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# Voltage Control Method of Distribution Networks Using PMU Based Sensitivity Estimation

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## Abstract

Application of phasor measurement unit (PMU) at distribution networks provide new options for voltage-to-power sensitivity estimation and voltage regulation. A novel voltage control method for distribution networks using PMU based sensitivity estimation is proposed in this paper. The voltage control records are extracted from the historical synchronized phasor measurements. The voltage-to-power sensitivities to reflect the relation of voltage change and power fluctuation are estimated with the obtained voltage control records. In addition to linear parameter, parameters to match the nonlinear relation between voltage and power variation and to track the operation conditions are introduced in the fitting model for sensitivity estimation to improve the accuracy of the voltage control strategy. A voltage control scheme is proposed based on the sensitivities estimated in which the measurements of partial nodes at the distribution network are the only needed data. Case studies on IEEE 33-node test feeder verify the correctness and effectiveness of the proposed method.

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## 1. Introduction

The increasing penetration of distributed generators (DGs) and flexible loads poses new challenges to the operation control and energy management of distribution networks [1]. Voltage problem is a key issue raised by the integration

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of DGs [2]. Moreover, the intermittency and volatility of the new electrical equipment as DGs and electric vehicles (EVs) make the voltage fluctuation of distribution network more violent and more frequent to be out of limit [3]. Although the access of distributed energy resources and flexible loads increases the risk of voltage issues, it also provides ways to mitigate the corresponding problem [4], [5].

The optimization model of voltage control should be established to handle the voltage problem of distribution networks. The complex nonlinear relations of power and voltage are modeled in the optimization problem and the voltage control strategy is obtained from the solution [6]. However, the optimization model for voltage control is naturally too complex to get effective solution [7]. Linearizing the nonlinear optimization problem into a linear programming problem with the help of voltage-to-power sensitivity is an effective way to overcome this difficulty. In reference [8], sensitivities are extracted by inverting the power flow Jacobian matrix with which the voltage control model is simplified. However, the linear sensitivities vary with the change of the distribution system operation status. The authors in reference [9] obtained the sensitivities from the line parameters directly that are constant under different operation status. But their approximation error is quite large. The accuracy in fitting the relation of voltage and power variations is improved in [10] as the equivalent impedance and nodal voltage are considered simultaneously.

By model linearization, the efficiency to solve the voltage control problem is highly improved, but the aforementioned methods are all rely on accurate line parameters. It is always difficult to acquire accurate line parameters in geographic information system of distribution networks which has adverse effect to voltage control. To address this issue, identical sensitivities to reference [8] are obtained in reference [11] from recorded voltage change and the power adjustment in voltage control. The historical measurements of power and voltage in various scenarios are exploited in reference [12] to fitting the relation of power and voltage via high-dimension functions and the sensitivities are obtained from the partial derivative functions. However, the actual operational data are hardly to meet the requirements.

The application of synchronized phasor measurements can greatly facilitate the online monitoring and boost the state estimation, fault location and topology identification of distribution networks [13]. The synchronism of the PMU measurements also makes it possible to record the output of controllable resources and the voltage changes of the nodes concerned in voltage control.

In this paper, the historical records of voltage control are exacted from the PMUs installed on partial nodes of the distribution network, and the voltage-to-power sensitivities are further estimated with these records. The quadric term and term associated with the nodal voltage measurements are introduced in the fitting function of power and voltage variations to match their nonlinear relations and track the change of operation status. Compared with linear sensitivity, the proposed fitting function enhanced the accuracy by exploiting the redundant measurements. Voltage control strategy is proposed based on the estimated sensitivity parameters in which the priority of schedulable resources to participate in the voltage control is confirmed and the quantity of the required resources is calculated. Case studies on IEEE 33-node test feeder verify the correctness and effectiveness of the proposed method.

## 2. PMU-based estimation of voltage-to-power sensitivity

The historical measurements of PMU for analysis can be acquired via phasor data concentrator (PDC) [14]. The records of voltage and power before and after the voltage control can be extracted from the historical measurements with time stamps which are the base data for sensitivity estimation.

### 2.1. Linear sensitivity estimation

Suppose that  $V_{b,i}$  and  $V_{e,i}$  denote the voltage of node  $i$  before and after voltage control respectively. Consider that the powers of other nodes have little change, the voltage change of node  $i$  is caused by the power change of node  $j$ , denoted by  $\Delta Q_j$ . Let  $\Delta V_i = V_{e,i} - V_{b,i}$ , the sensitivity of node  $i$  to node  $j$  can be approximated by the following equation.

$$S_{i,j} \approx \Delta V_i / \Delta Q_j \quad (1)$$

Suppose that  $\Delta \mathbf{V}_i = [\Delta V_i[1], \dots, \Delta V_i[C]]^T$  and  $\Delta \mathbf{Q}_j = [\Delta Q_j[1], \dots, \Delta Q_j[C]]^T$  denote the voltage and power variation vector of  $C$  sets of voltage control records respectively. Then, the optimal linear approximation can be obtained via least-

squares estimation (LSE) with the measurement redundancy to handle the adverse effects of the power variations of other nodes and measurement errors of PMU.

$$\hat{S}_{i,j} = (\Delta Q_j^T \Delta Q_j)^{-1} \Delta Q_j^T \Delta V_i \quad (2)$$

## 2.2. High dimension expression for sensitivity relation of voltage to power

Linear sensitivities are not accurate when the operation status changed or the power change is considerable large. To address this problem, the fitting function of power variation with voltage variation is further expressed as follow.

$$\Delta V_i = S_{i,j,1} \Delta Q_j / V_{bi} + S_{i,j,2} \Delta Q_j + S_{i,j,3} \Delta Q_j^2 \quad (3)$$

where  $S_{i,j,1}$ ,  $S_{i,j,2}$ , and  $S_{i,j,3}$  are sensitivity parameters to be estimated.  $S_{i,j,1}$  is the parameter to track the system operation status change.  $S_{i,j,2}$  is the linear fitting parameter.  $S_{i,j,3}$  is the parameter to match the nonlinear relation.

Supposed that  $A_1 = [\Delta Q_j[1]/V_{bi}[1], \dots, \Delta Q_j[C]/V_{bi}[C]]^T$ ,  $A_2 = [\Delta Q_j[1], \dots, \Delta Q_j[C]]^T$ ,  $A_3 = [\Delta Q_j^2[1], \dots, \Delta Q_j^2[C]]^T$ ,  $A = [A_1 A_2 A_3]$ . When  $C > 3$ , the optimal estimation of the sensitivity parameters via LSE is expressed as (4).

$$\begin{bmatrix} \hat{S}_{i,j,1} & \hat{S}_{i,j,2} & \hat{S}_{i,j,3} \end{bmatrix}^T = (A^T A)^{-1} A^T \Delta V_i \quad (4)$$

where  $\hat{S}_{i,j,1}$ ,  $\hat{S}_{i,j,2}$ , and  $\hat{S}_{i,j,3}$  are the optimal estimates of  $S_{i,j,1}$ ,  $S_{i,j,2}$ , and  $S_{i,j,3}$ .

## 3. Sensitivity based Voltage control method

It is constrained by the cost to install PMU on each node of distribution networks. In distribution networks, voltage problems usually arise in end nodes or the nodes with DGs and the voltage issues of the whole distribution network are eliminated after solving the problems in these nodes. Therefore, in this paper, we define that the nodes that are most likely to out of voltage limits as nodes with voltage observation. The nodes with schedulable reactive power resources are defined as controllable nodes. The voltage observation and controllable nodes are all configured with PMUs.

### 3.1. Priority of the voltage control

Priority of voltage control is confirmed to dispatch the most sensitive schedulable reactive power resources in voltage control to fulfill the purpose with least resources and ensure the optimality of the voltage control strategy. Without the knowledge of the line parameters, the voltage control priority is determined via the topology information. The flowchart to confirm the priority of voltage control is listed as follow.

- (1) Find the power supply path from the source node to the voltage observation nodes.
- (2) The priorities of the nodes on the power supply path are defined according to their electrical distance to the voltage observation nodes. The longer the electrical distance, the lower the voltage control priority.
- (3) For the nodes on the branch of the power supply path, their priorities are higher than the corresponding branch node and lower than the downstream node of this branch node. The longer electrical distance to the branch node, the higher priority of the node.
- (4) For the downstream nodes of the voltage observation nodes, their priorities are all higher than the corresponding voltage observation node. The longer electrical distance to the voltage observation node, the higher priority of the node.
- (5) The voltage control priorities of each controllable node to each voltage observation node are finally recorded according to their connection locations.

### 3.2. Voltage control scheme

The flowchart of the proposed voltage control scheme mainly includes following steps.

(1) Determine the priority list of voltage control according to the topology information and locations of controllable nodes and voltage observation nodes.

(2) Obtain the current voltage measurements of each voltage observation node. Calculate the voltage deviation of each node.

(3) Find the node with maximum voltage deviation and number the node with  $i$ , if its deviation is larger than the threshold value and its voltage measurement is out of the dead zone, continue to step (4), else skip to step (8).

(4) Obtain the priority list of node  $i$ , set  $n=1$ .

(5) If the  $n$ th controllable node with number  $j$  in the priority list has extra regulation capacity to participate in voltage control, continue to step (6); else if  $n$  is greater than the number of the controllable nodes, skip to step (8); else  $n=n+1$  and repeat step (5).

(6) Read measurements from the database of historical voltage control records and estimate the voltage-to-power sensitivity parameters of node  $i$  to node  $j$ . Determine the capacity scheduled, expressed as  $\Delta Q_j$ , of node  $j$  according to the sensitivity parameters, voltage deviation of node  $i$ , and extra capacity of node  $j$ , denoted by  $Q_{R,j}$ . The detailed procedure to calculate  $\Delta Q_j$  is shown below.

a. Calculate the difference between the current measured value and the voltage limit as  $\delta V_i = \min\{|V_{\min} - V_i|, |V_{\max} - V_i|\}$ ;

b. If the number of corresponding voltage control records in database is empty, let  $\Delta Q_j = 1.0 \text{ kVar}$  when  $V_i < V_{\min}$  or  $\Delta Q_j = -1.0 \text{ kVar}$  when  $V_i > V_{\max}$ ;

c. If the number of corresponding voltage control records is smaller than 3, the linear sensitivity parameter  $S_{i,j,2}$  is estimated to calculate  $\Delta Q_j$ . Let  $\Delta Q_j = \min\{\delta V_i / S_{i,j,2}, Q_{R,j}\}$  when  $V_i < V_{\min}$  and  $\Delta Q_j = \max\{-\delta V_i / S_{i,j,2}, -Q_{R,j}\}$  when  $V_i > V_{\max}$ ;

d. If the number of corresponding voltage control records is greater than 2, parameters  $S_{i,j,1}$ ,  $S_{i,j,2}$ , and  $S_{i,j,3}$  are estimated to calculate  $\Delta Q_j$ . Set  $K_1 = S_{i,j,1} / V_i + S_{i,j,2}$ ,  $K_2 = S_{i,j,3}$ , let  $\Delta Q_j = \min\left\{\left(-K_1 + \sqrt{K_1^2 + 4K_2 \delta V_i}\right) / (2K_2), Q_{R,j}\right\}$  when  $V_i < V_{\min}$  and  $\Delta Q_j = \max\left\{\left(-K_1 + \sqrt{K_1^2 - 4K_2 \delta V_i}\right) / (2K_2), -Q_{R,j}\right\}$  when  $V_i > V_{\max}$ .

(7) Obtain the current voltage measurements of voltage observation nodes when the output of the controllable node reach to its scheduled capacity. Store the voltage control records and return to step (2).

(8) Go to next time step, return to step (2).

## 4. Case studies and analysis

The proposed method is tested on IEEE 33-node test feeder [15]. The threshold values of voltage deviation are set as 0.95 p.u. and 1.05 p.u. The dead zones of voltage control are set as [0.949, 0.951] and [1.049, 1.051]. The measurements are simulated from the power flow calculation, where Gaussian noise with zero mean and standard deviation 1% is used to simulate the PMU measurement error.

1) Accuracy analysis of the sensitivity parameters

Table 1. Voltage-to-power sensitivities

Methods	Parameters
Power flow Jacobian matrix inversion	0.038908
Linear sensitivity	0.038909
Sensitivity with 3 parameters	0.061477, -0.028161, -0.002754

Table 2. Results of voltage control

Methods	Results of voltage control
Power flow Jacobian matrix inversion	0.9481
Linear sensitivity	0.9481
Sensitivity with 3 parameters	0.9501

PMU and the reactive power resource that is continuously adjustable are configured at node 33. Node 33 is set as the voltage observation node. Linear sensitivity and sensitivity with 3 parameters are obtained from the proposed

voltage control scheme. All of the linear sensitivity, sensitivity with 3 parameters and power flow Jacobian matrix inversion methods [8] are used to calculate the reactive power needed to handle the voltage problem. The estimated and calculated sensitivity parameters are shown in Table 1. The voltage of node 33 after voltage control with different methods are shown in Table 2.

As shown in Table 1, the linear sensitivity estimated with measurements is as accurate as the sensitivity calculated from power flow Jacobian matrix inversion which means sensitivity estimation with measurements is effective. As shown in Table 2, the sensitivity with 3 parameters are more accurate than the linear sensitivity. The capacity of reactive power required to maintain the voltage within limit is quite large, the linear relation assumptions of voltage change to power variation is invalid.

2) Time series voltage control with DGs

PMUs and DGs are connected at node 12, 14, 18, 30, 33. Parameters of DGs are shown in Table 3. Node 18 and node 33 are set as voltage observation nodes. Their priority lists of voltage control are {18, 14, 12, 33, 30} and {33, 30, 18, 14, 12} respectively. The reactive power of DGs can be adjusted continuously and constrained by their capacity and active power output. The voltage control time step is set as 15 minutes and there are 96 time slots one day. Output coefficient curves of load, wind turbine (WT), and photovoltaic (PV) are shown in Fig. 1.

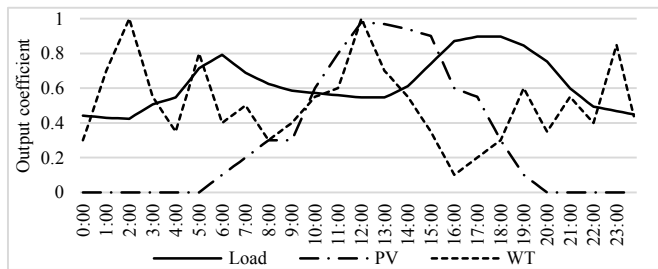


Fig. 1. Output coefficient curves of load, WT, and PV

Table 3. Parameters of DGs

Location	Type	Capacity (KVA)
12	PV	400
14	PV	300
18	WT	350
30	WT	150
33	WT	400

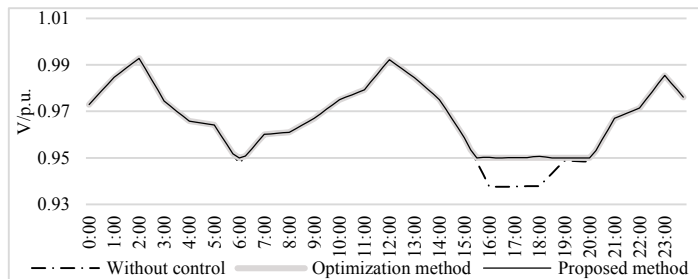


Fig. 2. Voltage control results of node 33

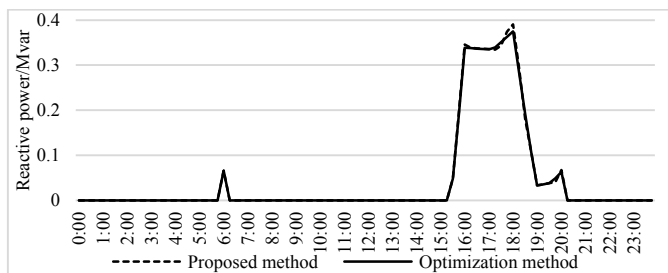


Fig. 3. Total reactive power of DGs at different time slot

To verify the optimality of the proposed method, the results of voltage control are compared with the optimization results of voltage control model with accurate line parameters. Voltage control results of node 33 are shown in Fig. 2. As shown in Fig. 2, the voltage problem of node 33 is settled with the proposed method. The results of the proposed method are fairly consistent with the optimization method with accurate line parameters. The total reactive power of DGs at various time slot is shown in Fig. 3. The total reactive power is almost identical with the two methods at various time slot. The total reactive power output in the whole day with the proposed method and the optimization method with accurate line parameters are 1.060Mvarh and 1.054Mvarh respectively. The relative error is 0.632%. Therefore, the optimality of the proposed method is guaranteed.

## 5. Conclusion

A novel voltage control method of distribution networks with PMU based sensitivity estimation is proposed in this paper. The historical measurements of PMU are used to estimate the voltage-to-power sensitivity parameters. The parameters to track the operation status change and to match the nonlinear relations between voltage variation and power variation are introduced in fitting function. Compared with the linear sensitivity, the proposed sensitivity parameters are more accurate. Voltage control scheme is proposed based on the estimated sensitivity parameters in which output of the schedulable resources are calculated directly according to the voltage measurements of the voltage observation nodes. The proposed voltage control method is completely based on the measurements which is agnostic to the errors in line parameters. By comparing with the optimization method with accurate line parameters, the proposed method can settle the voltage problem effectively. The proposed method provides new options of voltage control for active distribution networks.

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## References

- [1] Ehsan A, Yang Q. Optimal integration and planning of renewable distributed generation in the power distribution networks: a review of analytical techniques. *Applied Energy*, 2018; 210:44-59.
- [2] Farag HE, El-Saadany EF. Voltage regulation in distribution feeders with high DG penetration: From traditional to smart. *Power and Energy Society General Meeting*, July 2011.
- [3] Boynuegri AR, Vural B, Tascikaraoglu A, Yumurtacı R. Voltage regulation capability of a prototype Static VAR compensator for wind applications. *Applied Energy*, 2012; 93(5):422-431.
- [4] Zakariazadeh A, Homaei O, Jadid S, Siano P. A new approach for real time voltage control using demand response in an automated distribution system. *Applied Energy*, 2014; 117(6):157-166.
- [5] Elkhatib ME, El-Shatshat R, Salama MMA. Novel coordinated voltage control for smart distribution networks with DG. *IEEE Transactions on Smart Grid*, 2011; 2(4):598-605.
- [6] Kulmala A, Repo S, Jarventausta P. Coordinated voltage control in distribution networks including several distributed energy resources. *IEEE Transactions on Smart Grid*, 2014; 5(4):2010-2020.
- [7] Antoniadou-Plytaria KE, Kouveliotis-Lysikatos IN, Georgilakis PS. Distributed and decentralized voltage control of smart distribution networks: models, methods, and future research. *IEEE Transactions on Smart Grid*, 2017; 8(6):2999-3008.
- [8] Zhou Q, Bialek JW. Generation curtailment to manage voltage constraints in distribution networks. *IET Generation Transmission & Distribution*, 2007; 1(3):492-498.
- [9] Zad BB, Hasanvand H, Lobry J, Vallée F. Optimal reactive power control of DGs for voltage regulation of MV distribution systems using sensitivity analysis method and PSO algorithm. *International Journal of Electrical Power & Energy Systems*, 2015; 68:52-60.
- [10] Youssef KH. A new method for online sensitivity-based distributed voltage control and short circuit analysis of unbalanced distribution feeders. *IEEE Transactions on Smart Grid*, 2015; 6(3):1253-1260.
- [11] Sansawatt T, Ochoa LF, Harrison GP. Smart decentralized control of DG for voltage and thermal constraint management. *IEEE Transactions on power systems*, 2012; 27(3):1637-1645.
- [12] Zhang Z, Ochoa LF, Valverde G. A novel voltage sensitivity approach for the decentralized control of DG plants. *IEEE Transactions on Power Systems*, 2018; 33(2):1566-1576.
- [13] Von Meier A, Stewart E, McEachern A, Andersen M, Mehrmanesh L. Precision micro-synchrophasors for distribution systems: a summary of applications. *IEEE Transactions on Smart Grid*, 2017; 8(6):2926-2936.
- [14] Phadke AG, Thorp JS. *Synchronized phasor measurements and their applications*. New York, Springer, 2008.
- [15] Baran ME, Wu FF. Network reconfiguration in distribution systems for loss reduction and load balancing. *IEEE Transactions on Power Delivery*, 1989; 4(2):1401-1407.