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Thermal Dynamic Modelling and Temperature Controller Design for a House

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Abstract

Heat consumption management and effective temperature control strategies to meet heat demand in residential and office buildings have become an important aspect within energy management. A thermal dynamic model of a building is not only necessary to estimate the energy consumption under different operating conditions but also to design effective controllers. This paper presents a classical control approach for the indoor temperature regulation of buildings. State-space and transfer function models of house thermal behaviour are developed. These are obtained from first principles of heat transfer and their analogy with electrical systems. To capture a realistic behaviour of heat transfer, the proposed models consider parametric uncertainties. A frequency response-based approach is used to obtain a reduced order system that facilitates control system design. The models have been implemented in MATLAB/Simulink and a PI controller has been designed to maintain a comfortable indoor temperature in the building. Simulation results show that the controller effectively regulates temperature despite system disturbances. An energy saving of around 8% comparing the proposed controller to a traditional on/off controller is achieved.

Keywords: Building thermal control; building dynamic model; control system design; frequency response; heat transfer coefficient

1. Introduction

Efficient management of energy flows in buildings has an important economic impact in terms of energy savings. Building indoor temperature control not only requires effective control strategies, but also suitable building models. Inefficiency in heating operation could represent significant energy losses. Although a conventional on/off controller

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which does not consider the building's thermal dynamics could simplify the implementation of a temperature control system, a significant part of the energy supplied by the heating systems may be wasted. To prevent this shortcoming, it is necessary to both use accurate building models and more suitable control alternatives. The most common approach to model the thermal behaviour of a building is based on equivalent thermal parameters, where equivalent resistive and capacitive (RC) networks are employed to model the heat flow through a solid surface, such as a wall [1]. These elements are denoted as layers. Some references follow this approach for control system design [2-4]; however, the more building layers there are, the more complex the system becomes. It is thus important to obtain a reduced order model that simplifies the control system design process.

Using an RC networks approach, it is possible to reduce the order of the building model by analysing the influence of the resistance of each layer over the heat flow injected in each capacitor. This way, a dominant layer is obtained [5]. Another approach is to obtain the thermal conductivity average of a multi-layered element [6]. In terms of model description, the state-space model is the most popular representation. Nevertheless, it requires the measurement or estimation of every state variable in the system, which implies a higher implementation cost. Alternatively, the calculation of fundamental parameters, such as heat transfer coefficients, has been adopted [7]. Following this line, a single-input single-output (SISO) model of the dynamic thermal behaviour of a house is developed in this paper. This is based on equivalent RC networks and makes use of the calculation of heat transfer coefficients. A frequency response method borrowed from control engineering is applied to reduce the order of the model with sufficient accuracy. The method uses Bode diagrams, enabling control design in the frequency domain. A PI controller is then designed. The full control system is implemented in MATLAB/Simulink, with results demonstrating a good performance under system disturbances in addition to a better energy consumption than that of a traditional on-off controller.

Nomenclature

k	thermal conductivity [W/m°C]	Re	Reynolds number	m	mass [kg]
A	area [m^2]	Nu	Nusselt number	c_p	specific heat [J/kg°C]
U	heat transfer coefficient [W/m^2°C]	Pr	Prandtl number	C	thermal capacitance [J/°C]
L_t	thickness [m]	υ	the mean velocity [m/s]	R_v	convective resistance [°C/W]
L	length [m]	v	kinematic viscosity [m^2/s]	R_d	conduction resistance [°C/W]
ρ	density [kg/m^3]	α	absorption coefficient	I_s	solar radiation [W/m^2]

2. Dynamic thermal model of a house

A state-space dynamic thermal model of a building is developed using electro-thermal analogies. The calculation of convection and conduction heat transfer coefficients is described. Since the variables involved in the convection phenomenon depend on weather conditions, uncertainty ranges are established to capture different ambient conditions within the model during a specific year season.

2.1. Heat transfer coefficients

The indoor temperature of a building is determined by its heat transfer processes; *i.e.* convection, conduction, and radiation [8]. However, the thermal behaviour is mainly influenced by convection and conduction, with radiation being considered as a disturbance. Convection and conduction can be described, respectively, by

$$q_{cd} = (kA/L_t)(T_2 - T_1) = (U_d A)/(T_2 - T_1)\,, \tag{1}$$

$$q_{cv} = U_v A(T_2 - T_1)\,. \tag{2}$$

Using an electric circuit analogy, the heat flow can be modelled as a current, the temperature as a voltage source, and the thermal storage as a capacitance. According to (1) and (2), the convection and conduction resistances can be defined as $R_{cv}=1/U_vA$ and $R_{cd}=L/kA=1/U_dA$, respectively. To illustrate this concept, Fig. 1 shows the electric circuit representation of a simple wall and an insulated one. The convection heat transfer coefficient U depends on the mean velocity of the air. It is described by hydrodynamic and thermodynamic characteristics of the air and its value changes with temperature and speed. Although the accurate calculation of U is complicated, it is possible to compute an average value suitable for practical applications. In the work reported in this paper, U is obtained from the following equations: $U=\mathrm{Nu}k/L$, $\mathrm{Re}=\upsilon L/v$ and $\mathrm{Nu}=0.664\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}$ [8].

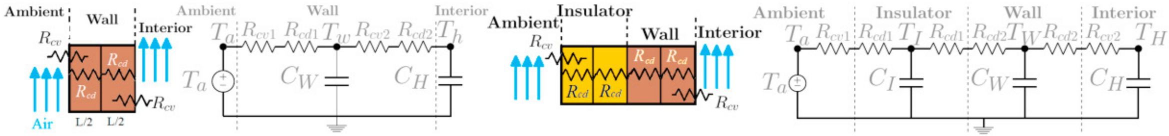
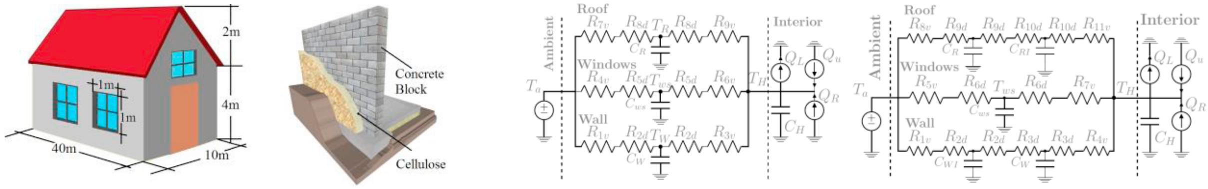


Fig. 1. Electric circuit analogy of convection and conduction heat transfer through a wall and an insulator.

2.2. Case study

The house geometry and the properties of the building materials are used to calculate the thermal storage capacity. The thermal capacitance is calculated as $C=mc_p$. The house dimensions are shown in Fig. 2. The energy balance equation for each house element can be obtained by applying Kirchhoff's laws over each grid node. The energy stored by the door is neglected. To exemplify the modelling approach, two different thermal models for the house shown in Fig. 2 are developed: with insulation and without an insulator. The insulator used is cellulose and it covers the roof and walls. The wall material is lightweight concrete and the roof is made with asbestos tiles. The electric circuit representations are shown in Figs. 2(c) and 2(d). The models include the energy loss (Q_L), energy inputs due to solar radiation and occupancy rate (Q_R), and heat input (Q_u).



(a) (b) (c) (d)

Fig. 2. (a) House dimensions. (b) Insulated wall by cellulose [9]. Electric circuit analogy of thermal model: (c) without and (d) with insulator.

The dashed lines in Figs. 2(c) and 2(d) separate the house elements schematically. Each energy storage element (capacitor) receives the heat transferred (current) by its surrounding elements. The total energy exchange depends on the element temperatures, which change dynamically. Table 1 shows the equations for both models. The calculation of U will be affected by the thermodynamic air properties, which in turn are determined by air temperature. These properties are provided in Table 2. Table 3 shows the values required to calculate the resistances and thermal capacitances, which are given in Table 4. These parameters are obtained with the equations provided in Section 2.1 and using $C=mc_p$, respectively. These values reflect the thermal dynamics of the system.

Table 1. Energy balance equations for both configuration models.

	House (without insulator)		Insulated House
Roof	$C_R\dot{T}_R=[R_{7v}+R_{8d}](T_a-T_R)+[R_{8d}+R_{9v}](T_H-T_R)$	Roof-Insulator	$C_R\dot{T}_R=[R_{8v}+R_{9d}](T_a-T_R)+[R_{9d}+R_{10d}](T_{RI}-T_R)$
		Insulator-Interior	$C_{RI}\dot{T}_{RI}=[R_{9d}+R_{10d}](T_R-T_{RI})+[R_{10d}+R_{11v}](T_H-T_{RI})$
Wall	$C_W\dot{T}_W=[R_{1v}+R_{2d}](T_a-T_W)+[R_{2d}+R_{3v}](T_H-T_W)$	Insulator-Wall	$C_{WI}\dot{T}_{WI}=[R_{1v}+R_{2d}](T_a-T_{WI})+[R_{2d}+R_{3d}](T_W-T_{WI})$
		Wall-Interior	$C_W\dot{T}_W=[R_{3d}+R_{2d}](T_{WI}-T_W)+[R_{3d}+R_{4v}](T_H-T_W)$
Windows	$C_{ws}\dot{T}_{ws}=[R_{4v}+R_{5d}](T_a-T_{ws})+[R_{5d}+R_{6v}](T_H-T_{ws})$	Windows	$C_{ws}\dot{T}_{ws}=[R_{5v}+R_{6d}](T_a-T_{ws})+[R_{6d}+R_{7v}](T_H-T_{ws})$
Interior	$C_H\dot{T}_H=[R_{8d}+R_{9v}](T_R-T_H)+[R_{2d}+R_{3v}](T_W-T_H)$ $+[R_{5d}+R_{6v}](T_{ws}-T_H)+Q_R-Q_L+Q_u$	Interior	$C_H\dot{T}_H=[R_{3d}+R_{4v}](T_W-T_H)+[R_{6d}+R_{7v}](T_{ws}-T_H)$ $+[R_{10d}+R_{11v}](T_{RI}-T_H)+Q_R-Q_L+Q_u$

Table 2. Air hydro and thermodynamic properties.

Variable	Min Value	Max Value	Unit		Min Value	Max	Value
External wind speed	3.65	5.7	m/s	Prandtl number *	0.720	0.701	
Internal wind speed	0.1	0.3	m/s	Thermal conductivity*	0.020	0.028	W/m°C
Kinematic viscosity*	12.43×10^{-6}	15.98×10^{-6}	m^2/s				

* Value ranges between −10°C to 30°C [10]

Table 3. House materials, areas and interior air properties

Element	Material	k	c_p	ρ	Thickness	Area
Wall	Lightweight concrete block	0.190	1000	600	0.2	435.95
Roof	Asbestos tiles	0.550	837	1900	0.025	522.16
Windows (6)	Glass	0.7	837	2500	0.01	6
Insulator	Cellulose fill IN13	0.043	837	10	0.2	-
Interior	Air	-	1005.4	1.225	-	-

Table 4. House with insulator resistance values [W/°C] and thermal capacitances [J/°C].

Equivalent R	Value	Equivalent R	Value	C	Value	C	Value
$R_{1v}+R_{2d}=R_1$	[134.54, 147.89]	$R_{6d}+R_{7v}=R_5$	[6.81, 13.76]	C_R	6.556×10^6	C_W	5.232×10^7
$R_{2d}+R_{3d}=R_2$	152.8649	$R_{8v}+R_{9d}=R_6$	[557.03, 809.56]	C_{RI}	8.741×10^5	C_{WI}	7.298×10^5
$R_{3d}+R_{4v}=R_3$	[72.03, 134.60]	$R_{9d}+R_{10d}=R_7$	222.3570	C_{ws}	125550	C_H	1.971×10^6
$R_{5v}+R_{6d}=R_4$	[39.53, 56.85]	$R_{10d}+R_{11v}=R_8$	[66.50, 103.65]				

Notice that the resistance values shown in Table 4 are given in ranges depending on the variations to the air hydro and thermodynamic properties (see Table 2). To simplify the model notation and the control design process, the equivalent resistances have been renamed. Using the information provided by Tables 1-4, a state-space representation of the form

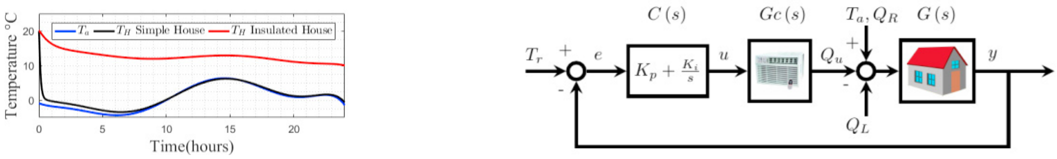
$$\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}u\,, \qquad y=\mathbf{C}\mathbf{x}+\mathbf{D}u\,, \tag{3}$$

has been obtained, where the system states are the element temperatures, the system input is the heat supplied by an actuator (such an HVAC or a radiator set Q_u), and the output is the house interior temperature (T_H). The ambient temperature (T_a), solar radiation, people transit and energy losses due to leaks are considered as disturbances. Using the equations to calculate U and the notation in Table 4, the insulated house state-space model is given by:

$$\begin{bmatrix}\dot{T}_{WI}\\ \dot{T}_{W}\\ \dot{T}_{ws}\\ \dot{T}_{R}\\ \dot{T}_{RI}\\ \dot{T}_{H}\end{bmatrix}=\begin{bmatrix}(-R_1-R_2)/C_{WI} & R_2/C_{WI} & 0 & 0 & 0 & 0\\ R_2/C_W & (-R_2-R_3)/C_W & 0 & 0 & 0 & R_3/C_W\\ 0 & 0 & (-R_4-R_5)/C_{ws} & 0 & 0 & R_5/C_{ws}\\ 0 & 0 & 0 & (-R_6-R_7)/C_R & R_7/C_R & 0\\ 0 & 0 & 0 & R_7/C_{RI} & (-R_7-R_8)/C_{RI} & R_8/C_{RI}\\ 0 & R_3/C_H & R_5/C_H & 0 & R_8/C_H & (-R_3-R_5-R_8)/C_H\end{bmatrix}\begin{bmatrix}T_{WI}\\ T_W\\ T_{ws}\\ T_R\\ T_{RI}\\ T_H\end{bmatrix}+\begin{bmatrix}0\\0\\0\\0\\0\\1/C_H\end{bmatrix}[Q_u]+\begin{bmatrix}R_1/C_{WI}\\0\\R_4/C_{ws}\\R_6/C_R\\0\\0\end{bmatrix}[T_a]\,, \tag{4}$$

$$y=[0\;\;0\;\;0\;\;0\;\;0\;\;1]\mathbf{x}+[0]Q_u\,. \tag{5}$$

Notice that the effect of T_a is explicitly shown as a disturbance. Under this consideration, it can be noticed that the system represented by (3)-(5) would be linear for each operating point. Fig. 3(a) shows the response of T_H for both house models given a cold ambient temperature T_a. As expected, T_H for a house with cellulose coverage is less affected by T_a than a non-insulated one. Additionally, T_H decreases more slowly in an insulated house.



(a) (b)

Fig. 3. (a) Thermal response of both models for a cold day without heat supply. (b) Closed-loop control scheme.

3. Model reduction and controller design

The state-space model in Section 2 offers information on the dynamic thermal behaviour of the house. However, its direct use requires the measurement/estimation of all state variables of the system. Instead of using a state feedback controller, a SISO model representation in the frequency domain is employed for control system design.

3.1. Model reduction

A transfer function relating the heat input (Q_u) and the temperature (T_H) can be established with

$$Y(s)/U(s)=T_H(s)/Q_u(s)=G(s)=\mathbf{C}(s\mathbf{I}\text{-}\mathbf{A})^{-1}\mathbf{B}\,. \tag{6}$$

Given that ambient conditions will affect the coefficients of system matrix **A**, obtaining an analytic representation for $G(s)$ becomes a complex and tedious process. Instead, a set of transfer functions is computed for all possible combinations of coefficient values. For instance, the wall convection coefficient U considering a maximum value of air speed and the air properties for temperatures −10°C and 30°C are calculated as $U_{-10°C}$=1.36 and $U_{30°C}$=1.6, which in turn can be used to define value ranges for R_{1v}. In general, the transfer function set is defined by:

$$G(s)=\frac{T_H(s)}{Q_u(s)}=\frac{5.075_{-7}s^5+[6.17_{-10},7.65_{-10}]s^4+[2.65_{-13},4.08_{-13}]s^3+[4.58_{-17},8.85_{-17}]s^2+[2.39_{-21},6.15_{-21}]s^1+[6.21_{-27},2.21_{-26}]}{s^6+[1.3_{-3},1.62_{-3}]_5s^5+[6.10_{-7},9.7_{-7}]_4s^4+[1.26_{-10},2.61_{-10}]s^3+[1.07_{-14},3.07_{-14}]s^2+[3.07_{-19},1.28_{-18}]s^1+[5.5_{-25},2.72_{-24}]}\,, \tag{7}$$

where X_{-y} stands for $X\times10^y$. It can be noticed that (7) is a 6th order transfer function, which may not be suitable for control system design. A frequency response method for system identification [11] is adopted to reduce the system order. It considers two steps: (i) plotting the Bode diagrams for each transfer function of the set described by (7), and (ii) proposing a system $g_a(s)$ that resembles $G(s)$ as much as possible. The last step is done firstly by establishing a 2nd order transfer function with a specific natural frequency ω_n and damping ratio ζ. This is given by

$$g_a(s) = \omega_n^2 / \left(s^2 + 2\zeta\omega_n s + \omega_n^2\right) . \tag{8}$$

A suitable zero is included to (8) to improve the approximation. This way, the model is approximated by a 2nd order system with $\omega_n = 0.9\times10^{-4}$, ζ=1.5, and a zero located in z=8.33×10^{-5}:

$$g_a(s) = \left(5.443\times10^{-7} s \ + \ 4.536\times10^{-11}\right) / \left(s^2 + 0.000135 s + 4.05\times10^{-9}\right) . \tag{9}$$

3.2. PI controller design

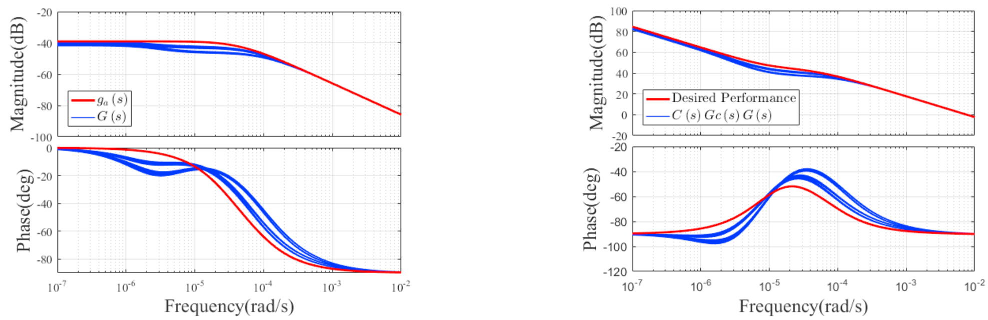
The closed-loop system for temperature regulation is shown in Fig. 3(b). It includes an actuator with a regulated output, which could be an air conditioner or a radiator supplying heat to the house. The modelling of such an actuator is out of the scope of this work. Instead, a simple transfer function representing an HVAC is adopted [12]:

$$G_C = (T_H m_a c_p) / 0.5s + 1 \ , \tag{10}$$

where T_H is the heater temperature [°C], m_a is the mass flow rate [kg/s] and c_p the specific heat of the air. Fig. 4(a) shows the open loop frequency responses for the set of plants $G(s)$. As it can be seen, the approximation given by $g_a(s)$ properly fits to $G(s)$, allowing a control design over a more tractable system. The desired closed-loop performance requirements are defined as a settling time of 3600 s and a maximum overshoot of 10%. These specifications are translated to the frequency domain as a damping ratio ζ=0.59, a phase margin of at least 58° and a minimum bandwidth ω_{bw}=0.007 rad/s. To meet these requirements, a PI controller is designed as follows:

$$C(s) = k_p + k_i / s = 0.065 + 2.7\times10^{-6} / s \ . \tag{11}$$

Controller (11) was obtained using Bode shaping techniques to achieve a phase margin of 90°. To avoid a large energy demand, a new bandwidth ω_{bw}=0.003 rad/s was adopted. The frequency responses of the controlled system are shown in Fig. 4(b). As it can be observed, a good performance for all plants $G(s)$ has been achieved.

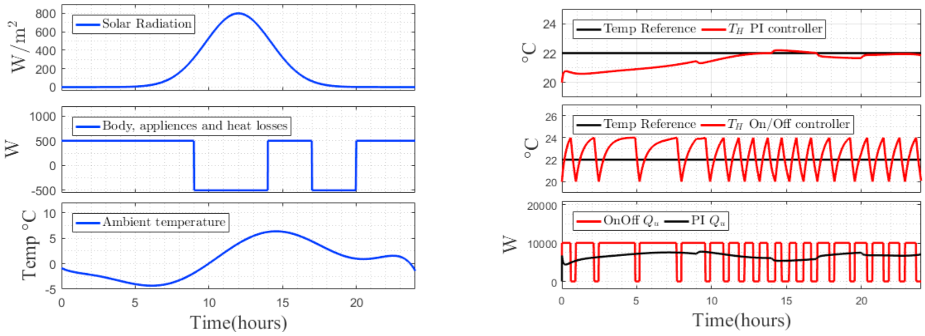


(a) (b)

Fig. 4. (a) Approximated model $g_a(s)$ and $G(s)$ open-loop Bode plots. (b) Bode plot of $C(s)G_C(s)G(s)$ and desired performance.

4. Results

A closed-loop simulation is performed in MATLAB/Simulink using the designed PI and conventional on/off controllers. Disturbances are applied as shown in Fig. 3(b). The heat injected to the house by solar radiation depends on the position of the windows with respect to the North and this is calculated by $Q_R = \alpha A_{ws} I_s$. The direction of I_s with respect to the windows determines the amount of radiation supplied. In this paper, this is considered as the maximum possible value for all windows to assess the worst disturbance to the system. For this condition, α = 0.2, A_{ws} =1 m^2 and I_s = 800 W/m^2. Additional disturbances due to appliances, house occupants and heat losses are included, as shown in Fig. 5(a). Fig. 5(b) shows the closed-loop responses during a cold day with an outdoor wind speed of 3.5 m/s and an internal wind speed of 0.1 m/s. A constant temperature reference is maintained throughout the day. As it can be observed, the use of the designed PI controller drastically improves system performance in terms of temperature regulation and energy consumption. Considering the outputs of the actuators, an energy consumption of 173.5 kWh is obtained when the on/off controller is used and 159.8 kWh with a PI controller. This represents a saving of 7.9% of energy when the proposed control scheme is adopted.



(a) (b)

Fig. 5. (a) Ambient temperature and system disturbances. (b) On/off and PI controller performances.

5. Conclusions

In this paper, a dynamic thermal model of a building was developed using electric circuit analogies. Heat transfer coefficients were computed based on energy transfer principles. An approach for building modelling which considers uncertainties in the calculation of the heat transfer coefficients was adopted. A state-space model was developed, which was used to obtain a transfer function equivalent. A frequency response approach was employed to define a reduced order model suitable for control system design. A PI controller was designed using Bode shaping techniques. The results show that the house temperature regulation was improved when the designed controller was adopted instead of a conventional on/off strategy. It is important to emphasise that the modelling approach adopted in this work can be applied for more complex buildings. Since the systems are described by 1st order ODEs, the computation cost to simulate these models is low. The methodology presented in this work may be expanded to the modelling of thermal loads for more comprehensive integrated energy systems.

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