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Citation for final published version:

Lin, Xiude, Zhu, Hanxing , Yuan, Xiaoli, Wang, Zuobin and Bordas, Stephane 2019. The elastic properties of composites reinforced by a transversely isotropic random fibre-network. *Composite Structures* 208 , pp. 33-44. 10.1016/j.compstruct.2018.09.097

Publishers page: <http://dx.doi.org/10.1016/j.compstruct.2018.09.097>

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Accepted Manuscript

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PII: S0263-8223(18)32463-2
DOI: <https://doi.org/10.1016/j.compstruct.2018.09.097>
Reference: COST 10244

To appear in: *Composite Structures*

Received Date: 12 July 2018
Revised Date: 14 August 2018
Accepted Date: 25 September 2018



Please cite this article as: Lin, X., Zhu, H., Yuan, X., Wang, Z., Bordas, S., The Elastic Properties of Composites Reinforced by a Transversely Isotropic Random Fibre-Network, *Composite Structures* (2018), doi: <https://doi.org/10.1016/j.compstruct.2018.09.097>

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The Elastic Properties of Composites Reinforced by a Transversely Isotropic Random Fibre-Network

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Abstract

This research stems from the idea of introducing a fibre-network structure into composites aiming to enhance the stiffness and strength of the composites. A novel new type of composites reinforced by a transversely isotropic fibre-network in which the fibres are divided into continuous segments and randomly distributed has been proposed and found to have improved elastic properties compared to other conventional fibre or particle composites mainly due to the introduction of cross-linkers among the fibres. Combining with the effects of Poisson's ratio of the constituent materials, the fibre network composite can exhibit extraordinary stiffness. A simplified analytical model has also been proposed for comparison with the numerical results, showing close prediction of the stiffness of the fibre-network composites. Moreover, as a plate structure, the thickness of the fibre network composite is adjustable and can be tailored according to the dimensions and mechanical properties as demanded in industry.

Key words: Fibre-network composites, transversely isotropic, elastic properties, cross-linker

1. Introduction

Fibre reinforced composites have been widely used in various fields for their attractive mechanical and physical properties with the wide choices of constituent materials and geometry structures. Numerous different structures of fibre composites, such as uni-directional fibre composites, cross-ply fibre composites, woven fabric composites and fibre laminates etc., have been designed and applied primarily for their advantages in directional mechanical properties. However, the superior properties are achieved by sacrificing the properties in other axial or planar directions. In addition, it is inevitable in engineering that loads are applied to the inferior directions of the structure. This may increase the risk of crack propagation and, even worse, fracture. For instance, delamination [1] is a common problem for laminate composites due to the weakly bonded interfaces between plies. The similar problem also exists even for the randomly distributed fibre composites which are mostly isotropic [2] or transversely isotropic [3]. Some three dimensional numerical models [2-4] of short fibre reinforced composites have been proposed by many researchers with the most frequently used method of random sequential adsorption (RSA). However, overlap between fibres are usually avoided which makes it difficult to generate a model with a high volume fraction. Besides, the constraints among fibres in the conventional fibre composites are weak since they are only, or at most, in contact but without bonding connection, thus rendering large deformation and easy pull-out [5] of fibres when subjected to load.

It has been found that interpenetrating composites reinforced by a self-connected fibre-network have significantly enhanced mechanical properties, such as stiffness and strength, compared to their counterparts with discontinuously reinforced phase structures [6-11]. Apart from the improved mechanical properties, good thermal and electrical conductivities [12, 13] can also be an advantage for fibre network composites owing to the connected network of fibres.

Therefore, we aim to construct a 3D fibre network reinforced composite. In terms of the fibre network, Clyne et al. have conducted a series of thorough investigations towards bonded metal fibre networks both experimentally and analytically, involving work in the characterisation of the network architecture and capture of independent elastic constants [14-19]. Some other research has also been done regarding to the mechanical properties of transversely isotropic fibre networks [20-23], such as metal fibre sintered sheet [24, 25]. However, when it comes to fibre network composites, much less has been conducted. A few experimental work was focused on metal matrix composites [26-28]. Jayanty et al. [10] have fabricated an auxetic stainless steel mat and a composite reinforced by the mat. Clyne et al. [14, 15, 19] have also included analysis of fibre network composites by introducing a strain reduction factor. However, no close form can be obtained from this analytical expression due to the complex architecture. Lake et al. [29] and Zhang et al. [30, 31] have proposed a 3D isotropic two-phase numerical model of collagen-agarose tissue in which a non-periodic Voronoi network is generated to represent collagen and a neo-Hookean solid to represent the matrix. The drawback of their model lies in that the fibres are assumed to be pin-jointed, the model is not periodic and the boundary conditions used in their model are not realistic.

To the best of our knowledge, there is few simulation or analytic research work to study the mechanical properties of interpenetrating composites reinforced by a transversely isotropic fibre network due to the combined complexity of fibre network architecture and coupling between the fibre network and matrix. This type of structure is frequently observed in bioscience such as cornea [32, 33] and cytoskeleton [34], and can be a promising structural material in engineering fields. Therefore, the main objective of this paper is to investigate the elastic properties of composites reinforced by a random transversely isotropic fibre network. In this paper, we have developed a code to automatically construct the periodic representative element (RVE) model for composites reinforced by a random transversely isotropic fibre

network, then use the commercial finite element software ABAQUS to simulate how the fibre volume fraction affects the in-plane and out-of-plane elastic properties. In addition, we have obtained analytical results from a simplified geometric model and compared the results of the transversely isotropic interpenetrating composites to those of the conventional composites.

2. Numerical implementation

2.1. Geometric model of transversely isotropic random fibre network

Before applying finite element analysis (FEA), a periodic representative volume element (RVE) with a size of $L \times L \times t$ is constructed for the interpenetrating composite. The periodic transversely isotropic random fibre-network model with N complete fibres is generated within the same domain (i.e. $L \times L \times t$) using a code similar to that developed to generate the 3D fibre-network with cross-linking in reference [21]. Figure 1 shows a periodic representative volume element (RVE) of the interpenetrating composite reinforced by a self-connected and transversely isotropic random fibre-network containing 50 complete fibres, in which the fibres on the front, left and bottom surfaces align with those on the back, right and top surfaces, respectively. Thus a large-size interpenetrating composite can be made up by a number of identical RVEs.

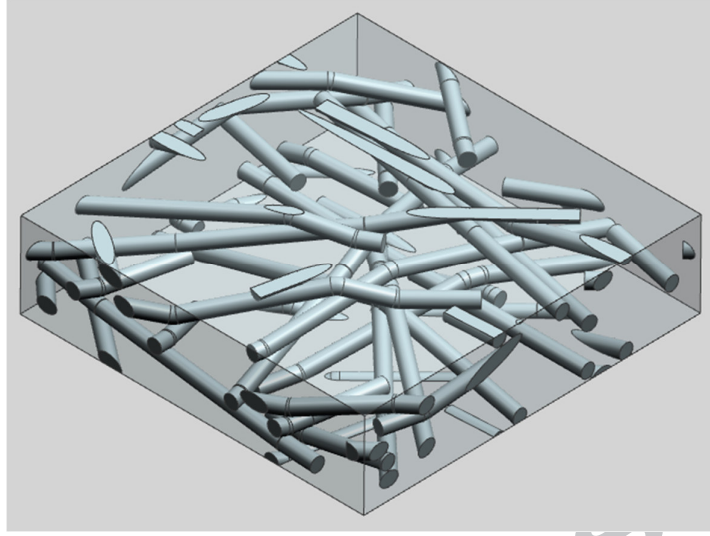


Fig. 1. A periodic representative volume element (RVE) of the composite reinforced by a transversely isotropic random fibre-network containing 50 complete fibres.

In the interpenetrating composite model shown in Fig. 1, the x - y plane projections of all the fibres are straight lines, and their x - z and y - z plane projections are polylines. For the projected straight lines of the fibres on the x - y plane, the coordinate of the centre point ($0 \leq x \leq L$, $0 \leq y \leq L$), the orientation ($0 \leq \theta \leq \pi$), and the length ($0.8L \leq L_i \leq 1.2L$) are all specified by random numbers (from 0 to 1) generated automatically by the computer. All the fibres are assumed to have the same diameter d . The z coordinates of the polylines are determined by the building-up process, see [27] for details. For two connected fibres, the overlap coefficient is defined as $c = 1 - \delta/d$, where δ is the distance between the centroidal lines of the two fibres.

The density of the cross-linkers is defined as the number of connections of a fibre with those below it, and given as $N_c = L/l_c$, where l_c is the mean distance between any two neighbouring connections of a fibre with those below it. The maximum inclination angle of the segments in a polyline is limited to be smaller than 21.5° . It is noted that in reference [27], only two fixed values of the fibre overlap coefficients, i.e. $c = 0.05$ and $c = 0.6$, are

considered; while in this paper, the value of fibre overlap coefficient is not a constant, but always increases with the volume fraction of the fibre-network or the density of cross-linkers as given by $c = 0.025(N_c + 1)$. For a RVE model containing N complete fibres, its thickness t depends on the density of crosslinkers and can be determined during the construction process of the fibre-network model. By taking account the overlap parts between the connected fibres, the volume fraction of the fibre-network can be obtained and given in Equation (1) and (2). For RVEs with 200 complete fibres (i.e. $N=200$) and a size of $L \times L \times t$, Figure 2 shows how the density of cross-linkers affects the thickness t and the volume fraction of the fibres, where $L=100\text{mm}$, $d=1\text{mm}$, and the mean length of complete fibres is L .

$$V_f = \frac{\sum_{i=1}^N \left(L_i \times \left(\frac{1}{4} \pi d^2 \right) - \sum_{j=1}^{N_c} V_{ij} \right)}{L \times L \times t} \quad (1)$$

Where,

$$V_{ij} = \frac{d^3 (\sqrt{c(2-c)} + 2\sqrt{c(1-c)})}{4 \sin \alpha_{ij}} \times \left\{ \arctan \left(\frac{\sqrt{c(2-c)}}{1-c} \right) - (1-c)\sqrt{c(2-c)} \right\} \quad (2)$$

where L_i are the fibre lengths, N is the number of fibres and d is the diameter of the circular cross section of a fibre. α_{ij} and V_{ij} are, respectively, the angle and the overlap volume between the two connected fibres at the j th crosslinker of fibre i .

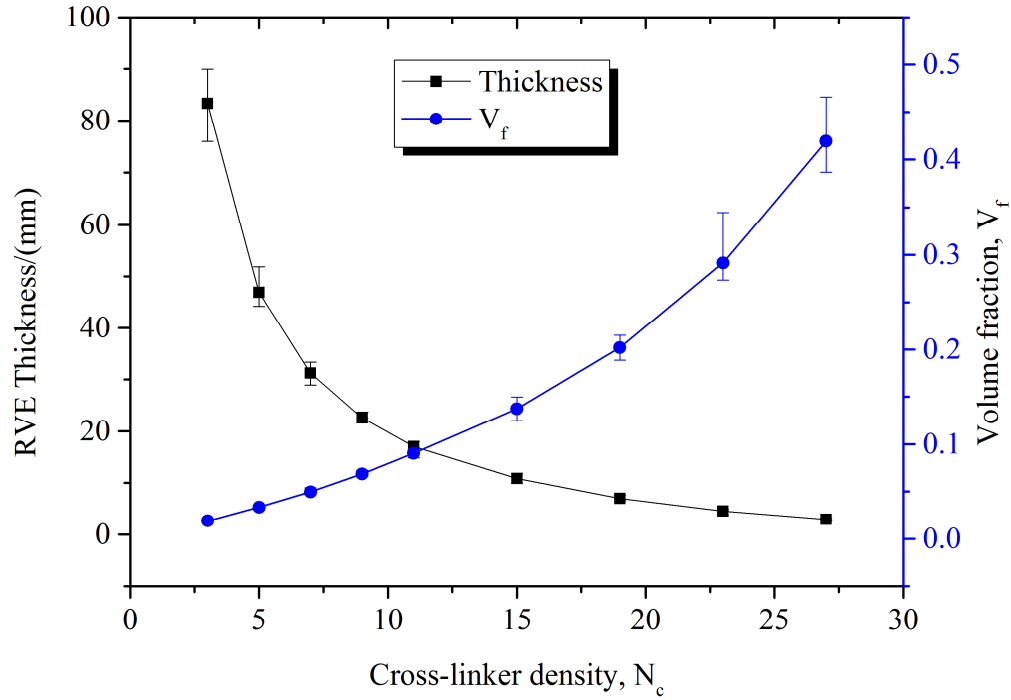


Fig. 2. Effects of cross-linker density, N_c , on RVE thickness t and fibre volume fraction V_f of fibre-network with aspect ratio $L/d = 100$.

2.2. RVE model of fibre-network reinforced composite

We have performed a large number of numerical tests and found that for each of periodic RVE models containing 50 complete fibres, as shown in Figure 1, its in-plane elastic properties are far from isotropic because the number of complete fibres is too small. To mesh both the matrix and fibres into solid tetrahedral elements for such a model, the total number of elements is between 1~2 millions. Because of the very complex interfaces between the fibres and matrix, it is very difficult to mesh the matrix and all the fibres into solid tetrahedral elements for the RVE models. What's worse, such a large number of elements dramatically increases the pre-processing time and slows down the computing speed in simulations. Due to the above reasons, we use periodic RVE models containing 200 complete fibres, as shown in Fig. 3, and mesh the

matrix into a large number (varying from 20000 to 230000 depending on the thickness of RVE) of 8-node solid brick C3D8R elements and the fibres into around 60000 Timoshenko 2-node beam B31 elements, and then use the commercial finite element software ABAQUS [28] to perform the simulations. The cross-linkers are represented by an inserted beam element and the diameter is assumed to be the same as that of the fibres.

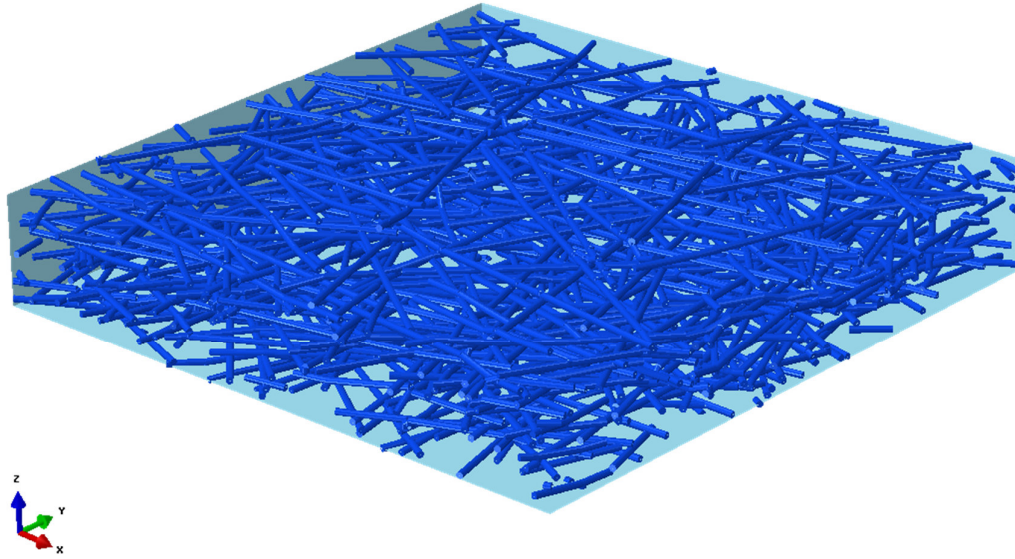


Fig. 3. Periodic RVE geometric model of composite reinforced by a transversely isotropic random fibre network containing 200 complete fibres, where the matrix is partitioned into brick elements and the fibres are partitioned into Timoshenko beam elements.

The periodic fibre-network RVE with 200 complete fibres (i.e. $N=200$) and a size of $100\text{mm} \times 100\text{mm} \times t$ (i.e. $L=100\text{mm}$ and t varies according to the cross-linker density N_c

for models with different volume fractions, see Fig. 2) is constructed in MATLAB and then imported into ABAQUS. In ABAQUS, a solid RVE with exactly the same size $100\text{mm} \times 100\text{mm} \times t$ is created to represent the matrix. To assemble the fibre-network and the matrix together, constraints are applied to the corresponding nodes in the matrix and the fibre-network to ensure that they have the same translation so as to transfer load between fibres and matrix. One common method is the Embedded Element Method (EEM), in which each node in fibre network will be coupled with the nodes of the coinciding element [35]. However, this method cannot be applied to our model because over-constraint occurs when both periodic boundary condition and embedded element method are applied to the matrix nodes on the the boundary of the RVE simultaneously. Therefore, another method, the automatic searching & coupling (ASC) technique proposed by Lu et al. [2], has been adopted in this model to avoid the conflict. The ASC technique involves node searching and coupling procedures, in which the closest matrix node is found out for each node on the fibre network and the translational freedom degrees of the corresponding fibre node and matrix node are coupled. By this way, all the corresponding nodes will be coupled and constrained for mechanical analysis. Another advantage of applying the ASC technique is reflected when it comes to meshing, that is no complex meshing is needed for the matrix thus saving the time in mesh generation and computing. As the RVE model of the fibre-network composite shown in Fig. 3 is periodic, periodic boundary conditions are applied to the RVE model in simulations. The mechanical properties of the matrix are exactly the same as what they are, while the Young's modulus of fibres is modified as $(E_f - E_m)$ because of the overlap between the fibre-network and the matrix, where E_f and E_m are the Young's moduli of the fibres and matrix, respectively [2].

2.3. Mesh size sensitivity

Different matrix mesh sizes have been tested for models with fibre volume fractions of 9% and 30% respectively, and the in-plane and out-of-plane Young's moduli and Poisson's ratios have been listed in Table 1. The convergence of both the in-plane and out-of-plane moduli in Fig. 4 gives us a more transparent vision of mesh sensitivity of the results. Taking the computing precision and efficiency into consideration, matrix mesh size of 1.5 mm×1.5 mm×0.6 mm through the x, y and z directions has been chosen for the following analysis. With this element mesh size and RVE size of 100mm × 100mm × t , the number of solid C3D8R elements in matrix varies from 20000 to 230000 depending on the thickness of RVE. Besides, the number of Timoshenko beam elements (B31) in fibres is around 60000 with the fibre mesh size of 1 mm.

Table 1. Mesh size effect on the in-plane and out-of-plane Young's moduli and Poisson's ratios of RVE.

Elastic properties	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Mesh 5	Mesh 6	
Size of elements (mm× mm×mm)	4×4×1	1.5×1.5× 0.8	1.5×1.5× 0.6	1.25×1.2 5×0.6	1×1×0.5	0.8×0.8× 0.4	
9%(V_f)	E_{11}	4.37	3.99	3.86	3.8	3.74	3.7
	E_{33}	1.89	1.71	1.565	1.54	1.5	1.48
	ν_{12}	0.339	0.337	0.336	0.335	0.335	0.334
	ν_{31}	0.094	0.096	0.097	0.098	0.1	0.101
30%(V_f)	E_{11}	12.4	12.01	11.9	11.88	11.86	11.87
	E_{33}	4.15	3.81	3.55	3.46	3.45	3.4
	ν_{12}	0.291	0.288	0.332	0.331	0.329	0.328
	ν_{31}	0.039	0.041	0.041	0.041	0.042	0.042

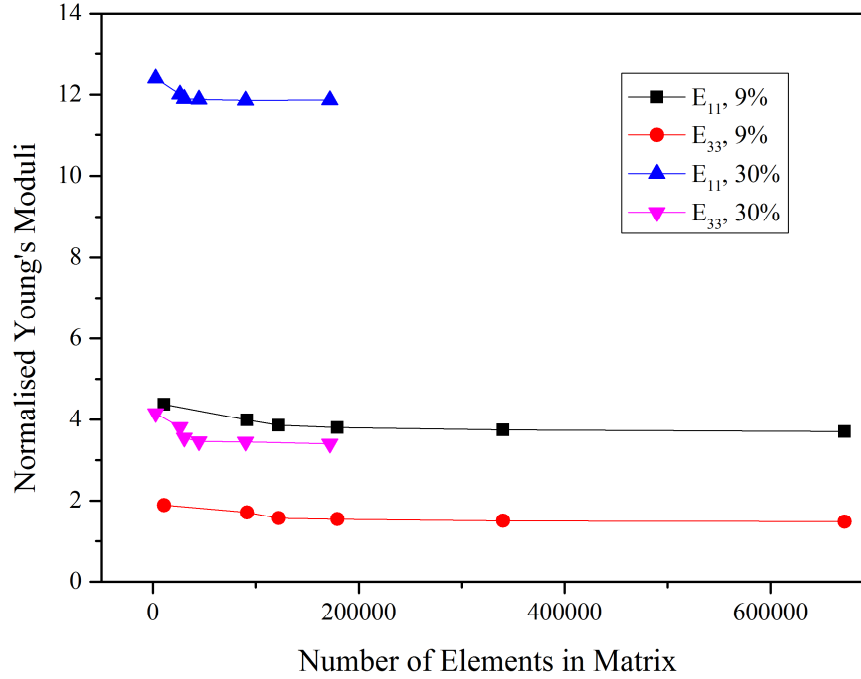


Fig. 4. Mesh size effects on the mean in-plane and out-of-plane Young's Moduli for 10 RVEs of composites with fibre volume fractions of 9% and 30%, respectively.

2.4. Fibre element type effect

The results in table 1 and Figure 4 are based on the analysis of RVEs with beam elements applied to the fibres and solid elements to the matrix. As mentioned before, the ASC Technique has been adopted to constrain every single node of the beam elements within the corresponding solid element in matrix. This method tremendously reduces the complexity of pairing the coincident nodes on fibres to those in the matrix. However, it has to be aware that there are limitations to this technique. The biggest concern lies in that additional stiffness/flexibility might be added to the RVE. Therefore, it is necessary to investigate the difference introduced by the application of beam elements to fibres compared to solid elements.

Ten RVEs which each contains 50 complete fibres were generated with the density of cross-linkers $N_c = 15$, overlap coefficient $c = 0.4$ and aspect ratio $L/d = 30$. Beam and solid elements were respectively applied to fibres in the the same RVE models while keeping the other conditions the same. The value of E_f/E_m is assumed as 100 and Poisson's ratios of fibres and matrix are kept the same as 0.3. A uniaxial tensile/shearing strain of 0.001 was applied to the RVE models and the corresponding reaction force was recorded. Table 1 lists the mean results of the five independent elastic constants with two different element types. E_{11} and G_{31} of RVEs with beam elements show smaller values than those of RVEs with solid elements whereas E_{33} of RVEs with beam elements is larger than that of RVEs with solid elements. It can be calculated that the difference between the stiffnesses with the two different element types is around 15%. One unavoidable problem is computing efficiency. When solid elements is adopted, the number of elements in a RVE reaches 1-2 millions or even larger depending on the dimensions of fibres, which is really time-consuming and unaffordable for a research involving several hundreds of such RVEs. Therefore, it can be a good choice to use beam elements in consideration of feasibility, efficiency and accuracy in computation. This is also how most other researchers deal with complex fibre reinforced composites.

Table 1. The independent elastic properties of RVE with beam and solid fibre element types, respectively, in which the density of cross-linkers $N_c = 15$, overlap coefficient $c = 0.4$ and aspect ratio $L/d = 30$. The values are averaged for 10 RVEs.

Fibre element type	E_{11}	ν_{12}	E_{33}	ν_{31}	G_{31}
Beam	2.496059	0.225184	1.383805	0.207891	0.466779
Solid	2.862772	0.190082	1.142895	0.127317	0.526806

2.5. Transverse isotropy of RVE

In order to evaluate the transverse isotropy, we simulated 10 models which each has 200 complete fibres, the density of cross-linkers $N_c = 11$, the overlap coefficient $c = 0.3$ and the aspect ratio $L/d = 100$.

Table 2. lists the Young's moduli, shear moduli and Poisson's ratios, and shows that the mean values of Young's moduli and the Poisson's ratio for 10 models are almost identical in the x and y directions (i.e. $E_{11} = E_{22}$ and $\nu_{12} = \nu_{21}$). In addition, the results also show that the shear modulus, Young's modulus and Poisson's ratio in the x-y plane meet the relationship $G_{12} = E_{11} / [2(1 + \nu_{12})]$. Moreover, $G_{13} = G_{23}$, $\nu_{13} = \nu_{23}$ and $\nu_{31} = \nu_{32}$ with the largest error less than 5%. These all suggest that the random fibre network composite structure is transversely isotropic and only five independent elastic constants, E_{11} , ν_{12} , E_{33} , ν_{31} and G_{31} , are needed for full elastic analysis.

Table 2. Young's moduli, Poisson's ratios and shear moduli of 10 RVEs with density of cross-linkers $N_c = 11$, overlap coefficient $c = 0.3$, number of complete fibres $N = 200$, and aspect ratio $L/d = 100$. The volume fraction is 9%.

	E_{11}	ν_{12}	ν_{13}	E_{22}	ν_{21}	ν_{23}	...
01	3.874956	0.313329	0.245138	3.833344	0.309964	0.248317	
02	3.972831	0.338440	0.235412	3.702278	0.315392	0.242851	
03	3.960358	0.373321	0.226916	3.374779	0.318121	0.243142	
04	3.777534	0.361487	0.227803	3.546943	0.339420	0.233979	
05	3.624164	0.329107	0.239986	3.838568	0.348577	0.234099	...
06	4.245049	0.354521	0.233621	3.498124	0.292142	0.249743	
07	3.549718	0.298000	0.254791	3.896780	0.327136	0.245870	
08	3.797864	0.310732	0.245548	3.941230	0.322462	0.240366	

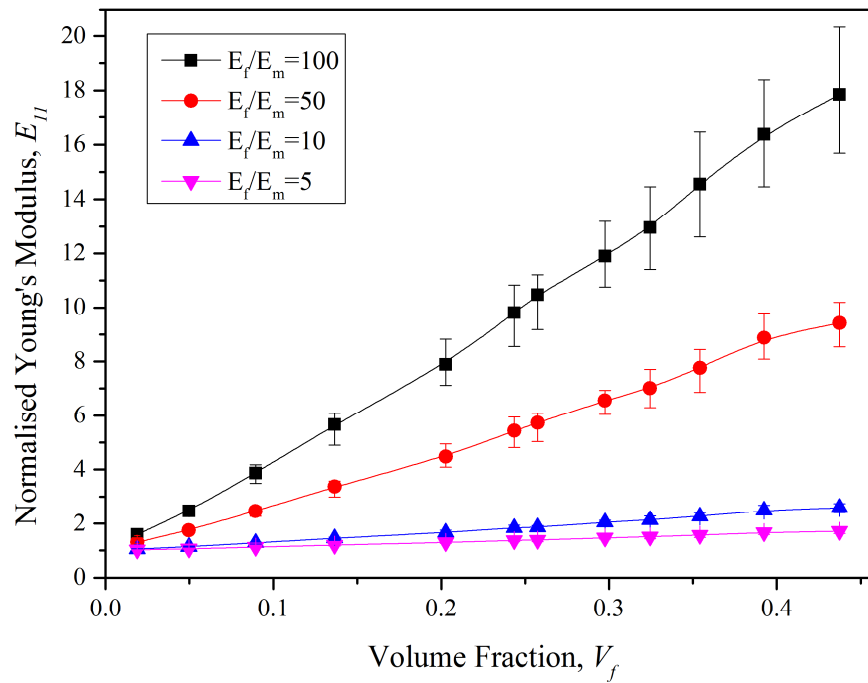
09	3.989732	0.324433	0.241150	3.779278	0.307320	0.246934
10	3.861258	0.360456	0.231369	3.452893	0.322335	0.240102
Mean	3.865346	0.336383	0.238173	3.686422	0.320287	0.242540
Std.	0.197312	0.025311	0.008820	0.202581	0.016048	0.005491
...						
	E_{33}	ν_{31}	ν_{32}	G_{12}	G_{23}	G_{31}
01	1.562067	0.098820	0.101188	1.392429	0.454912	0.452931
02	1.579220	0.093577	0.103589	1.439120	0.453450	0.456408
03	1.560705	0.089424	0.112444	1.484580	0.448540	0.457150
04	1.575962	0.095038	0.103961	1.525346	0.452607	0.456802
05	1.565345	0.103655	0.095464	1.468601	0.455882	0.452770
06	1.570637	0.086438	0.112133	1.433872	0.449030	0.460255
07	1.531894	0.109956	0.096656	1.344693	0.452540	0.447162
08	1.576973	0.101958	0.096176	1.410652	0.455451	0.453879
09	1.567528	0.094746	0.102420	1.406203	0.451545	0.453021
10	1.560764	0.093522	0.108530	1.459255	0.448647	0.456727
Mean	1.565110	0.096713	0.103256	1.436475	0.452260	0.454711
Std.	0.013519	0.006999	0.006245	0.051342	0.002785	0.003582

3. Numerical results

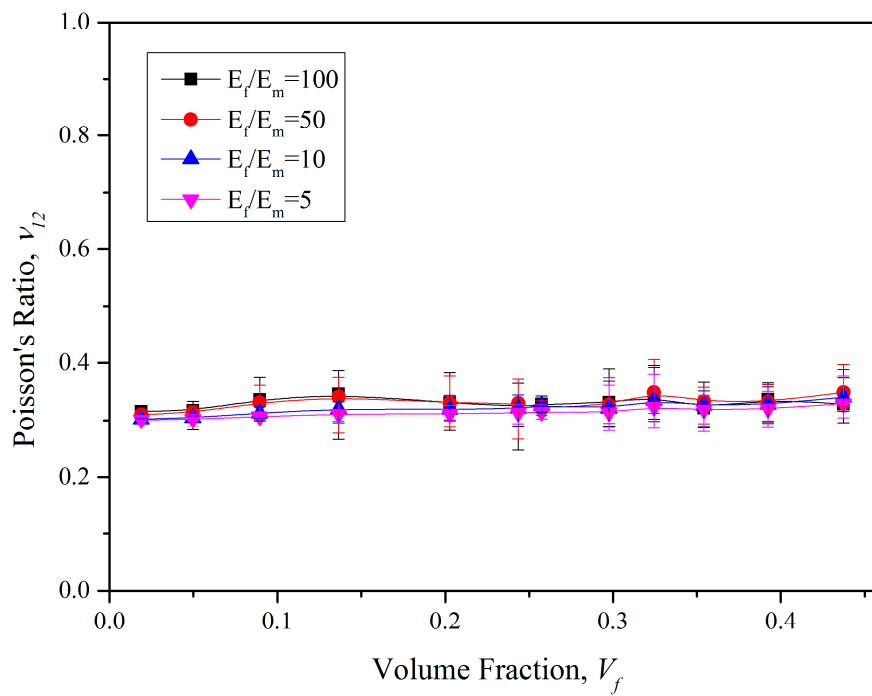
3.1. Elastic behaviours of fibre network composites

By using periodic boundary conditions and imposing a tensile or shear strain of 1‰ to the RVEs with 200 complete fibres, aspect ratio $L/d=100$, the same Poisson's ratios $\nu_f = \nu_m =$

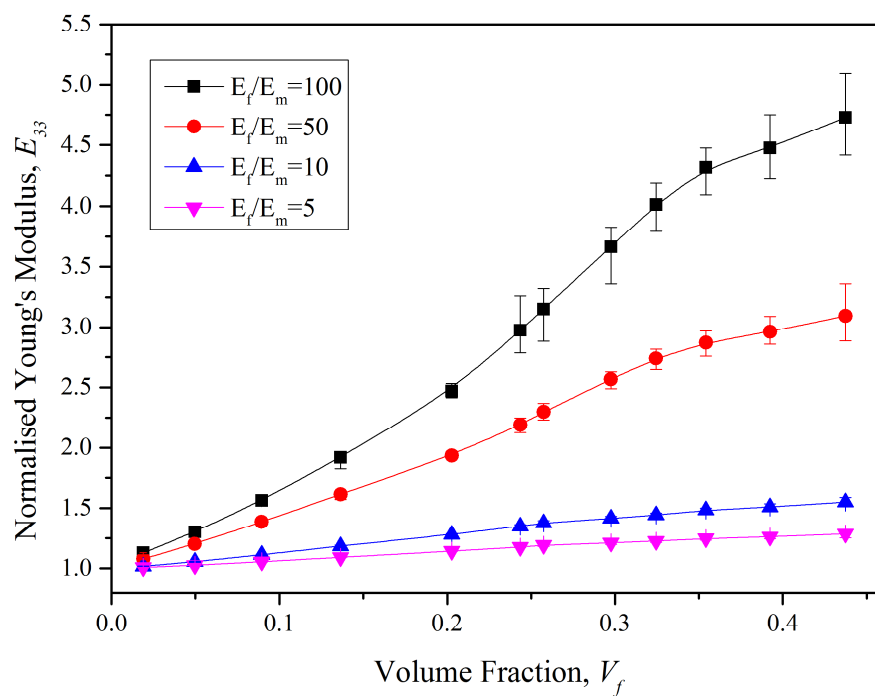
0.3 and various values of E_f/E_m (=100, 50, 10, 5), the results of the five independent elastic constants in terms of fibre volume fraction, respectively, have been obtained and shown in Fig. 5, where E_{11} , E_{33} and G_{31} are normalised by E_m . As can be seen, the in-plane Young's modulus E_{11} , out-of-plane Young's modulus E_{33} and shear modulus G_{31} all increase as the fibre volume fraction increases, which indicates that both tensile and shear stiffnesses can be improved by raising the volume fraction of the fibre network. Specifically, E_{11} shows a linear relation with the fibre volume fraction V_f while E_{33} appears as a quadratic function of V_f when the fibre volume fraction V_f is still less than 0.4, and then becomes a linear function of V_f . G_{13} indicates a similar relationship with the volume fraction as E_{33} . In terms of Poisson's ratio, it can be seen from Fig. 5 (b) and (d) that ν_{12} slightly fluctuates around 0.3, which is about the same value as ν_f or ν_m , for different fibre volume fractions while ν_{31} decreases as the fibre volume fraction increases. In addition, there is no doubt that E_{11} , E_{33} and G_{31} are increased with larger value of E_f/E_m . However, ν_{12} seems not affected by changing the value of E_f/E_m whereas ν_{31} decreases with the increase of E_f/E_m and V_f . In the case when both the value of E_f/E_m and volume fraction V_f become sufficiently large, ν_{31} tends to reach 0, which suggests that the out-of-plane tension/compression introduces almost no effect on in-plane expansion under this condition. However, this may not be true because if solid elements are used to model the fibres when V_f is very large, the value of ν_{31} should be largely dependent on the Poisson ratio of the fibre material ν_f .



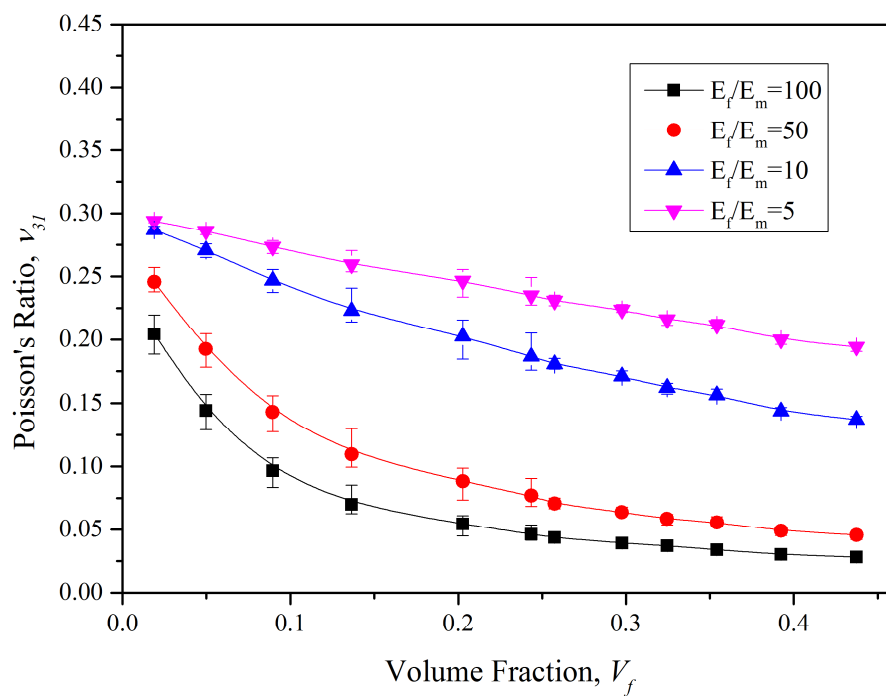
(a)



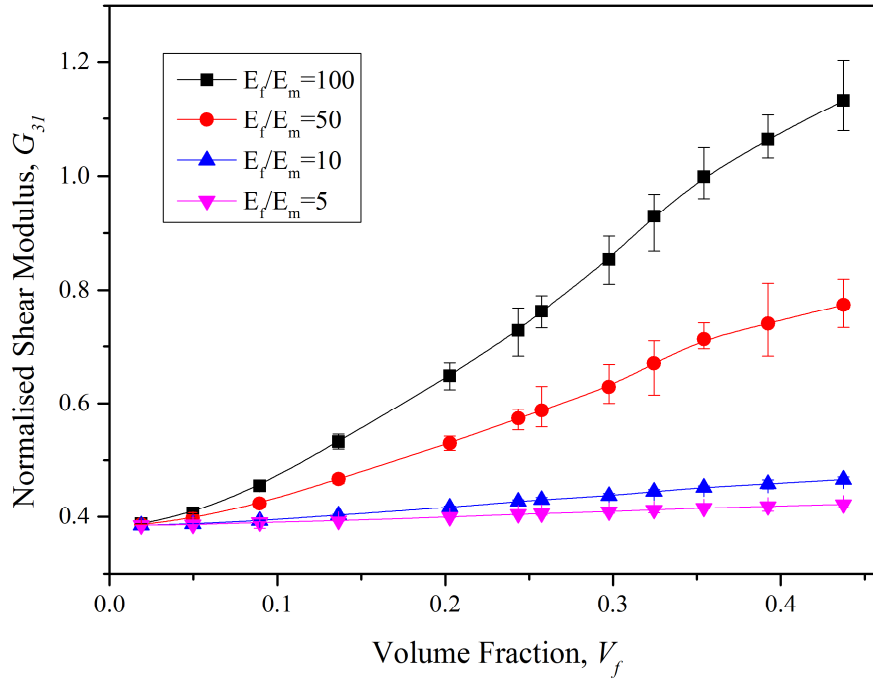
(b)



(c)



(d)



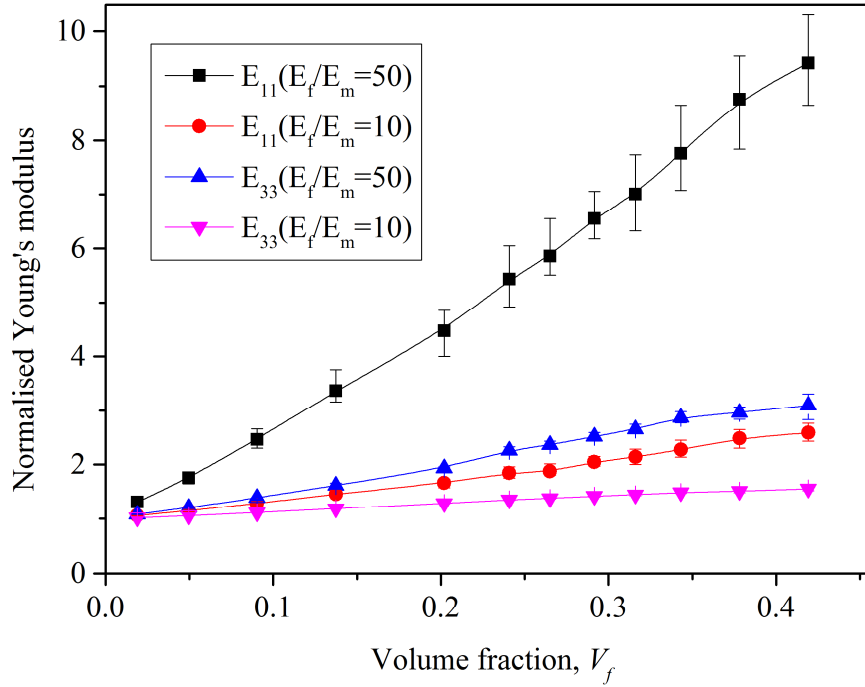
(e)

Fig. 5. Effects of fibre-network volume fraction on (a) in-plane Young's modulus E_{11} , (b) in-plane Poisson's ratio ν_{12} , (c) out-of-plane Young's modulus E_{33} , (d) out-of-plane Poisson's ratio ν_{31} and (e) out-of-plane shear modulus G_{31} of composites with the aspect ratio $L/d = 100$ and same Poisson's ratios $\nu_f = \nu_m = 0.3$. All the Young's moduli and shear moduli are normalised by E_m .

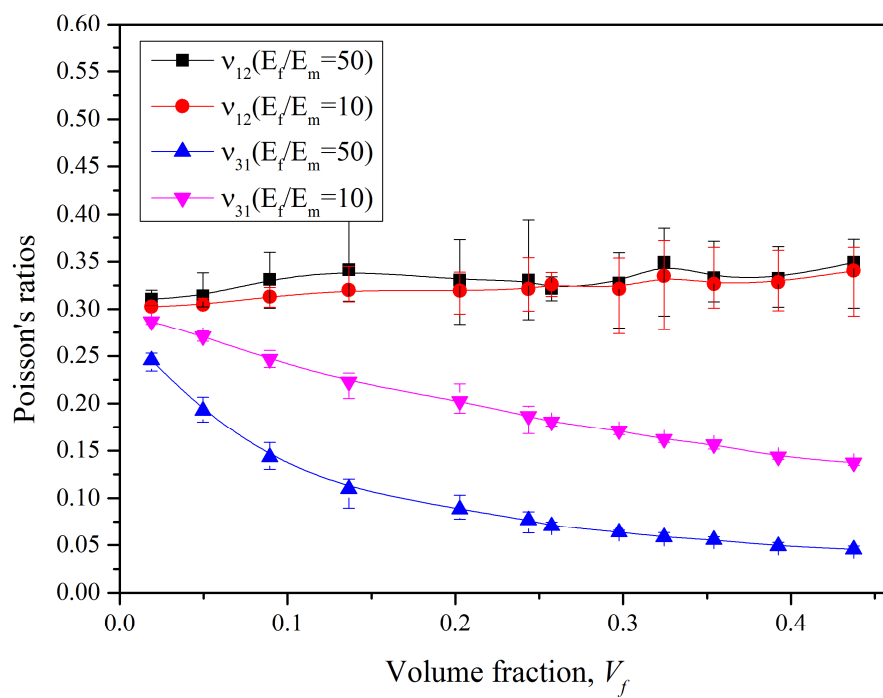
3.2. Comparison of the in-plane and out-of-plane elastic properties

Figure 6 presents the re-organised data from Fig. 5 for composites with Poisson's ratios $\nu_f = \nu_m = 0.3$ and the ratio of $E_f/E_m = 50$ and 10, respectively. Also, E_{11} , E_{33} and G_{31} are normalised by E_m . The results in Fig. 6 indicate that the in-plane Young's modulus E_{11} is higher than the out-of-plane Young's modulus E_{33} . Moreover, the larger the volume fraction is, the bigger the difference between the in-plane Young's modulus and out-of-plane Young's modulus is. The in-plane Young's modulus can be 3 times the out-of-plane Young's modulus

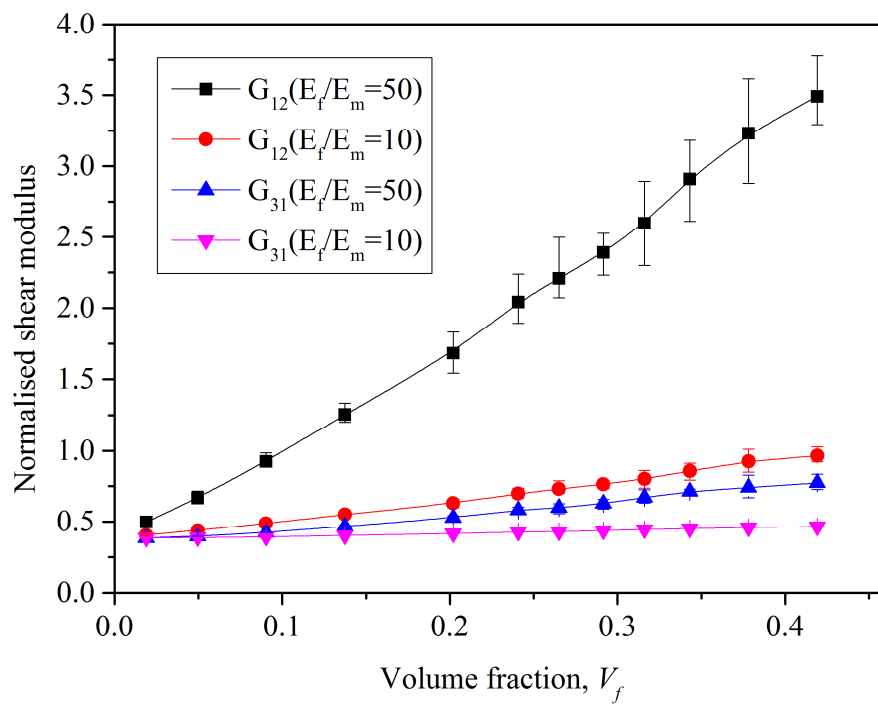
when the fibre volume fraction reaches approximately 50% and the ratio of $E_f/E_m = 50$ (see Fig. 6(a)). Fig. 6 (b) shows that the out-of-plane Poisson's ratio ν_{31} is always smaller than the in-plane Poisson's ratio ν_{12} and the difference between the out-of-plane and in-plane Poisson's ratios is getting larger as the volume fraction increases since the in-plane Poisson's ratio remains constants whereas the out-of-plane Poisson's ratio decreases with the increase of the fibre volume fraction. Besides, the in-plane shear modulus G_{12} and out-of-plane shear modulus G_{31} are also compared in Fig. 6(c). It can be seen that the in-plane shear modulus is also always larger than the out-of-plane shear modulus and, for instance, the in-plane shear modulus is almost 5 times the out-of-plane shear modulus when the fibre volume fraction reaches approximately 50% and the ratio of $E_f/E_m = 50$.



(a)



(b)



(c)

Fig. 6. Comparison of the in-plane and out-of-plane elastic properties of composites with $E_f/E_m = 10$ and 50: (a) in-plane and out-of-plane Young's moduli, (b) in-plane and out-of-plane Poisson's ratios, (c) in-plane and out-of-plane shear moduli. All the Young's moduli and shear moduli are normalised by E_m .

3.3. Effect of Poisson's ratio on the elastic properties

Poisson's ratio is a crucial parameter for the mechanical properties of composites [11, 36]. The effective elastic properties of fibre-reinforced composites are significantly dependent on the Poisson ratios of fibres and matrix. It is well known that the Poisson ratios of most conventional solid materials range from 0.1 to 0.4 and this range can be extended to (-1, 0.5) for some isotropic materials or designed structures. For instance, re-entrant open-celled foams could have a Poisson's ratio close to -1; rubber and low density open-celled foams possess a Poisson's ratio close to 0.5 [36].

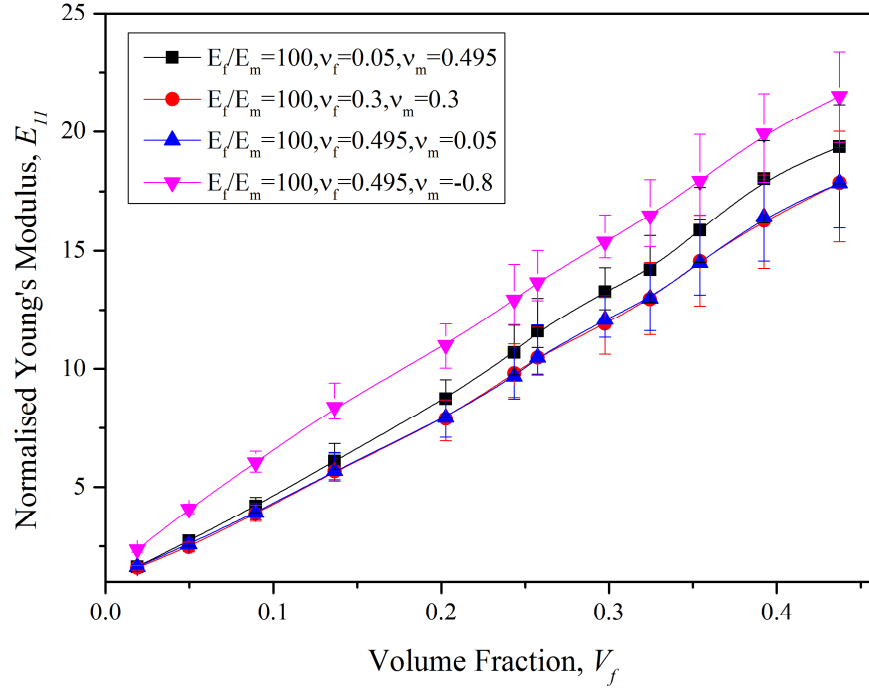
In order to explore the influence of Poisson's ratio alone on the elastic properties of the composites, the ratio of E_f/E_m is kept constant (e.g. 100 here) while different combinations of Poisson's ratios, either positive or negative, are adopted (i.e. $v_f = 0.05$ & $v_m = 0.495$, $v_f = 0.3$ & $v_m = 0.3$, $v_f = 0.495$ & $v_m = 0.05$ and $v_f = 0.495$ & $v_m = -0.8$). The effects of the Poisson ratios on the relationships between E_{11} , v_{12} , E_{33} , v_{31} and G_{31} , respectively, and the fibre volume fraction are shown in Fig. 7 (a)~(e), where E_{11} , E_{33} and G_{31} are normalised by E_m .

For the in-plane Young's modulus E_{11} , the proportional increasing tendency seems not affected by the choice of different combinations of the Poisson ratios. Specifically, there is no difference for the situations $v_f = 0.3$ & $v_m = 0.3$ and $v_f = 0.495$ & $v_m = 0.05$, whereas the combination between $v_f = 0.05$ & $v_m = 0.495$ shows a slightly higher value than the former

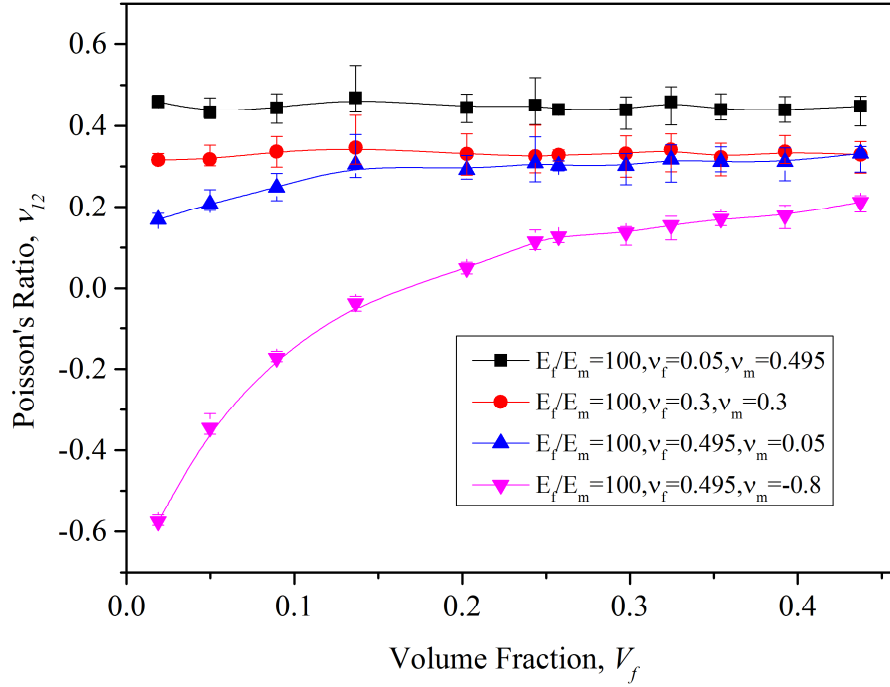
two situations. However, we have also noticed that the choice of negative Poisson's ratio (down triangle dot curve) can remarkably increase the in-plane Young's modulus compared to the combinations between positive Poisson's ratios. This inspires us of a method to enhance the elastic modulus during the material design.

As for the out-of-plane Young's modulus E_{33} , positive Poisson's ratios can also dramatically affect its magnitude, not to say negative Poisson's ratios. It can be seen from Fig. 7(c) that E_{33} with the case of $v_f = 0.05$ & $v_m = 0.495$ indicates a smaller value than that of $v_f = 0.495$ & $v_m = -0.8$ when the volume fraction is less than around 10% and then surpasses and increases faster than the later as the fibre volume fraction arises. Still, the situations when $v_f = 0.3$ & $v_m = 0.3$ and $v_f = 0.495$ & $v_m = 0.05$ demonstrate almost identical results in E_{33} .

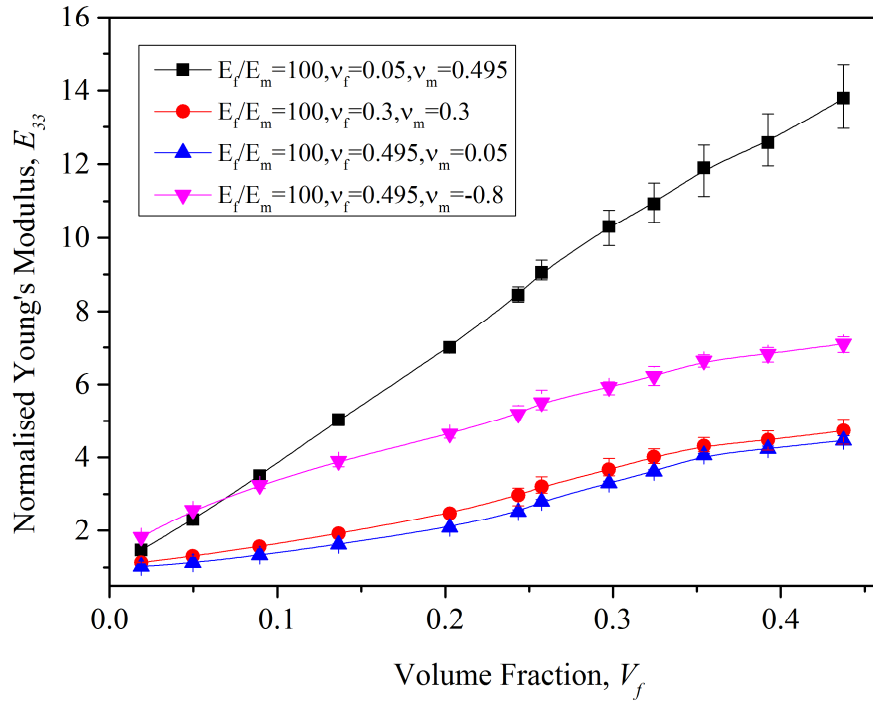
When the in-plane and out-of-plane Poisson's ratios (v_{12} and v_{31}) of the composites are compared, we can see that both are affected by different combinations of fibres and matrix Poisson's ratios. However, v_{12} shows a smaller variety (0.2~0.5) than v_{31} (0~0.5) for positive fibres and matrix Poisson's ratios. For the scenario of composites with negative matrix Poisson's ratio, v_{12} varies from -0.6 to 0.2 while v_{31} ranges from -0.6 to 0. Therefore, we can design the geometry with the expected effective in-plane and out-of-plane Poisson's ratios varying from negative to positive. It is also noticed that the out-of-plane shear modulus G_{31} does not change significantly as the Poisson's ratios change within the positive range whereas negative Poisson's ratios drastically improve G_{31} .



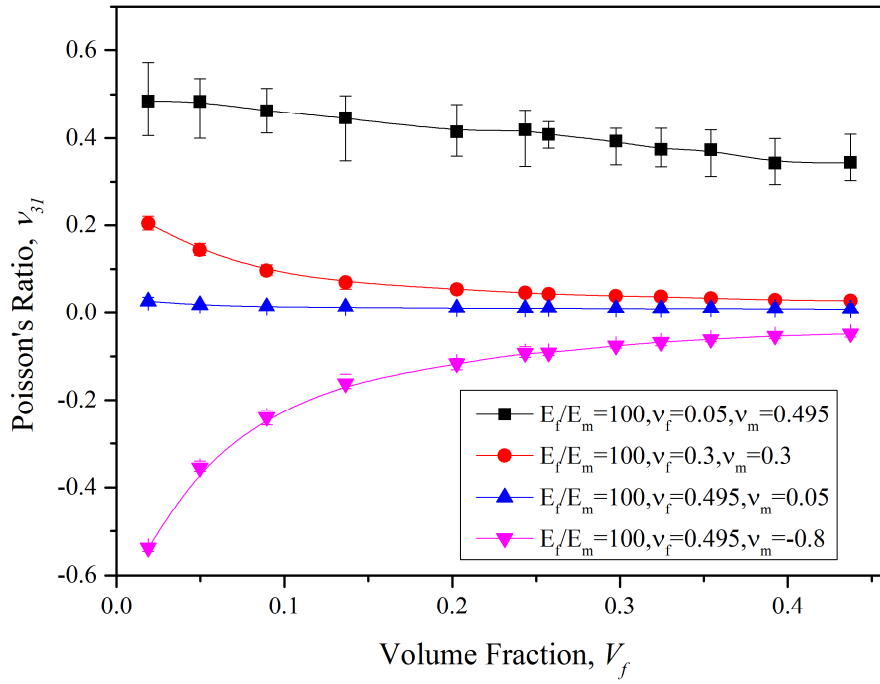
(a)



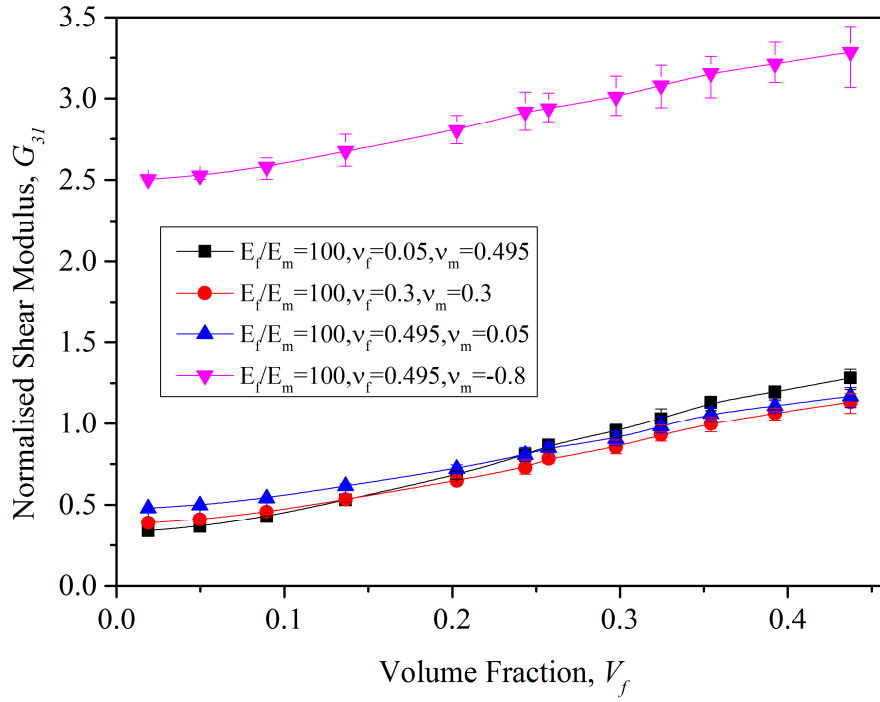
(b)



(c)



(d)



(e)

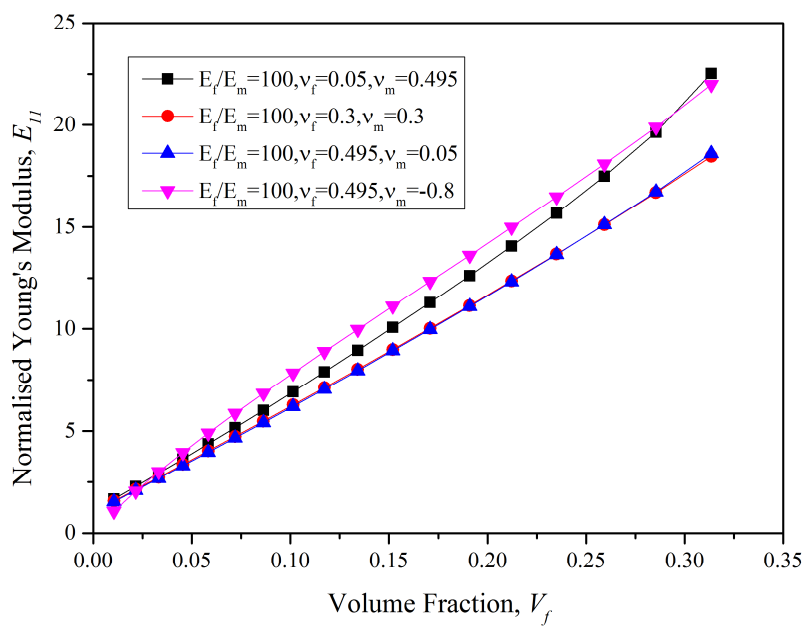
Fig. 7. Effects of fibre volume fraction on the elastic properties (a) E_{11} , (b) ν_{12} , (c) E_{33} , (d) ν_{31} and (e) G_{31} of composites with different combinations of Poisson's ratios. All the Young's moduli and shear moduli are normalised by E_m .

4. Analytical results

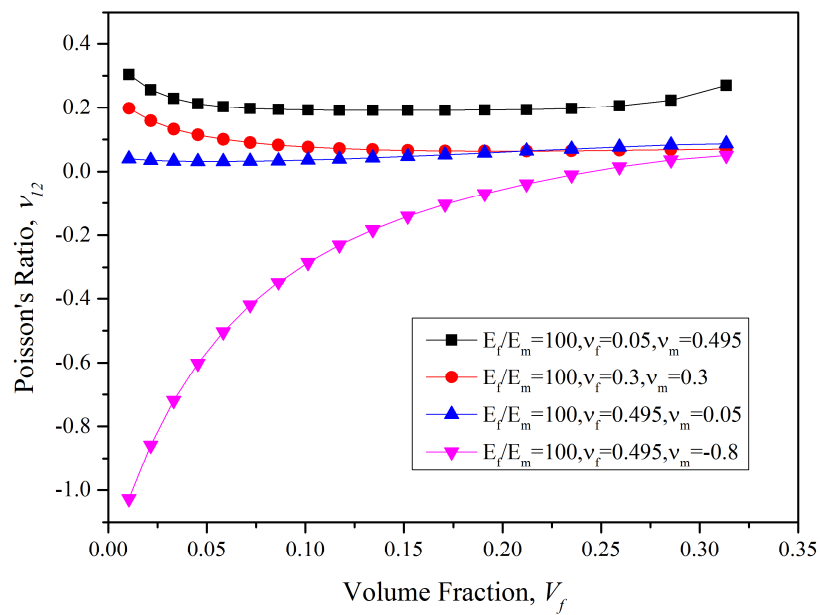
Based on the simplified geometry model (see Figures A1 and A2 in the Appendix) and by application of the fixed value of $E_f/E_m = 100$ and different combinations of the Poisson ratios (i.e. $\nu_f = 0.05$ & $\nu_m = 0.495$, $\nu_f = 0.3$ & $\nu_m = 0.3$, $\nu_f = 0.495$ & $\nu_m = 0.05$ and $\nu_f = 0.495$ & $\nu_m = -0.8$), the analytical results for the relationships of E_{11} , ν_{12} , E_{33} and ν_{31} are obtained, respectively, in terms of the fibre volume fraction in Fig. 8 (a)~(d).

On the whole, the analytical results in Fig. 8 agree well with the simulation results in Fig. 7 in respect of the trend of each curve and the relative relation among curves under different

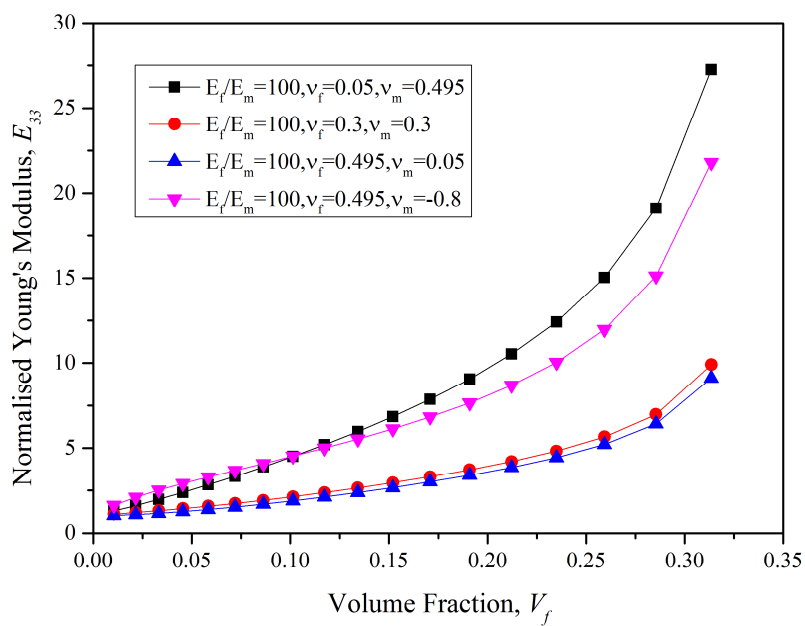
combinations of Poisson's ratios. For example, both E_{11} and E_{33} , when $v_f = 0.3$ & $v_m = 0.3$ and $v_f = 0.495$ & $v_m = 0.05$ are applied separately, have shown almost identical values; E_{33} under the case of $v_f = 0.05$ & $v_m = 0.495$ indicates a smaller value than that of $v_f = 0.495$ & $v_m = -0.8$ when the volume fraction is less than around 10% and then surpasses the later as the volume fraction arises; all the elastic moduli increase with the fibre volume fraction. However, it is also noted that the numerical and analytical results do have some disagreement, especially for the relative relations when the volume fraction is very large (i.e. larger than around 25%) or very small (i.e. less than around 5%). Besides, the analytical results in Fig. 8(c) have revealed that E_{33} increases as a linear relation with the fibre volume fraction when $v_f < 0.15$, and then becomes a parabolic function when v_f is larger, while the simulation results of E_{33} always remains an approximate linear relation with v_f . In general, the numerical results agree with the analytical results on condition that the volume fraction is neither too large nor too small and the numerical results can be reliable in predicting the trend and relation between the elastic properties and volume fraction under the influence of Poisson's ratios.



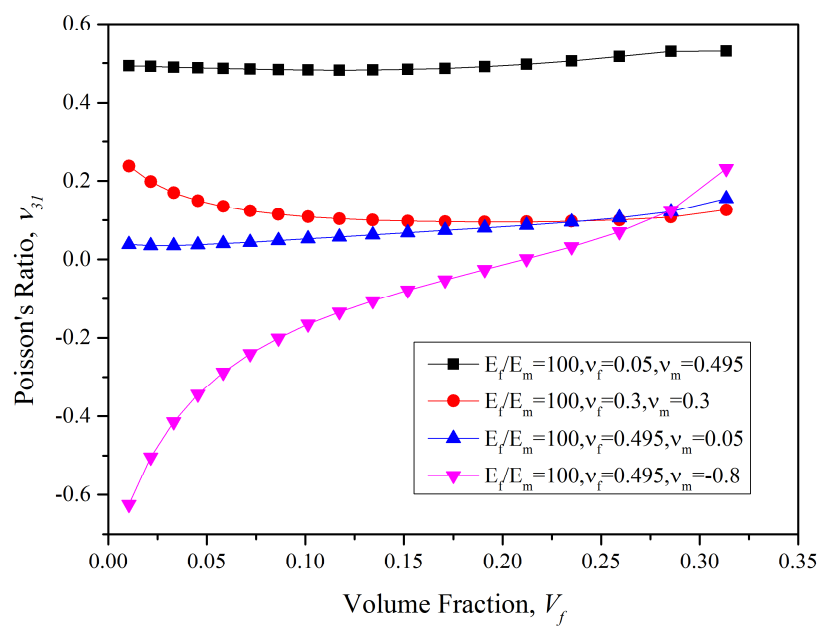
(a)



(b)



(c)



(d)

Fig. 8. Analytical results of the effects of Poisson's ratios on the effective elastic properties of composites (a) E_{11} ; (b) ν_{12} ; (c) E_{33} ; (d) ν_{31} . All the Young's moduli and shear moduli are normalised by E_m .

5. Discussion

In order to demonstrate the superior elastic properties of this new type of 3D transversely isotropic fibre-network reinforced composites, we compared the in-plane and out-of-plane Young's moduli with the experimental [10, 37-40] and numerical [2, 3, 41-43] results of other conventional fibre or particle composites (see Table 3 and Fig. 9). When compared to the simulation results of two transversely isotropic fibre composites without any intersections among the fibres, one with inclined randomly distributed short straight fibres [3] and the other with curved planar randomly distributed short fibres [41], both the in-plane and out-of-plane stiffnesses of the proposed composite indicate significantly larger values. Further comparison with the cross-ply composites [37] has been conducted and our designed composites still demonstrates superior in-plane stiffness to the later. Besides, the novel fibre-network composites demonstrates much larger in-plane stiffness than particle composites (Glass/epoxy [39] and Particle/matrix [42]). These results verified the expectation of the elastic properties of this novel structure, that is, with the intersections among fibres, the network can greatly enhance the stiffness of the composites.

The in-plane Young's modulus of our proposed composite is also compared with both the experimental results [38] and FEA results [2] where all the fibres in the composites are randomly distributed in parallel to the transverse plane (i.e. the x-y plane). By applying the same materials properties ($E_f=75\text{GPa}$, $E_m=1.6\text{GPa}$, $\nu_f=0.25$ and $\nu_m=0.35$) as given in [2], the relationship between E_{11} and the fibre volume fraction of our new type of composites has been obtained and demonstrated in Fig. 9 together with the experimental results and FEA results for

comparison. All the results have demonstrated an approximately proportional tendency, which is consistent with the numerical results of E_{11} shown in Fig. 5(a). As can be seen in Fig. 9, the values of the in-plane Young's modulus of our proposed composite are larger than the experimental results [38] and FEA results [2] under the same volume fraction. It should be noted that all fibres are straight and planar randomly distributed in [2] and [38] whereas the fibres are curved and the fibre segments are inclined out of the transverse plane in our fibre-network composite. Similarly, the transversely isotropic composite architecture studied in [40] (experimental study) and [43] (numerical analysis) is composed of fibres which are physically overlaid on each other [43] and intersections among fibres are ignored. The in-plane stiffness of our proposed composite also exhibits a larger value than both the experimental and numerical results. In addition, the proposed composite has been compared with the similar composite reinforced by a fibre network mat [10]. As shown in Fig. 9, the proposed fibre network composite still illustrates larger stiffnesses. This is possibly due to the difference in in-plane curvatures of fibres, which are straight in the proposed model and curved in [10]. This is consistent with a conclusion drawn in [43] that the Young's modulus decreases as the fibre curvature increases.

To conclude, the reason why our composite structure has a larger stiffness can be attributed to the introduction of cross-linkers between the fibres in the composites. Besides, there is no doubt that the cross-linkers along the out-of-plane direction in the fibre-network composites also render a superior out-of-plane stiffness to planar random fibre composites. Therefore, it is conjectured that both the in-plane and the out-of-plane stiffnesses of our new type of composite are superior to those of planar random fibre composites.

Table 3. Stiffness comparison between this research and others' experimental and numerical results.

Composites	$V_f(\%)$	E_f (GPa)	E_m (GPa)	ν_f	ν_m	Stiffness E_{11} (GPa)	Stiffness E_{33} (GPa)
Cross-ply [37]	43	193	0.7	0.3	0.3	29	-
This research	41.9	193	0.7	0.3	0.3	33.36	-
Short fibre [3]	13.5	70	3	0.2	0.35	6.8656	5.7658
This research	13.7	70	3	0.2	0.35	10.2261	7.1698
Short curved fibre [41]	35.1	70	3	0.2	0.35	14.47	9.49
This research	34.3	70	3	0.2	0.35	17.15	12.31
Glass/epoxy [39]	31	69	3	0.15	0.35	5.3	-
This research	32	69	3	0.15	0.35	10.3765	-
Particle/matrix [42]	20	450	70	0.17	0.3	96	-
This research	20.2	450	70	0.17	0.3	105.4307	-

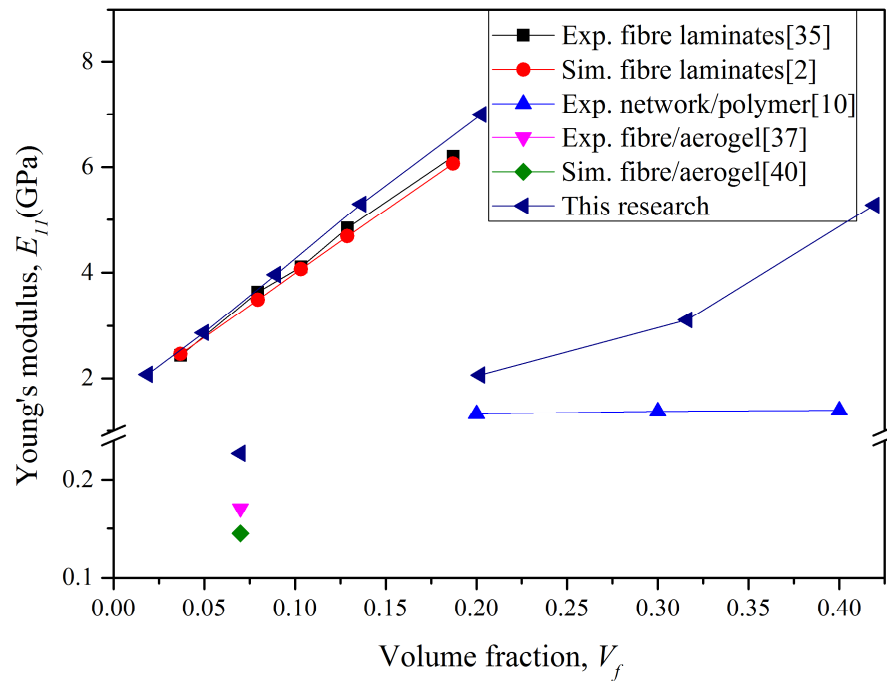


Fig. 9. Comparison of several results of Young's modulus E_{11} in terms of volume fraction.

6. Conclusions

A novel transversely isotropic composites reinforced by a self-connected fibre network has been successfully constructed and simulated to obtain the elastic properties. The simulation results are compared to the analytical results and other relevant experimental and FEA results. It is found that both the in-plane and out-of-plane stiffnesses of the new type of composites are superior to those of other types of transversely isotropic fibre-reinforced composites in which the fibres are not self-connected. It is also found that the combination of the Poisson ratios of the constituent materials could significantly affect the overall elastic modulus and Poisson's ratio of the composites. The analytical exploration of the simplified model has also shown a good agreement with the numerical results under moderate fibre volume fractions. Another advantage of this new type of composites lies in that the self-connected fibre-network, as a whole single ply, can dramatically minimise the delamination among fibres and thus prevent crack initiation and propagation. As a plate structure, the thickness of the fibre network composite is adjustable and can be tailored according to the dimensions and mechanical behaviours demanded in industry. The new structure can also simplify the manufacturing process while maintaining improved mechanical behaviours especially in the through-thickness direction.

Appendix: An Analytical Model

A1 Geometrical and mechanical model

Based on the simulation results of the elastic properties of the fibre network reinforced composites, we also aim to obtain analytic results for comparison. Since the fibres are randomly distributed, it increases the complexity and difficulty of deducing the theoretical expressions,

not to mention the structures with two phases. Therefore, for simplification and similarity, a simplified scaffold alike model has been proposed for analysis as shown in Fig. A1. The fibre network consists of several layers of fibres that are in parallel to the x-y plane, in which half of the fibres are oriented in the x direction and the other half in the y direction respectively. Moreover, the connected fibres are overlapped to some extent which is determined by the overlap coefficient c . Also, the cross section of each fibre is set as a square with side length of d for the sake of predigesting analysis and the error caused by the cross section difference is likely to be neglectable when the fibres are slender (i.e. the aspect ratio of fibre is sufficiently enough). Therefore, the overlap thickness between two fibres will be cd . For a geometry model with fibre length of L and cross-linking concentration of N_c , the length of each fibre segment will be $l_c = L/N_c$. By this way, a regular fibre network with cross-linking has been generated and the volume fraction of fibres can be controlled by adjusting the values of N_c and c . Then the matrix fills in the gap of the fibre network in three dimensions to make it a complete composite structure. Although the simplified geometry model is not strictly transversely isotropic as the fibres are along either the x direction or the y direction, the mechanism of deformation under axial loading is still similar and can be referential to this type of fibre network reinforced composites, including the geometry model we proposed with stochastic fibres.

A 3D schematic of a fibre-matrix composite unit cell. The unit cell is a rectangular prism with dimensions d (height), $d/2$ (width), and l_c (length). The top half is labeled "Fibre" and the bottom half is labeled "Matrix". The unit cell is divided into six layers by dashed yellow lines. The stress components are labeled as follows: σ_{z4} (top fibre surface), σ_{z2} (top fibre surface), σ_{z3} (fibre-matrix interface), σ_{z1} (fibre-matrix interface), σ_{x6} (left fibre surface), σ_{x4} (left fibre surface), σ_{x5} (left fibre surface), σ_{x4} (left fibre surface), σ_{x6} (left fibre surface), σ_{x1} (left matrix surface), σ_{x2} (left matrix surface), σ_{x3} (left matrix surface), σ_{x2} (left matrix surface), σ_{x1} (left matrix surface), σ_{y2} (right fibre surface), σ_{y5} (right fibre surface), σ_{y6} (right fibre surface), σ_{y5} (right fibre surface), σ_{y2} (right fibre surface), σ_{y1} (right matrix surface), σ_{y3} (right matrix surface), σ_{y4} (right matrix surface), σ_{y3} (right matrix surface), σ_{y1} (right matrix surface).

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with square cross section represent fibres and the rest light green block represents the matrix. Besides, due to the existing verlap between the connected fibres, which renders the cross section of fibres more complex at the joints, the whole RVE has to be devided into 20 blocks as indicated with dash lines in Fig. A2. The interfaces between fibres and matrix are assumed to be perfectly bonded and we only consider the normal stresses within the 20 cuboids and the compatibility conditions on the outer surfaces while ignoring the shear stresses and the compatibility conditions on the interfaces of the blocks [36]. Thus when a uniaxial load is applied in the x, or y, or z direction, only the three normal stresses on the surface of each bock will be taken into account and the three normal stresses inside of each cuboid are assumed to be constants. The RVE is not only periodic, but also symmetrical in the z direction. Therefore there are 6 different normal stresses (i.e. $\sigma_{x_1}, \sigma_{x_2}, \sigma_{x_3}, \sigma_{x_4}, \sigma_{x_5}$ and σ_{x_6}) in the x direction, 6 different normal stresses (i.e. $\sigma_{y_1}, \sigma_{y_2}, \sigma_{y_3}, \sigma_{y_4}, \sigma_{y_5}$ and σ_{y_6}) in the y direction and 4 different normal stresses (i.e. $\sigma_{z_1}, \sigma_{z_2}, \sigma_{z_3}$ and σ_{z_4}) in the z direction as labelled in Fig. A2. when an axial force/displacement is loaded, either in the x direction or in the z direction. In elastic study, the normal stress-strain relations for the blocks in series can be expressed as follows according to Hook's law

1) Normal stress-strain relations in the x direction:

$$\frac{(l_c - d)}{l_c E_m} (\sigma_{x_1} - v_m \sigma_{y_1} - v_m \sigma_{z_1}) + \frac{d}{l_c E_f} (\sigma_{x_1} - v_f \sigma_{y_2} - v_f \sigma_{z_2}) = \varepsilon_x \quad (A1)$$

$$\frac{(l_c - d)}{l_c E_m} (\sigma_{x_2} - v_m \sigma_{y_3} - v_m \sigma_{z_1}) + \frac{d}{l_c E_f} (\sigma_{x_2} - v_f \sigma_{y_5} - v_f \sigma_{z_2}) = \varepsilon_x \quad (A2)$$

$$\frac{(l_c - d)}{l_c E_m} (\sigma_{x_3} - v_m \sigma_{y_4} - v_m \sigma_{z_1}) + \frac{d}{l_c E_m} (\sigma_{x_3} - v_m \sigma_{y_6} - v_m \sigma_{z_2}) = \varepsilon_x \quad (A3)$$

$$\frac{(l_c - d)}{l_c E_f} (\sigma_{x_4} - v_f \sigma_{y_3} - v_f \sigma_{z_3}) + \frac{d}{l_c E_f} (\sigma_{x_4} - v_f \sigma_{y_5} - v_f \sigma_{z_4}) = \varepsilon_x \quad (A4)$$

$$\frac{(l_c - d)}{l_c E_f} (\sigma_{x_5} - v_f \sigma_{y_4} - v_f \sigma_{z_3}) + \frac{d}{l_c E_f} (\sigma_{x_5} - v_f \sigma_{y_6} - v_f \sigma_{z_4}) = \varepsilon_x \quad (A5)$$

$$\frac{(l_c - d)}{l_c E_m} (\sigma_{x_6} - v_m \sigma_{y_1} - v_m \sigma_{z_3}) + \frac{d}{l_c E_f} (\sigma_{x_6} - v_f \sigma_{y_2} - v_f \sigma_{z_4}) = \varepsilon_x \quad (A6)$$

2) Normal stress-strain relations in the y direction:

$$\frac{(l_c - d)}{l_c E_m} (\sigma_{y_1} - v_m \sigma_{x_1} - v_m \sigma_{z_1}) + \frac{d}{l_c E_m} (\sigma_{y_1} - v_m \sigma_{x_6} - v_m \sigma_{z_3}) = \varepsilon_y \quad (A7)$$

$$\frac{(l_c - d)}{l_c E_f} (\sigma_{y_2} - v_f \sigma_{x_1} - v_f \sigma_{z_2}) + \frac{d}{l_c E_f} (\sigma_{y_2} - v_f \sigma_{x_6} - v_f \sigma_{z_4}) = \varepsilon_y \quad (A8)$$

$$\frac{(l_c - d)}{l_c E_m} (\sigma_{y_3} - v_m \sigma_{x_1} - v_m \sigma_{z_1}) + \frac{d}{l_c E_f} (\sigma_{y_3} - v_f \sigma_{x_4} - v_f \sigma_{z_3}) = \varepsilon_y \quad (A9)$$

$$\frac{(l_c - d)}{l_c E_m} (\sigma_{y_4} - v_m \sigma_{x_3} - v_m \sigma_{z_1}) + \frac{d}{l_c E_f} (\sigma_{y_4} - v_f \sigma_{x_5} - v_f \sigma_{z_3}) = \varepsilon_y \quad (A10)$$

$$\frac{(l_c - d)}{l_c E_f} (\sigma_{y_5} - v_f \sigma_{x_2} - v_f \sigma_{z_6}) + \frac{d}{l_c E_f} (\sigma_{y_5} - v_f \sigma_{x_4} - v_f \sigma_{z_4}) = \varepsilon_y \quad (A11)$$

$$\frac{(l_c - d)}{l_c E_m} (\sigma_{y_6} - v_m \sigma_{x_3} - v_m \sigma_{z_2}) + \frac{d}{l_c E_f} (\sigma_{y_6} - v_f \sigma_{x_5} - v_f \sigma_{z_4}) = \varepsilon_y \quad (A12)$$

3) Normal stress-strain relations in the z direction:

$$\begin{aligned} & \frac{1}{E_m} [2d(1-c)\sigma_{z_1} - (d-2cd)v_m\sigma_{x_1} - 2cdv_m\sigma_{x_2} - (d-2cd)v_m\sigma_{x_3} \\ & - (d-2cd)v_m\sigma_{y_1} - 2cdv_m\sigma_{y_3} - (d-2cd)v_m\sigma_{y_4}] = 2d(1-c)\varepsilon_z \end{aligned} \quad (A13)$$

$$\begin{aligned} & \left(\frac{d}{E_f} + \frac{d-2cd}{E_m} \right) \sigma_{z_2} - (d-2cd) \frac{v_f \sigma_{x_1}}{E_f} - 2cd \frac{v_f \sigma_{x_2}}{E_f} - (d-2cd) \frac{v_m \sigma_{x_3}}{E_m} \\ & - (d-2cd) \frac{v_f \sigma_{y_2}}{E_f} - 2cd \frac{v_f \sigma_{y_5}}{E_f} - (d-2cd) \frac{v_m \sigma_{y_6}}{E_m} = 2d(1-c)\varepsilon_z \end{aligned} \quad (A14)$$

$$\left(\frac{d}{E_f} + \frac{d-2cd}{E_m} \right) \sigma_{z_3} - 2cd \frac{v_f \sigma_{x_4}}{E_f} - (d-2cd) \frac{v_f \sigma_{x_5}}{E_f} - (d-2cd) \frac{v_m \sigma_{x_6}}{E_m}$$

$$-(d - 2cd)\frac{v_m\sigma_{y_1}}{E_m} - 2cd\frac{v_f\sigma_{y_3}}{E_f} - (d - 2cd)\frac{v_f\sigma_{y_4}}{E_f} = 2d(1 - c)\varepsilon_z \quad (A15)$$

$$\frac{1}{2d(1 - c)E_f} [2d(1 - c)\sigma_{z_4} - 2cdv_f\sigma_{x_4} - (d - 2cd)v_f\sigma_{x_5} - (d - 2cd)v_f\sigma_{x_6} - (d - 2cd)v_f\sigma_{y_2} - 2cdv_f\sigma_{y_5} - (d - 2cd)v_f\sigma_{y_6}] = \varepsilon_z \quad (A16)$$

In the case of strain loading in the x direction, which means ε_x is given, periodic boundary conditions of the RVE require zero total force in the y and z directions, as given by

$$(l_c - d)(d - 2cd)\sigma_{y_1} + d(d - 2cd)\sigma_{y_2} + 2cd(l_c - d)\sigma_{y_3} + (l_c - d)(d - 2cd)\sigma_{y_4} + 2cd^2\sigma_{y_5} + d(d - 2cd)\sigma_{y_6} = 0 \quad (A17)$$

$$(l_c - d)^2\sigma_{z_1} + d(l_c - d)\sigma_{z_2} + d(l_c - d)\sigma_{z_3} + d^2\sigma_{z_4} = 0 \quad (A18)$$

Thus, the 18 unknown normal stresses and strains, i.e. $\sigma_{x_1}, \sigma_{x_2}, \sigma_{x_3}, \sigma_{x_4}, \sigma_{x_5}, \sigma_{x_6}, \sigma_{y_1}, \sigma_{y_2}, \sigma_{y_3}, \sigma_{y_4}, \sigma_{y_5}, \sigma_{y_6}, \sigma_{z_1}, \sigma_{z_2}, \sigma_{z_3}, \sigma_{z_4}, \varepsilon_y$ and ε_z , can be solved from the above 18 simultaneous equations. Accordingly, the Young's modulus in the x direction can be worked out through

$$E_x = \frac{\sigma_x}{\varepsilon_x} = \frac{(l_c - d)(d - 2cd)\sigma_{x_1} + 2cd(l_c - d)\sigma_{x_2} + (l_c - d)(d - 2cd)\sigma_{x_3}}{2d(1 - c)\varepsilon_x} + \frac{2cd^2\sigma_{x_4} + d(d - 2cd)\sigma_{x_5} + d(d - 2cd)\sigma_{x_6}}{2d(1 - c)\varepsilon_x} \quad (A19)$$

$$v_{xy} = \frac{\varepsilon_y}{\varepsilon_x} \quad (A20)$$

In the anner similar to the case of loading in the x direction, ε_z will be given when a strain load is applied in the z direction. Then the rest 18 unknown normal stresses and strains need to be

solved from 18 simultaneous equations, and the Young's modulus E_z and Poisson ratio

$\nu_{zx} = -\varepsilon_x / \varepsilon_z$ can accordingly be obtained.

Acknowledgements

XL is grateful for the PhD studentship from the School of Engineering, Cardiff University and the financial support from China Scholarship Council.

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