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Dynamic Modelling and Control of Thermal Energy Storage

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Abstract

Thermal energy storage (TES) is a critical element in district heating systems and having a good understanding of its dynamic behaviour is necessary for effective energy management. TES supports heat sources in achieving a steady power supply. Achieving heat and electric load demand translates into a discharging and charging control problem in terms of stored heat energy. To this end, an accurate dynamic model is essential to design effective controllers to improve district heating performance. A thermal dynamic model of a water tank, together with controllers for energy charging and discharging processes, are presented in this paper. The model is based on a computational fluid dynamics approach. It is developed using thermal stratification. The hot or cold-water stream which vertically crosses each tank section is considered to describe heat transfer. This is a non-linear process represented by partial differential equations, which are linearised to obtain a suitable model for control system design. State-space and transfer function representations of the system are obtained. The non-linear model is implemented in MATLAB/Simulink to design a linear controller that regulates the mass flow rate of cold and hot water to fill or empty the tank's energy according to performance specifications. The design of regulation and tracking controllers is explained. Simulation results show that a good performance in terms of the mass flow rate input demands is achieved with the proposed controllers.

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Keywords: Thermal energy storage; Non-linear dynamic model; Thermal stratification; Frequency domain; Control system design

1. Introduction

Energy loads in residential and industrial sectors vary from a daily to a seasonal basis. These loads can be supplied with the help of thermal energy storage (TES). TES is divided into seasonal and short-term depending on

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the storage method [1]. For instance, a seasonal TES method may use underground aquifers to store heat. Conversely, heat can be stored on an hourly or daily basis [2]. In district heating (DH) systems, short-term TES helps to manage loads that vary hourly. The main aim of TES is to temporarily recover the by-product generated by energy sources for a later use. The stored energy is used to move the loads from peak to lower demand periods, avoiding a mismatch between availability and demand of heat.

There are many options for the storage media. Water is the most common choice in DH systems due to its high specific heat and its ease of pumping to transport thermal energy. Storage water tanks are located within energy supply centres. Given that the charging and discharging processes of TES affect the efficiency of the overall system, a synchronised operation with the heat sources is necessary to improve the overall system performance. Thus, a good understanding of the dynamic behaviour of TES is essential not only for the design processes of energy supply centres, but also in the implementation of effective control strategies for DH systems.

Dynamic models of TES have been recently developed using computational fluid dynamics (CFD). This way, the thermal behaviour of a water tank can be described spatially. Three-dimensional complex models provide a detailed dynamic model [3]. However, it has been shown that one-dimensional models are sufficient to characterise temperature variations along the height of the tank with enough accuracy [4]. The most popular approach within CFD models is stratification. A thermal stratified tank is represented by horizontal layers. These layers have different densities due to temperature variations. Thus, each layer is modelled by a differential equation based on the energy conservation law. Fig. 1(a) shows the location of temperature sensors through the tank. These enable the monitoring of temperature in DH-based TES, justifying the use of stratification as a suitable approach to develop control-oriented models for energy management.

This paper proposes a non-linear dynamic TES water tank model describing its thermal behaviour. The model has been implemented in MATLAB. Despite of its non-linear nature, it is shown that the filling and emptying processes can be effectively managed by linear controllers following system linearisation. Two linear controllers have been designed for TES energy reference tracking and regulation. Both controllers are evaluated with regards to their mass flow rate demands to achieve the input references. A good closed-loop performance of the non-linear model is accomplished with both controllers. A control scheme acting on discharging and charging operation modes is proposed, which simplifies the controller implementation. It is shown that TES energy control can provide coordination between the oncoming heat and the load demands.

Nomenclature

ρ	density [kg/m^3]	U	heat transfer coefficient [W/m^2°C]	c_p	specific heat [J/kg°C]
$\dot{m}$	mass flow rate [kg/s]	T	temperature [°C]	A	Area [m^2]
h	height [m]	E_A	Average energy [Wh]	k	thermal conductivity [W/m°C]
T_{Av}	average temperature [°C]				

2. Dynamic model

The dynamics of a fluid mechanics system are normally described in terms of equations based on the conservation laws of mass, momentum and energy. Nevertheless, a very detailed model is not necessary to describe the thermal behaviour of TES. For instance, internal velocities and forces over the temperatures of the tank layers are negligible; therefore, the momentum conservation law is not required. Besides, the operating temperatures in a DH system fluctuate between 55°C to 95°C, which prevents a phase change in the water. Since the tank is maintained full, mass balance is not necessary. Therefore, only the energy conservation law is required. Fig. 1(b) shows the schematic of the energy conservation law and its application over a layer of a TES water tank.

The energy balance in volume Ω bounded by surface S is given by a partial differential equation (PDE):

$$\frac{\partial \rho E}{\partial t} + \vec{\nabla} \cdot \vec{F} = Q_v \cdot v + \vec{\nabla} \vec{Q}_S \,, \tag{1}$$

where ρE is the total energy per unit of mass, F is the energy crossing S, and Q_S and Q_v are the sources of energy coming from Ω and S, respectively [5]. There are two types of heat fluxes in each layer. The diffusive flux can be expressed using the Fourier's law of heat conduction as $F_D = -k\nabla T$ [6]. This flux in turn is represented in two forms:

the heat flux by the tank wall and the horizontal boundaries of the layer. The convective flux, given by $F_C=\rho vE$, is related to energy transport by a flow rate and, thus, has an associated velocity. Assuming there are neither internal volume sources as heat is released by a chemical reaction nor internal shear stresses, (1) is rewritten as:

$$\frac{\partial \rho E}{\partial t} - \vec{\nabla}\cdot(\rho\vec{v}E) = \vec{\nabla}\cdot\left(k\vec{\nabla}T\right). \tag{2}$$

Energy E can be defined as the enthalpy of the fluid, given by $E=c_p\Delta T$. The layer volume is the tank horizontal area (A_h) times the layer height (Δh). The convective flux is the input mass flow rate ($\dot{m}_F$), which can be supplied either at the top or bottom of the tank. It does not have a space variation as it is injected in a specific point. Applying the stratification method presented in [7,8], the PDE becomes an ordinary differential equation (ODE). Thus, solving equation (2) for the vertical dimension and neglecting the temperature difference along the horizontal direction, the conservation law equation for TES stratification can be rewritten as

$$\rho c_p A_h \Delta h \frac{dT_i}{dt} = c_p \dot{m}_F \frac{dT}{dt} + kA_h \frac{dT}{dt} - UA_l \frac{dT}{dt} = c_p \dot{m}_F (T_F - T_i) + kA_h (T_{i+1} - 2T_i + T_{i-1})/\Delta h - UA_l (T_a - T_i). \tag{3}$$

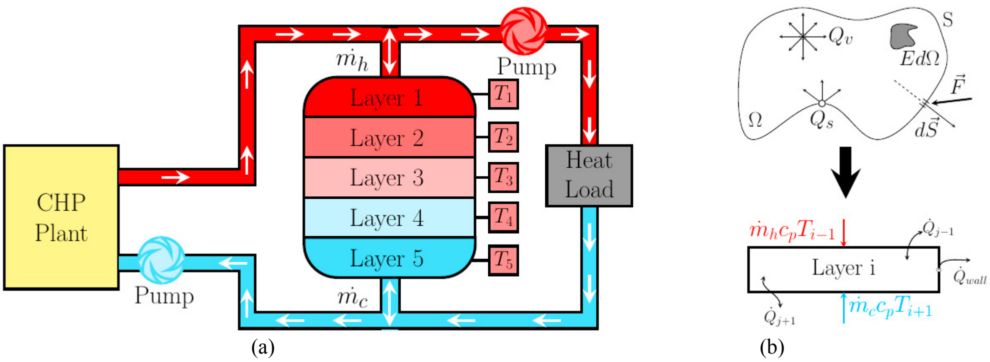


Fig. 1. (a) Schematic of TES configuration in DHS. (b) General form of conservation law and its application over TES layer.

Table 1. TES properties

Variable	Unit	Value	Variable	Unit	Value	Variable	Unit	Value
Volume capacity V_T	m^3	100	Diameter d	m	3.56	Tank height H	m	10
Horizontal Area A_h	m^2	9.95	Layer height h	m	2	Layer Lateral area A_l	m^2	22.37

2.1. Case study

Table 1 shows the dimensions of the water tank used in this paper. TES stratification is done by dividing the tank into five layers (see Fig. 1(a)). The tank surface used in TES for DH systems typically has excellent insulation properties that enable the heat flux between tank walls and ambient temperature to be neglected. TES receives hot water from energy sources and cold water from the DH system return flow. Therefore, the convective flow input is separated in two variables: a cold stream ($\dot{m}_c$) that is injected to the bottom layer and a hot stream ($\dot{m}_h$) that is injected to the top layer. Applying equation (3) to each layer, a TES thermal dynamic model is defined by:

$$\rho c_p A\Delta h \, {}^{dT_1}\!/_{dt} = c_p \dot{m}_h (T_h - T_1) + c_p \dot{m}_c (T_{i+1} - T_1) + kA(T_2 - T_1)/\Delta h, \tag{4}$$

$$\rho c_p A\Delta h \, {}^{dT_i}\!/_{dt} = c_p \dot{m}_h (T_{i-1} - T_i) + c_p \dot{m}_c (T_{i+1} + T_i) + kA(T_{i+1} - 2T_i + T_{i-1})/\Delta h, \tag{5}$$

$$\rho c_p A\Delta h \, {}^{dT_n}\!/_{dt} = c_p \dot{m}_c (T_c - T_n) + c_p \dot{m}_h (T_{n-1} + T_n) + kA(T_{n-1} - T_n)/\Delta h. \tag{6}$$

The operating mode of TES in a DH system helps to simplify the model as the charging and discharging processes are not carried out simultaneously. Therefore, even when the system has two inputs, both processes can be modelled as a single-input single-output system for control design purposes. Thus, a state-space representation of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \qquad y = \mathbf{C}\mathbf{x} + \mathbf{D}u, \tag{7}$$

is obtained, where the system states are the layer temperatures ($x_i=T_i$) and the system inputs are cold and hot stream mass flow rates ($u_1=\dot{m}_h$, $u_2=\dot{m}_c$). The control aims are to fill and to empty the tank's energy. For simplicity, an average temperature is considered as the system output ($y=T_3$). Equations (4), (5) and (6) are rewritten as

$$\dot{x}_1 = \left[c_p u_1 \left(T_h - x_1\right) + c_p u_2 \left(x_2 - x_1\right) + kA_h \left(x_2 - x_1\right) / \Delta h\right] / \rho c_p V_l \ , \tag{8}$$

$$\dot{x}_i = \left[c_p u_1 \left(x_{i-1} - x_i\right) + c_p u_2 \left(x_{i+1} - x_i\right) + kA_h \left(x_{i+1} - 2x_1 + x_{i-1}\right) / \Delta h\right] / \rho c_p V_l \ , \tag{9}$$

$$\dot{x}_5 = \left[c_p u_1 \left(x_4 - x_5\right) + c_p u_2 \left(T_c - x_5\right) + kA_h \left(x_4 - x_5\right) / \Delta h\right] / \rho c_p V_l \ , \tag{10}$$

Equations (8), (9) and (10) are non-linear. Following the linearisation method used in [9, 10], a linear state-space system is obtained. This is given by:

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \\ \Delta\dot{x}_3 \\ \Delta\dot{x}_4 \\ \Delta\dot{x}_5 \end{bmatrix} = \begin{bmatrix} a_1 & c_1 & 0 & 0 & 0 \\ b_1 & a_1 & c_2 & 0 & 0 \\ 0 & b_2 & a_1 & c_3 & 0 \\ 0 & 0 & b_3 & a_1 & c_4 \\ 0 & 0 & 0 & b_4 & a_1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \\ \Delta x_5 \end{bmatrix} + \begin{bmatrix} c_p \left(T_h - x_1\right)/n_1 & c_p \left(x_2 - x_1\right)/n_1 \\ c_p \left(x_1 - x_2\right)/n_2 & c_p \left(x_3 - x_2\right)/n_2 \\ c_p \left(x_2 - x_3\right)/n_3 & c_p \left(x_4 - x_3\right)/n_3 \\ c_p \left(x_3 - x_4\right)/n_4 & c_p \left(x_5 - x_4\right)/n_4 \\ c_p \left(x_4 - x_5\right)/n_5 & c_p \left(T_c - x_5\right)/n_5 \end{bmatrix} \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix} , \tag{11}$$

$$y = [0 \ \ 0 \ \ 1 \ \ 0 \ \ 0]\Delta\mathbf{x} + [0]\Delta\mathbf{u} \ , \tag{12}$$

where $n_i = \rho c_p V_i$, $a_i = (-c_p u_1 - c_p u_2 - k_i A_h)/n_i$, $b_i = (c_p u_1 + k_i A_h)/n_i$ and $c_i = (c_p u_2 + k_i A_h)/n_i$.

3. Control design

It is important to notice that the control variable is the energy that is injected to or taken from the tank. This is indirectly measured by temperature sensors along the vertical length of the tank and can be computed with the tank's enthalpy equation:

$$E_A = mc_p T/t = mc_p \left(\frac{1}{5}\sum_{i=1}^{5} T_i - T_c\right) \Big/ t = mc_p \left(T_{AV} - T_c\right)/t \ . \tag{13}$$

The non-linear model defined by (4), (5) and (6) was built in MATLAB/Simulink. Due to the fast change of the temperatures of the tank layers under input flows, selecting a specific operating point becomes a difficult task. Therefore, an average value of each layer's temperature is used to compute the coefficients of (11). This way, ten linearised systems are obtained, out of which a set of five systems G_1 is defined for the charging process, where the input is $u_1 = \dot{m}_h$, and another set of five systems G_2 is defined for the discharging process, where the input is $u_2 = \dot{m}_c$. Both are computed from a minimum to a maximum mass flow rate input using steps of 20%. Layer temperatures and stream values are shown in Table 2.

Table 2. Layer temperature average values and stream input ranges

T_1[°C]	T_2[°C]	T_3[°C]	T_4[°C]	T_5[°C]	$\dot{m}_h$ [kg/s] [min-max]	$\dot{m}_c$ [kg/s] [min-max]
89.09	85.76	80	74.4	64.7	[20-100]	[20-100]

The linearised systems given by (11) and (12) can be represented by a transfer function that relates the inputs ($u_1 = \dot{m}_h$, $u_2 = \dot{m}_c$) and the 3rd layer temperature ($y = T_3$) as

$$Y(s)/U_{1,2}(s) = G_{1,2}(s) = \mathbf{C}\left(s\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B} \ . \tag{14}$$

In (14), the system output is given by T_3. Nevertheless, once the system has been linearised, the implementation is done using the average temperature of the tank given by the average temperature of the layers (T_{Av}). Fig. 2 shows the control scheme proposed for the TES non-linear model, where the feedback temperature is T_{Av}.

A unique controller is necessary to simplify the control implementation. The control design method proposed in this paper involves the linearised systems (14) for both inputs. The system transfer function set is defined, in general, by:

$$G_{1,2}(s) = \frac{\left[2.86_{-4} \quad 2.95_{-4}\right]s^4 + \left[3_{-6} \quad 1.5_{-5}\right]s^3 + \left[1_{-8} \quad 2.8_{-7}\right]s^2 + \left[1.5_{-11} \quad 2.3_{-9}\right]s + \left[7_{-15} \quad 6.1_{-12}\right]}{s^5 + \left[1_{-2} \quad 5.1_{-2}\right]s^4 + \left[3.7_{-5} \quad 9.4_{-4}\right]s^3 + \left[6_{-8} \quad 7.5_{-6}\right]s^2 + \left[4_{-11} \quad 2.4_{-8}\right]s + \left[1_{-14} \quad 2.1_{-11}\right]} , \tag{15}$$

where X_{-y} stands for $X \times 10^y$. Fig. 2(b) shows the Bode plots of $G_{1,2}(s)$. Although the TES non-linear behaviour causes noticeable differences among transfer functions representing different operating points, a representative transfer function from $G_{1,2}(s)$ is selected to facilitate control system design. Once an adequate performance has been achieved for that transfer function, the performance for all $G_{1,2}(s)$ is assessed. In other words, a single plant is used to design regulation and tracking controllers via Bode-shaping. The selected plant is:

$$g_c(s) = \frac{2.948\times10^{-4}\, s^4 \ + \ 8.645\times10^{-6} s^3 + 9.068\times10^{-8}\, s^2 + 3.955\times10^{-10}\, s + 5.836\times10^{-13}}{s^5 + 3.07\times10^{-2} s^4 + 3.394\times10^{-4} s^3 + 1.621\times10^{-6} s^2 + 3.112\times10^{-9} s + 1.638\times10^{-12}} \ . \tag{16}$$

For temperature regulation a desired settling time of 1500 s is required. Given that the input flows are injected by different pumps, it is recommended to not use both during a single process (charging or discharging). For instance, during the charging process the filled energy reference should be reached without exhibiting an overshoot, as this would mean there is an opposite flow to compensate the negative error. Therefore, a damping value ζ=1 is imperative. The desired performance requirements are translated to the frequency domain as a phase margin of at least 78° and a minimum bandwidth $\omega_{bw}=3/t_s\zeta=0.002$ rad/s [11]. A PI controller satisfying these requirements is given by:

$$C_r(s) = k_p + k_i/s = (6s + 0.0001)/s \tag{17}$$

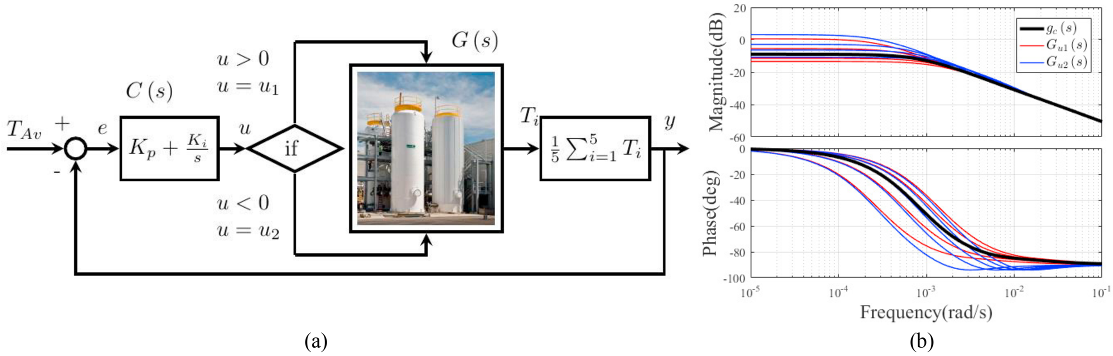


Fig. 2. (a) Closed loop control scheme. (b) Bode plot of the open loop plants.

For temperature tracking, it is necessary to approximately have 10 times the bandwidth of $g_c(s)$ to achieve a fast response. An integral action is required to avoid a significant error during reference tracking. As it is shown in Fig. 3(b), the following PI controller meets the specification requirements:

$$C_t(s) = k_p + k_i/s = (27.6s + 0.047)/s\ . \tag{18}$$

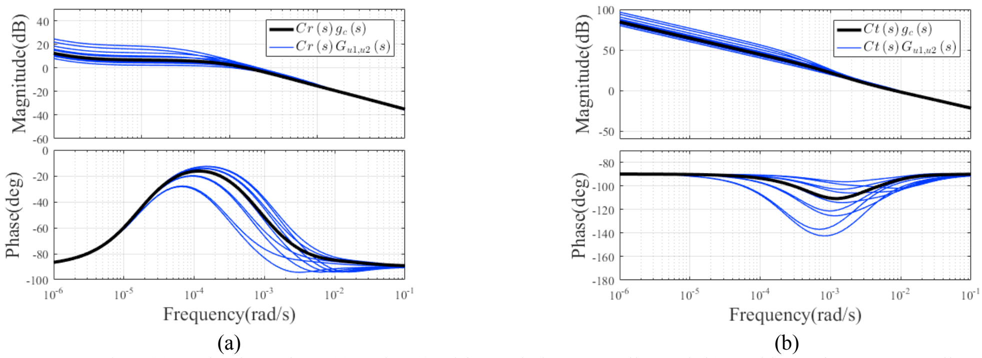
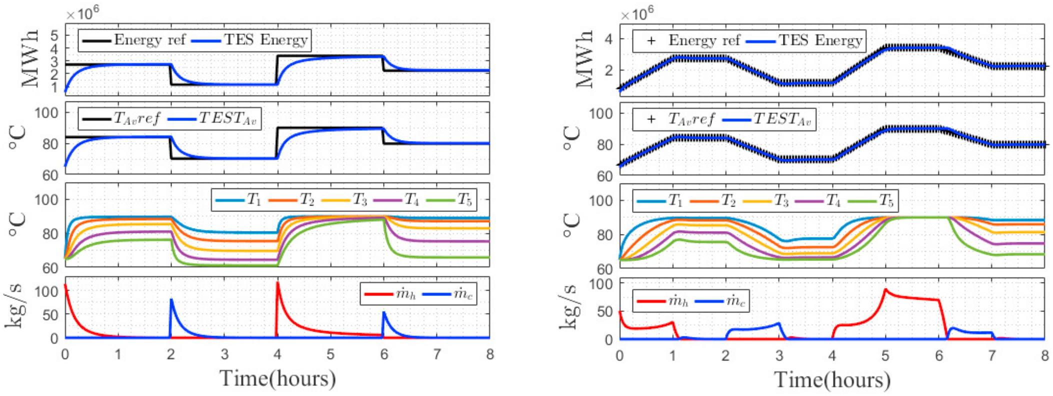


Fig. 3(a) Bode plots of $G_{1,2}(s)$ and $g_c(s)$ with regulation controller and (b) tracking reference controller.

4. Simulation results

The control system shown in Fig. 2(a) was implemented in MATLAB/Simulink using the non-linear model defined by (4)-(6). An energy reference is requested to the system. Using (13), the energy requirement is converted to a temperature reference, which controllers (17) and (18) attempt to regulate or track, respectively. The controller output charges or discharges the energy of the TES, injecting hot or cold water depending on the reference changes. The energy supplied by $\dot{m}_h$ or taken by $\dot{m}_c$ affects the layer temperatures, which are shown in Fig. 4. As it can be observed in Fig. 4(a), an effective regulation is achieved. Nevertheless, an abrupt and high demand of mass flow rate is required, which in practice may damage pipes in a TES. In addition, the pipe size might not be adequate for such mass flow rate peaks. Conversely, the reference tracking controller follows the energy reference without exhibiting noticeable errors. More importantly, the mass flow rate required by this controller does not present peaks during the energy tracking trajectory.



(a) (b)

Fig. 4. Simulation results (energy and temperature references, layer temperatures and streams) for (a) regulation and (b) tracking controllers.

5. Conclusions

In this paper, a dynamic non-linear model for TES was developed applying a CFD approach. Using system linearisation, transfer functions were obtained for different operating points. Using a closed-loop configuration, the two-input system was converted to a single-input system, simplifying the control design and its implementation. Reference tracking and regulation controllers were designed using Bode-shaping techniques. Simulation results show that these controllers achieve a good regulation and tracking performance despite system non-linearity. The mass flow rate demand is an important issue in DH systems since water streams are shared between TES and heating loads. Thus, a reference tracking controller provides the best option for TES charging and discharging processes. It should be highlighted that the work presented in this paper is part of ongoing research of the dynamic modelling and control of DH systems.

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