Energy harvesting using porous piezoelectric beam with impacts

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Abstract

An analytical model of impact energy harvester consisting of a cantilever beam with integrated piezoelectric patches and a ball is developed in this paper. The material chosen to extract the energy is porous PZT, a composite material made of two phases: air and PZT. This material offers good control of the capacitance and the stiffness of the resultant composite material and expands the design space for the harvester. The cantilever beam is modelled using a single degree-of-freedom approximation, and a load resistor is used to represent the external circuit. The response of the energy harvester and the power output is obtained for harmonic base excitation, and the effect of excitation frequency, boundary distance, load resistance and porosity of the PZT material. The results highlight the potential for the impact harvester and motivate further studies to optimize the harvester.

1. Introduction

The harvesting of energy from the environment has become important in powering systems where a wired system or the use of batteries is not appropriate, such as remote sensing systems. Piezoelectric materials have played an important role scavenging energy from vibrations in the environment for over a decade. Different authors have investigated the application of these materials under different conditions of excitation. For instance, Erturk and Inman[1] studied the performance of a cantilever beam energy harvester (EH) under harmonic base excitation. The importance of a correct optimization of the electrical circuit attached to the piezoelectric patch was highlighted. Adhikari et al.[2] studied random excitations using a stochastic approach on a single-degree-of-freedom model. The importance of a low mechanical damping and high electromechanical coupling to obtain the maximum power was shown. Also, different geometrical configurations have been analysed such as beams [1] and plates [3].

However, little attention has been paid to energy harvesters under impact excitations, although some interesting results have been published. One of the first contributions was made by Renaud et al.[4], where the performance of a cantilever beam EH with impact is explored. This harvester is modelled as single-degree-of-freedom system. To solve the dynamic equations, the authors establish an equivalence between the electro-mechanical model and an

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electrical model where the stiffness is modelled as capacitance, the damping as resistance and the mass as inductance, in addition to the electrical circuit coupled to the harvester. The impact is applied as an instant velocity load on the tip. This model gives a good insight of the dynamic behaviour of an EH, but it cannot represent the impact of a mass object on the EH. Jacquelin et al.[5] modelled the impact between two cantilever beams and a seismic mass. Using the anti-oscillator approach, the dynamic equations are solved for a limited number of degrees of freedom using the Rayleigh-Ritz procedure. The authors conclude that the maximum power obtained is due to the transient (impact) regime; this power is much higher than that obtained in the linear steady state (harmonic excitation) although the steady state power is more constant with time. Vijayan et al.[6] investigated two piezoelectric cantilever beams impacting with each other, using a Hertzian contact law [7]. In this harvester, the power is highly sensitive to the clearance and thickness ratio. The performance of an impact EH is based on exciting higher frequencies than the linear harvester with the same base excitation frequency and amplitude, using the impact. These last two approaches rely on the inertia of the beams to generate the impact between the beams and a seismic mass or the boundaries of the box enclosing the harvester. These approaches do not use the impact sources from the surrounding environment, and hence no optimization is performed on the colliding object.

Recent studies [8,9] suggest that the porous piezoelectric material, made of air and piezoelectric ceramics, may give important benefits by controlling the stiffness and the capacitance of the piezoelectric patch. This allows the improvement the power output and optimisation of the piezoelectric patch characteristics. The studies also shows that the effect of the porosity is associated with a decrease of the mass at a higher rate than the stiffness; hence, there is an increase of the natural frequency of the harvester. It is suggested that the porous piezoelectric material is potentially able to harvest more power per unit mass than the conventional dense piezoelectric material. The porous piezoelectric material is composed of two phases; air and piezoelectric material. The piezoelectric material is normally lead zirconate titanate (PZT) or Barium Titanate. For each phase the material constants are well known, but the set of homogenized material constants must be calculated for the composite. One of the most used approaches is to homogenize the material using analytical methods, for example the well-known and validated Mori-Tanaka method, which is based on *mean-field homogenization theory*. This method improves the Eshelby solution [10] given for ellipsoidal inclusions in elastic mediums. To perform a Mori-Tanaka homogenization, the authors refer to the existing literature [10–12].

Vijayan et al.[6] showed that impacts on the EH can increase the energy in the beam and excite higher modes. Hence, it seems reasonable to propose that an impact EH should have better performance when it is excited away from its resonance frequency (*off-resonance*) than a conventional linear EH. Here, this approach is used together with the advantages that the porous piezoelectric materials give, to propose an EH that is able to extract energy from low excitation frequencies far from resonance. The sensitivity of this porous cantilever EH under base excitation and impact from spherical particles is then studied to better understand the main parameters that affect its performance, in order to select the most sensible ones for a more comprehensive future study. The paper is organised as follows: first the model is presented and the basis of the dynamic equations of a single-degree-of-freedom impact EH approximation are explained. Then, example simulation results are given for different parameters values. Finally, the conclusions are presented.

2. Model Description

The system studied in this paper is a single-degree-of-freedom model of a cantilever beam with two piezoelectric patches, one on the top and one on the bottom surface. The piezoelectric patches cover both surfaces of the beam completely. The beam is impacted by a spherical ball that is characterized by its mass, initial speed and initial distance with respect to the beam. The contact forces are modelled using a non-linear spring derived from a *Herztian Contact Law*[7]. The ball displacement is limited by a boundary (*BC*) at distance BC_{gap}, and the stiffness of this contact is the same as the ball-beam contact. The ball is only subject to gravity and the forces from the contact with the beam or boundary.

The dynamic behaviour of the system is modelled using the coupled electro-mechanical equations [6]:

Geometry		Elastic Material Properties		Piezoelectric Material Properties	
Beam Length (mm)	250	Elastic Modulus (GPa)	70	Elastic Modulus (GPa)	61
Piezoelectric Thickness (mm)	0.15	Poisson's ratio	0.3	Poisson's ratio	0.3
Elastic Layer Thickness (mm)	0.05	Mass Density (kg/m^3)	2700	Mass Density (kg/m^3)	7750
Beam width (mm)	5			Piezoelectric constant e_{31} (C/m^2)	5.31
				Relative Dielectric constant ϵ_{33}/ϵ_0	1500

Table 1: Geometrical properties of the beam, material properties of the elastic support material and piezoelectric material.

$$m\,\ddot{x}_{beam} + C\,\dot{x}_{beam} + K\,x_{beam} - k_p\,v = -F_{Impact} + F_{BE} \quad (1a)$$

$$C_p\,\dot{v} + \frac{v}{R} + k_p\,\dot{x}_{beam} = 0 \quad (1b)$$

$$m_{ball}\,\ddot{x}_{ball} = F_{Impact} - F_{BC} - m_{ball}\,g \quad (1c)$$

$$F_{Impact} = \begin{cases} K_{Contact}\,(x_{beam} - x_{ball})^{\frac{3}{2}} & \text{if } x_{beam} > x_{ball} \\ 0 & \text{otherwise} \end{cases} \quad (2a)$$

$$F_{BC} = \begin{cases} K_{BC\ Contact}\,(x_{ball} - BC_{gap})^{\frac{3}{2}} & \text{if } x_{ball} > BC_{gap} \\ 0 & \text{otherwise} \end{cases} \quad (2b)$$
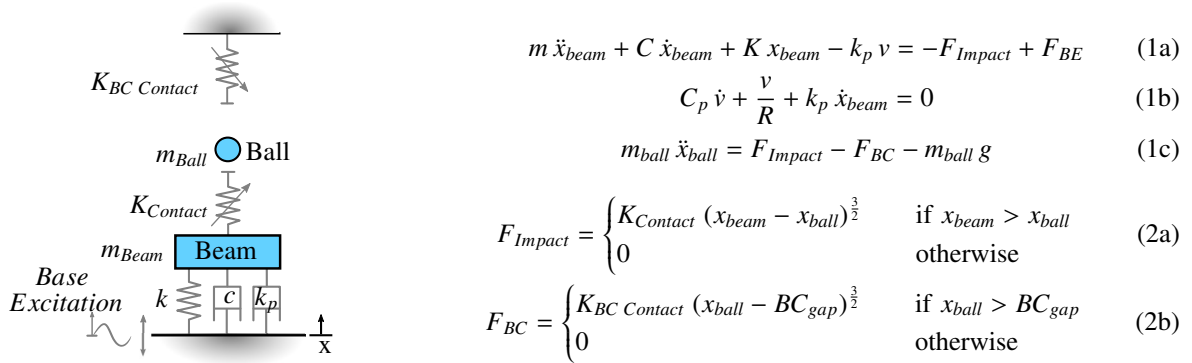
Fig. 1: Proposed energy harvester model.

where the dot represents the derivative with respect to time. The beam mass is m, the beam damping is C and the beam stiffness is K. The parameter k_p is the piezoelectric coupling between the electrical and mechanical fields. The coupling allows the energy in the mechanical system to be converted into electrical energy (voltage), to be harvested to power small devices. The effect is to add damping to the system. The parameter C_p represents the capacitance of the piezoelectric patches, and quantifies the energy lost by the induced electric field in the piezoelectric patches. The terms F_{Impact} and F_{BE} correspond to the forces that arise from the non-linear contact and the base excitation respectively. Equations (1a) and (1b) describe the beam dynamic behaviour and Equation (1c) describe the dynamics of the ball. Equation (2a) models the force from the contact between the ball and the beam. This force is non-linear and proportional to the indentation and a constant $K_{Contact}$, which defines the stiffness of the impact. The procedure to obtain this constant is discussed in [7,13]. Here the value is assumed to be $10^7\,\mathrm{N\,m^{-3/2}}$ [5]. Similarly, the force F_{BC} models the impact of the ball with the boundary. These differential equations are solved numerically using the *Runge-Kutta* method.

3. Numerical Results

In order to assess the sensitivity of the main parameters of the impact EH, a stochastic parametric study is performed using the presented model in order to identify the key features and range of values for a future optimum design study. In the present study, the initial speed of the ball is a stochastic variable with a uniform distribution between 0 and 1 m/s. At each EH the number of simulations performed with different initial ball velocities varied depending on the sensitivity of the output quantity to the initial velocity. Since the main focus is the performance of the device to harvest energy, the different models are compared based on the average power over a specific time or the maximum instantaneous power generated. The time simulation was sufficiently long to ensure the steady state had been reached, which was typically after 15s. The impact energy harvester (IEH) is compared to a non-impact energy harvester (NIEH) with identical geometrical properties. Unless mentioned otherwise, the main properties of the impact energy harvester study are: base excitation frequency is 2.2 Hz, contact stiffness ($K_{Contact}$) is $10^{E}7$, boundary distance (BC_{gap}) is 1m and the ratio m_{ball}/m is 0.2. The load resistance R is optimized for each simulation using the equation $\omega R C_p = 1$ which has been derived for the linear case; it will be shown later that the optimum resistance is not affected by the impacts. Table 1 gives the other harvesters properties. Different variables have been included in this parametric study, such as the frequency of the base excitation, the boundary distance, the load resistance and percentage of porosity.

As it stated in the introduction, a good off-resonance performance is one of the key objectives in this study. With the parameters given in Table 1 the harvester has its first natural frequency at 2.7 Hz. Figure 2 shows the output power for a range of excitation frequencies, for both the linear EH and the impact EH, and shows that the peak in the power output occurs when the harvester is resonant. At the resonance frequency, the performance of the IEH in terms of the average power harvested, is not as good as the NIEH. However, the performance of the IEH is superior to that of the NIEH slightly away from the natural frequency (2.65Hz and 2.75Hz). The maximum instantaneous power (see figure 2a) is much higher for the IEH than for the NIEH, because of the energy spikes provided by the impacts. Since the highest power from the EH is obtained during the impact, optimization can be performed to ensure the best frequency of impacts to harvest the maximum power possible.

Given the nature of the model, when a ball is bouncing between the beam and a boundary, it is reasonable to assume that the location of the boundary with respect to the beam affects the frequency of impacts between the ball and the beam. Figure 3 shows the effect of the boundary distance on the EH performance. There are two main peaks in the power response around 0.3 and 0.7, which seems to indicate a coupling between the frequency of impact and the resonance frequency of the beam. To explore this relationship further, the individual responses for two different values of BC_{gap} are shown in Figure 4, where the importance of the number of impacts can be assessed. In Figure 4a there are many impacts, which provides more energy to the beam, than in Figure 4b where the beam is excited much less by the ball impacts.

Another important characteristic of any energy harvester is the electric load connected to it. Different authors [2,14] have pointed out the importance of the correct optimization of the circuit attached to the piezoelectric patches. In Figure 5, the response of the IEH for different values of resistance is presented. Here, the IEH performance is higher than the NIEH with almost twice harvested power. It should be noted that this study is performed at an off-resonance
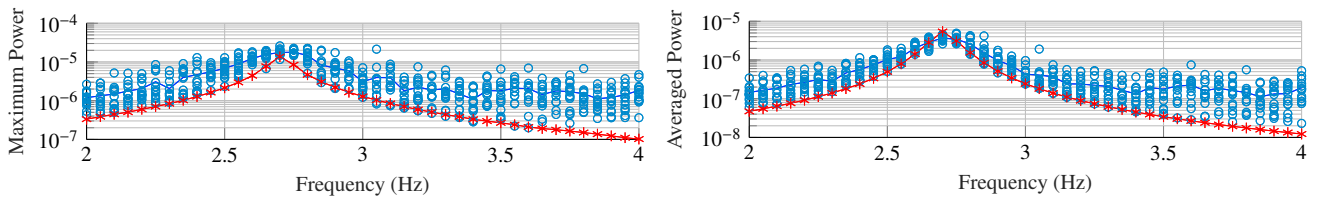

Fig. 2: The maximum instantaneous power (left, figure 2a) and aaveraged power (right, figure 2b) for a range of excitation frequencies. The impact energy harvester is in blue, and the non-impact (linear) energy harvester is in red.
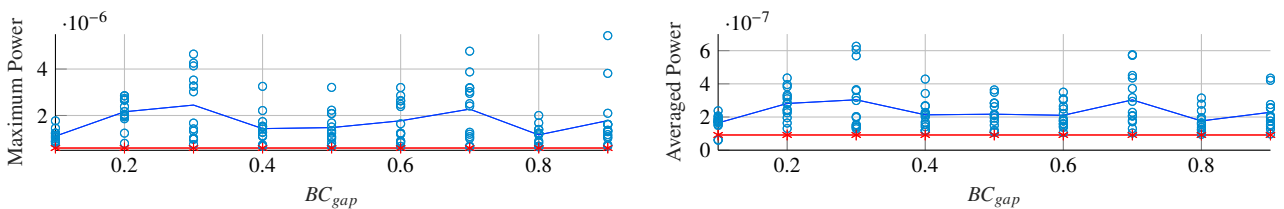

Fig. 3: The effect of different boundary distances (BC_{gap}) on the maximum instantaneous power (left, figure 3a) and the averaged power (right, figure 3b). The impact energy harvester is in blue, and the non-impact (linear) energy harvester is in red.
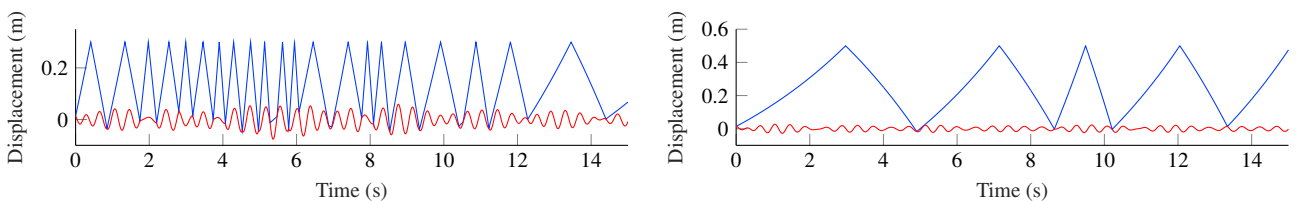

Fig. 4: The time response of the impact energy harvester for two different values of boundary distance (BC_{gap}). The beam is represented by the red line and the ball by the blue line. The left figure (4a) correspond to $BC_{gap} = 0.3$m and the right figure (4b) to $BC_{gap} = 0.5$m
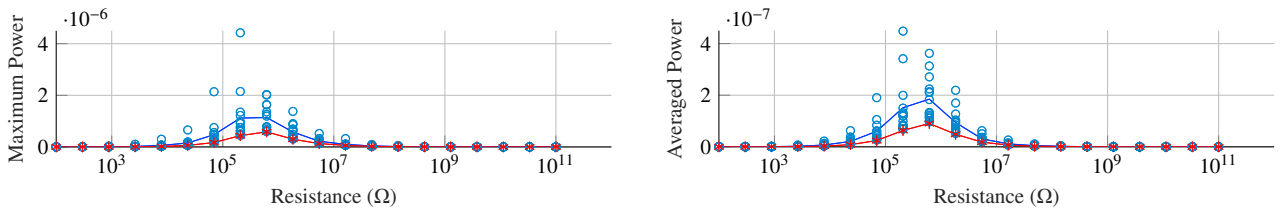
Fig. 5: The maximum instantaneous power (left, figure 5a) and averaged power (right, figure 5b) for a range of load resistances, excited off-resonance at 2.2 Hz. The impact energy harvester is in blue, and the non-impact (linear) energy harvester is in red.
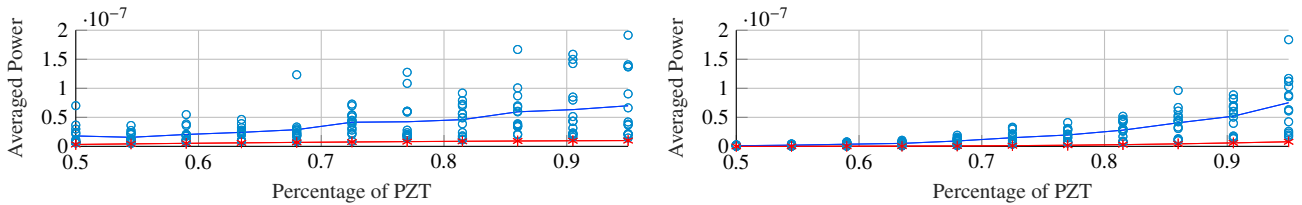

Fig. 6: The averaged power for a range of piezoelectric porosity, for a model with constant thickness (left, figure 6a) and a model with a constant mass of piezoelectric material (right, figure 6b). The impact energy harvester is in blue, and the non-impact (linear) energy harvester is in red.

frequency (2.2 Hz) where the impacts provide more energy to the system that the linear model. Figure 5b shows that the addition of the impacts to the energy harvester does not affect the optimal resistance. Hence the same electrical load can be used as for a linear EH, which simplifies the design of a combined impact and non-impact energy harvester that can work over a range of harmonic base excitations.

Finally, the effect of the porosity of the piezoelectric material on the EH performance is studied. In Figure 6 two different porous distributions are compared. The first model (6a) keeps the geometry constant, and hence the mass of piezoelectric material decreases as the porosity increases. The second model (6b) keeps the mass of piezoelectric material constant, which increases the beam thickness and hence the natural frequency, but is a fairer comparation between porous or non-porous materials.

4. Conclusions

A parametric study of the main impact energy harvester (EH) parameters has been performed in this paper. The initial velocity of the ball, and hence its energy, is considered as a stochastic variable. The purpose of this study is to identify the key features and parameter ranges for a future optimization. It was found that an impact EH can extract more energy than a linear energy harvester, for off-resonance base excitation frequencies with an appropriate optimization. This off-resonance frequency range is very close to the first mode frequency (about 1̃Hz) so an impact energy harvester can be very efficient when there is uncertainty about the base excitation frequency. In case of the external circuit, the impacts have only a limited effect on the optimal resistance of the EH; therefore, the resistance can be assumed from its linear counterpart, which it is very convenient for coupled impact / base excited energy harvesters.

Acknowledgement

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