Delay-time modelling of a critical system subject to random inspections

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Abstract

We model the inspection-maintenance of a critical system in which the execution of inspections is random. The models we develop are interesting because they mimic realities in which production is prioritised over maintenance, so that inspections might be impeded or they might be opportunistic. Random maintenance has been modelled by others but there is little in the literature that relates to inspection of a critical system. We suppose that the critical system can be good, defective or failed, and that failure impacts on production, so that a failure is immediately revealed, but a defect does not. A defect, if revealed at inspection, is a trigger for replacement. We compare the cost and reliability of random inspections with scheduled periodic inspections and discuss the implications for practice. Our results indicate that inspections that are performed opportunistically rather than scheduled periodically may offer an economic advantage provided opportunities are sufficiently frequent and convenient. A hybrid inspection and replacement policy, with inspections subject to impediments, is robust to departure from its inspection schedule.

Keywords**:** Maintenance; reliability; random inspection; production; quality

1. Introduction

The modelling and optimization of maintenance intervention offers significant economic benefit (Alsyouf, 2007) and therefore maintenance is increasingly highlighted in the literature as an integral part of production and business (e.g. Ding et al., 2015). The impacts of maintenance on production and production on maintenance must be considered if modelling is to be useful (Scarf, 1997). We model inspection-maintenance of a critical system, in which inspections are impacted by production, so that inspections are carried out at random times. The state of the system is modelled using the delay-time concept (Christer, 1987), so that the system can be in one of three states, good, defective or failed. The defective state is followed by the failed state, and the system functions in the defective state. Failures are immediately revealed, and so the purpose of inspection is to establish whether or not the system is defective, and if so to carry out some preventive (prior to failure) maintenance.

In this paper, in particular, we suppose that inspections are carried out at random times. This is a departure from the standard assumption in the delay-time modelling (Christer, 1999), although inspection at failures is considered in Wang and Christer (2003). Random maintenance has been studied by others (Nakagawa, 2014), and random inspection-maintenance has been studied, but typically in the case when a protection or cold-stand-by system may be in either the good or failed state, so that therein the purpose of inspection is to establish if the system would operate in the event of a demand for its function (Zhao and Nakagawa, 2015). Our models are novel because we consider random inspections of a critical system, defining a critical system as one in which failure impacts upon production output (however that is defined), so that a failure is immediately revealed.

In practice, inspections may occur at random times for various reasons. For example, inspections may be carried out at opportunities that arise due to stoppages to a larger system of which the critical system of interest is some part (Dekker and Smeitink, 1991; Zheng, 1995; Dagpunar, 1996; Nilsson et al., 2009; Laggoune et al., 2010; Ding and Tian, 2011; Hu and Zhang, 2014; Cavalcante and Lopes, 2015; Peng and Zhu, 2017; Xia et al., 2017; Do et al., 2019). Dynamic grouping (Wildeman et al., 1997; Vu et al., 2015) might have a similar effect. Alternatively, inspections may be scheduled, but their execution may depart from the schedule in a way that is random or has some element of randomness. For example, inspections may be advanced by breaks in production in the manner described in Li et al. (2016), or inspections may be delayed by production (e.g. Tan and Kramer, 1997; Budai et al., 2008; Xia et al., 2015, 2016; Li et at., 2016) or by unavailability of spares (e.g. Zahedi-Hosseini et al., 2017, 2018; Zhang and Zeng, 2017) or by lack of resources (e.g. Berrade et al., 2017; Sleptchenko et al., 2018) or by overrun on maintenance for other systems in a fleet (e.g. Durazo-Cardenas et al., 2018) or by mission constraints (e.g. Yang et al., 2016; Diallo et al., 2018; Liu et al., 2018). Delayed replacement following a positive inspection (defect found) is different but somewhat related (e.g. van Oosterom et al., 2014; Yang et al., 2018).

We call the events that delay inspections impediments. When an inspection cannot be performed at its scheduled time, the maintainer may postpone inspection until the next scheduled inspection time. Such postponements can arise, for example, when a maintenance vessel has a limited time-window for maintenance of an offshore wind-farm (Irawan, 2017), and the durations of individual maintenance actions for individual turbines are themselves random to some extent. Impediments may arise because maintainers act in such a way as to reduce the quality of maintenance (Scarf and Cavalcante, 2012; Alberti et al., 2018). Such quality reduction may be the result of cost-cutting or supplier-switching or simply neglect of older, legacy systems. Impediments are an example of imperfect inspection, although this is different to the type of imperfection studied in Berrade et al. (2012, 2013) and Cavalcante et al. (2019), for example, wherein inspections are subject to false positives and false negatives.

It is important therefore that the modelling of inspection, and the consequent optimization, encapsulates such practical issues. Indeed, recent modelling developments in maintenance (Elodie et al., 2018) indicate a move away from the notion that system-state is the trigger for maintenance towards the notion that the system-state must be monitored and managed until the next opportunity for maintenance presents itself. In this way, maintenance is more responsive to production operations and logistics requirements (Garambaki et al., 2016; Yildrim et al., 2017). Thus, both production (usage) and maintenance (stoppage) have to be jointly managed in a way that maximises system performance.

In this paper we model random inspections in three parts, and study in each part the behaviour of the long-run cost per unit time (cost-rate) and the long-run mean time between failures using a numerical example. In the first part, we take the classical delay-time model for a one-component system and study a policy in which inspections arise purely at random (according to a Poisson process with a fixed rate). In the second part, we model a policy with inspections that are scheduled every Δ time units but which are subject to impediments, wherein any particular inspection is carried out with probability $1-q$ and not carried out with probability q, independently of all other inspections. In the third part, we model a hybrid policy (inspection and replacement) in which inspections are subject to impediments in a similar manner to the second part.

The three cases are considered in Sections 3, 4 and 5 of the paper respectively. In the next section we present modelling assumptions and notation that is common to the three parts. We conclude the paper with a discussion of the implications of our findings for practice.

2. Assumptions and notation

2.1. *Notation*

X	The sojourn in the good state (the time to defect arrival)
H	The delay-time (the time from defect arrival to failure)
$f_X, F_X, \bar{F}_X$	The density, distribution and reliability (survival) functions of X
μ_X	The mean time to defect arrival
η	The characteristic life of X when we specify that X follows a Weibull distribution
β	The shape parameter said Weibull distribution
$f_H, F_H, \bar{F}_H$	The density, distribution and reliability (survival) functions of H
μ_H	The mean delay-time
λ	The characteristic life of H; when $H \sim \mathrm{Ex}(\lambda)$, $\lambda = \mu_{\mathrm{H}}$
β_H	The shape parameter when H follows a Weibull distribution
Z	The time from defect arrival until the subsequent scheduled inspection
$f_Z ,. F_Z .$	The density and distribution functions of Z
Δ	The inspection interval
q	The probability that an inspection is impeded
c_{I}	The cost of an inspection
c_{P}	The cost of replacement when the system is in the defective state
c_{F}	The cost of replacement of the system when it is in the failed state, $c_{\mathrm{F}} > c_{\mathrm{P}}$
C_∞	The long-run total cost per unit time (cost-rate)
μ_{F}	The long-run mean time between failures
p	The mixing parameter when X follows a Weibull mixture distribution
$\eta_1, \beta_1, \eta_2, \beta_2$	The parameters of this Weibull mixture

2.2. *Assumptions*

1) The system comprises a component in a socket that together perform an operational function. This is the standard description of a one-component system (Ascher and Feingold, 1984).
2) The system can be in one of three states: good, defective, failed.
3) The defective state can only be revealed at an inspection.
4) The system operates in the defective state.
5) The system does not operate in the failed state.
6) A failure is immediately revealed.
7) The sojourns in the good state and the defective state are random variables that are independent.
8) The system is renewed on failure and when a defect is found at inspection.
9) Renewal corresponds to replacement of the (failed or defective) component, and replacements are instantaneous.
10) Inspections are perfect, so that there are no misclassification errors or defects induced by inspection.

3. Inspections at opportunities

3.1. *Motivation*

Mathematical models of inspection-maintenance generally assume that inspections are periodic or that they occur at pre-specified times (aperiodic inspections). The reality is that departures from the inspection schedule are highly likely, for many of the reasons that we discuss above, and collectively because production takes priority over maintenance. Even where strict regulations apply (e.g. commercial aviation), inspections may occur early (ahead of schedule). In this section, we consider the case in which inspections arise purely at random, and study the economic (cost) and safety (reliability) implications. In particular, we are interested in comparing a periodic policy with a policy with random inspections that occur at the same frequency. Also we will suppose: a) the inspection cost is the same for each policy; b) the inspection cost for the random policy is much less than for the periodic policy. The latter is justified because departures from a periodic schedule are likely to be cost-motivated.

3.2 *The model*

In addition to the assumptions of Section 2.2, we assume that inspections arise according to a Poisson process with rate $1/\Delta > 0$.

Renewal occurs on failure or when a defect is found at inspection. Let U be cost of a renewal cycle and V the length of a renewal cycle. Then

$$E(V) = \int_0^\infty \int_0^\infty (x+h)\Pr(Z>h) f_H(h) f_X(x) dh dx + \int_0^\infty \int_0^\infty \int_0^h (x+z) f_Z(z) f_H(h) f_X(x) dz dh dx . \quad (1)$$

The first term corresponds to defect arrival at x, delay-time h, and Z (the time to next opportunity following the defect arrival point) exceeding h, so that the result is indeed failure at $x+h$. The second term corresponds to defect arrival at x, delay-time h, and Z not exceeding h, so that the result is preventive replacement at $x+z$.

The expression (1) is general, but $\Pr(Z>h)$ is difficult to specify generally. When opportunities arise according to a Poisson process, the lack of memory property implies that Z is exponentially distributed with mean Δ, so that $f_Z(z) = \frac{1}{\Delta} e^{-z/\Delta}$. Then we get

$$E(V) = \int_0^\infty \int_0^\infty (x+h) e^{-h/\Delta} f_H(h) f_X(x) dh dx + \int_0^\infty \int_0^\infty \int_0^h (x+z) \tfrac{1}{\Delta} e^{-z/\Delta} f_H(h) f_X(x) dz dh dx .$$

Collecting the terms in the integrand that involve x, we can simplify this to obtain

$$E(V) = \mu_X + \int_0^\infty (h e^{-h/\Delta} + \int_0^h \frac{z}{\Delta} e^{-z/\Delta} dz) f_H(h) dh ,$$

where $\mu_X = E(X)$. Now $\int_0^h \frac{z}{\Delta} e^{-z/\Delta} dz = -h e^{-h/\Delta} + \Delta(1-e^{-h/\Delta})$ (integration by parts), so we get

$$E(V) = \mu_X + \int_0^\infty \Delta(1-e^{-h/\Delta}) f_H(h) dh .$$

Now if H is exponential with mean λ, we get

$$E(V) = \mu_X + \int_0^\infty \Delta(1-e^{-h/\Delta}) \frac{1}{\lambda} e^{-h/\lambda} dh = \mu_X + \frac{\Delta\lambda}{(\Delta+\lambda)} .$$

This simple result has the limiting properties that we would expect. When $\Delta = 0$, renewal will occur immediately upon defect arrival (because opportunities arise infinitely often) and so the mean cycle length is just the mean of *X*. When $\lambda = 0$, then again $E(V) = E(X)$ because failure is immediate. When $\Delta = \infty$ (no opportunities), $E(V) = \mu_X + \lambda$. When $\lambda = \infty$ (so renewal will occur at the next opportunity with probability 1), $E(V) = \mu_X + \Delta$.

To obtain the expected cost $E(U)$, we proceed as follows. Conditional on $X = x$, the expected (expectation here is with respect to the Poisson process of opportunities) number of inspections in $[0, x]$ (the time the system is in the good state) is x / Δ. Taking the expectation of this with respect to *X* we get that the unconditional number of inspections during the sojourn in the good state is μ_X / Δ. Then

$$E(U) = c_\mathrm{I} \frac{\mu_X}{\Delta} + (c_\mathrm{I} + c_\mathrm{P}) \times P_\mathrm{I} + c_\mathrm{F} \times P_\mathrm{F},$$

where P_I and P_F are the probability that the renewal cycle ends with inspection and failure respectively. Notice intuitively that these probabilities will not involve *X*, since they can only depend on *H* (the delay-time) and *Z* (the time to next opportunity following a defect arrival), and *Z* does not depend on *X*. Thus,

$$P_\mathrm{F} = P(Z > H) = \int_0^\infty \Pr(Z > h) f_H(h) dh = \int_0^\infty e^{-h/\Delta} \frac{1}{\lambda} e^{-h/\lambda} dh = \frac{1}{\lambda} \int_0^\infty e^{-h(\frac{1}{\lambda}+\frac{1}{\Delta})} dh = \frac{1}{\lambda} . \frac{1}{(\frac{1}{\lambda}+\frac{1}{\Delta})} = \frac{\Delta}{\lambda + \Delta},$$

and

$$P_\mathrm{I} = P(Z \le H) = 1 - P(Z > H) = 1 - \frac{\Delta}{\lambda + \Delta} = \frac{\lambda}{\lambda + \Delta},$$

so that

$$E(U) = c_\mathrm{I} \times \frac{\mu_x}{\Delta} + (c_\mathrm{I} + c_\mathrm{P}) \times \frac{\lambda}{\lambda + \Delta} + c_\mathrm{F} \times \frac{\Delta}{\lambda + \Delta},$$

and so

$$C_{\infty,\mathrm{random}} = \frac{E(U)}{E(V)} = \frac{c_\mathrm{I} \dfrac{\mu_X}{\Delta} + (c_\mathrm{I} + c_\mathrm{P}) \dfrac{\lambda}{\lambda + \Delta} + c_\mathrm{F} \dfrac{\Delta}{\lambda + \Delta}}{\mu_X + \Delta\lambda / (\Delta + \lambda)} = \frac{(\mu\lambda + \mu\Delta + \lambda\Delta) c_\mathrm{I} + \lambda\Delta c_\mathrm{P} + \Delta^2 c_\mathrm{F}}{\mu(\lambda + \Delta)\Delta + \lambda\Delta^2}. \quad (2)$$

For comparison purposes, next we need to find the cost-rate when inspection is periodic with interval Δ. This is obtained as follows. Conditional on a defect arrival at *x* in the *i*-th inspection interval, the renewal cycle length is $\int_0^{i\Delta - x} (x + h) f_H(h) dh + i\Delta(1 - F_H(i\Delta - x))$ and the cost of the renewal cycle is $((i-1)c_\mathrm{I} + c_\mathrm{F}) F_H(i\Delta - x) + (ic_\mathrm{I} + c_\mathrm{P})(1 - F_H(i\Delta - x))$. The first terms in each of these expressions correspond to failure in the interval and the second terms to preventive replacement at the end of the interval. Relaxing the conditioning on *x*, we obtain

$$C_{\infty,\mathrm{periodic}} = \frac{E(U)}{E(V)} = \frac{\sum_{i=1}^\infty \int_{(i-1)\Delta}^{i\Delta} \left\{((i-1)c_\mathrm{I} + c_\mathrm{F}) F_H(i\Delta - x) + (ic_\mathrm{I} + c_\mathrm{P})(1 - F_H(i\Delta - x))\right\} f_X(x) dx}{\sum_{i=1}^\infty \int_{(i-1)\Delta}^{i\Delta} \left\{\int_0^{i\Delta - x} (x + h) f_H(h) dh + i\Delta(1 - F_H(i\Delta - x))\right\} f_X(x) dx}.$$

These expressions can also be developed using general results about the maximum and minimum of two independent, positive-valued, continuous random variables.

Consider the cost-rate for the general model of inspections, for which both of the above are special cases, in the following way. Let inspections occur such that the expected number of inspections in $[0, t]$ is t / Δ. Note that we have defined *Z* as the time from defect arrival to next inspection. Inspection may be random or deterministic. Then

$$E(U) = c_\mathrm{I} \frac{\mu_X}{\Delta} + (c_\mathrm{I} + c_\mathrm{P}) \Pr(H > Z) + c_\mathrm{F} P(H \le Z) = c_\mathrm{I} \frac{\mu_x}{\Delta} + c_\mathrm{F} + (c_\mathrm{I} + c_\mathrm{P} - c_\mathrm{F}) \Pr(H > Z),$$

and

$$E(V) = \mu_X + E\{\min(H, Z)\}.$$

Then

$$C_\infty = \frac{c_{\mathrm{I}}\mu_X / \Delta + c_{\mathrm{F}} + (c_{\mathrm{I}} + c_{\mathrm{P}} - c_{\mathrm{F}})\Pr(H > Z)}{\mu_X + E\{\min(H,Z)\}}, \tag{3}$$

and for two independent, positive-valued, continuous random variables

$$\Pr(H > Z) = \int_0^\infty (1 - F_H(z)) f_Z(z)\mathrm{d}z,$$

and

$$E\{\min(H,Z)\} = \int_0^\infty \int_0^z (1 - F_H(h)) f_Z(z)\mathrm{d}h\mathrm{d}z.$$

When inspection is periodic, $f_Z(z) = \sum_{i=1}^\infty f_X(i\Delta - z)$, defined on $(0,\Delta)$, so that 0 and Δ are the required limits of the integral with respect to z in the above. When inspections arise according to a Poisson process with rate $1/\Delta$, $F_Z(z) = 1 - e^{-z/\Delta}$ (exponential), defined on $(0,\infty)$, so that 0 and ∞ are the required limits of the integral with respect to z in the above, albeit with the upper limit truncated at some convenient point in numerical calculation. When H and Z are exponential, then (2) is obtained from (3).

3.3. *Mean time between failures*

We quantify the reliability of the maintained system using $E(V)/\Pr(\text{cycle ends in failure})$, the "long-run mean time between failures" (Scarf et al. 2005), which we denote μ_{F}. Thus, in general,

$$\mu_{\mathrm{F}} = \frac{\mu_X + E\{\min(H,Z)\}}{1 - \Pr(H > Z)}. \tag{4}$$

For random inspections and exponentially distributed delay-time we have

$$\mu_{\mathrm{F}} = \left(\mu_X + \frac{\Delta\lambda}{(\Delta + \lambda)}\right) \Big/ \frac{\Delta}{\lambda + \Delta} = \mu_X + \frac{\lambda}{\Delta}\mu_X + \lambda.$$

For periodic inspections and generally distributed delay-time we have

$$\mu_{\mathrm{F}} = \frac{\sum_{i=1}^\infty \int_{(i-1)\Delta}^{i\Delta} \left\{ \int_0^{i\Delta - x} (x+h) f_H(h)dh + i\Delta(1 - F_H(i\Delta - x)) \right\} f_X(x)dx}{1 - \left(\sum_{i=1}^\infty \int_{(i-1)\Delta}^{i\Delta} (1 - F_H(i\Delta - x)) f_X(x)dx \right)}.$$

Thus, we can simply report this value, or solve a constrained optimization problem (Driessen et al, 2017) when a reliability requirement might be imposed by a regulator (e.g. Aven, 2017) or in a contract (e.g. Murthy et al., 2015).

3.4. *Numerical study*

As typical of studies of this kind, we suppose that the sojourn in the good state has a Weibull distribution: $\bar{F}_X = \exp(-(x/\eta)^\beta), (x > 0, \eta > 0, \beta \ge 1)$. Throughout, $\eta = 10$ in an arbitrary unit of time. In the base case, $\beta = 4$. We set $c_{\mathrm{P}} = 1$, so c_{P} that is the unit of cost. In the base case, we set $c_{\mathrm{I}} = 0.04$ and $c_{\mathrm{F}} = 5$. We use a crude search to minimise the cost-rate.

In Figure 1 and Table 1, we show results for cases in which the delay-time is exponentially distributed. In Table 2, the delay-time is Weibull-distributed.

Figure 1 in particular shows how the minimum cost-rate varies with the cost of inspection and compares periodic inspection with random inspection at the same frequency. At each value of c_{I}, the minimum-cost periodic inspection interval is determined, $\Delta^*_{\text{periodic}}$. Then for the random inspection, the cost-rate at this inspection interval is determined when a) the cost of inspection is the same as for the periodic inspection policy, and b) when the cost of inspection is zero.

Figure 2 shows the cost-rate against the (mean) inspection interval for each policy.

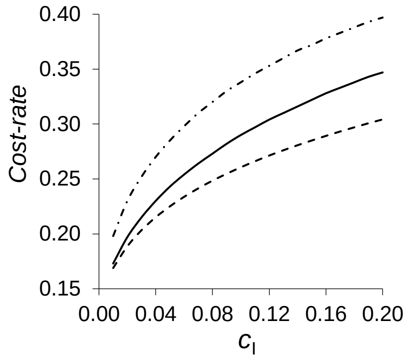

Figure 1. Cost-rate versus cost of inspection: periodic inspection at optimum inspection interval $\Delta^*_{\text{periodic}}$ (——); random inspection with $\Delta_{\text{random}} = \Delta^*_{\text{periodic}}$ and $c_{\text{I,random}} = c_{\text{I,periodic}}$ (– ▪ –) ; and random inspection with $\Delta_{\text{random}} = \Delta^*_{\text{periodic}}$ and $c_{\text{I,random}} = 0$ (----). Weibull time to defect arrival ($\eta = 10$, $\beta = 4$); exponential delay-time ($\lambda = 2$); $c_{\text{P}} = 1$, $c_{\text{F}} = 5$.

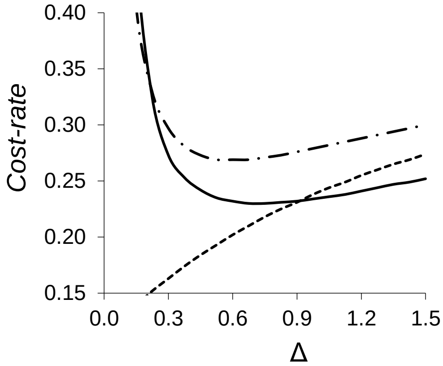

Figure 2. Cost-rate versus Δ for: periodic inspection (——) with $c_{\text{I}} = 0.04$; random inspection with (– ▪ –) with $c_{\text{I}} = 0.04$; and random inspection (----) with $c_{\text{I}} = 0$. Weibull time to defect arrival ($\eta = 10$, $\beta = 4$); exponential delay-time ($\lambda = 2$); $c_{\text{P}} = 1$, $c_{\text{F}} = 5$.

Table 1. Cases with Weibull time to defect arrival ($\eta = 10$, β) and exponential delay-time distribution (mean λ). $c_P = 1$.

β	λ	$c_{I,periodic}$	$c_{I,random}$	c_F	$\Delta^*_{periodic}$	$C^*_{\infty,periodic}$	μ_F	$100 \times C_{\infty,random} / C^*_{\infty,periodic}$ at $\frac{\Delta^*_{periodic}}{2}$	at $\Delta^*_{periodic}$	at $2\Delta^*_{periodic}$	μ_F at $\frac{\Delta^*_{periodic}}{2}$	at $\Delta^*_{periodic}$	at $2\Delta^*_{periodic}$
4	2	0.04	0.04	5	0.725	0.229	58.3	124	118	130	61.1	36.1	23.6
4	2	0.04	0	5	0.725	0.229		75	94	118			
4	1	0.04	0.04	5	0.527	0.279	41.7	125	118	128	44.5	27.3	18.7
4	1	0.04	0	5	0.527	0.279		71	91	114			
4	4	0.04	0.04	5	1.039	0.193	80.0	119	116	127	82.9	48.0	30.5
4	4	0.04	0	5	1.039	0.193		79	96	118			
4	2	0.02	0.02	5	0.487	0.197	82.6	120	116	131	85.5	48.3	29.7
4	2	0.02	0	5	0.487	0.197		79	96	121			
4	2	0.08	0.08	5	1.111	0.273	40.9	125	117	126	43.7	27.4	19.2
4	2	0.08	0	5	1.111	0.273		72	91	112			
4	2	0.04	0.04	2.5	1.444	0.175	33.5	115	110	113	36.2	23.6	17.3
		0.04	0	2.5	1.444	0.175		83	94	105			
4	2	0.04	0.04	10	0.448	0.298	89.1	129	124	146	92.0	51.5	31.3
		0.04	0	10	0.448	0.298		69	94	131			
2	2	0.04	0.04	5	0.717	0.234	57.5	123	118	129	60.3	35.6	23.2
2	2	0.04	0	5	0.717	0.234		75	94	118			

Table 2. Cases with Weibull time to defect arrival ($\eta = 10$, $\beta = 4$) and Weibull delay-time distribution (λ, β_H, $\mu_H = \lambda\Gamma(1+1/\beta_H)$). $c_F = 5$, $c_P = 1$.

β_H	λ	μ_H	$c_{I,periodic}$	$c_{I,random}$	$\Delta^*_{periodic}$	$C^*_{\infty,periodic}$	μ_F	$100 \times C_{\infty,random} / C^*_{\infty,periodic}$ at $\frac{\Delta^*_{periodic}}{2}$	at $\Delta^*_{periodic}$	at $2\Delta^*_{periodic}$	μ_F at $\frac{\Delta^*_{periodic}}{2}$	at $\Delta^*_{periodic}$	at $2\Delta^*_{periodic}$
1	2	2	0.04	0.04	0.725	0.229	58.2	124	118	130	61.1	36.1	23.6
1	2	2	0.04	0	0.725	0.229		75	94	118			
2	2.257	2	0.04	0.04	0.980	0.170	160.4	128	133	163	126.1	48.0	25.0
2	2.257	2	0.04	0	0.980	0.170		80	109	151			
4	2.207	2	0.04	0.04	1.309	0.143	405.7	134	157	205	141.9	42.3	21.9
4	2.207	2	0.04	0	1.309	0.143		92	135	194			
1	1	1	0.04	0.04	0.527	0.279	41.7	125	118	128	44.5	27.3	18.7
1	1	1	0.04	0	0.527	0.279		71	91	114			
2	1.128	1	0.04	0.04	0.610	0.210	104.6	135	134	158	87.4	36.0	20.2
2	1.128	1	0.04	0	0.610	0.210		73	103	143			
4	1.103	1	0.04	0.04	0.733	0.176	255.1	143	155	193	107.1	34.9	19.0
4	1.103	1	0.04	0	0.733	0.176		82	124	178			
1	4	4	0.04	0.04	1.039	0.193	80.0	119	116	127	82.9	48.0	30.5
1	4	4	0.04	0	1.039	0.193		79	96	118			
2	4.514	4	0.04	0.04	1.659	0.142	228.3	121	128	160	172.5	63.1	31.4
2	4.514	4	0.04	0	1.659	0.142		87	111	152			
4	4.413	4	0.04	0.04	2.462	0.120	545.4	127	155	206	173.5	50.2	25.8
4	4.413	4	0.04	0	2.462	0.127		100	141	199			

3.5. *Discussion of the results*

We can see that, when the delay-time is unpredictable (exponential), random inspection, at opportunities, has a lower cost-rate than periodic inspection when the inspection frequencies are the same and the cost of inspection for opportunistic inspection is zero (Figure 1). This advantage of the opportunistic policy is not evident when delay-times are more predictable (Weibull). This is as we would expect. Random inspections are suited to random delay-times. However, the random policy has more frequent failures. Figure 2 further indicates that the random inspection policy with zero inspection-cost has a cost advantage provided the inspection frequency (interval) is sufficiently large (small), noting that in this policy Δ is not a decision variable and so is not within the control of the decision-maker. On the other hand, if opportunities occur very frequently (say twice as often as in the best periodic policy), and the inspection cost is zero at these opportunities, then it appears the random policy beats the periodic policy both in cost terms (cost-rate) and reliability terms (mean time between failures. Thus, the nature of the delay time, the frequency of opportunities and cost of inspection at these opportunities, which is unlikely to be zero in practice, will determine whether opportunistic inspection is preferred. A thorough investigation of the effects of these factors on the globally optimal policy would be interesting but is omitted.

3.6. *Implications for practice*

Our analysis suggests that an opportunistic inspection policy will work well provided opportunities arise at a rate that is more frequent than inspections in the minimum cost-rate periodic policy. Careful monitoring of the frequency of opportunities will be necessary. One might then expect a practical policy to be: inspect at random opportunities and carry out a scheduled inspection when and only when the time since the last inspection reaches some critical value, $\Delta_{\max}$. We do not study this modified random inspection policy, although it may be interesting to do so in future. Achieving the necessary inspection frequency, on the other hand, might be managed by classifying stoppages due to different causes as triggers for inspection and ensuring that there are sufficient of them. Furthermore, for a multi-component system, reciprocal arrangements (inspect A while B is failed and inspect B while A is failed) may be sensible.

4. Inspections subject to impediments

4.1. *Motivation*

Now we consider inspections with a different source of randomness. We suppose that inspections are scheduled to occur periodically, but that a scheduled inspection may not be executed due to some impediment. Maintenance policies with fixed intervals implicitly assume that the preventive action will be executed at the planned time. However, in reality, circumstances may arise that are not within the control of the maintainer and that necessitate postponement of maintenance. Thus, bad weather at sea may disrupt maintenance schedules for off-shore wind-farms (Li et al., 2016; Scheu et. al., 2018) and underwater systems (Uyiomendo and Markeset, 2010). High winds (Halvorsen-Weare et al., 2017) or lack of resources (Stock-Williams and Swamy, 2018) may impede maintenance of onshore systems.

Our purpose is then to study the effect of such impediments in an abstract setting, and to consider what might be an appropriate management response when it is known that there is likely to be variation from the prescribed inspection schedule.

4.2. *The model*

The system we study is as described earlier (see the assumptions in Section 2.2). Inspections are scheduled to occur periodically every Δ time units. Additionally, we assume that a scheduled inspection is impeded (is not executed) with probability *q*, a constant, independently of other inspections and the state of the system. Thus the sequence of values of the indicator variable for impediments is a Bernoulli process. We also assume that when an inspection is impeded no cost of inspection is incurred.

We can obtain a result similar to (3) for the cost-rate. The expected number of inspections executed in $[0,t]$ is $(1-q)t/\Delta$. Thus, the first term in the numerator of (3), which corresponds to the cost inspections that occur during the phase of the renewal cycle in which the system is in the good state, becomes $c_{\mathrm{I}}(1-q)\mu_X/\Delta$. Then we only need to determine $\Pr(H>Z)$ and $E\{\min(H,Z)\}$, since the other cost incurred remains exactly the same: $c_{\mathrm{I}}+c_{\mathrm{P}}$ if $H>Z$ and c_{F} if $H\le Z$, where Z is the time from defect arrival to the next scheduled inspection that is not impeded and recalling that H is the delay-time.

Now, $(k-1)\Delta<Z\le k\Delta$ if a defect arises at time *x* and the subsequent $k-1$ inspections are impeded and the *k*-th is not impeded. Using this logic it follows that the density function of *Z* is

$$f_Z(z)=\sum_{i=k}^{\infty}(1-q)q^{k-1}f_X(i\Delta-z),\quad (k-1)\Delta<z\le k\Delta,\quad k=1,2,\dots .$$

Thus, if $z\in(0,\Delta)$, then the defect can arise in any inspection interval (this is the summation over all *i*) and the subsequent scheduled inspection must be unimpeded. Further if $z\in(\Delta,2\Delta)$, then the defect can arise in any inspection interval and the period that ends with an unimpeded inspection must span exactly one impeded inspection. And so on.

Then,

$$\Pr(H>Z)=\int_0^{\infty}(1-F_H(z))f_Z(z)\mathrm{d}z=\sum_{k=1}^{\infty}(1-q)q^{k-1}\sum_{i=k}^{\infty}\int_{(k-1)\Delta}^{k\Delta}(1-F_H(z))f_X(i\Delta-z)\mathrm{d}z\,, \tag{5}$$

and

$$E\{\min(H,Z)\}=\int_0^{\infty}\int_0^{z}(1-F_H(h))f_Z(z)\mathrm{d}h\mathrm{d}z=\sum_{k=1}^{\infty}(1-q)q^{k-1}\sum_{i=k}^{\infty}\int_{(k-1)\Delta}^{k\Delta}\left\{\int_0^{z}(1-F_H(h))\mathrm{d}h\right\}f_X(i\Delta-z)\mathrm{d}z\,. \tag{6}$$

In practical calculations, these summations must be truncated at a point that provides sufficient accuracy in the cost-rate.

These expressions (5) and (6) are then used to evaluate the cost-rate, which is explicitly

$$C_\infty=\frac{c_{\mathrm{I}}\mu_X(1-q)/\Delta+c_{\mathrm{F}}+(c_{\mathrm{I}}+c_{\mathrm{P}}-c_{\mathrm{F}})\Pr(H>Z)}{\mu_X+E\{\min(H,Z)\}}.$$

The long-run mean time between failures, which quantifies the maintained-system reliability, is given by (4).

4.3 *Impeded opportunities*

If inspections at opportunities, which we model in Section 3 as a Poisson process with rate $1/\Delta$, are impeded by the same mechanism (with constant probability *q* and independently), then it follows that the inspections that are not impeded arise according to a thinned Poisson process with rate $(1-q)/\Delta$. Therefore, we can replace for Δ by $\Delta/(1-q)$ in (2) to obtain the cost-rate for this case, provided $q<1$.

If $q = 1$, or equivalently $1/\Delta = 0$, then there are no inspections and the policy is failure-based maintenance. We do not study this case in detail because varying q is equivalent to varying Δ.

4.4. *Numerical study*

Chosen distributions and parameter values are as in Section 3.4: . $\bar{F}_X = \exp(-(x/\eta)^{\beta})$. with $\eta = 10$ and $\beta = 4$, and $c_{\mathrm{P}} = 1$. In Table 3 we show the results for some particular cases. In the table, we also show the cost-rate for a sub-optimal policy. This sub-optimal policy has an inspection interval Δ that is set equal to the inspection interval for the optimum policy when $q = 0$, $\Delta^*_{q=0}$. In this way, we show how the cost-rate and mean time between failures would be affected when impediments occur but they are ignored in modelling by the decision-maker.

For $q = 0$, Δ^* and C^*_{∞} are determined to accuracies of 0.0001 and 0.00001 respectively when the first summation in each of (5) and (6) is truncated at an upper limit of $k = 25$ and the second summation is truncated at $i = k + 30$. For $q = 0.4$, larger upper limits are required to achieve the same accuracy ($k = 25$, $i = k + 30$). Optima are found using a crude search.

Figure 3 shows how the optimum is influenced by the impediment probability q. Figure 4 demonstrates the sensitivity of the cost-rate to the inspection interval for different specifications of the delay-time distribution.

Table 3. Cost-rate and "reliability" for the optimum policy and for a sub-optimal policy with $\Delta = \Delta^*_{q=0}$. Weibull distributed time to defect arrival ($\eta = 10$, $\beta = 4$) and delay-times. $c_{\mathrm{P}} = 1$.

Case	β_H	λ	μ_H	c_{I}	q	c_{F}	Δ^*	C^*_{∞}	μ^*_{F}	$C_{\infty}(\Delta^*_{q=0})$	$\mu_{\mathrm{F}}(\Delta^*_{q=0})$
1	1	2	2	0.04	0	5	0.725	0.232	58.2	--	--
2	1	2	2	0.04	0.2	5	0.555	0.240	53.3	0.243	43.1
3	1	2	2	0.04	0.4	5	0.401	0.247	49.3	0.263	32.0
4	2	2.2568	2	0.04	0	5	0.981	0.172	160.1	--	--
5	2	2.2568	2	0.04	0.2	5	0.686	0.184	128.5	0.191	73.3
6	2	2.2568	2	0.04	0.4	5	0.464	0.193	111.9	0.224	40.6
7	4	2.2066	2	0.04	0	5	1.310	0.145	404.6	--	--
8	4	2.2066	2	0.04	0.2	5	0.793	0.162	236.3	0.181	73.9
9	4	2.2066	2	0.04	0.4	5	0.513	0.172	193.3	0.229	35.6
10	1	2	2	0.02	0	5	0.487	0.198	82.6	--	--
11	1	2	2	0.02	0.2	5	0.368	0.204	75.4	0.208	59.4
12	1	2	2	0.02	0.4	5	0.281	0.211	65.6	0.225	42.4
13	1	2	2	0.08	0	5	1.113	0.278	40.9	--	--
14	1	2	2	0.08	0.2	5	0.871	0.286	37.5	0.289	31.5
15	1	2	2	0.08	0.4	5	0.639	0.296	34.8	0.309	24.5
16	1	1	2	0.04	0	5	0.527	0.281	41.7	--	--
17	1	1	2	0.04	0.2	5	0.411	0.291	38.2	0.295	31.7
18	1	1	2	0.04	0.4	5	0.301	0.301	35.3	0.317	24.3
19	1	4	2	0.04	0	5	1.041	0.195	79.8	--	--
20	1	4	2	0.04	0.2	5	0.788	0.201	73.1	0.204	58.2
21	1	4	2	0.04	0.4	5	0.563	0.207	67.7	0.218	42.4
22	1	2	2	0.04	0	2.5	1.447	0.178	33.4	--	--
23	1	2	2	0.04	0.2	2.5	1.154	0.181	30.8	0.182	26.5
24	1	2	2	0.04	0.4	2.5	0.861	0.184	28.6	0.188	21.3
25	1	2	2	0.04	0	10	0.448	0.300	89.1	--	--
26	1	2	2	0.04	0.2	10	0.337	0.314	81.5	0.321	63.7
27	1	2	2	0.04	0.4	10	0.274	0.330	68.7	0.359	45.2

4.5. *Discussion of results*

Broadly, we can see (Figure 3) that as the chance of impediments increases the frequency of inspections increases. The cost also increases but less so when the delay-time is more random (figure 3a). So again, random inspections appear most suited to random delay-times. However, when the chance of impediments is higher, departure from the optimum frequency of inspection is more costly—there is greater curvature in the cost-rate functions for smaller Δ^* (greater *q*) in Figure 3a. These effects are more marked at when the cost of failure is larger (cases 22-27 in Table 3) and somewhat unaffected by the cost of inspection (cases 10-15).

In the base case, the random inspection policy costs 18% more than the periodic policy at the same frequency and cost of inspection (case 1 in Table 1), whereas impediments with $q = 0.4$ increase the cost rate by only 6% (cases 1 and 3 in Table 3). When delay-times are less random, the corresponding cost increases are 57% and 19% respectively. Thus some random variation from a periodic policy seems preferable to a purely random policy, and this preference is more acute when delay-times are more predictable. In fact, one might be prepared to put up with an impediments probability that is much larger than $q = 0.4$ before preferring a random inspection policy.

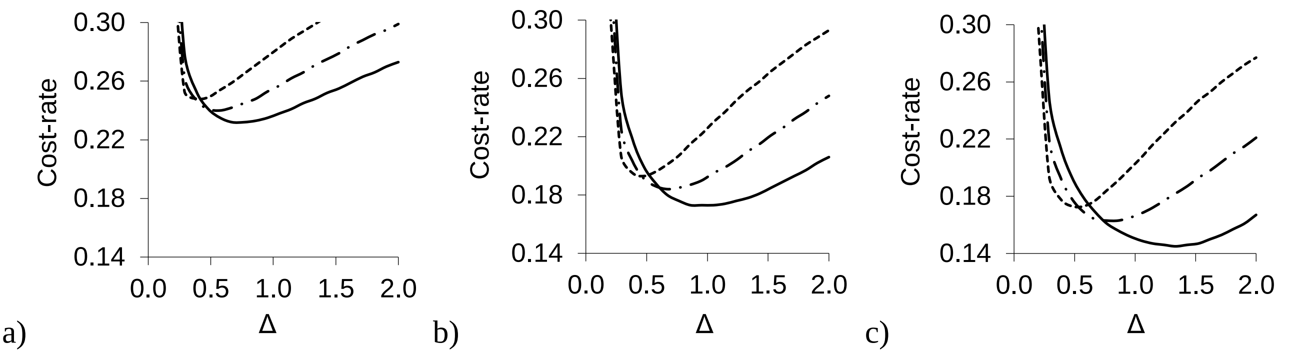

Figure 3: Cost-rate versus Δ with $q = 0$ (——); $q = 0.2$ (–·–); $q = 0.4$ (----) and Weibull distributed delay-time with $\mu_H = 2$ and a) $\beta_H = 1$, b) $\beta_H = 2$, c) $\beta_H = 4$. Weibull ($\eta = 10$, $\beta = 4$) time to defect arrival throughout. $c_P = 1$, $c_I = 0.04$, $c_F = 5$.

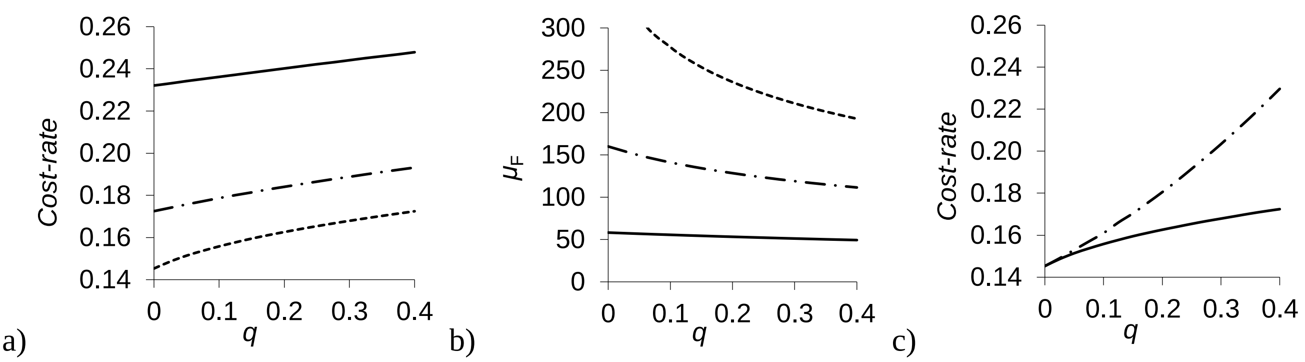

Figure 4. (a) Minimum cost-rate versus impediment probability for periodic inspection policy with impediments; (b) mean time between failures of minimum cost-rate policy versus impediment probability: Weibull delay-time with $\mu_H = 2$ and $\beta_H = 1$ (——) ; $\beta_H = 2$ (–·–) ; $\beta_H = 4$ (----); (c) Minimum cost-rate for the optimal (——) and cost-rate for sub-optimal (– –) policy for Weibull delay-time ($\mu_H = 2$, $\beta_H = 4$). Weibull ($\eta = 10$, $\beta = 4$) defect time to arrival. $c_P = 1$, $c_I = 0.04$, $c_F = 5$.

Ignorance of impediments is quite costly (Figure 4c). Thus, if impediments are frequent, then the analysis suggests that inspections should be scheduled more frequently.

While the mean time between failures provides additional information for the decision-maker (De Almeida et al., 2015), its behaviour over the range of cases studied is as expected. For the management implications below, we note the significant reduction in the "reliability" of the sub-optimal policy (last column of Table 3), particularly for predictable delay-times.

4.6. *Management implications*

We distil the following key points from the discussion of the results:

1) If impediments that postpone or cancel inspections are likely, then the frequency of planned, scheduled inspections should be adjusted upwards.
2) Moving to a purely random policy that utilises opportunities for inspections is more costly than enduring frequent postponements, particularly if failures have somewhat predictable warning times. Note here, we are interpreting the delay-time as period of warning of an impending failure, although this is not strictly true because in the model without inspection there is no warning.
3) Where impediments do arise then ignoring them in the planning of maintenance requirements is likely to prove quite costly and risky (decreased reliability).

5. Hybrid inspection and replacement with impediments

5.1 *Motivation*

In this final section we study impediments when the reliability of components is heterogeneous so that a hybrid inspection and replacement policy is natural. So we suppose that times to defect arrival arise from a mixture distribution that models a population of components with two sub-populations: a weak population with short lives and a strong population with long lives (e.g. Attardi et al., 2005; Castet and Saleh, 2010). The hybrid policy in which inspection is scheduled during the early life of the system and replacement is scheduled in later life was proposed by Scarf et al. (2009) and further extended to include replacements at opportunities by Cavalcante et al. (2018). Our question of interest is: what should be the appropriate maintenance planning response when inspections are impeded?

5.2 *The model*

Model assumptions are as described in Section 2.2. The maintenance policy is as follows. Inspections are scheduled to occur times $\Delta, 2\Delta, ..., K\Delta$ from renewal and then replacement of the system is scheduled at time *T* from renewal. The policy has three decision variables: K, Δ, T. We suppose that an inspection is impeded with probability *q*, independently of other inspections and the state of the system. In this way, the number of inspections that are executed in the inspection phase of a renewal cycle has a $\text{bin}(K, q)$ distribution. We assume also that there is no possibility of impediment at *T*, although in practice one might imagine some delay to replacement is possible.

Calculation of the cost-rate proceeds in a different way to Sections 3 and 4. We first condition on the defect arising in the *i*-th inspection interval. We shall call the interval between the last inspection at $K\Delta$ and T the $K+1$th interval for convenience. Then, we consider the cases in which the system fails. It could fail in interval $j = i, ..., K+1$. If so, all intervening inspections must be impeded, noting that when $j = i$ there is no intervening inspection. So conditional on a defect arising at time *x* in the *i*-th inspection interval and failure occurring subsequently at any time $x + h < T$, the expected cycle length is:

$$E_{\mathrm{F}}(V\,|(i-1)\Delta < x < i\Delta) = \int_0^{i\Delta-x}(x+h)\mathrm{d}F_H + \sum_{j=i+1}^{K} q^{j-i}\int_{(j-1)\Delta-x}^{j\Delta-x}(x+h)\mathrm{d}F_H + q^{K-i+1}\int_{K\Delta-x}^{T-x}(x+h)\mathrm{d}F_H \;.$$

Note, $\mathrm{d}F_H$ is just shorthand for $f_H(h)\mathrm{d}h$ here. The first integral corresponds to failure in the same interval as the defect arrival, and there is no q term because there are no intervening inspections. The integral inside the summation corresponds to failure in the *j*-th interval, so that the intervening inspections are all impeded (with probability q^{j-i}). The final integral corresponds to failure in the $K+1$ th interval. It follows that this expression is valid only for $i=1,...,K-1$ ($K \ge 2$). When $i=K$, we have

$$E_{\mathrm{F}}(V\,|(K-1)\Delta < x < K\Delta) = \int_0^{K\Delta-x}(x+h)\mathrm{d}F_H + q\int_{K\Delta-x}^{T-x}(x+h)\mathrm{d}F_H \;,$$

and when $i=K+1$

$$E_{\mathrm{F}}(V\,|K\Delta < x < T) = \int_0^{T-x}(x+h)\mathrm{d}F_H \;.$$

A similar argument can be used when conditioning on a defect arising at time *x* in the *i*-th inspection interval and preventive replacement occurring subsequently. In a similar notation we have for $i=1,...,K$, $(K \ge 1)$,

$$E_{\mathrm{D}}(V\,|(i-1)\Delta < x < i\Delta) = (1-q)\sum_{j=i}^{K} q^{j-i}\, j\Delta\bar{F}_H(j\Delta - x) + q^{K-i+1}T\bar{F}_H(T-x)\,.$$

The first term corresponds to preventive replacement at a subsequent inspection and the second term corresponds to preventive replacement at *T*. Notice in the latter case, the delay-time spans all intervening inspections of which there are $K-i+1$ in number. The other case, when, $i=K+1$, is

$$E_{\mathrm{D}}(V\,|K\Delta < x < T) = T\bar{F}_H(T-x)\,.$$

Now writing $E_{\mathrm{F}|i}(V)$ as a shorthand for $E_{\mathrm{F}}(V\,|(i-1)\Delta < x < i\Delta)$ for $i=1,...,K$ and $E_{\mathrm{F}|K+1}(V)$ for $E_{\mathrm{F}}(V\,|K\Delta < x < T)$, and $E_{\mathrm{D}|i}(V)$ as a shorthand for $E_{\mathrm{D}}(V\,|(i-1)\Delta < x < i\Delta)$ for $i=1,...,K$ and $E_{\mathrm{D}|K+1}(V)$ for $E_{\mathrm{D}}(V\,|K\Delta < x < T)$, relaxing the conditioning we have

$$E(V) = \sum_{i=1}^{K}\int_{(i-1)\Delta}^{i\Delta}\left\{E_{\mathrm{F}|i}(V) + E_{\mathrm{D}|i}(V)\right\}\mathrm{d}F_X + \int_{K\Delta}^{T}\left\{E_{\mathrm{F}|K+1}(V) + E_{\mathrm{D}|K+1}(V)\right\}\mathrm{d}F_X + T\bar{F}_X(T)$$

using $\mathrm{d}F_X$ as shorthand for $f_X(x)\mathrm{d}x$. The final term here accounts for the case when there is no defect arrival before *T*.

The expect cost per cycle can be derived using the same logic. Thus

$$E(U) = \sum_{i=1}^{K}\int_{(i-1)\Delta}^{i\Delta}\left\{E_{\mathrm{F}|i}(U) + E_{\mathrm{D}|i}(U)\right\}\mathrm{d}F_X + \int_{K\Delta}^{T}\left\{E_{\mathrm{F}|K+1}(U) + E_{\mathrm{D}|K+1}(U)\right\}\mathrm{d}F_X + \left\{(1-q)Kc_{\mathrm{I}} + c_{\mathrm{P}}\right\}\bar{F}_X(T)\,.$$

where the final term here accounts for the case when there is no defect arrival before *T*, when one expects to pay for a proportion $(1-q)$ of the *K* inspections at a cost of c_{I} each, and the failure-related terms in this expression are defined by

$$\begin{aligned} E_{\mathrm{F}|i}(U) &= E_{\mathrm{F}}(U\,|(i-1)\Delta < x < i\Delta) \\ &= \left\{(i-1)(1-q)c_{\mathrm{I}} + c_{\mathrm{F}}\right\}\left\{\int_0^{i\Delta-x}\mathrm{d}F_H + \sum_{j=i+1}^{K} q^{j-i}\int_{(j-1)\Delta-x}^{j\Delta-x}\mathrm{d}F_H + q^{K-i+1}\int_{K\Delta-x}^{T-x}\mathrm{d}F_H\right\} \end{aligned}$$

for $i=1,...,K-1$, $(K \ge 2)$, and

$$E_{\mathrm{F}|K}(U) = E_{\mathrm{F}}(U\,|(K-1)\Delta < x < K\Delta)$$
$$= \{(K-1)(1-q)c_{\mathrm{I}} + c_{\mathrm{F}}\}\left\{\int_0^{K\Delta-x} \mathrm{d}F_H + q\int_{K\Delta-x}^{T-x} \mathrm{d}F_H\right\}$$
$$= \{(K-1)(1-q)c_{\mathrm{I}} + c_{\mathrm{F}}\}\{F_H(K\Delta-x) + q\{F_H(T-x) - F_H(K\Delta-x)\}\}$$

and

$$E_{\mathrm{F}|K+1}(U) = E_{\mathrm{F}}(U\,|K\Delta < x < T) = \{(K-1)(1-q)c_{\mathrm{I}} + c_{\mathrm{F}}\}F_H(T-x)\,.$$

Notice here that the cost is the same in every case: the cost of the inspections prior to the defect arrival plus the cost of the failure. Any inspections scheduled between the defect arrival and failure are impeded and so do not incur a cost.

The preventive replacement-related terms in the expected cost per cycle expression are defined by

$$E_{\mathrm{D}|i}(U) = E_{\mathrm{D}}(U\,|(i-1)\Delta < x < i\Delta)$$
$$= (1-q)\sum_{j=i}^{K} q^{j-i}\{(i-1)(1-q)c_{\mathrm{I}} + c_{\mathrm{I}} + c_{\mathrm{P}}\}\bar{F}_H(j\Delta-x) + q^{K-i+1}\{(i-1)(1-q)c_{\mathrm{I}} + c_{\mathrm{P}}\}\bar{F}_H(T-x)$$

for $i = 1,\ldots,K$, $(K \geq 1)$ and

$$E_{\mathrm{D}|K+1}(U) = E_{\mathrm{D}}(U\,|K\Delta < x < T) = \{K(1-q)c_{\mathrm{I}} + c_{\mathrm{P}}\}\bar{F}_H(T-x)\,.$$

Note, different, simpler expressions must be used when $K = 0$ and when $K = 1$, although when $K = 1$ only the failure related expressions need to be adapted.

Then

$$C_\infty(K,\Delta,T) = E(U)/E(V)\,,$$

and

$$\mu_{\mathrm{F}} = \frac{E(V)}{\sum_{i=1}^{K}\int_{(i-1)\Delta}^{i\Delta}\left(\int_0^{i\Delta-x}\mathrm{d}F_H + \sum_{j=i+1}^{K} q^{j-i}\int_{(j-1)\Delta-x}^{j\Delta-x}\mathrm{d}F_H + q^{K-i+1}\int_{K\Delta-x}^{T-x}\mathrm{d}F_H\right)\mathrm{d}F_X + \int_{K\Delta}^{T} F_H(T-x)\mathrm{d}F_X}\,.$$

A further, related model might be developed by supposing instead that during the inspection phase inspections arise opportunistically according to a Poisson process (as in the model in Section 3). We do not pursue this here, but it may make an interesting study.

5.3. *Numerical study*

We introduce, in Table 4, a Weibull mixture for the time to defect arrival distribution so that $\bar{F}_X = p\exp(-(x/\eta_1)^{\beta_1}) + (1-p)\exp(-(x/\eta)^{\beta})$ for $x > 0$, $\eta,\eta_1 > 0$, and $\beta,\beta_1 \geq 1$. Thus *p* is the mixing parameter. We use different (to the numerical examples in Sections 3 and 4) parameter values for the strong-subpopulation in this Weibull mixture. This is because the component distributions have to be well-separated (large shape parameters) and the delay-time has to be short for the hybrid policy to be interesting $(K^*\Delta^* << T^*)$ (Scarf et al., 2009). The delay-time is exponentially distributed. The cost of replacement acts as the unit of cost as before ($c_{\mathrm{P}} = 1$). Other cost-parameter values are unchanged. The cost-rate is minimised using a crude search over successively finer lattices until accuracy in the minimum cost-rate of 0.0001 is achieved.

Table 4. Minimum cost-rate policy and sub-optimal policy ($\Delta = \Delta^*_{q=0}$).Weibull mixture ($\eta_1 = 2$, $\beta_1 = 3$, $\eta = 10$, $\beta = 5$) for time to defect arrival and exponential (mean λ) delay-time. $c_I = 0.04$.

Case	p	λ	q	c_I	c_F	K^*	Δ^*	T^*	C^*_∞	μ^*_F	$C_\infty(\Delta^*_{q=0})$	$\mu_F(\Delta^*_{q=0})$
1	0.1	0.2	0	0.04	5	2	1.111	6.399	0.293	36.02	--	--
2	0.1	0.2	0.2	0.04	5	2	1.111	6.395	0.294	35.34	0.295	35.31
3	0.1	0.2	0.4	0.04	5	3	0.803	6.398	0.295	35.39	0.296	34.63
4	0.2	0.2	0	0.04	5	5	0.523	6.756	0.367	25.12	--	--
5	0.2	0.2	0.2	0.04	5	6	0.447	6.761	0.369	24.59	0.370	23.82
6	0.2	0.2	0.4	0.04	5	7	0.386	6.768	0.371	23.69	0.372	22.62
7	0.1	0.4	0	0.04	5	2	1.200	6.488	0.277	41.09	--	--
8	0.1	0.4	0.2	0.04	5	2	1.199	6.492	0.278	39.49	0.279	39.54
9	0.1	0.4	0.4	0.04	5	3	0.869	6.497	0.280	39.47	0.281	38.08
10	0.2	0.4	0	0.04	5	6	0.488	6.772	0.331	33.67	--	--
11	0.2	0.4	0.2	0.04	5	7	0.422	6.789	0.335	31.78	0.336	30.56
12	0.2	0.4	0.4	0.04	5	9	0.334	6.805	0.339	30.38	0.342	27.69

5.4 *Discussion of the results*

It is apparent from Table 4 that if the chance of an impediment is relatively large, then the maintainer can simply schedule more inspections than would otherwise be the case. The cost-rate increases only slightly (of the order of 1%) when $q = 0.4$. The optimum replacement interval T^* is broadly the same across the cases. Note that the discrete nature of the policy in respect of K, the number of scheduled inspections in the inspection phase, means we would not expect T^* to vary smoothly across the cases.

In this policy, as opposed to the pure inspection policies considered in Sections 3 and 4, it seems that the replacement acts as an insurance against low-quality maintenance (high chance of impediment) during the inspection phase, so that the effect of impediments on the cost-rate is only small. Thus, replacement acts as a back-stop. Even when impediments are ignored, so that a sub-optimal policy might be used (final columns of Table 4), the increase in cost-rate is only 3% in the worst case; compare the cost rate in case 10 with the sub-optimal cost-rate in case 12. The "reliability" is only marginally reduced (in comparison to the effects in the pure inspection policy).

Thus, we might conclude from this, taking a view across the three models in this section and Sections 3 and 4, that if impediments are likely then a good policy is a hybrid of inspection and replacement regardless of whether there is heterogeneity in the reliability of components.

In practice, if inspections can be impeded then replacements may be likewise. Then, a block-replacement policy—in which replacements are scheduled every T time units regardless of events (failures, inspections) in between—might be a sensible policy. Models of block replacement with impediments and age-based replacement with defaulting or postponement may be worthy of study.

6. Conclusions

In this paper, we study inspection-maintenance policies in which the execution of inspections is subject to some randomness. The models mimic realities in which production is prioritised over maintenance, so that inspections might be impeded or they might be executed at opportunities. Three models are developed. In the first, inspections are executed only at opportunities, which arise according to a Poisson process. In particular, we consider effect of the frequency of opportunities and the cost-advantage of inspection at opportunities over scheduled inspection on the long-run cost of maintenance and the mean time between failures. In the second model, which is a pure inspection policy with scheduled inspections that are subject to cancellation at random (impediment), we are again interested to study the appropriate policy response to a large impediment probability. The third model offers one possible response: schedule a binding replacement after an initial inspection phase. Throughout, we suppose a three-state system in which the defective state is revealed only on inspection and the failed state is immediately revealed. The applicability of these models is summarised in Table 5.

Table 5. Summary of model applicability.

	Model description	Practical circumstances for use
1	Purely random inspection	Tactical decision because opportunities arise often, or *laissez faire* attitude to inspection-maintenance
2	Periodic inspections with impediments	Inspections scheduled but production prioritized, or spare-parts stock-outs possible, or manpower resources unavailable at random, or mission over-run
3	Periodic inspection phase with impediments plus scheduled replacement	Inspections scheduled intensively in early life following e.g. installation or overhaul (“burn-in” or “break-in” period), and subject to impediments as in Model 2

Random inspection-maintenance has been little studied in the literature. The reality is that the timing of the execution inspections is quite likely to depart from a deterministic, planned schedule because broader objectives of the system owner or circumstances outside the control of the maintainer may intervene, for example, priority for production, spare-parts’ stock-outs, overrun on other maintenance activities, and personnel constraints. Thus, its study is timely and useful.

Our results indicate that inspections that are performed opportunistically rather than scheduled periodically may offer an economic advantage (lower cost-rate) provided opportunities are sufficiently frequent and that an inspection at an opportunity has a lower cost than a scheduled inspection. This cost-discount is justified when production takes priority over maintenance, presumably because satisfying demand for production is a more immediate goal than determining the state of the production system, which may be good anyway. When impediments induce randomness in the inspection process, analysis of our model suggests that this randomness is most problematic when maintenance requirements analysis ignores the possibility of postponements and cancellations of maintenance actions.

On implications for practice, we reflect on these in the discussion of each model (Sections 3.6, 4.5, and 5.4). We summarise them again here, briefly, while noting that our primary purpose is to study how the violations of an inspection-maintenance plan influence performance. Thus, our analysis suggests that there exist circumstances in which strict adherence to fixed inspection schedules is not cost-effective. However, care must be taken to ensure that opportunistic inspection does not lead to zero inspection. Where postponement or cancellation of scheduled inspections is likely, inspections should be scheduled more frequently than they would be when assuming otherwise, and recording of impeded inspections would be good practice. Also, if inspection times are random then scheduled replacement may provide a good back-stop. Finally, a hybrid policy (inspection and scheduled replacement) appears to be effective when there are different sources of uncertainty, intrinsic ones and external ones.

We have confined our analysis of the models to numerical studies. Therefore, we cannot make general claims about the optimality or preference for one particular policy. However, the non-simplicity of cost-rate functions for the models makes obtaining general results extremely difficult. We recognise this as a limitation of this work.

Further development of this study might include the development of a model of a modified random inspection policy, whereby the trigger for inspection is an opportunity or crossing of a threshold, Δ_{max}, for the time since the last inspection. Multi-component extensions will be interesting when there are interactions between components. For example, when failure of A is an opportunity for inspection of B and vice-versa, a reliability increase of one may imply a reliability decrease of the other when the components are considered independently. Finally, research on impeded replacements in standard policies such as age-based and block replacement would be timely.

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