# INNOVATION, R&D SPILLOVERS, AND THE VARIETY AND CONCENTRATION OF THE LOCAL INDUSTRY STRUCTURE

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**Abstract**

This paper presents a Cournot oligopoly model with R&D spillovers both within and across industries. The aim is to provide a theoretical foundation for the main hypotheses regarding the impact of the local industry structure on innovation and output. Depending on the spillover rates and the degree of product differentiation between the industries, the firms respond differently to changes in variety and concentration, which can explain the mixed empirical results. Furthermore, innovation and output may react in opposite ways, which makes the choice of the performance measure critical.

*JEL classification*: L13, O31, R11.

*Keywords*: concentration, innovation, knowledge spillover, regional economy, variety.

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# 1 Introduction

Since Glaeser et al. (1992), numerous empirical studies have attempted to determine which local industry structures are most conducive to innovation, in terms of their variety and concentration. That is, whether regional diversity or specialisation, and similarly local competition or concentration, encourages innovation and growth. However, the empirical results have been inconclusive and ambiguous, since the main competing hypotheses have all found some support (Beaudry and Schiffauerova, 2009; De Groot et al., 2009, 2016).

A further difficulty when trying to make sense of these mixed findings is the lack of fully developed theories about how the local industry structure affects innovation. The theoretical foundations may be sought from industrial economics, where there have been many theoretical studies of R&D spillovers and innovation incentives. However, most of these studies have focused on spillovers within a single industry and the desirability of R&D cooperation in this context (see, De Bondt, 1997).

Few studies have considered the simultaneous occurrence of intra- and inter-industry knowledge spillovers. Among them, Steurs (1995) examined the case of a two-industry, two-firm-per-industry model and allowed R&D spillovers to occur both within and between industries. Steurs' model demonstrates that the two spillover channels have different but interdependent effects on R&D, but not how the local industry structure affects the outcome. As noted by several authors, among the micro-foundations of urban agglomeration, learning and knowledge spillovers are the least understood, and thus there is an urgent need to advance theoretical research into localised knowledge spillovers, which should inform empirical research rather than lagging behind it (Duranton and Puga, 2004; Fujita and Krugman, 2004; Puga, 2010).

The present study builds on previous theoretical research by considering the case of non-cooperative Cournot firms, which invest in cost-reducing or demand-enhancing technologies that may spill within and across industries. Steurs' (1995) model is extended to consider several industries, the products of which may be complements, independent, or

imperfect substitutes, as well as several firms within each of industry. The main theoretical contribution is showing how the level of concentration within local industries and the variety of these industries affects innovation and output. The aim is to bring some clarity to the mixed empirical results and the overall implications for regional economic policy.

The results show that whether variety and concentration increase or decrease effective R&D and the industry output depends, in particular, on both spillover rates and the degree of product differentiation. I also show that the inverted-U relationship between competition and innovation may arise due to the combination of inter- and intra-industry spillovers. The marginal effects of variety and concentration on innovation and output may be opposite, which indicates why the choice of the performance measure can affect the empirical results. Similarly, the outcome is shown to differ when a relative rather than an absolute measure of variety is used, both of which are common in empirical research.

The remainder of this paper is organised as follows. Section 2 provides a brief review of theoretical and empirical research into inter- and intra-industry knowledge spillovers. Section 3 presents the theoretical model and its equilibrium analysis. The main research questions concern the comparative statics, particularly how the effective R&D and total industry output respond to changes in the variety and concentration of local industries, which are studied in Section 4. Section 5 considers extensions to the baseline model in terms of relative variety and industry-specific concentration. The modelling choices and heterogeneity between industries are discussed in Section 6, which is followed by the conclusions. All proofs are in the online appendix.

## 2  Literature Review

While Bernstein (1988) was the first empirical study of both intra- and inter-industry R&D spillovers to consider these as proper externalities[1], Glaeser et al. (1992) marked the true

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[1]The effect of inter-industry spillovers was found to be small and the same for all industries. Thus, the extent of intra-industry spillovers was the key factor according to Bernstein (1988).

Table 1: The hypotheses of the factors conducive to innovation and economic growth

|  | Competition | Concentration |
|---|---|---|
| Specialisation | Porter | Marshall-Arrow-Romer |
| Diversity | Jacobs | ? |

beginning of empirical research into this issue and the role of the local industry structure in it. Based on previous theoretical research, they formulated three hypotheses regarding localised knowledge spillovers: Marshall-Arrow-Romer (MAR) externalities, Porter externalities, and Jacobs externalities. As illustrated in Table 1, the differences between the hypotheses concern whether it is the combination of regional specialisation or diversity and competition or concentration within the industries that fosters innovation and growth. Interestingly, Glaeser et al. (1992) did not present a hypothesis, according to which diverse *and* concentrated industry structures would be the best combination. As we will later see, this fourth, missing hypothesis is also a viable perspective.

Glaeser et al. (1992) proceeded to test the hypotheses with U.S. city-level data and found support for Jacobs externalities. Many studies followed and there has been a continuous stream of related empirical research up to the present. For example, a survey by Beaudry and Schiffauerova (2009) analysed the results of 67 studies in this area. Similarly, De Groot et al. (2016) performed a meta-analysis of 73 empirical studies of agglomeration externalities. A major issue identified by these surveys is that the empirical results are very mixed. Depending on the context (e.g., the data, time period and empirical model), support was found for each of the three hypotheses. In particular, the choice of the dependent variable tends to lead to different results.

Glaeser et al. (1992) formulated their hypotheses loosely based on earlier ideas but to the best of my knowledge, a formal model of the underlying mechanisms has never been developed. This is an interesting issue in itself, but the lack of a formal theory also makes it difficult to interpret the mixed empirical findings because we have no clear idea

of how particular circumstances affect the spillover mechanisms. Arguably, these issues have contributed to the dissatisfaction with the specialisation–diversity framework and the subsequent search for alternatives (van Oort, 2015).

Since the geography of innovation literature is largely empirically orientated, the theoretical basis must be sought elsewhere. Following Spence (1984), d'Aspremont and Jacquemin (1988) and Kamien et al. (1992), there has been much theoretical research into (intra-industry) R&D spillovers and innovation incentives in industrial economics. De Bondt et al. (1992) found that effective R&D is increasing in concentration in the case of a single, homogeneous industry. Interestingly, the presence of inter-industry R&D spillovers affects this outcome, as shown in the present study. As such, the combination of the two spillover channels presents a novel context that can generate the inverted-U relationship between innovation and competition, which has generated interest elsewhere (Kamien and Schwartz, 1976; Aghion et al., 2005; Belleflamme and Vergari, 2011; Barbos, 2015).

Even in industrial economics, research into inter-industry spillovers is almost non-existent and most extensions concern R&D cooperation in the context of a single industry. Extra-industry sources of R&D were included in the model proposed by Cohen and Levinthal (1989) but they were taken as exogenous. Later, Katsoutacos and Ulph (1998) and Leahy and Neary (1999) compared cases where firms operate either in the same or different industries, but did not study both inter- and intra-industry spillovers simultaneously. Some studies have also considered spillovers between vertically related firms (Atallah, 2002; Ishii, 2004).

To the best of my knowledge, Steurs (1995) and König et al. (2014) are the only previous studies to consider spillovers between industries, as well as within them. Steurs' model considers the case of two segmented and identical duopolies with different inter- and intra-industry spillover rates. Steurs showed that inter-industry spillovers always increase the effective R&D, but they also decrease the rate of intra-industry spillovers, which

maximises the effective R&D. Thus, inter- and intra-industry spillovers are strategically interdependent. König, Liu and Zenou study R&D networks that include both rival firms and others producing independent products, and in which the spillover rates are the same. They show how a key trade-off between the spillover and rivalry effects arises in the network, and proceed to study optimal network design and R&D subsidies among other issues.

To analyse the effect of the variety and concentration of the local industry structure, the main differences between the earlier studies and the present work is that we consider several firms as well as industries, different inter- and intra-industry spillover rates, and products that may be either complements, substitutes or independent products. R&D cooperation is not of direct interest in this case because the argument behind the three hypotheses is based on involuntary leakage of R&D rather than its deliberate transfer.[2] Thus, instead of addressing R&D cooperation in the present study, the reader is directed to Steurs (1995) and König et al. (2014) for further details. In addition, we do not consider the relationship between location choice and R&D strategies, which has been studied in Gersbach and Schmutzler (1999); Mai and Peng (1999); Baranes and Tropeano (2003); Fosfuri and Rønde (2004); Piga and Poyago-Theotoky (2005) and Leppälä (2016), among others.

## 3   The Model

We consider an agglomeration of $m$ industries, each of which consists of $n$ identical single-product firms that produce a homogeneous output. $q_{ij}$ is the output produced and sold by firm $i$ in industry $j$, and the total output of industry $j$ is $Q_j = \sum_{i=1}^{n} q_{ij}$. In each industry, the firms face a linear inverse demand curve with similar characteristics:

$$P_j = a_j - Q_j - b \sum_{l \neq j} Q_l,$$

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[2]While cooperation is likely to increase the spillover rates and R&D efforts, the empirical evidence suggests that on aggregate formal R&D cooperation accounts only for a small share of all knowledge transfers (Brenner, 2007).

$\forall j, l \in m.$[3] The parameter $b \in (-\frac{1}{m-1}, 1)$ measures the substitutability between the goods produced by the different industries. A negative $b$ implies that the industries produce complements, $b = 0$ implies that the markets are perfectly segmented and that the final outputs are independent, and a positive $b$ implies that the the industries produce imperfect substitutes.

The (initial) unit cost of all firms in industry $j$ is $c_j$. $X_{ij}$ is the firm's effective R&D (output), with $a_j > c_j > X_{ij} \geq 0$. We let $v_j \equiv a_j - c_j$ denote the industry's initial market size. In this context, R&D output can be considered as a cost reduction or equally well as a demand-enhancing invention (De Bondt and Veugelers, 1991). Effective R&D is given by

$$X_{ij} = x_{ij} + \beta \sum_{k \neq i} x_{kj} + \sigma \sum_{l \neq j} \sum x_{il},$$

where $x_{ij}$ is the firm's own R&D output, $\beta \sum_{k \neq i} x_{kj}$ are the output spillovers from the other firms in the same industry, and $\sigma \sum_{l \neq j} \sum x_{il}$ are the spillovers from firms in the other industries. $\beta \in [0, 1]$ is the intra-industry spillover rate and $\sigma \in [0, 1]$ is the inter-industry spillover rate.

For tractability and to follow the earlier studies, the cost of the firm's own R&D output $x_{ij}$ is assumed to be quadratic and given by

$$R(x_{ij}) = \frac{1}{2} \gamma x_{ij}^2,$$

where $\gamma > 0$ is an inverse measure of the efficiency of R&D. I assume that the values of $\beta$ and $\sigma$ are exogenous, where they reflect the extent to which R&D is leaked and useful across firms and different industries. It is further assumed that the $m$ industries are technologically related, such that some beneficial spillovers exist between them (Frenken et al., 2007). As shown by Bernstein and Nadiri (1988), the set of industries bound by

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[3]The inverse demand function can be obtained from a quadratic utility function, which is here strictly concave for the stated range of $b$ (Singh and Vives, 1984; Amir et al., 2017).

spillovers may not be large. Thus, there may be other industries in the same location as well, but they are not related in this sense. It is a highly stylised assumption that the industries are identical in terms of the spillover rates, number of firms and product market relationships, but it facilitates our analysis with respect to the impact of variety and concentration. A departure from the symmetricity is made with respect to industry-specific concentration in subsection 5.2 and other issues are discussed in Section 6.

The firms play a two-stage game. In the first stage, the firms in all industries simultaneously decide their R&D outputs, $x_{ij}$. In the second stage, the firms engage in Cournot competition and choose their final good outputs, $q_{ij}$. For expository reasons, I assume that there is no uncertainty with respect to the R&D output, and discounting between the stages is also ignored. These assumptions are not expected to affect the qualitative properties of the model. We derive the quasi-symmetric subgame-perfect Nash equilibria, where in each industry the firms play the same strategy, by backward induction.

## 3.1 Quasi-Symmetric Equilibria

In the final production stage, firm $i$ in industry $j$ maximises its operative profit given by

$$\pi_{ij} = (P_j - c_j + X_{ij})q_{ij} = (v_j - Q_j - b\sum_{l \neq j} Q_l + X_{ij})q_{ij}. \tag{1}$$

The Cournot equilibrium output is

$$q_{ij} = \left[ u_j + AX_{ij} - B\sum_{k \neq i} X_{kj} - b\sum_{l \neq j}\sum X_{il} \right] \times C^{-1} \tag{2}$$

8

with

$$u_j = (n+1+bn(m-1))v_j - bn\sum_{j=1}^{m} v_j,$$

$$A = n(n(1-b)+1) + bn(m-1)(n(1-b)+b) > 0,$$

$$B = n(1-b)+1+bn(m-1)(1-b) > 0,$$ (3)

$$C = (n(1-b)+1)(n+1+bn(m-1)) > 0,$$

and the equilibrium operative profit becomes

$$\pi_{ij} = (q_{ij})^2.$$ (4)

The derivation of Equations (2) and (4) is given in the online appendix.

In the first stage, the firms choose their R&D levels. Given the subsequent output levels, firm $i$ in industry $j$ chooses $x_{ij}$ in order to maximise its overall profit

$$\Pi_{ij} = \pi_{ij} - R(x_{ij}) = (q_{ij})^2 - \frac{1}{2}\gamma x_{ij}^2,$$

where $q_{ij}$ is given by Equation (2).

The first-order conditions for subgame-perfect Nash equilibria are given by

$$\frac{\partial \Pi_{ij}}{\partial x_{ij}} = 2q_{ij}\frac{D}{C} - \gamma x_{ij} = 0 \text{ with}$$

$$D = A - \beta(n-1)B - \sigma bn(m-1) > 0$$ (5)

and $A, B$ and $C$ defined earlier in (3).[4]

The second-order conditions in the R&D stage require that $\gamma C^2 > 2D^2$. The strategic

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[4]To verify the positivity of $D$, note that it is decreasing in both $\beta$ and $\sigma$ if $b > 0$ and non-decreasing in $\sigma$ otherwise. When $\beta, \sigma = 1$, $D$ reduces to $n(1-b)+1 > 0$. When $\beta = 1$ and $\sigma = 0$, $D$ is equal to $n(1-b)+1+bn(m-1) > 0$.

effects within and across industries are given by

$$\frac{\partial^2 \Pi_{ij}}{\partial x_{ij} \partial x_{kj}} = 2\frac{DF}{C^2} \gtreqless 0 \text{ with}$$

$$F = \beta A - (1 + \beta(n-2))B - \sigma bn(m-1)$$

and

$$\frac{\partial^2 \Pi_{ij}}{\partial x_{ij} \partial x_{ik}} = 2\frac{DG}{C^2} \gtreqless 0 \text{ with}$$

$$G = (A - (n-1)B)\sigma - b(1 + \beta(n-1) + \sigma n(m-2))$$

$$= -b(1 + \beta(n-1)) + \sigma(n+1).$$

(6)

(7)

Clearly, the signs of Equations (6) and (7) are the same as the signs of $F$ and $G$. As is typically the case, the magnitudes of the spillover rates therefore determine whether the R&D choices are strategic substitutes or complements. However, note that $G < 0$ and the inter-industry R&D choices are strategic substitutes only if $b > 0$ and the goods are also substitutes.

The stability condition[5], which requires that the best response functions cross correctly (Henriques, 1990), is given by

$$\gamma C^2 - 2D^2 - 2D \left| (F(n-1) + Gn(m-1)) \right| > 0,$$

(8)

which is a stronger restriction than the second-order condition.

When the firms in each industry make a symmetric choice, $x_{ij} = x_j$, $\forall i \in n, \forall j \in m$, the

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[5]The stability condition is calculated from $\left| (\sum_{kl \neq ij} \partial^2 \Pi_{ij}/(\partial x_{ij} \partial x_{kl}))/(\partial^2 \Pi_{ij}/\partial^2 x_{ij}) \right| = \left| \sum_{kl \neq ij} \partial x_{ij}/\partial x_{kl} \right| < 1$ using the second-order condition and Equations (6) and (7).

best response function (5) yields the following equilibrium R&D output:

$$x_j = \frac{2D}{\gamma C^2 - 2(n(1-b)+1)DE}\left(u_j + \frac{2DGnm}{Z}(\bar{u} - u_j)\right) \text{ with}$$

$$E = 1 + \beta(n-1) + \sigma n(m-1),$$

$$Z = \gamma C^2 - 2D(1 + \beta(n-1) - \sigma n)(n+1+bn(m-1)), \quad (9)$$

$$\bar{u} = \sum u_j/m = (n(1-b)+1)\sum v_j/m,$$

and $D$ and $G$ as defined earlier. The derivation of Equation (9) is given in the online appendix. Given the effective R&D, $X_j = (1 + \beta(n-1))x_j + \sigma n \sum_{l \neq j} x_l$, the equilibrium output in industry $j$ is

$$q_j = \left[u_j + (A - (n-1)B)X_j - bn\sum_{l \neq j} X_l\right] \times C^{-1}. \quad (10)$$

To guarantee that all industries are active in both stages and $q_j, x_j > 0$ in the equilibrium, I make the following assumptions:

**Assumption 1** *The initial size of each market satisfies*

$$v_j > \frac{bn\left(\sum a_j - c_j\right)}{n + 1 + bn(m-1)}$$

*and*

$$u_j \lessgtr \frac{2DGnm}{2DGnm - Z}\bar{u} \leftrightarrow \frac{2DGnm - Z}{Z} \gtrless 0$$

*for all $j \in m$.*

**Assumption 2** *The R&D efficiency is sufficiently low such that $\gamma > \tilde{\gamma}$, where*

$$\tilde{\gamma} \equiv \max\left\{(n(1-b)+1)E, D + |F(n-1) + Gn(m-1)|\right\} \times \frac{2D}{C^2}.$$

Suppose that there exists an industry, the initial market size of which is exactly equal to

11

the average market size among the industries: $\bar{v} = \sum v_j/m$. Using Equation (9), we get the equilibrium R&D output in this representative industry:

$$\bar{x} = \frac{2\bar{v}(n(1-b)+1)D}{\gamma C^2 - 2(n(1-b)+1)DE}.\tag{11}$$

By multiplying Equation (11) by $E$ we find that the average effective R&D is

$$\bar{X} = \frac{2\bar{v}(n(1-b)+1)DE}{\gamma C^2 - 2(n(1-b)+1)DE}.\tag{12}$$

Furthermore, the firm-level equilibrium output is

$$\bar{q} = \frac{(\bar{v}+\bar{X})(n(1-b)+1)}{C} = \frac{\bar{v}+\bar{X}}{n+1+bn(m-1)},\tag{13}$$

where $\bar{X}$ is given by Equation (12). The total industry output is then $\bar{Q} = n\bar{q}$.

While the following analysis focuses on the representative industry, heterogeneity in the initial market size among other possible asymmetries is discussed in Section 6. Besides its analytical convenience, the focus on the representative industry allows us to extend the result to the overall local economy as the aggregate output across the industries is simply $\sum Q_j = m\bar{Q}$.

# 4    Comparative Statics

Rather than to perform a normative analysis, the aim of the paper is to address the mixed empirical findings. A sketch of a welfare analysis is presented in the online appendix and, therefore, this section concentrates on studying the changes in the representative industry's effective R&D and total output. These two are the most relevant with regard to the dependent variables used in empirical research, which are typically measures of innovation, economic growth or productivity (Beaudry and Schiffauerova, 2009).

## 4.1 Effective R&D

The first interesting issue regarding comparative statics concerns the impact of the two spillover rates on the effective R&D (of the representative industry).

**Proposition 1** *Effective R&D increases in the inter-industry spillover rate, $\sigma$, if the industries produce complements or independent goods. For imperfect substitutes, the effective R&D maximising inter-industry spillover rate is given by*

$$\sigma^* = \min\left\{\frac{A - b - \beta(n-1)(B+b)}{2bn(m-1)}, 1\right\}.$$

**Proposition 2** *The intra-industry spillover rate that maximises effective R&D is given by* $\beta^* = \min\left\{\max\left\{0, \tilde{\beta}\right\}, 1\right\}$ *where*

$$\tilde{\beta} = \frac{A - B - \sigma n(m-1)(B+b)}{2B(n-1)}.$$

As noted by Steurs (1995), inter-industry spillovers reinforce the disincentive effect of intra-industry spillovers. However, as may be expected, this depends on the number of industries and firms as well as on the inter-industry spillover rate $\sigma$. It is easy to verify that without inter-industry spillovers, the effective R&D maximising intra-industry spillover rate corresponds to previous results, i.e., $\beta^* = 1/2$, if $b = 0$ but is larger otherwise. However, the effective R&D maximising $\beta$ approaches zero as the inter-industry spillover rate increases. $\beta$ has the same effect on the effective R&D maximising $\sigma$ if the products are imperfect substitutes.

In the case of complements and independent products in particular, the maximal inter-industry spillover rate, $\sigma = 1$, and no intra-industry spillovers, $\beta = 0$, yield the highest effective R&D. Yet, it may be doubtful that the spillover rates can differ from each other so greatly, in particular that the inter-industry spillover rate is (substantially) higher than the intra-industry rate. That is, we might consider it unlikely that firms could benefit more from

external R&D derived from a different industry. To further analyse this trade-off between the spillover sources, we consider the following special case where the two spillover rates are the same.

**Proposition 3** *If the intra- and inter-industry spillover rates are equal, $\beta = \sigma = \phi$, then the common spillover rate that maximises effective R&D is*

  *i*   $\phi^* = \frac{1}{2} \frac{mn^2 - 2n + 1}{(mn-1)(n-1)} \in \left( \frac{1}{2}, 1 \right)$, *with* $\frac{\partial \phi^*}{\partial m} > 0, \frac{\partial \phi^*}{\partial n} < 0$, *for independent products;*

  *ii*   $\phi_c^* \in (\phi^*, 1]$ *for complements; and*

  *iii*   $\phi_s^* \in \left( \frac{1}{2}, \phi^* \right)$ *for imperfect substitutes.*

It is interesting that the optimal spillover rate in this case is on the higher range of spillovers, which is bounded below by $1/2$. Unsurprisingly, the optimal rate approaches 1 as the variety increases because this makes inter-industry spillovers relatively more important. The number of firms has the opposite effect because rivalry becomes more intense[6].

**Proposition 4** *If the products are independent or complements, effective R&D is non-decreasing in the number of industries, m, and strictly increasing if either $b < 0$ or $\sigma > 0$. If the products are imperfect substitutes, then effective R&D can be increasing, decreasing, or inverse U-shaped with respect to m.*

This proposition shows that, in the case of complements and independent products in particular, diversity has a positive impact on innovation. However, it should be noted that even when effective R&D is increasing in all *m*, the set of such technologically related industries and/or complementary products may be quite limited. Furthermore, the case of imperfect substitutes shows that ambiguity with respect to the impact of variety can rise quite naturally. Together with the other factors, the degree of product differentiation can be a cause of the mixed empirical results regarding the variety of industries.

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[6]For analytical convenience, the fact that both *m* and *n* must be an integer has been ignored. On the latter issue, see Amir and Lambson (2007).

**Proposition 5** *If the products are independent or complements, effective R&D is increasing or inverse U-shaped with respect to the number of firms, n. If the products are independent and either $\beta = 1$ or $\sigma = 0$ and $\beta \neq 1/2$, effective R&D is maximised for $n = 1$ (monopoly). If the products are imperfect substitutes, then effective R&D can be increasing, decreasing, or inverse U-shaped with respect to n.*

With respect to competition, the results are even more mixed as various patterns are possible for all product types. Intuitively, an increase in the number of firms has two effects as there is both an increase in rivalry as well as in spillovers. Which of these effects dominates depends on the relative strength of inter- and intra-industry spillovers. This result can be compared with De Bondt et al. (1992), where entry reduces effective R&D in a homogeneous oligopoly but has an ambiguous effect if the products are differentiated.

Figure 1 presents a contour plot, which illustrates how the effective R&D maximising number of firms depends on the spillover rates $\beta$ and $\sigma$. Consider, for example, the mid-right area of the diagram, where the intra-industry rate is very high and the inter-industry rate is moderate. There, as well as in the low-left corner where both spillover rates are low, it is the monopoly market structure that maximises the effective R&D.

Now, if $\beta$ starts to decrease from its maximal rate then the number of firms that maximises the effective R&D increases from one to an intermediate level. These spillover rate combinations generate a similar inverted-U relationship between competition and innovation that has been found in some empirical studies (e.g. Aghion et al., 2005). There is an exponential increase in the effective R&D maximising number of firms until we finally reach the area $n^* = +\infty$ where there is no finite effective R&D maximising $n$. Given these combinations of spillover rates we should observe a positive relationship between competition and innovation.

Alternatively, if the variety is low or intra-industry spillovers are high, then a more concentrated industry maximises the effective R&D. In this case, the negative effect of increased competition on innovation incentives dominates the positive inter-industry spillover
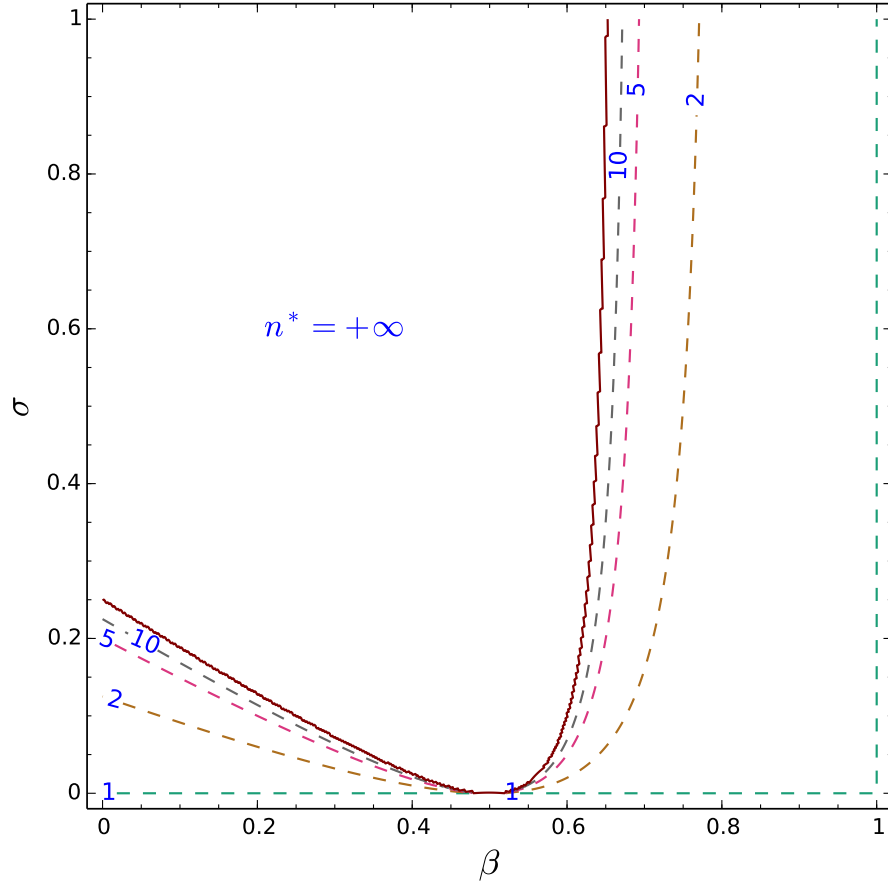
Figure 1: The effective R&D maximising number of firms, $n^*$ (with $b = 0$, $m = 3$).

effect. Under the same conditions, however, effective R&D can increase in the number of industries. As such, the missing hypothesis of diversity-concentration combination is also a relevant alternative to consider. Be that as it may, this result also shows that innovation proxies might not always be the most important performance measure to consider.

## 4.2 Industry Output

The dependent variable in empirical research has not always been a measure of innovation or R&D. Instead, some papers have studied the impact of variety and concentration on variables such as employment and output. While there is an obvious link between R&D and output, the local industry structure can affect them differently. Therefore, it is important to determine how the total industry output is affected because this is more relevant

with respect to some of the empirical studies. Again, the focus in the analysis is on the representative industry with its average initial market size.

Both intra- and inter-industry spillover rates affect the equilibrium output, at the firm and industry level, through effective R&D alone, as we can observe from Equation (13). Since $\text{sign}(\partial \bar{X}/\partial \beta) = \text{sign}(\partial \bar{Q}/\partial \beta)$ and $\text{sign}(\partial \bar{X}/\partial \sigma) = \text{sign}(\partial \bar{Q}/\partial \sigma)$, this allows us to directly conclude that

**Proposition 6** *Intra- and inter-industry spillover rates that maximise effective R&D also maximise the total industry output.*

While the choice between R&D- and output-related performance measures does not matter in the case of spillover rates, this is typically not true for the variety and concentration of industries. Totally differentiating $\bar{Q}(m,(\bar{X}(m)))$ with respect to $m$ yields

$$\frac{\mathrm{d}\bar{Q}}{\mathrm{d}m} = \frac{\partial \bar{Q}}{\partial m} + \frac{\partial \bar{Q}}{\partial \bar{X}}\frac{\mathrm{d}\bar{X}}{\mathrm{d}m} \tag{14}$$

with

$$\frac{\partial \bar{Q}}{\partial m} = \frac{-bn^2(\bar{v}+\bar{X})}{(n+1+bn(m-1))^2} \gtreqqless 0 \leftrightarrow b \lesseqqgtr 0,$$

$$\frac{\partial \bar{Q}}{\partial \bar{X}} = \frac{n}{n+1+bn(m-1)} > 0,$$

and $\mathrm{d}\bar{X}/\mathrm{d}m$ given by Equation (A21) in the online appendix. In the view of Proposition 4 it is now easy to see that

**Proposition 7** *The total industry output is i) increasing in m for industries producing complements; ii) increasing for independent industries if and only if $\sigma > 0$ and constant otherwise; and iii) increasing for industries producing imperfect substitutes only if effective R&D is increasing as well.*

The marginal effect of variety on the total industry output and effective R&D have the same sign for industries that are independent or produce complements. In the case of

17

imperfect substitutes, however, the marginal effects can have different signs. For example, from Equation (14) we can see that $m$, which maximises effective R&D ($\mathrm{d}\bar{X}/\mathrm{d}m = 0$), has a negative marginal effect on the total industry output ($\mathrm{d}\bar{Q}/\mathrm{d}m < 0$).

Similarly for uncovering the effect of concentration, totally differentiating $\bar{Q}(n,(\bar{X}(n))$ with respect to $n$ yields

$$\frac{\mathrm{d}\bar{Q}}{\mathrm{d}n} = \frac{\partial\bar{Q}}{\partial n} + \frac{\partial\bar{Q}}{\partial\bar{X}}\frac{\mathrm{d}\bar{X}}{\mathrm{d}n} \tag{15}$$

with

$$\frac{\partial\bar{Q}}{\partial n} = \frac{\bar{v}+\bar{X}}{(n+1+bn(m-1))^2} > 0,$$

$$\frac{\partial\bar{Q}}{\partial\bar{X}} = \frac{n}{n+1+bn(m-1)} > 0,$$

and $\mathrm{d}\bar{X}/\mathrm{d}n$ given by Equation (A25) in the online appendix. In comparison with Proposition 5 it becomes clear that

**Proposition 8** *Irrespective of the degree of product differentiation, the total industry output is increasing in n if effective R&D is increasing as well.*

An implication of Proposition 8 is that the marginal effect of $n$ on the total industry output may have a positive sign while on effective R&D the sign of its marginal effect is negative. In addition, if such interior solutions exist, $n$ that maximises the total industry output is larger than $n$ that maximises effective R&D. It seems plausible that the difference, which is due to the direct effect $\partial\bar{Q}/\partial n$, is decreasing in $b$, but this is hard to verify since $\bar{X}$ is highly non-linear in $b$ and the effect depends on the spillover rates as well. The surveys of empirical research show that when the independent variable is economic growth, the impact of MAR externalities with its combination of specialised and concentrated local industry structure is far less often positive and occasionally even negative (Beaudry and Schiffauerova, 2009; De Groot et al., 2016).

Figure 2 presents a contour plot, which illustrates how the industry output maximising number of firms depends on the spillover rates $\beta$ and $\sigma$. In comparison to Figure 1, here a concentrated market structure is optimal for maximising the effective R&D only in the upper right corner, where both spillover rates are high. Nevertheless, this presents an interesting inverted-U relationship between competition and industry output. For most, more moderate spillover combinations, though, the industry output is always increasing in $n$. This shows that cases exists, where concentration can have opposite effects on different performance measures.
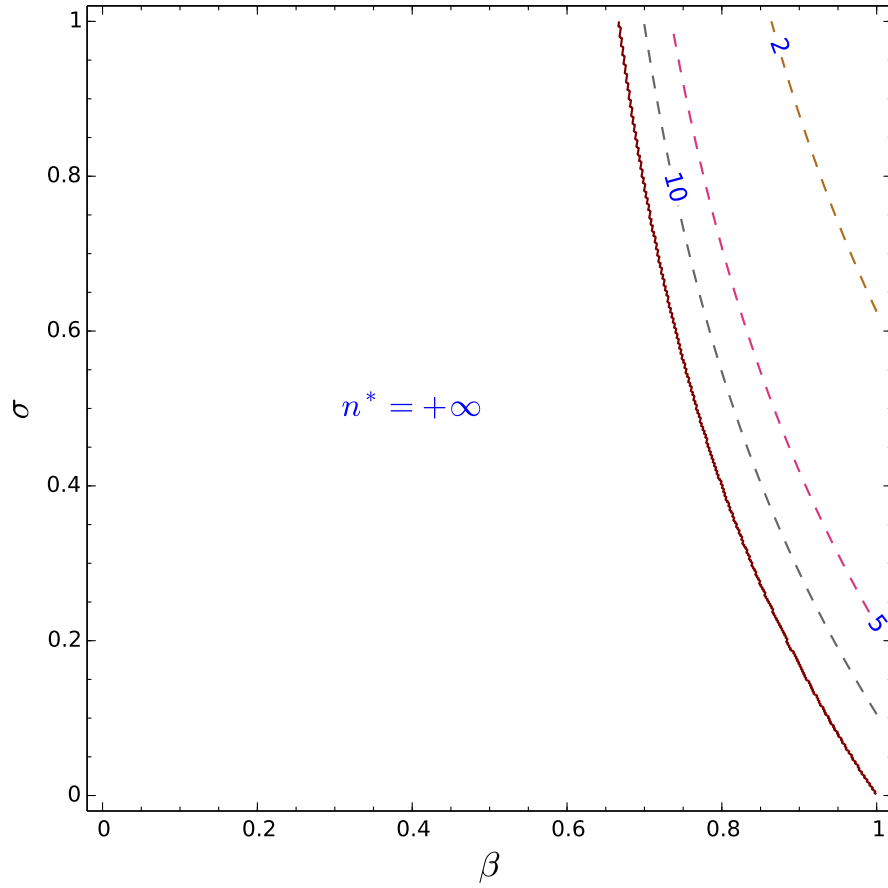


Figure 2: The industry output maximising number of firms, $n^*$ (with $b = 0$, $m = 3$, $\gamma = 2$).

# 5 Extensions

## 5.1 Relative Variety of Industries

In the studied baseline model, an increase in the variety of industries leads also to an increase in the size of the local economy. This contributes partly to the outcome that both the effective R&D and industry output are always increasing in variety in the case of complements and independent products. Thus, there is a natural link between the size and variety of the local economic base, but it also combines two different effects, which we may wish to analyse separately.

Similarly, empirical papers have studied the effect of relative variety as well as that of absolute variety. Unfortunately, these papers rarely consider the theoretical case for the use of relative or absolute measure (Kemeny and Storper, 2015). A survey of the literature reveals that this choice has an impact on the empirical findings (De Groot et al., 2009).

To study the impact of variety in purely relative terms, I consider a modified version of the Shubik-Levitan demand equation (Shubik and Levitan, 1980; Kagitani et al., 2016). As such, the inverse demand in industry $j$ is given by $P_j = a - dQ_j - b\sum_{l \neq j} Q_l, \forall j, l \in m$, where $d \equiv m - (m-1)b$ and $b$ defined as earlier. The initial size of the each market, $v = a - c$, is now assumed to be the same for each industry.

With this modification the aggregate local demand is independent of the number of industries as well the degree of product differentiation: $\sum_{j=1}^m Q_j = a - \bar{P}$, where $\bar{P}$ is the average price across the industries. As such, instead of fixing the size of the individual market, as previously, we now fix the size of the aggregate local demand and maximum consumer surplus, of which each market has an equal share.

Following the same procedure as earlier and now solving for the fully symmetric sub-game-perfect Nash equilibria, the equilibrium R&D output becomes

$$\hat{x} = \frac{2vd(n(d-b)+d)\hat{D}}{\gamma\hat{C}^2 - 2d(n(d-b)+d)\hat{D}\hat{E}}. \tag{16}$$

with

$$\hat{A} = dn(n(d-b)+d) + bn(m-1)(n(d-b)+b) > 0,$$

$$\hat{B} = d(n(d-b)+d) + bn(m-1)(d-b) > 0,$$

$$\hat{C} = d(n(d-b)+d)(d(n+1)+bn(m-1)) > 0, \tag{17}$$

$$\hat{D} = \hat{A} - \beta(n-1)\hat{B} - \sigma dbn(m-1) > 0,$$

$$\hat{E} = 1 + \beta(n-1) + \sigma n(m-1) > 0.$$

Given the steeper slope of the demand function, Assumption 2 is sufficient to guarantee the second-order conditions, stability, and interior solutions. Subsequently, the firm-level equilibrium output is

$$\hat{q} = \frac{v + \hat{X}}{d(n+1) + bn(m-1)}, \tag{18}$$

where $\hat{X} = \hat{x}\hat{E}$ is the effective R&D. The total industry output is $\hat{Q} = n\hat{q}$ and the aggregate local output is $\sum Q = mn\hat{q}$. By combining analytic arguments with numerical simulation, we obtain the following result.

**Proposition 9** *If the products are independent, then effective R&D can be increasing in relative variety but not for any number of industries larger than 2. Furthermore, the effective R&D maximising number of industries tends to be smaller in the case of complements and larger when the products are imperfect substitutes.*

Proposition 9 shows that, in contrast to the case of absolute variety, the positive effect of relative variety on effective R&D is rather limited for complements and independent products. The underlying intuition is that an increase in relative variety implies a decrease in each market, which has a negative multiplier effect when the products are complements. A high degree of substitutability and a large number of firms is typically required for a positive effect of relative variety. An example of such a case is presented in Figure 3. We can see that the effective R&D maximising $m$ is larger when $\beta$ is low and $\sigma$ high. However,

the range of *m* for which the effect is positive is still rather limited as the highest *m* in this example is approximately 4.2.
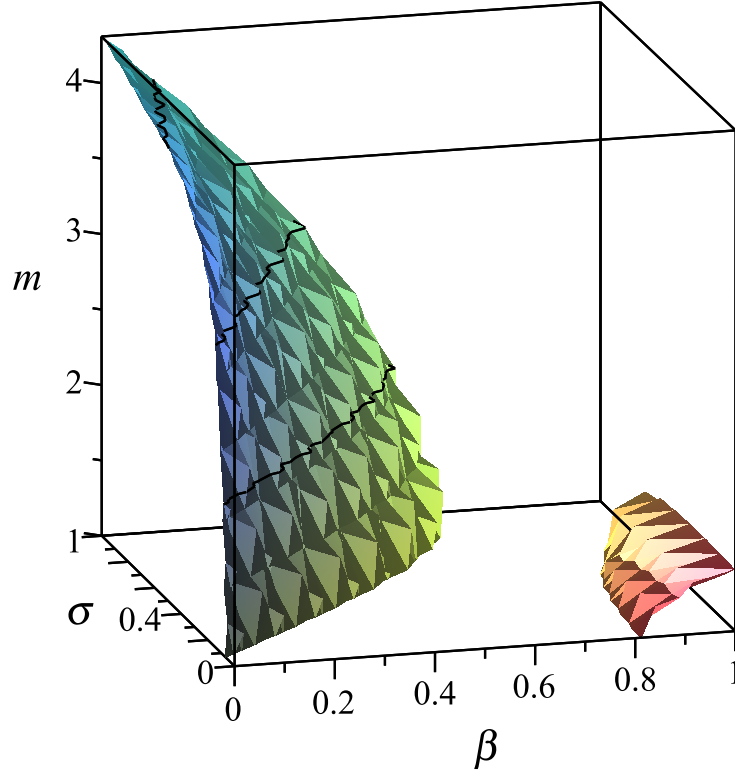


Figure 3: The effective R&D maximising relative variety (with $b = 9/10$, $n = 15$).

Totally differentiating the aggregate local output $\sum Q$ with respect to *m* yields

$$\frac{\mathrm{d}\sum Q}{\mathrm{d}m} = \hat{Q} + m\frac{\mathrm{d}\hat{Q}}{\mathrm{d}m} \tag{19}$$

with

$$\frac{\mathrm{d}\hat{Q}}{\mathrm{d}m} = \frac{\partial\hat{Q}}{\partial m} + \frac{\partial\hat{Q}}{\partial\hat{X}}\frac{\mathrm{d}\hat{X}}{\mathrm{d}m},$$

$$\frac{\partial\hat{Q}}{\partial m} = \frac{-n(n+1-b)(v+\hat{X})}{(m(n+1)-b(m-1))^2} < 0,$$

22

$$\frac{\partial \hat{Q}}{\partial \hat{X}} = \frac{n}{n+1+bn(m-1)} > 0,$$

and $\mathrm{d}\hat{X}/\mathrm{d}m$ given by Equation (A29) in the online appendix. Furthermore, it is easily checked that

$$\hat{Q} \gtreqqless -m\frac{\partial \hat{Q}}{\partial m} \leftrightarrow b \gtreqqless 0.$$

It follows directly that

**Proposition 10** *Irrespective of the degree of substitutability, only if effective R&D is increasing in relative variety, then the total industry output is increasing as well. For the aggregate local output to increase in relative variety, it is i) necessary that effective R&D is increasing if the products are complements; ii) necessary and sufficient if the products are independent; and iii) sufficient if the products are imperfect substitutes.*

This result implies that in the case complements or independent products it is even less likely that relative variety has a positive effect when we consider the industry output or the aggregate local output. Again, the relative measure of variety reverses the results for the different product types. As illustrated in Figure 4, for close substitutes the aggregate local output is increasing under a range of different circumstances. Interestingly, the positive effect seems to be the strongest when either both spillover rates are close to 1 or close to zero.

## 5.2   Industry-Specific Concentration

While an increase in the number of firms reduces the market shares and, consequently the incentive to innovate, there are also more spillover sources. The presence of inter-industry spillovers may imply that on the balance effective R&D increases, which does not happen in the case of a homogeneous oligopoly and intra-industry spillovers alone. However, this raises the question of whether industry-specific concentration can have the same effect.
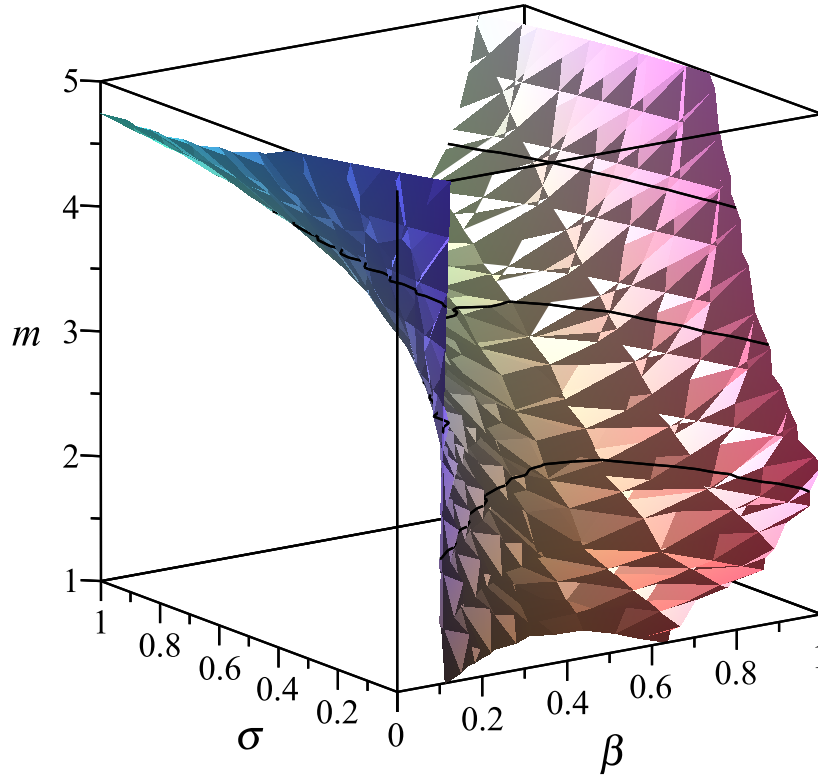
Figure 4: The aggregate local output maximising relative variety (with $b = 9/10$, $n = 15$, $\gamma = 5$).

Rather than the average concentration of the local economy, industry-specific measures of it may also be more relevant in empirical research.

The earlier analysis of concentration was facilitated by the symmetry between the industries. Since with industry-specific concentration this is no longer the case, we need to simplify the analysis in other ways. As such, we restrict here to the case of independent industries, $b = 0$, each of which has the same initial market size $v = a - c$. In this case the strategic interdependence between the industries is limited to inter-industry spillovers and excludes the output effects.

Suppose that the number of firms in industry $j$ is $n_j$, whereas in all the other industries it is $n$. As such, we study how effective R&D $X_j$ and total output $Q_j$ in industry $j$ respond

to changes in $n_j$. Following the same procedure as earlier to solve for the sub-game perfect Nash equilibria[7], the symmetric best-response R&D outputs are

$$x_j = \frac{2(n_j - (n_j - 1)\beta)(v + \sigma(m-1)nx)}{\gamma(n_j + 1)^2 - 2(n_j - (n_j - 1)\beta)(1 + \beta(n_j - 1))} \tag{20}$$

for firms industry $j$ and

$$x = \frac{2(n - (n-1)\beta)(v + \sigma n_j x_j)}{\gamma(n+1)^2 - 2(n - (n-1)\beta)(1 + \beta(n-1) + \sigma(m-2)n)} \tag{21}$$

for firms in other industries. Assumption 2 is sufficient to guarantee that the second-order conditions are met and that the slopes of the best response functions, while positive, are less than 1. Therefore, Equations (20) and (21) jointly determine the equilibrium R&D outputs. Consequently, the firm-level equilibrium output in industry $j$ is given by

$$q_j = \frac{v + X_j}{n_j + 1}, \tag{22}$$

where $X_j = (1 + \beta(n_j - 1))x_j + \sigma(m-1)nx$ is the effective R&D of firms in the industry.

**Proposition 11** *Considering independent industries, the effective R&D in an industry can increase when the number of firms in that industry alone increases, but only if the industry's aggregate R&D output increases, which excludes some high intra-industry spillover rates.*

This result shows that the potential positive effect of competition on innovation found earlier was not merely an artefact of the assumed symmetry between the industries. Critical for this effect is that the intra-industry spillover rate is not too high. However, this is not sufficient as the inter-industry spillovers need to be extensive enough to compensate for the (likely) decrease in the effective intra-industry R&D.

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[7]Details available upon request.

Totally differentiating the output in industry $j$ with respect to $n_j$ yields

$$\frac{\mathrm{d}Q_j}{\mathrm{d}n_j} = \frac{\partial Q_j}{\partial n_j} + \frac{\partial Q_j}{\partial X_j}\frac{\mathrm{d}X_j}{\mathrm{d}n_j}$$

with

$$\frac{\partial Q_j}{\partial n_j} = \frac{v+X_j}{(n_j+1)^2} > 0 \text{ and } \frac{\partial Q_j}{\partial X_j} = \frac{n_j}{n_j+1} > 0.$$

Again, the implication is that

**Proposition 12** *Considering independent industries, the total output in industry $j$ is increasing in $n_j$ if effective R&D is increasing as well.*

Similarly as the effect of overall concentration across the industries, it is therefore possible that the marginal effect of industry-specific concentration has different signs on effective R&D and the total industry output. The latter can be increasing in the number of firms in that industry even if the former is not.

# 6 Discussion of the Modelling Choices and Heterogeneity

The focus in the main analysis (Section 4) has been on the representative industry, which both simplifies the case as well allows to extend the results directly to the local economy. However, as Equations (9) and (10) reveal, the initial market size will also lead to different outcomes between the industries. Consider, for example, term $(2DGnm)/Z$ in Equation (9), which determines whether a larger than average market size leads to a larger or lower than average R&D output. In particular, the latter outcome holds when $G \geq 0$ and the products are independent or complements. Conversely, $G < 0$ holds for imperfect substitutes with high values of $\beta$ and low values of $\sigma$.

It also less straightforward whether higher than average R&D output will also imply higher than average effective R&D. It can be shown that this is the case if and only if $1 + \beta(n-1) - \sigma n > 0$, which again depends on the two spillover rates. It follows that

26

the ambiguous relationship between market size and effective R&D also complicates the relationship between market size and equilibrium output.

Naturally, the real world industries are heterogeneous in many other ways in addition to their initial market size. For example, a good can be an imperfect substitute to one product while a complement to another. In addition, the industries are likely to differ in terms of both intra- and inter-industry spillovers rates. While extending the model in the direction of heterogeneous product relationships and spillover rates would make it more realistic, unfortunately that also makes it intractable. However, the main results of the analysis are not affected by the simplified assumptions. Since the impact of the local industry structure on innovation and output is ambiguous even with common parameters of $b, \beta$ and $\sigma$, the ambiguity would also be there when the parameters possibly differ between the industries.

Another issue to discuss concerns the relationship between the local industry structure and the spillover rates. The comparative static analysis studies how the equilibrium changes with variations in one variable while the others stay constant. In reality, when new firms or industries enter, for example, the spillover rates between the incumbents and the entrants may differ from the earlier ones. The comparative static analysis essentially separates these effects from each other using a ceteris paribus assumption, which is what empirical analysis typically aims to do as well. However, from the perspective of regional economic policy these factors cannot be considered fully independent.

With respect to the main hypotheses examined in the empirical literature, the underlying idea of Porter externalities is that competition fosters innovation because the firms would not survive otherwise. This aspect of competition is missing from the model, so it may not fully capture Porter externalities. Similarly, the original idea behind Jacobs externalities has more to do with the development of new inventions and applications rather than R&D spillovers (Jacobs, 1969). The model could be extended to consider product innovations, for example, to capture this aspect of Jacobs externalities.

The model could also be extended by including endogenous absorptive capacity, the

27

relevance of which is often emphasised in innovation studies. However, although the absorptive capacity has been shown to increase the R&D investments of firms, its effect has been quantitative rather than qualitative (Martin, 2002). As such, it is not expected that the incorporation of absorptive capacity would reverse the trends with respect to variety and concentration.

Lastly, I would like to note two neglected areas of research, which may have some interesting implications. If some of the firms are multi-product producers as in Eckel and Neary (2010) and Eckel et al. (2015), for example, the strategic R&D incentives of these firms are different and their impact on the markets may be significant. This seems relevant for empirical research as well. Furthermore, spillovers from local universities are often studied in empirical research (e.g. Veugelers and Cassiman, 2005), but rarely considered in theoretical research. Hence, extending the model to consider industry-university linkages could help to bridge this gap.

# 7  Conclusion

This study had two main aims: to provide appropriate theoretical foundations for MAR, Porter and Jacobs externalities; and to address the mixed empirical results with respect to these three hypotheses. As we have seen, the impact of variety and concentration is rather less straightforward than what the main hypotheses suggest. How firms respond to these two factors varies with the circumstances, which include, in particular, the product-type linkages between the industries and intra- and inter-industry spillover rates.

There exists different conceivable circumstances under which evidence in support of the three main hypothesis, as well as the missing, fourth hypothesis, can be expected. For example, regional diversity is likely to be found beneficial when the products are complements and an absolute measure of variety is used, whereas the opposite holds for relative measures of variety. Likewise, concentration may increase effective R&D but decrease the

28

total industry output. As such, even in the same context the choice of the performance measure can be critical.

While we considered industries between which there are spillovers, this is often not the same as the total number of local industries. Following Frenken et al. (2007), more recent empirical studies have similarly aimed to capture the "related variety" of co-located industries. In addition, Bloom et al. (2013) have studied the spillover-rivalry trade-off using measures of a firm's position in technology and product market space. Similarly, the results of this study suggest that it would be important to control for the intra- and inter-industry spillover rates and the degree of product differentiation when assessing the impact of the local industry structure.

The examined model is based on the standard output spillover model in the industrial organization literature, which may be worth reconsidering in subsequent study. As such, the present study is merely an early step in this area where there is surprisingly little theoretical work. Nevertheless, it highlights several critical issues that should be considered when building empirical models to study localised knowledge spillovers and interpreting their results.

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