

ORCA - Online Research @ Cardiff

This is an Open Access document downloaded from ORCA, Cardiff University's institutional repository:https://orca.cardiff.ac.uk/id/eprint/125743/

This is the author's version of a work that was submitted to / accepted for publication.

Citation for final published version:

Dixon, Huw, Easaw, Joshy and Heravi, Saeed 2020. Forecasting inflation gap persistence: do financial sector professionals differ from non-financial sector ones? International Journal of Finance and Economics 25 (3), pp. 461-474. 10.1002/ijfe.1762

Publishers page: http://dx.doi.org/10.1002/ijfe.1762

Please note:

Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher's version if you wish to cite this paper.

This version is being made available in accordance with publisher policies. See http://orca.cf.ac.uk/policies.html for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.



Forecasting Inflation Gap Persistence: Do Financial Sector Professionals differ from Non-Financial Sector Ones?

Huw Dixon, Joshy Easaw*, Saeed Heravi,

Cardiff University Business School, Cardiff, UK

September 2019

Forthcoming in International Journal of Finance and Economics

We grateful acknowledge the comments and suggestions of the anonymous referees. We also thank Bo Guan for the research assistance. Any omissions or mistakes are entirely ours.

^{*} Corresponding author: :EasawJ1@cardiff.ac.uk

Forecasting Inflation Gap Persistence: Do Financial Sector Professionals differ from Non-Financial Sector Ones?

Abstract

The purpose of the present paper is to investigate forecasted inflation gap persistence using professionals' survey-based data, differentiating between financial and non-financial sectors professionals. We derive the forecasted inflation gap persistence and using a state dependent model, we estimate the non-linear persistence coefficient of the inflation gap. We distinguish between the pre-Great Moderation, Great Moderation and Great Recession periods. Our main results indicate that whilst the estimates of persistence for GDP inflation largely confirm the results obtained using a linear model, for CPI inflation we find that there is strong evidence for state-dependence and time variation. By and large, the results are consistent with the price stability policy pursued in the Great Moderation period and perceived disinflationary pressures during the Great Recession period.

Keywords: Forecasted Inflation Gaps and Persistence, Monetary Policy Regimes and Price Stability, Financial and Non-Financial Sector Professionals Survey-based Forecasts, State-Dependent Models

JEL classification: E31, E37, E52, E58.

I: Introduction

The main purpose of the present paper is to understand how professional forecasters in the financial and non-financial sectors form their forecast of inflation persistence and their perception of inflation gap persistence. Cogley et al (2010) suggest that inflation comprises of a slow trending local mean and stationary component defined as the inflation gap. Inflation gap, therefore, is the difference between *actual* inflation and *trend* inflation. A key question is: what is the persistence of the inflation gap and is this persistence influenced by monetary policy and macroeconomic shocks? While a number of papers have reported a decline in the persistence of shocks to inflation rates in the US (notably, Cogley and Sargent, 2001, and Levin and Piger, 2004), we wish to assess whether professionals' forecasts are consistent with this decline in persistence.

As argued by Jain (2019), assessing and comparing the difference between perceived inflation persistence and actual inflation persistence has valuable implications for central bank's price stability policy. Indeed, as Chan et al (2018) point out, the tolerance for any temporary deviation of inflation from its trend or target may vary with monetary authorities, and this was especially low during the Volker era. Coibion and Gorodnichenko (2015) also suggested that during the Volker era, forecasters' attention moved from macro to micro shocks, thereby increasing their inattentiveness. To what extent do professionals' forecasts of inflation gap persistence reflect the underlying persistence in the US inflation data and conduct of monetary policy?

We look at the behavior of professionals' forecasts in three sub-periods: pre-Great Moderation (pre-GM), the Great Moderation (GM) which began in the mid-1980s, and the Great Recession (GR) following the sub-prime financial crisis of 2007/08. The paper provides insights into the important issue of actual inflation persistence and the implications for monetary policy. The findings also contribute to a burgeoning literature investigating the

structure and nature of professional forecast errors (see, Coibion and Gorodnichenko (2015) and references therein).

The present paper contributes to the existing literature on inflation persistence in three respects. Firstly, we derive the inflation gap using the forecaster's short-horizon forecast of inflation, which is available over a longer period, for both the GDP deflator (hereafter: GDP inflation) and CPI (hereafter: CPI inflation). Importantly, the present paper extends the literature by allowing for non-linear persistence of the perceived inflation gap which is consistent with forward-looking behavior. The empirical analysis takes a novel approach as it considers both time-varying and state-dependent behavior. We argue that both need to be considered concurrently to understand the effect of monetary policy, especially the pursuit of price stability, on perceived inflation gap persistence. Secondly, and in addition to the existing literature, we derive the perceived inflation gap using the difference between the forecasters' long and short-horizon forecasts for GDP inflation, thus enabling us to compare both methods and their robustness. Thirdly, for the GDP inflation we consider and assess the difference between predictions of financial and non-financial sector forecasters.

Our main results are as follows. Firstly, financial sector forecasters consistently predict higher inflation persistence than their non-financial sector counterparts, but the difference is only statistically significant following the financial crisis of 2007-08, or GR period. Secondly, the forecasted inflation gap persistence for the GDP inflation (using the short horizon forecast) is slightly lower in pre-GM period and then becomes almost constant during the GM and GR periods. Thirdly, for the CPI inflation (using the short horizon forecast), the forecasted inflation gap persistence is much lower during the pre-GM and early GM period than most of the GM period (1990-2008). There is also a drop in perceived persistence in the GR period, though still higher than the pre-GM era. Fourthly, when we compare the short and the long-horizon method over the period 1994 to 2013, there is little evidence of time variation or state dependence.

The paper is outlined as follows. The next section introduces and discusses related literature. Section III outlines the simple theoretical framework which forms the basis for empirical analysis. Section IV reports and discussed the empirical analysis and results. Section V outlines the summary of the key results and draws the concluding remarks. Finally, Appendix A.1 and A.2 expounds the non-linear state-dependent model (SDM) and econometric technique used in the analysis, while Appendix A.2 outlines the dataset.

II: Related Literature:

The present paper tackles issues that are related to three strands of the exiting literature. Firstly, there has been a heightened research interest in the key area of survey forecasts. Recent interest in professionals' inflation forecasts is largely motivated by the need to understand the drivers of macroeconomic fluctuations. In an innovative paper Fuhrer (2017) includes actual survey expectations of professional forecasters in a DGSE model, rather than the usual stylized rational expectations. He finds that using actual expectations performs considerably better, exhibiting strong correlations to key macroeconomic variables. Consequently, he proposes methods for endogenizing survey expectations in general equilibrium macro models for improving monetary policy. Fuhrer (2015) also argues that expectations are both an important source and an explanation of macroeconomic persistence.

The interest in survey forecast has paralleled the rational inattentive literature. A number of theoretical models attempt to explain deviations from full-information rational expectations through informational rigidities or inattentiveness (for example, Mankiw and Reis (2002), Woodford (2003) and Sims (2003), Mackowiak, B.A., el at (2019)). The inattentiveness literature can be broadly divided into experts' and non-experts' forecast, where the former pertained to professional forecasters and the latter to both households and firms. Coibion and Gorodnichenko (2015), focusing on experts, adapt and extend the Nordhaus' concept of 'weak efficiency' (Nordhaus (1987)) forecast into the contemporary inattentiveness literature.

Importantly, they point out that the different types of models predict quantitatively similar forecast errors. Likewise, Andrade and Le Bihan (2013) find empirical evidence to support professional inattentiveness being consistent with informational rigidities but find the frequency of updating differs between them. The professionals' forecasting behavior based on survey forecasts have been an important source of understanding forecaster disagreements (Mankiw et al, 2004) and its relation to aggregate uncertainty (Lahiri and Sheng, 2010) and understanding macroeconomic persistence (Fuhrer, 2017).

Carroll (2004) and (2006) focusing on households' forecasting behavior introduced an innovative social learning model based on epidemiology. Here the agent, or household, absorbs the professionals' forecasts via the news media. Easaw et al (2013), while using the social learning framework, undertook a comprehensive study of how households form macroeconomic forecasts at individual household level. Also, Easaw and Mossay (2015) extended the social learning model to a spatial context for studying how non-experts form macroeconomic expectations.

The second strand of the related literature looks at how to model inflation forecasts, starting with the Stock and Watson (2007) Unobservable Component (UC) framework. The UC approach distinguished between the stationary and non-stationary components of inflation. This notion of forecasting the stationary 'inflation gap' around some slowly-varying local mean has been fairly effective (see Kozicki and Tinsley (2001), Stock and Watson (2010), Cogley, Primiceri and Sargent (2010) and Clark (2011)). In a recent comprehensive review, Faust and Wright (2013) argue strongly for inflation forecast models that account for a slowly-varying trend. We adopt the approach of using long-horizon (10-years ahead) survey forecasts to measure the permanent component in the UC model to derive the non-stationary trend component of inflation (see Clark (2011), Kozicki and Tinsley (2012), Wright (2012)).

An important aspect of the UC model is its stochastic volatility (SV). Recent examples of estimating the UC model with SV are Berger et al (2016), Mertens (2016), Stock and Watson (2016) and Chan et al (2018). Also, Merten and Nason (2018) use the UC approach which incorporates stochastic volatility process to access professionals' inflation forecast dynamics; notably inattentiveness due to information rigidities. They extend the UC-SV framework to account for time-varying behavior (TV UC-SV).

The third strand of the literature considers the issue of estimating inflation gap persistence and its importance for monetary policy. In an influential paper Cogley et al. (2010) study US inflation persistence interpreting trend inflation as determined by slowly changing long-term inflation targets. In a recent paper, Jain (2019) takes a novel approach to study professionals' inflation forecasts. Jain (2019) uses a stationary version of the UC approach which is extended to a state-space model to consider US inflation persistence by estimating 'perceived inflation persistence'. It is based on the idea that if professional forecasters assume an inflationary shock to be transitory, they would not make substantial revisions for their multiperiod forecasts and perceive shock persistence to be low and vice versa if they perceive high persistence. During the GM period monetary authorities have shown a low tolerance for the persistence of the inflation gap. Hence, forecasters' perceived persistence of inflation gap has direct implications for monetary policy and price stability.

III: Perceived State-Dependent Inflation Gap Persistence: Theoretical Issues

Faust and Wright (2013) provide a recent and comprehensive review of the prevalent ways to model and forecast inflation. They capture the varying local mean by measuring the trend level of inflation. In addition, they define 'inflation gap', which is a stationary component, as the difference between actual inflation and the stochastic trend level. Faust and Wright (2013) maintain that the forecasting of inflation gap around some slow varying local mean has proved to have successful predictive abilities (for example, Stock and Watson (2010), Cogley et al

(2010) and Nason and Smith (2013)). Therefore, following the seminal work of Stock and Watson (2007), we define a general unobservable components with stochastic volatility (UC-SV) representation of actual inflation rate (π_t), whose components are a stochastic trend (τ_t), and a stationary factor, or inflation gap (ξ_t):

$$\pi_{t} = \tau_{t} + \xi_{t} \tag{1}$$

$$\tau_t = \tau_{t-1} + \eta_t$$
 where $\eta_t = \sigma_{\eta,t} \zeta_{\eta,t}$ (2)

$$\xi_t = \rho_{t-1} \xi_{t-1} + \nu_t \qquad \text{where } \nu_t = \sigma_{\nu,t} \zeta_{\nu,t}$$
 (3)

$$\ln \sigma_{\eta,t}^2 = \ln \sigma_{\eta,t-1}^2 + \nu_{\eta,t}$$
 (4)

$$\ln \sigma_{v,t}^2 = \ln \sigma_{v,t-1}^2 + \nu_{v,t} \tag{5}$$

The stochastic trend is assumed to follow a random walk process without a drift, where η_t is the innovation with stochastic volatility (SV) $\sigma_{\eta,t}$. Cogley et al (2010) using the UC model to derive the inflation gap which they define as the stationary component ξ_t , that is the difference between current inflation rates from its trend. The inflation gap, in turn, is assumed to be persistent, where ν_t denotes the inflation gap shock with SV, $\sigma_{\nu,t}$. The covariance of the two innovations τ_t and ξ_t may be non-zero while the AR coefficient ρ_t captures the persistence of the inflation gap. In both Stock and Watson (2007) and Nason and Smith (2013) inflation persistence is assumed to be constant over time, $\rho_t = \rho$. Cogley et al (2010) estimate the value of a time-invariant ρ to be around 0.5, well below unity. Therefore, we specifically assume that the two unobservable variables state to evolve accordingly:

$$\begin{bmatrix} \tau_t \\ \xi_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \rho_{t-1} \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ \xi_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ v_t \end{bmatrix}$$
 (6)

In this section, we consider two ways to derive perceived inflation gap. Firstly, we assume that professional forecasters form their inflation forecasts by estimating the stochastic trend $F_t(\tau_t)$ and their inflation forecasts for *h-periods ahead* is as follows:

$$F_{t}(\pi_{t+h}) = F_{t}(\tau_{t+h}) + F_{t}(\xi_{t+h}) \tag{7}$$

and:

$$F_{t}(\tau_{t+h}) = F_{t}(\tau_{t}) + \eta_{h,t} \qquad \text{where } \eta_{h,t} = \sigma_{\eta,h} \zeta_{h,t}$$

$$F_{t}(\xi_{t+h}) = (\rho_{t}....\rho_{t+h-1})(F_{t}\xi_{t})$$

$$F_{t}(\xi_{t}) = \xi_{t} + v_{t}$$

For large h and ρ_t bounded away from 1 we have the approximation and limit as the time horizon h tends to infinity:

$$(\rho_{t}\rho_{t-1}....\rho_{t+h-1})\xi_{t} + v_{t} \approx 0$$

$$\lim_{h \to \infty} (\rho_{t}\rho_{t-1}....\rho_{t+h-1})\xi_{t} + v_{t} = 0$$

Hence the long-term forecast (large h) can be seen as capturing the "trend" element:

$$F_{t}(\pi_{t+h}) \approx F_{t}(\pi_{t+\infty}) = F_{t}(\tau_{t}) + \eta_{ht} \tag{8}$$

Forecasters form short-horizon inflation forecasts (small h) according to (7). We assume that forecasters make the approximation $\rho_t \rho_{t+1...} \rho_{t+h-1} \approx \rho_t^h$ based on information available in period t, so that (7) becomes:

$$F_{t}(\pi_{t+h}) = F_{t}(\tau_{t}) + \eta_{h,t} + \rho_{t}^{h} F_{t}(\xi_{t})$$
(9)

Forecasters, when focusing on short-horizon forecasts, may also wish to form multiperiod forecasts. In this case, agents are interested in the momentum of future inflation and the persistence of any transitory shock and perceived inflation gap is crucial. The differences of any multi-period forecasts across the horizons are:

$$F_{t}(\pi_{t+h+1}) - F_{t}(\pi_{t+h}) = F_{t} \Delta(\pi_{t+h+1}) = \rho_{t}^{h}(\rho_{t} - 1)F_{t}(\xi_{t}) + \Delta\eta_{h+1,t}$$
(10)

Following Mertens and Nason (2018), the SV is assumed to evolve as a random walk and $\Delta \eta_{h+1,t} = 0$. Multiplying equation (10) by $(1 - \rho_t L)$ we derive the professionals' forecast of *inflation momentum*¹ over time:

¹ By *inflation momentum* we mean the persistence of changes in inflation as represented by (11). The key point is that the autoregressive coefficient for inflation momentum is exactly the same as the coefficient for inflation gap persistence.

$$F_{t} \Delta(\pi_{t+h+1}) = \rho_{t-1} F_{t-1} \Delta(\pi_{t+h}) + \rho_{t}^{h} (\rho_{t} - 1) \nu_{t}$$
(11)

Following the assumption in (3), the inflation gap shock v_t is an unforecastable white noise, so equation (11) can be re-written as:

$$F_t \Delta(\pi_{t+h+1}) = \rho_{t-1} F_{t-1} \Delta(\pi_{t+h}) + \varepsilon_t \tag{12}$$

where ε_t satisfies $E_t[\varepsilon_t] = E_t[s_i s_j] = 0$ ($i \neq j$) but may be heteroskedastic when ρ_t varies over time. The advantage of equation (12) is that we can use the data on changes in short term forecasts (inflation momentum) to estimate the inflation gap persistence parameter ρ_t . This is useful because the data on short term forecasts goes back much further than the data on long-term forecasts. We will allow for the variance of the error term to vary over time in our estimation procedure. We will also evaluate the accuracy of the approximation $\rho_t \rho_{t+1\dots} \rho_{t+h-1} \approx \rho_t^h$.

So far, we have considered the derivation of perceived inflation gap using the forecaster's short-horizon inflation forecasts. Professional forecasters' perceived inflation gap can also be derived using their long-horizon forecast which is available from 1992 in our dataset. Recalling: $F_t(\pi_{t+h}) \approx F_t(\pi_{t+\infty}) = F_t(\tau_t) + \eta_{h,t}$, we assume the long-horizon forecasts such as ten-year ahead forecasts proxies the infinite-horizon forecasts²:

$$F_{t}(\pi_{t+10}) = F_{t}(\tau_{t}) + \eta_{10t} \tag{13}$$

On the other hand, a short-horizon inflation forecast will be influenced by the current inflation gap:

$$F_{t}(\pi_{t+1}) = F_{t}(\tau_{t}) + \eta_{1,t} + \rho_{t} F_{t}(\xi_{t})$$
(14)

Subsequently, both short and long-horizon forecasts are related via the stochastic trend component with their difference being:

$$F_{t}(\pi_{t+1}) - F_{t}(\pi_{t+10}) = \rho_{t} F_{t}(\xi) + \Delta \eta_{10,t}$$
(15)

² We assume: $F_t(\pi_{t+10}) = F_t(\tau_t) + \rho_{t|t+10}F_t(\xi_t) = F_t(\tau_t)$ where $\rho_{t|t+10} = \rho_t^{10} \approx 0$. Also, see the recent literature that have assumed that the 10-year forecast proxies trend inflation (for example, Faust and Wright (2013), Mertens (2015) and Chan et al (2018)).

as before: $\Delta \eta_{10,t} = 0$, which is simply forecasted one-year ahead inflation gap or:

$$F_{t}(\pi_{t+1}) - F_{t}(\pi_{t+10}) = \rho_{t-1}(F_{t-1}(\pi_{t+1}) - F_{t-1}(\pi_{t+10}))$$
(16)

In this paper, we consider a general non-linear model where the parameters ρ_t is time-varying due to state-dependence. As professional forecasters update their information set and revise their forecast and the forecast of inflation momentum, they will also revise their estimate of the persistence of inflation gap or ρ . It depends on the update of the forecast which embodies the relevant information. It will specifically depend on the forecasters perceived varying inflation gap. In other words, it could depend on their expected inflation relative to its trend. If it is negative (forecasted inflation is below the trend), positive (above its trend) or zero (when it is close to its trend). A state-dependent model in a general form AR(k) model can be depicted as follows:

$$X_{t} = \mu(x_{t-1}) + \rho_{1}(x_{t-1})X_{t-1} + \dots \rho_{k}(x_{t-1})X_{t-k} + \varepsilon_{t}$$
(17)

where $X_t = F_t \Delta(\pi_{t+h})$ or $= F_t(\pi_{t+1}) - F_t(\pi_{t+10})$ and ε_t is a sequence of independent zero-mean random error terms and, as noted earlier, may be heteroskedastic. At the time when the forecast is made (t-1) the development of the forecast X_t is determine by the values $\{X_{t-1},....,X_{t-k}\}$, together with the future values of ε_t . Hence, $x_{t-1} = \{X_{t-1},....,X_{t-k}\}'$ may be regarded as the 'state vector' of the process X_t . The only information, therefore, in the 'past' relevant to the future development of the process is contained in the state-vector. For the purpose of the present paper, this is can be simplified to an AR (1) as follows:

$$X_{t} = \mu(x_{t-1}) + \rho(x_{t-1})X_{t-1} + \varepsilon_{t}$$
(18)

We have now derived the non-linear version of ρ using two distinct methods and two different professionals' forecast of inflation. The remainder of the paper focuses on the empirical analyses.

Section IV: Empirical Analysis and Results: Professionals' Inflation Gap Persistence Forecast A State-Dependent Approach

In this section, we apply the state-dependent model fitting technique derived from the Kalman algorithm as outlined in Appendix 1³ to the inflation gap forecast data described in Appendix 2. We first report results of the forecasted, or perceived, inflation gap persistence using the short-horizon forecast (equation 12 above), followed by the second method considering the difference between the long and short-horizon forecasts (equation 16 above). The latter method is used while differentiating between the financial and non-financial sector professionals. The figures also show the graphs of the statistical significance and 90% confidence interval estimate for the estimates of the persistence parameter $\hat{\rho}_t$ in (12) or (16).

Figures 1 and 2 below outline the forecasted inflation gap persistence based on the short-horizon forecast of GDP inflation. We take as our starting point the simple linear model with a constant ρ and a constant term μ , estimated on the initial stretch, taking the first 20 periods of the data (1970q3 – 1975q4). Aggregating over all forecasters, the initial estimates over this initial period are:

$$\hat{\mu} = -0.135 \text{ and } \hat{\rho} = 0.604 \text{ and } \hat{R} = \begin{bmatrix} 0.008 & -0.012 \\ -0.012 & 0.047 \end{bmatrix}$$

We then estimate the SDM model as outlined in Appendix 1, iteratively starting from half-way through the initial stretch, 10 quarters from the beginning of the dataset at 1973q4. For comparison, the dotted line represents the linear estimates taken across them for the whole period which is depicted by the dotted line in Figure 1 ($\hat{\rho} = 0.65$). The estimates of ρ_i are shown both against time and against the previous lag of inflation gap in Figures 1 and 2 respectively.

_

³ We also undertook estimations following two particle filter methodologies as suggested by Carvalho et al, (2010) and Strovik (2002). The results are similar to our extended Kalman filter approach and available from the authors on requests.

Figures 1 and 2[about here]

As suggested in the Introductory section, the estimates allow us to consider the forecasted inflation gap persistence distinguishing between the pre-Great Moderation (pre-GM), Great Moderation (GM) and Great Recession (GR) periods. In the pre-GM period, the forecasted persistence is more variable and below the linear estimates: from 1975 to 1985, there was even lower persistence with values in the range 0.5-0.6. The GM period, on the other hand, displays little variation and approximates the linear estimate. It is higher that the preceding pre-GM period. The estimated inflation gap persistence forecasts for the GR period virtually mimics the GM period. There is some state dependence (Figure 2) with a higher forecasted persistence for a positive gap than for a negative gap.

It is interesting that the estimated inflation gap persistence forecast for GPD inflation is higher in the GM and GR periods and fairly time invariant. The results outlined in Figures 1 and 2 need to be considered together. The state-varying results show that the estimated ρ_t is highest ($\hat{\rho}_t \approx 0.7$) when the forecasted inflation gap was close to zero. Clearly, the professional forecasters perceived actual inflation to be close to its trend during the GM and GR periods. This is consistent with the pursuit of price stability policies of the GM period.

Figures 3 and 4 below outline the forecasted inflation gap persistence based on the short-horizon forecast of CPI inflation. Once again, we take as our starting point the simple linear model with a constant ρ and a constant term μ , fitted on the first 20 observations (starting from 1984q4) yielding initial estimates:

$$\hat{\mu} = -0.218 \text{ and } \hat{\rho} = 0.21 \hat{\sigma}_{\varepsilon}^2 = 0.056 \text{ and } \hat{R} = \begin{bmatrix} 0.005 & 0.011 \\ 0.011 & 0.061 \end{bmatrix}$$

The estimates of ρ are shown both against time and against the lag of inflation gap in Figures 3 and 4 respectively (again, the dotted line represents the simple linear estimate over the whole period, with $\hat{\rho} = 0.34$):

Figures 3 and 4[about here]

The non-linear model yields very different results from the linear model for CPI inflation. During the pre-GM period, the estimated $\hat{\rho}_t$ is lower than the linear estimate. During the GM period, starting in the mid-80s, $\hat{\rho}_t$ starts rising, reaching its peak in the end of 1990 ($\hat{\rho}_t = 0.55$). The reminder of the GM $\hat{\rho}_t$ was above its linear estimate and fairly time invariant. In the GM period, the autocorrelation coefficient is only 0.5: for other periods (pre-GM and GR) it is lower. In the GR era, $\hat{\rho}_t$ rises sharply in the second half of 2008, falling back to its linear estimate a year later. The CPI inflation gap is less persistent than the GDP inflation gap. The state dependence (Figure 4) estimates are the reverse of GDP inflation forecast with a lower forecasted persistence for a positive gap than for a negative gap.

As with the preceding GDP inflation forecast, the results outlined in Figures 3 and 4 need considered in together. The state-varying results show that the estimated $\hat{\rho}_t = 0.45$ when the forecasted inflation gap approaches zero. As before, the professional forecasters perceived actual inflation to be close to its trend during the GM and, therefore, is consistent with the pursuit of price stability policies of the GM period. Nevertheless, unlike the GDP inflation forecast, the professionals perceive a divergence of actual inflation from its trend at the start of the GR era. Indeed, the higher value $\hat{\rho}_t = 0.48$ is consistent with a perceived negative inflation gap. This is consistent with the view that disinflationary pressures were prevalent in the GR period.

We now turn to the estimated inflation gap using the difference between the long and short-horizon forecast, while distinguishing between professionals from the financial and non-financial sectors. Here, due to the availability of data, we are only able to consider the GM and GR periods (and not the pre-GM period).

Starting with the financial sector, the simple model with a constant ρ and a constant term μ , estimated on the initial stretch (1991q4-1995q3) yields the starting estimates:

$$\hat{\mu} = 0.034 \text{ and } \hat{\rho} = 0.658, \ \hat{\sigma}_{\varepsilon}^2 = 0.033 \text{ and } \hat{R} = \begin{bmatrix} 0.002 & -0.004 \\ -0.004 & 0.032 \end{bmatrix}$$

Starting the recursion from 1994q1, the estimates of ρ_t are shown both against time and against the previous lag of inflation gap in figures 5 and 6 respectively:

Figures 5 and 6[about here]

In the case of the financial sector, we can see that the non-linear model predicts a consistently lower persistence level than the linear model for most of the GM period with a trough around 2000. The non-linear estimates are fairly stable, although the degree of persistence rises slightly in the GR period at the end of the sample, and falls slightly from 1994-2000. In the GR period, the estimated ρ_t reaches the linear estimate of 0.74. In terms of state-dependence (Figure 6), $\hat{\rho}_t$ is around 0.6 when the forecasted inflation gap is zero. This corresponds to the estimates between the mid-1990s and mid-2000s. During this period, the professional forecasters perceived actual inflation to be close to its trend. The higher estimate (around 0.75) in the GR is consistent with professionals' perception of a negative inflation gap and, as before, this is consistent with anticipated disinflationary pressures.

Turning to the non-financial sector professionals' forecast, the linear model with a constant ρ and a constant term μ , is estimated using the initial twenty observations of the dataset gives the following estimates:

$$\hat{\mu} = 0.04 \text{ and } \hat{\rho} = 0.627, \ \hat{\sigma}_{\varepsilon}^2 = 0.017 \text{ and } \hat{R} = \begin{bmatrix} 0.002 & -0.005 \\ -0.005 & 0.025 \end{bmatrix}$$

Starting the recursion from 1994Q1, the estimates of $\hat{\rho}$ is shown both against time and against the previous lag of inflation gap in figures 7 and 8 respectively:

Figures 7 and 8 [about here]

During the GM period, similar to the financial sector professionals, $\hat{\rho}_i$ is below the linear estimates. Nevertheless, forecasted inflation gap persistence is lower than in the financial sector: the non-linear estimates are mostly in the region 0.5-0.6. Unlike their financial sector counterpart, the non-financial sector professionals forecast of inflation gap persistence does not alter for the GR period. Similar to the preceding estimates, the non-financial sector professionals forecast of inflation gap persistence is consistent with the perception that during these periods actual inflation approximates its trend. Furthermore, the linear and non-linear methods show little difference throughout the whole period. This is reflected in little or no state dependence, varying within a narrow band around 0.54 to 0.57 (see Figure 8). Overall the forecast of both sector professionals is consistent with the pursuit of price stability policies by the monetary authorities following the GM period.

Figure 9 below outlines the difference between the inflation persistence forecast of professionals from the financial and non-financial sector, which confirms the preceding findings. Two notable points can be made. Firstly, forecasters from the financial sector consistently predict/forecast higher inflation persistence than their non-financial sector counterparts but the difference is insignificant. Secondly, in contrast the difference is statistically significant in the GR period.

Figures 9 [about here]

Clearly, both sector professionals perceived different inflation gaps in the GR period.

Why might financial sector professionals predict more persistence to inflation? The

difference between the inflation forecasts of two sectors must be in terms of their information. The main difference between the two sets of forecasts occurs after the crisis. One possible explanation of the difference was that the impact of "extraordinary monetary policy" measures was better known and understood in the financial sector. Indeed, inflation turned out to be more persistent post-2008 than it had been previously. One of the reasons for this was that the Federal Reserve ceased to use the interest rate actively to control inflation, with the result that inflation remained above 3% for the period April to December 2011 despite weak output growth. In addition, for many who were not financial experts, the ramifications of Quantitative Easing (large scale asset purchases) were something of a mystery and were probably better understood by forecasters based in the financial sector.

Finally, in analyzing the forecast data, we made the assumption that agents used the approximation $\rho_i \, \rho_{i+1} - \rho_{i+k-1} \approx \rho_i^k$. Our results will only be valid if this is in general a good approximation. We can use the estimates of ρ_i to generate both $\rho_i \, \rho_{i+1} \rho_{i+2} \rho_{i+3}$ and ρ_i^4 . We can then see how these two are different as a percentage of the average value of $\rho_i \, \rho_{i+1} \rho_{i+2} \rho_{i+3}$ taken over the whole sample (which is 0.0400). Clearly, in cases where ρ_i varies little, the approximation must be good. Therefore, we look at the case where ρ_i varies the most: the case of the CPI inflation gap in Figure 3. Taken over the whole period, the approximation is good: the average error is -1.5%. If we take the absolute error and do not allow positives to cancel negatives, the average absolute error is 8.60%. If we look more closely, the approximation breaks down in a few brief time periods when there is a big change in inflation gap persistence. The biggest error is in 2008q4 when ρ_i is falling and the approximation overestimates the actual by 130%. Similarly, in 1990q1 when ρ_i is increasing rapidly, the approximation underestimates by 93%. However, for over two thirds of the 120 quarters the error is less than 5%. Moreover, given that the average value of ρ_i

is about 0.4, the average values of both $\rho_t \rho_{t+1} \rho_{t+2} \rho_{t+3}$ and ρ_t^4 are small relative to the value of the parameter being estimated.

Section V: Concluding Remarks and Summary

The main purpose of the present paper is to understand how professional forecasters form their forecast of inflation gap persistence. We wish to assess the dynamic nature of perceived inflation persistence, which has valuable implications for central bank's price stability policy. Focussing on the perceived persistence of inflation gap will enable us to assess the reaction of professional forecasters to transitory shocks to inflation or, specifically, their views about how tolerant monetary authorities were to the deviation of actual inflation from its trend. The paper considers how the perceived persistence of inflation gap evolves distinguishing between three periods: pre-Great Moderation, the Great Moderation (GM), beginning in the mid-1980s, and the Great Recession (GR) following the sub-prime financial crisis of 2007/08. We also assess, where possible, the difference between forecasters from the financial and non-financial sectors.

In brief, the professional forecasters perceived inflation to be close to its trend during the GM period. Hence, forecasted inflation gap to be close (equal) to zero. This is consistent with the pursuit of price stability policies of the GM period. During the GR period the professionals' forecasted inflation gap is consistent with perceived disinflationary pressures, and during this period there is a significant difference between the forecasts of professionals from the financial and non-financial sectors. Finally, as expected, during pre-GM perceived inflation gap persistence is more variable.

REFERENCES

Andrade, P. and H. Le Bihan (2013), "Inattentiveness Professional Forecasters", *Journal of Monetary Economics*, Vol. 60, No. 4, pp. 967-982.

Benati, Luca and Paolo Surico, 2008. "Evolving U.S. Monetary Policy and The Decline of Inflation Predictability," *Journal of the European Economic Association*, vol. 6(2-3), pages 634-646, 04-05.

Berger. T, Gerdie Everaert and Hauke Vierke (2016) "Testing for time variation in an unobserved components model for the U.S. economy", *Journal of Economic Dynamics and Control* Vol. 69, No.3, pp.179-208

Carvalho, C.M., M.S. Johannes, H.F. Lopes, and N.G. Polson (2010), "Particle learning and smoothing". *Statistical Science* 25, pp. 88–106.

Chan., J, T Clark and G Koop (2018), "A New Model of Inflation, Trend Inflation, and Long-Run Inflation Expectations" *Journal of Money, Credit and Banking*, Vol 50, (1), pp. 5-53

Clark, Todd E. (2011), "Real-Time Density Forecasts from VARs with Stochastic Volatility", *Journal of Business and Economic Statistics*, 29, pp.327-341.

Coibion, O. and Y. Gorodnichenko (2015), "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts", *American Economic Review*, Vol. 105, No. 8, pp. 2644-2678.

Cogley.,T and Thomas J. Sargent, (2001) "Evolving Post-World War II U.S. Inflation Dynamics," *NBER Macroeconomics Annual* 16, pp.331-373.

Cogley, T., G. E. Primiceri and T. J. Sargent (2010), "Inflation-gap Persistence in the U.S.", *American Economic Journal: Macroeconomics*, Vol. 2, pp. 43-69.

Easaw. J, R. Golinelli, M. Malgrini, (2013) "What Determines Households Inflation Expectations? Theory and Evidence from a Household Survey", *European Economic Review*, $61 \, (July), pp. 1 - 13$

Easaw. J, and Mossay, P, (2015) "Households Forming Macroeconomic Expectations: Rational Inattentive Behavior with Social Learning" *BE Contributions to Macroeconomic, Vol. 15(1)*

Faust, J. and J. Wright (2013), "Forecasting Inflation", in G. Elliott and A. Timmermann (eds.), *Handbook of Economic Forecasting*, Volume 2A, Elsevier.

Fuhrer, J. (2015), "Expectations as a source of macroeconomic persistence: an exploration of firms' and households' expectation formation,", *Federal Reserve Bank of Boston Working Papers*, No. 15-5.

Fuhrer, J. (2017), "Expectations as a Source of Macroeconomic Persistence: Evidence from Survey Expectations in a Dynamic Macro Model", *Journal of Monetary Economics*, Vol. 86, pp. 22-35.

Haggan, V, S.M. Heravi and M.B. Priestley (1984), A Study Of the Application Of State-dependent Models in Non-linear Time Series Analysis, *Journal of Time Series Analysis*, vol.5, No. 2, 69-102

Jain, M., (2019), "Perceived Inflation Persistence", *Journal of Business and Economic Statistics*, 37:1 pp. 110-120.

Kozicki, S and P A. Tinsley (2001), "Shifting Endpoints in the Term Structure of Interest Rates, *Journal of Monetary Economics*, 47, pp.613-652.

Kozicki, S and P A. Tinsley (2012), "Effective Use of Survey Information in Estimating the Evolution of Expected Inflation", *Journal of Money, Credit and Banking*, 44, pp.145-169.

Lahiri., K and Xuguang Sheng (2010) "Measuring forecast uncertainty by disagreement: The missing link", *Journal of Applied Econometrics*, Vol 25 (4), pp. 514-538

Levin, A. T. and J. M. Piger (2004), "Is Inflation Persistence Intrinsic in Industrial Economies?", *ECB Working Paper Series*, No. 334.

Lopes, H.F., and R.S. Tsay (2011), "Particle filters and Bayesian inference in financial econometrics". *Journal of Forecasting* 30, pp. 168–209.

Mackowiak, B., F. Matejka, and M. Wiederholt, (2018), "Survey: Rational Inattention, a Disciplined Behavioral Model." *CEPR Discussion Paper No. DP13243*

Mankiw, N. G. and R. Reis (2002), "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve", *The Quarterly Journal of Economics*, Vol. 117, No. 4, pp. 1295-1328.

Mankiw, N. Gregory, Ricardo Reis and Justin Wolfers, (2003) "Disagreement About Inflation Expectations", *NBER Macroeconomics Annual*, 18 pp. 209-248.

Mertens, E. (2016). "Measuring the level and uncertainty of trend inflation". *The Review of Economics and Statistics* 98, pp. 950–967.

Mertens, E. and J. Nason, (2018) "Inflation and professional forecast dynamics: an evaluation of stickiness, persistence, and volatility," *BIS Working Papers 713, Bank for International Settlements*.

Nason, J. M. and G. W. Smith (2013), "Measuring the Slowly Evolving Trend in US Inflation with Professional Forecasts", *Queen's Economics Department Working Paper*, No. 1316.

Nordhaus, W. D. (1987), "Forecasting Efficiency: Concepts and Applications", *The Review of Economics and Statistics*, Vol. 69, No. 4, pp. 667-674.

Priestley, M.B and S.M. Heravi, (1986), Identification of Non-linear Systems Using General State-Dependent Models., *Journal of Applied Probability*, Vol. 23A, 257-272

Priestley, M.B. (1981), State-dependent Models: A General Approach to Non-linear Time Series Analysis. *Journal of Time Series Analysis*, vol. 1,47-71

- Sims, C. A. (2003), "Implications of Rational Inattention", *Journal of Monetary Economics*, Vol. 50, No. 3, pp. 665-696.
- Stock, J. H. and M. W. Watson (2007), "Why Has US Inflation Become Harder to Forecast?", *Journal of Money, Credit and Banking*, Vol. 39, No. 1, pp. 3-33.
- Stock, J. H. and M.W. Watson (2010), "Modeling Inflation After the Crisis, in Macroeconomic Challenges: The Decade Ahead", Federal Reserve Bank of Kansas City Economic Policy Symposium.
- Stock, J. H. and M.W. Watson, (2016), "Dynamic Factor Models, Factor-Augmented Vector Autoregressions, and Structural Vector Autoregressions in Macroeconomics", *Handbook of Macroeconomics* Vol. 2, pp. 415-525
- Storvik, G. (2002). Particle filters for state-space models with the presence of unknown static parameters. *IEEE Transactions on Signal Processing* 50, pp. 281–289.
- Woodford, M. (2003), "Imperfect Common Knowledge and the Effects of Monetary Policy" in P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford (eds.), *Knowledge, Information, and Expectations in Modern Macroeconomics*, Princeton University Press.
- Wright, J. H. (2013) "Evaluating Real-Time VAR Forecasts with an Informative democratic Prior", *Journal of Applied Econometrics*, Vol 28 (5) pp. 762-776.

Figure 1: Time-varying Inflation Gap Persistence: GDP Inflation (Short Horizon Forecasts)

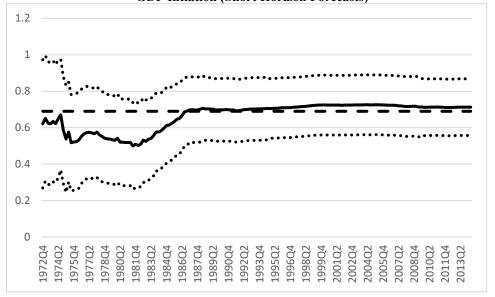


Figure 2: State-varying Inflation Gap Persistence: GDP Inflation (Short Horizon Forecasts)

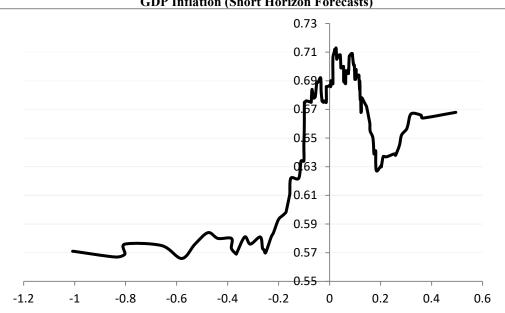


Figure 3: Time-varying Inflation Gap Persistence: CPI Inflation (Short Horizon Forecasts)

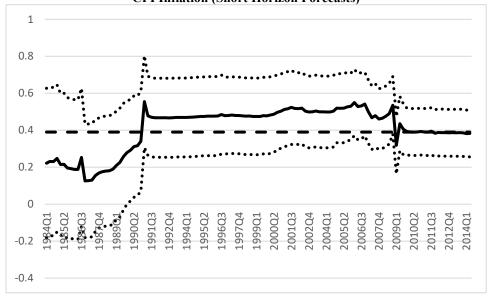


Figure 4: State-varying Inflation Gap Persistence: CPI Inflation (Short Horizon Forecasts)

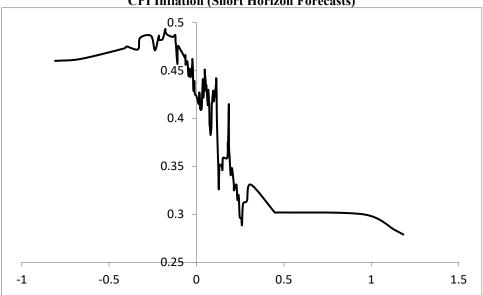


Figure 5: Time-varying Inflation Gap Persistence: Financial Sector (Long – Short Horizon Forecasts) GDP Inflation

1 | 0.9 | 0.9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.

Figure 6: State-varying Inflation Gap Persistence: Financial Sector (Long – Short Horizon Forecasts) GDP Inflation

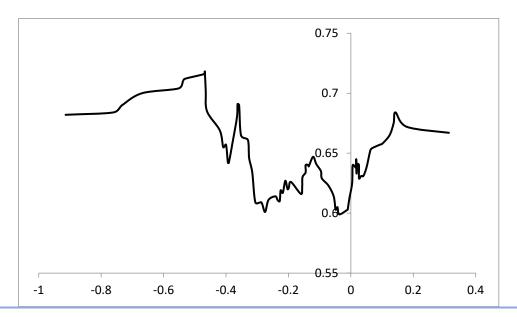


Figure 7: Time-varying Inflation Gap Persistence ($\hat{\rho}$): Non-Financial Sector (Long – Short Horizon Forecasts) GDP Inflation

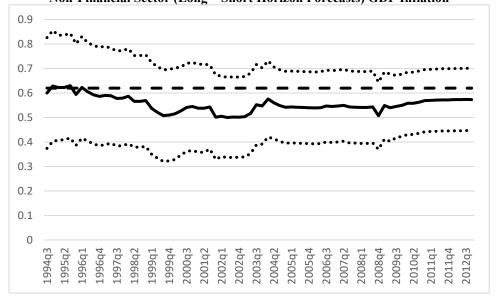


Figure 8: State-varying Inflation Gap Persistence ($\hat{\rho}$): Non-Financial Sector (Long – Short Horizon Forecasts) GDP Inflation

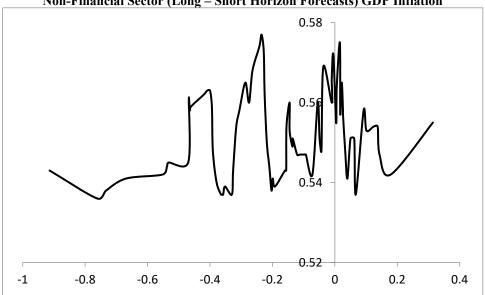
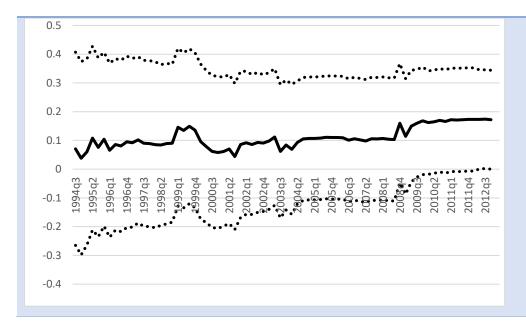


Figure 9: Time varying Difference Between Financial and Non-Financial Sectors (Long-Short Horizon Forecasts) GDP Inflation



Appendix 1: Estimation of State-Dependent Models (SDM):

This appendix outlines an estimation technique based on a general class of non-linear time series, called State Dependent Models (SDM)⁴. The principal advantage of the SDM is that it allows for a general form of non-linearity and this enables fitting without any specific prior assumption about the form of the non-linearity. We describe the estimation of the SDM's and give a precise formulation of this approach, focusing on how to estimate the parameter ρ in the inflation gap model as outlined in equation (18). We also extend the estimation procedure to consider the heteroskedasticity inflation gap model by allowing the variance of the residuals to change from one point to the next.

If the state-vector x_t , is augmented with the constant unity to include the mean parameter μ , we can then write the state-vector for model (18) which is an AR of order 1 as:

$$x_{t} = (1, X_{t})'$$
 (A1.1)

The SDM may be given a formal state-space representation as follows:

$$x_{t} = \{\mathbf{F}(x_{t-1})\}x_{t-1} + \varepsilon_{t}$$

$$X_{t} = \mathbf{H}.x_{t}$$
(A1.2)

where the transition matrix **F** is given by $\mathbf{F} = \begin{bmatrix} 1 & 0 \\ \mu & \rho \end{bmatrix}$ with $\mathbf{H} = (0:1)$ and $\varepsilon_t = \varepsilon_t (0:1)'$

In fitting the SDM model for this case, we are concerned with the estimation of the parameters μ and ρ . However, these coefficients depend on the state vector x_{t-1} , and, thus, the estimation problem becomes the estimation of the functional form of this dependency. A recursive method is used to estimate these coefficients. Priestly (1981) has shown it is

27

⁴ This based on the models originally developed in Priestley (1981) which includes non-linear time series models and linear ARMA as a special case. Also, see Lopes and Tsay (2011)

possible to base the estimation procedure on the extended Kalman Filter algorithm provided some assumptions are made about the parameters⁵. The simplest assumption that can be made is that the parameters are locally linear functions of the state-vector x_t as follows:

$$\rho(x_t) = \rho_0 + x_t' \gamma = \rho_0 + X_t \gamma$$
 (A1.3)

$$\mu(x_t) = \mu_0 + x_t' \alpha = \mu_0 + X_t \alpha \tag{A1.4}$$

where ρ_0 and μ_0 are constants, α and γ are 'gradient' vectors. This assumption is valid, provided μ and ρ are slowly changing functions of X_t . Using these assumptions, 'updating' equations for the coefficients μ and ρ can be written as follows:

$$\rho(x_{t+1}) = \rho(x_t) + \Delta x_t' \gamma_{t+1} = \rho(x_t) + (X_t - X_{t-1}) \gamma_{t+1}$$
(A1.5)

$$\mu(x_{t+1}) = \mu(x_t) + \Delta x_t' \alpha_{t+1} = \mu(x_t) + (X_t - X_{t-1}) \alpha_{t+1}$$

where $\Delta x_t = x_t - x_{t-1} = (X_t - X_{t-1})$. The 'gradient' parameters α_t and γ_t are unknowns, which must be estimated. The basic strategy in the estimation procedure is to allow these parameters to wander in the form of 'random walks. In the matrix form, the random walk model for the gradient parameters may be written in matrix form as

$$\mathbf{B}_{t+1} = \mathbf{B}_t + \mathbf{V}_{t+1} \tag{A1.6}$$

where $\mathbf{B}_{t+1} = (\alpha_t, \gamma_t)$ and \mathbf{V}_{t+1} is a sequence of independent matrix-valued random variables such that $\mathbf{V}_{t+1} \sim \mathbf{N}(\mathbf{0}, \mathbf{\Sigma}_{\mathbf{v}})$

In each period (t), the estimation procedure then determines the values of $\mathbf{B}_{\mathbf{t}}$ which minimizes the difference between the observed value of X_{t+1} and its forecast, \hat{X}_{t+1} computed from the model fitted at time t.

_

⁵ Lopes and Tsay (2011) are unequivocal that when using the local mean level, "the traditional Kalman filter is available to produce the 'optimal' estimate of the filtered state vector." (pp. 174)

Priestley (1981) has shown that the SDM can be reformulated by re-writing the model in a state-space form where the state-vector represents the current state of all the parameters of the model, i.e.

$$\theta_{t} = (\mu_{t-1}, \rho_{t-1}, \alpha_{t}, \gamma_{t})$$
 (A1.7)

$$\mathbf{H_{t}} = (1, -X_{t-1}, 0, 0) \tag{A1.8}$$

We may now express the state-dependent model (18) in the form

$$X_{t} = H_{t} \theta_{t} + \varepsilon_{t} \tag{A1.9}$$

And the evolution of θ_t over time is given by

$$\theta_t = F_{t-1}\theta_{t-1} + W_t \tag{A1.10}$$

Where

$$\mathbf{F_t} = \begin{bmatrix} 1 & 0 & (X_{t-1} - X_{t-2}) & 0 \\ 0 & 1 & 0 & (X_{t-1} - X_{t-2}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The two equations (A1.9) and (A1.10) are now of the standard form and the extended Kalman filter can be applied directly. Applying the Kalman algorithm to the equations (A1.9) and (A1.10), the updating equations for the parameters takes the form

$$\hat{\theta}_{t} = \mathbf{F}_{t-1}\hat{\theta}_{t-1} + \mathbf{K}_{t} \{ \mathbf{X}_{t} - [\mathbf{H}_{t} \cdot \mathbf{F}_{t-1} \cdot \hat{\theta}_{t-1}] \}$$
 (A1.11)

Where \mathbf{K}_{t} is the 'Kalman gain' matrix.

In practice, this recursive procedure has to start at time $t=t_0$ with initial values for the parameters and the variance and covariance matrix of the parameters and the residual variance of the model, i.e. $\hat{\theta}_{t_0-1}$, $\hat{\mathbf{C}}_{t_0-1}$ and $\hat{\sigma}_{\varepsilon}^2$.

To establish these initial values, we apply the same procedure used in Haggan et al (1986), and formulate a practical estimation procedure as follows:

- i. take an initial stretch of the data, say the first 2m observations, and fit a standard linear AR(1) model. This will provide initial values $\hat{\mu}$, $\hat{\rho}$ and the residual variance of the model, $\hat{\sigma}_s^2$.
- ii. start the recursion midway along the initial stretch of data at $t_o=m$, and set

$$\hat{\theta}_{t_0-1} = (\hat{\mu}, \hat{\rho}, 0, 0)'$$
 and $\hat{\mathbf{C}}_{t_0-1} = \begin{pmatrix} \hat{R}_{\mu, \rho} & 0 \\ 0 & 0 \end{pmatrix}$

where $\hat{R}_{\mu,\rho}$ is the estimated variance-covariance matrix of $\hat{\mu}$ and $\hat{\rho}$ obtained from the initial AR(1) model fitting. It also seems reasonable to set all the initial gradients to zero, assuming that the initial values are reasonably accurate at: t_0 =m. We also extended the SDM estimation procedure to allow the residual variance to change from one point to the next. In this case, we compute and update the variance of the residuals, $\hat{\sigma}_{\varepsilon}^2$ using the current information on the residuals so far obtained.

In this study we estimated our starting values using the first 20 quarters of the data. We start the actual recursive estimation halfway through the initial period at 10 quarters. The parameters were smoothed using a non-parametric function fitting technique which employs a rectangular smoothing kernel. The results show the parameters plotted against the state-vector and also, as the algorithm is sequential, we can present the parameters against time scale. The resulting computer graphs of the parameters give a clearer idea of the type of non-linearity present in the model. A detailed account of the theory of SDM and Monte Carlo simulation studies are given in Priestley (1981) and Haggan, Heravi and Priestley (1984) and Priestley and Heravi (1986).

Appendix 2 - Variables Definition and Their Preliminary Analysis

The Survey of Professional Forecasters (SPF) regression data ⁶

The two forecasts $F_t(\pi_{t,t+3})$ and $F_t(\pi_{t+1,t+4})$ for both GDP and CPI inflation rates are computed by defining P = GDP inflation or P = CPI Inflation in the following formulas, written in terms of the SPF labels (for details, see also the SPF survey documentation):

$$one step P = 100*(((1+P2/100)*(1+P3/100)*(1+P4/100)*(1+P5/100))^{.}.25-1)$$

$$multistep P = 100*(((1+P3/100)*(1+P4/100)*(1+P5/100)*(1+P6/100))^{.}.25-1)$$

More explicitly, the one-year-ahead forecast of P is defined as the mean of the SPF forecasts released in quarter t for the current and the next three quarters (i.e. t,t+1, t+2, t+3), in symbols F_t ($\pi_{t,t+3}$); the corresponding multi-step forecast is the mean of the SPF forecasts - again released in quarter t - for the next four quarters (i.e. t+1, t+2, t+3, t+4), in symbols: F_t ($\pi_{t+1,t+4}$). Therefore, the one-step- and the multistep-ahead horizons of the SPF forecasts overlap for three quarters. The professionals' forecasts for ten-years ahead are derived in a similar way.

Figure 1 depicts the professional forecasters perceived inflation gap for GDP inflation and CPI inflation. The data for former covers the period 1970q3-2014q2 and the latter begins in 1981q4. Figure 2 depicts the professional forecasters perceived inflation gap for GDP inflation using the difference between long and short-horizon forecasts. The data is disaggregated into forecasters belonging to either the financial or non-financial sectors. The ten-year ahead or long-horizon forecasts are available from 1991q4:

Figures A1 and A2[about here]

⁶ For further information, see the following link to the SPF site at the Federal Reserve Bank of Philadelphia: http://www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/

Figure A1: Inflation gap (GDP Inflation) and Inflation gap (CPI Inflation) Figure A2: Financial and Non-Financial Sectors Inflation gap (GDP Inflation) (Difference between Long and Short Horizon Forecasts) (Short-Horizon Forecast) 0.6 1 0.4 0.2 0 0.5 -0.2 -0.4 Inflation gap (GDP) Financial -0.6 · Inflation gap(CPI) ······ Non-Financial -0.8 -1 -1.2 -1.4 1970Q3 1973Q1 1980Q3 1983Q1 1985Q3 1988Q1 1990Q3 1993Q1 1995Q3 1998Q1 2000Q3 2003Q1 2005Q3 2008Q1 1995q3 1999q2 2000q3 2001q4 2003q1 2006q4 2008q1 1996q4 2004q2 2005q3 2009q2 1994q2 1998q1