# Inspection and replacement policy with a fixed periodic schedule

C.A.V. Cavalcante*
*Department of Production Engineering, Federal University of Pernambuco, Recife, Brazil.*
R.S. Lopes
*Department of Production Engineering, Federal University of Pernambuco, Recife, Brazil.*
P.A. Scarf
*Cardiff Business School, Cardiff University, UK (scarfp@cardiff.ac.uk) and RANDOM - Research Group on Risk and Decision Analysis in Operations and Maintenance, UFPE, Recife, Brazil.*
**corresponding author.*

ABSTRACT: We study an inspection and replacement policy for a system when maintenance actions are executed at times or "visits" with a fixed, periodic schedule. Then, the novel focus of the paper is not when to schedule maintenance but what maintenance to schedule at known times. Violations (defaults) may occur because time or resources is limited. The model is motivated by maintenance policy typically used for high-value, engineered systems e.g. windfarms, transportation systems, manufacturing lines. This motivation underlies the importance of the work. Inspections are modelled using the delay time concept. System lifetime is heterogeneous, so that early defects, due to say poor quality parts or installation, may occur. There is a lag between failure and corrective replacement and so we model the cost of downtime. We determine the cost-rate, system reliability and average availability of a policy with an initial inspection (preventive maintenance) phase followed by a wear-out (corrective maintenance) phase. A numerical example illustrates and investigates characteristics of the policy. Results indicate that the scheduled time for preventive replacement and the effective visit-frequency have the largest effect on policy performance. The policy is scalable to multi-component systems in a way that facilitates the study of grouping policies.

Keywords: *maintenance, replacement, inspection, delay time, opportunistic maintenance*

## 1. INTRODUCTION

There are significant costs associated with logistics for the maintenance of dispersed systems (Shafiee, 2015; Hedjazi et. al. 2019; Nguyen et al., 2019). Thus, immediate attention to failures of such systems (corrective maintenance) and even the actions defined to avoid failures (preventive maintenance) are constrained by the transportation of personnel, equipment and spare parts (Arvesen et al., 2013; Rodrigues et al., 2020; Kafiabad et. al., 2020). Also, even when logistics is not an issue, operations may have priority over maintenance (Scarf et al., 2019; Broek et al., 2020; Sinisterra and Cavalcante, 2020). These factors mean that there are significant challenges for research in maintenance modelling and optimization, and recent studies of strategies that rationalize maintenance logistics in distribution systems (Duarte et al., 2020), in renewable energy systems (Shafiee and Sørensen, 2019), for transportation systems (Zhang et al., 2020) have been published. Furthermore, simultaneous planning of maintenance and the logistics for maintenance is receiving increasing attention (De Jonge and Scarf, 2020; Topan et al., 2020). Strategies include combining maintenance actions (Shahraki et al., 2020), opportunistic actions (Canh Vu et al., 2020; Shen et al, 2021), multi-echelon resourcing (Wang, et. al. 2019), periodic operational stoppages (Hu et al., 2020; Ruiz et al., 2021). Notwithstanding such planning, defaulting may occur. Such violations of a maintenance plan can include postponement (Yang et al. 2019; Berrade et al., 2017, Cavalcante et al., 2019) or even cancellation of maintenance actions (Alotaibi et al., 2020). Our work here contributes to this research agenda and, in particular, is most closely related to the study of opportunistic policies.

There has been much research on the modelling of opportunistic maintenance policies, both in theory (e.g. Dekker & Smeitink, 1991; Budai et al., 2006; Laggoune et al., 2010; Xia et al., 2017b,c; Zhang and Zeng, 2017) and for practice (e.g Ding and Tian, 2011; Hu and Zhang, 2014; Cavalcante and Lopes, 2015; Shafiee et al., 2015; Garambaki et. al., 2016; Xia et al., 2017a; Yildirim et al., 2017; Xia et al., 2021). Typically, opportunities arise at random from economic and structural interdependencies among components or parts (Dekker & Smeitink, 1991). Grouped maintenance policies also exploit such dependencies to maintain collections of parts (Do et al., 2013; Vu et al. 2015; Peng & Zhu, 2017), but opportunistic maintenance is different because it aims to maintain a part or parts when another part of the system causes a stop.

However, in our paper here, we take a different approach and suppose that maintenance actions can be executed only at pre-specified times. We suppose these times are periodic and denote them by $is$, $i = 1, 2, \ldots$. Such periodic "opportunities" may arise when operations cease, for example, at seasonal shutdowns (Zhang et al., 2020) or when maintenance resources are available, for example, at scheduled visits of a maintenance vessel to an offshore windfarm. This pre-specification makes policy both simpler to study (fewer decision variables) and easier to implement in practice, and akin to block replacement rather than age replacement, but nonetheless applicable to maintenance of windfarms (e.g. Shafiee, 2015), transportation systems (e.g. Corman et al., 2017), and manufacturing systems (e.g. Zahedi-Hosseini et al., 2018). This applicability to many types of high-value, technical systems underlies the importance of the work in our paper. At the "visits", we suppose that a maintainer of the system chooses either to inspect the system to determine its state or replace the system, or to do both these actions. Thus, the novel contribution of this paper is that it models not *when* to schedule the execution of maintenance but *what* maintenance to schedule at known times. We also suppose that there may occur violations (defaults) because time and resources at periodic visits may be limited or conditions may vary.

The choice of inspection and/or replacement at fixed times is formalized by supposing that, from new, inspections of the system are scheduled for times $is$, $i = 1, \ldots, K$, preventive replacement is scheduled at time $Ms$, and corrective replacement (on failure) occurs at a visit subsequent to failure. The notion of inspection is based on the delay-time model in which a failure is immediately revealed and preceded by a defective state that extends for some random time, the delay time, during which the system operates. The defective state is only revealed by inspection. The model we develop in this paper is related to the models of hybrid inspection and replacement (Scarf et al., 2009; Scarf and Cavalcante, 2010; Arts and Basten, 2018) and develops the connection between opportunistic maintenance and inspection that few papers consider (Wang and Christer, 2003; Cavalcante and Lopes, 2015; Berrade et al. 2017; Scarf et al., 2019). Thus, the first phase $[0, Ks]$ (of the life of the system) is the inspection phase, and the second phase $(Ks, Ms]$ is a wear-out (or failure correction) phase. The phases arise in this order because we suppose that replacements may be heterogeneous, so that on replacement new systems may be weak or strong. The two-phase maintenance policy modulates different levels of care needed during the life of a critical component. The management of a two-phase policy is more onerous than a one-phase policy, but despite this there is increasing evidence of the benefit of the additional complication (e.g. Yeh et al., 2010; Cavalcante et al., 2011; Alberti et al., 2019; Yang et al., 2019), particularly when component lives are heterogeneous. Such heterogeneity may arise as a result of variations in quality (Scarf and Cavalcante, 2012) or variations in exogenous factors (e.g. weather) at the times of replacements. The latter may be particularly relevant given the fixed times of maintenance interventions. Early failures have been observed for wind turbine components (Hahn et al., 2007; Spinato et al., 2009; Faulstich et al., 2011).

Exogenous factors may also lead to the postponement of maintenance (Van Oosterom et al., 2014; De Jonge et al., 2015; Zhang et al., 2019; Wang et al., 2020). Many mechanisms can be envisaged here: weather conditions may place additional strain on maintenance resource or may be too bad for any maintenance; a supply vessel may run-out of spare parts; maintenance actions on other assets may overrun. We model these mechanisms by supposing the possibility of defaulting on replacement. This possibility to default makes it particularly important to consider the cost of unavailability (downtime). In the model, a downtime cost arises when the system fails and remains so until replacement, either at the next scheduled maintenance intervention (visit) or beyond it if there is a default. An earlier, simpler version of the model without defaulting or downtime is developed in Scarf et al. (2018).

The paper focuses on a single-component system (non-repairable), although the model provides a structure for aggregating the isolated maintenance plans of one-component systems with a common time-base, $s$. Thus, importantly, the common time-base provides a framework for combining maintenance inventions for a multi-component system or a fleet of multi-component systems, so that chosen groups of components or systems would be maintained at chosen multiples of $s$. This could simplify the grouping problem significantly.

The precise specification of the model (assumptions, system, and policy) is described in the next section. To optimize the policy, we consider a long-run cost per unit time criterion. This “cost-rate”, and also the average availability and the approximate system reliability, quantified by the mean time between operational failures, are developed in Section 3. Section 4 presents a numerical example to illustrate the policy. And we conclude with a discussion in the final section.

## 2. SPECIFICATION OF THE MODEL

### 2.1. *Notation*

| | |
|---|---|
| $X$ | Time to defect arrival |
| $f$, $F$, $\bar{F}$ | Density, cumulative distribution and reliability (survival) functions of $X$ |
| $r$ | Mixture parameter |
| $H$ | Delay time |
| $g$, $G$, $\bar{G}$ | Density, cumulative distribution, reliability (survival) functions of $H$ |
| $s$ | Time between visits |
| $p$ | Probability of default |

| | |
|---|---|
| $c_F, c_P, c_I$ | Cost of failure replacement, cost of preventive replacement and cost of inspection |
| $c_D$ | Downtime cost per unit time |
| $K$ | Number of inspections in a renewal cycle (a decision variable) |
| $M$ | Number of visits between scheduled preventive replacements (a decision variable) |
| $U(K,M)$ | Expected cost of a renewal cycle |
| $V(K,M)$ | Expected length of a renewal cycle |
| $Q(K,M)$ | Long-run total cost per unit time (cost-rate) |
| $\mu(K,M)$ | Mean time between operational failures |
| $A(K,M)$ | Average availability |

2.2. *The system*

We consider a component that when in its socket performs an operational function and we assume that replacement of a component, whether preventive or corrective, is the renewal of the system. This is the notional non-repairable, one-component system of Ascher and Feingold (1984). This setup defines a proper renewal process where the replacement times are renewal epochs. In this way, we can calculate the expected cost and expected length of a renewal cycle (Section 3), and use their ratio, the long-run cost per unit time (cost-rate), as a suitable criterion to optimize the decision variables.

At any particular time, the system can be in one of three states: good, defective or failed, and the system operates in both the good and the defective states and the distinction between the good and defective states is revealed only by inspection. This is the classic delay time model, see Werbińska-Wojciechowska (2019) for a recent review. The time in the good state G (the defect arrival time), *X*, is a random variable with a mixture distribution, with mixing parameter *r*, so that effectively components lives arise from one of two populations, a weak population and a strong population. The time in the defective state D (the delay time), *H*, is a random variable and is independent of *X*.

2.3. *The policy*

We suppose that maintenance can only be carried out fixed, periodic times $is$ $(i = 1, 2, ...)$. No actions can occur at instants other than these times, which we term visits. Thus, the interval between visits is *s*. We consider an inspection and replacement policy with two phases. In the first phase, inspections are scheduled to occur at each of the first $K$ visits, that is, at times $is$ $(i = 1, ..., K)$. In the second phase, there are no inspections at visits and the system is scheduled for replacement at the *M*-th visit, $M > K$, that is, when the system is aged $Ms$. Thus, the policy has two decision variables: $K$ and $M$.

On inspection, if the system is defective, the component is replaced if there is no default, or at the subsequent visit if there is a default. This replacement of a defective component is termed a preventive replacement. Inspections are perfect: there is no misclassification of component-state and no defect induction.

A failed component is replaced at the first visit following failure if there is no default, or at the subsequent visit if there is a default. This replacement of a failed component is termed a corrective replacement. The probability of a default is *p*, regardless of whether the defaulted replacement is preventive or corrective.

We assume that two consecutive defaults cannot occur. Therefore, there is at most one default in a renewal cycle. This is because notionally there is no defaulting of inspection; defaults occur only in respect of preventive or corrective replacements. Thus, a default occurs only if either a component that is failed at $is$ is replaced at $(i+1)s$ or a component that is found to be defective at $is$ is replaced at $(i+1)s$. We also assume that no default is possible at the preventive replacement at time $Ms$.

The costs that contribute to the cost-rate are the cost of inspection, $c_I$, the cost of a preventive replacement, $c_P$, and the cost of a corrective replacement, $c_F$, with $c_I < c_P < c_F$. These costs are per event. Downtime is costed per unit time at rate $c_D$.

2.4. *Justifications*

While the fixed time between visits is convenient for model development, there are important practical circumstances that motivate such periodic times for maintenance. In the case of an offshore windfarm, we envisage the visit of a maintenance vessel every *s* time units because the windfarm is large and resources are limited. For energy, transportation and manufacturing systems we envisage situations in which shutdowns are periodic and between shutdowns operation has primary priority.

Defaults may arise because resources are limited and the consumption of resources, either time or spare parts, is variable and unknown. Thus, the very nature of preventive replacement at a positive inspection is random because the system state varies at random. Therefore, the demand for resources is not known in advance with certainty, and, for example, when a maintenance vessel has $\tau$ hours to visit *n* turbines to do an unknown number of maintenance interventions, time and spare parts may run out. Also, the external environment may affect the consumption of resources in a way that is random. For example, in bad weather, installation of new components may take longer than in good weather.

Resource limitations may also influence the performance of new, replacement components. Thus, component heterogeneity may be the result not only of variable quality of parts (Scarf and Cavalcante, 2012) but also of maintenance carried out in difficult circumstances. Therefore, a short component-life may be the result of a poorly executed installation. With the possibility of short component-lives, early inspection may be justified, so that great care is taken of the system during early life (Scarf et al., 2009; Scarf and Cavalcante, 2010). Component heterogeneity is a special case of heterogeneity of maintenance interventions, a topic that is attracting increasing interest (e.g. Zhao et al., 2019; Zhang et al., 2020).

Variability in the environment (exogenous factors) can also explain defaulting. For example, the weather may be too bad for execution of maintenance, and the maintainer may postpone maintenance to the next visit. However, at this subsequent visit, we might expect a postponed replacement to be prioritized. This in some way justifies our assumption that at most one default occurs. Also, the assumption that there can be no default at $Ms$ is justified because we would expect the resources required for this maintenance intervention to take priority over other actions.

The notion of whether failure is immediately revealed or not is not relevant because in the model there always exists some time lag between failure and subsequent replacement at a scheduled visit. However, we do suppose that, at a visit, if the system is failed this failure is known to the maintainer. So, in this sense, while failures are not likely to be safety-critical, they are not soft failures (e.g. Taghipour and Banjevic, 2012) because failure prevents the operational function of the system.

Nonetheless, in some sense, the model we study unifies maintenance models for critical systems with those of protection systems (Vaurio, 1999). Thus, the costing of downtime may be justified by, say, loss of production revenue (in the case of a critical system) or the potential for loss due to unavailability in the event of a demand (in the case of a protection system). The cost-rate of downtime will vary considerably depending on the context, from a low value in the case of a wind turbine, whereby the cost of lost power generation is small relative to the cost of damage to the turbine as a result of failure, to a moderate value in the case of a manufacturing system. Very high values of downtime-cost would preclude the suitability of a maintenance policy with fixed times for maintenance.

Even though downtime is costed, the cost of failure is larger than the cost of preventive replacement. The additional cost of failure may arise because: a) a failure may cause damage to the wider system of which the component of interest is some important part; b) refurbishment of a good or a defective component would cost significantly less than that for a failed component.

Notice that replacements are always synchronized with visits, and so renewals occur only at visits. This provides an important simplification, so that the policy unifies aged-based replacement and block replacement. We might study a more general policy in which inspections are scheduled every *N*-th visit, that is, at times $Ns, 2Ns, ..., KNs$ (from new). This policy may be appropriate when visits occur very frequently (e.g. at shutdowns or under reduced timetables of transportation systems that occur on a weekly basis). We might study a policy with three phases in which inspections are scheduled to occur when the system is aged $is$, for $i = 1, ..., K$ and $i = L, ..., M$ ($K < L < M$), that is in early life and in late life but not when the system is middle-aged. However, in this paper, we focus on the simpler two-variable policy.

## 3. CALCULATION OF THE POLICY CRITERIA

The notion of reliability is the foundation of the model, but we focus on criteria that are themselves derivative: the average availability and the cost-rate. This is because we suppose that: a corrective maintenance action is not immediate so that the system is unavailable between failure and subsequent repair; in reality there are likely to be many replicates of the system so that only average availability (over a fleet of assets) is important; and the system is not safety-critical. While the concepts of risk (Selvik and Aven, 2011; Aven, 2020) and resilience (Cai et al., 2018) may be interesting to apply to the system (or fleet of systems), we suppose that the economic cost over the life-cycle is the primary focus of the notional decision-maker.

### 3.1. *Description of the renewal cases*

To present the calculations of the policy criteria concisely, we use the functions $\phi_{l,c}$, $l = 1, ..., 4$, $c = 1, ..., 15$ (Table 1). Therein: $\phi_{1,c}$ is the function appropriate for calculating the probability of occurrence for case *c*; $\phi_{2,c}$ is the function appropriate for calculating the expected length of a renewal cycle for case *c*; $\phi_{3,c}$ is the function appropriate for calculating the downtime in a renewal cycle for case *c*; and $\phi_{4,c}$ is the function appropriate for calculating the expected cost of a renewal cycle for case *c*. The cases themselves are described one by one in the following and illustrated in Figures 1 and 2.

Table 1. The functions $\phi_{l,c}$ in the terms contributing to the decision criteria.

| | probability | cycle length | downtime | cost per cycle |
|---|---|---|---|---|
| Case, $c$ | $\phi_{1,c}$ | $\phi_{2,c}$ | $\phi_{3,c}$ | $\phi_{4,c}$ |
| 1 | 1 | $is$ | $is-(x+h)$ | $(i-1)c_I+c_F$ |
| 2 | 1 | $is$ | $is-(x+h)$ | $(i-1)c_I+c_F$ |
| 3 | 1 | $is$ | $is-(x+h)$ | $(i-1)c_I+c_F$ |
| 4 | 1 | $is$ | $is-(x+h)$ | $Kc_I+c_F$ |
| 5 | 1 | $is$ | $is-(x+h)$ | $Kc_I+c_F$ |
| 6 | 1 | $is$ | $is-(x+h)$ | $Kc_I+c_F$ |
| 7 | 1 | $Ms$ | $Ms-(x+h)$ | $Kc_I+c_F$ |
| 8 | 1 | $is$ | $is-(x+h)$ | $Kc_I+c_F$ |
| 9 | 1 | $is$ | $is-(x+h)$ | $Kc_I+c_F$ |
| 10 | 1 | $Ms$ | $Ms-(x+h)$ | $Kc_I+c_F$ |
| 11 | 1 | $is$ | 0 | $ic_I+c_P$ |
| 12 | 1 | $is$ | 0 | $ic_I+c_P$ |
| 13 | 1 | $Ms$ | 0 | $Kc_I+c_P$ |
| 14 | 1 | $Ms$ | 0 | $Kc_I+c_P$ |
| 15 | 1 | $Ms$ | 0 | $Kc_I+c_P$ |

The cost-rate is determined by calculating the probabilities of the three types of renewal scenario that arise: related to corrective replacement following failure; related to preventive replacement of a defective component at inspection; and related to preventive replacement at $Ms$.

A failure can arise as a result of a defect that itself arises either in early-life during the inspection phase or in later-life during the corrective replacement phase. The costs are different in each case, so we develop the calculations of each case in an exhaustive way. The cases related to failure and corrective replacement are illustrated in Figure 1.

In all the following expressions for brevity we write $\mathrm{d}F$ for $f(x)\mathrm{d}x$ and $\mathrm{d}G$ for $g(h)\mathrm{d}h$. While we will suppose, in the numerical example, that $f$ has a particular form (mixture), the calculations below are presented in a general way.

We begin with case 1 (Figure 1). Here, for $i=1,\ldots,K,\ K=1,\ldots,M-1$, renewal occurs at $is$ when a defect and failure occur in the interval $[(i-1)s, is]$ and there is no default, so that (corrective) replacement occurs at $is$. The corresponding term, which contributes to outcome probability or the expected cost of a renewal cycle, $E(U)$, or the downtime, or the expected length of a renewal cycle, $E(V)$, depending on the specification of $\phi_{l,1}$ in Table 1, is

$$T_{l,1}=(1-p)\sum_{i=1}^{K}\int_{(i-1)s}^{is}\int_{0}^{is-x}\phi_{l,1}\mathrm{d}G\mathrm{d}F\,.$$

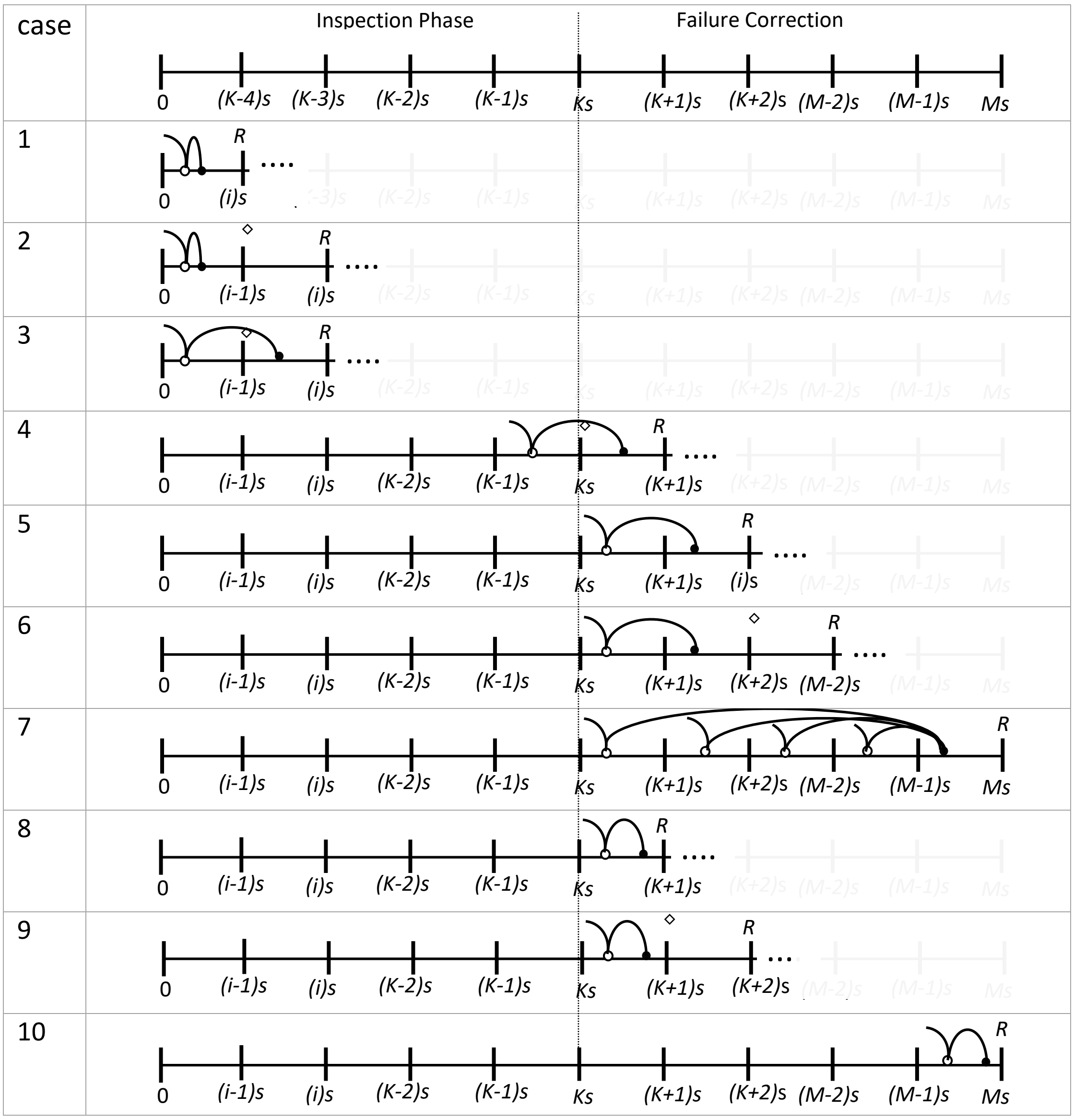


Figure 1. Cases with renewal at corrective replacement (on failure). Symbol key: inspection (┼); defect arrival ( ○ ); failure( •) ; default ( ◇ ); replacement (R).

In case 2, for $i=2,...,K,\ \ K=1,...,M-1$, renewal occurs at $is$ when a defect and failure occur in $[(i-2)s,(i-1)s]$ and there is a default, so that (corrective) replacement is at $is$. The corresponding term is

$$T_{l,2}=p\sum_{i=2}^{K+1}\int_{(i-2)s}^{(i-1)s}\int_{0}^{(i-1)s-x}\phi_{l,2}\mathrm{d}G\mathrm{d}F\ .$$

In case 3, for $i=2,...,K,\ \ K=1,...,M-1$, renewal occurs at $is$ when a defect occurs in $[(i-2)s,(i-1)s]$, there is a default (on the preventive replacement) at $(i-1)s$, and failure occurs in $[(i-1)s,is]$ followed by corrective replacement at $is$. Note, default of this corrective replacement cannot occur because two consecutive defaults are not permitted. The corresponding term is

$$T_{l,3}=p\sum_{i=2}^{K}\int_{(i-2)s}^{(i-1)s}\int_{(i-1)s-x}^{is-x}\phi_{l,3}\mathrm{d}G\mathrm{d}F\ \ .$$

In case 4, a defect arises in $[(K-1)s, Ks]$ , $K=1,...,M-1$, and a default of the preventive replacement occurs. This is similar to case 3 but considered separately because when there is a default at $Ks$ there are no subsequent inspections, so failure can occur in any subsequent interval $[(i-1)s, is]$, $i=K+1,...,M$ . The corresponding term is

$$T_{l,4} = p\int_{(K-1)s}^{Ks}\sum_{i=K+1}^{M}\int_{(i-1)s-x}^{is-x}\phi_{l,4}\mathrm{d}G\mathrm{d}F ,$$

Case 5 is renewal in the wear-out phase at $is$, $i=K+2,...,M-1$, $K=1,...,M-3$, following failure in $[(i-1)s, is]$ and defect arrival in a previous interval $[(j-1)s, js]$, $j=K+1,...,i-1$, noting that this need not be an adjacent interval because there is no inspection in the wear-out phase, and no default. The corresponding term is

$$T_{l,5} = (1-p)\sum_{i=K+2}^{M-1}\sum_{j=K+1}^{i-1}\int_{(j-1)s}^{js}\int_{(i-1)s-x}^{is-x}\phi_{l,5}\mathrm{d}G\mathrm{d}F .$$

The similar case when renewal is at $Ms$ is treated separately (case 7) because we assume there is no possibility to default $Ms$.

Case 6 is similar to case 5 except there is a default on the corrective replacement. Renewal is at $is$, $i=K+3,...,M$ , $K=1,...,M-3$, default is at $(i-1)s$, failure occurs in $[(i-2)s,(i-1)s]$, and the defect arises in a previous but not necessarily adjacent interval in the wear-out phase. The corresponding term is

$$T_{l,6} = p\sum_{i=K+3}^{M}\sum_{j=K+1}^{i-2}\int_{(j-1)s}^{js}\int_{(i-2)s-x}^{(i-1)s-x}\phi_{l,6}\ \mathrm{d}G\mathrm{d}F .$$

As mentioned above, if failure occurs in $[(M-1)s, Ms]$, then a default on the corrective replacement is not possible. Defect arrival before $Ks$ is part of case 4. Defect arrival after $Ks$, but in a preceding interval, is the next case, case 7, with corresponding term

$$T_{l,7} = \sum_{j=K+1}^{M-1}\int_{(j-1)s}^{js}\int_{(M-1)s-x}^{Ms-x}\phi_{l,7}\ \mathrm{d}G\mathrm{d}F ,$$

valid for $K=1,...,M-2$.

The last three cases correspond to defect arrival and failure in the same interval in the wear-out phase. In case 8 there is no default and the corresponding term, valid for $i=K+1,...,M-1$ and $K=1,...,M-2$, is

$$T_{l,8} = (1-p)\sum_{i=K+1}^{M-1}\int_{(i-1)s}^{is}\int_{0}^{is-x}\phi_{l,8}\mathrm{d}G\mathrm{d}F ,$$

and in case 9 there is a default (on the corrective replacement) and the corresponding term, valid for $i=K+2,...,M$ and $K=1,...,M-2$, is

$$T_{l,9} = p\sum_{i=K+2}^{M}\int_{(i-2)s}^{(i-1)s}\int_{0}^{(i-1)s-x}\phi_{l,9}\mathrm{d}G\mathrm{d}F .$$

Finally, in case 10 defect and failure both occur in $[(M-1)s, Ms]$. This case is treated in isolation because there is no possibility of default. The term is

$$T_{l,10} = \int_{(M-1)s}^{Ms}\int_{0}^{Ms-x}\phi_{l,10}\mathrm{d}G\mathrm{d}F$$

Special cases of these terms arise for values of $K$ and $M$ that preclude some cases. Thus, when $M=1$ or $M=2$ or $M=3$ these terms simplify or disappear altogether. We omit their detailed description for brevity.

The cases above deal with those renewals that are corrective replacements (on failure). Next, we consider cases with preventive replacement (Figure 2), and we use the same structure. The cases are simpler because preventive replacement can only occur during the inspection phase or at $Ms$.

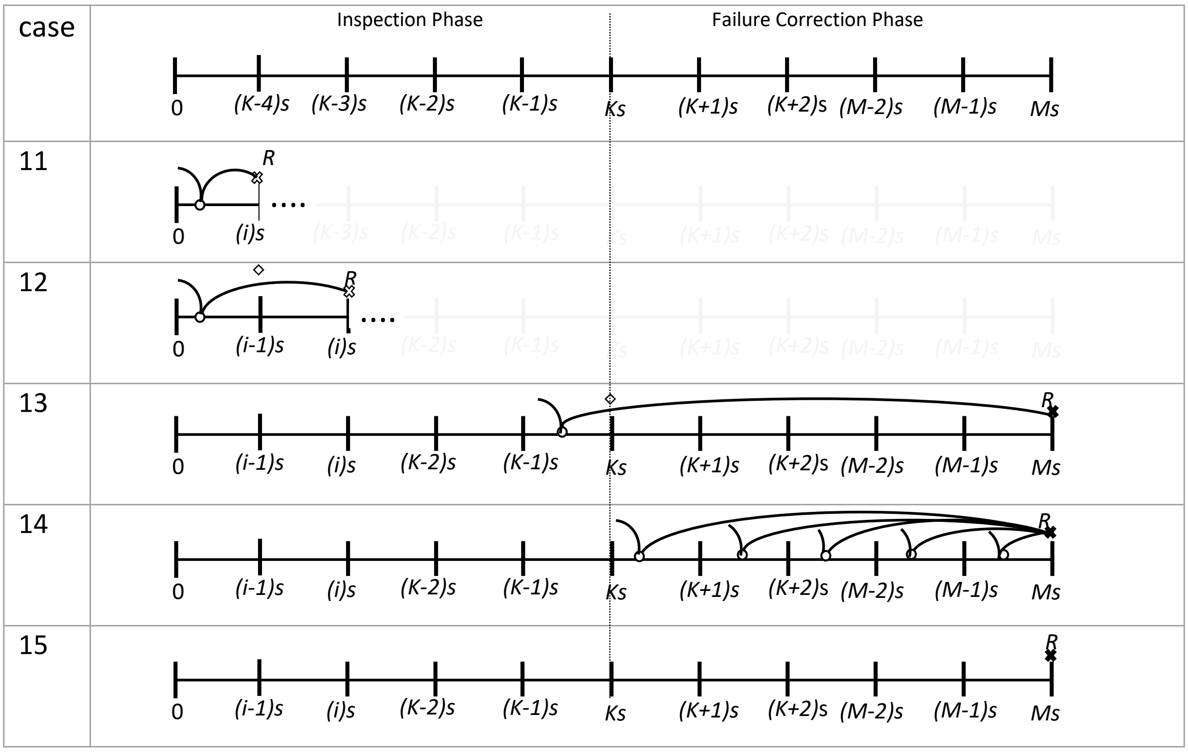



Figure 2. Cases with renewal at preventive replacement. Symbol key: inspection (┼); defect arrival ( ○ ); preventive replacement ( ✖); default ( ◇ ); replacement (R).

So, renewal on preventive replacement at $is$, during the inspection phase, with no default (case 11) has corresponding term

$$T_{l,11} = (1-p)\sum_{i=1}^{K}\int_{(i-1)s}^{is}\int_{is-x}^{\infty}\phi_{l,11}\ \mathrm{d}G\mathrm{d}F\ ,$$

$i=1,...,K$, $K=2,...,M-1$, $K<M$, and with default (case 12) has corresponding term

$$T_{l,12} = p\sum_{i=2}^{K}\int_{(i-2)s}^{(i-1)s}\int_{is-x}^{\infty}\phi_{l,12}\ \mathrm{d}G\mathrm{d}F\ ,$$

$i=2,...,K$, $K=2,...,M-1$, $K<M$. The last three cases relate to renewal on preventive replacement at $Ms$. In case 13, the defect arises in $[(K-1)s, Ks]$, $K=1,...,M-1$, there is a default at $Ks$ and the defect survives to $Ms$, whence the term is

$$T_{l,13} = p\int_{(K-1)s}^{Ks}\int_{Ms-x}^{\infty}\phi_{l,13}\ \mathrm{d}G\mathrm{d}F\ .$$

In case 14, the defect arises in the wear-out phase, in the interval $[(i-1)s, is]$, $i = K+1,...,M$ , $K < M$, and survives to $Ms$. No default can occur because there is no inspection in the wear-out phase and no default at $Ms$. So, the corresponding term is

$$T_{l,14} = \sum_{i=K+1}^{M} \int_{(i-1)s}^{is} \int_{Ms-x}^{\infty} \phi_{l,14} \mathrm{d}G\mathrm{d}F .$$

Finally, in case 15 there is no defect arrival in $[0, Ms]$, so no possibility of failure or default, and the corresponding term is

$$T_{l,15} = \phi_{l,15} \left[1 - F(Ms)\right].$$

3.2. *Calculation of the decision criteria*

We now derive the cost-rate, the average availability and the mean time between operational failures, using the appropriate function $\phi$ in each term.

Using the function in column 2 of Table 1, so that $\phi_{1,c} = 1$ for all cases, $c = 1,...,15$, the corresponding term $T_{1,c}$ is the probability of occurrence for case *c*. Therefore, we must obtain $\sum_{c=1}^{15} T_{1,c} = 1$. This provides a check on the exhaustiveness of the cases.

Using the function $\phi_{2,c}$ in column 3 of Table 1, the corresponding term $T_{2,c}$ is the contribution to the expected length of a renewal cycle for each case. Column 4 in Table 1 gives the functions $\phi_{3,c}$ appropriate for the terms that are the contributions to the downtime. Finally, column 5 of Table 1 gives the functions $\phi_{4,c}$ appropriate for the terms that are the contributions to the expected cost of inspection and replacement in a renewal cycle. Then, it follows that the expected downtime in a cycle is

$$E(W) = \sum_{c=1}^{15} T_{3,c} ,$$

and the expected cost of a renewal cycle is given by

$$E(U) = \sum_{c=1}^{15} T_{4,c} + c_D E(W) ,$$

and the expected length of a renewal cycle is given by

$$E(V) = \sum_{c=1}^{15} T_{2,c} ,$$

and the cost-rate is given by

$$Q(K, M) = E(U) / E(V) .$$

and average availability is given by

$$A(K, M) = \{E(V) - E(W)\} / E(V) .$$

To approximate the system reliability, we use the mean time between operational failures (MTBOF). This is simply the ratio of the expected length of a renewal cycle to the probability that a renewal cycle ends in failure (Scarf et al., 2005). It is the reciprocal of the long-run mean number of failures per unit time. Thus,

$$\mu(K, M) = E(V) / \sum_{c=1}^{10} T_{1,c} .$$

## 4. NUMERICAL EXAMPLE

We specify the distribution of the time to defect arrival, *X*, as a mixture of Weibull distributions, $W_1(\eta_1,\beta_1)$ and $W_2(\eta_2,\beta_2)$, with mixing parameter *r*, so that the reliability function of *X* is $\bar{F}(x) = r\exp\{-(x/\eta_1)^{\beta_1}\} + (1-r)\exp\{-(x/\eta_2)^{\beta_2}\}$. We specify the distribution of the delay time (time in the defective state), *H*, as an exponential distribution with rate $\lambda$ (mean $1/\lambda$). In the base case, we set the $\eta_1 = 1$, $\beta_1 = 3$, $\eta_2 = 10$, $\beta_2 = 3$, $r = 0.2$, $\lambda = 0.5$, and $s = 1$, so that the interval between visits is the unit of time and equal to the characteristic "life" of the weak population.

The cost of inspection, preventive replacement and failure are respectively $c_I = 0.1$, $c_P = 1$, $c_F = 4$, noting that these costs are the cost per event. The cost of downtime is $c_D = 2$ per unit of time that the system is failed (down).

All numerical calculations are implemented in Mathcad (version 15), which provides a convenient platform for calculating multiple integrals. However, the computation burden is heavy, particularly when *K* and *M* are large because many double integrals must be evaluated and the search space is large. It would be interesting to seek a more efficient computational approach by using a semi-Markov decision process framework (see e.g. Salari and Makis, 2020), although we do not pursue this further here.

In the tables of results (Tables 2 and 3), the first row corresponds to the base parameter case. In subsequent rows, parameter values that are different to the base case have a grey highlight. Finally, note that all the results presented were calculated using the formulae given in Section 3. However, for the purpose of verification, we also simulated the policy and obtained results that were within 0.0005 (in absolute terms) of the calculated results (based on $10^6$ repetitions).

### 4.1. *Cost-rate, system reliability and average availability without defaulting*

We consider first results without defaulting and we make some observations that these results indicate, bearing in mind that we cannot claim that any findings apply generally, because this numerical study considers a limited region of the parameter space. Nonetheless, we can see that $K^*$ and $M^*$ are well separated, so that the two phases are distinct, when the characteristic lives of the populations are well separated (c.f. row 1 and row 5 6), and when the downtime cost-rate is not large (c.f. rows 1, 7, 8). This is provided $r > 0$. The separation of the phases persists for a range of values of *s* (c.f. rows 1, 10, 11), and that while $K^*$ and $M^*$ depend on *s* the lengths of the inspection and failure-correction phases, $sK^*$ and $s(M^* - K^*)$ do not. So, we might tentatively conclude that changing *s* changes the cost-rate and the average availability but does not change the policy. We can also see that a moderate value of the visit interval is cost-minimising, presumably because more frequent visiting increases the inspection cost-rate and less frequent visiting increases the failure cost-rate. This claim is supported by the apparent relationship between availability and *s* (average availability decreases as *s* increases). Note finally, although *s* is not a decision variable in the policy, a decision-maker would in practice be expected to have some control of it.

We also present the values of the mean time of operational failures (MTBOF) calculated at the cost-optimum point, $\mu(K^*, M^*)$. Observe that the MTBOF behaves as we would expect: the less heterogeneity of components the greater is the system reliability (c.f. rows 1-3); when the downtime cost ($c_D$) increases, a more conservative policy has a higher MTBOF ( c.f. rows 1, 8 and 9 ); and as the mean delay time ($1/\lambda$) decreases the MTBOF decreases substantially (c.f. rows 1, 4 and 5).

Table 2. Optimal policy without defaulting for various values of $\eta_1$, $\beta_1$, $r$, $\lambda$, $c_D$, and $s$. Other parameters are fixed at base-case values: $\eta_2 = 10$, $\beta_2 = 3$, $c_I = 0.1$, $c_P = 1$, $c_F = 4$.

| row | | | | | | | cost-rate | | | | average availability | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\eta_1$ | $\beta_1$ | $r$ | $\lambda$ | $c_D$ | $s$ | $K^*$ | $M^*$ | $Q$ | $\mu$ | $K^*$ | $M^*$ | $A_\infty$ |
| 1 | 1 | 3 | 0.2 | 0.5 | 2 | 1 | 2 | 7 | 0.313 | 36.4 | 2 | 4 | 0.994 |
| 2 | 1 | 3 | 0 | 0.5 | 2 | 1 | 0 | 6 | 0.229 | 64.7 | 0 | 3 | 0.999 |
| 3 | 1 | 3 | 0.1 | 0.5 | 2 | 1 | 2 | 7 | 0.284 | 41.4 | 2 | 4 | 0.996 |
| 4 | 1 | 3 | 0.2 | 1 | 2 | 1 | 2 | 7 | 0.364 | 24.8 | 2 | 4 | 0.990 |
| 5 | 1 | 3 | 0.2 | 2 | 2 | 1 | 1 | 7 | 0.414 | 17.7 | 2 | 4 | 0.984 |
| 6 | 2 | 3 | 0.2 | 0.5 | 2 | 1 | 7 | 9 | 0.314 | 44.1 | 3 | 5 | 0.994 |
| 7 | 1 | 5 | 0.2 | 0.5 | 2 | 1 | 2 | 7 | 0.311 | 37.1 | 2 | 4 | 0.994 |
| 8 | 1 | 3 | 0.2 | 0.5 | 1 | 1 | 2 | 7 | 0.301 | 36.4 | - | - | - |
| 9 | 1 | 3 | 0.2 | 0.5 | 4 | 1 | 6 | 8 | 0.335 | 47.6 | - | - | - |
| 10 | 1 | 3 | 0.2 | 0.5 | 2 | 0.5 | 6 | 15 | 0.346 | 38.1 | 7 | 9 | 0.998 |
| 11 | 1 | 3 | 0.2 | 0.5 | 2 | 2 | 1 | 3 | 0.343 | 33.8 | 1 | 3 | 0.979 |

An interesting point is that availability can be maximised (Figure 3a), although not simultaneously to minimising the cost (Figure 3b). This is the weak-strong heterogeneity effect, so that infrequent preventive replacement is sub-optimal—higher probability of failure of the strong (surviving) components and so lower average availability—and very frequent replacement is sub-optimal—early replacement may result in a strong component being replaced by a weak one. Also, we can see that from a cost-rate point of view, *K* and *M* act somewhat independently—$M^* = 7$ whatever the value of *K*, whereas with regard to availability, *K* and *M* interact so that the optimum length of the wear-out phase is constant $M^* - K^* = 3$ whatever the value of *K*. When $r = 0$, so that components are not heterogeneous, matters are simpler and there is no inspection, although we might anticipate inspection at every visit would be optimal if the cost of inspection is sufficiently small.

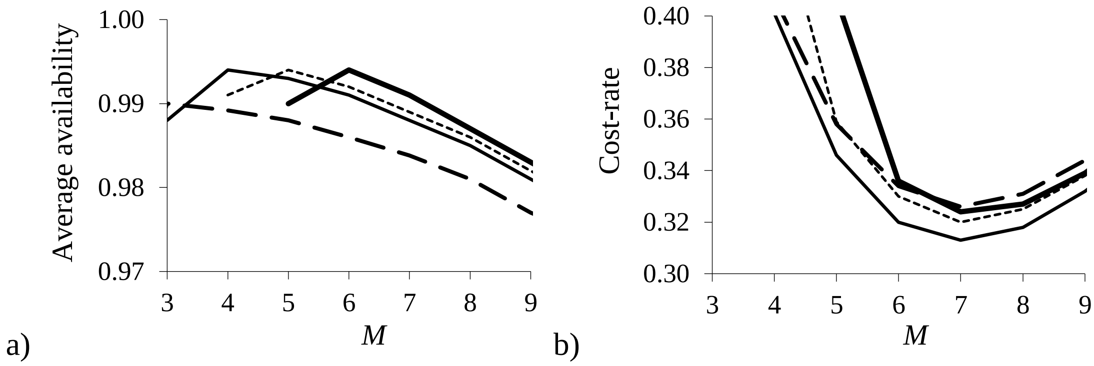



Figure 3. In the base case, without defaulting, (a) average availability and (b) cost-rate versus $M$ for various values of $K$: $K = 1$ (— — —); $K = 2$ (———); $K = 3$ (------); $K = 4$ (━━━).

4.2. *Cost-rate, system reliability and average availability with defaulting*

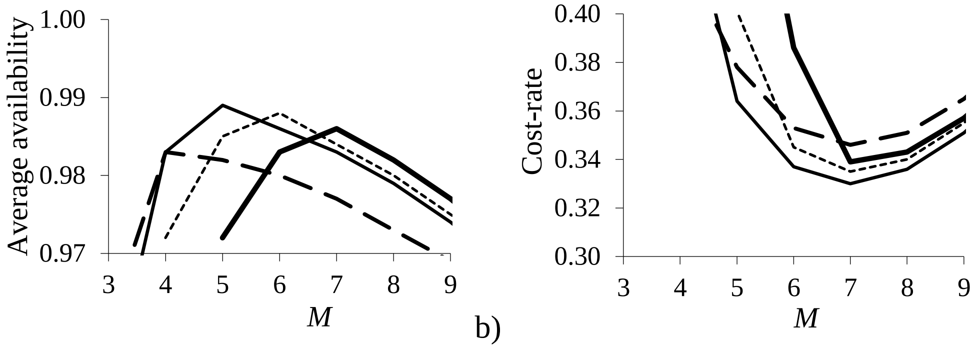
Now we consider results of the model when defaulting can occur, Table 3. Here, it appears that defaulting increases the cost-rate and decreases the availability but does not change the policy (c.f. rows 1 of Table 2 with rows 3 and 4 of Table 3). The average unavailability more than doubles (from 0.6% to 1.6%) as the default probability increases (from 0 to 0.4), while the cost-rate increases by 11% (from 0.313 to 0.347) over the same range. When $s$ is larger ( $s = 2$ ), this effect is more dramatic (c.f. row 11 of Table 2 with row 11 of Table 3), where the cost-rate increases by 30% (0.343 to 0.446) over the same range. When $s$ is smaller ( $s = 0.5$ ), the cost-rate increase is 5% (0.346 to 0.363). Thus, as we might have anticipated, defaulting is significant problem if scheduled visits are infrequent, and other policy adjustments (to $K$ and $M$ ) cannot compensate.

In both defaulting scenarios (with and without), inspection appears to be cost-effective only when there is heterogeneity, that is, if there is a tendency to early failure of the system. When there exists defaulting, the ineffectiveness of inspection is perhaps because there will often be a delay between a positive inspection (defect identification) and the consequent preventive replacement, so that inspection is not very effective for preventing failure. When there is no defaulting, this ineffectiveness appears to persist.

Comparison of Figure 4 with Figure 3 suggests that the cost-rate and average availability are less sensitive to departures of $K$ and $M$ from their optimal values when there is the possibility to default than when there is not. This makes sense because if there is the possibility to default on an action (replacement), as a result of a shortage of spare-parts say, then the policy itself is less effective and implementation of an action at its prescribed moment is less important.

Table 3. Optimal policy with defaulting for various values of $\eta_1$, $\beta_1$, $r$, $p$, $\lambda$, $c_D$, and $s$. Other parameters are fixed at base-case values: $\eta_2 = 10$, $\beta_2 = 3$, $c_I = 0.1$, $c_P = 1$, $c_F = 4$.

| row | | | | | | | | cost-rate | | | | average availability | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\eta_1$ | $\beta_1$ | $r$ | $p$ | $\lambda$ | $c_D$ | $s$ | $K^*$ | $M^*$ | $Q$ | $\mu$ | $K^*$ | $M^*$ | $A_\infty$ |
| 1 | 1 | 3 | 0.2 | 0.2 | 0.5 | 2 | 1 | 2 | 7 | 0.330 | 32.9 | 2 | 5 | 0.989 |
| 2 | 1 | 3 | 0 | 0.2 | 0.5 | 2 | 1 | 0 | 6 | 0.232 | 64.7 | 0 | 3 | 0.999 |
| 3 | 1 | 3 | 0.2 | 0.4 | 0.5 | 2 | 1 | 2 | 7 | 0.347 | 30.1 | 2 | 5 | 0.984 |
| 4 | 2 | 3 | 0.2 | 0.4 | 0.5 | 2 | 1 | 4 | 7 | 0.348 | 35.8 | 3 | 6 | 0.983 |
| 5 | 2 | 3 | 0.2 | 0.2 | 0.5 | 2 | 1 | 4 | 7 | 0.333 | 39.7 | 3 | 6 | 0.987 |
| 6 | 1 | 3 | 0.2 | 0.2 | 2 | 4 | 1 | 2 | 6 | 0.498 | 20.2 | 2 | 5 | 0.974 |
| 7 | 1 | 3 | 0.2 | 0.4 | 2 | 4 | 1 | 2 | 6 | 0.540 | 19.2 | 2 | 4 | 0.965 |
| 8 | 1 | 3 | 0.2 | 0.2 | 0.5 | 2 | 0.5 | 6 | 15 | 0.354 | 36.1 | 6 | 9 | 0.997 |
| 9 | 1 | 3 | 0.2 | 0.4 | 0.5 | 2 | 0.5 | 6 | 15 | 0.363 | 34.3 | 6 | 9 | 0.995 |
| 10 | 1 | 3 | 0.2 | 0.2 | 0.5 | 2 | 2 | 2 | 5 | 0.417 | 19.3 | 2 | 5 | 0.944 |
| 11 | 1 | 3 | 0.2 | 0.4 | 0.5 | 2 | 2 | 2 | 5 | 0.446 | 18.7 | 2 | 5 | 0.930 |




a) b)

Figure 4. In the base case, with defaulting ( $p = 0.2$ ), (a) average availability and b) cost-rate versus $M$ for various values of $K$: $K = 1$ (— — —); $K = 2$ (——); $K = 3$ (------); $K = 4$ (▬▬).

Comparing Table 2 with Table 3, it appears that when components are not heterogeneous, defaulting has little effect on the MTBOF (c.f. rows 2 in the tables). However, interestingly, with heterogeneity, when the interval between visits is large defaulting has a greater effect on reliability than when it is small (c.f. rows 8, 10, Table 3, and rows 10, 11, Table 2).

### 4.3. *Required availability analysis with defaulting*

Defaulting reflects uncertainty that exists at the time of execution of maintenance action. This uncertainty can arise from events (e.g. bad weather, shortages of spares, absence of personnel) that are difficult to predict but possible to quantify (see e.g. Energy Institute, 2020). Thus, a default analysis can be interpreted as a maintenance risk analysis. One possible way to conduct such an analysis is through a constrained-availability approach. Here, for a given visit-interval $s$, the maximum allowable default probability $p$ that meets a required average availability can be determined (Figure 5). Thus, if visits are sufficiently frequent, then default occurrence may be acceptable. This indication, although not a general result, has important practical implications for the management of maintenance. Such an analysis will also be useful for contract negotiation when maintenance is outsourced, wherein targets for availability may be specified.

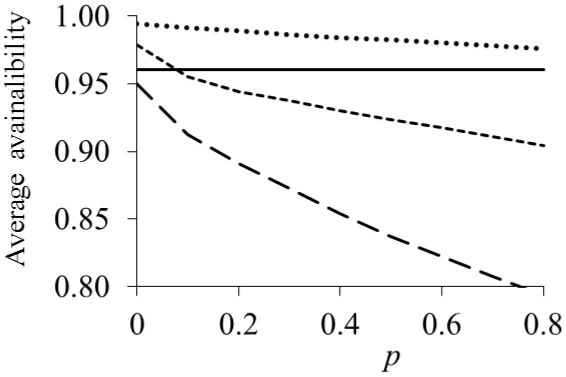



Figure 5. In the base case, average availability of the cost-minimum policy versus $p$ for various $s = 1$ (•••••), $s = 2$ (----), $s = 3$ (— —); example average availability constraint at 0.96 (——).

4.4. *Implications for practice*

The policy, and the corresponding model, is motivated by issues in the maintenance and logistics for maintenance of high-value engineered systems such as windfarms, and systems used in e.g. transportation, manufacturing, and defence. Thus, in our view, this work is applicable in a way that addresses the recommendation of Scarf (1997). Therefore, we restate the implications for the planning and control of maintenance as follows.

- Significant variance from planned actions (defaulting) has an important, negative effect on the maintenance performance only when the periodic visits are infrequent. Otherwise, the fixed-frequency policy is relatively robust to defaulting. It follows that the "effective visit frequency", $(1-p)/s$, is a key quantity of interest for planning. Thus, planners should be aware that an appropriate maintenance-frequency depends on the risk of default.
- This interaction between visit-frequency and default probability is interesting for practice because the one can compensate for the other, although for a small visit-frequency and high default probability the timing of actual interventions becomes somewhat akin to random inspections (e.g. Scarf et al., 2019).
- Weak-strong heterogeneity implies that average availability can be maximised. This is because early replacement may result in a component in good condition, with a long expected residual life, being replaced by a weak one, with a short expected life (Scarf et al., 2009). This is different to the case of homogeneously failing systems, for which increasing the frequency of maintenance generally increases the reliability of a system (see e.g Alotaibi et al., 2020). Thus, it is important that maintainers are aware that early replacement of a component may be doing harm rather than good to a system performance. Broadly, this is the maintenance-quality effect (Scarf and Cavalcante, 2012; Dourado and Viana, 2021). Also, this is useful for out-sourcing contracts where availability may be specified.
- Finally, when the visit frequency is fixed, defaulting, as a result of a shortage of spare-parts or personnel or lack of access or priority of production, makes the policy less effective and policy optimization is less relevant. Thus, we suggest that managers need to solve issues related to defaulting as a first priority and carefully plan interventions as a second priority.

## 5. CONCLUSIONS

We derive exact expressions for the long run cost per unit time (cost-rate) and average availability for a maintenance policy in which inspections and replacements of a non-repairable system can only be executed at scheduled visits that occur every $s$ time units. The policy has two phases, an initial inspection phase followed by a failure-correction phase. The policy considers *what* maintenance to schedule at known times rather than *when* to schedule maintenance. The phases are adapted to the possible heterogeneity of the lifetime of the system. The policy is important because the pre-specification of maintenance epochs mimics the reality in which immediate attention to failures (corrective maintenance) and even the actions defined to avoid failures (preventive maintenance) are constrained by operations and/or logistics. The policy is also simple to study because the age of the system is always in-phase with the periodic visits. It is easy to implement in practice, and suitable for extending to multi-components systems and the grouping of preventive maintenance. The paper also considers various factors in imperfection in maintenance, e.g. early defects, poor installation, spare-part shortages, overrun on work schedules. These factors increase uncertainty about the effectiveness of maintenance interventions. Thus, the model we propose supports the decision maker when it is not clear if the adoption of an action is effective.

The results indicate a number of interesting characteristics of the policy. The visit interval *s* does not have a dramatic effect on the policy, so that the cost-optimal lengths of the phases (the inspection phase and the failure-correction phase) do not vary with *s*. This is not surprising. Also, neither very frequent visits nor infrequent visits are helpful, in the sense that in both cases the cost-rate increases. Thus, the cost-rate appears to be non-linear in *s*. On the other hand, average availability increases as visits become more frequent. Defaulting has a strong effect on cost, reliability, and unavailability if visits are infrequent, but not so otherwise. Inspection is relatively ineffective unless there is a tendency to early failure.

Our principal observation relevant for maintenance planning in practice is that visit frequency and default probability are important factors and assessment of the first should take account of the second. Thus, we might point to the "effective visit frequency", $(1-p)/s$, as a key quantity of interest when a maintenance schedule is fixed.

A limitation of the paper is that we use a numerical study so we cannot make general conclusions about the optimality of policy decision variables or the validity of our conclusions. However, we would claim that our findings are reasonable and valid for the range of parameter values we consider. Furthermore, the purpose of the numerical analysis is not so much to determine general characteristics of the policy but more to indicate what is possible. When broader behaviours are sort, more cases can be investigated.

In future work, it would be interesting to use the model as a framework for analysing grouping policies for multi-component systems. In such policies, components are assigned to groups (in an optimal manner), each group having the same maintenance interval. A specified visit-interval will significantly simplify this assignment problem. Studies that consider, say, more flexible inspection scenarios, for example, inspections in later life, or the effect of emergency (immediate) corrective response to failure may also be useful.

*Acknowledgements*: The work of Phil Scarf was carried out with funding for a research project by the Coordination for the Improvement of Higher Education Personnel in Brazil (Capes), within the scope of the Capes-PrInt Program. The work of Cristiano Cavalcante and Rodrigo Lopes has been supported by the CNPq (Brazilian Research Council).

REFERENCES

Alberti, A. R., Cavalcante, C. A., Scarf, P. & Silva, A. L. (2018). Modelling inspection and replacement quality for a protection system. Reliability Engineering & System Safety, 176, 145-153.

Alotaibi, N. M., Cavalcante, C. A. V., Lopes, R. S. & Scarf, P. A. (2020) Preventive replacement with defaulting. IMA Journal of Management Mathematics 31, 491–504.

Arts, J. & Basten, R. (2018) Design of multi-component periodic maintenance programs with single-component models. IISE Transactions 50, 606-615.

Arvesen, A., Birkeland, C. & Hertwich, E.G. (2013) The importance of ships and spare parts in LCAs of offshore wind power. Environmental Science & Technology 47, 2948-2956.

Ascher H. & Feingold, F. (1984) Repairable Systems Reliability. Marcel Dekker.

Aven, T. (2020) Three influential risk foundation papers from the 80s and 90s: Are they still state-of-the-art? Reliability Engineering & System Safety, 193, 106680.

Berrade, M.D., Scarf, P.A. & Cavalcante, C.A.V. (2017) A study of postponed replacement in a delay time model. Reliability Engineering & System Safety 168, 70-79.

Broek, uit het M.A.J., Teunter, R.H., de Jonge, B., Veldman, J. & Van Foreest N.D. (2020) Condition-based production planning: adjusting production rates to balance output and failure risk. Manufacturing & Service Operations Management (in press).

Budai, G., Dekker, R. & Nicolai, R.P. (2008) Maintenance and production: A review of planning models. In Kobbacy K., Murthy D.N.P. (eds.) Complex System Maintenance Handbook, Springer, pp.321-344.

Cai, B., Xie, M., Liu, Y., Liu, Y. & Feng, Q. (2018) Availability-based engineering resilience metric and its corresponding evaluation methodology. Reliability Engineering & System Safety, 172, 216-224.

Cavalcante, C. A., Scarf, P. A. & De Almeida, A. T. (2011). A study of a two-phase inspection policy for a preparedness system with a defective state and heterogeneous lifetime. Reliability Engineering & System Safety 96, 627-635.

Cavalcante, C.A.V. & Lopes, R.S. (2015). Multi-criteria model to support the definition of opportunistic maintenance policy: A study in a cogeneration system. Energy 80, 32-40.

Cavalcante, C.A.V., Lopes, R.S. & Scarf, P.A. (2018) A general inspection and opportunistic replacement policy for one-component systems of variable quality. European Journal of Operational Research 266, 911-919.

Cavalcante, C.A.V., Scarf, P.A., & Berrade, M. D. (2019). Imperfect inspection of a system with unrevealed failure and an unrevealed defective state. IEEE Transactions on Reliability, 68(2), 764-775.

Corman, F., Kraijema, S., Godjevac, M. & Lodewijks, G. (2017) Optimizing preventive maintenance policy: A data-driven application for a light rail braking system. Journal of Risk and Reliability 231, 534-545.

De Jonge B., Dijkstra A.S. & Romeijnders W. (2015) Cost benefits of postponing time-based maintenance under lifetime distribution uncertainty. Reliability Engineering & System Safety 140, 15-21.

De Jonge, B & Scarf P.A. (2020) A review on maintenance optimisation. European Journal of Operational Research 285, 805-824.

Dekker, R. & Smeitink, E. (1991) Opportunity-based block replacement. European Journal of Operational Research, 53(1), 46-63.

Ding, F. & Tian, Z. (2011). Opportunistic maintenance optimization for wind turbine systems considering imperfect maintenance actions. International Journal of Reliability, Quality and Safety Engineering 18, 463–481.

Do, P., Barros, A., Bérenguer, C., Bouvard, K. & Brissaud F (2013) Dynamic grouping maintenance with time limited opportunities. Reliability Engineering & System Safety 120, 51-59.

Dourado, A. & Viana, F.A.C. (2021) Early life failures and services of industrial asset fleets, Reliability Engineering & System Safety 205, 107225.

Duarte, Y.S., Szpytko, J. & del Castillo Serpa, A.M. (2020) Monte Carlo simulation model to coordinate the preventive maintenance scheduling of generating units in isolated distributed Power Systems. Electric Power Systems Research 182, 06237.

Energy Institute (2020) Health and safety statistics. https://www.gplusoffshorewind.com/work-programme/statistics (Accessed 4/8/2020).

Faulstich, S., Hahn, B. & Tavner, P.J. (2011) Wind turbine downtime and its importance for offshore deployment. Wind Energy 14, 327-337.

Garambaki, A.H.S., Thaduri, A., Seneviratne, A.M.N.D.B. & Kumar, U. (2016) Opportunistic inspection planning for railway emaintenance. IFAC-PapersOnLine 49(28), 197-202.

Hahn, B., Durstewitz, M. & Rohrig, K. (2007) Reliability of wind turbines. In Wind energy (pp. 329-332). Springer, Berlin, Heidelberg.

Hedjazi, D., Layachi, F. & Boubiche, D.E. (2019) A multi-agent system for distributed maintenance scheduling. Computers & Electrical Engineering 77, 1-11.

Hu, J. & Zhang, L. (2014) Risk based opportunistic maintenance model for complex mechanical systems. Expert Systems with Applications 41, 3105-3115.

Hu, J., Shen, J. & Shen, L. (2020). Periodic preventive maintenance planning for systems working under a Markovian operating condition. Computers & Industrial Engineering 142, 106291.

Kafiabad, S.T., Zanjani, M.Z. & Nourelfath, M. (2020) Integrated planning of operations and on-job training in maintenance logistics networks. Reliability Engineering & System Safety 199, 106922.

Laggoune, R., Chateauneuf, A. & Aissani, D. (2010) Impact of few failure data on the opportunistic replacement policy for multi-component systems. Reliability Engineering & System Safety 95(2): 108–119.

Nguyen, H.S.H., Do, P., Vu, H-C. & Iung, B. (2019) Dynamic maintenance grouping and routing for geographically dispersed production systems. Reliability Engineering & System Safety 185, 392-404.

Peng, H. & Zhu, Q. (2017) Approximate evaluation of average downtime under an integrated approach of opportunistic maintenance for multi-component systems. Computers & Industrial Engineering 109, 335-346.

Rodrigues, T.F., Dantas, M.M. & Cavalcante, C.A.V (2020) A dynamic inventory rationing policy for business-to-consumer e-tail stores in a supply disruption context. Computers & Industrial Engineering 142, 106379.

Ruiz, C., Pohl, E. & Liao, H. (2021) Bayesian degradation modelling for spare parts inventory management. IMA Journal of Management Mathematics 32, 31–49.

Salari, N. & Makis, V. (2020) Application of Markov renewal theory and semi-Markov decision processes in maintenance modeling and optimization of multi-unit systems. Naval Research Logistics 67, 548– 558.

Scarf, P.A. (1997) On the application of mathematical models in maintenance, European Journal of Operational Research 99, 493-506.

Scarf, P.A. & Cavalcante, C.A.V. (2010) Hybrid block replacement and inspection policies for a multi-component system with heterogeneous component lives. European Journal of Operational Research 206, 384-394.

Scarf, P.A. & Cavalcante, C.A.V. (2012) Modelling quality in replacement and inspection maintenance. International Journal of Production Economics 135, 372-381.

Scarf, P.A., Cavalcante, C.A.V. & Lopes, R.S. (2018) An opportunistic maintenance policy for heterogeneous components. In Safety and Reliability: Safe Societies in a Changing World (Stein Haugen, Anne Barros, Coen van Gulijk, Trond Kongsvik, Jan Erik Vinnem, Eds.), Taylor and Francis, pp. 693-699.

Scarf P.A., Cavalcante C.A.V. & Lopes, R.S. (2019) Delay-time modelling of a critical system subject to random inspections. European Journal of Operational Research 278, 772-782.

Scarf, P.A., Cavalcante, C.A.V., Dwight, R. & Gordon, P. (2009) An age based inspection and replacement policy for heterogeneous components. IEEE Transactions on Reliability 58, 641–648.

Scarf, P.A., Dwight, R. & Al-Musrati, A. (2005) On reliability criteria and the implied cost of failure for a maintained component. Reliability Engineering and System Safety 89, 199-207.

Selvik, J. T. & Aven, T. (2011) A framework for reliability and risk centered maintenance. Reliability Engineering & System Safety, 96, 324-331.

Shafiee, M. & Sørensen, J.D. (2019) Maintenance optimization and inspection planning of wind energy assets: Models, methods and strategies. Reliability Engineering & System Safety 192, 105993.

Shafiee, M. (2015) Maintenance logistics organization for offshore wind energy: current progress and future perspectives, Renewable Energy 77, 182-193.

Shafiee, M., Finkelstein, M. & Bérenguer, C. (2015) An opportunistic condition-based maintenance policy for offshore wind turbine blades subjected to degradation and environmental shocks. Reliability Engineering & System Safety 142: 463-471.

Shahraki, A.F., Yadav, O.P. & Vogiatzis, C. (2020) Selective maintenance optimization for multi-state systems considering stochastically dependent components and stochastic imperfect maintenance actions. Reliability Engineering & System Safety 196, 106738.

Shen, J., Zhang, Y., Ma, Y. & Lin, C. (2021) A novel opportunistic maintenance strategy for systems with dependent main and auxiliary components, IMA Journal of Management Mathematics 32, 69–90.

Sinisterra, W.Q. & Cavalcante, C.A.V. (2020) An integrated model of production scheduling and inspection planning for resumable jobs. International Journal of Production Economics 227, 107668

Spinato, F., Tavner, P. J., Van Bussel, G. J. & Koutoulakos, E. (2009). Reliability of wind turbine subassemblies. IET Renewable Power Generation 3, 387-401.

Taghipour, S. & Banjevic, D. (2012) Optimal inspection of a complex system subject to periodic and opportunistic inspections and preventive replacements. European Journal of Operational Research 220, 649–660.

Topan, E., Eruguz, A.S., Ma, W., van der Heijden, M.C. & Dekker, R. (2020) A review of operational spare parts service logistics in service control towers. European Journal of Operational Research, 282, 401-404.

Van Oosterom, C.D., Elwany, A.H., Çelebi, D. & Van Houtum, G.J. (2014) Optimal policies for a delay time model with postponed replacement. European Journal of Operational Research 232, 186-197.

Vaurio, J.K. (1999) Availability and cost functions for periodically inspected preventively maintained units. Reliability Engineering & System Safety 63, 133-140.

Vu, H.C., Do, P., Barros, A. & Bérenguer, C. (2015) Maintenance planning and dynamic grouping for multi-component systems with positive and negative economic dependencies. IMA Journal of Management Mathematics 26, 145-170.

Vu, H.C., Do, P., Fouladirad, M. & Grall, A. (2020) Dynamic opportunistic maintenance planning for multi-component redundant systems with various types of opportunities. Reliability Engineering & System Safety 198, 106854.

Wang, W. & Christer, A.H. (2003) Solution algorithms for a nonhomogeneous multi-component inspection model. Computers & Operations Research 30, 19-34.

Wang, X., Zhou, H. Parlikad A.K. & Xie, M. (2020) Imperfect preventive maintenance policies with unpunctual execution. IEEE Transactions on Reliability (in press) doi: 10.1109/TR.2020.2983415.

Wang, Z., Cui, B., Feng, Q., Huang, B., Ren, Y., Sun, B., Yang, D. & Zuo, Z. (2019) An agent-based approach for resources' joint planning in a multi-echelon inventory system considering lateral transshipment. Computers & Industrial Engineering 138, 106098.

Werbińska-Wojciechowska, S. (2019) Delay-time-based maintenance modeling for technical systems—theory and practice. In Advances in System Reliability Engineering (Mangey Ram, J. Paulo Davim, Eds.), Academic Press, 2019, pp.1-42.

Xia, T. B., Tao, X. Y. & Xi, L. F. (2017a) Operation process rebuilding (OPR)-oriented maintenance policy for changeable system structures. IEEE Transactions on Automation Science and Engineering 14, 139-148.

Xia, T., Sun, B., Chen, Z., Pan, E., Wang, H. & Xi, L. (2021) Opportunistic maintenance policy integrating leasing profit and capacity balancing for serial-parallel leased systems. Reliability Engineering & System Safety 205, 107233.

Xia, T., Xi, L., Pan, E. & Ni, J. (2017c) Reconfiguration-oriented opportunistic maintenance policy for re-configurable manufacturing systems. Reliability Engineering & System Safety 166, 87-98.

Xia, T., Xi, L., Pan, E., Fang, X. & Gebraeel, N. (2017b). Lease-oriented opportunistic maintenance for multi-unit leased systems under product-service paradigm. Journal of Manufacturing Science and Engineering 139, 071005.

Yang, L., Ye, Z-S., Lee, C-G., Yang, S-F. & Peng, R. (2019) A two-phase preventive maintenance policy considering imperfect repair and postponed replacement, European Journal of Operational Research 274, 966-977.

Yeh, R. H., Chang, W. L. & Lo, H. C. (2010). Optimal threshold values of age and two-phase maintenance policy for leased equipments using age reduction method. Annals of Operations Research 181, 171-183.

Yildirim, M., Gebraeel, N. & Sun, X. (2017) Integrated predictive analytics & optimization for opportunistic maintenance and operations in wind farms. IEEE Transactions on Power Systems 32, 4319-4328.

Zahedi-Hosseini, F., Syntetos, A. & Scarf P.A. (2018) Optimisation of inspection policy for multi-line production systems. European Journal of Industrial Engineering 12, 233-251.

Zhao, X., Gaudoin, O., Doyen, L. & Xie, M. (2019) Optimal inspection and replacement policy based on experimental degradation data with covariates. IISE Transactions 51, 322-336.

Zhang, C., Gao, W., Yang, T. & Guo, S. (2019) Opportunistic maintenance strategy for wind turbines considering weather conditions and spare parts inventory management. Renewable Energy 133, 703-711.

Zhang, C., Gao, Y., Yang, L., Gao, Z. & Qi, J. (2020) Joint optimization of train scheduling and maintenance planning in a railway network: A heuristic algorithm using Lagrangian relaxation. Transportation Research Part B: Methodological 134, 64-92.

Zhang, F., Shen, J. & Ma, Y. (2020) Optimal maintenance policy considering imperfect repairs and non-constant probabilities of inspection errors. Reliability Engineering & System Safety 193, 106615.

Zhang, X. & Zeng, J. (2017) Joint optimization of condition-based opportunistic maintenance and spare parts provisioning policy in multiunit systems. European Journal of Operational Research 262, 479-498.