,5]$.

Table 5.1. Input parameters for Example 5.2.

h_g	h_d	С	d	а	b
0.4	0.1	10	0.2	500	20000
Dollars/unit/week	Dollars/unit/week	Dollars/unit	Dollars/unit	Units/week	Units/week
α	r	l	Z	β	k
50	700	1	20	2	300
Units/week	Units/week	Units/week	Units/week	Units/week	Dollars/cycle
τ	π	γ	m	Δ	
1	49	0	5	1	
Units/week	Units/week	Units/week	days	days	

The optimal values of $Q_j^*, T_{1j}^*, T_{2j}^*, \omega_j^*$, and the corresponding total minimum cost is obtained. The system parameters specified in Table 5.1 yield a lot size of $Q_1^*\cong 518$ units, which is screened by time $T_{11}^*=0.026\cong 0.18$ days and consumed by time $T_{21}^*=0.695\cong 4.8$ days. The total minimum cost per week is $W_1^*=8136$ dollars and the total minimum cost per cycle is $w_1^*=5650$ dollars. The number of defective items is $p_1Q_1^*=10.4$ units and the amount of outdated (spoiled) and/or deteriorated items is $\omega_1^*=9$ units. If it is beneficial to operate on a discrete cycle length of a complete period, then $W_1^*=\min\left[W_1=\frac{w}{0.5714},W_1=\frac{w}{0.7143}\right]$. The optimal order quantity in this case is $Q_1^*=533$ units, which is given by $W_1^*=\frac{w_1^*}{0.7143}=8136$ dollars per week to satisfy the demand, defects and deterioration for five days. This quantity is screened by time $T_{11}^*=0.027\cong 0.19$ days and the total minimum cost per cycle is $w_1^*=5812$ dollars. The number of defective items is $p_1Q_1^*=10.6$ units and the number of outdated (spoiled) and/or deteriorated items is $\omega_1^*=9.6$ units.

Suppose that after a 100 per cent screening the lot size is classified based on a FEFO policy and is found to be on the set $\underline{Q_1}=(120,114,134,91,67,7)$, which corresponds to a 5-day policy, i.e. $Q_1^*=533$ units. Now, let us assume that at the end of the first day the relevant information gathered indicates that $D_{11}=63$, $\omega_{11}=3$ and $\sum_{y=1}^5 d_{y1}=1+0+1+0+0=2$, then $I_{g1}(0.1429)=526-63-3-2=458$ units. The necessary condition to place an order is $I_{g1}(t_{11}) \leq \left(D_{11}+\omega_{11}+\sum_{y=1}^5 d_{y1}-D_{01}\right)(T_{21}-t_{11})$, but $I_{g1}(0.1429)>68(4)$ and consequently we do not place an order. Suppose that after the third day we have:

 $D_{21}=151,\ D_{31}=276,\ \omega_{31}=7,\ \sum_{i=1}^3\omega_{i1}=12,\ \sum_{y=3}^5d_{y1}=0+1+0=1$ and $\sum_{x=1}^3\sum_{y=x}^5d_{y1}=1+1+2+2+1=7$, then $I_{g1}(0.429)=526-276-12-7=231$ units and $I_{g1}(0.429)<133(2)$. Thus, an order must be placed in which Eqs. (4.21) and (5.8) can be used to obtain the optimal replenishment quantity that takes into account a suitable adjustment to avoid lost sales. As such, approximations for the demand and deteriorating rates, say $\hat{r}=\widehat{D}_J=\frac{\sum_{i=1}^m(D_{ij}-D_{i-1j})}{m},\ \hat{\alpha}=\max\left[0,\alpha\left(\frac{Q_{j+1}}{Q_j}\right)\right],\ \text{ and }\ \hat{z}=z\frac{\sum_{i=1}^md_{ij}+\sum_{i=1}^m\omega_{ij}}{\omega_j^*}$ may be employed. Note that if a record is kept for the actual demand requested, then the unsatisfied demand is given by D_{lj} , where φD_{lj} is backordered and the rest $(1-\varphi)D_{lj}$ is lost.

5.3.3. Time and temperature history (TTH)

In this section, we consider VOI such as TTH of an item as it moves through a supply chain is transferable within that supply chain. In this case, the remaining shelf lifetime can be readily calculated. For example, Bremner (1984) and Ronsivalli and Charm (1975) developed a shelf

lifetime model for fresh fish that links the spoilage rate to a given temperature.

Now, let ${}^{\circ}C_y$ and t_y denote respectively, the temperature and time elapsed of an item in a supply chain entity y, then the remaining shelf lifetime is given by

$$L = M - s(^{\circ}C_a)t_a - s(^{\circ}C_b)t_b,$$

where

$$M = m + t_a + t_b$$
 and $s({}^{\circ}C_v) = (0.1 {}^{\circ}C_v + 1)^2$.

If this VOI is available to the next supply chain entity x, then a significant reduction in the cost per cycle can be achieved (Ketzenberg et al., 2015). In our model, the VOI can be perceived at external and/or internal domains of coordination. At the domain of external coordination, this model addresses the VOI to capture a safe remaining shelf lifetime and acknowledges the potential impact of transporting and handling a product at both external and internal levels. Hence, the reflection of the VOI can result in a reduction of the percentage of imperfect items that may arrive already perished and/or defective, i.e.

$$f_{2j} \le L$$
, $\delta(t) = \frac{l}{z(1+\gamma)-\beta t}$, $p_j = \frac{\tau}{\pi + e^{\gamma j}}$, $\gamma = \frac{\phi}{M}$ and $\phi = [s(^{\circ}C_a)t_a + s(^{\circ}C_b)t_b]$.

To illustrate this, we introduce an example, (Example 5.3) where the same set of values as in the previous example (Example 5.2) is considered.

Now, let $M=m+t_a+t_b$, ${}^{\circ}\mathrm{C}_a=3$, $t_a=2$, ${}^{\circ}\mathrm{C}_b=0$ and $t_b=2$, then $L=9-3.38-2=3.62\cong 4$ days. if this information is available, then the optimal quantity with shelf lifetime $i\in[0,4]$ is $Q_1^*=420$ units, which is consumed in 4 days. The total minimum cost per week is $W_1^*=8108$ dollars and the total minimum cost per cycle is $w_1^*=4633$ dollars. The

number of defective items is $p_1Q_1^*=8.3$ units and the number of outdated (spoiled) and/or deteriorated items is $\omega_1^*=3.7$ units. Thus, with the VOI, a reduction of $c_s=1460$ dollars/year can be achieved for this single item.

Remark 5.2

The proposed model is viable for the case in which items are classified based on their quality, size, appearance, freshness, etc. In this case, a distinct selling price s_i may be linked to its corresponding quantity q_{ij} , i.e. $\underline{S} = (s_m, s_{m-1}, ..., s_0)$ is applied for the set $\underline{Q_j} = (q_{mj}, q_{m-1j}, ..., q_{0j})$. Further, it is still applicable if an item partially loses its value based on its perceived actuality (obsolescence). Here ω_{ij} can be kept in store at a discounted price and $s({}^{\circ}C_y)$ is based on the shelf lifetime model suitable for the item ordered.

5.4. Renewal theory

With regard to defective items, according to the academic literature, a random proportion of such items is usually assumed with a known probability distribution. Hence, from (4.21) we have

$$E(W) = \frac{E(w)}{E(f_{2j})} = \frac{E(w'_{Q_j})}{E(f'_{2j,Q_j})},$$
(5.10)

where $(1 - E[p_j])x(t) > D(t)$.

Eqs. (4.2), (4.14), (4.15), (4.19) and (4.20) can be used to find Eq. (5.10).

If
$$\delta(t) = \frac{l}{z} = \theta$$
, $D(t) = r$, and $x(t) = b$, then we have

$$E\big[f_{2j}\big] = \frac{\log\left(\frac{E[K]\theta}{r}\right)}{\theta}, E[K] = \left(bf_{1j} - \frac{E[p_j]e^{\theta f_{1j}}b}{\theta}\right) + \frac{E[p_j]b}{\theta} + \frac{r}{\theta}.$$

In this thesis, we have used the assumption that defective items are stored in a different warehouse. This assumption relaxes the behaviour of the inventory level that is presented by Jaggi and Mittal (2011) and Moussawi-Haidar et al. (2014). This is because not every defective item can be sold at a salvage price; rather, defective items may engender a disposal cost.

For comparison purposes, Table 5.2 presents the input parameters of the example used by Jaggi and Mittal (2011). Their model leads to $Q^{JM}=1283$ units, which is larger than the optimal quantity obtained using Eq. (4.21) in our model, i.e. $Q^*=1167$ units.

Table 5.2. Input parameters for comparison examples for renewal theory $(p \sim U[0,\ell], E[p] = \int_0^\ell pf(p) \, dp = \frac{\ell}{2})$.

h	С	d	k	r	b
5	25	0.25	100	50000	175200
Dollars/unit/year	Dollars/unit	Dollars/unit	Dollars/cycle	Units/year	Units/year
l	Z	l			
1	10	0.04			
Units/year	Units/year	Units/year			

Similarly, Moussawi-Haidar et al. (2014) use the same set of values from Jaggi and Mittal (2011) except from the following parameters: d=0.5 dollars, z=20, where $Q^{M\ et\ al}=1280$ units, which is greater than our optimal $Q^*=1278$ units. In both papers, the objective is to maximise the total profit per unit time, where s=50 dollars (v=20 dollars) is the selling price for a good (defective) item. Therefore, the maximum total profit per year, is set equal to:

$$TPU^*(Q) = \frac{s(Q^*(1-p)-\omega^*) + vpQ^*}{T_2^*} - W^* = \frac{50 \times (1166.8 \times 0.98 - 1.3) + 20 \times (1166.8) \times 0.02}{0.0228} - 1297016 = \frac{1297016}{1297016} = \frac{12$$

 $1228200 \text{ dollars} > TPU(Q)^{JM} = 1224183 \text{ dollars}.$

On the other hand, when d=0.5 dollars, z=20, the corresponding maximum total profit per year is reduced to $TPU^*(Q)=1211415$ dollars $\cong TPU(Q)^{M\ et\ al}=1211414$ dollars. Although the difference in the order quantities is negligible between the two compared papers, our model produces a larger quantity when the deterioration rate decreases, which supports the findings presented by Moussawi-Haidar et al. (2014).

Now, if
$$\delta(t) = 0$$
, then $\frac{E(w)}{(\alpha E[f_{2j}] + r)E[f_{2j}]} = \frac{E(w'_{Q_j})}{(1 - E[p_j])}$, $E[f_{2j}] = \frac{-r + (r^2 + 2\alpha Q_j(1 - E[p_j]))^{\frac{1}{2}}}{\alpha}$.

For simplicity, let D(t)=r, x(t)=b, and $\delta(t)=0$, then Eq. (5.10) reduces to the model of Wahab and Jaber (2010) as follows: $Q_j^*=\sqrt{\frac{2rk}{h_g E\left[\left(1-p_j\right)^2\right]+\frac{rE\left[p_j\right]}{b}\left[h_g+h_d\right]}}.$

For $h_g=h_d=h$, it reduces to the work of Jaber et al. (2008) and with $p_j=p$ it yields the optimal order quantity presented by Salameh and Jaber (2000) and (Maddah and Jaber (2008)

as follows:
$$Q_j^* = \sqrt{\frac{2rk}{hE[(1-p)^2] + \frac{2hrE[p]}{b}}}$$
. Finally, if $p_j = 0$, then, $Q_j^* = \sqrt{\frac{2rk}{h}} = \text{EOQ}$.

5.5. Coordination mechanisms

The generic nature of the model explores various coordination mechanisms that may improve inventory management as shown below.

Let $\dot{D}(t)=(q-c_1e^{-c_dg})D(t)>0$, where $\dot{D}(t)$ is the demand based on an acceptance quality level $q(q_{min}\leq q\leq 1)$ and a discount rate $g(0\leq g\leq 1)$ for a cut-price c_d offered by

the supplier for a single purchased item and c_1 is a positive parameter. The case of $c_1=0$ implies that $\dot{D}(t)=qD(t)$, and the case of $c_1=0$ and q=1 reflects an independent demand function, where $\dot{D}(t)=D(t) \ \forall t\geq 0$. Any item that does not satisfy the minimum acceptance quality level q_{min} is considered a defective item.

This function may apply for a demand-driven pricing model assumed by the supplier for which a unit purchasing price $\dot{c}=ce$, where e can take the form $e=q-c_d g>0$ or $e=q-\frac{c_d g}{q}>0$. The case q=1 applies for a discounted purchasing price, where $e=1-c_d g$, and the case of g=0 and q=1 reflects an independent purchasing cost, where $\dot{c}=c$. Note that $\dot{D}(t)$ increases (decreases) as the acceptance quality level and/or discount rate increases (decreases).

Such a contract unifies three managerial decision strategies that govern both the supplier and the retailer, i.e. the acceptance quality level, unit discount rate and unit purchasing price. Moreover, it encourages the supplier to invest in quality innovation to maintain sustainable product quality levels that may reduce defects per shipment in order to maximise its discounted stream of net revenue. Further, for the case of $e=q-c_d \mathcal{G}$, or $e=q-\frac{c_d \mathcal{G}}{q}$, the supplier would benefit from improving quality levels by increasing the purchase price, while simultaneously the retailer would incur an additional charge payable to the supplier in order to receive better quality items.

Another demand function that can be integrated by improving the quality level and decreased by increasing the unit purchasing price may be implemented, where $\ddot{D}(t) = D(t) - c_1 \ddot{c} e^{-c_2 q} > 0$ and c_2 is a positive parameter (Vörös, 2002).

A similar demand function that depends on price and quality may take the form $\ddot{D}(t)=$ Special cases of the general EOQ model - 77 -

 $D(t)-c_1\ddot{c}+c_2q$, where $\ddot{D}(t)$ increases with an acceptance quality level $q(q_{min} \le q \le 1)$ and decreases with a unit purchasing price \ddot{c} (Chenavaz, 2012).

For all the above scenarios, a unit purchasing price $\dot{c}=\ddot{c}=\ddot{c}=cq$ can also be incorporated. In the last two demand functions, the case of $c_1=c_2=0$ reflects an independent arbitrary demand function.

In a decentralised coordinated scenario, where the supplier and the retailer cooperate in order to render the total minimum (maximum) cost (revenue) closer to that associated with a centralised one, a selling price for the retailer, say $c_3se^{-c_4s}$, can be assumed with $s \ge \dot{c}$ and c_3 and c_4 being positive parameters (Smith and Achabal, 1998; Roy et al., 2015). It is worth noting here that c_4 must be chosen such that $\frac{1}{c_4} > \dot{c}$.

5.6. Stochastic parameters

It is often the case that input parameters are randomly distributed. The versatile nature of our model accommodates such randomness as shown below.

Let D_j be a random variable of the demand that is predetermined according to the information gained by the supplier due to its coordination as an output of the j^{th} inspection process. For example, suppose that $D_j \sim U\left[\mu_j - \sqrt{3}\sigma_j, \mu_j + \sqrt{3}\sigma_j\right]$. It is clear that $E(D_j) = \mu_j = D(t) = r$ (Roy et al., 2013; Modaka et al., 2016). Similarly, $E(x_j) = x_j = x(t) = b$, and $E\left(\delta_j\right) = \delta_j = \delta(t) = \frac{l}{z'}$ which is the case provided in Section 5.2.3. Note that D_j , and hence the actual yield, may vary from one cycle to another (e.g. the parameters are nonstationary).

If q is assumed to be a random variable with known mean and variance, then the yield at the supplier site would be represented by a random draw from a quality distribution. If this is the case, then the yield is simultaneously influenced by internal and external randomness.

5.7.A 100 per cent inspection and sampling test

There is no doubt that many products require inspection, so as to guarantee an appropriate service to customers. In addition, such inspection is essential to update the Information System records with good items that are actually available in stock in order to satisfy demand. Further, when new components are required in a production setting, their ordering policy depends on the production batch size of the products that require such components. Therefore, the presence of defective components has a direct impact on the production batch size. Moreover, there exists a plethora of factors that may force supply chain management to initiate both an inspection process and periodic review to enhance productivity, improve profitability, meet total product demand and avoid the tarnished reputation associated with product recalls (Klassen and Vereecke 2012). A 100 per cent inspection may eliminate the return service cost caused by defective items. However, the assumption that each lot undergoes a 100 per cent inspection implicitly applies to any smaller amount of the lot. For example, let ϵ be a fraction of the amount ordered representing a random sample size drawn from the batch. It is clear that $\epsilon x(t)$ can also be implemented in the model.

5.8. Further implications

If safety issues arise from keeping defective items in store, then the model formulation allows for an immediate disposal of defective items, i.e. $h_d=0$.

In practice, the actual consumption period is random and, consequently, $t_{rj}(T_{1j-1} \le t_{rj} \le T_{2j-1})$ can be used to represent the actual cycle length. If $I_{gj-1}(t_{rj}) > 0$, then the subsequent replenishment is cycle dependent, where Eq. (5.6) can be used to derive the optimal lot size.

Remark 5.3

To avoid repetition, it is important to note here that the findings presented in Chapter 4 apply for every single case that are addressed in this chapter.

In the next section, we summarise the implications and managerial insights addressed in this chapter. In addition, we relate the research contributions to the general body of knowledge in the discipline and emphasise the fact that the model may trigger other applications that can be disseminated from the general formulation.

5.9. Summary of implications and managerial insights

- The generic nature of our model enables the decision maker to incorporate the desired functions that are suitable to a system.
- A general step-by-step solution procedure to determine the optimal policy for

continuous intra-cycle periodic review applications is presented. The suggested procedure is valid for generalised models and the proposed idea can be further extended to be implemented in inventory mathematical modelling. The structure of the model allows for both continuous and discrete periodic review. The proposed solution procedure considers different inventory fluctuations during the planning horizon.

- Clearer definitions associated with the terms deterioration, perishability and obsolescence to purify and distinguish the role of each term for the model are presented.
- A detailed method is provided that underpins and portrays how deterioration,
 perishability and obsolescence may collectively apply to an item.
- The proposed model intersects the areas of fixed and random lifetimes of perishable products, where unsatisfied demand may or may not be lost, products may arrive already perished, and a product may not retain the same utility throughout its shelf lifetime.
- The proposed model is viable for the case in which items are classified based on their quality, size, appearance, freshness, etc.
- The accuracy of RFID temperature tags that capture the TTH, and the use of that TTH
 data are adopted to model the shelf lifetime of an item. In this case, the VOI can be
 quite valuable in reducing the total system cost.
- The VOI can be perceived at external and/or internal domains of coordination. At the
 domain of external coordination, the VOI captures a safe remaining shelf lifetime and
 acknowledges the potential impact of transporting and handling a product at both
 external and internal levels.

- The mathematical formulation is linked to the renewal theory to show that previously published models in this area constitute special cases of our model.
- Coordination mechanisms that may affect the supplier and the retailer are explored to improve inventory control at both echelons.
- The versatile nature of our model accommodates stochastic process, where input
 parameters are randomly distributed. It also accounts for the case when subsequent
 replenishment is cycle dependent.
- The dis-location of good and defective items allows for an immediate disposal of defective and/or already perished items in case of any potential safety issues.
- The proposed model is a viable solution for a 100 per cent inspection and for any smaller amount of the lot, i.e. a random sample size drawn from the batch.

5.10. Conclusion and further research

In this chapter, we have extended the financial implications and managerial insights of our work whereby we offered a number of special cases to illustrate the theoretical application of our general model. The special cases covered respond to many real-world challenges/opportunities for inventory improvements. Product quality related issues, including defective and/or already perished items received from suppliers, dis-location of good and defective items, tracking the quality of perishable products in a supply chain and transfer of knowledge from one inventory cycle to another, were modelled.

The mathematical formulation of our model intersects the areas of fixed and random lifetimes of perishable products, whereby the value of the temperature history and flow time through

the supply chain is also used to determine an appropriate policy. Furthermore, it provides a general procedure for continuous intra-cycle periodic reviews so as to adjust and control the flow of raw materials, component parts and finished goods to maintain sustainable competitive advantage. Coordination mechanisms and managerial decision strategies that govern both the supplier and the retailer were also addressed to improve inventory management at both echelons.

We tested and observed the behaviour of varying demand, screening, defective and deterioration rates, VOI and perishable and non-perishable (infinite shelf lifetime) items that are subject to deterioration while in storage. The resulting insights offered to inventory managers are thought to be of great value since many of these issues have not been investigated before. Moreover, we underpinned and portrayed how the terms deterioration, perishability and obsolescence may collectively apply to an item.

This study unifies and extends the academic literature that accounts for product quality related issues, whereby the validity of the proposed model is ascertained, i.e. the solution is the same as in published sources or in some cases produces better results. To the best of our knowledge, this appears to be the first time that such special cases are addressed, formulated, investigated, and numerically verified.

As illustrated in this chapter, the generic nature of the model enables decision makers to incorporate other forms of varying demand, screening, defective and deterioration rates for the above special cases to evaluate different scenarios.

The implications and managerial insights addressed in this chapter, when further coupled with the findings discussed in Chapter 4, reveal that the model emerges as a viable solution

that manages and controls the flow of perishable and non-perishable products.

It should be noted that it is impossible for a single existing theory to adequately capture all aspects of the relevant processes and the inventory problems associated with them. For example, the classical EOQ model is often based on the assumption that the OW has unlimited capacity. However, there is a plethora of factors that may allure inventory managers to purchase more goods that may exceed the limited capacity of OW, resulting in excessive units being stored in another RW. This problem necessitates the formulation of a two-warehouse inventory system. This further line of inquiry is addressed in Part C below.

Part C: Lot size inventory model with two levels of storage

This part consists of three chapters. These chapters introduce three general two-warehouse inventory models. Chapter 6 underlies the LIFO policy, Chapter 7 underlies the FIFO policy and Chapter 8 relates to the AIFO policy.

The classical formulation of a two-warehouse inventory model is often based on the LIFO or FIFO dispatching policy. The LIFO policy relies upon inventory stored in a RW, with ample capacity, being consumed first, before depleting inventory of an OW that has a limited capacity. Consumption works the opposite way around for the FIFO policy. In this PhD thesis, a new policy entitled "Allocation-In-Fraction-Out (AIFO)" is proposed. Unlike LIFO and FIFO, AIFO implies simultaneous consumption fractions associated with RW and OW. In that respect, the goods at both warehouses are depleted by the end of the same cycle. Moreover, three general two-warehouse inventory models for items that are subject to inspection for imperfect quality are developed and compared – each underlying one of the dispatching policies considered. Each sub-replenishment that is delivered to OW and RW incurs a distinct transportation cost and undergoes a 100 per cent screening. The percentage of defective items per lot may reduce according to a learning curve and the OW may not be fulfilled with its maximum capacity. The mathematical formulation considers arbitrary functions of time in order to reflect a diverse range of strategies. In this part, we provide illustrative examples that analyse the behaviour of deterioration, VOI and perishability in different settings. We find that considering different transportation costs associated with OW and RW and the incorporation of varying demand, screening, defectiveness and deterioration rates significantly impact on the optimal order quantity.

Part C

Part C is based on the paper entitled "Beyond LIFO and FIFO: Exploring an Allocation-In-Fraction-Out (AIFO) policy in a two-warehouse inventory model" by Adel Alamri and Aris Syntetos, which is, at the time of writing, under a 3rd review round by the *International Journal of Production Economics*.

6. A general EOQ model for imperfect quality items under LIFO dispatching policy

6.1.Introduction

In this chapter, we present a general two-warehouse EOQ model for items with imperfect quality under varying demand, defective items, a screening process and deterioration rates for an infinite planning horizon. The general model developed considers the LIFO dispatching policy, i.e. the items are stocked into OW, with limited capacity, first and then in RW that has ample capacity. Under a LIFO dispatching policy, items stored in RW are consumed first, before depleting items in OW. Therefore, the generality of the model extends beyond academic interests to allow the decision maker to determine the optimum order quantity that minimises total system cost.

Each lot that is delivered to the inventory system undergoes a 100 per cent screening where defective items are stored in different warehouses. Thus, different holding costs for the good and defective items are modelled. Items deteriorate while they are effectively in storage. We consider demand, screening and deterioration rates being arbitrary functions of time to allow the decision maker to evaluate the consequences of a diverse range of strategies by employing a single inventory model. The percentage of imperfect quality items per lot reduces according to a learning curve. After a 100 per cent screening, defective items may be sold at a salvage price as a single batch at the end of the screening process or incur a disposal penalty charge.

A rigorous method is utilised to show that the solution, if it exists, is unique and global

optimal. Practical examples that are published in the literature for generalised models in this area are shown to be special cases of our models. We provide illustrative examples that analyse the behaviour of deterioration and VOI in different settings.

The remainder of the chapter is organised as follows: The need for the research is presented in Section 6.2. Our general two-warehouse EOQ model for items with imperfect quality, the assumptions and notation of the inventory system are presented in Section 6.3. Section 6.4 presents the solution procedures, followed, in Section 6.5, by illustrative examples that demonstrate the application of the theoretical results in practice. Managerial insights and concluding remarks are provided in Section 6.6. The proof of the optimality and uniqueness of our solution is presented in Appendix B.

6.2. Need for the research

The classical EOQ model is often based on the assumption that a single OW has an infinite capacity. In practice, however, there exists a plethora of factors that may entice decision makers to purchase a number of units that may exceed the limited capacity of OW. From an economical point of view, it is perhaps cost effective if the excess units are stored in another, rented, warehouse (RW), which is assumed to be of ample capacity (Hartley, 1976). Such factors may include price discounts for bulk purchase offered by the supplier, revenue being higher than the holding cost in RW, and evading high inflation rates (Hsieh et al., 2008; Lee and Hsu; 2009; Liang and Zhou, 2011; Yang, 2004; 2006; 2012; Zhong and Zhou, 2013; Zhou and Yang, 2005).

The classical formulation of a two-warehouse inventory model assumes that the lot size

entering the system first fulfils the maximum storage capacity of the OW with the remaining quantity, over and above that maximum capacity, being kept at the RW. Subsequently, when the LIFO dispatching policy is employed, then the goods of the RW are consumed first before considering the OW inventory. Researchers advocating such a policy assume a higher (lower) holding cost (deterioration rate) in RW due to the availability of better preserving environmental conditions (e.g. Chung et al. (2009) and Jaggi et al. (2015)).

A survey of the inventory literature reveals that there is no published work that investigates the model of Chung et al. (2009) for time-varying demand and product deterioration.

6.3. Formulation of the general model under LIFO dispatching policy

6.3.1. Assumptions and notation

We will use throughout this chapter the subscript "o(r)" to indicate the quantity related to the OW (RW). We will also employ the subscript "g(d)" to refer to good (defective) items. So, for example, and denoting the cycle index by j, $I_{rgj}(t)$ denotes the inventory level of good items at time t in RW, and $I_{odj}(t)$ refers to the inventory level of defective items at time t in OW. We will also use the subscript (L) to refer to the LIFO dispatching policy.

Our model is developed under the following assumptions and notation:

- 1. A single item is held in stock.
- 2. The lead-time is negligible, i.e. any replenishment ordered at the beginning of a cycle arrives just prior to the end of that same cycle.

- 3. The demand, screening and deterioration rates are arbitrary functions of time denoted by D(t), x(t) and $\delta_v(t)$ respectively.
- 4. The OW has a fixed limited capacity and the RW has unlimited capacity.
- 5. The percentage of defective items per lot reduces according to a learning curve denoted by p_i , where j is the cycle index.
- 6. Shortages are not allowed, i.e. we require that $(1-p_j)x(t) \ge D(t) \ \forall \ t \ge 0$.
- 7. The cost parameters are as follows:

c =Unit purchasing cost;

d =Unit screening cost;

 T_{rj} = Screening time of items stored in RW;

 T_{oj} = Screening time of items stored in OW;

 T_{Ri} = Depleting time of items stored in RW;

 T_i = Depleting time of items stored in OW (cycle length);

 $h_{rg} = \text{Holding cost of good items per unit per unit time for RW};$

 $h_{rd} = \text{Holding cost of defective items per unit per unit time for RW;}$

 $h_{og} = \text{Holding cost of good items per unit per unit time for OW;}$

 h_{od} = Holding cost of defective items per unit per unit time for OW;

k = Cost of placing an order.

6.3.2. The model

At the beginning of each cycle j(j=1,2,...), a lot of size Q_{Lj} enters the inventory system such that a quantity of size q_{oj} is kept in the OW and the quantity of size $q_{rj}=Q_{Lj}-q_{oj}$ is kept in the RW. The general model developed considers the LIFO dispatching policy, i.e. items stored in a RW, being consumed first, before depleting items in an OW. Each subreplenishment that enters the OW (RW) undergoes a 100 per cent screening process at a rate of x(t) that starts at the beginning of the cycle and ceases by time T_{oj} (T_{rj}), by which point T_{oj} (T_{rj}) units have been screened and T_{oj} (T_{rj}) units have been consumed. Each subreplenishment covers the actual demand and deterioration during both the first phase (screening) and the second phase (non-screening). During the screening phase, items not conforming to certain quality standards (defective items) are stored in different warehouses.

The goal is to formulate a general inventory model for the LIFO dispatching policy and then prove the existence, uniqueness and global optimality of the solution.

When applying a LIFO dispatching policy, items stored in the RW are depleted first by time T_{Rj} . In this model, we distinguish two cases:

Case 1. $T_{oj} \leq T_{Rj}$. The behaviour of such a model is depicted in Fig. 6.1.

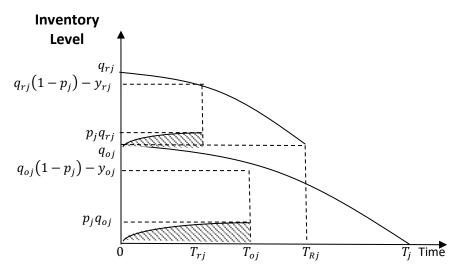


Fig. 6.1. Inventory variation of the two-warehouse model during one cycle when $T_{oi} \leq T_{Ri}$ (LIFO).

The variations in the inventory levels depicted in Fig. 6.1 are given by the following differential equations:

$$\frac{dI_{rgj}(t)}{dt} = -D(t) - p_j x(t) - \delta_r I_{rgj}(t), \qquad 0 \le t < T_{rj} \quad (6.1)$$

$$\frac{dI_{rgj}(t)}{dt} = -D(t) - \delta_r I_{rgj}(t), \qquad T_{rj} \le t \le T_{Rj} \quad (6.2)$$

$$\frac{dI_{ogj}(t)}{dt} = -p_j x(t) - \delta_o I_{ogj}(t), \qquad 0 \le t < T_{oj} \quad (6.3)$$

$$\frac{dI_{ogj}(t)}{dt} = -\delta_o I_{ogj}(t), \qquad T_{oj} \le t < T_{Rj} \quad (6.4)$$

$$\frac{dI_{ogj}(t)}{dt} = -D(t) - \delta_o I_{ogj}(t), \qquad T_{Rj} \le t \le T_j \qquad (6.5)$$

with the boundary conditions: $I_{rgj}(0)=q_{rj}$, $I_{rgj}(T_{Rj})=0$, $I_{ogj}(0)=q_{oj}$, $I_{ogj}(T_{oj}^+)=I_{ogj}(T_{oj})e^{g_o(T_{oj})}$ and $I_{ogj}(T_j)=0$,

where

$$Q_{Lj} = q_{oj} + q_{rj} = \int_0^{T_{oj}} x(u) du + \int_0^{T_{rj}} x(u) du.$$
 (6.6)

Finally, the variations in the inventory levels for defective items (shaded area) depicted in Fig. 6.1 are given by the following differential equations:

$$\frac{dI_{rdj}(t)}{dt} = p_j x(t), \qquad 0 \le t \le T_{rj} \quad (6.7)$$

$$\frac{dI_{odj}(t)}{dt} = p_j x(t), \qquad 0 \le t \le T_{oj} \quad (6.8)$$

with the boundary conditions $I_{rdj}(0)=0$, $I_{odj}(0)=0$, $I_{rdj}(T_{rj})=p_jq_{rj}$ and $I_{odj}(T_{oj})=p_jq_{oj}$.

Considering the boundary conditions, the solutions of the above differential equations are given by:

$$I_{rgj}(t) = e^{-(g_r(t) - g_r(0))} \int_0^{T_{rj}} x(u) du - e^{-g_r(t)} \int_0^t [D(u) + p_j x(u)] e^{g_r(u)} du,$$

$$0 \le t < T_{rj} \quad (6.9)$$

$$I_{rgj}(t) = e^{-g_r(t)} \int_t^{T_{Rj}} D(u) e^{g_r(u)} du,$$
 $T_{rj} \le t \le T_{Rj}$ (6.10)

$$I_{ogj}(t) = e^{-\left(g_o(t) - g_o(0)\right)} \int_0^{T_{oj}} x(u) du - e^{-g_o(t)} \int_0^t p_j x(u) e^{g_o(u)} du, \qquad 0 \le t < T_{oj} \quad \text{(6.11)}$$

$$I_{ogj}(t) = e^{-\left(g_o(t) - g_o(0)\right)} \int_0^{T_{oj}} x(u) du - e^{-g_o(t)} \int_0^{T_{oj}} p_j x(u) e^{g_o(u)} du, \ T_{oj} \le t < T_{Rj} \ \ (6.12)$$

$$I_{ogj}(t) = e^{-g_o(t)} \int_t^{T_j} D(u) e^{g_o(u)} du, \qquad T_{Rj} \le t \le T_j \quad (6.13)$$

respectively.

Now, the per cycle cost components for the given inventory model are as follows:

Purchasing cost + Screening cost = $(c+d)\int_0^{T_{oj}} x(u)du + (c+d)\int_0^{T_{rj}} x(u)du$.

Note that the purchasing cost includes the defective and deterioration costs.

Holding cost for the RW = $h_{rg}[I_{rgj}(0,T_{rj})+I_{rgj}(T_{rj},T_j)]+h_{rd}I_{rdj}(0,T_{rj})$.

Holding cost for the OW = $h_{og}[I_{ogj}(0,T_{oj}) + I_{ogj}(T_{oj},T_j)] + h_{od}I_{odj}(0,T_{oj})$.

Thus, the total cost per unit time of the underlying inventory model during the cycle $[0, T_j]$, as a function of T_{rj} , T_{Rj} and T_j , say $Z_L(T_{rj}, T_{Rj}, T_j)$, is given by:

$$Z_{L}(T_{rj}, T_{Rj}, T_{j}) = \frac{1}{T_{j}} \left\{ (c+d) \int_{0}^{T_{oj}} x(u) du + (c+d) \int_{0}^{T_{rj}} x(u) du + h_{rg} \left[\left[G_{r}(T_{rj}) - G_{r}(0) \right] e^{g_{r}(0)} \int_{0}^{T_{rj}} x(u) du - \int_{0}^{T_{rj}} \left[G_{r}(T_{rj}) - G_{r}(u) \right] \left[D(u) + p_{j}x(u) \right] e^{g_{r}(u)} du + \int_{T_{rj}}^{T_{Rj}} \left[G_{r}(u) - G_{r}(T_{rj}) \right] D(u) e^{g_{r}(u)} du + h_{rd} \left[\int_{0}^{T_{rj}} \left[T_{rj} - u \right] p_{j}x(u) du \right] + h_{og} \left[\left[G_{o}(T_{oj}) - G_{o}(0) \right] e^{g_{o}(0)} \int_{0}^{T_{oj}} x(u) du - \int_{0}^{T_{oj}} \left[G_{o}(T_{oj}) - G_{o}(u) \right] p_{j}x(u) e^{g_{o}(u)} du + \left[G_{o}(T_{Rj}) - G_{o}(T_{oj}) \right] e^{g_{o}(0)} \int_{0}^{T_{oj}} x(u) du - \int_{0}^{T_{oj}} \left[G_{o}(T_{Rj}) - G_{o}(T_{oj}) \right] p_{j}x(u) e^{g_{o}(u)} du + \int_{T_{Rj}}^{T_{j}} \left[G_{o}(u) - G_{o}(T_{Rj}) \right] D(u) e^{g_{o}(u)} du + h_{od} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] p_{j}x(u) du \right] + k \right\}.$$

$$(6.14)$$

Case 2. $T_{oj} > T_{Rj}$. The behaviour of such a model is depicted in Fig. 6.2.

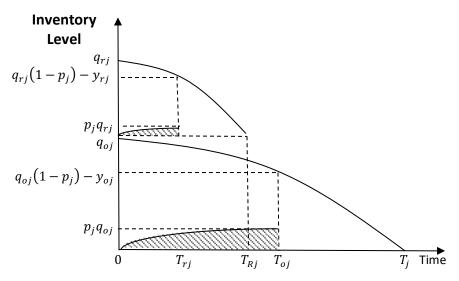


Fig. 6.2. Inventory variation of the two-warehouse model during one cycle when $T_{Rj} < T_{oj}$ (LIFO).

First, we note from Fig. 6.2 that the inventory levels for good and defective items of the RW and for defective items of the OW are the same as in Case 1.

Now, the variations in the inventory level for good items of the OW depicted in Fig. 6.2 are given by the following differential equations:

$$\frac{dI_{ogj}(t)}{dt} = -p_j x(t) - \delta_o I_{ogj}(t), \qquad 0 \le t < T_{Rj}$$
 (6.15)

$$\frac{dI_{ogj}(t)}{dt} = -D(t) - p_j x(t) - \delta_o I_{ogj}(t),, T_{Rj} \le t < T_{oj} (6.16)$$

$$\frac{dI_{ogj}(t)}{dt} = -D(t) - \delta_o I_{ogj}(t), \qquad T_{oj} \le t \le T_j \quad (6.17)$$

with the boundary conditions:

$$I_{ogj}(0) = q_{oj}, \ I_{ogj}(T_{oj}^{+}) = I_{ogj}(T_{oj})e^{g_o(T_{oj})} \text{ and } I_{ogj}(T_j) = 0.$$

The solutions of the above differential equations are:

$$I_{ogj}(t) = e^{-\left(g_o(t) - g_o(0)\right)} \int_0^{T_{oj}} x(u) du - e^{-g_o(t)} \int_0^t p_j x(u) e^{g_o(u)} du, \qquad 0 \le t < T_{Rj}$$
 (6.18)

$$\begin{split} I_{ogj}(t) &= e^{-\left(g_o(t) - g_o(0)\right)} \int_0^{T_{oj}} x(u) du - e^{-g_o(t)} \int_0^{T_{Rj}} p_j x(u) e^{g_o(u)} du - e^{-g_o(t)} \int_{T_{Rj}}^t \left[D(u) + p_j x(u)\right] e^{g_o(u)} du, \end{split}$$

$$T_{Rj} \leq t < T_{oj} \quad (6.19)$$

$$I_{ogj}(t) = e^{-g_o(t)} \int_t^{T_j} D(u) e^{g_o(u)} du,$$
 $T_{oj} \le t \le T_j$ (6.20)

respectively.

Thus, the total cost per unit time of the underlying inventory model during the cycle $[0, T_j]$, as a function of T_{rj} , T_{Rj} and T_j , say $H_L(T_{rj}, T_{Rj}, T_j)$, is given by:

$$H_{L}(T_{rj}, T_{Rj}, T_{j}) = \frac{1}{T_{j}} \left\{ (c+d) \int_{0}^{T_{oj}} x(u) du + (c+d) \int_{0}^{T_{rj}} x(u) du + h_{rg} \left[\left[G_{r}(T_{rj}) - G_{r}(0) \right] e^{g_{r}(0)} \int_{0}^{T_{rj}} x(u) du - \int_{0}^{T_{rj}} \left[G_{r}(T_{rj}) - G_{r}(u) \right] \left[D(u) + p_{j}x(u) \right] e^{g_{r}(u)} du + \int_{T_{rj}}^{T_{Rj}} \left[G_{r}(u) - G_{r}(T_{rj}) \right] D(u) e^{g_{r}(u)} du \right] + h_{rd} \left[\int_{0}^{T_{rj}} \left[T_{rj} - u \right] p_{j}x(u) du \right] + h_{og} \left[\left[G_{o}(T_{Rj}) - G_{o}(0) \right] e^{g_{o}(0)} \int_{0}^{T_{oj}} x(u) du - \int_{0}^{T_{Rj}} \left[G_{o}(T_{Rj}) - G_{o}(u) \right] p_{j}x(u) e^{g_{o}(u)} du + \left[G_{o}(T_{oj}) - G_{o}(u) \right] \left[e^{g_{o}(0)} \int_{0}^{T_{oj}} x(u) du - \int_{0}^{T_{Rj}} p_{j}x(u) e^{g_{o}(u)} du \right] - \int_{T_{Rj}}^{T_{oj}} \left[G_{o}(T_{oj}) - G_{o}(u) \right] \left[D(u) + p_{j}x(u) \right] e^{g_{o}(u)} du + \int_{0}^{T_{oj}} \left[G_{o}(u) - G_{o}(T_{oj}) \right] D(u) e^{g_{o}(u)} du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] + h_{od} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] e^{g_{o}(u)} du \right] du + h_{rd} \left[\int_{0}^$$

It is noted that the total cost per unit time for Case 2 given by (6.21) is identical with that for Case 1 given by (6.14).

Moreover, the variables T_{rj} , T_{Rj} and T_j that minimise $Z_L(T_{rj}, T_{Rj}, T_j)$ or $H_L(T_{rj}, T_{Rj}, T_j)$ given by (6.14) or (6.21) are related to each other through the following relations:

$$0 < T_{ri} < T_{Ri} < T_i, (6.22)$$

$$e^{g_r(0)} \int_0^{T_{rj}} x(u) du = \int_0^{T_{Rj}} D(u) e^{g_r(u)} du + \int_0^{T_{rj}} p_j x(u) e^{g_r(u)} du, \tag{6.23}$$

$$e^{g_o(0)} \int_0^{T_{0j}} x(u) du = \int_{T_{Rj}}^{T_j} D(u) e^{g_o(u)} du + \int_0^{T_{0j}} p_j x(u) e^{g_o(u)} du.$$
 (6.24)

Thus, our goal is to solve the following optimisation problem, which we shall call problem (m_L) :

$$(m_L) = \begin{cases} \text{minimise } H_L(T_{rj}, T_{Rj}, T_j) \text{ given by (6.14) or (6.21)} \\ \text{subject to (6.22 - 6.24)} \end{cases}$$

From Eqs. (6.23) and (6.24), $T_{rj}=0 \Rightarrow T_{Rj}=0$ and $T_{rj}>0 \Rightarrow T_{rj}< T_{Rj}< T_{j}$. Thus, Eqs. (6.23) and (6.24) imply constraint (6.22). Consequently, if we temporarily ignore the monotony constraint (6.22) and call the resulting problem (m_{L1}) , then relation (6.22) does satisfy any solution of (m_{L1}) . Hence (m_{L}) and (m_{L1}) are equivalent.

6.4. Solution procedures

From Eqs. (6.6), (6.23) and (6.24) we note that T_{rj} , T_{Rj} and T_j can be determined as functions of q_{rj} , say

$$T_{ri} = f_{ri}(q_{ri}), (6.25)$$

$$T_{Rj} = f_{Rj}(q_{rj}), (6.26)$$

$$T_i = f_i(q_{ri}).$$
 (6.27)

Thus, if we substitute (6.23)-(6.27) in (6.14) or (6.21) then problem (m_{L1}) will be converted to the following unconstrained problem with the variable q_{rj} (which we shall call problem (m_{L2})):

$$W_{L}(Q_{Lj}) = \frac{1}{f_{j}} \Big\{ (c+d) \int_{0}^{f_{oj}} x(u) du + (c+d) \int_{0}^{f_{rj}} x(u) du + h_{rg} \Big[-G_{r}(0) e^{g_{r}(0)} \int_{0}^{f_{rj}} x(u) du + \int_{0}^{f_{rj}} p_{j} x(u) G_{r}(u) e^{g_{r}(u)} du + \int_{0}^{f_{Rj}} D(u) G_{r}(u) e^{g_{r}(u)} du \Big] + h_{rd} \Big[\int_{0}^{f_{rj}} [f_{rj} - u] p_{j} x(u) du \Big] + h_{og} \Big[-G_{o}(0) e^{g_{o}(0)} \int_{0}^{f_{oj}} x(u) du + \int_{0}^{f_{oj}} p_{j} x(u) G_{o}(u) e^{g_{o}(u)} du + \int_{f_{Rj}}^{f_{j}} D(u) G_{o}(u) e^{g_{o}(u)} du \Big] + h_{od} \Big[\int_{0}^{f_{oj}} [f_{oj} - u] p_{j} x(u) du \Big] + k \Big\}.$$

$$(6.28)$$

If we let $W_L = \frac{w_L}{f_i}$, then the necessary condition for having a minimum for problem (m_{L2}) is

$$w'_{q_{rj}}f_j = f'_{j,q_{rj}}w_L, (6.29)$$

where $w_{q_{rj}}'$ and $f_{j,q_{rj}}'$ are the derivatives of w_L and f_j with respect to q_{rj} , respectively.

Also, from (6.23) and (6.24) we obtain

$$e^{g_r(0)} - p_j e^{g_r(f_{rj})} = f'_{Rj,q_{rj}} D(f_{Rj}) e^{g_r(f_{Rj})}.$$
(6.30)

$$f'_{j,q_{rj}}D(f_j)e^{g_o(f_j)} = f'_{Rj,q_{rj}}D(f_{Rj})e^{g_o(f_{Rj})}.$$
(6.31)

Considering (6.30) and (6.31), but also (6.25)-(6.28) we have:

$$w'_{q_{rj}} = h_{rg} \left[\left(G_r(f_{Rj}) - G_r(0) \right) e^{g_r(0)} + \left(G_r(f_{rj}) - G_r(f_{Rj}) \right) p_j e^{g_r(f_{rj})} \right] + \frac{h_{rd}}{x(f_{rj})} \int_0^{f_{rj}} p_j x(u) du + h_{og} \left[\left(G_o(f_j) - G_o(f_{Rj}) \right) f'_{j,q_{Rj}} D(f_j) e^{g_o(f_j)} \right] + (c + d)$$
(6.32)

Also, (6.29)
$$\Leftrightarrow W_L = \frac{w_L}{f_j} = \frac{w'_{q_{rj}}}{f'_{j,q_{rj}}}.$$
 (6.33)

Eq. (6.33) can be used to determine the optimal value of Q_{Lj} and its corresponding total minimum cost. Then the optimal values of T_{rj} , T_{Rj} and T_j can be found from (6.25), (6.26) and (6.27), respectively.

6.5. Numerical analysis

In this section, we present illustrative examples to support the application of our mathematical model and solution procedures in different realistic situations. The versatile nature of our model allows the incorporation of the desired functions that are suitable for a system. For example, the variation of demand, screening, deterioration and defectiveness rates with time (or due to any other factors) is a quite natural phenomenon (Alamri, 2011; Benkherouf et al., 2014; Datta et al. 1998; Hariga and Benkherouf, 1994; Grosse et al., 2013; Jaber et al., 2008; Karmarkar and Pitbladdo, 1997; Murdeshwar, 1988; Sana, 2010).

6.5.1. Varying rates

In this example (Example 6.1), we consider the following functions for varying demand, screening, defectiveness and deterioration rates:

$$x(t) = at + b, \ D(t) = \alpha t + r, \ p_j = \frac{\tau}{\pi + e^{\gamma j}}, \ \delta_o(t) = \frac{l_o}{z_o - \beta_o t} \text{ and } \delta_r(t) = \frac{l_r}{z_r - \beta_r t'}$$

where $b, r, \pi, z_y > 0$; $a, \alpha, l_y, \tau, \gamma, \beta_y, t \ge 0$ and $\beta_y t < z_y$.

Note that $\delta_y(t)$ is an increasing function of time and p_j reduces according to an S-shaped logistic learning curve (Jordan, 1958; Carlson, 1973).

Problem (m_{L2}) has been coded in *MATLAB* for the above functions and solutions were obtained for a wide range of the control parameter values. Here, and for comparison purposes, we thematically consider situations with parameters that are presented in Table 6.1 below.

Table 6.1. Input parameters for example 6.1.

h_{og}	h_{od}	h_{rg}	h_{rd}	q_o	k
20	5	25	5	2000	3000
Dollars/unit/year	Dollars/unit/year	Dollars/unit/year	Dollars/unit/year	Units	Dollars/cycle
а	b	α	r	С	d
1000	100200	500	50000	100	0.50
Units/year	Units/year	Units/year	Units/year	Dollars/unit	Dollars/unit
l_o	l_r	Z_{0}	z_r	eta_o	β_r
1	1	20	33.33	25	20
Units/year	Units/year	Units/year	Units/year	Units/year	Units/year
τ	π	γ			
70.067	819.76	0.7932			
Units/year	Units/year	Units/year			

In this example (Example 6.1), we have taken $p_1=0.08524$ resulting in a total number of $Q_1^*=q_{o1}+q_{r1}^*=2000+1495=3495$ units, which is screened by time $T_{o1}=0.020\cong 7$ days and consumed by time $T_1^*=0.0638\cong 23$ days. Note that the sub-replenishment $q_{o1}(q_{r1}^*)$ is screened by time $T_{o1}(T_{r1}^*)$ and counsumed by $T_{R1}^*(T_1^*)$. The total minimum cost per year is $W_1^*=5585101$ dollars and the total minimum cost per cycle is $w_1^*=356330$ dollars. The number of defective items is $p_1Q_1^*=298$ units and the number of deteriorated items is $\omega_1^*=\omega_{o1}^*+\omega_{r1}^*=4.38+0.59=4.97$ units, which is the difference between the actual demand and the amount held in stock at the beginning of the cycle excluding the number of defective items. The amount $p_1Q_1^*$ may be sold at a salvage price at times T_{o1} and T_{r1}^* or incur a disposal penalty charge.

In the next section, we test and observe the behaviour of the theoretical model in different realistic scenarios. Table 6.2 depicts the effect of all model parameters on the optimal values. Table 6.3 shows the effect of Wright's learning curve on the optimal values when the deterioration rates increase.

6.5.2. Sensitivity analysis

The results presented in Table 6.2 summarise the sensitivity analysis of the optimal order quantity and total minimum cost per unit time with respect to all model parameters. The first row denotes the original values of the proposed model (Example 6.1) and the last one yields the values of the EOQ model presented in Chapter 4. Table 6.3 compares two consecutive cycles to observe the effect of Wright's learning curve, i.e. $p_j = \frac{\tau}{\pi+1} j^{-\gamma}$ on the optimal values.

Table 6.2. Sensitivity analysis for the general model.

Parameter	f_o	f_r^*	f_R^*	f^*	q_o^*	q_r^*	ω_o	ω_r^*	W^*
Original model	0.020	0.0149	0.0273	0.0638	2000	1495	4.38	0.59	5585101
$h_{og} = h_{od} = 20$ $h_{rg} = h_{rd} = 25$	0.020	0.0147	0.0269	0.0634	2000	1472	4.34	0.57	5585796
$h_{og} = h_{rg} = 20$	0.020	0.0172	0.0315	0.0680	2000	1724	4.79	0.79	5583382
$\beta_o = \beta_r = 0$	0.020	0.0151	0.0276	0.0641	2000	1510	4.28	0.60	5584866
$l_o = l_r = 0$	0.020	0.0175	0.0321	0.0687	2000	1756	0	0	5576235
$l_o = l_r = 0$ $\alpha = a = 0$	0.020	0.0183	0.0335	0.0701	2000	1833	0	0	5574310
$h_{og} = h_{rg} = 20$									
$z_r = z_o = 20$ $\beta_o = \beta_r = 25$	0	0.0354	0.0648	0.0648	0	3550 ª	0	5.4	5585464

^a The order quantity as in Example 4.1 (Table 4.2).

Table 6.3. The effect of Wright's learning curve on the optimal values of the general model.

Parameter	j	p_{j}	f_{oj}	f_{rj}^*	f_{Rj}^*	f_{rj}^*	q_{oj}^*	q_{rj}^*	ω_{oj}^*	ω_{rj}^*	W_j^*
$z_o = 10$ $z_r = 20$	1	0.08537	0.020	0.0128	0.0235	0.0128	2000	1285	8.24	0.73	5593672
$z_o = 10$ $z_r = 20$	2	0.04926	0.020	0.0119	0.0226	0.0119	2000	1189	8.45	0.66	5384686

In the next section, we summarise some key findings in order to highlight the behaviour of the theoretical model in different settings and to relate the results addressed in this chapter to the general body of knowledge in the discipline.

6.5.3. Findings

- The incorporation of varying demand, screening, defectiveness and deterioration rates significantly impact on the optimal quantity that is allocated to the RW (Table 6.2).
- The lot size with the same assumed holding costs for the good and defective items is
 less than the one with differing holding costs (Table 6.2). Such finding is consistence
 with that presented by Wahab and Jaber (2010) and the finding addressed in Chapter
 4.
- The optimal order quantity with the same assumed holding costs for the OW and RW is less than the one with differing holding costs (Table 6.2).
- The effect of α , the rate of change in the demand significantly influences the optimal amount that is allocated to the RW (Table 6.2).
- The results obtained in Table 6.2 reveal that the reduction of the optimal quantity that
 is allocated to the RW does not imply that the total minimum cost per year decreases;
 in fact, it may increase.
- As learning increases, i.e. the percentage of defective items per lot decreases, the total minimum cost per year decreases (Table 6.3). Such finding is consistence with that presented by Wahab and Jaber (2010) and the finding addressed in Chapter 4.
- All optimal quantities decrease as learning increases, except for the number of deteriorated items in the OW which experiences a minor increase that can be justified by the slight increase in the cycle length (Table 6.3). Such finding is consistence with that presented in Chapter 4.
- Previously published models in this area are shown to be special cases of our model (Table 6.2 and Appendix B).

Remark 6.1

The above findings support the findings presented in Chapter 4. Moreover, this model is a viable solution for all special cases that are suggested in Chapter 5. Further investigations related to this model will be addressed in Chapter 8.

6.6. Conclusion and further research

In this chapter, we have been concerned with the implications of the LIFO dispatching policy associated with a two-level storage, where each sub-replenishment that is allocated to OW (RW) undergoes a 100 per cent screening. In particular, a general two-warehouse EOQ model for items with imperfect quality was presented. Items not conforming to certain quality standards are isolated in separate facilities with different holding costs for the good and defective items being considered.

The general model developed considers the LIFO dispatching policy, i.e. the items are stocked in OW first and then in RW. However, inventory stored in RW is consumed first, before depleting inventory of OW. The generality of our model stems from the fact that the demand, screening and product deterioration rates are arbitrary functions of time. Therefore, the generality of the model unifies and extends the academic literature relating to imperfect quality items to allow the decision maker to determine the optimum order quantity that minimises total system cost.

The analytical results illustrate the impact of incorporating learning curve and varying demand, deterioration, defectiveness and screening rates on the optimal order quantity. The

obtained numerical results indicate that a reduction in the optimal quantity that is allocated to the RW does not necessarily imply a lower total minimum cost per unit time.

This model is viable for fixed and random lifetimes of perishable products, where VOI may be used to model the shelf lifetime of an item (see Chapter 8). The versatile nature of our model and the fact that it may reflect a diverse range of strategies has been emphasised whereby the validity of the general model is ascertained. Mathematical proof was presented (Appendix B), which shows that the solution to the underlying inventory system, if it exists, is unique and global optimal. To the best of our knowledge, this appears to be the first time that such a general formulation of a two-level storage inventory model under LIFO dispatching policy is presented, investigated and numerically verified.

Further research can be addressed for finite or infinite planning horizons that may include extensions, such as allowing for shortages, considering that the screening rate follows learning and forgetting curves and the risk of failure during screening (Type I and Type II errors). In addition, it appears plausible to formulate an EPQ model, to assess the formulation of EOQ model considering multiple items or to study the effect of different supplier trade credit practices.

As illustrated in Chapters 4, 5 and 6, our mathematical formulation allows inventory managers to incorporate other forms of varying demand, screening, defectiveness and deterioration rates so they can establish the optimum order quantity that minimises total system cost.

It should be noted here that the implication of the LIFO dispatching policy may not be practically or economically attainable, i.e. the FIFO policy is more in line with realistic operating conditions. This implies the formulation of a general two-warehouse inventory

model that considers the FIFO dispatching policy, i.e. the goods of the OW are consumed first before considering the RW inventory. Chapter 7 below will address this line of research.

7. A general EOQ model for imperfect quality items under FIFO dispatching policy

7.1. Introduction

In this chapter, we propose a general two-warehouse EOQ model for items with imperfect quality, considering varying demand, defective items, a screening process and deterioration rates for an infinite planning horizon. Unlike the LIFO dispatching policy, the general model developed in this chapter adopts the FIFO dispatching policy. That is to say, the items are first stocked in OW that has limited capacity and then in RW that has ample capacity. However, under FIFO dispatching policy, items stored in OW are consumed first, before depleting items in RW. Therefore, the proposed general model enables interested readers to determine the optimum order quantity that minimises total system cost.

The remainder of the chapter is organised as follows: Section 7.2 introduces the need for considering FIFO dispatching policy in inventory management. Our general two-warehouse EOQ model for items with imperfect quality, the assumptions and notation of the inventory system are presented in Section 7.3. Section 7.4 presents the solution procedures, followed, in Section 7.5, by illustrative examples that demonstrate the application of the theoretical results in practice. Managerial insights and concluding remarks are provided in Section 7.6. The proof of the optimality and uniqueness of our solution is presented in Appendix C.

7.2. Need for the research

As discussed in Chapter 6, the traditional EOQ model is often based on the assumption that a single OW has unlimited capacity. In practice, decision makers may purchase a number of units that may exceed the limited capacity of OW, resulting in excess units being stored in another RW, which is assumed to be of ample capacity. Such situations emerge, for example, if a discounted price of goods is offered by the supplier, the item under consideration is a seasonal product, revenue is higher than the holding cost in RW, and attempts are made to avoid high inflation rates (Hsieh et al., 2008; Lee and Hsu; 2009; Liang and Zhou, 2011; Yang, 2004; 2006; 2012; Zhong and Zhou, 2013; Zhou and Yang, 2005).

As the literature suggests, the classical formulation of a two-warehouse inventory model assumes that the lot size entering the system first fulfils the maximum storage capacity of the OW with the remaining quantity, over and above that maximum capacity, being kept at the RW. Unlike the LIFO dispatching policy, the employment of the FIFO dispatching policy implies that the goods of the OW are consumed first, before considering the RW inventory. This case is usually justified by holding cost reduction, especially when the holding cost in RW is lower than that in OW due to competition, i.e. various offers are available in the market (e.g. Lee, 2006; Niu and Xie, 2008). Moreover, in some practical situations, the implication of the FIFO dispatching policy may fit well with the operational and financial performance of an inventory system.

A survey of the inventory literature reveals that there is no published work that investigates the model of Salameh and Jaber (2000) considering FIFO dispatching policy and time-varying demand and product deterioration.

7.3. Formulation of the general model under FIFO dispatching policy

7.3.1. Assumptions and notation

In this chapter, we adopt the same assumptions and notation as those used in Chapter 6. Specifically, we use the subscript "o(r)" to indicate the quantity relating to the OW (RW) and the subscript "g(d)" to refer to good (defective) items. We will also use the subscript (F) to refer to the FIFO dispatching policy. It should be noted that under a FIFO dispatching policy, the time by which the OW (RW) is depleted is given by:

 T_i = Depleting time of items stored in OW;

 $T_{Rj} =$ Depleting time of items stored in RW (cycle length).

7.3.2. The model

At the beginning of each cycle j(j=1,2,...), a lot of size Q_{Fj} is delivered such that a quantity of size q_{oj} is kept in the OW and the remaining amount of size $q_{rj}=Q_{Fj}-q_{oj}$ is kept in the RW. The employment of the FIFO dispatching policy implies that the goods of the OW are consumed first, before considering the RW inventory. The quantity q_{oj} (q_{rj}) that enters the OW (RW) undergoes a 100 per cent screening process at a rate of x(t) that starts at the beginning of the cycle and ceases by time T_{oj} (T_{rj}), by which point in time y_{oj} (y_{rj}) units have been consumed. The quantity q_{oj} (q_{rj}) covers the actual demand and deterioration during both the first phase (screening) and the second phase (non-screening). During the screening phase, items not conforming to certain quality standards (defective items) are stored in different warehouses.

The goal is to formulate a general inventory model for the FIFO dispatching policy and then prove the existence, uniqueness and global optimality of the solution.

When applying a FIFO dispatching policy, the goods of the RW are consumed only after depleting the goods of OW, i.e. q_{oj} is consumed first, which implies that the cycle length for the OW is a predetermined value. The behaviour of such a model is depicted in Figs. 7.1 and 7.2.

Case 1. $T_{rj} \leq T_j$.

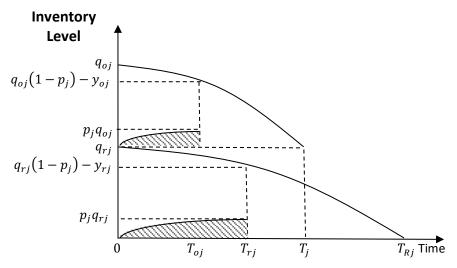


Fig. 7.1. Inventory variation of the two-warehouse model during one cycle when $T_{rj} \leq T_i$ (FIFO).

Case 2. $T_{rj} > T_j$.

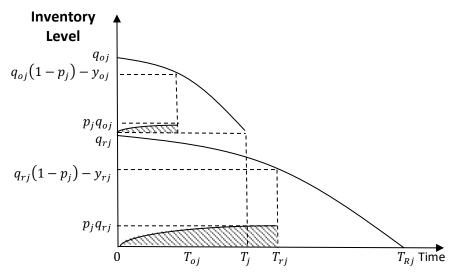


Fig. 7.2. Inventory variation of the two-warehouse model during one cycle when $T_i < T_{ri}$ (FIFO).

As shown above for the LIFO dispatching policy, the formulation of Case 1 or Case 2 would lead to an identical objective function. Thus, only Case 1 will be considered here. Also, from Figs. 7.1 and 7.2 it is clear that inventory levels for defective items of the RW and OW are the same as those of the LIFO policy.

Now, the variations in the inventory levels depicted in Fig. 7.1 are given by the following differential equations:

$$\frac{dI_{ogj}(t)}{dt} = -D(t) - p_j x(t) - \delta_o I_{ogj}(t), \qquad 0 \le t < T_{oj} \quad (7.1)$$

$$\frac{dI_{ogj}(t)}{dt} = -D(t) - \delta_o I_{ogj}(t), \qquad T_{oj} \le t \le T_j \qquad (7.2)$$

$$\frac{dI_{rgj}(t)}{dt} = -p_j x(t) - \delta_r I_{rgj}(t), \qquad 0 \le t < T_{rj} \quad (7.3)$$

$$\frac{dI_{rgj}(t)}{dt} = -\delta_r I_{rgj}(t), \qquad T_{rj} \le t < T_j \qquad (7.4)$$

$$\frac{dI_{rgj}(t)}{dt} = -D(t) - \delta_r I_{rgj}(t), \qquad T_j \le t \le T_{Rj} \quad (7.5)$$

with the boundary conditions: $I_{rgj}(0) = q_{rj}$, $I_{rgj}(T_{Rj}) = 0$, $I_{ogj}(0) = q_{oj}$, $I_{rgj}(T_{rj}^+) = I_{rgj}(T_{rj})e^{gr(T_{rj})}$ and $I_{ogj}(T_j) = 0$.

Solving the above differential equations we obtain:

$$I_{ogj}(t) = e^{-\left(g_o(t) - g_o(0)\right)} \int_0^{T_{oj}} x(u) du - e^{-g_o(t)} \int_0^t \left[D(u) + p_j x(u)\right] e^{g_o(u)} du,$$

$$0 \le t < T_{oj}$$
 (7.6)

$$I_{ogj}(t) = e^{-g_o(t)} \int_t^{T_j} D(u) e^{g_o(u)} du,$$
 $T_{oj} \le t \le T_j$ (7.7)

$$I_{rgj}(t) = e^{-\left(g_r(t) - g_r(0)\right)} \int_0^{T_{rj}} x(u) du - e^{-g_r(t)} \int_0^t p_j x(u) e^{g_r(u)} du, \qquad 0 \le t < T_{rj} \quad (7.8)$$

$$I_{rgj}(t) = e^{-(g_r(t) - g_r(0))} \int_0^{T_{rj}} x(u) du - e^{-g_r(t)} \int_0^{T_{rj}} p_j x(u) e^{g_r(u)} du, \quad T_{rj} \le t < T_j$$
 (7.9)

$$I_{rgj}(t) = e^{-g_r(t)} \int_t^{T_{Rj}} D(u) e^{g_r(u)} du,$$
 $T_j \le t \le T_{Rj}$ (7.10)

respectively.

Now, the total cost per unit time of the underlying inventory model during the cycle $[0, T_{Rj}]$, as a function of T_{rj} and T_{Rj} , say $H_F(T_{rj}, T_{Rj})$, is given by:

$$H_{F}(T_{rj}, T_{Rj}) = \frac{1}{T_{Rj}} \left\{ (c+d) \int_{0}^{T_{oj}} x(u) du + (c+d) \int_{0}^{T_{rj}} x(u) du + h_{og} \left[\left[G_{o}(T_{oj}) - G_{o}(0) \right] e^{g_{o}(0)} \int_{0}^{T_{oj}} x(u) du - \int_{0}^{T_{oj}} \left[G_{o}(T_{oj}) - G_{o}(u) \right] \left[D(u) + p_{j}x(u) \right] e^{g_{o}(u)} du + \int_{T_{oj}}^{T_{j}} \left[G_{o}(u) - G_{o}(T_{oj}) \right] D(u) e^{g_{o}(u)} du + h_{od} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] p_{j}x(u) du \right] + h_{rg} \left[\left[G_{r}(T_{rj}) - G_{r}(0) \right] e^{g_{r}(0)} \int_{0}^{T_{rj}} x(u) du - \int_{0}^{T_{rj}} \left[G_{r}(T_{rj}) - G_{r}(u) \right] p_{j}x(u) e^{g_{r}(u)} du + \left[G_{r}(T_{j}) - G_{r}(T_{rj}) \right] e^{g_{r}(0)} \int_{0}^{T_{rj}} x(u) du - \int_{0}^{T_{rj}} \left[G_{r}(T_{j}) - G_{r}(T_{rj}) \right] p_{j}x(u) e^{g_{r}(u)} du + \int_{T_{j}}^{T_{Rj}} \left[G_{r}(u) - G_{r}(T_{j}) \right] D(u) e^{g_{r}(u)} du + h_{rd} \left[\int_{0}^{T_{rj}} \left[T_{rj} - u \right] p_{j}x(u) du \right] + k \right\}.$$

$$(7.11)$$

Note that T_{rj} and T_{Rj} that minimise $H_F(T_{rj}, T_{Rj})$ are related to each other as follows:

$$0 < T_{rj} < T_{Rj}, (7.12)$$

$$e^{g_r(0)} \int_0^{T_{rj}} x(u) du = \int_{T_j}^{T_{Rj}} D(u) e^{g_r(u)} du + \int_0^{T_{rj}} p_j x(u) e^{g_r(u)} du.$$
 (7.13)

Thus, our goal is to solve the following optimisation problem, which we shall call problem (m_F) :

$$(m_F) = \begin{cases} \text{minimise } H_F(T_{rj}, T_{Rj}) \text{ given by (7.11)} \\ \text{subject to (7.12)and (7.13)} \end{cases}.$$

From Eq. (7.13), $T_{rj}=0 \Rightarrow T_{Rj}=T_j=0$ and $T_{rj}>0 \Rightarrow T_{rj}< T_{Rj}$. Thus, if we ignore constraint (7.12) and call the resulting problem (m_{F1}) then relation (7.12) does satisfy any solution of (m_{F1}) . Hence (m_F) and (m_{F1}) are equivalent.

7.4. Solution procedures

First, we note from (7.12) that T_{rj} and T_{Rj} can be determined as functions of q_{rj} , say

$$T_{rj} = f_{rj}(q_{rj}),$$
 (7.14)

$$T_{Rj} = f_{Rj}(q_{rj}),$$
 (7.15)

respectively.

Thus, replacing (7.13)-(7.15) in (7.11) then problem (m_F) will be reduced to the following unconstrained problem with the variable q_{rj} , say (m_{F2}) :

$$W_{F}(Q_{j}) = \frac{1}{f_{Rj}} \Big\{ (c+d) \int_{0}^{f_{oj}} x(u) du + (c+d) \int_{0}^{f_{rj}} x(u) du + h_{og} \Big[-G_{o}(0) e^{g_{o}(0)} \int_{0}^{f_{oj}} x(u) du + \int_{0}^{f_{oj}} p_{j} x(u) G_{o}(u) e^{g_{o}(u)} du + \int_{0}^{f_{j}} D(u) G_{o}(u) e^{g_{o}(u)} du \Big] + h_{od} \Big[\int_{0}^{f_{oj}} [f_{oj} - u] p_{j} x(u) du \Big] + h_{rg} \Big[-G_{r}(0) e^{g_{r}(0)} \int_{0}^{f_{rj}} x(u) du + \int_{0}^{f_{rj}} p_{j} x(u) G_{r}(u) e^{g_{r}(u)} du + \int_{f_{j}}^{f_{Rj}} D(u) G_{r}(u) e^{g_{r}(u)} du \Big] + h_{rd} \Big[\int_{0}^{f_{rj}} [f_{rj} - u] p_{j} x(u) du \Big] + k \Big\}.$$

$$(7.16)$$

If we let $W_F = \frac{w_F}{f_{Ri}}$, then the necessary condition for having a minimum for problem (m_{F2}) is

$$w'_{q_{ri}}f_{Rj} = f'_{Rj,q_{ri}}w_{L}, (7.17)$$

Also, (7.13) yields

$$e^{g_r(0)} - p_j e^{g_r(f_{rj})} = f'_{Rj,q_{rj}} D(f_{Rj}) e^{g_r(f_{Rj})}.$$
(7.18)

From (7.18) and (7.14)-(7.16) we have

$$w'_{q_{rj}} = h_{rg} \left[\left(G_r(f_{Rj}) - G_r(0) \right) e^{g_r(0)} + \left(G_r(f_{rj}) - G_r(f_{Rj}) \right) p_j e^{g_r(f_{rj})} \right] + \frac{h_{rd}}{x(f_{rj})} \int_0^{f_{rj}} p_j x(u) du + (c+d).$$
(7.19)

Also, (7.17)
$$\Leftrightarrow W_F = \frac{w_F}{f_{Rj}} = \frac{w'_{q_{rj}}}{f'_{Rj,q_{rj}}}.$$
 (7.20)

Eq. (7.20) can be used to determine the optimal value of q_{rj} and its corresponding total minimum cost. Then the optimal values of T_{rj} and T_{Rj} can be found from (7.14) and (7.15), respectively.

7.5. Illustrative examples for different settings

In this section, we present examples to illustrate the theoretical application of our mathematical model and solution procedures in different realistic situations.

7.5.1. Varying demand, screening, defectiveness, and deterioration rates

In this example (Example 7.1), we consider the following functions for varying demand, screening, defectiveness and deterioration rates:

$$x(t) = at + b$$
, $D(t) = \alpha t + r$, $p_j = \frac{\tau}{\pi + e^{\gamma j}}$, $\delta_o(t) = \frac{l_o}{z_o - \beta_o t}$ and $\delta_r(t) = \frac{l_r}{z_r - \beta_r t}$,

where $b, r, \pi, z_y > 0$; $a, \alpha, l_y, \tau, \gamma, \beta_y, t \ge 0$ and $\beta_y t < z_y$.

Note that the above functions are the same as those used for Chapter 6, where $\delta_{
m V}(t)$ is an

increasing function of time and p_j reduces according to an S-shaped logistic learning curve (Jordan, 1958; Carlson, 1973).

Problem (m_{F2}) has been coded in *MATLAB* for the above functions and solutions were obtained for a wide range of the control parameter values. Here, and for comparison purposes, we thematically consider situations with parameters that are presented in Table 7.1 below.

Table 7.1. Input parameters for example 7.1.

h_{og}	h_{od}	h_{rg}	h_{rd}	q_o	k
20	5	25	5	2000	3000
Dollars/unit/year	Dollars/unit/year	Dollars/unit/year	Dollars/unit/year	Units	Dollars/cycle
а	b	α	r	С	d
1000	100200	500	50000	100	0.50
Units/year	Units/year	Units/year	Units/year	Dollars/unit	Dollars/unit
l_o	l_r	Z_{o}	z_r	eta_o	β_r
1	1	20	33.33	25	20
Units/year	Units/year	Units/year	Units/year	Units/year	Units/year
τ	π	γ			
70.067	819.76	0.7932			
Units/year	Units/year	Units/year			

In this example (Example 7.1), we have taken $p_1=0.08524$ resulting in a total number of $Q_1^*=q_{o1}+q_{r1}^*=2000+1402=3402$ units, which is screened by time $T_{o1}=0.020\cong 7$ days and consumed by time $T_{R1}^*=0.0621\cong 23$ days. Note that the sub-replenishment $q_{o1}(q_{r1}^*)$ is screened by time $T_{o1}(T_{r1}^*)$ and counsumed by $T_1^*(T_{R1}^*)$. The total minimum cost per year is $W_1^*=5587111$ dollars and the total minimum cost per cycle is $w_1^*=346960$ dollars. The number of defective items is $p_1Q_1^*=290$ units and the number of deteriorated items is $\omega_1^*=\omega_{o1}^*+\omega_{r1}^*=1.78+1.95=3.73$ units, which is the difference

between the actual demand and the amount held in stock at the beginning of the cycle, excluding the number of defective items. The amount $p_1Q_1^*$ may be sold at a salvage price at times T_{o1} and T_{r1}^* or incur a disposal penalty charge.

In the next section, we analyse the behaviour of the theoretical model in different settings.

Table 7.2 illustrates the effect of changing all model parameters on the optimal values. The effect of Wright's learning curve on the optimal values is depicted in Table 7.3.

7.5.2. Sensitivity analysis

Table 7.2 summarises the sensitivity analysis of the optimal order quantity and total minimum cost per unit time with respect to all model parameters. The first row of Table 7.2 represents the original values of the proposed model (Example 7.1) and the last one denotes the values of the EOQ model presented in Chapter 4. Table 7.3 compares two consecutive cycles to observe the effect of Wright's learning curve, i.e. $p_j = \frac{\tau}{\pi+1} j^{-\gamma}$ on the optimal values when the deterioration rates increase.

Table 7.2. Sensitivity analysis for the general model.

Parameter	f_o	f_r^*	f^*	f_R^*	q_o^*	q_r^*	ω_o	ω_r^*	<i>W</i> *
Original model	0.020	0.0140	0.0365	0.0621	2000	1402	1.78	1.95	5587111
$h_{og} = h_{od} = 20$ $h_{rg} = h_{rd} = 25$	0.020	0.0138	0.0365	0.0618	2000	1381	1.78	1.91	5587789
$h_{og} = h_{rg} = 20$	0.020	0.0182	0.0365	0.0699	2000	1827	1.78	2.76	5581324
$\beta_o = \beta_r = 0$	0.020	0.0140	0.0365	0.0622	2000	1406	1.76	1.93	5587011
$l_o = l_r = 0$	0.020	0.0155	0.0366	0.0651	2000	1558	0	0	5580380
$l_o = l_r = 0$ $\alpha = a = 0$	0.020	0.0163	0.0365	0.0664	2000	1631	0	0	5578555
$h_{og} = h_{rg} = 20$	0	0.0354	0.0648	0.0648	0	3550 °	0	5.4	5585464
$z_r = z_o = 20$ $\beta_o = \beta_r = 25$	U	0.0554	0.0048	0.0048	U	3330 *	U	5.4	<i>3</i> 363404

^a The order quantity as in Example 4.1 (Table 4.2).

Table 7.3. The effect of Wright's learning curve on the optimal values of the general model.

Parameter	j	p_{j}	f_{oj}	f_{rj}^*	f_j^*	f_{Rj}^*	q_{oj}^*	q_{rj}^*	ω_{oj}^*	ω_{rj}^*	W_j^*
$z_o = 10$ $z_r = 20$	1	0.08537	0.020	0.0135	0.0365	0.0611	2000	1348	3.62	3.12	5592737
$z_o = 10$ $z_r = 20$	2	0.04926	0.020	0.0124	0.0379	0.0616	2000	1246	3.83	3.04	5383925

In the next section, we summarise some key findings of our work and relate the results to the general body of knowledge in the discipline.

7.5.3. Findings

- The tabulated results indicate that varying demand, screening, defectiveness and deterioration rates significantly impact on the optimal quantity that is allocated to the RW (Table 7.2).
- When the same assumed holding costs for the good and defective items are considered, then the optimal order quantity is less than the one with differing holding costs (Table 7.2).
- The lot size with the same assumed holding costs for the OW and RW is less than the one with differing holding costs (Table 7.2).
- Increasing (decreasing) the rate of change in the demand significantly impacts the optimal amount that is allocated to the RW (Table 7.2).
- The tabulated results indicate that the reduction of the optimal quantity that is allocated to the RW does not necessarily decrease the total minimum cost per year, i.e. it may increase it.
- The total minimum cost per year decreases as learning increases (Table 7.3).
- As learning increases, all optimal quantities decrease except for the amount of deteriorated items in the OW that encounter a minor increase due to the slight increase in the cycle length (Table 7.3).
- Previously published models in this area are shown to be special cases of our model (Table 7.2 and Appendix C).

Remark 7.1

The above findings support the findings presented in Chapters 4 and 6. Moreover, this model is a viable solution for all special cases that are suggested in Chapter 5. Further investigations related to this model will be addressed in Chapter 8.

Remark 7.2

Although Table 7.3 assumes a higher (lower) holding cost (deterioration rate) in RW (e.g. Chung et al. (2009) and Jaggi et al. (2015)), the total minimum cost per year is less than that obtained under the LIFO dispatching policy (Table 6.3). This implies that the trade-off between LIFO and FIFO dispatching policies constitutes a key business objective in supply chain management.

7.6. Conclusion and further research

In this chapter, a general two-warehouse EOQ model for items with imperfect quality has been presented. Each sub-replenishment that is allocated to OW (RW) undergoes a 100 per cent screening and the percentage of defective items per lot reduces according to a learning curve. The general model developed in this chapter reflects the FIFO dispatching policy, i.e. the items are stocked into OW first and then in RW. However, the goods of the OW are consumed first, before considering the RW inventory.

The mathematical formulation reflects several practical concerns with regard to product

quality related issues, with the demand, screening and product deterioration rates being arbitrary functions of time. Therefore, the generality of the model may assist operations managers to determine the optimum order quantity that minimises total system cost. It has been shown that the solution to the underlying inventory model, if it exists, is unique and global optimal (Appendix C). Items deteriorate while they are effectively in stock and items not conforming to certain quality standards are stored in separate facilities with different holding costs of the good and defective items being considered.

Illustrative examples to support application of the model and solution procedure in different realistic situations have been presented. The analytical results reflect the incorporation of learning effects and varying demand, deterioration, defectiveness and screening rates on the proposed model. We observed the effect of changing all model parameters and found that a reduction in the optimal order size does not necessarily lead to a lower total minimum cost per unit time.

The proposed model is viable for fixed and random lifetimes of perishable products, where VOI may be used to model the shelf lifetime of an item (see Chapter 8). The versatile nature of our model and the fact that it may accommodate many real-world concerns has been emphasised, whereby the results obtained are compatible with the behaviour observed in many real-life settings. To the best of our knowledge, this appears to be the first time that such a general formulation of a two-level storage inventory model under FIFO dispatching policy is presented, investigated and numerically verified.

Several interesting extensions for finite or infinite planning horizons are possible, such as allowing for shortages, considering that the screening rate follows learning and forgetting curves and the risk of failure during screening (Type I and Type II errors). In addition, it seems

plausible to formulate an EPQ model considering FIFO dispatching policy, to assess the formulation of an EOQ model considering multiple items or to study the effect of different supplier trade credit practices.

The mathematical formulation allows the incorporation of other forms of varying demand, screening, defective items and deterioration rates so that interested readers can determine the optimum order quantity that minimises total system cost.

At this point, it is important to note that the terms LIFO and FIFO are often associated with cost accounting, and indeed there is a considerable amount of research conducted in this area. However, these terms are solely used, for the purposes of this PhD thesis, to indicate which warehouse is being used first.

It should also be noted that a time gap exists between consecutive sub-replenishments that are delivered to OW and RW. Under the LIFO policy, the time gap may affect availability of RW and the FIFO policy renders the OW unusable during the consumption period of RW. Subsequently, this necessitates introducing costs associated with the OW or RW being idle when formulating a two-warehouse inventory model. This line of research will be addressed in Chapter 8 below.

8. General EOQ models for imperfect quality items under LIFO, FIFO and AIFO dispatching policies

8.1. Introduction

As the literature suggests, the classical formulation of a two-warehouse inventory model is often based on the LIFO or FIFO dispatching policies. The LIFO policy relies upon inventory stored in the RW, with ample capacity, being consumed first, before depleting inventory of the OW that has limited capacity. Consumption works the opposite way around for the FIFO policy. In this chapter, a new policy entitled "Allocation-In-Fraction-Out (AIFO)" is proposed. Unlike LIFO and FIFO, AIFO implies simultaneous consumption fractions associated with RW and OW. That said, the goods at both warehouses are depleted by the end of the same cycle. However, the LIFO and FIFO policies assume no cost effect while the initially used warehouse is idle. Subsequently, this necessitates introducing costs associated with the OW or RW being idle when formulating a two-warehouse inventory model, i.e. a KPI to trade-off the costs associated with AIFO, LIFO and FIFO. Therefore, three general two-warehouse inventory models for items that are subject to inspection for imperfect quality are developed and compared – each underlying one of the dispatching policies considered. A rigorous method is utilised to show that the solution to the underlying inventory model for the AIFO policy, if it exists, is unique and global optimal (Appendix D).

Practical examples that are published in the literature for generalised models in this area are shown to be special cases of our FIFO, LIFO and AIFO models. We observe and test the behaviour of the theoretical models in different settings (e.g. different transportation costs associated with OW and RW, functions for varying demand, screening, defectiveness and

deterioration rates, VOI, perishable items that are subject to deterioration while in storage and by means of relaxing the inherent determinism related to the maximum fulfilment of the capacity of OW).

The remainder of this chapter is organised as follows: Section 8.2 emphasises the need for the research. In Section 8.3, we present our three EOQ models for items with imperfect quality and the solution procedures related to the AIFO dispatching policy. Illustrative examples, a comparison between the three models and special cases are offered in Section 8.4. Concluding remarks and opportunities for further research are provided in Section 8.5. The proof of the optimality and uniqueness of our solution is presented in Appendix D.

8.2. Need for the research

The classical formulation of a two-warehouse inventory model assumes that the lot size entering the system first fulfils the maximum storage capacity of the OW with the remaining quantity, over and above that maximum capacity, being kept at the RW. Subsequently, this entails two types of dispatching policies. The first is to consume the goods of the RW at the earliest, which is termed LIFO dispatching policy. Researchers advocating such a policy assume a higher (lower) holding cost (deterioration rate) in RW due to the availability of better preserving environmental conditions (e.g. Jaggi et al., 2015). Conversely, when the FIFO dispatching policy is employed, the goods of the OW are consumed first before considering the RW inventory. This case is usually justified by holding cost reduction, especially when the holding cost in RW is lower than that in OW due to competition, i.e. various offers are available in the market (e.g. Lee, 2006; Niu and Xie, 2008).

The above assumption embedded in the EOQ models with a two-level of storage implies that the lot size is delivered to the inventory system in one batch. This assumption ignores the cost effects of transporting items to distinct warehouses, and whether those items are transported to OW first and then to RW, or vice versa. It is important to note that if no penalty charges are payable to the supplier when a replenishment (bulk quantity) is divided into two sub-replenishments, then there is no reason why the second sub-replenishment is not delivered at or just before the stored items in either warehouse are completely consumed. That is, the mathematical formulation of a two-level storage has no meaning. Therefore, considering differing unit transportation costs among supply chain levels may have a considerable effect on the optimal order quantity. This can be justified by the distinct location of each warehouse, i.e. there exists at least a marginal difference in distance that incurs an additional transportation cost payable for inventory movements.

From a managerial point of view, there is indeed a time gap between consecutive sub-replenishments that are delivered to OW and RW. The LIFO policy may influence the warehouse rental contract, i.e. the time gap may affect the availability of RW (Fig. 8.1). On the other hand, the FIFO policy renders the OW unusable during the consumption period of RW (Fig. 8.2). Because of this, both LIFO and FIFO assume no cost effect while the initially used warehouse is idle. Finally, in the case of managing perishable products, LIFO and FIFO may not be the right choices, given that the order quantity needs to be consumed based on a FEFO policy.

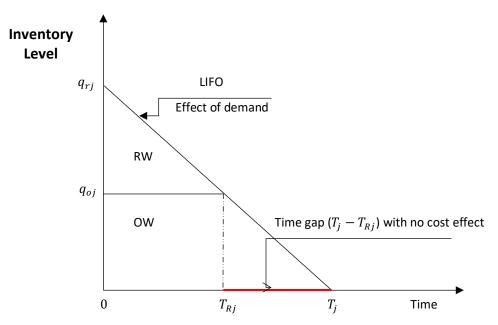


Fig. 8.1. Inventory variation of the two-warehouse model during one cycle (LIFO).

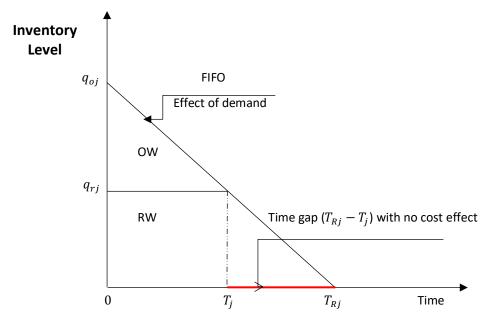


Fig. 8.2. Inventory variation of the two-warehouse model during one cycle (FIFO).

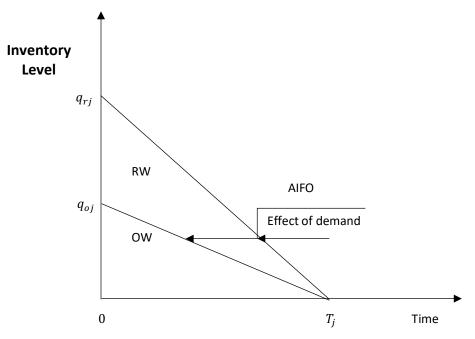


Fig. 8.3. Inventory variation of the two-warehouse model during one cycle (AIFO).

In this chapter, a new policy entitled "Allocation-In-Fraction-Out (AIFO)" is developed. Under an AIFO dispatching policy the goods at RW and OW experience simultaneous consumption fractions, which implies that the inventories at both warehouses are depleted by the end of the same cycle (Fig.8.3). On the other hand, the LIFO and FIFO policies assume no cost effect while the initially used warehouse is idle, which is unrealistic and a rare scenario to encounter in practice. Subsequently, this necessitates introducing costs associated with the OW or RW being idle when formulating a two-warehouse inventory model, i.e. a key performance indicator to trade-off the costs associated with AIFO, LIFO and FIFO. Therefore, three general EOQ models for items with imperfect quality are presented and compared. The first model underlies the LIFO policy, the second model underlies the FIFO policy and the third model relates to the AIFO policy.

It becomes apparent that the trade-off between the three policies constitutes a key business

objective in supply chain management. Under both the LIFO and FIFO dispatching policies, the cost associated with the OW or RW being idle is treated as an input parameter as well as a decision variable. If the cost is a decision variable, then it constitutes KPI, i.e. an upper-bound (cost associated with OW (RW) being idle) that renders AIFO the optimal dispatching policy.

To the best of our knowledge, the maximum capacity of the OW is invariably treated in the academic literature as an input parameter. Relaxing the inherent determinism related to the maximum fulfilment of the capacity of OW may lead to maximizing net revenue. In addition, if the system is subject to learning, then the lot size may reduce for each successive replenishment. However, such reduction only affects the amount allocated to the RW, and the amount allocated to the OW remains at the maximum capacity. Relaxing the inherent determinism of this assumption implies comprehensive learning that can be achieved simultaneously, i.e. the amounts that are allocated to both the OW and RW are affected.

The proposed models may be viewed as realistic in today's competitive markets and reflective of a number of practical concerns with regard to product quality related issues. These issues relate to imperfect items received from suppliers, deterioration of goods during storage, potential dis-location of good and defective items, tracking the quality of perishable products in a supply chain and transfer of knowledge from one inventory cycle to another.

8.3. Formulation of the general models

8.3.1. Assumptions and notation

We will use throughout the chapter the subscript "o(r)" to indicate the quantity related to the OW (RW). We will also employ the subscript "g(d)" to refer to good (defective) items. Thus, for example, denoting the cycle index by j, $I_{rgj}(t)$ denotes the inventory level of good items at time t in RW, and $I_{odj}(t)$ refers to the inventory level of defective items at time t in OW. We will also use the subscript i(i=A,L,F) to refer to the AIFO, LIFO and FIFO dispatching policies, respectively.

Our models are developed under the following assumptions and notation:

- 1. A single item is held in stock.
- 2. The lead-time is negligible, i.e. any replenishment ordered at the beginning of a cycle arrives just prior to the end of that same cycle.
- 3. The demand, screening and deterioration rates are arbitrary functions of time denoted by D(t), x(t) and $\delta_{\nu}(t)$ respectively.
- 4. The OW has a fixed limited capacity and the RW has unlimited capacity.
- 5. The percentage of defective items per lot reduces according to a learning curve denoted by p_i , where j is the cycle index.
- 6. Shortages are not allowed, i.e. we require that $(1-p_j)x(t) \ge D(t) \ \forall \ t \ge 0$.
- 7. The cost parameters are as follows:

c =Unit purchasing cost;

d =Unit screening cost;

 $c_L =$ Charge payable per unit time if RW remains idle for the LIFO model;

 c_F = Cost incurred per unit time if OW remains idle for the FIFO model;

 s_o = Unit transportation cost for OW;

 s_r = Unit transportation cost for RW;

 $h_{rg} = \text{Holding cost of good items per unit per unit time for RW;}$

 h_{rd} = Holding cost of defective items per unit per unit time for RW;

 $h_{og} = \text{Holding cost of good items per unit per unit time for OW;}$

 $h_{od} = \text{Holding cost of defective items per unit per unit time for OW;}$

k = Cost of placing an order.

At the beginning of each cycle j(j=1,2,...), a lot of size Q_{ij} is delivered such that a quantity of size q_{oij} is allocated to the OW and the remaining amount of size $q_{rij} = Q_{ij} - q_{oij}$ is allocated to the RW. Each sub-replenishment that enters the OW (RW) undertakes a 100 per cent screening process at a rate of x(t) that starts at the beginning of the cycle and ceases by time T_{oij} (T_{rij}), by which point, q_{oij} (q_{rij}) units have been screened and y_{oij} (y_{rij}) units have been consumed. Each sub-replenishment covers the actual demand and deterioration during both the first phase (screening) and the second phase (non-screening). During the screening phase, items not conforming to certain quality standards (defective items) are

stored in different warehouses.

8.4. AIFO dispatching policy

As an application of an AIFO dispatching policy, items are simultaneously depleted from the RW and OW at rates \emptyset_{oj} D(t) and \emptyset_{rj} D(t) respectively, where $\emptyset_{rj}=1-\emptyset_{oj}$. Unlike LIFO and FIFO, the analysis of AIFO is limited to one case, i.e. the cycle length for the RW and OW is the same, i.e. T_j . The behaviour of such a model is depicted in Fig. 8.4.

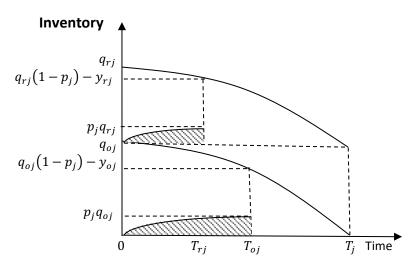


Fig. 8.4. Inventory variation of the two-warehouse model during one cycle (AIFO).

The variations in the inventory levels depicted in Fig. 8.4 for the OW and RW are given by the following differential equations:

$$\frac{dI_{ogj}(t)}{dt} = -\phi_{oj}D(t) - p_{j}x(t) - \delta_{o}I_{ogj}(t), \qquad 0 \le t < T_{oj} \quad (8.1)$$

$$\frac{dI_{ogj}(t)}{dt} = -\emptyset_{oj}D(t) - \delta_o I_{ogj}(t), \qquad T_{oj} \le t \le T_j \qquad (8.2)$$

$$\frac{dI_{rgj}(t)}{dt} = -\phi_{rj}D(t) - p_{j}x(t) - \delta_{r}I_{rgj}(t), \qquad 0 \le t < T_{rj} \quad (8.3)$$

$$\frac{dI_{rgj}(t)}{dt} = -\emptyset_{rj}D(t) - \delta_r I_{rgj}(t), \qquad T_{rj} \le t \le T_j \qquad (8.4)$$

with the boundary conditions $I_{ogj}(0)=q_{oj}$, $I_{ogj}(T_j)=0$, $I_{rgj}(0)=q_{rj}$ and $I_{rgj}(T_j)=0$

where

$$Q_{ij} = \int_0^{T_{oj}} x(u) du + \int_0^{T_{rj}} x(u) du.$$
 (8.5)

Finally, the variations in the inventory levels for defective items (shaded area) depicted in Fig. 8.4 are given by the following differential equations:

$$\frac{dI_{rdj}(t)}{dt} = p_j x(t), \qquad 0 \le t \le T_{rj} \quad (8.6)$$

$$\frac{dI_{odj}(t)}{dt} = p_j x(t), \qquad 0 \le t \le T_{oj} \quad (8.7)$$

with the boundary conditions $I_{rdj}(0)=0$, $I_{odj}(0)=0$, $I_{rdj}(T_{rj})=p_jq_{rj}$ and $I_{odj}(T_{oj})=p_jq_{oj}$.

Solving the above differential equations, we get

$$I_{ogj}(t) = e^{-(g_o(t) - g_o(0))} \int_0^{T_{oj}} x(u) du - e^{-g_o(t)} \int_0^t [\emptyset_{oj} D(u) + p_j x(u)] e^{g_o(u)} du,$$

$$0 \le t < T_{oj}$$
 (8.8)

$$I_{ogj}(t) = e^{-g_o(t)} \int_t^{T_j} \phi_{oj} D(u) e^{g_o(u)} du,$$
 $T_{oj} \le t \le T_j$ (8.9)

$$I_{odj}(t) = \int_0^t p_j x(u) du,$$
 $0 \le t \le T_{oj}$ (8.10)

$$I_{rgj}(t) = e^{-(g_r(t) - g_r(0))} \int_0^{T_{rj}} x(u) du - e^{-g_r(t)} \int_0^t [\phi_{rj} D(u) + p_j x(u)] e^{g_r(u)} du,$$

$$0 \le t < T_{ri}$$
 (8.11)

$$I_{rgj}(t) = e^{-g_r(t)} \int_t^{T_j} \phi_{rj} D(u) e^{g_r(u)} du,$$
 $T_{rj} \le t \le T_j$ (8.12)

$$I_{rdj}(t) = \int_0^t p_j x(u) du,$$
 $0 \le t \le T_{rj}$ (8.13)

respectively.

Now, the per cycle cost components for the given inventory model are as follows:

Purchasing cost + Screening cost + Transportation cost = $(c+d+s_o)\int_0^{T_{oj}} x(u)du + (c+d+s_r)\int_0^{T_{rj}} x(u)du$. Note that the purchasing cost includes the defectiveness and deterioration costs.

 $\text{Holding cost for the RW} = h_{rg} \big[I_{rgj} \big(0, T_{rj} \big) + I_{rgj} \big(T_{rj}, T_j \big) \big] + h_{rd} I_{rdj} \big(0, T_{rj} \big).$

Holding cost for the OW = $h_{og}[I_{ogj}(0,T_{oj}) + I_{ogj}(T_{oj},T_j)] + h_{od}I_{odj}(0,T_{oj})$.

Thus, the total cost per unit time of the underlying inventory model during the cycle $[0, T_j]$, as a function of T_{rj} , T_j and \emptyset_{oj} say $Z_A(T_{rj}, T_j, \emptyset_{oj})$ is given by

$$Z_{A}(T_{rj}, T_{j}, \emptyset_{oj}) = \frac{1}{T_{j}} \left\{ (c + d + s_{o}) \int_{0}^{T_{oj}} x(u) du + (c + d + s_{r}) \int_{0}^{T_{rj}} x(u) du + h_{og} \left[\left[G_{o}(T_{oj}) - G_{o}(0) \right] e^{g_{o}(0)} \int_{0}^{T_{oj}} x(u) du - \int_{0}^{T_{oj}} \left[G_{o}(T_{oj}) - G_{o}(u) \right] \left[\emptyset_{oj} D(u) + h_{og} \left[\left[G_{o}(T_{oj}) - G_{o}(0) \right] e^{g_{o}(0)} du + \int_{T_{oj}}^{T_{j}} \left[G_{o}(u) - G_{o}(T_{oj}) \right] \phi_{oj} D(u) e^{g_{o}(u)} du \right] h_{od} \left[\int_{0}^{T_{oj}} \left[T_{oj} - u \right] p_{j} x(u) du \right] + h_{rg} \left[\left[G_{r}(T_{rj}) - G_{r}(0) \right] e^{g_{r}(0)} \int_{0}^{T_{rj}} x(u) du - \int_{0}^{T_{rj}} \left[G_{r}(T_{rj}) - G_{r}(0) \right] e^{g_{r}(0)} du \right] + h_{rg} \left[\left[G_{r}(T_{rj}) - G_{r}(0) \right] e^{g_{r}(0)} du + \int_{T_{rj}}^{T_{j}} \left[G_{r}(u) - G_{r}(T_{rj}) \right] \phi_{rj} D(u) e^{g_{r}(u)} du \right] + h_{rd} \left[\int_{0}^{T_{rj}} \left[T_{rj} - u \right] p_{j} x(u) du \right] + k \right\}.$$

$$(8.14)$$

Our objective is to find T_{rj} , T_j and \emptyset_{oj} that minimise $Z_A(T_{rj}, T_j, \emptyset_{oj})$, where $Z_A(T_{rj}, T_j, \emptyset_{oj})$ is given by (8.14). But the variables T_{rj} , T_j and \emptyset_{oj} are associated with each other through the

following relations:

$$0 < T_{rj} < T_j, (8.15)$$

$$e^{g_o(0)} \int_0^{T_{oj}} x(u) du = \int_0^{T_j} \phi_{oj} D(u) e^{g_o(u)} du + \int_0^{T_{oj}} p_j x(u) e^{g_o(u)} du,$$
 (8.16)

$$e^{g_r(0)} \int_0^{T_{rj}} x(u) du = \int_0^{T_j} \phi_{rj} D(u) e^{g_r(u)} du + \int_0^{T_{rj}} p_j x(u) e^{g_r(u)} du.$$
 (8.17)

Thus, our goal is to solve the following optimisation problem, which we shall call problem (m_A)

$$(m_A) = \begin{cases} \text{minimise } Z_A(T_{rj}, T_j, \emptyset_{oj}) \text{ given by } (8.14) \\ \text{subject to } (8.15 - 8.17) \text{ and } 0 \le \emptyset_{oj} \le 1 \end{cases}$$

From Eq. (8.17), $T_{rj}=0 \Rightarrow T_j=0$ and $T_{rj}>0 \Rightarrow T_{rj}< T_j$. Thus Eq. (8.17) implies constraint (8.15). Hence, if we temporarily ignore the monotony constraint (8.15) and call the resulting problem (m_{A1}) then (8.15) does satisfy any solution of (m_{A1}) . Therefore, (m_A) and (m_{A1}) are equivalent. Moreover, $T_{rj}>0 \Rightarrow {\rm RHS}$ of (8.9) and (8.12) >0, i.e. Eqs. (8.16) and (8.17) guarantee that the number of good items is at least equal to the demand and deterioration during screening.

8.5. Solution procedures

First, we note from (8.16) and (8.17) that T_{rj} , T_j and \emptyset_{oj} can be determined as functions of q_{rj} , say

$$T_{rj} = f_{rj}(q_{rj}),$$
 (8.18)

$$T_j = f_j(q_{rj}), (8.19)$$

$$\emptyset_{oj} = \emptyset_j(q_{rj}). \tag{8.20}$$

Thus, considering Eqs. (8.16)-(8.20) then the problem (m_A) is converted to the following unconstrained problem with the variable Q_{Aj} (which we shall call problem (m_{A2})).

$$W_{A}(Q_{Aj}) = \frac{1}{f_{j}} \Big\{ (c + d + s_{o}) \int_{0}^{f_{oj}} x(u) du + (c + d + s_{r}) \int_{0}^{f_{rj}} x(u) du + h_{og} \Big[-G_{o}(0)e^{g_{o}(0)} \int_{0}^{f_{oj}} x(u) du + \int_{0}^{f_{oj}} p_{j}x(u) G_{o}(u)e^{g_{o}(u)} du + h_{og} \Big[-G_{o}(0)e^{g_{o}(u)} du \Big] + h_{od} \Big[\int_{0}^{f_{oj}} [f_{oj} - u]p_{j}x(u) du \Big] + h_{rg} \Big[-G_{r}(0)e^{g_{r}(0)} \int_{0}^{f_{rj}} x(u) du + \int_{0}^{f_{rj}} p_{j}x(u) G_{r}(u)e^{g_{r}(u)} du + (1 - u)g_{j} \Big] \Big[-G_{o}(u)G_{o}(u)e^{g_{r}(u)} du + h_{rd} \Big[\int_{0}^{f_{rj}} [f_{rj} - u]p_{j}x(u) du \Big] + k \Big\}.$$

$$(8.21)$$

If we let $W_A = \frac{w_A}{f_i}$, then the necessary condition for having a minimum for problem (m_{A2}) is

$$w'_{q_{rj}}f_j = f'_{j,q_{rj}}w_A, (8.22)$$

where $w_{q_{rj}}^\prime$ and $f_{j,q_{rj}}^\prime$ are the derivatives of $w_{\!\scriptscriptstyle A}$ and $f_{\!\scriptscriptstyle J}$ with respect to q_{rj} , respectively.

Also, (8.16) and (8.17) yield

$$f'_{j,q_{rj}} = \frac{\left(e^{gr(0)} - p_{j}e^{gr(f_{rj})}\right) \left(\int_{0}^{f_{j}} D(u)e^{g_{o}(u)}du\right)^{2}}{D(f_{j})e^{gr(f_{j})} \left(\left(\int_{0}^{f_{j}} D(u)e^{g_{o}(u)}du\right)^{2} - S\left(\int_{0}^{f_{j}} D(u)e^{g_{o}(u)}du - e^{\left(g_{o}(f_{j}) - g_{r}(f_{j})\right)}\int_{0}^{f_{j}} D(u)e^{g_{r}(u)}du\right)\right)},$$
(8.23)

$$\emptyset'_{j,q_{rj}} = -\frac{sf'_{j,q_{rj}}D(f_j)e^{g_o(f_j)}}{\left(\int_0^{f_j}D(u)e^{g_o(u)}du\right)^2},$$
(8.24)

where $S = e^{g_0(0)} \int_0^{f_{oj}} x(u) du - \int_0^{f_{oj}} p_j x(u) e^{g_0(u)} du$.

Considering the above and also Eqs. (8.18)-(8.21) we have

$$w'_{q_{rj}} = (c + d + s_r) + h_{rg} \left[\left(G_r(f_j) - G_r(0) \right) e^{g_r(0)} + \left(G_r(f_{rj}) - G_r(f_j) \right) p_j e^{g_r(f_{rj})} + \right.$$

$$\phi'_{j,q_{rj}} \left(G_r(f_j) \int_0^{f_j} D(u) e^{g_r(u)} du - \int_0^{f_j} D(u) G_r(u) e^{g_r(u)} du \right) \right] + \frac{h_{rd}}{x(f_{rj})} \int_0^{f_{rj}} p_j x(u) du +$$

$$h_{og} \left[\phi'_{j,q_{rj}} \int_0^{f_j} D(u) G_o(u) e^{g_o(u)} du + \phi_j f'_{j,q_{rj}} D(f_j) G_o(f_j) e^{g_o(f_j)} \right].$$
(8.25)

Also, (24)
$$\Leftrightarrow W_A = \frac{w_A}{f_j} = \frac{w'_{q_{rj}}}{f'_{j,q_{rj}}}.$$
 (8.26)

Eq. (8.26) can be used to determine the optimal value of Q_{Aj} and its corresponding total minimum cost. Then the optimal values of T_{rj} , T_j and \emptyset_j can be found from Eqs. (8.18), (8.19) and (8.20), respectively.

8.6. LIFO dispatching policy

When applying a LIFO dispatching policy, items stored in the RW are depleted first by time T_{Rj} . In this model, we distinguish two cases:

The mathematical formulation for Cases 1 and 2 can be obtained in a similar way as that for Chapter 6, where the total cost per unit time of the underlying inventory model is identical for Cases 1 and 2 and is given by

$$W_{L}(Q_{Lj}) = \frac{1}{f_{j}} \Big\{ (c + d + s_{o}) \int_{0}^{f_{oj}} x(u) du + (c + d + s_{r}) \int_{0}^{f_{rj}} x(u) du + h_{rg} \Big[-G_{r}(0)e^{g_{r}(0)} \int_{0}^{f_{rj}} x(u) du + \int_{0}^{f_{rj}} p_{j}x(u)G_{r}(u)e^{g_{r}(u)} du + \int_{0}^{f_{Rj}} D(u)G_{r}(u)e^{g_{r}(u)} du \Big] + h_{rd} \Big[\int_{0}^{f_{0j}} [f_{rj} - u]p_{j}x(u) du \Big] + h_{og} \Big[-G_{o}(0)e^{g_{o}(0)} \int_{0}^{f_{oj}} x(u) du + \int_{0}^{f_{oj}} p_{j}x(u)G_{o}(u)e^{g_{o}(u)} du + \int_{f_{Rj}}^{f_{j}} D(u)G_{o}(u)e^{g_{o}(u)} du \Big] + h_{od} \Big[\int_{0}^{f_{oj}} [f_{oj} - u]p_{j}x(u) du \Big] + k + c_{Lj} \Big(f_{j} - f_{Rj} \Big) \Big\}.$$

$$(8.27)$$

8.7. FIFO dispatching policy

When applying a FIFO dispatching policy, the goods of the RW are consumed only after depleting the goods of OW, i.e. q_{oj} is consumed first, which implies that the cycle length for the OW is a predetermined value. In this model, we distinguish two cases:

The mathematical formulation for Cases 1 and 2 can be obtained in a similar way as that for Chapter 7, where the total cost per unit time of the underlying inventory model is identical for Cases 1 and 2 and is given by

$$W_{F}(Q_{Fj}) = \frac{1}{f_{Rj}} \Big\{ (c + d + s_{o}) \int_{0}^{f_{oj}} x(u) du + (c + d + s_{r}) \int_{0}^{f_{rj}} x(u) du + h_{og} \Big[-G_{o}(0)e^{g_{o}(0)} \int_{0}^{f_{oj}} x(u) du + \int_{0}^{f_{oj}} p_{j}x(u)G_{o}(u)e^{g_{o}(u)} du + \int_{0}^{f_{j}} D(u)G_{o}(u)e^{g_{o}(u)} du \Big] + h_{od} \Big[\int_{0}^{f_{oj}} \Big[f_{oj} - u \Big] p_{j}x(u) du \Big] + h_{rg} \Big[-G_{r}(0)e^{g_{r}(0)} \int_{0}^{f_{rj}} x(u) du + \int_{0}^{f_{rj}} p_{j}x(u)G_{r}(u)e^{g_{r}(u)} du + \int_{f_{j}}^{f_{Rj}} D(u)G_{r}(u)e^{g_{r}(u)} du \Big] + h_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big] + k_{rd} \Big[\int_{0}^{f_{rj}} \Big[f_{rj} - u \Big] p_{j}x(u) du \Big]$$

8.8. Numerical analysis and special cases

In this section, we present illustrative examples and special cases to support the application of our mathematical models and solution procedures in different realistic situations. First, we formulate the upper-bound (cost applied if OW (RW) is idle). Then we test the behaviour of the theoretical models in different settings. For example, we consider different transportation costs associated with OW and RW, time-varying rates, VOI and perishable items. Moreover, we observe the impact of relaxing the inherent determinism related to the maximum fulfilment of the capacity of OW.

8.8.1. Formulation of the upper-bound

Even though c_{ij} is formulated for the LIFO and FIFO models, its associated value is set to be equal to zero. That is, rather than assigning c_{ij} a specific value that would render the AIFO policy to perform better than LIFO and FIFO, ignorance of such a value implies that AIFO is optimal unless $c_{ij} \leq \Delta_{ij}$.

Now, let

$$\Delta_{Lj} = \varepsilon h_{rg} + \frac{T_{Lj}^* \left(W_{Aj}^* - W_{Lj}^*\right)}{T_{Lj}^* - T_{LRj}^*},$$

$$\Delta_{Fj} = \varepsilon h_{og} + \frac{T_{FRj}^* \left(W_{Aj}^* - W_{Fj}^*\right)}{T_{FRj}^* - T_{Fj}^*},$$

where arepsilon=0.5 denotes the minimum average inventory of one unit that can be stored in either unused warehouse.

This appears realistic, since we either store at least one unit, in the unused warehouse, or do not keep any (e.g. EOQ). On the other hand, an AIFO policy implies simultaneous consumption fractions associated with RW and OW, where the goods at both warehouses are depleted by the end of the same cycle, i.e. $\varepsilon=0$.

Thus, Δ_{ij} constitutes KPI, i.e. an upper-bound (cost applied if OW (RW) is idle) that renders AIFO the optimal dispatching policy.

Note that as $T^*_{ij} \to T^*_{iRj}$ then $\varepsilon \to 0 \Rightarrow W^*_{Aj} = W^*_{Lj} = W^*_{Fj} \Rightarrow \mathrm{EOQ} \Rightarrow c_{ij} = 0 = \Delta_{ij}$ (recall m_{i2}). Therefore, if $W^*_{Lj} > W^*_{Fj}$ then c_{Fj} represents the cost incurred per year when the OW remains unusable (empty). Conversely, if $W^*_{Lj} < W^*_{Fj}$ then c_{Lj} denotes the charge payable per year if the RW remains idle or the charge incurred per year in order to guarantee that the RW is available.

As can been seen in the next section, this cost is typically very small with respect to the minimum average holding cost per year incurred to store items in either warehouse.

8.8.2. Varying rates

In this example (Example 8.1), we consider the following functions for varying demand, screening, defectiveness and deterioration rates:

$$x(t) = at + b$$
, $D(t) = \alpha t + r$, $p_j = \frac{\tau}{\pi + e^{\gamma j}}$, $\delta_o(t) = \frac{l_o}{z_o - \beta_o t}$ and $\delta_r(t) = \frac{l_r}{z_r - \beta_r t}$

where $b, r, \pi, z_y > 0$; $a, \alpha, l_y, \tau, \gamma, \beta_y, t \ge 0$ and $\beta_y t < z_y$.

Note that $\delta_y(t)$ is an increasing function of time and p_j reduces according to an S-shaped logistic learning curve (Jordan, 1958; Carlson, 1973).

Each problem (m_{i2}) has been coded in *MATLAB* for the above functions and solutions were obtained for a wide range of the control parameter values. Here, and for comparison purposes, we thematically consider situations with parameters that are presented in Table 8.1 below.

Table 8.1. Input parameters for example 8.1.

h_{og}	h_{od}	h_{rg}	h_{rd}	q_o	k
20	5	25	5	2000	3000
Dollars/unit/year	Dollars/unit/year	Dollars/unit/year	Dollars/unit/year	Units	Dollars/cycle
а	b	α	r	С	d
1000	100200	500	50000	100	0.50
Units/year	Units/year	Units/year	Units/year	Dollars/unit	Dollars/unit
l_o	l_r	Z_{o}	z_r	eta_o	eta_r
1	1	20	33.33	25	20
Units/year	Units/year	Units/year	Units/year	Units/year	Units/year
τ	π	γ	S_{o}	s_r	
70.067	819.76	0.7932	0.50	0.75	
Units/year	Units/year	Units/year	Dollars/unit	Dollars/unit	

Table 8.2. Optimal results for varying demand, screening, defectiveness and deterioration rates.

j	p_{j}	f_{rj}^*	f_j^*	f_{Rj}^*	Q_j^*	ω_{oj}^*	ω_{rj}^*	W_j^*	w_j^*	Policy	c_{ij} vs. \emptyset_j^*
1	0.08524	0.0121	0.0586	0.0221	3209	3.88	0.39	5618002	329280	LIFO	$c_{Lj} \leq 1446$
1	0.08524	0.0110	0.0365	0.0568	3107	1.78	1.45	5619757	318980	FIFO	$c_{Fj}=0$
1	0.08524	0.0116	0.0577	-	3158	2.79	0.95	5618896	324160	AIFO	$\emptyset_j^* = 0.63$
2	0.08497	0.0121	0.0586	0.0221	3209	3.88	0.39	5616361	329210	LIFO	$c_{Lj} \leq 1446$
2	0.08497	0.0110	0.0366	0.0568	3106	1.78	1.45	5618116	318910	FIFO	$c_{Fj}=0$
2	0.08497	0.0116	0.0577	-	3158	2.79	0.95	5617256	324120	AIFO	$\emptyset_j^* = 0.63$
3	0.08436	0.0120	0.0586	0.0221	3207	3.88	0.39	5612658	329050	LIFO	$c_{Lj} \leq 1446$
3	0.08436	0.0110	0.0366	0.0568	3104	1.79	1.45	5614413	318720	FIFO	$c_{Fj}=0$
3	0.08436	0.0115	0.0577	-	3156	2.79	0.94	5613552	323920	AIFO	$\emptyset_j^* = 0.63$
4	0.08305	0.0120	0.0586	0.0221	3204	3.88	0.39	5604720	328700	LIFO	$c_{Lj} \leq 1446$
4	0.08305	0.0110	0.0366	0.0568	3100	1.79	1.45	5606477	318330	FIFO	$c_{Fj}=0$
4	0.08305	0.0115	0.0577	-	3152	2.80	0.94	5605616	323540	AIFO	$\emptyset_j^* = 0.63$
5	0.08030	0.0119	0.0587	0.0220	3196	3.89	0.38	5588132	327970	LIFO	$c_{Lj} \leq 1445$
5	0.08030	0.0109	0.0367	0.0568	3092	1.80	1.44	5589892	317510	FIFO	$c_{Fj}=0$
5	0.08030	0.0114	0.0577	-	3144	2.80	0.94	5589030	322760	AIFO	$\emptyset_j^* = 0.64$

Table 8.2 shows the effect of learning on the optimal values of Q_{ij}^* , T_{rij}^* , T_{Rij}^* , T_{ij}^* , ω_{ij}^* , \emptyset_j^* and the corresponding total minimum costs for five successive cycles. In the first cycle, the optimal order quantities for the three models are $Q_{L1}^*=3209$ units, $Q_{F1}^*=3107$ units and $Q_{A1}^*=3209$

3158 units (and $\emptyset_1^*=0.63$), respectively. The corresponding total minimum costs per year are $W_{L1}^*=5618002$ dollars, $W_{F1}^*=5619757$ dollars and $W_{A1}^*=5618896$ dollars and the total minimum costs per cycle are $w_{L1}^*=329280$ dollars, $w_{F1}^*=318980$ dollars and $w_{A1}^*=324160$ dollars, respectively. The number of deteriorated items is ω_{11}^* , which signifies the difference between the actual demand and the amount held in either warehouse at the beginning of the cycle, excluding the number of defective items. The number of defective items is $p_1Q_{11}^*$, which can be sold at a salvage price at times T_{o11}^* and T_{r11}^* or incur a disposal penalty charge. As learning increases, i.e. the percentage of defective items per lot decreases, all optimal quantities for the three models decrease, except the number of deteriorated items in the OW that experiences a minor increase due to the slight increase in the cycle length (Table 8.2).

8.8.3. AIFO vs. LIFO/FIFO

Although Table 8.2 indicates that the LIFO dispatching policy performs better than the AIFO policy, the fact remains that the former policy ignores cost effects during the time elapsed between consuming the goods of the RW and the time by which the next sub-replenishment is delivered. On the other hand, the latter operates in a simultaneous consumption fashion at the OW and RW, i.e. the goods at both warehouses are depleted by the end of the same cycle. It should be noted that when a LIFO policy is considered, the idle time has been found to be significant for a wide range of the control parameter values. In this example (Example 8.1), $T_{L1}^* - T_{LR1}^* = 0.0586 - 0.0221 = 0.0365 \cong 13$ days, which constitutes more than 62 per cent of the cycle length. That is to say, the RW remains idle for more than 227 days per year and free of charge, which is unrealistic and rare to encounter in practice.

Thus, a LIFO dispatching policy is optimal if, and only if, the charge payable to keep the RW available is less than or equals the upper-bound, i.e. $c_{L1} \leq \Delta_{L1} = 0.5 h_{rg} + \frac{T_{L1}^*(W_{A1}^* - W_{L1}^*)}{T_{L1}^* - T_{LR1}^*} = 10 + 1436 = 1446$ dollars per year.

Note that this cost is typically very small with respect to the average holding cost per year incurred to store items in the RW, which is given by $\frac{T_{LR1}^*h_{rg}}{2T_{L1}^*} \times 1209 = 5702$ dollars per year, assuming also that $h_{rd} = 0$. This is so, since $T_{L1}^* > T_{LR1}^*$, i.e. there is a time gap (free of charge) between consecutive sub-replenishments that are delivered to the RW. If for instance $T_{L1}^* = T_{LR1}^* \Rightarrow \text{EOQ}$, then this cost increases to 15113 dollars per year. Therefore, c_{L1} denotes the cost per year incurred if no items are stored in the RW. Considering m_{L2} and Table 8.2, this cost is less than 53 dollars per cycle or less than 900 dollars per year. This can be further justified if, for instance, this cost (e.g. 53 dollars) is included in the ordering cost applied for LIFO and setting $c_{L1} = 0$, then $W_{A1}^* < W_{L1}^*$.

For a FIFO dispatching policy, the time elapsed for the OW to remain unusable is more than 7 days, which constitutes more than 36 per cent of the cycle length, i.e. 130 days per year of an empty space. In many industrial situations, a substantial portion of holding cost also applies to an empty space.

It should be noted that the AIFO dispatching policy not only overcomes this issue, but may also lead to a discounted holding cost that can be gained if a continuous and long-term rental contract is beneficial and, hence, further reduction in the total minimum cost per year can be achieved.

As illustrated in Table 8.2, other forms of varying demand, screening, defectiveness and

deterioration rates may be incorporated in each model in order to allow managers to assess the consequences of a diverse range of strategies.

In the next section, we analyse the behaviour of the theoretical models in different settings, taking into account that the associated value of c_i is set to be equal to zero for every single case. Tables 8.3, 8.4, 8.5 and 8.6 depict the effect of each model parameter on the optimal values. Table 8.7 tests and compares the effect of learning when the maximum fulfilment of the capacity of OW is relaxed. Fig. 8.5 compares LIFO and AIFO for consecutive cycles in order to observe the effect of different learning curves on the optimal order quantities. Finally, Fig. 8.6 indicates the effect of different learning curves on the maximum rental cost associated with the RW, i.e. c_{Li} (upper-bound).

8.8.4. Sensitivity analysis

In this section, further interesting insights can be obtained. For example, Tables 8.3, 8.4, 8.5 and 8.6 summarise the effect of each model parameter on the optimal values. Table 8.7 replicates the first two rows of Table 8.5 for two consecutive cycles in order to observe the effect of Wright's learning curve, i.e. $p_j = \frac{\tau}{\pi+1} j^{-\gamma}$ on the optimal order quantity when the capacity of the OW is a decision variable with that of fixed value. Fig. 8.5 compares the optimal order quantity of AIFO with that of LIFO for 15 consecutive cycles with respect to $p_j = \frac{\tau}{\pi+e^{\gamma j}}$ (Jordan, 1958; Carlson, 1973) and $p_j = \frac{\tau}{\pi+1} j^{-\gamma}$ (Wright, 1936). Finally, Fig. 8.6 indicates the effect of different learning curves on the maximum rental cost associated with the RW, i.e. c_{Lj} (upper-bound).

Table 8.3. Sensitivity analysis for transportation costs.

Parameter	f_o^*	f_r^*	f^*	f_R^*	q_o^*	q_r^*	ω_o^*	ω_r^*	W*	Policy	<i>c</i> _i vs. Ø*
	0.020	0.0121	0.0586	0.0221	2000	1209	3.88	0.39	5618002	LIFO	$c_L \leq 1446$
$s_o = 0.50$	0.020	0.0110	0.0365	0.0568	2000	1107	1.78	1.45	5619757	FIFO	$c_F=0$
$s_r = 0.75$	0.020	0.0116	0.0577	-	2000	1158	2.79	0.95	5618896	AIFO	$Ø^* = 0.63$
	0.020	0.0149	0.0638	0.0273	2000	1494	4.38	0.59	5612482	LIFO	$c_L \leq 1781$
$s_o = 0.50$	0.020	0.0140	0.0365	0.0621	2000	1402	1.78	1.95	5614482	FIFO	$c_F = 0$
$s_r = 0.50$	0.020	0.0145	0.0630	-	2000	1449	3.04	1.30	5613495	AIFO	$Ø^* = 0.58$
	0.020	0.0149	0.0638	0.0273	2000	1495	4.38	0.59	5585101	LIFO	$c_L \leq 1781$
$s_o = s_r = 0$	0.020	0.0140	0.0365	0.0621	2000	1402	1.78	1.95	5587111	FIFO	$c_F=0$
	0.020	0.0145	0.0630	-	2000	1450	3.04	1.30	5586119	AIFO	$Ø^* = 0.58$
	0.020	0.0152	0.0644	0.0279	2000	1527	4.44	0.62	5610115	LIFO	$c_L \leq 1828$
$s_o = 0.50$	0.020	0.0151	0.0365	0.0642	2000	1513	1.78	2.15	5612175	FIFO	$c_F = 0$
$s_r = 0.40$	0.020	0.0155	0.0650	-	2000	1558	3.14	1.44	5611145	AIFO	$Ø^* = 0.56$
q_o^*	0.016	0.0193	0.0647	0.0354	1609	1935	3.86	0.99	5609991	LIFO	$c_L \leq 2271$
$s_o = 0.50$	0	0.0338	0.0620	0.0620	0	3391	0	3.07	5611828	FIFO	\Rightarrow EOQ
$s_r = 0.40$	0.014	0.0270	0.0639	-	1420	2078	2.17	1.90	5611016	AIFO	$Ø^* = 0.41$

Table 8.4. Sensitivity analysis for holding costs with $\boldsymbol{s}_o = \boldsymbol{s}_r = \mathbf{0}$.

Parameter	f_o^*	f_r^*	f^*	f_R^*	q_o^*	q_r^*	ω_o^*	ω_r^*	<i>W</i> *	Policy	c_i vs. \emptyset^*
	0.020	0.0147	0.0634	0.0269	2000	1472	4.34	0.57	5585796	LIFO	$c_L \leq 1764$
$h_{og} = h_{od} = 20$	0.020	0.0138	0.0365	0.0618	2000	1381	1.78	1.91	5587789	FIFO	$c_F = 0$
$h_{rg} = h_{rd} = 25$	0.020	0.0142	0.0626	-	2000	1428	3.03	1.27	5586805	AIFO	$Ø^* = 0.58$
	0.020	0.0172	0.0680	0.0315	2000	1724	4.79	0.79	5583382	LIFO	$c_L \leq 2198$
$h_{og} = h_{rg} = 20$	0.020	0.0182	0.0365	0.0699	2000	1827	1.78	2.76	5581324	FIFO	$c_F = 0$
	0.020	0.0177	0.0690	-	2000	1777	3.33	1.75	5582364	AIFO	$\phi^* = 0.58$
q_o^*	0	0.0371	0.0679	0.0679	0	3719	0	3.69	5581411	LIFO	\Rightarrow EOQ
$h_{og} = h_{rg} = 20$	0.0103	0.0271	0.0188	0.0685	1030	2718	0.47	3.40	5580732	FIFO	$c_F \leq 886$
	0.0035	0.0337	0.0680	-	348	3373	0.56	3.34	5581367	AIFO	$0^* = 0.09$

Table 8.5. Sensitivity analysis for deterioration rates with $\boldsymbol{s}_o = \boldsymbol{s}_r = \mathbf{0}$.

Parameter	f_o^*	f_r^*	f^*	f_R^*	q_o^*	q_r^*	ω_o^*	ω_r^*	<i>W</i> *	Policy	<i>c</i> _i vs. Ø*
	0.020	0.0128	0.0599	0.0235	2000	1285	8.24	0.73	5593672	LIFO	$c_L = 0$
$z_o = 10$	0.020	0.0135	0.0365	0.0611	2000	1348	3.62	3.12	5592737	FIFO	$c_F \leq 1236$
$z_r = 20$	0.020	0.0132	0.0605	-	2000	1318	6.01	1.90	5593230	AIFO	$Ø^* = 0.60$
q_o^*	0.0097	0.0228	0.0595	0.0418	971	2288	4.84	2.33	5593342	LIFO	$c_L = 0$
$z_o = 10$	0.0134	0.0196	0.0245	0.0603	1343	1959	1.62	3.97	5592454	FIFO	$c_F \leq 741$
$z_r = 20$	0.0117	0.0210	0.0599	-	1173	2108	3.44	3.04	5592887	AIFO	$Ø^* = 0.36$
	0.020	0.0151	0.0641	0.0276	2000	1510	4.28	0.60	5584866	LIFO	$c_L \leq 1914$
$\beta_o = \beta_r = 0$	0.020	0.0140	0.0365	0.0622	2000	1406	1.76	1.93	5587011	FIFO	$c_F = 0$
	0.020	0.0146	0.0632	-	2000	1458	2.97	1.29	5585950	AIFO	$Ø^* = 0.58$
	0.020	0.0175	0.0687	0.0321	2000	1756	0	0	5576235	LIFO	$c_L \leq 3965$
$l_o = l_r = 0$	0.020	0.0155	0.0366	0.0651	2000	1558	0	0	5580380	FIFO	$c_F = 0$
	0.020	0.0165	0.0669	-	2000	1658	0	0	5578342	AIFO	$Ø^* = 0.55$

Table 8.6. Sensitivity analysis for special cases of the general models with $s_o=s_r={f 0}.$

Parameter	f_o^*	f_r^*	f^*	f_R^*	q_o^*	q_r^*	ω_o^*	ω_r^*	W*	Policy	<i>c</i> _i vs. Ø*
	0.020	0.0162	0.0662	0.0297	2000	1525	4.61	0.70	5581490	LIFO	$c_L \leq 1939$
$\alpha = -500$	0.020	0.0153	0.0366	0.0645	2000	1530	1.78	2.19	5583594	FIFO	$c_F = 0$
	0.020	0.0158	0.0654	-	2000	1579	3.16	1.47	5586805	AIFO	$Ø^* = 0.56$
$l_o = l_r = 0$	0.020	0.0183	0.0701	0.0335	2000	1833	0	0	5574310	LIFO	$c_L \leq 4142$
$\alpha = 0$	0.020	0.0163	0.0365	0.0664	2000	1631	0	0	5578555	FIFO	$c_F = 0$
a = 0	0.020	0.0173	0.0683	-	2000	1733	0	0	5576466	AIFO	$0^* = 0.54$
$h_{og} = h_{rg} = 20$	0.020	0.0161	0.0659	0.0295	2000	1611	4.59	1.15	5584203	LIFO	$c_L = 0$
$z_r = z_o = 20$	0.020	0.0161	0.0365	0.0659	2000	1611	1.78	3.96	5584203	FIFO	$c_F = 0$
$\beta_o = \beta_r = 25$	0.020	0.0161	0.0659	-	2000	1611	3.19	2.55	5584203	AIFO	$\emptyset^* = 0.55$
$h_{og} = h_{rg} = 20$	0	0.0354	0.0648	0.0648	0	3550 ª	0	5.4	5585464	LIFO	$c_L = 0$
$z_r = z_o = 20$	0	0.0354	0.0648	0.0648	0	3550 a	0	5.4	5585464	FIFO	$c_F = 0$
$\beta_o = \beta_r = 25$	0	0.0354	0.0648	-	0	3550 a	0	5.4	5585464	AIFO	$\emptyset^* = 0$
$f_o = 0 \Longrightarrow EOQ$											

^a The order quantity as in Example 4.1 (Table 4.2).

Table 8.7. The effect of Wright's learning curve on variable capacity of the OW with $s_o=s_r={f 0}.$

Parameter	j	f_{oj}^*	f_{rj}^*	f_j^*	f_{Rj}^*	q_{oj}^*	q_{rj}^*	ω_{oj}^*	ω_{rj}^*	W_j^*	Policy	c_{ij} vs. \emptyset_j^*
	1	0.020	0.0128	0.0599	0.0235	2000	1285	8.24	0.73	5593672	LIFO	$c_{Lj}=0$
$z_o = 10$	1	0.020	0.0135	0.0365	0.0611	2000	1348	3.62	3.12	5592737	FIFO	$c_{Fj} \leq 1236$
$z_r = 20$	1	0.020	0.0132	0.0605	-	2000	1318	6.01	1.90	5593230	AIFO	$\emptyset_j^* = 0.60$
	2	0.020	0.0119	0.0604	0.0226	2000	1189	8.45	0.66	5384686	LIFO	$c_{Lj}=0$
$z_{o} = 10$	2	0.020	0.0124	0.0379	0.0616	2000	1246	3.83	3.04	5383925	FIFO	$c_{Fj} \leq 1075$
$z_r = 20$	2	0.020	0.0122	0.0610	-	2000	1219	6.22	1.82	5384334	AIFO	$\emptyset_j^* = 0.62$
q_o^*	1	0.0097	0.0228	0.0595	0.0418	971	2288	4.84	2.33	5593342	LIFO	$c_{Lj}=0$
$z_{o} = 10$	1	0.0134	0.0196	0.0245	0.0603	1343	1959	1.62	3.97	5592454	FIFO	$c_{Fj} \leq 741$
$z_r = 20$	1	0.0117	0.0210	0.0599	-	1173	2108	3.44	3.04	5592887	AIFO	$\emptyset_j^* = 0.36$
q_o^*	2	0	0.0319	0.0606	0.0606	0	3196	0	4.83	5384265	LIFO	\Rightarrow EOQ
$z_o = 10$	2	0.0123	0.0197	0.0235	0.0610	1237	1977	1.45	4.11	5383677	FIFO	$c_{Fj} \leq 552$
$z_r = 20$	2	0.0093	0.0226	0.0607	-	936	2262	2.86	3.39	5384011	AIFO	$\emptyset_j^* = 0.29$

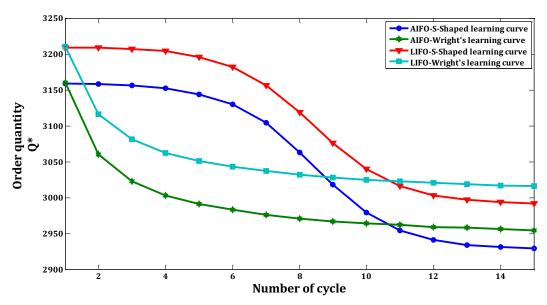


Fig. 8.5. A comparison of the optimal lot sizes of AIFO and LIFO for S-shaped and Power learning curves.

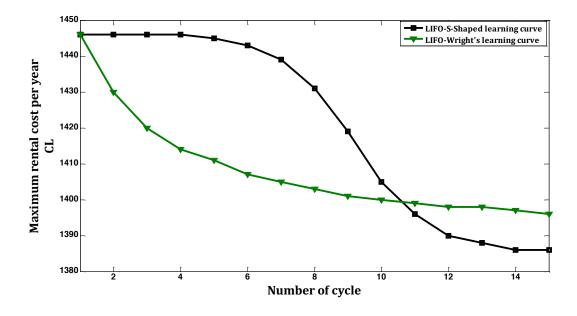


Fig. 8.6. A comparison of the maximum rental cost per year for S-shaped and Power learning curves.

In the next section, we list some key findings that depict the behaviour of the theoretical models in different realistic scenarios and we further relate the results of our work to the general body of knowledge in the discipline.

8.8.5. Findings

- For any $s_o = s_r \ge 0$, the optimal order quantity is identical for each model, which signifies the importance of considering differing transportation costs in the mathematical formulation of two-warehouse inventory models (Table 8.3).
- The dis-location of good and defective items significantly influences the optimal order quantity (Tables 8.3 and 8.4).
- The assumption that the OW is fulfilled with the maximum capacity is indeed not the optimal choice for specific input parameters (Tables 8.3, 8.4, 8.5 and 8.7). Although such finding may appear to be counterintuitive, it is indeed an important finding for practitioners, since the objective is to minimise the total system cost.
- Under FIFO and LIFO policies, it may become optimal that no items are stored in the
 OW, i.e. the problem reduces to the EOQ (Tables 8.3, 8.4 and 8.7), which is consistent
 with the behaviour of outsourcing inventory holding through a vendor managed
 inventory (VMI) or other inventory intermediary arrangement (Table 8.6).
- Relaxing the inherent determinism of the maximum capacity associated with OW, not only produces better results, but may also reduce the value of the upper-bound significantly (Tables 8.4, 8.5 and 8.7).
- For equal holding costs and deterioration rates, the optimal order quantity for the three models is identical, i.e. $c_i = 0$, which implies that LIFO (AIFO) is optimal if and

only if $c_i=0$ (Table 8.6). This finding is fundamental, since it not only shows the validity and robustness of the proposed models, but also underpins and portrays the value added for integrating the upper-bound in the mathematical formulations.

- Table 8.6 reveals that when $f_o=0$, all models are reduced to a single-level warehouse base model presented in Chapter 4 (Alamri et al. (2016). In this case, $c_i=0$, which also shows the validity and robustness of the proposed models.
- It should be noted that the results presented in Table 8.2 reveal that the reduction in the optimal order quantity does not affect the OW. That said, although the OW may benefit from the VOI that reduces the defective items per lot, it still retains the maximum capacity of goods and, consequently, the effect of learning does not really apply here. On the other hand, this is not the case when relaxing the inherent determinism of the maximum capacity associated with OW. In particular, such relaxation implies comprehensive learning that can be achieved simultaneously, i.e. the amounts that are allocated to both the OW and RW are affected (Table 8.7). However, and despite the fact the lot size may reduce for each successive replenishment, the amount that is allocated to the OW either remains static (due to capacity restriction) or experiences further reduction, but the amount that is allocated to the RW may decrease (increase) subject to the input parameter. This is a key finding, which demonstrates the impact of learning on the operational and financial performance of an inventory system with a two-level storage.
- ullet Fig. 8.5 indicates that the optimal order quantity of AIFO and LIFO follows the same fashion as that of p_i .
- The same behaviour observed in Fig. 8.5 holds true for the total minimum costs per year, which can be further justified by the reduction gained in the maximum rental

cost per year (upper-bound) (Fig. 8.6).

- The S-shaped logistic learning curve generates greater quantities in the incipient phase, which is consistent with the behaviour of slow improvement that is observed in practice (Dar-El, 2000).
- Previously published models in this area are shown to be special cases of our models (Table 8.6, Appendix B, Appendix C and Appendix D).

Remark 8.1

The proposed models are not limited to the above contributions; the versatile nature of our models and the fact that they may trigger other applications that can be disseminated from the general formulation is shown in Section 8.9 below.

8.9. Special cases of the general EOQ models

In this section, we offer a number of special cases to illustrate the theoretical application of our mathematical models. We aim to address the quality related issues discussed in Chapters 2 and 5 when formulating LIFO and FIFO EOQ models for items that require 100 per cent screening. In particular, we show that the special cases covered in Chapter 5 can be easily addressed for LIFO and FIFO, although many of these issues have neither been recognised nor analytically examined before. Moreover, the interface between different transportation costs associated with OW and RW and the upper-bound (cost applied if OW (RW) is idle) allows inventory managers to compare between LIFO, FIFO and AIFO dispatching policies to establish

the optimum order quantities that minimise the total system cost.

The remainder of the section is organised as follows: First, we provide a brief summary of general solution procedures for managing perishable products and lifetime constraints, followed by linking our mathematical formulations to the renewal theory, whereby we show that previously published models in this area constitute special cases of our models.

8.9.1. Perishable products and lifetime constraints

The implication of the inspection process into inventory decision-making can be further explored to accommodate an inventory system with a two-level storage. Specifically, a lot of size $Q_{ij}=q_{oij}+q_{rij}$ is delivered such that a quantity of size q_{oij} is allocated to the OW and the remaining amount of size $q_{rij}=Q_{ij}-q_{oij}$ is allocated to the RW. The assumption that each sub-replenishment that enters the RW undertakes a 100 per cent screening would imply that $\underline{q_{rij}}=\left(q_{rmj},q_{rm-1j},\ldots,q_{r0j}\right)$ where q_{rkj} is the number of units with $k(k=0,1,\ldots,m)$ useful periods of shelf lifetime. Here, q_{r0j} denotes newly replenished items that have arrived already perished or items not satisfying certain quality standards (defective items). A similar argument holds true for the quantity q_{oij} that is allocated to the OW. Although no buyer would pay for defective and already perished items, they would surely be interested in seeing a reduction in the presence of such quantities in subsequent replenishments. Our formulation allows for an immediate disposal of the amount $q_{r0j}+q_{o0j}$ in case of any potential safety issues, i.e. $h_{od}=h_{rd}=0$. Now, let ω_{rkj} denote the quantity of the on-hand inventory of shelf lifetime k that perishes by the end of period k in RW. Thus, we have

$$\omega_{rkj} = \begin{cases} q_{rkj} - \left[D_{kj} - \left(\sum_{n=1}^{k-1} q_{rnj} - \sum_{n=1}^{k-1} \omega_{rnj} - \sum_{n=1}^{k} d_{rnj} \right) \right]^+ \\ 0 & \text{otherwise,} \end{cases}$$

where $D_{kj}<\left(\sum_{n=1}^k q_{rnj}-\sum_{n=1}^{k-1}\omega_{rnj}-\sum_{n=1}^k d_{rnj}\right)$ is the actual demand observed up to the periodic review k, and d_{rkj} is the number of items of shelf lifetime k that deteriorate in RW while on storage. Hence, $\sum_{y=k}^m d_{ryj}$ denotes the total sum of deteriorated items in RW in period k, i.e. an item may not retain the same utility throughout its shelf lifetime, and consequently $\sum_{k=1}^m \omega_{rkj}$ refers to the total sum of inventory in RW that perishes in cycle j, excluding any replenished items that have arrived already perished. A similar argument holds true for the quantity ω_{okj} that perishes in OW. It is important to note that if LIFO or FIFO are considered, then $\sum_{k=1}^m \omega_{rkj} + \sum_{k=1}^m \omega_{okj}$ is likely greater than that experienced under the AIFO policy. This can be justified by the marginal difference in cycle length (Sections 8.8.2 and 8.8.4) and the fact that under the LIFO and FIFO policies, only one warehouse is utilised at a time. This is an important issue, especially in the case when a distinct selling price v_k may be linked to its corresponding quantity q_{kj} , i.e. $\underline{V}=(v_m,v_{m-1},\dots,v_0)$ is applied for the set $\underline{Q}_{ij}=(q_{mj},q_{m-1j},\dots,q_{0j})$. Therefore, our formulation is viable if, for instance, an item partially loses its value based on its perceived actuality (obsolescence).

The above discussion further underpins and demonstrates how the terms "deterioration", "perishability" and "obsolescence" may collectively affect inventories in a two-level storage. Note that m_{i2} can still be used to drive the optimal quantity that needs to be added to the on-hand inventory for the next replenishment, i.e. $q_{ikj} = Q_{ij+1} - I_{ogij}(t_{kj}) - I_{rgij}(t_{kj})$, where t_{kj} denotes the time up to the periodic review. This relation holds true for Sections 6.5.1, 7.5.1, 8.8.2 and 8.8.4, i.e. $q_j = Q_j - I_{ogj-1}(T_{j-1}) - I_{rgj-1}(T_{j-1})$ for AIFO and LIFO and $q_j = Q_j - I_{ogj-1}(T_{Rj-1}) - I_{rgj-1}(T_{Rj-1})$ for FIFO. Therefore, similar arguments as those for

Sections 5.2 and 5.3 can be implemented here.

As an example of lifetime constraint, we can assume that T denotes the remaining shelf lifetime of an item and ${}^{\circ}\mathrm{C}_y$ and t_y represent, respectively, the temperature and time elapsed of an item in a supply chain entity y. Then we have $\mathrm{T} = M - \zeta({}^{\circ}\mathrm{C}_a)t_a - \zeta({}^{\circ}\mathrm{C}_b)t_b$, where $\zeta({}^{\circ}\mathrm{C}_y) = (0.1{}^{\circ}\mathrm{C}_y + 1)^2$ and $M = m + t_a + t_b$ (Bremner, 1984; Ronsivalli and Charm, 1975). In this case, $f_j \leq \mathrm{T}$ for AIFO and LIFO and $f_{Rj} \leq \mathrm{T}$ for FIFO, and consequently, the VOI can be quite valuable in reducing the cost per cycle (Chapter 5). Note that $z_y = 1 + \mathrm{T}$ can also fit here.

8.9.2. Stochastic parameters

Let $\underline{X_j}$ refer to a set of random variables that are predetermined according to the VOI gained by the system due to its coordination as an output of the j^{th} inspection process. Suppose that $\underline{X_j} \sim U\left[\underline{Y_j} - \sqrt{3}\underline{Z_j}, \underline{Y_j} + \sqrt{3}\underline{Z_j}\right].$ Note that $E\left(\underline{X_j}\right) = \underline{Y_j}$, i.e. if $D_j \sim U\left[\mu_j - \sqrt{3}\sigma_j, \mu_j + \sqrt{3}\sigma_j\right]$, then $E(D_j) = \mu_j = D(t) = r$. A similar argument holds true for other input parameters.

Note that $\underline{X_j}$ and hence the actual yield may vary from one cycle to another (e.g. the parameters are nonstationary). Thus, we have $E(W_i) = \frac{E(w_i)}{E(f_{ij})} = \frac{E(w'_{q_{rj}})}{E(f'_{j,q_{rj}})}$, for AIFO and LIFO,

and
$$E(W_F) = \frac{E(w_F)}{E(f_{Rj})} = \frac{E\left(w_{q_{rj}}'\right)}{E\left(f_{Rj,q_{rj}}'\right)}$$
, for FIFO, where $\left(1 - E\left[p_j\right]\right)x(t) > D(t)$.

In the example provided by Jaggi et al. (2015), it is assumed $E[p]=0.02, q_o=800, h_{og}=h_{od}=5, h_{rg}=h_{rd}=7, \beta_o=\beta_r=0, c=25, k=100, r=50000, \ l_o=l_r=1, z_o=100, l_o=100, l_o=1000, l_o=10000, l_o=1000, l_o=10000, l_o=100000, l_o=10000, l_o=100000, l_o=10000, l_o=100000, l_o=10000, l_o=10000, l_$

 $3.33, z_r=5, \ \alpha=0 \ ext{and} \ b=175200, \ ext{resulting in} \ Q^{J\ et\ al}=915 \ ext{units}.$ This quantity is greater than our optimal $Q_L^*=909 \ ext{units}$ and $w_L^*=1312381 \ ext{dollars}.$ However, $Q_F^*=943 \ ext{units}$ and $w_F^*=1312315,$ and consequently FIFO performs better than LIFO. Moreover, if q_o is taken as a decision variable, then $q_o^*=0, Q_L^*=920 \ ext{units}$ and $w_L^*=1312126 \ ext{dollars} \Longrightarrow$ the solution of FIFO does not exist. This result is consistent with the results obtained in Tables 8.3, 8.4, 8.5 and 8.7 and seems realistic, given that the objective is to minimise the total system cost. Setting $l_o=l_r=0$, the result is identical with that of Chung et al. (2009) and Jaggi et al. (2015), where $Q_L^*=Q^{C\ et\ al}=Q^{J\ et\ al}=1290 \ ext{units}.$

In the next section, we summarise the implications and managerial insights of our work and relate the results of the study to the general body of knowledge in the discipline. Further, we emphasise the fact that each model may trigger other applications that can be disseminated from the general formulation.

8.10. Summary of implications and managerial insights

- The versatile nature of each model allows the incorporation of the desired functions that are suitable to a system. Consequently, the list of implications and managerial insights outlined in Section 5.9 fits our models as well.
- Each model emerges as a viable solution that manages and controls the flow of perishable and non-perishable products so as to reduce cost and/or waste for the benefit of economy, environment and society.
- General solution procedures for LIFO, FIFO and AIFO to determine the optimal dispatching policy for continuous intra-cycle periodic review applications for two-level

- storage EOQ models are presented. The proposed solution procedures consider different inventory fluctuations during the planning horizon.
- A detailed method to illustrate how deterioration, perishability and obsolescence may collectively affect inventories in a two-level storage is explored.
- The accuracy of RFID temperature tags that capture the TTH, and the use of that TTH
 data are adopted to model the shelf lifetime of an item under LIFO, FIFO and AIFO
 dispatching policies.
- The mathematical formulations are linked to the renewal theory to show that previously published models in this area constitute special cases of our models.
- The importance of considering differing transportation costs in the mathematical formulation of two-warehouse inventory models is illustrated.
- The assumption that the OW is fulfilled with the maximum capacity is indeed not the
 optimal choice for specific input parameters, which constitutes an important decision
 for practitioners, since the objective is to minimise the total system cost.
- Relaxing the inherent determinism of the maximum capacity associated with OW, not only produces better results, but may also suggest an alternative policy and reduce the value of the upper-bound significantly. Moreover, it implies comprehensive learning that can be achieved simultaneously, i.e. the amounts that are allocated to both the OW and RW are affected, which demonstrates the impact of learning on the operational and financial performance of an inventory system with a two-level storage.
- Integrating the upper-bound in the mathematical formulations of LIFO and FIFO models, i.e. the introduction of a KPI to trade-off the costs associated with AIFO, LIFO and FIFO policies constitutes a key business objective in supply chain management.

- Under FIFO and LIFO policies, it may become optimal that no items are stored in the OW, which suggests an outsourcing inventory holding through a VMI or other inventory intermediary arrangement.
- In the case of managing perishable products, LIFO and FIFO may not be the right dispatching policies, given that the order quantity needs to be consumed based on a FEFO policy. This is so, since under the LIFO or FIFO dispatching polices, the total sum of inventory that perishes in each cycle is likely greater than that experienced under the AIFO policy.
- The dimension of risk influencing the management of perishable products may increase if, for instance, a distinct selling price is linked to its corresponding quantity with a distinct useful period of shelf lifetime.
- Under an AIFO dispatching policy, a discounted holding cost can be gained if a continuous and long-term rental contract is beneficial and hence further reduction in the total minimum cost per year can be achieved.

8.11. Conclusion and further research

In this chapter, we have been concerned with the implications of dispatching policies associated with a two-level storage, where each lot is subjected to a 100 per cent screening. Three general EOQ models for items with imperfect quality were presented and compared. The first model underlies LIFO, the second model underlies FIFO and the third model relates to simultaneous consumption fractions associated with OW and RW and is entitled Allocation-In-Fraction-Out (AIFO). Under an AIFO dispatching policy, the cycle length is the same for both OW and RW, and consequently the upper-bound (cost applied if OW (RW) is idle) that renders

AIFO the optimal dispatching policy has also been provided. It has been shown that the solution to AIFO inventory model, if it exists, is unique and global optimal. Items not conforming to certain quality standards are isolated in separate facilities with different holding costs of the good and defective items being considered.

The analytical results illustrate the impact of considering different transportation costs associated with OW and RW, as well as the incorporation of varying demand, deterioration, defectiveness and screening rates on the optimal order quantity. Relaxing the inherent determinism related to the maximum capacity of OW not only produces better results, but may also reduce the value of the upper-bound significantly. If the system is subject to learning, then this relaxation implies further reduction in the quantity allocated to the OW for each successive sub-replenishment that is delivered to OW, i.e. the system experiences comprehensive and simultaneous learning.

This study is viable for fixed and random lifetimes of perishable products, where VOI is adopted to model the shelf lifetime of an item. The versatile nature of our models and the fact that they may reflect a diverse range of strategies has been emphasised, whereby the validity of the general models is ascertained, i.e. the solution is the same as in published sources or in some cases produces better results. To the best of our knowledge, this appears to be the first time that such an Allocation-In-Fraction-Out (AIFO) policy is presented, which necessitates a general formulation of LIFO and FIFO EOQ models for investigation and numerical comparison purposes.

Further research can be addressed for finite or infinite planning horizons that may include extensions, such as allowing for shortages, considering that the screening rate follows learning and forgetting curves, and the risk of failure during screening (Type I and Type II

errors). In addition, it appears plausible to assess the formulation of EOQ models for a two-level storage to consider multiple items or to study the effect of different supplier trade credit practices.

Part D: Conclusion

This part consists of the last chapter of our work. In this chapter (Chapter 9), we highlight the overall contributions, findings and implications, and managerial insights presented in this PhD thesis. This chapter closes with a discussion of further research.

9. Summary of contributions and further research

9.1.Introduction

In this chapter, we highlight the overall contributions and findings presented in this PhD thesis, summarise the implications and managerial insights of our work, relate the results of the study to the general body of knowledge in the discipline and provide a discussion of natural avenues for further research.

Inventory management is a field which has been relatively mature for several decades. Moreover, there is no single existing theory that can adequately capture all aspects of the relevant processes and the inventory problems associated with them. Therefore, making a contribution that scholars would deem "significant" is not an easy task.

In this PhD thesis, we have attempted to advance the current state of knowledge in the field of inventory mathematical modelling and management by means of providing theoretically valid and empirically viable generalised inventory frameworks to assist inventory managers towards the determination of optimum order/production quantities that minimise the total system cost. From a theoretical perspective, the generic nature of our models enables the decision maker to incorporate a diverse range of strategies that may reflect many real-world concerns. From a practitioner's perspective, this PhD thesis generated interesting managerial insights. Despite the obvious implications that are triggered by the general formulations and the fact that many practical examples published in the literature for generalised models in this area constitute special cases of our proposed models, the following objectives were also considered, for the first time, in this work:

- 1) To explore the implications of the inspection process on inventory decision-making and link such process with the management of perishable inventories;
- 2) To derive a general, step-by-step solution procedure for continuous intra-cycle periodic review applications;
- 3) To demonstrate how the terms "deterioration", "perishability" and "obsolescence" may collectively apply to an item;
- 4) To develop a new dispatching policy that is associated with simultaneous consumption fractions from an owned warehouse (OW) and a rented warehouse (RW). The policy developed is entitled "Allocation-In-Fraction-Out (AIFO)";
- 5) To relax the inherent determinism related to the maximum fulfilment of the capacity of OW to maximising net revenue; and
- 6) To assess the impact of learning on the operational and financial performance of an inventory system with a single-level storage and a two-level storage.

9.2. Models overview

In this section, we emphasise the practical application of our work, where each model is based on a single item held in stock, whether this item is perishable or non-perishable. This means that each model is applicable in contexts such as grocery retailing, armed forces, hospitals, wholesaling, manufacturing, etc.

Below, 3 single item examples (grocery retailer, hospital and manufacturer) are given

illustrating the applicability and importance of the models.

Case 1: Suppose that a grocery retailer is purchasing a seasonal product say, Indian mango. Assume that the demand rate for such product is considered, respectively, to be linearly increasing with time in the first three replenishments (what would more generally correspond to the growth stage of the life cycle of the product, i.e. customers are likely to buy mango), constant for the following two replenishments (in the mature stage of the life cycle of the product) and linearly decreasing with time in the last two replenishments (in the declining stage of the life cycle of the product, i.e. few customers are likely to buy mango).

The lot size shipment that enters the sorting facility consists of cartons of mangos worth 8 dollars each and contains six pieces. Since customers normally check many cartons before they pick the right one, the lot size shipment undertakes 100 per cent inspection. In this case, inspection reduces the potential rejection that could happen because of one defective piece of mango that could be easily replaced with a good one in the same carton. In addition, inspection can be used to classify those cartons based on some sort of presentation, price or freshness in order to reduce waste and/or maximise revenue. Moreover, any defective piece of mango may affect other pieces and thus increases potential waste. All defective complete cartons and/or pieces of mangos can be sold at a salvage price or even incur a disposal cost.

As time goes on, the rate of deterioration increases due to the change in the physical status of mangos. In addition, customers randomly, check many cartons before they pick the right one, i.e. they may touch some mangos in the cartons, which in turn increases the rate of deterioration as many more costumers have checked the same cartons.

In this case, i.e. Case 1, an inventory system with a two-level storage (OW and RW) applies, if for example, the grocery retailer may have been enticed with price discounts to purchase a number of cartons of mangos that exceed the limited capacity of OW. In this case, the grocery retailer can trade-off between AIFO, LIFO and FIFO to find the best dispatching policy.

Case 2: Suppose that a hospital receives a seasonal flu medicine, say drug X (Vaccine), which is liquid in nature. Assume that the demand rate for such medicine is considered to be linearly increasing with time. The lot size shipment that enters the warehouse consists of cartons of drug X, each of which may have a different expiration date as a result of a dispatching policy embraced by the supplier. In this case, and to reduce any potential risk of perishability, the lot size shipment undertakes 100 per cent inspection so as to classify those cartons into two categories, say long-term and short-term dispatching policies (FEFO). Any cartons with remaining shelf life time less than, say six months are considered to be defective as per the regulation of say, Ministry of Health that the supplier agrees upon. All defective complete cartons are sent back to the supplier and all damaged bottles of drug X are also considered defective and may incur a disposal cost.

Demand, say for three different pharmacies in the hospital is satisfied from inventory of short-term category first, then demand is satisfied from inventory of long-term category. As time goes on, some bottles of drug X may get damaged while they are on storage and some others may experience misplacement by pharmacists, nurses or doctors such that the minimum remaining shelf life time of six months is no longer valid when such bottles are found. All bottles of drug X that do not meet the minimum remaining shelf life time of six months are considered deteriorated and must sent back to the supplier. All damaged bottles of drug X

are also considered deteriorated and may incur a disposal cost.

In this case, i.e. Case 2, classifying cartons into two categories, i.e. long-term and short-term and, the fact that the demand is satisfied from inventory of short-term category first, then demand is satisfied from inventory of long-term category involves a straightforward application of either LIFO or FIFO dispatching policies. This implies that inventory of short-term category (theoretically stored in (OW or RW)) is consumed first, before considering inventory of long-term category (theoretically stored in (RW or OW)), i.e. an inventory system with a two-level storage applies here. Another application of an inventory system with a two-level storage occurs, if for instance, the lot size shipment may be allocated to two warehouses due to capacity restriction of one warehouse.

Case 3: Suppose that a manufacturer purchases returned products, say product Y (lithium battery) as well as three different new components that are required in a production setting. Inventory of returned products is stored in a separate shop (warehouse). Assume that the demand rate for such product follows a production rate that is considered to be linearly increasing with time.

The lot size shipment of returned products that enters the warehouse consists of batteries each of which has three different usable functional components. In this case, the lot size shipment of returned products undertakes 100 per cent inspection, where each battery is disassembled to isolate usable functional components to be transferred in batches to the production shop. All defective batteries and/or unusual functional components can be sold as scrap or even incur a disposal cost.

Each transferred batch contains distinct usable functional components, which represent a fraction of the demand and the rest is satisfied from inventory of new components. As time goes on, some batteries may get totally damaged while they are on storage and some damage may also affect some functional components. All totally damaged batteries and unusual functional components can be sold as scrap or even incur a disposal cost.

In this case, i.e. Case 3, an inventory system with a two-level storage (OW and RW) applies, if for example, it is cost effective for the manufacturer to purchase a number of batteries that exceed the limited capacity of OW. In this case, the manufacturer can trade-off between AIFO, LIFO and FIFO to find the best dispatching policy.

9.3. Lot size inventory model with one level of storage

9.3.1. Research contribution

In this section, we summarise and emphasise the financial implications and managerial aspects of our work in the case of a single storage. Opportunities and challenges in this case include poor supplier service levels (imperfect items received from suppliers), potential dislocation of good and defective items (different warehouses for the good and defective items), goods' deterioration during storage and transfer of knowledge from one inventory cycle to another.

The combination of such challenges/opportunities is important in many industries and may significantly influence the optimal order quantity. This is an important issue especially in the case of managing perishable products where inspection would imply that products may be classified according to quality, size, appearance, freshness, etc., and where a distinct selling

price may be linked to its corresponding quantity. Moreover, inspection may eliminate the return service cost associated with product recalls. In real-life settings, the impact of allowing through defective items could be severe (Klassen and Vereecke 2012). The service cost may include goodwill cost, transportation cost, and re-processing cost, and that may affect all supply chain members. Inspection may also reduce holding costs due to the deployment of less preserving environmental conditions, i.e. the defective items are not usually stored in the same warehouse where the good items are stored (e.g. Wahab and Jaber, 2010). Inspection may also be presumed essential for updating the Information System records with good items that are actually available in stock, so as to avoid shortages. Moreover, a 100 per cent inspection will render a potential random lifetime of a product deterministic, i.e. it intersects the areas of fixed and random lifetimes of an item. Finally, inspection not only isolates defective and/or already perished items, but also leads to the consumption of the order quantity based on a FEFO policy. For example, isolation, i.e. dis-location of good and defective items, allows for an immediate disposal of defective and/or already perished items in case of any potential safety issues.

The generality of our model stems from the fact that the demand, screening, and product deterioration rates are arbitrary functions of time. The proposed model unifies and extends the academic literature related to imperfect quality items, which is comparatively diverse in nature. Mathematical proof was presented, which shows that the solution to the underlying inventory model, if it exists, is unique and global optimal. It has been shown that practical examples published in the literature for generalised models constitute special cases of our model. This implies that the solution is the same as in published sources or in some cases produces better results, which indicates that the validity of the general model is ascertained.

The versatile nature of our model, and the fact that it may accommodate many real-world concerns, has been emphasised in that the results obtained are compatible with the behaviour observed in many real-life settings. The obtained numerical results support application of the model and solution procedure in different realistic situations. For example, the presence of product deterioration and varying demand and screening rates significantly impact on the optimal order quantity. However, a reduction in the optimal order size does not necessarily lead to a lower total minimum cost per unit time. Moreover, we extended upon the financial implications and managerial insights of our work by offering a number of special cases to illustrate the theoretical application of our general model.

The mathematical formulation of our model intersects the areas of fixed and random lifetimes of perishable products, whereby the value of the temperature history and flow time through the supply chain is also used to determine an appropriate policy. Furthermore, it provides a general procedure for continuous intra-cycle periodic reviews so as to adjust and control the flow of raw materials, component parts and finished goods to maintain sustainable competitive advantage. Coordination mechanisms and managerial decision strategies that govern both the supplier and the retailer were also addressed to improve inventory management at both echelons.

We tested and observed the behaviour of varying demand, screening, defectiveness and deterioration rates, VOI and perishable and non-perishable (infinite shelf lifetime) items that are subject to deterioration while in storage. The resulting insights offered to inventory managers are considered to be of significant value, since many of these issues have not been previously investigated. Therefore, the model emerges as a viable solution that manages and controls the flow of perishable and non-perishable products.

Below, we list some key findings that depict and emphasise the behaviour of the theoretical model in different settings and relate the results of the study to the general body of knowledge in the discipline.

9.3.2. Key Findings

- The total minimum cost per unit time and the total minimum cost per cycle decrease
 as learning increases, which supports the findings presented by Wahab and Jaber
 (2010).
- As learning increases, all optimal quantities decrease, except for the amount of deteriorated items that incur a minor increase that can be justified by the slight increase in the cycle length.
- The presence of defects and varying demand and deterioration rates significantly impact on the optimal order quantity.
- The lot size with the same assumed holding costs for the good and defective items is less than the one with differing holding costs. However, the difference between the two quantities vanishes as p_j takes on relatively small values. Such finding is consistence with that presented by Wahab and Jaber (2010).
- The results show a slight decrease in the total minimum cost per unit time due to a slight decrease in p_j . This is true in the incipient phase when an S-shaped logistic learning curve is assumed, which is consistent with the behaviour of slow improvement observed in this short phase, making the S-shaped learning curve an appropriate model to use (Dar-El, 2000). On the other hand, this is not the case when Wright's learning curve is considered, which then leads to smaller quantities in the incipient phase and hence the total minimum cost per unit time behaves similarly.

- The reduction in the total minimum cost per unit time and the optimal order quantities follow the same fashion as that of p_i .
- The effect of α , the rate of change in the demand significantly influences the optimal order quantity and the total minimum cost per unit time.
- The presence of deterioration has a significant impact on the optimal order quantity and the total minimum cost per unit time. Such finding is consistence with that presented by Moussawi-Haidar et al. (2014).

9.3.3. Implications and managerial insights

In this section, we summarise the implications and managerial insights of our work and relate the results of the study to the general body of knowledge in the discipline.

- The generic nature of our model allows incorporation of the desired functions that are suitable to a system.
- A general step-by-step solution procedure to determine the optimal policy for continuous intra-cycle periodic review applications is presented, which is valid for the generalised models and can be further extended to be implemented in inventory mathematical modelling. Moreover, the structure of the model allows for both continuous and discrete periodic review.
- Clearer definitions that are associated with the terms deterioration, perishability and obsolescence are suggested in order to refine and distinguish the role of each term for the model. In addition, a detailed method was also provided, which underpins and portrays how these terms may collectively apply to an item.

- The proposed model intersects the areas of fixed and random lifetimes of perishable products. Moreover, it is viable for the case in which items are classified based on their quality, size, appearance, freshness, etc., whereby the dis-location of good and defective items allows for an immediate disposal of defective and/or already perished items in case of any potential safety issues.
- The proposed model emerges as a viable solution that manages and controls the flow
 of perishable and non-perishable products in order to reduce cost and/or waste for
 the benefit of economy, environment and society.
- The VOI can be perceived at external and/or internal domains of coordination. At the domain of external coordination, the integration of the VOI captures a safe remaining shelf lifetime. For example, the accuracy of RFID temperature tags that capture the TTH data are adopted to model the shelf lifetime of an item. At the domain of external and internal coordination, the presence of the VOI acknowledges the potential impact of transporting and handling a product through the supply chain. In this case, any information gained from previous replenishments can be incorporated to reduce the presence of both defective and already perished items in subsequent replenishments.
- The versatile nature of our model accommodates renewal theory, stochastic process and inspecting a random sample size drawn from the batch, where input parameters are randomly distributed. It also accounts for the case when subsequent replenishment is cycle dependent. This line of research was also enhanced, whereby coordination mechanisms and managerial decision strategies that govern both the supplier and the retailer are addressed to improve inventory management at both echelons.

9.4. Lot size inventory model with two levels of storage

9.4.1. Research contribution

In this section, we highlight research contribution and summarise the theoretical application, financial implications of dispatching policies associated with a two-level storage and managerial aspects related to our findings.

The contribution of this work goes beyond addressing the practical concerns with regards to product quality related issues that are linked with the opportunities raised in Section 9.2.1 when formulating a two-warehouse inventory model for items that require 100 per cent screening. In particular, a new policy entitled "Allocation-In-Fraction-Out (AIFO)" was developed. Under an AIFO dispatching policy the goods at RW and OW experience simultaneous consumption fractions, which implies that the inventories at both warehouses are depleted by the end of the same cycle. On the other hand, the LIFO and FIFO policies assume no cost effect while the initially used warehouse is idle, which is unrealistic and a rare scenario to encounter in practice. Subsequently, this necessitated introducing costs associated with the OW or RW being idle when formulating a two-warehouse inventory model. Therefore, three general EOQ models for items with imperfect quality were presented and compared. The first model underlies the LIFO policy, the second model underlies the FIFO policy and the third model relates to the AIFO policy.

It becomes apparent that the tradeoff between the three policies constitutes a key business objective in supply chain management. Under both the LIFO and FIFO dispatching policy, the cost associated with the OW or RW being idle is treated as an input parameter as well as a decision variable. If the cost is a decision variable, then it constitutes a KPI, i.e. an upper-

bound (cost associated with OW (RW) being idle) that renders AIFO the optimal dispatching policy.

To the best of our knowledge, the maximum capacity of the OW is invariably treated in the academic literature as an input parameter. Relaxing the inherent determinism related to the maximum fulfilment of the capacity of OW may lead to maximizing net revenue. In addition, if the system is subject to learning, then the lot size may reduce for each successive replenishment. However, such reduction affects the amount allocated to the RW only, and the amount allocated to the OW remains at the maximum capacity. Relaxing the inherent determinism of this assumption implies comprehensive learning that can be achieved simultaneously, i.e. the amounts that are allocated to both the OW and RW are affected.

We have shown that the solution to each underlying inventory model, if it exists, is unique and global optimal. Practical examples that are published in the literature for generalised models in this area are shown to be special cases of our FIFO, LIFO and AIFO models. We observed and tested the behaviour of the theoretical models in different settings (e.g. different transportation costs associated with OW and RW, functions for varying demand, screening, defective and deterioration rates, VOI, perishable items that are subject to deterioration while in storage and by means of relaxing the inherent determinism related to the maximum fulfilment of the capacity of OW). Items not conforming to certain quality standards are isolated in separate facilities with different holding costs of the good and defective items being considered.

To avoid repetition, it is important to note here that the contributions presented in Sections 9.2.1 and 9.2.2 hold true for the three models proposed for LIFO, FIFO and AIFO. Moreover, the analytical results illustrate the impact of considering different transportation costs

associated with OW and RW on the optimal order quantity. Relaxing the inherent determinism related to the maximum capacity of OW not only produces better results but may also significantly reduce the value of the upper-bound. If the system is subject to learning, then this relaxation implies further reduction in the quantity allocated to the OW for each successive sub-replenishment that is delivered to OW, i.e. the system experiences comprehensive and simultaneous learning. The mathematical formulations are linked to the renewal theory, which have led to further interesting insights.

The generic nature of our models and the fact that they may reflect a diverse range of strategies has been emphasised, whereby the validity of the general models is ascertained, i.e. the solution is the same as in published sources or, in some cases, produces better results.

In the next section, we highlight the key findings of our work and emphasise the behaviour of the theoretical models in different settings in order to relate the findings of the study to the general body of knowledge in the discipline.

9.4.2. Key Findings

- For any $s_o=s_r\geq 0$, the optimal order quantity is identical for each model, which signifies the importance of considering differing transportation costs in the mathematical formulation of two-warehouse inventory models.
- The dis-location of good and defective items significantly influences the optimal order quantity.
- The assumption that the OW is fulfilled with the maximum capacity may not be the optimal choice for specific input parameters.

- When a LIFO (FIFO) policy is considered, the idle time has been found to be significant
 for a wide range of the control parameter values and free of charge, which is
 unrealistic and rare to encounter in practice. An AIFO dispatching policy not only
 overcomes this issue, but may also lead to a discounted holding cost.
- It has been shown that the upper-bound is typically very small with respect to the minimum average holding cost per year incurred to store items in either warehouse.
- Under FIFO and LIFO policies, it may become optimal that no items are stored in the OW, i.e. the problem reduces to the EOQ, which is consistent with the behaviour of outsourcing inventory holding through a VMI or other inventory intermediary arrangement.
- For equal holding costs and deterioration rates, the optimal order quantity for the three models is identical, i.e. $c_i=0$, which implies that LIFO (AIFO) is optimal if and only if $c_i=0$. This finding is fundamental, since it not only shows the validity and robustness of the proposed models, but also underpins and portrays the value added for integrating the upper-bound in the mathematical formulations.
- When $f_o=0$, all models are reduced to a single-level warehouse base model presented in Chapter 4. In this case, $c_i=0$, which also shows the validity and robustness of the proposed models.
- Relaxing the inherent determinism of the maximum capacity associated with OW implies comprehensive learning that can be achieved simultaneously.
- The optimal order quantity and the total minimum costs per year follow the same fashion as that of p_i .
- Previously published models in this area are shown to be special cases of our models (Table 6 and Appendices B, C and D).

9.4.3. Implications and managerial insights

In this section, we summarise the implications and managerial insights of our work and relate the results of the study to the general body of knowledge in the discipline.

- The general solution procedure for continuous intra-cycle periodic review applications introduced for an inventory system with one level of storage is valid for LIFO, FIFO and AIFO to determine the optimal dispatching policy for two-level storage EOQ models. Likewise, the implications and managerial insights presented in Section 9.3.2 hold true for the three models proposed for LIFO, FIFO and AIFO. Moreover, the impact of deterioration, perishability and obsolescence was investigated to illustrate how these terms may collectively affect inventories in a two-level storage.
- The importance of considering differing transportation costs in the mathematical formulation of two-warehouse inventory models was emphasised, which led to further interesting insights.
- For specific input parameters, it may not be an optimal choice if, for instance, the OW
 is fulfilled with the maximum capacity, which supports the objective that aims to
 minimise the total system cost.
- Relaxing the inherent determinism of the maximum capacity associated with OW not only produces better results but may also suggest an alternative policy and significantly reduce the value of the upper-bound. Moreover, it implies comprehensive learning that can be achieved simultaneously, i.e. the amounts that are allocated to both the OW and RW are affected, which demonstrates the impact of learning on the operational and financial performance of an inventory system with a two-level storage.

- Integrating the upper-bound in the mathematical formulations of LIFO and FIFO models shows that the trade-off between the three policies constitutes a key business objective in supply chain management.
- In the case of managing perishable products, LIFO and FIFO may not be the right dispatching policies, given that the order quantity needs to be consumed based on a FEFO policy. This is so, since under the LIFO or FIFO dispatching polices, the total sum of inventory that perishes in each cycle is likely to be greater than that experienced under the AIFO policy.
- The dimension of risk influences the management of perishable products may increase, if for instance, a distinct selling price is linked to its corresponding quantity with distinct useful periods of shelf lifetime.
- Under an AIFO policy, a discounted holding cost can be gained if a continuous and long-term rental contract is used and hence further reduction in the total minimum cost can be achieved.

9.5. Further research

Based on the findings of the proposed EOQ models, several interesting lines of further inquiry can be addressed for finite or infinite planning horizons; for example:

- To consider the screening rate follows learning and forgetting curves with allowed shortages.
- To allow for the risk of failure during screening (Type I and Type II errors).
- To consider different supplier trade credit practices, such as a permissible delay in payment.

 To formulate an EPQ model in which product quality levels depend on an instantaneous cost of investing in product innovation.

All the subsequent steps of research suggested above can be addressed for an inventory system with one level of storage, as well as for two levels of storage, associated with AIFO, LIFO or FIFO dispatching policy.

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References - 210 -

Appendix A. EOQ model with one level of storage

The goal is to prove the existence, uniqueness and global optimality of the solution to the general inventory model with one level of storage.

Lemma 1.

$$f'_{2j,Q_i} \geq 0.$$

Proof.

From (4.12),
$$T_{1j} > 0 \Longrightarrow \left(e^{g(0)} - p_j e^{g(f_{1j})}\right) x(f_{1j}) \ge D(f_{1j}) e^{g(f_{1j})}$$
 (A.1)

First, we note from (A.1) that $e^{g(0)} \ge p_j e^{g(f_{1j})} \Longrightarrow f'_{2j,Q_j} \ge 0$.

Also, (A.1)

$$\Leftrightarrow 0 \le p_j \le e^{-g(f_{1j})} \left(e^{g(0)} - \frac{D(f_{1j})e^{g(f_{1j})}}{x(f_{1j})} \right). \tag{A.2}$$

Moreover, if $\delta(\cdot)=0$, i.e. for the without deterioration case, (A.2) implies Assumption 5, Chapter 4. This completes the proof of the Lemma. \Box

Theorem 1. Any existing solution of (m_2) is a minimising solution to (m_2) if

$$L'(Q_j) > 0, (A.3)$$

i.e. $L\!\left(Q_{j}\right)$ is an increasing function of Q_{j} , where

$$L(Q_j) = w'_{Q_j} f_{2j} - f'_{2j,Q_j} w. (A.4)$$

Proof.

From (4.17), (4.18), and (A.4) we have

$$\frac{d^2W}{dQ_i^2} = \frac{L'(Q_j)f_{2j}^2 - 2f_{2j}f_{2j,Q_j}'L(Q_j)}{f_{2j}^4} > 0.$$
(A.5)

This is so, since the value of Eq. (A.5) at this solution (where $L(Q_j)=0$), is equal to $\frac{L'(Q_j)}{f_{2j}^2}>0$. This completes the proof of the Theorem. \Box

Theorem 2. If condition (A.3) holds, then any existing solution of (m_2) is the unique and global optimal solution to (m_2) .

Proof.

Recall that $Q_j=0\Leftrightarrow f_{1j}=f_{2j}=0$, then $L(0)=-f'_{2j,Q_j}w<0$. Thus, Eq. (4.21) has solution(s) if $L(Q_j)\geq 0$ for some $Q_j>0$. Now, let $\xi(Q_j)=L(Q_j)-L(0)>0$, then $\xi'(Q_j)=L'(Q_j)>0$ for any $Q_j>0$. Since L(0) is constant, f_{2j} is an increasing function of Q_j , and $f'_{2j,Q_j}\geq 0$ (recall Lemma 1) we can find $Q_j>0$ so that $\xi(Q_j)$ dominates L(0) in which case $L(Q_j)=\xi(Q_j)+L(0)$ has a nonnegative value. Hence, the solution of Eq. (4.21) ($\Leftrightarrow L(Q_j)=0$) does exist and it is the unique global optimal solution. This completes the proof of the Theorem. \square

Appendix A

Next, we shall give the conditions under which the global optimal solution can be attained.

From Eqs. (4.19), (4.20), and (A.4) we obtain

$$f_{2j,Q_j}^{"} = \frac{-p_j \delta(f_{1j}) e^{g(f_{1j})} - f_{2j,Q_j}^{'} e^{g(f_{2j})} \left(D'(f_{2j}) + \delta(f_{2j}) D(f_{2j}) \right)}{D(f_{2j}) e^{g(f_{2j})}}, \tag{A.6}$$

$$\begin{split} w_{Q_j}^{\prime\prime} &= h_g \left[e^{g(0) - g(f_{2j})} + \left(1 - e^{g(f_{1j}) - g(f_{2j})} \right) p_j + \delta(f_{1j}) \left(G(f_{1j}) - G(f_{2j}) \right) p_j e^{g(f_{1j})} \right] + \\ h_d \left[\frac{-x'(f_{1j})}{\left(x(f_{1j}) \right)^2} \int_0^{f_{1j}} p_j x(u) du + \frac{p_j}{x(f_{1j})} \right], \end{split}$$

$$L'(Q_j) = w_{Q_j}'' f_{2j} - f_{2j,Q_j}'' w, (A.7)$$

respectively, where w_{Q_j}'' and f_{2j,Q_j}'' are the derivatives of w_{Q_j}' and f_{2j,Q_j}' (w.r.t) Q_j . Here we need only to add the following restrictions for the case $D'(\cdot) < 0$ and $x'(\cdot) < 0$.

$$\delta(\cdot)x(\cdot) \ge |x'(\cdot)|,\tag{A.8}$$

$$\delta(\cdot)D(\cdot) \ge |D'(\cdot)|. \tag{A.9}$$

From Lemma 1 and (A.9), $f_{2j,Q_j}^{\prime\prime} \leq 0$.

Appendix A - 213 -

Now, assume that $w_{Q_j}''>0$, then we are sure that condition (A.3) (given by Eq. (A.7)) holds, which guarantees the existence and uniqueness of solution to Eq. (4.21). Conversely, let $w_{Q_j}''\leq 0$, and recall that L(0)<0, then we have two possibilities.

$$1. w_{Q_j}'' f_{2j} - f_{2j,Q_j}'' w \le 0.$$

Then Eq. (A.7) is a non-increasing function of Q_j . Therefore, $L(Q_j) < 0, \forall Q_j \ge 0$. Hence Eq. (21) is infeasible in which case the solution does not exist.

$$2. w_{Q_j}'' f_{2j} - f_{2j,Q_j}'' w > 0.$$

Then Eq. (A.7) is an increasing function of Q_j . Therefore, condition (A.3) holds, i.e. the solution to problem (m_2) does exist and it is the unique and global optimal solution.

Appendix A

Appendix B. EOQ model with two levels of storage (LIFO)

The goal is to prove the existence, uniqueness and global optimality of the solution to a general inventory model for the LIFO dispatching policy. It should be noted that the general model presented in Chapter 8 is considered here, i.e. Eq. (8.27).

Lemma.

$$0 \leq f_{j,q_{rj}}' \leq f_{Rj,q_{rj}}'.$$

Proof.

From (6.23),
$$T_{rj} > 0 \Longrightarrow (e^{g_r(0)} - p_j e^{g_r(f_{rj})}) x(f_{rj}) \ge D(f_{rj}) e^{g_r(f_{rj})}.$$
 (B.31)

From (6.24),
$$T_{oj} > 0 \Rightarrow (e^{g_o(0)} - p_j e^{g_o(f_{oj})}) x(f_{oj}) \ge D(f_{oj}) e^{g_o(f_{oj})}$$
. (B.32)

First, we note from (B.31) that $e^{g_r(0)} \ge p_j e^{g_r(f_{rj})} \Longrightarrow f'_{Rj,q_{rj}} \ge 0 \Longrightarrow f'_{j,q_{rj}} \le f'_{Rj,q_{rj}}$.

Also, (B.31) and (B.32) \Leftrightarrow

$$0 \le p_j \le \min\left(e^{-g_r(f_{rj})}\left(e^{g_r(0)} - \frac{D(f_{rj})e^{g_r(f_{rj})}}{x(f_{rj})}\right), e^{-g_o(f_{oj})}\left(e^{g_o(0)} - \frac{D(f_{oj})e^{g_o(f_{oj})}}{x(f_{oj})}\right)\right).$$
(B.33)

Moreover, if $\delta_y(\cdot)=0$, i.e. for the case without deterioration, (B.33) implies Assumption 6, Chapter 6. This completes the proof of the Lemma. \Box

The existence, uniqueness and global optimality of the solution can be obtained in a similar fashion as that for the general inventory system with one level of storage presented in Appendix A.

Appendix B - 215 -

Next, we shall give the conditions under which the global optimal solution can be attained.

From Eqs. (6.30),(6.31), (8.27) we obtain

$$f_{j,q_{rj}}^{"} = \frac{-p_{j}\delta_{r}(f_{rj})e^{g_{o}(f_{Rj})-g_{r}(f_{Rj})+g_{r}(f_{rj})} + f_{Rj,q_{rj}}^{"}D(f_{Rj})(\delta_{o}(f_{Rj})-\delta_{r}(f_{Rj}))e^{g_{o}(f_{Rj})} - f_{j,q_{rj}}^{"}(D'(f_{j})+\delta_{o}(f_{j})D(f_{j}))e^{g_{o}(f_{j})}}{D(f_{j})e^{g_{o}(f_{j})}}$$
(B.34)

$$w_{q_{rj}}^{"} = +h_{rg} \left[e^{g_{r}(0)-g_{r}(f_{Rj})} + \left(1 - e^{g_{r}(f_{rj})-g_{r}(f_{Rj})}\right) p_{j} + \delta_{r}(f_{rj}) \left(G_{r}(f_{rj}) - G_{r}(f_{Rj})\right) p_{j} e^{g_{r}(f_{rj})} \right] + \frac{h_{rd}p_{j}}{x(f_{rj})} \left[1 - \frac{x'(f_{rj})}{x(f_{rj})} \int_{0}^{f_{rj}} x(u) du \right] + c_{Lj} \left(f''_{j,q_{rj}} - f''_{Rj,q_{rj}}\right) + h_{og} \left[\left(1 - e^{g_{o}(f_{j}) - g_{o}(f_{Rj})}\right) f'_{j,q_{rj}} D(f_{j}) + \left(G_{o}(f_{j}) - G_{o}(f_{Rj})\right) \left(f''_{j,q_{rj}} D(f_{j}) e^{g_{o}(f_{j})} + f'_{j,q_{rj}} D(f_{j}) \delta_{o}(f_{j}) e^{g_{o}(f_{j})} \right) \right],$$

$$(B.35)$$

$$\psi'_{L}(q_{rj}) = w''_{q_{rj}} f_{j} - f''_{j,q_{rj}} w_{L} > 0,$$
(B.36)

respectively.

As can be seen from (6.24), (6.30), (6.31), (8.27), (B.33)-(B.35), $f_{oj}=0 \Rightarrow f_{Rj}=f_{j}$. In this case, problem (m_{L2}) reduces to that of (m_{2}) , i.e. the general inventory model with one level of storage. This shows the validity and robustness of the proposed formulation.

Appendix B - 216 -

Appendix C. EOQ model with two levels of storage (FIFO)

The goal is to prove the existence, uniqueness and global optimality of the solution to a general inventory model for the FIFO dispatching policy presented in Chapter 8, i.e. Eq. (8.28).

Lemma.

$$f'_{Rj,q_{rj}} \geq 0.$$

Proof.

From (7.13),
$$T_{rj} > 0 \Longrightarrow (e^{g_r(0)} - p_j e^{g_r(f_{rj})}) x(f_{rj}) \ge D(f_{rj}) e^{g_r(f_{rj})}.$$
 (C.1)

Recall that
$$T_{oj} > 0 \Rightarrow (e^{g_o(0)} - p_j e^{g_o(f_{oj})}) x(f_{oj}) \ge D(f_{oj}) e^{g_o(f_{oj})}$$
. (C.2)

Hence, condition (B.33) holds.

As can be seen from (C.1): $e^{g_r(0)} \ge p_j e^{g_r(f_{rj})} \Longrightarrow f'_{Rj,q_{ri}} \ge 0$.

This completes the proof of Lemma. □

The existence, uniqueness and global optimality of the solution can be obtained in a similar fashion as that for the general inventory model with one level of storage presented in Appendix A, where the conditions under which the global optimal solution can be attained are given by:

$$f_{Rj,q_{rj}}^{"} = \frac{-p_j \delta_r(f_{rj}) e^{gr(f_{rj})} - f_{Rj,q_{rj}}^{'} \left(D'(f_{Rj}) + \delta_r(f_{Rj})D(f_{Rj})\right) e^{gr(f_{Rj})}}{D(f_{Rj}) e^{gr(f_{Rj})}}, \tag{C.3}$$

Appendix C - 217 -

$$w_{q_{rj}}^{"} = +h_{rg} \left[e^{g_r(0) - g_r(f_{Rj})} + \left(1 - e^{g_r(f_{rj}) - g_r(f_{Rj})} \right) p_j + \delta_r(f_{rj}) \left(G_r(f_{rj}) - G_r(f_{Rj}) \right) p_j e^{g_r(f_{rj})} \right] + \frac{h_{rd}p_j}{x(f_{rj})} \left[1 - \frac{x'(f_{rj})}{x(f_{rj})} \int_0^{f_{rj}} x(u) du \right] + c_{Fj} \left(f_{Rj,q_{rj}}^{"} - f_{j,q_{rj}}^{"} \right), \tag{C.4}$$

$$\psi_F'(q_{rj}) = w_{q_{rj}}'' f_{Rj} - f_{Rj,q_{rj}}'' w_F > 0.$$
(C.5)

Note that if $f_{oj}=0 \Rightarrow f_j=0$ then we can deduce from Eqs. (7.13)-(7.16) that problem (m_{F2}) reduces to that of (m_2) , i.e. the general inventory model with one level of storage. This can be further clarified from (7.18), (7.19), (C.3) and (C.4).

Appendix C - 218 -

Appendix D. EOQ model with two levels of storage (AIFO)

Lemma.

$$f'_{j,q_{Rj}} \geq 0.$$

Proof.

From (8.16),

$$T_{oj} > 0 \Longrightarrow e^{g_o(0)} - p_j e^{g_o(f_{oj})} \ge 0.$$
 (D.1)

From (8.17),

$$T_{rj} > 0 \Longrightarrow \left(e^{g_r(0)} - p_j e^{g_r(f_{rj})} \right) x(f_{rj}) \ge \left(1 - \emptyset_j \right) D(f_{rj}) e^{g_r(f_{rj})} -$$

$$\emptyset'_{j,q_{rj}} \int_0^{f_{rj}} D(u) e^{g_r(u)} du.$$
(D.2)

As can be seen from (D.2) and (8.24): $e^{g_r(0)} \ge p_j e^{g_r(f_{rj})} \Longrightarrow f'_{j,q_{rj}} \ge 0$.

Also, (D.1) and (D.2) \Leftrightarrow

$$0 \le p_j \le \min\left(e^{-g_r(f_{rj})} \left(e^{g_r(0)} - \frac{(1-\phi_j)D(f_{rj})e^{gr(f_{rj})} - \phi'_{j,q_{rj}} \int_0^{f_{rj}} D(u)e^{gr(u)}du}{x(f_{rj})}\right), e^{g_o(0) - g_o(f_{oj})}\right). \tag{D.3}$$

From (D.3),
$$\emptyset_j = 0 \Rightarrow T_{oj} = 0 \Rightarrow 0 \leq p_j \leq \min\left(e^{-g_R(f_{rj})}\left(e^{g_R(0)} - \frac{D(f_{rj})e^{g_R(f_{rj})}}{x(f_{rj})}\right), 1\right)$$

which is consistent with that of (A.2). Moreover, if $\delta_y(\cdot)=0$, i.e. for the case without deterioration, (D.3) implies Assumption 6, Chapter 8. This completes the proof of the Lemma.

The existence, uniqueness and global optimality of the solution can be obtained in a similar fashion as for the LIFO and FIFO policies, where the conditions under which the global optimal solution can be attained are given by:

$$f_{j,q_{rj}}^{"} = \frac{-\left(\int_{0}^{f_{j}} D(u)e^{g_{o}(u)}du\right)^{3} \delta_{r}(f_{rj})p_{j}e^{g_{r}(f_{rj})} - AS - CX + D}{D(f_{j})e^{g_{r}(f_{j})}\left[\left(\int_{0}^{f_{j}} D(u)e^{g_{o}(u)}du\right)^{3} - BS\right]},\tag{D.4}$$

$$\emptyset_{j,q_{rj}}^{"} = \frac{\left(2\left(f_{j,q_{rj}}^{'}D(f_{j})e^{g_{o}(f_{j})}\right)^{2} - \int_{0}^{f_{j}}D(u)e^{g_{o}(u)}du\left(f_{j,q_{rj}}^{"}D(f_{j})e^{g_{o}(f_{j})} + Y\right)\right)S}{\left(\int_{0}^{f_{j}}D(u)e^{g_{o}(u)}du\right)^{3}},\tag{D.5}$$

where

$$A = 2\left(f'_{j,q_{rj}}D(f_j)\right)^2 e^{\left(g_o(f_j) + g_r(f_j)\right)} \int_0^{f_j} D(u)e^{g_o(u)} du,$$

$$B = \left(\int_0^{f_j} D(u) e^{g_o(u)} du \right)^2 - e^{\left(g_o(f_j) - g_r(f_j)\right)} \int_0^{f_j} D(u) e^{g_o(u)} du \int_0^{f_j} D(u) e^{g_r(u)} du,$$

$$C = \left(\int_0^{f_j} D(u)e^{g_o(u)}du\right)^3 - S\left(\int_0^{f_j} D(u)e^{g_o(u)}du\right)^2,$$

$$X = f'_{j,q_{rj}} e^{g_r(f_j)} \left(D'(f_j) + \delta_r(f_j) D(f_j) \right),$$

$$Y = f'_{j,q_{rj}} e^{g_o(f_j)} \left(D'(f_j) + \delta_o(f_j) D(f_j) \right),$$

$$D = S\left(2\left(f'_{j,q_{rj}}D(f_{j})e^{g_{o}(f_{j})}\right)^{2} - Y\int_{0}^{f_{j}}D(u)e^{g_{o}(u)}du\right)\int_{0}^{f_{j}}D(u)e^{g_{r}(u)}du,$$

$$\begin{split} w_{qrj}'' &= +h_{rg} \left[e^{g_{r}(0)-g_{r}(f_{j})} + \left(1 - e^{g_{r}(f_{rj})-g_{r}(f_{j})} \right) p_{j} + \delta_{r}(f_{rj}) \left(G_{r}(f_{rj}) - G_{r}(f_{j}) \right) p_{j} e^{g_{r}(f_{rj})} + \emptyset_{j,q_{rj}}'' \left(G_{r}(f_{j}) \int_{0}^{f_{j}} D(u) e^{g_{r}(u)} du - \int_{0}^{f_{j}} D(u) G_{r}(u) e^{g_{r}(u)} du \right) + \\ \emptyset_{j,q_{rj}}' e^{-g_{r}(f_{j})} \int_{0}^{f_{j}} D(u) e^{g_{r}(u)} du \right] + \frac{h_{rd}p_{j}}{x(f_{rj})} \left[1 - \frac{x'(f_{rj})}{x(f_{rj})} \int_{0}^{f_{rj}} x(u) du \right] + \\ h_{og} \left[\emptyset_{j,q_{rj}}'' \int_{0}^{f_{j}} D(u) G_{o}(u) e^{g_{o}(u)} du + \left(\emptyset_{j,q_{rj}}'' + 2\emptyset_{j,q_{rj}}' \right) f_{j,q_{rj}}' D(f_{j}) G_{o}(f_{j}) e^{g_{o}(f_{j})} \right], \end{split}$$
 (D.6)

$$\psi_A'(q_{rj}) = w_{q_{ri}}'' f_j - f_{j,q_{ri}}'' w_A > 0.$$
(D.7)

Note that
$$\delta_r(\cdot) = \delta_o(\cdot) \Longrightarrow (8.23) \Leftrightarrow f'_{j,q_{rj}} = \frac{e^{g_r(0)} - p_j e^{g_r(f_{rj})}}{D(f_j) e^{g_r(f_j)}} \Longrightarrow (\mathrm{D}.4)$$

$$\Leftrightarrow f_{j,q_{rj}}^{"} = \frac{-p_j \delta_r(f_{rj}) e^{gr(f_{rj})} - f_{j,q_{rj}}^{"} \left(D^{"}(f_j) + \delta_r(f_j)D(f_j)\right) e^{gr(f_j)}}{D(f_j) e^{gr(f_j)}}.$$

Considering the above and (8.21), (8.25) and (D.6), $\emptyset_j = 0 \Longrightarrow T_{oj} = 0$, the model reduces to that of of (m_2) , i.e. the general inventory system with one level of storage.

Appendix D - 221 -