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Inequality and Economic Growth in the UK

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Abstract This paper analyses the effect of wealth inequality on UK economic growth in recent decades with a heterogeneous-agent growth model where agents can enhance individual productivity growth by undertaking entrepreneurship. The model assumes wealthy people are more able to afford the costs of entrepreneurship. Wealth concentration therefore stimulates entrepreneurship among the rich and so aggregate growth, whose fruits in turn are largely captured by the rich. This process creates a mechanism by which inequality and growth are correlated. The model is estimated and tested by Indirect Inference and is not rejected. Policy-makers face a trade-off between redistribution and growth.

Keywords Inequality · Growth · Heterogeneous-agent · Entrepreneurship · Indirect Inference

JEL Classification E10 · O30 · O40

1 Introduction

This paper investigates the relationship between capital inequality and aggregate economic growth in the UK during recent decades. This was a period when UK growth was greatly strengthened by a variety of supply-side reforms, while at the same time inequality rose substantially and commentators on the

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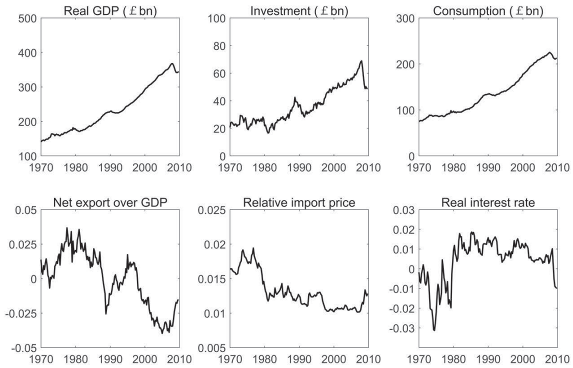

Fig. 1 Facts of the postwar economy in the UK

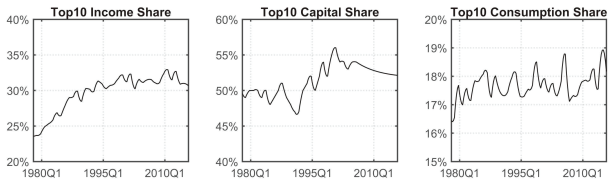

Fig. 2 Inequality Indicators

political left (e.g. Hutton, 1995) widely blamed the reforms for a 'culture of greed' and for the rise in inequality.

Minford and Meenagh (2020) explored a DSGE model linking growth to these supply-side reforms, testing it by Indirect Inference on the facts of the episode — set out in Figure 1.

The facts of inequality also show substantial movement during this period: see Figure 2 for the shares of the top 10% of the population in income, wealth and consumption. For example, the income share of the top 10% rose from 1980 to the mid-1990s before levelling off, while their wealth share rose to a peak at the end of the 1990s before falling back almost to its starting point. Their consumption share fluctuated around a slightly rising trend.

In this paper we build on Minford and Meenagh (2019) and ask whether there was indeed also some relationship between growth and inequality at work in this episode. We supplement their model with a heterogeneous agent set-up in which random shocks have distributional effects and higher wealth increases the incentive to innovate as an entrepreneur, the idea being that the costs of entrepreneurial entry are more easily absorbed. We test this model too by Indirect Inference against the same facts but now including inequality. To anticipate our main results, we find that this model of growth and inequality is not rejected as a match to the facts of data behaviour. The model implies that there are trade-offs between growth and inequality for policymakers to explore via redistribution in a developed economy like the UK; we show that these trade-offs face diminishing returns, with a rising sacrifice of growth required for a rising reduction of inequality.

These trade-offs have become a central concern of policy-makers, highlighted by Piketty and Zucman (2014); this reviewed a large amount of evidence for the fluctuations in inequality and growth over the industrial era in a wide range of countries. Many have concluded from this evidence that there must be some link between inequality and growth; however, a convincing link both in theory and in empirical studies has proved elusive, as we shall see in the literature reviewed in the next section. Here we propose a simple link of relevance to developed economies in which higher wealth makes it easier to take entrepreneurial risks; the link works formally through the marginal utility cost of barriers to entrepreneurial action becoming lower with rising wealth. However, this formal link in the abstract set-up of a DSGE model can be thought of as representing a wide practical menu of advantages in entrepreneurship accruing from higher wealth: these could include better contacts/networks, more parental financial backing and cross-holdings of shares across sectors, to mention just a few possible channels.

We introduce heterogeneity by classifying the population into two groups for simplicity, the rich (the top 10%) who own higher capital holdings and the rest. During our sample period these two groups already have been formed and behave according to their endowments. However, since all individuals have the same idiosyncratic characteristics, how they came to arrive in these two groups we also aim to explain in terms of the model.

The central mechanism in the model is Meenagh et al. (2007)'s endogenous growth mechanism. In this, individuals have entrepreneurship incentives which drive individual productivity growth and further aggregate growth. In addition, we relate individual entrepreneurship incentives to the wealth distribution so that the rich have larger entrepreneurship incentives than the poor, their wealth reducing the costs of entrepreneurial entry. History affords many examples of successful entrepreneurs born into rich families or the middle class, such as John Pierpont Morgan, Rupert Murdoch, Warren Edward Buffett, William Henry Gates III and Steven Paul Jobs etc. Levine and Rubinstein (2017) also provide some supporting evidence: using NLSY79 data in the US, they find that more entrepreneurs come from well-educated and high-income families. This mechanism reinforces wealth inequality since productivity growth tends to originate mainly with the rich, who in turn reap larger rewards. The mechanism causes wealth to be gradually concentrated on the rich while also gradually raising the growth rate. Nevertheless, this process can be interrupted and even temporarily reversed by aggregate shocks, such as crises and wars, and also by idiosyncratic shocks to income groups; furthermore, it can be, and often is, modified by redistributive policies. In this paper we test for the presence of such a mechanism in recent UK history which, as we have seen, shows interesting variation in both growth and inequality.

This paper has the following structure. The current introduction is followed by a literature review in the next section. Our model is set out in section

3. Then section 4 describes the Indirect Inference method for model testing and estimation. Section 5 introduces the data we use. Empirical results are described in section 6. Section 7 concludes.

2 Relevant Literature

2.1 Inequality and growth

Other theoretical work that has proposed ways in which inequality would affect growth includes: Galor and Tsiddon (1997) via parents' ability; Bhattacharya (1998) via bequests; Aghion and Bolton (1997) and Galor and Moav (2004) via borrowing restrictions biting more on the less wealthy; Acemoglu and Robinson (2001) via median voter effects of inequality causing political disruption to the economy. Our theory here is chosen for its potential relevance to a developed economy like the UK experiencing reforms designed specifically to benefit entrepreneurs; it focuses on an asymmetry between entrepreneurs according to wealth, an asymmetry that both causes inequality to boost growth and growth to boost inequality. However, as noted in the introduction, this asymmetry could also spring from some of the wealth-related advantages proposed in these earlier theories.

The main empirical effort to investigate more broadly the effect of inequality on growth, as asserted by an even wider range of theories than all these, has been through panel data methods, with multi-country time-series samples. These studies face a besetting identification problem: when an economy grows fast, whatever the reason that is triggering it, many accompanying features occur as well that enter the list of suggested causal/control factors: there is much R&D, more government spending on infrastructure, more education spending both public and private, better institutions and so on, including inequality. How is the panel econometrician to identify which causes are driving growth and which are driven by it? It is not easy to find convincing methods of identification.

Furthermore, we find in these panel data results a wide variation in the relationships between inequality and growth. This sensitivity in the results is partly related to the lack of identification, since the possible specification of variables and causal direction is not clearly pinned down, leaving a wide scope for econometric choice. Thus there is still no consensus on whether inequality stimulates or impedes economic growth. For example, Alesina and Rodrik (1994), Deininger and Olinto (2000) and Bagchi and Svejnar (2015) find a negative effect of inequality (income or wealth) on economic growth. By contrast, Perotti (1996), Barro (2000), Forbes (2000) and Berg et al. (2018) find a positive inequality effect (some only in developed countries). Overall, as Halter et al. (2014) pointed out, both the estimation method and the sample employed have a considerable influence on the estimated inequality effects.

Barro (2000), after an exhaustive examination of the panel data, sums his findings up as follows: 'Evidence from a broad panel of countries shows little overall relation between income inequality and rates of growth and investment. However, for growth, higher inequality tends to retard growth in poor countries and encourage growth in richer places. The Kuznets curve — whereby inequality first increases and later decreases during the process of economic development — emerges as a clear empirical regularity. However, this relation does not explain the bulk of variations in inequality across countries or over time.'

This leaves us with the more painstaking methods of testing particular theories such as ours in particular episodes, as we will do here. In this, we are able to deploy powerful methods available to time-series econometricians. A relatively unfamiliar method recently developed is that of Indirect Inference — Le et al. (2016) and Meenagh et al. (2019). In this, one sets out a fully identified structural model derived from the optimising theory and checks whether it can match the facts of an episode; this match is checked by simulation. The episode facts are described in some summary way, for example by a VECM, and the coefficients of this VECM are then compared with their statistical distribution, as obtained from repeated simulations of the model by bootstrapping the model's errors. If the probability of the actual episode facts coming from this distribution is below a critical threshold, the model is rejected. It turns out from our Monte Carlo experiment detailed below that in growth models like the one here the power of this test in small samples is extremely high against parameter inaccuracy and against serious mis-specification it is as much as 100%.

2.2 Heterogeneous Agent Models

Our research relates secondly to the literature on heterogeneous-agent models (HAMs), of which Bewley (1981, 1983) are early examples. These study variety in individual behaviour by introducing idiosyncratic shocks (individual income endowments) and incomplete capital asset markets (represented by a borrowing constraint). In the early stage, only idiosyncratic shocks (like individual income uncertainty and employment uncertainty) were employed for heterogeneity and the numerical algorithm concentrates on solving for equilibrium market prices like the real interest rate (Hansen, 1985; Aiyagari and Gertler, 1991; Aiyagari, 1994). Diaz-Gimenez and Prescott (1992), and Krusell and Smith (1998, 2006) develop a new method to solve models by searching for an equilibrium law of motion for the wealth distribution around which some new numerical algorithms are developed. The distribution is generally described by finite order moments for simplicity and individual decisions are assumed to be made based on the distribution of moments. However, there might be an infinite-dimensionality issue if high order moments are considered for individual optimal decisions (Algan et al. 2014). The new generation of HAMs attempts to remove the dependence on aggregate laws of motion when solving for individual behaviour by searching for equilibrium cross-agent distributions in each period described by density functions (Algan et al. 2008; Reiter, 2009; Young, 2010; Boháček and Kejak, 2018).

Our solution method here belongs to the latest generation. We solve the model via the Extended Path algorithm for nonlinear rational expectations models (Fair and Taylor, 1983; Matthews et al. 1994), simultaneously computing the optimal strategies of our two chosen population groups and the aggregate outcomes for the economy.

2.3 Entrepreneurship

Our paper also connects to the literature on entrepreneurship. There is a huge literature on modelling how entrepreneurship boosts growth via innovations of entrepreneurs. Most assumes inelastic labour supply and risky entrepreneurship and thus households have to make an occupational choice to be a regular worker or an entrepreneur (Boadway et al. 1991; Banerjee and Newman, 1993; Grossmann, 2009). There are also studies that investigate the relationship between entrepreneurship and wealth, and the impact of the redistribution policies on entrepreneurship (Cagetti and De Nardi, 2006, 2009; García-Peñalosa and Wen, 2008; Atolia and Prasad, 2011; Doepke and Zilibotti, 2014). In all these models, risk insurance is vital because of the uncertain cost of Schumpeterian entrepreneurship and their analyses focus on the static equilibria. This paper instead considers the deterministic cost of entrepreneurship such as market regulatory barriers and government barriers like taxes, and the dynamic interactions of entrepreneurship, wealth and growth. The role of entrepreneurship in our model is closely related to Minford and Meenagh (2019, 2020) which is described in detail in the next section.

3 The Model

3.1 An outline of the model

At the heart of this model is the idea that individual households own their own firm, in which they engage their own labour. They own previously invested capital, and they can deploy their scarce time either in leisure or in two forms of productive activity. They can profit from undertaking entrepreneurship and paying the costs of entrepreneurial entry; or they can pursue regular work on their existing capital stock, providing them with a normal income. In these labour choices they can compute an expected return from this entrepreneurship; at the margin they equate this expected return with the income they would get from working in this regular way. Individuals' optimal behaviour is affected by their current wealth: someone who is poor will have a high marginal utility of current consumption from regular work; someone who is rich will have a low one. Hence the relative marginal utility provided by expected returns on entrepreneurship will be higher for the rich than the poor.

We assume in this that the marginal utility of expected future returns will be similar for both since if the entrepreneurship is successful both will expect to be rich and if not both will expect to be poor; rising or falling future returns are accompanied by respectively falling and rising marginal utility as income and consumption move together. The key difference is in the marginal utility of the consumption sacrificed to pay the costs of entrepreneurship against which the expected return to entrepreneurship is measured. For a poor person whose income is low the marginal utility of these costs is higher, whereas for a rich person with high income the marginal utility of these costs is low. As all poor households are identical, similarly all rich households, we can think of poor working for a wage in other poor households — a labour market among the poor and similarly rich people working for other rich households — a rich household labour market. But the two labour markets are segmented.

In a world of complete equality, wealth is shared equally and is therefore modest for all households. For all of them the marginal utility of entrepreneurship costs is high relative to expected future returns from entrepreneurship and little entrepreneurship is done. As wealth is redistributed to one set of households, the rich, from the others, the relative expected returns rise for the rich as the marginal utility of entrepreneurship costs falls rapidly with the concavity of the utility function: meanwhile the marginal utility of these costs rises rapidly for the others for the same reason. As a result, the rich have a much larger incentive to undertake entrepreneurship, while the others lose their incentive. However, as the rich undertake more entrepreneurship, they augment their future income and wealth, which leads to further increases in the amount of entrepreneurship they undertake. This in turn induces further increases in their wealth so that the process tends upwards without limit, with both wealth and growth rising among the rich. As the poor reduce their entrepreneurship, it gradually falls off asymptotically towards zero and with it the growth in their wealth. So the total amount of entrepreneurship rises as this process continues, because of the asymmetry between the growing rise in entrepreneurship and growth among the rich on the one hand and the fall in both among the poor. Ultimately, as wealth is redistributed more and more the percentage rise in the richer group is applied to a large volume of entrepreneurship while the percentage fall in the poorer group is applied to a very small amount of entrepreneurship and is consequently trivial in absolute terms. This is the mechanism at work in the model, randomly triggered by individual shocks. Thus randomly some will be fortunate and acquire more wealth, others unfortunate and lose wealth. This will create more absolute entrepreneurship, with the fortunate carrying out most of it. Hence it will create rising growth. At the same time it will create rising inequality.

Finally, we have governmental intervention in this process via its tax/subsidy/regulation systems. We can think of this as a political process driven by the fact that the two groups, rich and poor, share the society and must live with each other in a sustainable way. We do not attempt to create a political economy model of this: we simply assume there is some exogenously given redistributive system in place and investigate its consequences for growth and inequality. Plainly there is a trade-off here for society and government: by intervening with redistribution the government will dilute the two mechanisms at work. There will be less inequality and less growth. We could think of the underlying political economy model as one in which the poor, who will be numerically most powerful, recognise this trade-off: they want growth but also want its fruits to be spread to them. At the one extreme they could be totally spread and then there would be no growth; at the other not spread at all and then there would be maximum growth but only enjoyed by the rich. This would be an interesting strand of future work to pursue in understanding the full dynamics of growth and inequality; however, in this paper we leave it to one side.

The set-up we have just set out is embedded in a standard and simplified DSGE model of two agents, who consume and save in the form of capital; there is standard borrowing and lending, and the economy is closed. One might then ask why entrepreneurs without wealth could not borrow to finance their ideas. The answer in this model is that they must post collateral which for them is too costly to contemplate.

3.2 Individual behaviour

Assume the population in an economy is comprised of two groups with constant population weights $\mu_i; i = 1, 2$. Both groups consider the same utility as follows where $N_{i,t}$ and $Z_{i,t}$ are labour input and entrepreneurship time respectively; there are idiosyncratic shocks to consumption ($\epsilon_{i,t}$) and labour ($\nu_{i,t}$) utility.

$$U(C_{i,t}, N_{i,t}, Z_{i,t}) = \Phi \frac{(C_{i,t}\epsilon_{it})^{1-\Psi_1}}{1-\Psi_1} + (1-\Phi)\frac{(1 - N_{i,t}\nu_{it} - Z_{i,t})^{1-\Psi_2}}{1-\Psi_2} \quad (1)$$

We assume both the capital market and each segmented labour market are perfect. Entrepreneurship has a unit cost π_t and the total cost of entrepreneurship for individual i is $\pi_t Z_{i,t}$. This cost is assumed to be the result of government taxes and regulations on entrepreneurial activity; the former will yield revenue to the government, either explicit (the taxes) or implicit (the regulations). In addition the government levies a general consumption tax at the rate, τ. We assume that these revenues are offset by lump sum transfer payments to households; in what follows we will assume that both the entrepreneurial cost and the lump sum transfers are indexed to general income, i.e. overall GDP, so that in effect the government decides on π_t to yield a ratio of the entrepreneurship cost to GDP each period. Agents have the following budget constraint where $b_{i,t}$ is individual bonds and T_t is the average lump sum transfer to households of the proceeds of the entrepreneurial tax.

$$(1-\tau)Y_{i,t} + (1+r_{t-1})b_{i,t} - \pi_t Z_{i,t} + T_t = C_{i,t} + b_{i,t+1} + K_{i,t} - (1-\delta)K_{i,t-1} \quad (2)$$

Individuals have a Cobb-Douglas production function (3) where the non-stationary individual productivity $A_{i,t}$ evolves as the process (4) which depends on individual time spent on entrepreneurship as well as the aggregate productivity shock $\upsilon_{A,t}$.

$$Y_{i,t} = A_{i,t}(K_{i,t-1})^{\alpha}(N_{i,t})^{1-\alpha} \tag{3}$$

$$\frac{A_{i,t+1}}{A_{i,t}} = \theta_1 + \theta_2 Z_{i,t} + \upsilon_{A,t} \tag{4}$$

As the model is deliberately simple, one can easily obtain the following optimal decisions from first order conditions.

$$(C_{i,t}\epsilon_{it})^{-\Psi_1} = (1+r_t)\beta E_t\left[(C_{i,t+1}\epsilon_{it+1})^{-\Psi_1}\right] \tag{5}$$

$$(C_{i,t}\epsilon_{it})^{-\Psi_1} = \beta\left\{E_t\left[(C_{i,t+1}\epsilon_{it+1})^{-\Psi_1}\right]\left[\alpha(1-\tau)\frac{Y_{i,t+1}}{K_{i,t}} + 1 - \delta\right]\right\} \tag{6}$$

$$\frac{1-\Phi}{(1-N_{i,t}\nu_{it}-Z_{i,t})^{\Psi_2}} = \Phi\,(C_{i,t})^{-\Psi_1}(1-\tau)(1-\alpha)\frac{Y_{i,t}}{N_{i,t}} \tag{7}$$

$$\frac{1-\Phi}{(1-N_{i,t}\nu_{it}-Z_{i,t})^{\Psi_2}} + \Phi\frac{\pi_t}{(C_{i,t})^{\Psi_1}} = \Phi\theta_2\frac{A_{i,t}}{A_{i,t+1}}E_t\left[\sum_{s=1}^{\infty}\beta^s\frac{(1-\tau)Y_{i,t+s}}{(C_{i,t+1})^{\Psi_1}}\right] \tag{8}$$

Equation (8) is an accurately optimal decision rule for $Z_{i,t}$ and can be turned into equation (9) by approximating $Y_{i,t}/C_{i,t}$ as a random walk before the steady state (shown in Appendix A).

$$(1-\tau)(1-\alpha)\frac{Y_{i,t}}{N_{i,t}} + \pi_t = (1-\tau)\frac{A_{i,t}}{A_{i,t+1}}Y_{i,t}\frac{\beta\theta_2}{1-\beta} \tag{9}$$

where Ψ_1 is set to unity for simplicity and this value is also used in our empirical study. Equation (9) gives an approximately optimal decision rule for $Z_{i,t}$. According to the perfect labour market assumption, $(1-\alpha)\,Y_{i,t}/N_{i,t}$ is the individual implied real wage rate $w_{i,t}$. As individual entrepreneurship has the unit cost π_t as well as the unit opportunity cost $w_{i,t}$, we define an "entrepreneurship penalty rate" $\pi'_{i,t} = \pi_t/w_{i,t}$ to reflect the total cost and rewrite (9) as the following

$$\frac{A_{i,t+1}}{A_{i,t}} = \frac{(1-\tau)\beta\theta_2\frac{Y_{i,t}}{w_{i,t}}}{(1-\beta)(1-\tau+\pi'_{i,t})} \tag{10}$$

Notice how the costs of entrepreneurship relative to current productivity, $\pi'_{i,t}$, and so to current income, reduce productivity growth. As current income and consumption fall, the marginal utility of these costs rises, raising the disincentive to entrepreneurship.

We can easily linearise (10) as (11) by relegating $Y_{i,t}/w_{i,t}$ into the error term.

$$\ln A_{i,t+1} - \ln A_{i,t} = \phi_{1,i} - \phi_{2,i}\ln\pi'_{i,t} + \varepsilon_{A,t} \tag{11}$$

We specify $\pi'_{i,t}$ as a function of the economy-wide money costs of entrepreneurship relative to the average wage ($\pi'_t = \pi_t/w_t$) and so to the *ith* individual's wealth through the curvature of the utility function; our exact specification is given below.

3.3 Entrepreneurship penalty rate

Via (10) the individual penalty rate falls with rising income. Income rises directly with capital input. We capture this effect by assuming it to be negatively related to the lagged individual-aggregate capital per capita ratio $K_{i,t-2}/K_{t-2}$ which is a measure of relative wealth and income. This assumes, as noted earlier, that entrepreneurial costs are indexed to the general rise in income (and so wealth). Lastly we assume a lagged adjustment term, $\pi'_{i,t-1}$. $\pi'_{i,t}$ thus evolves as follows.

$$\ln \pi'_{i,t} = \rho_0^\pi + \rho_1^\pi \ln \pi'_{i,t-1} - \rho_2^\pi \cdot Q(K_{i,t-2}/K_{t-2}) + \ln \pi'_t \tag{12}$$

where $\rho_1^\pi \geq 0$; $\rho_0^\pi, \rho_2^\pi > 0$. We assume that our function (effectively the inverse of consumption marginal utility, rising with income and so capital) takes the form $Q(K_{i,t}/K_t) = (\mu_i/\omega_{Y,i})\,(K_{i,t}/K_t)^2$ where capital and income are normalised at unity if they are equal to the average. $K_{i,t}/K_t = 1$ represents perfect equality while either greater or less than 1 implies inequality.[1] In Appendix B we show that $Q(K_{i,t}/K_t)$ has a minimum at perfect equality where $K_{i,t}/K_t = \mu_i/\omega_{Y,i} = 1$, and penalty rates are identical across agents.

3.4 Aggregate economy

Each aggregate variable is the weighted sum of individual ones.

$$Y_t = \mu_1 Y_{1,t} + \mu_2 Y_{2,t} \tag{13}$$

$$K_t = \mu_1 K_{1,t} + \mu_2 K_{2,t} \tag{14}$$

$$C_t = \mu_1 C_{1,t} + \mu_2 C_{2,t} \tag{15}$$

Market clearing in goods can be written as

$$Y_t = K_t + C_t - (1-\delta)K_{t-1} + G_t + \epsilon_{m,t} \tag{16}$$

where $G_t = \tau Y_t$ represents government consumption spending and the error $\epsilon_{m,t}$ represents other miscellaneous government spending. The government returns the entrepreneur tax cost minus this miscellaneous spending as a lump sum transfer to all households; and it spends its general tax revenue on consumption which, like its other spending, has no effect on the economy's productivity or household utility. It thus pursues a balanced primary budget and

[1] $\mu_i/\omega_{Y,i}$ aims to avoid the penalty policy too beneficial to the rich as the poor generally has a greater population weight relative to their average income share.

for simplicity we assume has no debt. The bond market clears via Walras' Law. We can therefore rewrite equation (16) in terms of private disposable income as

$$(1-\tau)Y_t = K_t + C_t - (1-\delta)K_{t-1} + \epsilon_{m,t} \tag{17}$$

The list of linearised model equations is shown in Appendix C.

4 Indirect Inference

In this section, we set out and explain our methodology of model testing and parameter estimation: Indirect Inference (II), developed by Le et al. (2011). II is based on the idea that if the structural model is true in terms of both specification and parameters, the properties of the actual data should come from the distribution of the properties of the simulated data with some critical minimum probability. The data properties can be captured by a simple 'auxiliary model' such as a VAR, impulse response functions or moments. Define the parameters of the structural model and the auxiliary model as θ and β respectively. We first use the actual data to estimate the auxiliary parameters, say $\widehat{\beta}$. Given the null hypothesis $H_0 : \theta = \theta_0$, we simulate S samples using the structural model and estimate the auxiliary parameters using each simulated sample to obtain estimators $\widetilde{\beta}_s(\theta_0); s = 1, \cdots, S$. To evaluate whether $\widehat{\beta}$ comes from the distribution of $\left\{\widetilde{\beta}_s(\theta_0)\right\}$, we compute the Wald statistic

$$Wald_a = \left[\widehat{\beta} - \overline{\widetilde{\beta}_s(\theta_0)}\right]' \sum(\theta_0)^{-1} \left[\widehat{\beta} - \overline{\widetilde{\beta}_s(\theta_0)}\right]$$

which asymptotically follows a $\chi^2(k)$ distribution where k is the number of elements in β and $\sum(\theta_0)$ is the variance-covariance matrix of $\widehat{\beta}_s - \widetilde{\beta}_s(\theta_0)$. We can check the allocation of $Wald_a$ in the distribution of simulated $Wald_s$; $s = 1, \cdots, S$ where $Wald_s$ is computed when using the s^{th} simulated sample to estimate $\widehat{\beta}$. If $Wald_a$ is less than the c^{th} percentile value of $\{Wald_s\}$ sorted from smallest to largest, H_0 cannot be rejected in a $c\%$ confidence interval; otherwise the model is false. An alternative way is to compute the transformed Mahalanobis Distance (TMD) and compare it with the critical value of t distribution on the $c\%$ confidence interval.

$$Z = T_c \left[\frac{\sqrt{2Wald_a} - \sqrt{2k-1}}{\sqrt{2Wald_c} - \sqrt{2k-1}}\right]$$

where T_c is the critical value of a one-tail t distribution on the $c\%$ confidence interval.

Generally, a (linearised) DSGE model can be represented as a VARMA or a VAR(∞) which can be further represented to a VAR(p) with a finite order or even a VAR(1) (Dave and De Jong 2007; Wickens 2014). However, the long-run solution of our model can only be approximated as a VARX with non-stationary lagged endogenous variables X due to nonstationary productivities. Le et al. (2011), Le et al. (2016) and Meenagh et al. (2019) conduct Monte

Table 1 Power test against numerical falsity of parameters

Parameter Falseness	True	1%	3%	5%	7%	9%
Rejection Rate with 95% Confidence	5%	10%	20%	28%	44%	97%

Carlo experiments to find the power of the test as the variables included and the order of the VAR vary. They find that a VAR(1) in 3 endogenous variables typically has good power, while raising the order or the variable number further can boost the power too far for any hope of finding a tractable model that can pass the test. Hence, we use a VARX(1) with 3 variables, aggregate output, aggregate capital and capital inequality, combined with the lagged individual productivities as the "X". The auxiliary parameter vector β contains 9 VAR coefficients and 3 variances of the VAR residuals.

Given the null hypothesis that the structural model is true, one can back out the structural errors from the model and the actual data and then bootstrap these structural errors to obtain simulated samples. II is also used to estimate the parameters by searching for the parameter values such that the relevant Wald or TMD is smallest.

Le et al. (2011) and Le et al. (2016) conduct Monte Carlo power tests on three testing methods on different models: II, the Likelihood ratio test; and the "unrestricted Wald" test (in which the reduced form VAR on the data sample rather than the VAR from the structural model is bootstrapped). II is found to be give far more potential power than the other classical testing methods.

To evaluate the power of II on our model here, we use Monte Carlo experiments to compute the power of the test both against parameter mis-estimation, and more importantly, against model mis-specification, including models with different causal sequencing and capable apparently of providing 'observationally equivalent' data.

4.0.1 The power of the test against numerical inaccuracy

We first generate 500 samples from the true model and the actual data. Then treating each simulated sample from the true model as the observation, we test the false model by II and calculate the rejection rate out of the 500 Monte Carlo experiments. Table 1 shows the result of our power test against the false models with mis-estimation where both structural parameters and the AR coefficients of the errors are steadily falsified by $+/-x\%$ each time. The probability of rejecting the false models rises sharply with an increase in the falsity of parameters.

4.0.2 The power of the test against mis-specification

We then test the power of II against a mis-specified model in which the basic mechanism of wealth inequality on entrepreneurship is turned off: the equa-

Table 2 Frequency of rejection of mis-specified model when estimated model is true

Mis-specified model	Bootstrap samples	Rejected samples	Frequency of rejection
Replace (C15)-(C16) by AR processes	500	497	99.4%

tions of the penalty rate (40)-(41) in Appendix C are replaced by a simple AR(1) process. Wealth inequality in this false model continues to be generated by randomness but does not in turn generate more innovation. We keep the parameters the same as the fully-estimated values in the benchmark model. Growth therefore continues to occur as a result of shocks to the aggregate penalty rate. But there is no longer any mechanism linking growth to inequality. On the same 500 samples the rejection rate of this mis-specified model at 95% confidence is no less than 99.4% and hence we can conclude that II provides huge power against mis-specified models which attempt to mimic the results from the true model.

5 Model Data

We use seasonally adjusted quarterly data in the UK from 1978Q1 to 2015Q4 without any filtering. The aggregate output and consumption are GDP and household final consumption expenditure respectively, measured by chain volume with base year 2012 from UK national statistics. The nominal interest rate is the 90-day rate reported by the Bank of England expressed as a rate per quarter. The real interest rate is the difference between this and the expected next-quarter inflation rate. Aggregate labour is the number employed in the UK aged 16 and over relative to the size of the labour force. The aggregate capital stock is estimated by the perpetual inventory method.

For data on the entrepreneurship penalty rate $\pi'_{i,t}$, we follow Minford and Meenagh (2020): disincentives are determined by two factors, "labour market regulation" (LMR) and the tax rate. LMR describes the degree of intervention in the labour market: this is measured by the average of two indicators "centralised collective bargaining" (CCB) and "mandated cost of worker dismissal" (MCD) reported by the Fraser Institute. CCB describes the procedure for both employers and employees to make a collective agreement on contracts (Gernigon et al. 2000). It is measured by a value from 0 (hardest to agreements) to 10 (easiest), so is MCD. For the tax rate, we use here the corporation tax

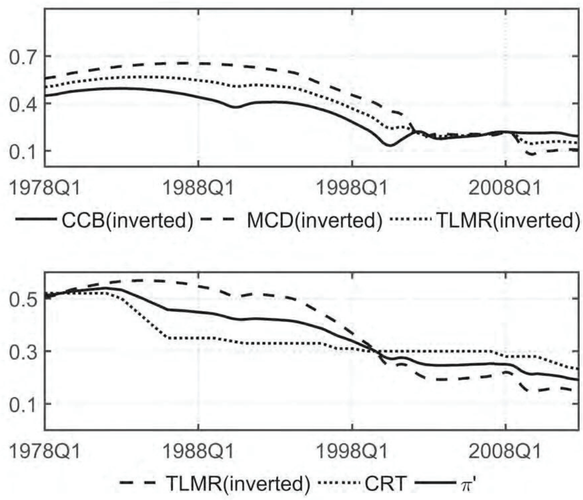

Fig. 3 Generated Penalty Rate

rate (CTR). [2] The quarterly π'_t is then the average of the transformed LMR (TLMR) and the CTR as shown in Figure 3.

The two groups we consider are the two income deciles, the top 10% and the bottom 90%. Data on income and wealth distributions come from the World Wealth and Income Database (WID). We use the distribution of taxes on final goods and services reported by UK ONS as our proxy for the consumption distribution. Group labour supply is estimated as group labour income divided by its wage. We find the individual labour ratio is quite close to the group population ratio so that one can also simply assume both inner-group representative agents have the same labour supply. Individual $\pi'_{i,t}{=}\pi'_t/w_{it}$. Individual productivities are the individual Solow residuals. We measure inequality by the share of the top decile plotted in Figure 2, rather than the Gini coefficient (D'Ambrosio and Wolff, 2011).

[2] As the available data on both CCB and MCD are annual ones and are incomplete in our time horizon, we firstly supplement the omitted annual observations using 3-points quadratic estimating interpolation. Then to generate quarterly series, we follow Minford and Meenagh (2020) to use a Denton method with a highly frequent instrument, Trade Union Membership rate (TUM) which is the fraction of the number of employees who are trade union members out of the total number of employees. Since both CCB and MCD describe how lax the regulation is, we define the final instrument by the inverted "1-TUM". Afterwards, we can yield quarterly data of CCB and MCD using Denton method. Lastly, these quarterly data are inverted (in order to describe costs instead of benefits) and scaled less than unity (to keep consistent with the magnitude of the tax rate).

6 Empirical Results

6.1 Model tendencies over time

This section examines the workings of the interactive growth-inequality mechanism of the model. We start from two identical groups in our two deciles; there is no inequality. Inequality is then generated solely by innovations from given distributions. Since backed-out innovations from the model are the same across identical agents, bootstrapping is not used to generate initial inequality. Instead, innovations of individual labour input are randomly drawn from an independent normal distribution with equal mean for both groups so that each group has an equal chance to become the richer one.

As we expect the tendency to be parameter-independent if it exists, parameter values are calibrated at illustrative values. We see how growth and inequality move over a period of 250 quarters, or 62 years. To carry out this experiment, we assume an initial 'base-line' economy in which wealth shares remain equal from the start, with the effect of capital shares on productivity over-ridden, to reproduce aggregate observed data trends over 250 quarters. We then draw innovations from identical idiosyncratic distributions (of labour input — i.e. 'working hard' — each stationary distribution has a positive mean and variance), and generate the simulated long sample shown. Each shock draw is added to the lagged effect of the previous shocks, so steadily raising the level of each group's input over time, as the mean level gradually converges on a long run level, simulating a very low exogenous fixed growth rate but with the input varying around it in a stationary manner. It can be seen that these repeated shocks create rising capital inequality and rising growth, even though the shocks simply impart a small fixed deterministic growth rate (Figure 4).

It might be thought that random draws would generate initial inequality that further sets of random draws would tend to offset, causing a reversal. However, the draws create a self-perpetuating effect via the model's mechanism; once an unequal wealth allocation has been established, this triggers asymmetric growth favouring the more wealthy. This is self-feeding so that future sets of draws reducing inequality merely act temporarily to dampen the self-feeding process. What we see here is similar to a first-mover advantage in sports of children born early in a year who therefore stand out as bigger and stronger initially in their year group; these children get more training/encouragement in consequence, making them stand out more in later stages of education, and so on. This gives rise to the birth-date effect in sports/athletic prowess (e.g. Stracciolini et al. 2016).

It turns out that a key factor required to initialise the mechanism is the small deterministic growth rate due to the mean shock. When this is added to the random variation, it triggers a decisively larger response among those who by chance have become rich than among those who by chance have become poor; this is due to the nonlinearity of the response of innovation to capital inequality. It is this interaction that triggers the growth-inequality mechanism.

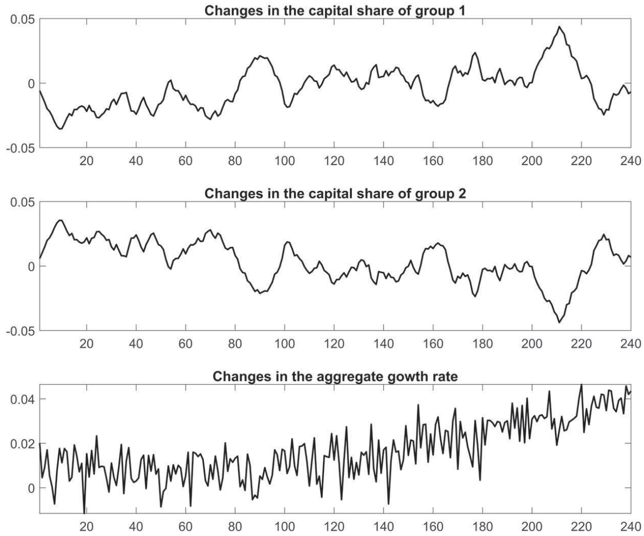

Fig. 4 Tendency caused by labour input innovations with a positive mean

Without this mean shock, there is merely fluctuation in inequality in both groups and no growth-inequality tendency as revealed in Figure 5.

It is by adding to this a simple equal upward trend in labour productivity in both groups, such as is found in the UK as in most economies — such as is given by the mean 0.01 shock for both groups assumed in our earlier chart (Figure 4) — we obtain the previous steady rise in the aggregate growth rate over time.

The addition of further shocks, aggregate or group, either affects the growth rate (equal shocks to π_{it}, or aggregate shocks to productivity) or inequality (differential shocks to π_{it} or to productivity), or both (interest rate shocks). For example the next chart (Figure 6) is what we obtain when we add aggregate interest rate innovations (due to demand shocks) to these individual labour productivity shocks. These added demand shocks to the cost of capital change investment differentially because output from the richer group is growing faster than for the poorer group. So the richer group invests more and gets richer faster than the poorer group. Inequality increases over time by more as does the growth rate.

When all shocks as estimated for the model are applied to a neutral baseline, we obtain rising inequality and growth on a substantial scale (Figure 7).

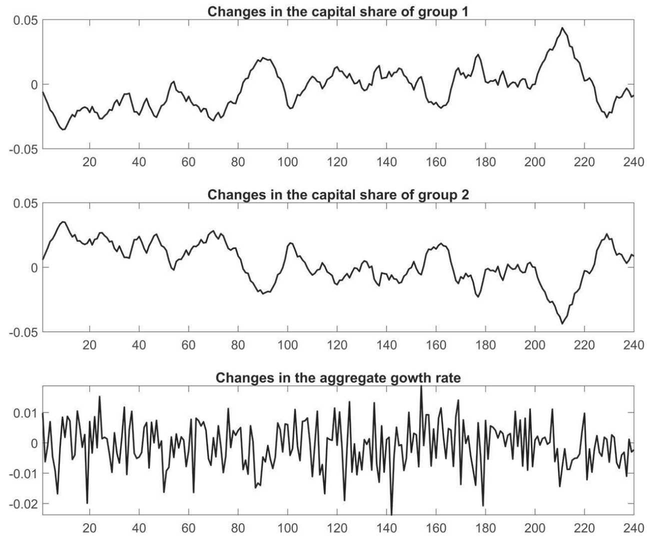

Fig. 5 Tendency caused by labour input innovations with zero mean

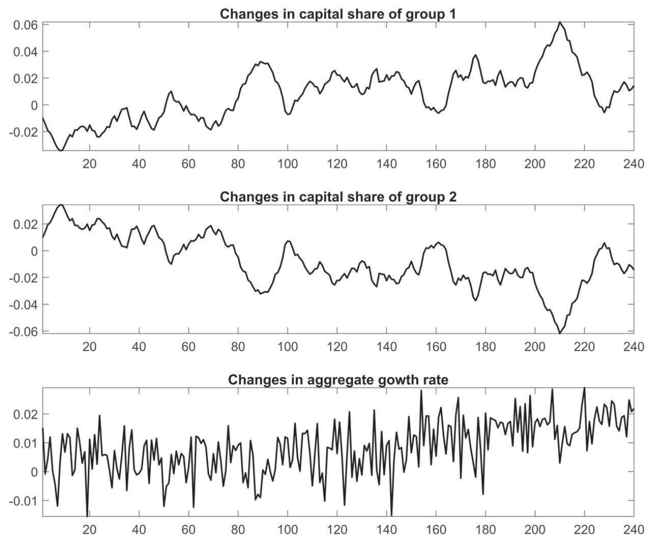

Fig. 6 Tendency caused by innovations of both labour input and interest rate

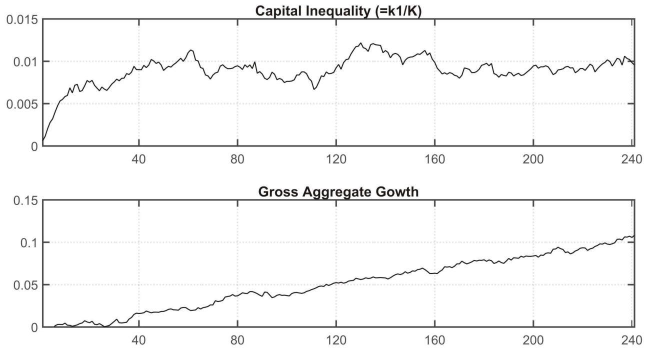

Fig. 7 Tendency of Aggregate Growth and Capital Inequality

Table 3 Calibrated Parameters

Share of capital in Cobb-Douglas production	α	0.3000
Utility discount rate	β	0.9975
Capital depreciation rate	δ	0.0034
Share of conumption preference in CRRA utility	Φ	0.5000
Elasticity of consumption in CRRA utility	Ψ_1	1.0000
drift in individual entrepreneurship penalty equations	ρ_0^π	0.3690
Steady-state consumption share by top 10% income decile	$\omega_{C,1}$	0.2000
Steady-state aggregate output/consumption ratio	Y/C	1.7100
Steady-state aggregate capital/consumption ratio	K/C	33.535

Table 4 Estimated Parameters

Elasticity of labour in CRRA utility	Ψ_2	1.0000
Marginal effect of entrepreneurship time on individual productivity growth	θ_2	0.5100
(Negative) Marginal effect of capital on individual entrepreneurship penalty rate	ρ_2^π	0.0012
(Negative) Marginal effect of penalty rate on productivity growth for the rich	$\phi_{2,1}$	0.5479
(Negative) Marginal effect of penalty rate on productivity growth for the poor	$\phi_{2,2}$	0.2195

6.2 Empirical results on actual data

In this section, we consider whether the model can match the actual behaviour of UK quarterly data since 1978. Table 3 shows our calibration by either theoretical considerations or observed data such as shares. The elasticity of consumption in utility Ψ_1 is set to 1 following the proof in Appendix A. The constant term ρ_0^π in the individual equation of $\pi'_{i,t}$ is set to the sample average of the aggregate penalty rate.

Our estimation is focused on the entrepreneurship process: ρ_2^π measures how capital distribution affects entrepreneurship by reducing the effect of the penalty rate and the two $\phi_{2,i}$ parameters tell us how individual productivity responds to changes in penalty rate for the rich and the poor respectively and the estimated parameters are shown in Table 4.

Table 5 AR coefficients of structural errors

$ln\pi'_t$	$\varepsilon_{A,t}$	$\varepsilon_{C,t}$	$\varepsilon_{C1,t}$	$\varepsilon_{C2,t}$	$\varepsilon_{N1,t}$	$\varepsilon_{N2,t}$
0.973	0.938	0.833	0.888	0.866	0.973	0.968

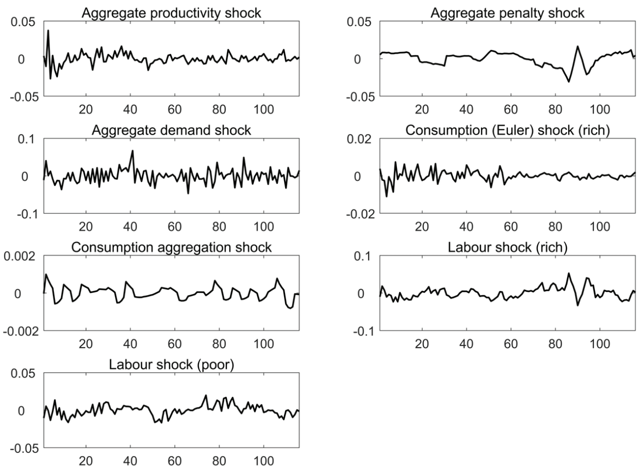

Fig. 8 Backed-out errors with estimated parameters

Table 6 II wald test results

Number of samples	Wald Statistic	Transformed MD (t-stat)	P-value
1000	30.93	1.608	5.4%

Given these parameters, AR coefficients of the backed-out structural errors are summarised in Table 5.

Figure 8 shows the backed-out errors with estimated parameters.

The II testing result in Table 6 indicates that the null hypothesis that the structural parameters are equal to our estimators cannot be rejected on the 5% confidence interval.

As illustrated in the II section, our auxiliary model VARX takes the form of

$$\begin{bmatrix} Y_t \\ K_t \\ IQ_t \end{bmatrix} = \beta \begin{bmatrix} Y_{t-1} \\ K_{t-1} \\ IQ_{t-1} \end{bmatrix} + \alpha X_t + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}$$

Table 7 Coefficients of the auxiliary model

Auxiliary Coefficients	Actual	Simulated		
		Mean	Lower 2.5%	Upper 2.5%
β_1	0.64977	0.47635	0.21154	0.76607
β_2	0.29530	0.17139	−0.00842	0.31557
β_3	0.03509	−0.17624	−0.92794	0.30629
β_4	0.01055	0.01372	−0.00594	0.03553
β_5	1.00728	0.99974	0.98835	1.01358
β_6	−0.00388	−0.00477	−0.05051	0.03764
β_7	−0.03665	0.03303	−0.06125	0.11580
β_8	0.03412	−0.02949	−0.10705	0.01613
β_9	0.97499	0.92975	0.69342	1.06173
$Var(\widehat{\varepsilon}_1)$	0.0000245	0.0001205	0.0000587	0.0002094
$Var(\widehat{\varepsilon}_2)$	0.0000003	0.0000008	0.0000005	0.0000015
$Var(\widehat{\varepsilon}_3)$	0.0000131	0.0000115	0.0000062	0.0000200

where X_t is a vector of the exogenous non-stationary variables. Table 7 shows the coefficients of the auxiliary model with the actual data compared to those with the simulated data. [3]

As the table shows, the VAR coefficients all lie within the simulated 95% bounds, while two out of three error variances lie outside. This general tendency of data coefficients to lie inside the model-simulated range gives a useful insight into why the test is passed, though it leaves out the simulated covariation which can be important in the joint test.

6.2.1 Impulse response functions

Now we analyse how the major shock, to the penalty rate, affects both the aggregate economy and individual behaviours — the full range of IRFs is shown in the figure section at the end. If $\pi'_{i,t}$ falls due to a negative shock like in Figure 9, both groups will undertake more entrepreneurship. However, the distributions of capital will gradually become more unequal because the group innovation rate, and so growth rate, is more sensitive to changes in the penalty rate for the rich. Meanwhile, the aggregate output will also rise as does aggregate growth. This IRF shows rather clearly how an aggregate shock raising all groups nevertheless spurs inequality as well as growth.

The example demand shock — not important in the model's behaviour, as shown by the variance decomposition below — triggers the model's business cycle behaviour, and has no effect on inequality in the short term, and only a temporary effect on growth. However, it has a permanent effect on output and productivity as both groups react to higher income by innovating for a time (See more IRF figures in Appendix D).

[3] The auxiliary coefficient vector to compute the Wald statistic contains the elements in $\beta \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 \\ \beta_4 & \beta_5 & \beta_6 \\ \beta_7 & \beta_8 & \beta_9 \end{bmatrix}$ and the variances of the VARX regression residuals $Var(\widehat{\varepsilon}_i); i = 1, 2, 3$.

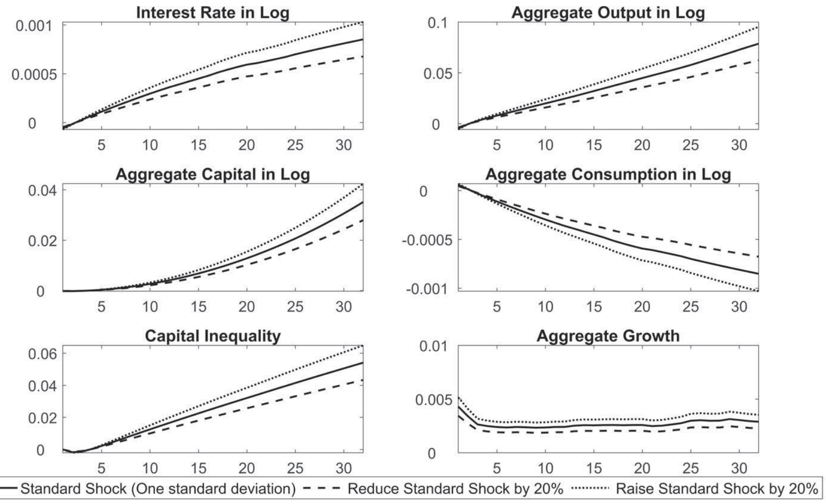

Fig. 9 IRF to the negative aggregate penalty rate shock

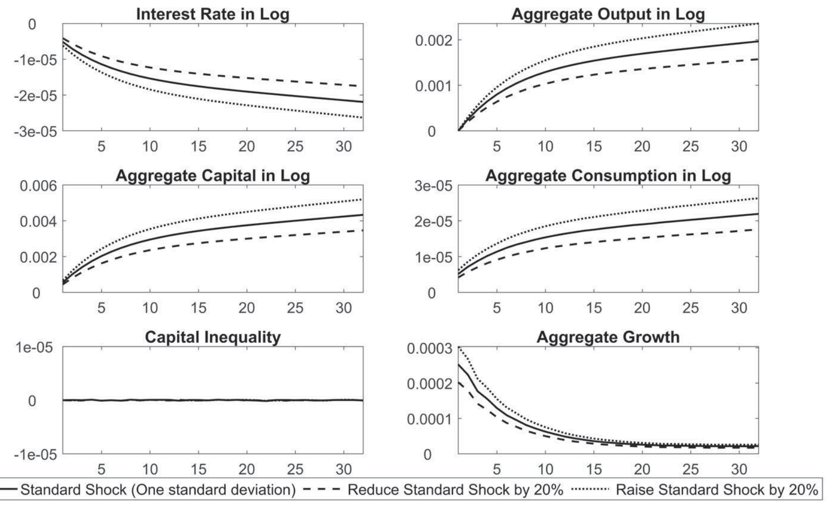

Fig. 10 IRF to the demand shock

Next, the variance decomposition.[4] Table 8 indicates that all variables' variation is dominated by innovations in the penalty rate and aggregate pro-

[4] We first draw random innovations from normal distributions with model-implied standard deviations on one structural error individually to obtain 1,000 simulated samples. Then we calculate the variance of each endogenous variable along the time horizon and average variances over samples for each variable. Repeat this on each structural error and calculate the proportions corresponding to different errors for each variable. This decomposition tells us whether the change in a variable caused by a certain innovation is stable across time.

Table 8 Variance decomposition of key variables (static proportions)

Variable	Penalty shock	Prod. shock	Demand shock	Euler shock (rich)	Cons. shock (poor)	Labour shock (rich)	Labour shock (poor)
Interest rate	74.0%	11.5%	2.50%	0.30%	0.00%	6.00%	5.60%
Aggregate output	86.4%	8.90%	0.90%	0.10%	0.00%	1.50%	2.20%
Aggregate capital	83.5%	10.4%	1.90%	0.10%	0.00%	1.60%	2.40%
Aggregate cons.	86.7%	2.60%	0.50%	5.20%	2.40%	1.40%	1.20%
Income of the rich	87.4%	5.30%	0.50%	0.20%	0.00%	5.50%	1.00%
Income of the poor	66.8%	18.4%	1.90%	0.20%	0.00%	4.80%	7.90%
Capital of the rich	85.8%	7.10%	1.30%	0.20%	0.00%	4.40%	1.10%
Capital of the poor	63.7%	17.1%	3.20%	0.30%	0.10%	7.70%	8.00%
Cons. of the rich	73.4%	2.20%	0.50%	21.7%	0.00%	1.20%	1.10%
Cons. of the poor	93.7%	2.80%	0.60%	0.10%	0.00%	1.50%	1.30%
Labour of the rich	83.8%	5.10%	0.50%	0.50%	0.00%	9.20%	0.90%
Labour of the poor	62.6%	17.0%	1.70%	0.20%	0.00%	4.30%	14.2%
Capital inequality	78.9%	0.00%	0.00%	0.60%	0.00%	14.0%	6.50%
Growth rate	31.7%	17.6%	9.20%	9.20%	9.10%	11.3%	11.9%

ductivity: since both these shocks feed into the model through their effects on productivity, this reveals that variation in productivity dominates the model. Other shocks play a minor role, since they do not affect productivity. Productivity is non-stationary, hence the penalty rate and innovation shocks to its growth rate have a permanent effect on non-stationary output. Their effects on the growth rate and capital inequality reveal that the penalty rate is the major driver of these two stationary variables.

6.3 Redistribution policy

As the rising wealth concentration will not spontaneously die out due to the interactive relation between inequality and economic growth — as illustrated above for a long lasting bootstrap simulation of the model — a redistribution policy is needed if policy-makers are concerned with social equality. Rao and Coelli (2002) also propose this policy dilemma that governments may apply deregulation to improve competitiveness and production efficiency which generally raises inequality at the same time. If social welfare is reduced by inequality, then, governments may wish to trade off inequality and output. Now we consider what such policy intervention might achieve.

Consider a policy of redistribution, income transfer from the rich to the poor. For simplicity, we assume that the government transfers a certain proportion τ_{Y1} of the income of the rich to the poor; we assume there is no government spending for which income tax revenue needs to be raised so that government here is purely acting on redistribution, not goods provision. The realisation of the income transfer needs an approximation in our model.[5] Av-

[5] The realisation of the income transfer needs approximation in our model; otherwise the transferred income to the poor will not affect their individual capital accumulation and

Table 9 Summary of the redistributive effects by income transfer

Fall in Growth rate	Fall in Inequality	Transfer rate
0.5%	1.0%	0.1
1.0%	1.5%	0.2
2.0%	2.5%	0.3

eraging 500 bootstraps, we compare the model outcomes for aggregate output, growth and inequality with these varying income transfer rates. The comparisons are shown in Figure 11. As we can see, compared with a zero transfer rate, the income transfer, as would be expected, reduces inequality but at the cost of lowering growth and so long term output. As the redistribution rate is raised, we see that inequality falls further but at a rising cost in terms of growth. The marginal ratio of loss of growth to reduction in inequality rises with the rising transfer rate. Thus, an initial movement towards redistribution (from none to a rate of 0.1) lowers inequality by nearly 1% in capital share of top income group, compared with no-transfer case but reduces growth by about 0.5% — a ratio of 0.5. Pushing the redistributive rate up further to 0.2 reduces inequality by a further 0.5% in capital share of top income group, compared with no-transfer case but lowers growth by a further 0.5% — a ratio of 1.0. Pushing the transfer rate up to 0.3 reduces inequality by another 1% but lowers growth by a further 1% — also a ratio of 1.0 (Table 9). Hence the marginal trade-off between inequality and growth becomes worse at higher rates of redistribution.

The way these transfers impact on the economy over time is also instructive, as shown in Figure 12 for accumulating output loss over time and evolving inequality. The reduction in inequality comes quickly and is then partially reversed as output steadily falls with the lower growth rate, so gradually reducing the value of the transfer as well as steadily impoverishing the economy as a whole.

7 Conclusions

This paper constructs a theoretical framework to investigate the relationship between capital inequality and economic growth in the UK during the recent decades. In our model, wealth inequality enhances entrepreneurship incentives of the rich to stimulate growth and the growth in turn aggravates inequality.

consumption because the individual budget constraints themselves are not used as model equations. We use an approximation to avoid this trap. Suppose a constant income tax rate τ_Y is enforced on the rich. The tax revenue per capita across the whole population now is $\tau_Y \mu_1 Y_{1,t}$ which is transferred to the poor at the end of period t. Since the original linearised individual capital equation can be written as $\ln(K_{i,t}/Y_{i,t}) = \exp[1/\Psi_1 - K/((1-\tau)\alpha Y)] \equiv f(r_t)$, individual capital per capita $K'_{i,t}$ after income transfer can be written as $K'_{i,t} = f(r_t) Y_{i,t}(1-x_i)$ where x_i is the equivalent income tax rate for group i. Given the population weights μ_i, $x_1 = \tau_Y$ and $x_2 = -\tau_Y(\mu_1 Y_{1,t})/(\mu_2 Y_{2,t}) \approx -\tau_Y \omega_{Y1}/\omega_{Y2}$.

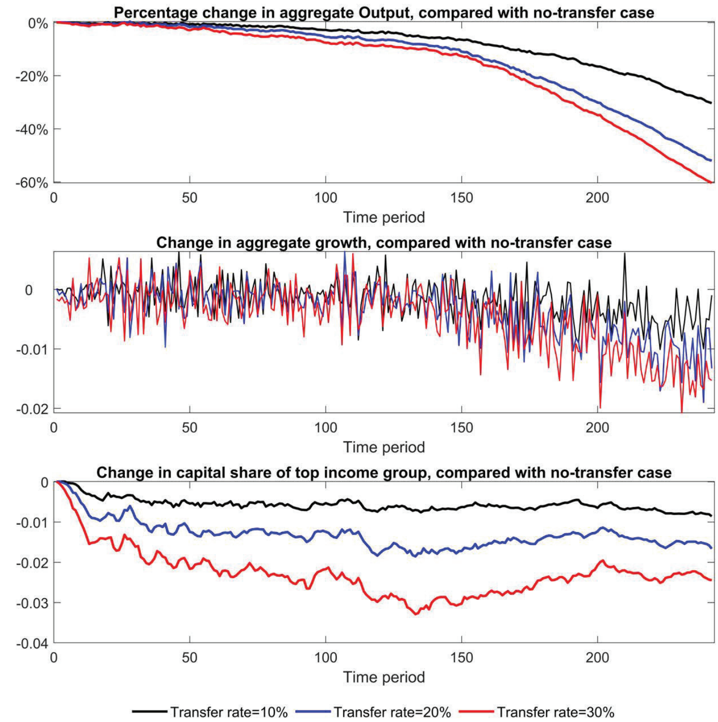

Fig. 11 Redistributive effects by different income transfer rates

When considering the 10%-90% income segmentation, our benchmark model cannot be rejected by the Indirect Inference test and can fit the main characteristics of the UK data. Policy-makers have to face a trade-off between wealth equalisation and economic growth when redistribution policy is conducted. For this, an income transfer regime provides the best trade-off between growth and inequality. The trade-off worsens as the transfer rate rises which suggests that governments will limit the transfer rate. While our model is deliberately simplified and other complicated settings could be added to in various ways, it is striking that it can match UK data behaviour closely, while offering a theoretically attractive mechanism for explaining the broad international correlations between inequality and growth.

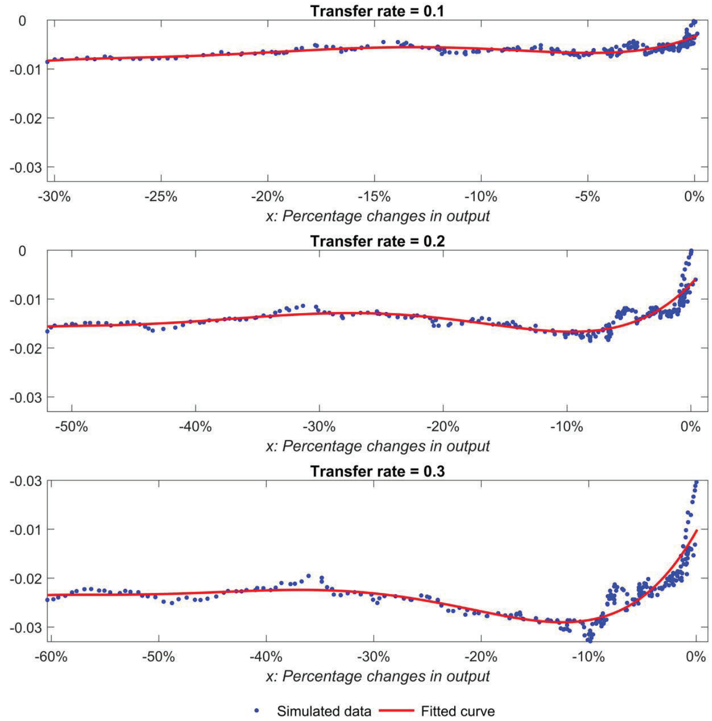

Fig. 12 Relation between changes in output and changes in capital share of top income group

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Appendix A: Approximating C-Y ratio by a random walk

Define $\widetilde{Y}_{i,t} = Y_{i,t} + (1-\delta)K_{i,t-1} - K_{i,t} - \pi_t Z_{i,t}$. Rewrite individual budget constraint as

$$(1 + r_{t-1})b_{i,t} = C_{i,t} + b_{i,t+1} - \widetilde{Y}_{i,t} \tag{18}$$

The condition of no Ponzi that present value of all the future increment of bonds should be equal to $(1 + r_{t-1})b_{i,t}$ implies

$$(1 + r_{t-1})b_{i,t} = C_{i,t} - \widetilde{Y}_{i,t} + E_t \sum_{j=1}^{\infty} \frac{C_{i,t+j} - \widetilde{Y}_{i,t+j}}{\varPi_{s=1}^{j}(1 + r_{t+s-1})} \tag{19}$$

Euler equation (5) can be approximated by

$$(C_{i,t})^{\Psi_1} = \frac{1}{\beta} E_t \left[\frac{(C_{i,t+1})^{\Psi_1}}{(1 + r_t)} \right] \approx \frac{1}{\beta^j} E_t \left[\frac{(C_{i,t+j})^{\Psi_1}}{\varPi_{s=1}^{j}(1 + r_{t+s-1})} \right] \tag{20}$$

For simplicity, we set $\Psi_1 = 1$ which is also employed in the empirical study. Then (20) can be simplified to $E_t \left[(C_{i,t+j})^{\Psi_1} / \left(\beta^j \varPi_{s=1}^{j}(1 + r_{t+s-1}) \right) \right] = C_{i,t}$. In fact, we can also use this simplified form of (20) as long as Ψ_1 is close to unity which is true in many empirical papers. Rewrite (19) as

$$\frac{C_{i,t}}{Y_{i,t}} = (1-\beta) \left[(1 + r_{t-1}) \frac{b_{i,t}}{Y_{i,t}} + \frac{\overbrace{Y_{i,t}}}{Y_{i,t}} + \frac{1}{Y_{i,t}} E_t \sum_{j=1}^{\infty} \frac{\widetilde{Y}_{i,t+j}}{\varPi_{s=1}^{j}(1 + r_{t+s-1})} \right] \tag{21}$$

where (21) means that current consumption should equal the sum of current bond gross return and present value of permanent income in all the future, denoted by $\overbrace{Y_{i,t}} = \widetilde{Y}_{i,t} + E_t \sum_{j=1}^{\infty} \left[\widetilde{Y}_{i,t+j} / \left(\varPi_{s=1}^{j}(1 + r_{t+s-1}) \right) \right]$ discounted by the rate $(1-\beta)$. The steady state of bonds is $(1 + r_{T-1})b_{i,T} = b_{i,T+1}$. Bonds generally follow an AR process, $b_{i,t+1} = (1 + r_{t-1})b_{i,t} + x_{i,t}$, before the steady state, which is nonstationary. This can be transformed to $(b_{i,t+1}/Y_{i,t+1})(Y_{i,t+1}/Y_{i,t}) = (1 + r_{t-1})(b_{i,t}/Y_{i,t}) + x_{i,t}/Y_{i,t}$ which implies that $b_{i,t}/Y_{i,t}$ before the steady state approximately has a unit root because the random growth rate $Y_{i,t+1}/Y_{i,t}$ is generally close to r_{t-1}. Hence, (21) implies that $C_{i,t}/Y_{i,t}$ can also be approximated to a random walk.

Appendix B: Derive the relationship between the aggregate growth and inequality

Given the linearised aggregate output equation, aggregate growth is

$$g_{t+1} \equiv \varDelta \ln Y_{t+1} = \omega_{Y1} \varDelta \ln Y_{1,t+1} + \omega_{Y2} \varDelta \ln Y_{2,t+1} + \varepsilon_{Y,t+1}$$

Individual output growth is yielded using the linearised equations (30) and (31) in Appendix C.

$$\varDelta \ln Y_{i,t+1} = \alpha \varDelta \ln K_{i,t} + (1-\alpha) \varDelta \ln N_{i,t+1} + \varDelta \ln A_{i,t+1}$$

Using (32-(33) and (36)-(39) yields

$$\begin{aligned} \varDelta \ln Y_{i,t+1} &= \alpha \left(\ln Y_{i,t} - \varphi r_t - \ln K_{i,t-1} \right) - \phi_{2,i} \pi'_{i,t} \\ &+ \frac{(1-\alpha)}{(1+\Psi_2)} \left[\varDelta \ln Y_{i,t+1} - \Psi_1 \varDelta \ln C_{i,t+1} + \frac{2\Psi_2 \phi_{2,i}}{\theta_2} \varDelta \pi'_{i,t+1} \right] + error \end{aligned} \tag{22}$$

Taking E_t on (22) and Substituting out $\Delta E_t \ln C_{i,t+1}, \pi'_{i,t+1}$ and $\pi'_{i,t}$ using (12) and (34) yields

$$\left(\frac{\alpha+\Psi_2}{1+\Psi_2}\right) E_t \Delta \ln Y_{i,t+1} = \alpha(\ln Y_{i,t} - \ln K_{i,t-1}) - \left(\alpha\varphi + \frac{1-\alpha}{1+\Psi_2}\right) r_t$$
$$- \phi_{2,i}\left[\frac{2(1-\alpha)\Psi_2}{(1+\Psi_2)\,\theta_2}+1\right]\left[\rho_1^\pi \pi'_{i,t-1} - \rho_2^\pi Q(\frac{K_{i,t-2}}{K_{t-2}})\right]$$
$$+ \frac{2(1-\alpha)\Psi_2\phi_{2,i}}{(1+\Psi_2)\,\theta_2}\left[\rho_1^\pi \pi'_{i,t} - \rho_2^\pi Q(\frac{K_{i,t-1}}{K_{t-1}})\right] + error$$

To get rid of the past $\pi'_{i,t-s}; s > 1$, set $\rho_1^\pi = 0$. Aggregating the equations above across individuals with assumption that $\phi_{2,i}$ is same dnoted by $\phi_{2,2}$ across groups for simplicity yields

$$\left(\frac{\alpha+\Psi_2}{1+\Psi_2}\right) E_t g_{t+1} = \alpha(\ln Y_t - \ln K_{t-1}) - \left(\alpha\varphi + \frac{1-\alpha}{1+\Psi_2}\right) r_t + \rho_2^\pi \phi_{2,2} \tag{23}$$
$$\left\{\left[\frac{2(1-\alpha)\Psi_2}{(1+\Psi_2)\,\theta_2}+1\right]\Sigma\omega_{Yi}Q(\frac{K_{i,t-2}}{K_{t-2}}) - \frac{2(1-\alpha)\Psi_2}{(1+\Psi_2)\,\theta_2}\Sigma\omega_{Yi}Q(\frac{K_{i,t-1}}{K_{t-1}})\right\}$$
$$+ error$$

We set $Q(K_{i,t-1}/K_{t-1}) = \dfrac{\mu_i}{\omega_{Yi}}(K_{i,t-1}/K_{t-1})^2$ where $q_t = K_{i,t-1}/K_{t-1}$ also measures capital inequality and define the aggregate term $Q_t \equiv \Sigma\omega_{Yi}Q(K_{i,t-1}/K_{t-1})$. Then

$$Q_t = \sum \omega_{Yi}\frac{\mu_i}{\omega_{Yi}}(\frac{K_{i,t-1}}{K_{t-1}})^2 = \mu_1(q_t)^2 + \mu_2\left[\frac{1}{\mu_2}(1-\mu_1 q_t)\right]^2$$
$$\frac{dQ_t}{dq_t} = 2\mu_1\left[q_t\left(1+\frac{\mu_1}{\mu_2}\right) - \frac{1}{\mu_2}\right] = \frac{2\mu_1}{\mu_2}(q_t - 1)$$

Note $dQ_t/dq_t > 0$ if $q_t > 1$ while $dQ_t/dq_t < 0$ if $q_t < 1$. Hence, Q_t has has minimum at perfect equality ($q_t = 1$). Equation (B2) now can be rewritten as

$$\left(\frac{\alpha+\Psi_2}{1+\Psi_2}\right) E_t g_{t+1} = \rho_2^\pi \phi_{2,2}\left\{\left[\frac{2(1-\alpha)\Psi_2}{(1+\Psi_2)\,\theta_2}+1\right] Q_{t-1} - \frac{2(1-\alpha)\Psi_2}{(1+\Psi_2)\,\theta_2}Q_t\right\} + \cdots \tag{24}$$

The aggregate growth above is still complicated due to both current and lagged inequality terms. However, if we consider a mid-term or a long-term growth, g_L, by summing up temporary growth rates within a long period, we yield the following

$$\left(\frac{\alpha+\Psi_2}{1+\Psi_2}\right) g_L \approx \rho_2^\pi \phi_{2,2}\left\{\left[\frac{2(1-\alpha)\Psi_2}{(1+\Psi_2)\,\theta_2}+1\right] \Sigma_{t=0}^T Q_{t-1} - \frac{2(1-\alpha)\Psi_2}{(1+\Psi_2)\,\theta_2}\Sigma_{t=0}^T Q_t\right\} + \cdots \tag{25}$$

Since the other growth determinant real interest rate endogenously depends on lagged output and capital, both short-run and long-run growth rate in (24) and (25) can be approximated as a reduced form of lagged output, capital and inequality which is the usual form of existing empirical regression studies. Note that $Q_t > 0$ and $\Sigma_{t=0}^T Q_{t-1} \approx \Sigma_{t=0}^T Q_t$ for a long term. Since $[2(1-\alpha)\Psi_2]\,/\,[(1+\Psi_2)\,\theta_2] + 1 > [2(1-\alpha)\Psi_2]\,/\,[(1+\Psi_2)\,\theta_2]$, the long-run growth rate is minimised when capital distribution is perfectly equal. Importantly, the stimulating effect in very short term is not as clear as that in long term because when inequality stimulates entrepreneurship incentives, labour input will have a decline which implies a negative but temporary effect on growth.

Appendix C: Model list

Equations (26) and (34) are obtained from individual Euler equation (5) and the consumption aggregation equation. ω_{Y1} and ω_{K1} in (27) and (28) are the steady-state income share and capital share for the rich. To linearise individual capital equation (6), we use (5) to rewrite it as $K_{i,t} = \alpha(1-\tau)E_t Y_{i,t+1}/\left[\delta + r_t\right]$. We use the approximation $E_t\left(Y_{i,t+1}/C_{i,t+1}\right) \approx Y_{i,t}/C_{i,t}$ to linearise it as

$$\ln K_{i,t} \approx \left[\alpha(1-\tau)\left(Y_i/K_i\right)\left(\ln Y_{i,t} + E_t \ln C_{i,t+1} - \ln C_{i,t}\right) - r_t\right]/\left[\delta + r\right]$$

regardless of some constant terms. Finally use (5) to obtain (32) and (33). To linearise labour equation (7), we firstly take the 1^{st} order Taylor expansion with the approximation $1 - N_i - Z_i \approx 0.5$ and $N_i \approx 0.5$ and then substitute $Z_{i,t}$ out using (4) and (11). Individual bonds are removed from equation list because they take small share over individual capital resource which we are not interested in. The list of model equations is

$$r_t = \Psi_1\left(E_t \ln C_{2,t+1} - \ln C_{2,t}\right) - \ln \beta \tag{26}$$

$$\ln Y_t = \ln\left[\mu_1 \exp\left(\ln Y_{1,t}\right) + \mu_2 \exp\left(\ln Y_{2,t}\right)\right] \tag{27}$$

$$\ln K_t = \ln\left[\mu_1 \exp\left(\ln K_{1,t}\right) + \mu_2 \exp\left(\ln K_{2,t}\right)\right] \tag{28}$$

$$\ln C_t = (1-\tau)\frac{Y}{C}\ln Y_t - \frac{K}{C}\left[\ln K_t - (1-\delta)\ln K_{t-1}\right] + \varepsilon_{m,t} \tag{29}$$

$$\ln Y_{1,t} = \alpha \ln K_{1,t-1} + (1-\alpha)\ln N_{1,t} + \ln A_{1,t} \tag{30}$$

$$\ln Y_{2,t} = \alpha \ln K_{2,t-1} + (1-\alpha)\ln N_{2,t} + \ln A_{2,t} \tag{31}$$

$$\ln K_{1,t} = \ln Y_{1,t} - \left[\frac{K}{(1-\tau)\alpha Y} - \frac{1}{\Psi_1}\right] r_t \tag{32}$$

$$\ln K_{2,t} = \ln Y_{2,t} - \left[\frac{K}{(1-\tau)\alpha Y} - \frac{1}{\Psi_1}\right] r_t \tag{33}$$

$$\ln C_{1,t} = E_t \ln C_{1,t+1} - \frac{1}{\Psi_1}\left(r_t + \ln \beta\right) + \varepsilon_{C1,t} \tag{34}$$

$$\ln C_{2,t} = \frac{1}{\omega_{C2}}\left(\ln C_t - \omega_{C1}\ln C_{1,t}\right) + \varepsilon_{C2,t} \tag{35}$$

$$\ln N_{1,t} = \frac{1}{(1+\Psi_2)}\left(\ln Y_{1,t} - \Psi_1 \ln C_{1,t} + 2\frac{\Psi_2 \phi_{2,1}}{\theta_2}\ln \pi'_{1,t}\right) + \varepsilon_{N1,t} \tag{36}$$

$$\ln N_{2,t} = \frac{1}{(1+\Psi_2)}\left(\ln Y_{2,t} - \Psi_1 \ln C_{2,t} + 2\frac{\Psi_2 \phi_{2,1}}{\theta_2}\ln \pi'_{2,t}\right) + \varepsilon_{N2,t} \tag{37}$$

$$\ln A_{1,t+1} = \ln A_{1,t} + \phi_{1,1} - \phi_{2,1}\ln \pi'_{1,t} + \varepsilon_{A,t} \tag{38}$$

$$\ln A_{2,t+1} = \ln A_{2,t} + \phi_{1,1} - \phi_{2,1}\ln \pi'_{2,t} + \varepsilon_{A,t} \tag{39}$$

$$\ln \pi'_{1,t} = \rho_1^\pi \ln \pi'_{1,t-1} - \rho_2^\pi \frac{\mu_1}{\omega_{Y1}}\left(\frac{K_{1,t-2}}{K_{t-2}}\right)^2 + \ln \pi'_t \tag{40}$$

$$\ln \pi'_{2,t} = \rho_1^\pi \ln \pi'_{2,t-1} - \rho_2^\pi \frac{\mu_2}{\omega_{Y2}}\left(\frac{K_{2,t-2}}{K_{t-2}}\right)^2 + \ln \pi'_t \tag{41}$$

Appendix D: The rest IRF figures

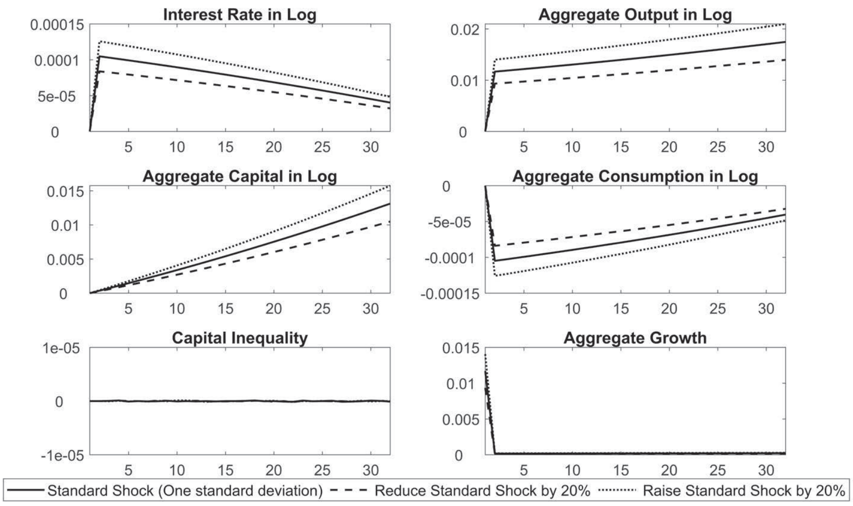

Fig. 13 IRF to the aggregate productivity shock

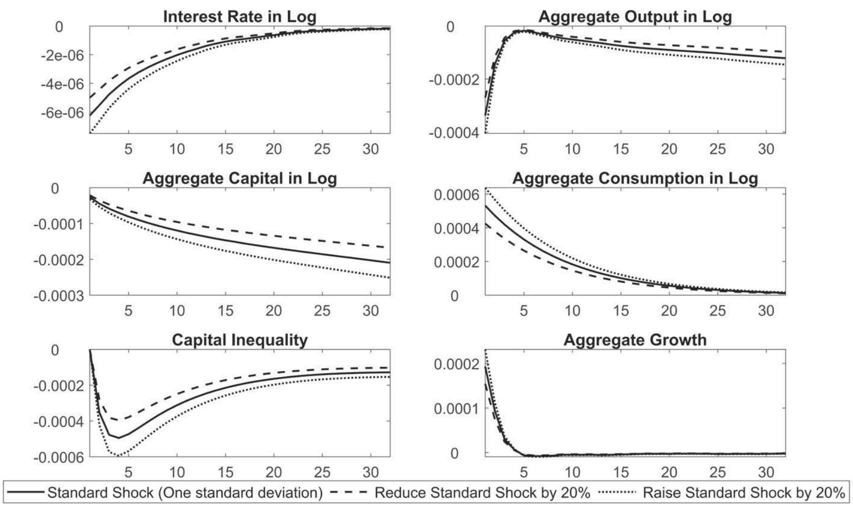

Fig. 14 IRF to individual consumption shock (Euler shock) for the rich

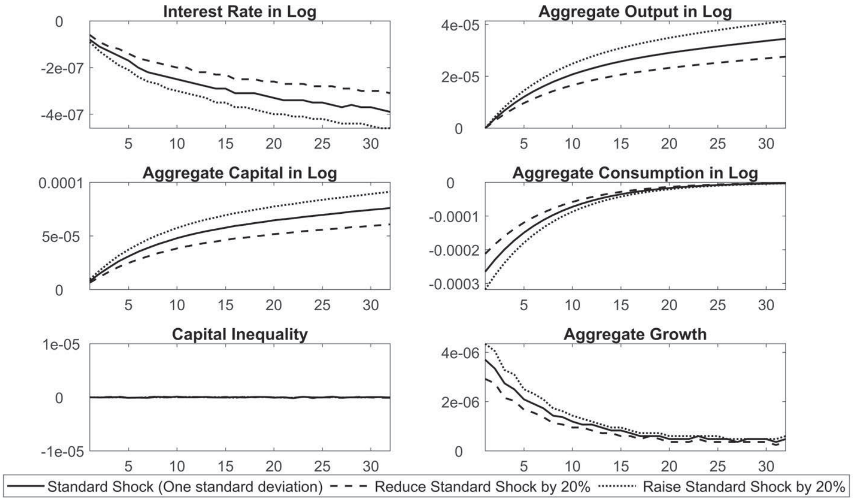

Fig. 15 IRF to the consumption aggregation shock

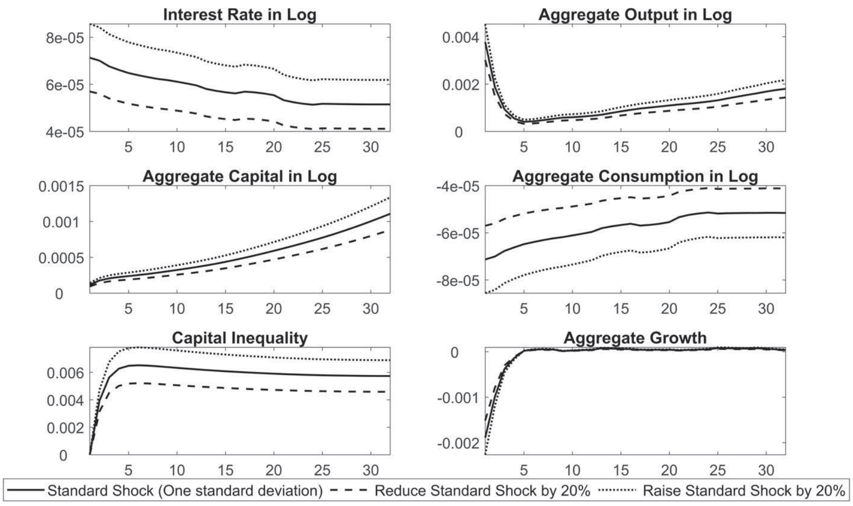

Fig. 16 IRF to the labour input shock for the rich

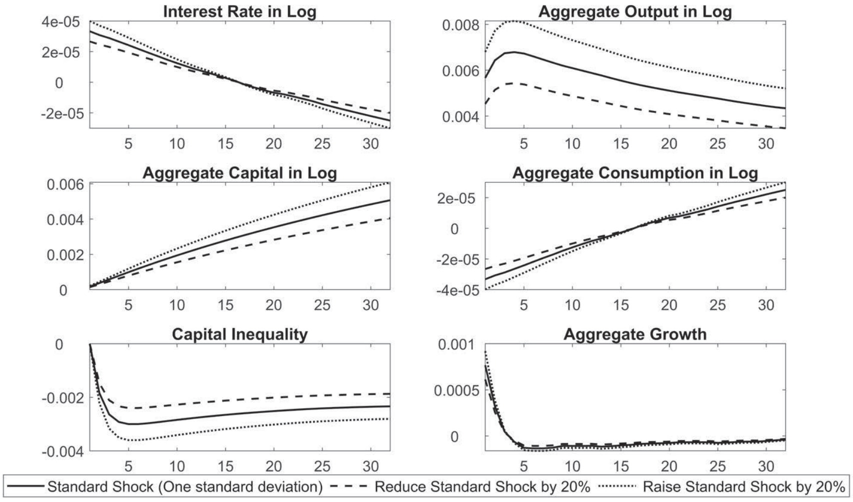

Fig. 17 IRF to the labour input shock for the poor