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Forecasting industrial production indices with a new singular spectrum analysis forecasting algorithm

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Existing time series analysis and forecasting approaches struggle to produce accurate results in application to time series with complex trend, such as those commonly displayed by indices of industrial production (IIPs). In this study, a new version of the Singular Spectrum Analysis (SSA) technique is developed, namely the Separate Trend and Seasonality (SSA-STS) forecasting algorithm. Its performance is compared to those of benchmark, classical times series forecasting methods, including Basic SSA (the core version of SSA), ARIMA, Exponential Smoothing (ETS) and Neural Network (NN). The methods in this study are applied to both simulated and real data. The latter includes twenty four monthly series of seasonally unadjusted IIPs of various sectors for the UK, Germany and France. Using the out-of-sample forecasts, the results of this newly developed SSA-STS algorithm were compared to the other aforementioned forecasting schemes by the means of pooled Root-Mean-Square-Error (RMSE). The pooling is done based on the number of steps ahead the forecasts extend, allowing for the performance of the methods to be evaluated on short and long horizons. The Kolmogorov-Smirnov Predictive Accuracy (KSPA) statistical test is applied to certify whether the errors produced by SSA-STS are statistically significantly smaller than those of all the benchmark methods. Since this new technique is based on separate trend and seasonality forecasting, it overcomes the difficulties in forecasting series with complex trends and seasonality, thus demonstrating a clear advantage over other methods in such particular cases.

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1. INTRODUCTION

Time series are thought to be composed of a combination of a trend, a periodic (referred to as seasonality if the frequency of oscillations follows a seasonal/monthly schedule), and random residuals. Components are attributed to these groups based on their frequencies. However, identifying these components correctly for real life time series (made up of ordered observations over time), that have not been generated from a linear recurrence relation (LRR), is not as evident as it may seem. Hence, the purpose of the Singular Spectrum Analysis (SSA) technique is to decompose time series into a sum of components with simple interpretable shapes and create a grouping that may be utilized for further actions, such as forecasting. There are many different modifications to the original Basic SSA methodology [12, 11]. Implementing the appropriate method, based on the particularities displayed by the time series under consideration, allows for a tailored approach, leading to better results.

A popular application for SSA are indices of industrial production (IIPs). These series are especially interesting, not only from an economic point of view, as they can be used as a proxy to evaluate the current state of the economy by policymakers, banks and economists in general, but also due to their particular structure. They often display complex trends with strong seasonal sub-cycles. This means that standard forecasting methods, such as ARIMA, struggle to accurately predict them, whereas SSA has an advantage due to its non-parametric nature. As shown in [16, 17], and further in this study, Basic SSA is simple to implement and indeed outperforms previously applied methods. However, applying it to such series can result in a poor reconstruction, as it fails to extract the tail-end seasonality correctly and does not capture the complexity of such trends [14], and therefore SSA with derivatives, or DerivSSA, is preferred [13]. The DerivSSA method improves separability of components by changing their contributions based on their frequency. This conveniently groups components into those that constitute to the trend and the seasonality, hence facilitating the creation of an accurate reconstruction, which could reflect as an improvement in subsequent forecasts via SSA, since they rely heavily on the quality of the reconstruction.

However, this approach is not tailored for forecasting. One may still encounter a similar problem when creating forecasts directly from the reconstructed series, as using a method that evaluates the series as a whole will not capture the eccentricities of a complex trend and seasonality components, thus failing to predict their different behaviors. Hence, there is some room for improvement.

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Therefore, the present paper is devoted to the development of a new version of SSA-type forecasting algorithm to address the need for a highly flexible method of accurately forecasting time series with complex trends and strong seasonality, such as the ones commonly seen in industrial production data. The new SSA-STS forecasting algorithm is based on utilising the SSA technique to separately forecast the trend and the periodic component of a series with the required form, then the combination should provide an accurate forecasts for both long and short horizons. The greater flexibility stems from the fact that users can select four parameters, as supposed to the two in the traditional SSA forecasting algorithms.

It is important to note that the conventional algorithms applied to the trend and seasonal component separately do not give benefits compared to the proposed algorithm.

This paper is organized as follows. In the following section, a preliminary review of the pertinent literature, necessary for the understanding of the research, is given. Section 3 details the new SSA-STS forecasting algorithm, as well as the ideas and methods used in its creation. In Section 4, a description of the data being utilised is provided. In Section 5, the algorithm is applied to simulated and real industrial production data, along with benchmark methods, and a comparison is made. Finally, Section 6 contains the conclusions drawn from the results of the research.

2. DEVELOPMENTS OF SSA AND RELEVANT LITERATURE

SSA is a novel and powerful non-parametric time series analysis and forecasting technique. This flexible methodology is based on the decomposition of the series into a sum of simple components, allowing for easy interpretation and facilitating the further construction of forecasts of periodic patterns in the presence of trends, see [12, 11]. It has been successfully employed in various areas, ranging from mathematics and signal processing to meteorology, genetics, economics and tourism research [20, 27]. A full review of the application of SSA for economic and financial time series is presented in [19]. The main advantage of SSA is that it can be used without making statistical assumptions, such as normality and stationarity of the data.

The evolution of ideas leading to SSA as we know it, started in the 18th century with a chain of key advancements in signal processing and time series analysis [8]. Some notable examples are, the spectral decomposition of time series [23, 22] and the embedding theorem [24, 29]. In general, [4] and [9] are regarded as the initial, official publications of SSA, although many findings in previous papers contributed to this development, and their authors may also be credited [6, 1, 32]. The work of Ghil and Vautard throughout the late 80s and early 90s further established the theory of SSA and its applications in climatology, [30, 31].

A completely separate approach and chain of developments took place in Russia during this time, culminating in a variant of SSA known as Caterpillar. This peculiar name originated from the analogy between a caterpillar's movement and the moving window utilised in this analysis. Although this was only publicised after the fall of the Soviet Union in [7], the precedent historical influences indicate that consistent work was carried out throughout the 70s, 80s and 90s (see, e.g. [3, 2, 5]).

In the following years up until the present, major developments on the methodology of SSA were achieved, detailed in [12, 14, 11]. Including, the development of the Rssa package in programming language R for fast implementation of the techniques has also widened the application of SSA to a variety of problems. Further modifications of the SSA technique have been steadily researched and introduced to cope with different problems encountered, for example the SSA-AMUSE method [15], which again facilitates the separation of components.

Some of the most widely studied and forecasted time series are IIPs, to which SSA and the benchmark methods used in this study, among others, have been applied. Industrial production data is commonly used by policymakers and economists as it is an important contributing factor to the gross domestic product (GDP) as a whole. Hence, great importance is given to the accuracy of IIP forecasts, since the ability to correctly predict their movement can prove to be highly beneficial. Extensive research has been carried out into the optimal forecasting method for IIPs.

For instance, the seasonality patterns of seasonally unadjusted series for eight components of real industrial production in Germany, France and the UK are investigated in [25]. Their findings show that seasonality accounts for over 80% of the variations in all series for the UK and Germany, except for vehicles. They also found stronger seasonality in France, due to declines in production in the summer in traditional industrial sectors.

Using the same data, the accuracy of NN forecasts was studied in [21] and it was concluded that, in general, NN models dominate linear models in the ability to predict the direction of change, but not in actual forecasting performance. Again, with the same data, SSA and ARIMA forecasts are compared in [16], where it was found that SSA produces more accurate forecasts than the other benchmark models and performs very well in predicting the direction of change.

In [26] vector ETS was applied to forecast several series, including that of aggregate industrial production and it was shown that the forecasting power quickly deteriorates as the horizon extends. This is unsurprising, since ETS relies on the recent history of the series, hence the error of this method increases with the horizon.

The proposed SSA-STS forecasting algorithm grasps ideas from the literature mentioned above, in particular from the works of [13, 14, 11] and extends upon it. Hence, contributing a method for forecasting specific series, with complex trends and strong seasonalities, by considering these components separately throughout the reconstruction and forecasting steps, allowing for different parameters to be utilised, thus improving the precision of results. This is particularly applicable to the forecasting of industrial production series, which typically display such characteristics.

We apply the new forecasting algorithm to the 24 time series of seasonally unadjusted monthly indices of industrial production (IIP) in Germany, France and the UK, and proceed to compare its performance against other benchmark methods mentioned in this review. The results are presented in a way that the forecasting accuracy can be evaluated for short and long horizons.

3. METHODOLOGY

The SSA methodology consists of a family of methods, such that the most suitable method can be chosen, depending on the structure of a given time series and researchers' needs. The general scheme of SSA consists of four steps: embedding, decomposition, grouping and reconstruction. The core version of the SSA methodology is called Basic SSA, which is briefly described below, along with other SSA modifications, including the new proposed SSA-STS forecasting algorithm.

3.1 Basic SSA

Let $x_1, \ldots, x_T$ be a time series of length T. Given a window length L $(1<L<T)$, we construct the L-lagged vectors $X_i=(x_i, \ldots, x_{i+L-1})^T$, $i = 1, 2, \ldots, K=T-L+1$, and compose these vectors into the matrix $\mathbf{X} = (x_{i+j-1})_{i,j=1}^{L,K} = [X_1 : \ldots : X_K]$. $\mathbf{X}$ is a Hankel matrix, meaning all the elements along its diagonal $i+j=$constant are equal.

The columns X_j of $\mathbf{X}$ can be considered as vectors belonging to the L-dimensional space $\mathbb{R}^L$. The singular-value decomposition (SVD) of the matrix $\mathbf{X}\mathbf{X}^T$ yields a collection of L eigenvalues and eigenvectors. For a given integer r, $r < L$, we choose the r largest eigenvalues and corresponding eigenvectors of $\mathbf{X}\mathbf{X}^T$. These r components of the SVD decomposition can be separated into several groups, e.g. a trend group and a periodic group. The chosen eigenvectors determine an r-dimensional subspace in $\mathbb{R}^L$; call this subspace S_r. The L-dimensional data $\{X_1, \ldots, X_K\}$ is then projected onto this r-dimensional subspace S_r and the subsequent averaging over the diagonals gives us some Hankel matrix $\tilde{\mathbf{X}}$, which we consider as an SSA reconstruction of $\mathbf{X}$. The time series corresponding to $\tilde{\mathbf{X}}$ is called the reconstructed series and usually serves as an estimator of the signal when the observed time series is noisy.

3.2 SSA forecasting

There are two ways of constructing forecasts based on the SSA decomposition of the series described above, see [12, Ch 2]. The most obvious way is to use the linear recurrent formula which the last terms of the series reconstructed from $\tilde{\mathbf{X}}$ satisfy. We however prefer to use the so-called 'SSA vector forecast' [12, Sect. 2.3.1]. The main idea of this forecasting algorithm is as follows. A selection of r eigenvectors of $\mathbf{X}\mathbf{X}^T$ leads to the creation of the subspace S_r. SVD properties allow us to assert that the L-dimensional vectors $\{X_1, \ldots, X_K\}$ lie close to this subspace. Consider the vectors $Z_1, \ldots, Z_K$, where Z_i is defined as the projection of X_i onto the subspace S_r. The vector forecasting algorithm then sequentially constructs the vectors $\{Z_{K+1}, Z_{K+2}, \ldots\}$ so that they stay in the chosen subspace S_r and the hankelization of the matrix $(Z_1, \ldots, Z_K, Z_{K+1}, Z_{K+2}, \ldots)$ gives the vector forecast.

3.3 DerivSSA

The application of Basic SSA to a given time series may lead to a poor separation [especially, at the tail-ends] of individual components in the SVD decomposition. To resolve this problem, DerivSSA was developed, which improves the separability of trend components from periodic ones, [11, Sect. 2.5]. Specifically, DerivSSA arranges the components based on their derivatives, such that periodic components with high frequencies are assigned a larger contribution and the trend components are placed to the end of the decomposition. The algorithm of DerivSSA is similar to that of Basic SSA, with the replacement of the matrix $\mathbf{X}$ by the matrix $\mathbf{X}_D = [X_1 : \ldots : X_K, X_2 - X_1 : \ldots : X_K - X_{K-1}]$.

3.4 SSA-STS forecasting algorithm

The new SSA-STS forecasting algorithm proposed in this study is based on separate trend and seasonality forecasting. Initially, perform DerivSSA on a time series with some choice of parameters L and r. The application of DerivSSA produces a list of components, where the first r_s components are grouped into the seasonal component and the following r_t are grouped into the trend component. Then, conducting separate forecasts for the seasonal component, by Basic SSA with some L_s and the same r_s, and for the trend component, by SSA with double centering with some L_t and r_t, [11, Sect. 2.3]. Finally, the sum of the two forecasts is taken as the final forecast of the given time series. The above description of the SSA-STS algorithm contains six parameters L, r, L_t, r_s, L_s, r_s, which provides a lot of freedom and flexibility. For reducing the computational complexity of the search for optimal values of parameters, we introduce the R code of the SSA-STS algorithm with four parameters `Lt,rt,Ls,rs` as follows.

```
s=ssa(TimeSeries,Ls)
      # Perform the SSA decomposition
deriv=fossa(s, nested.groups=c(1:(rs+rt)))
      # Perform DerivSSA
rec=reconstruct(deriv,
                groups=list(seasonality=1:rs,
                trend=((rs+1):(rs+rt))))
      #Obtain the trend and seasonal component
s_trend = ssa(rec$trend,Lt,
          column.projector=1,row.projector=1)
      # Perform trend decomposition by SSA
        with projections
f_trend=forecast(s_trend,groups=list(1:Lt),
                 method='vector',len=Nahead)
      # Forecasting the trend
s_seas = ssa(rec$seasonality,Ls)
      # Perform the SSA decomposition of the
        seasonal component
f_seas=forecast(s_seas,groups=list(1:rs),
                method='vector',len=Nahead)
      # Forecasting the seasonal component
ssa_sts_forecast=f_trend$mean+f_seas$mean
      # the sum of forecasts of the trend and
        the seasonal component
```

The individual steps of the SSA-STS algorithm are well-known, see e.g. [14, 11], however, to the best of our knowledge, such an algorithm has not been suggested earlier in the literature.

Our algorithm relies on the improved separability of components provided by DerivSSA to correctly extract the trend and periodic/seasonality. However, in the forecasting step, since there is no need to separate any further and sufficiently many components are used, Basic SSA can be applied without negative consequences. Note that, if a given time series has a relatively simple trend, Basic SSA yields a good decomposition, thus the forecasts produced by SSA-STS and Basic SSA are similar. We would expect the SSA-STS forecasts to be superior only in the presence of a complex trend in a time series.

3.5 Parameter choice

For successful application of SSA methodology, the choice of parameters L (window length) and r (number of components considered) is of utmost importance, as an appropriate selection can improve the separability of components of the SVD decomposition and further generate more accurate forecasts [10]. Hence, to compare the results of Basic SSA and SSA-STS forecasting algorithms it is critical that optimal parameters are selected for both methods. For the purpose of forecasting IIPs, it is a good idea to find parameters for both SSA-STS and Basic SSA algorithms on the basis of accuracy of retrospective forecasts, that is, cutting the time series at different points and minimizing the aggregated RMSE of residuals between the forecasts and the given time series.

3.6 Other benchmark methods

The benchmark forecasting algorithms to which SSA-STS is compared to in this paper are well-known methods which have been applied to a wide range of time series, including those of industrial production.

The traditional forecasting methods are autoregressive integrated moving average (ARIMA), exponential smoothing (ETS) and neural network (NN) models. ARIMA is a parametric method and makes several assumptions about the data (e.g. stationarity, normality etc.) which are often too stringent for the reality.

ETS is motivated by the idea that older observations have less forecasting power than newer ones, hence the weights used in the weighted sum for the prediction shrink exponentially with time. This works well for series without complex, changing trends or seasonality, however there is also a great risk of errors accumulating when using this technique.

In contrast, NN is a highly flexible machine learning algorithm which allows for non-linear relationships, however also requires a large amount of previously observed data to perform well. This, along with Basic SSA, may prove to be the best rivals to SSA-STS for forecasting IIPs, due to their adaptability.

4. THE DATA

Data availability statement: the data used in this study is taken from Eurostat, the official statistical agency of the European Community.

Eight major components of real industrial production in three European countries, France, Germany and the UK, are considered in this study. The time series data are seasonally unadjusted monthly indices in Food Products, Chemicals, Basic Metals, Fabricated Metals, Machinery, Electrical Machinery, Vehicles and Electricity/Gas (Utilities) industries. The selected industries are important and diverse, seen as the eight time series included account for at least half of total industrial production in each country. In all cases, the sample period ends in July 2019. However, the starting dates for data in the UK, Germany and France data are different and reflect the availability of consistent data from Eurostat. The same time series data, with smaller sample periods, which ended earlier, have been previously employed and studied by [25, 21, 16, 28]. Note that data after August 2019 is not included in the sample, since forecasting during the COVID-19 pandemic is an untractable task and would not provide fair comparisons.

Figure 1 presents the time series data used in this study. As can be seen from these graphs, the movements of all these series are dominated by seasonality. However, except Utilities (Electricity and Gas supply), they all have complex trend pattern with different periods of expansions and contractions, as well as a sharp decline in production in 2008/2009 due to the financial and sub-prime mortgage banking crisis.

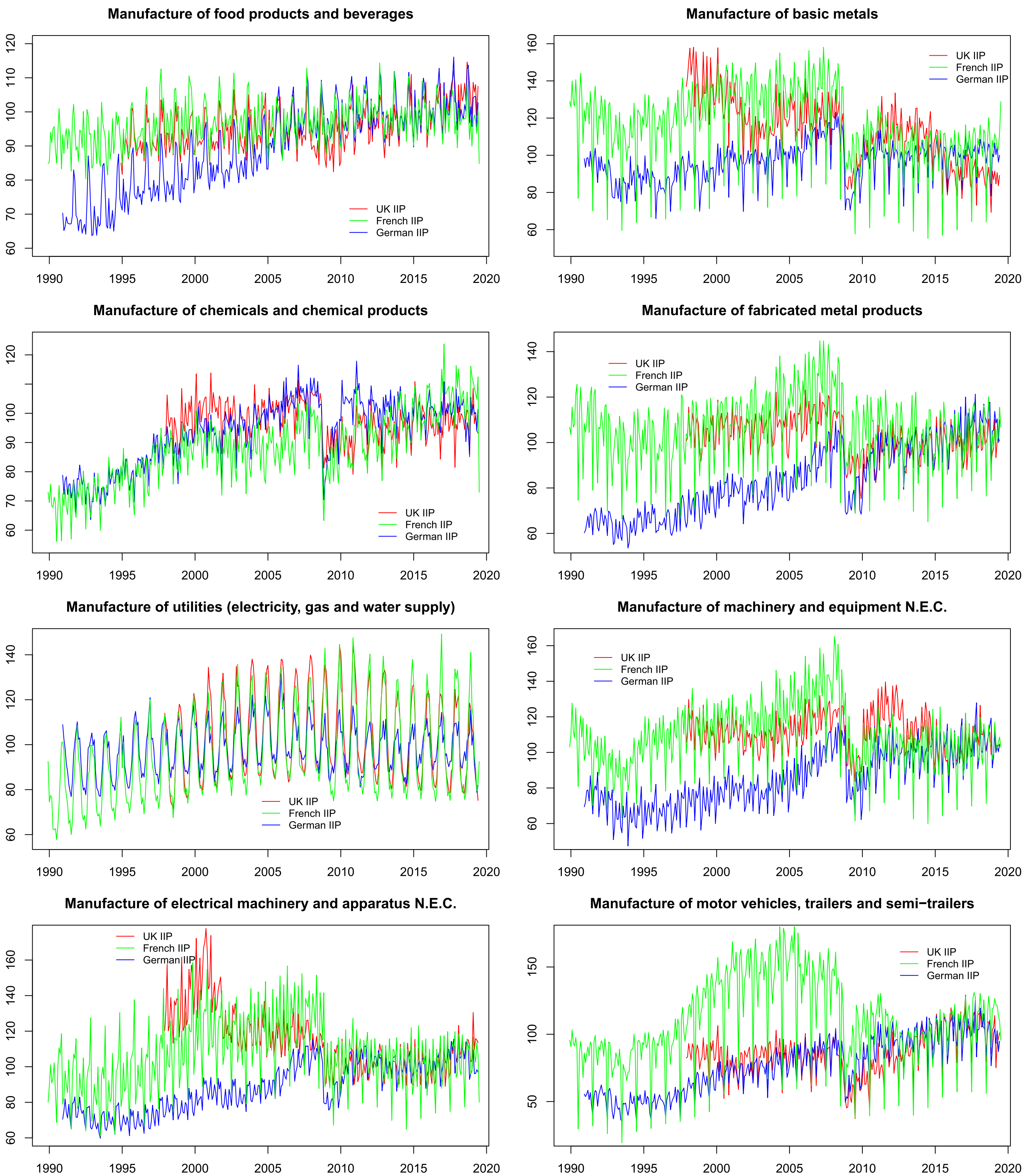

Figure 1. IIPs for Germany, France and the UK.

Table 1. Descriptive statistics of the production data

Series	Mean	Geom Mean	*S.D.*	R^2
UK				
Food	0.65	0.57	3.89	0.60
Basic Metals	-2.45	-3.22	11.71	0.50
Chemicals	0.13	-0.06	5.99	0.64
Fabricated Metal	-0.24	-0.49	6.88	0.64
Utilities	0.13	-0.02	5.46	0.84
Machinery	-0.46	-0.96	9.85	0.72
Electrical Equipment	-0.77	-1.08	7.78	0.76
Vehicles	0.84	-0.65	14.77	0.78
Germany				
Food	1.39	1.32	3.63	0.60
Basic Metals	0.13	-0.49	10.38	0.71
Chemicals	0.99	0.70	7.40	0.67
Fabricated Metal	1.82	1.42	8.58	0.58
Utilities	0.14	0.02	4.93	0.78
Machinery	1.12	0.59	9.82	0.82
Electrical Equipment	1.11	0.78	7.83	0.86
Vehicles	2.16	1.21	12.79	0.71
France				
Food	0.25	0.18	3.67	0.58
Basic Metals	-0.69	-1.05	8.01	0.96
Chemicals	1.55	1.34	6.31	0.75
Fabricated Metal	-0.18	-0.47	7.45	0.92
Utilities	1.03	0.86	5.93	0.81
Machinery	-0.28	-0.86	9.98	0.92
Electrical Equipment	0.34	0.04	7.40	0.93
Vehicles	1.01	-0.32	14.26	0.94

Mean, Geom Mean and S.D. are the arithmetic mean, geometric mean and standard deviation of the annual percentage change in the series over the sample period.

Table 1 contains information for each annual difference series and therefore refer to percentage changes in the original series. The first three columns are the sample arithmetic mean, geometric mean and standard deviation.

The sample means indicate the different growth/decline in production for these industrial sectors over the sample period, and it can be seen that some experienced a substantial rise during this period. In particular, production of Vehicles and Fabricated Metal in Germany show increase of 2.16% and 1.82% per year (based on arithmetic mean). However, others suffered declines, for instance, Basic Metal in the UK shows a sharp fall, with arithmetic average of 2.5% and geometric average of 3.2%.

The sample standard deviations indicate higher volatility for all the three countries in production for the Vehicles industrial sector, with the lowest volatility in Food products. This is expected and aligns with the economic theory of consumer elasticity of demand.

The fourth column in Table 1 shows the seasonal coefficient of determination, which is obtained by regressing the monthly changes in production data against twelve monthly dummies, computed via a regression. The results generally show stronger seasonality for the French industrial production series than those for the corresponding series of Germany and the UK. In particular, monthly dummy variables account for over 90% of the variations in Basic Metals, Fabricated Metals, Machinery and Vehicles in France, which are associated with declines in production during the summer for the traditional industrial sectors. The results are in line with those reported in [25] for the data period ending December 1995.

5. APPLICATION

This section tests the new SSA-STS forecasting algorithm against well known forecasting methods, including Basic SSA, ARIMA, ETS and NN, on simulated and real data.

5.1 Numerical examples on simulated data

We now turn to the main purpose of this paper, evaluating the performance of forecasts. We use the out-of-sample forecast Root Mean Square Error (RMSE) to measure performance and support the findings via the statistical Kolmogorov-Smirnov Predictive Accuracy (KSPA) test [18].

First, we apply the SSA-STS forecasting algorithm to artificially generated data that emulates the movement of an IIP, to ensure that the algorithms performance is satisfactory and indeed produces exceedingly accurate results in regard to the benchmark methods it is being compared to. Our interest centers on the treatment of the trend in forecasting. The simulated series consists of the sum of an appropriate trend with differing complexity, periodic pattern and noise.

Consider an artificial time series mimicking UK industrial production data, with observations spanning from January 1998 to July 2019. More specifically, the time series Iα, which was generated as the sum of a trend, the seasonal component of the form $5.7\cos(2\pi t/12)+6.4\cos(2\pi t/4+1.2)$ and the Gaussian white noise with variance of 1. The trend is defined by the vector of length 259, mirroring the number of observations in the IIP series for the UK, with the following form

$$100 + \alpha \text{ cumsum}(S_{-0.17,61}, S_{0.35,68}, S_{-3,11}, S_{1.6,25}, S_{-0.6,52}, S_{0.2,42}),$$

where $S_{z,k}$ is the vector of length k with all elements equal to z and 'cumsum' is the cumulative sum operator. Note that, the considered model of the trend has spline form, which is a very flexible model since the spline can nicely approximate any nonlinear trend. The parameter α, ranging from 0.0 to 1.0, indicates the degree to which the trend differs from the constant value of 100. The higher the value of α, the more complex the trend will be.

As previously mentioned, in our study we compare the forecasts obtained by five methods: SSA-STS, Basic SSA, ARIMA, ETS and NN. To asses these methods, out-of-sample forecasts are computed for the last 60 observations. That is, we compare the forecasts by computing the aggregated RMSE of retrospective 12-month ahead forecasts with cutting points from July 2014 to July 2018. We re-estimate parameters of ARIMA, ETS and NN forecasts at each cutting point using the functions `auto.arima`, `ets` and `nnetar` from the R package `forecast`. However, the same parameters were selected for SSA-STS and Basic SSA forecasting algorithms in the post-sample period, these parameters are chosen optimally on the basis of retrospective forecast accuracy. They are shown in Table 2, as well as that, we also choose parameters $L_t = 12$ and $r_t = 3$ for the SSA-STS forecasting algorithm and maintain throughout.

The sensitivity analysis [which is not presented here due to lack of space, but is available upon request] shows the accuracy of SSA-STS and Basic SSA forecasts weakly depends on the parameters chosen for these methods.

Table 2. Parameters of the SSA-STS and Basic SSA forecasting algorithms for the artificial time series $I\alpha$ with $\alpha = 0.1, 0.2, 0.4, 0.6, 0.8, 1$

	SSA-STS				Basic SSA	
	L	r	L_s	r_s	L	r
I0.1	132	6	36	4	96	6
I0.2	36	7	84	4	120	6
I0.4	48	9	84	4	36	8
I0.6	48	9	60	4	36	8
I0.8	60	10	36	4	36	8
I1.0	60	10	48	4	36	8

The aggregated RMSE for these forecasts are presented in Table 3. We can see that for $\alpha = 0.1$ or 0.2 [i.e. series with relatively constant trends] the Basic SSA and SSA-STS forecasts perform similarly, both better than the rest of the other methods, for the whole range of step ahead forecasts considered. However, as α increases and the trend becomes more complex, the SSA-STS forecast develops an advantage. The RMSE values for the other methods clearly growing at a faster rate than those of the SSA-STS forecasts, hence for the case of $\alpha \geq 0.4$ the SSA-STS forecast has a noticeably smaller RMSE, indicating that it is indeed more accurate than the other forecasting methods. However, a contender for best forecasting results is the ETS approach, for shorter (1 through 6) steps ahead. This method acquires a lower aggregate RMSE that SSA-STS for $\alpha = 0.8$ and 1. Although, it fails to outperform SSA-STS consistently, as it can be seen that the new SSA-STS algorithm dominates ETS for other α values, as well as for all longer horizon forecasts

Table 3. Aggregated RMSE for different step ahead forecasts for the artificial time series $I\alpha$ with $\alpha = 0.1, 0.2, 0.4, 0.6, 0.8, 1$ *by 5 forecasting algorithms*

	SSA-STS	Basic SSA	ARIMA	ETS	NN
Aggregated RMSE for $1, 2, \ldots, 6$-step ahead forecasts					
I0.1	1.10	1.09	1.28	1.16	1.20
I0.2	1.23	1.34	1.36	1.26	1.57
I0.4	1.34	1.62	1.55	1.48	1.78
I0.6	1.51	1.94	1.91	1.57	2.57
I0.8	1.75	2.29	2.10	1.69	3.71
I1.0	1.93	2.67	2.22	1.84	4.67
Aggregated RMSE for $7, 8, \ldots, 12$-step ahead forecasts					
I0.1	1.19	1.11	1.40	1.23	1.24
I0.2	1.34	1.26	1.56	1.41	1.54
I0.4	1.55	2.17	2.29	1.84	2.33
I0.6	1.78*	2.79	3.50	2.18	3.37
I0.8	2.02*	3.47	4.34	2.50	5.57
I1.0	2.31*	4.20	4.42	2.91	6.89

Note:* indicates that forecasting errors for a method is statistically significantly smaller than forecasting errors for all other methods by the one-sided KSPA test based on a p-value of 0.1.

(7 through 12 steps ahead). On the other hand, forecasts produced using ARIMA are considerably worse than the other methods, as ARIMA does not deal effectively with periodic patterns and nonlinear trends.

The KSPA test is carried out to verify whether the SSA-STS out-performance in terms of accuracy is statistically significant. We use this test instead of the traditional Diebold-Mariano (DM) test, due to the lack of necessary underlying assumptions. The KSPA test has been developed and applied in [18] to successfully test the predictive accuracy of SSA and ARIMA.

Indeed, as expected the KSPA one-sided test showed that for series with complex trends ($\alpha = 0.6, 0.8, 1.0$) in long horizons, the SSA-STS algorithm produces significantly smaller errors than the rest of the benchmark methods applied. These results from the statistical test corroborate the claim that SSA-STS dominates forecasting in such cases.

5.2 Forecasting of IIPs

We now consider eight time series of real industrial production for Germany, France and the UK. All series end in July 2019. However, the time series data for the UK starts in January 1998, January 1991 for Germany and in January 1990 for France. Data up to July 2014 is used in the estimation, and the post-sample forecast RMSE is used to measure performance, which is then tested by the one-sided KSPA test. We estimate parameters of ARIMA, ETS and NN forecasts at each cutting point, as mentioned in the previous section. However, only one choice of parameters for the SSA-STS and Basic SSA forecasting algorithms is maintained for all cutting points. These parameters are shown in Table 4, additionally parameters $L_t = 12$ and $r_t = 3$ were chosen for the analysis of the trend in the SSA-STS forecasting algorithm.

Table 4. Parameters of the SSA-STS and Basic SSA forecasting algorithms for 8 IPIs for 3 countries

	SSA-STS				Basic SSA	
	L	r	L_s	r_s	L	r
UK						
Food	144	20	84	18	84	20
Basic Metals	108	15	60	13	60	12
Chemicals	60	22	60	20	72	23
Fabricated Metals	48	22	48	20	48	19
Utilities	48	6	36	5	36	6
Machinery	120	20	48	16	36	13
Electrical Equipment	120	26	24	17	60	18
Vehicles	144	20	36	16	60	17
Germany						
Food	84	28	84	26	84	20
Basic Metals	48	18	48	16	60	17
Chemicals	48	21	60	19	48	21
Fabricated Metals	36	27	72	22	48	16
Utilities	60	24	60	23	36	16
Machinery	36	16	84	14	36	16
Electrical Equipment	48	18	36	16	36	13
Vehicles	96	24	72	17	108	8
France						
Food	36	21	72	18	84	24
Basic Metals	48	19	72	16	72	22
Chemicals	60	20	72	18	72	20
Fabricated Metals	72	20	24	16	72	15
Utilities	36	9	36	8	36	9
Machinery	60	21	48	19	60	16
Electrical Equipment	60	20	36	17	72	14
Vehicles	48	17	48	14	120	14

We compute the aggregated RMSE of retrospective 12-month ahead forecasts with cutting points at each month between July 2014 and July 2018 to evaluate the forecasting performance of the five methods at different horizons. Table 5 and Table 6 present the RMSE results of the five methods for all the three countries and eight sectors, for short horizon (1 to 6 months ahead) and long horizon (7 to 12 months ahead) forecasts, respectively.

A summary RMSE Ratio (RRMSE) is also computed as the average ratio of the RMSE of the SSA-STS to that of other models for each country, and overall. Thus, a ratio of less than one is an indication that the SSA-STS model produces less errors on average for each country. RRMSE (overall) is also computed for all 24 series. The Score (overall) is a score of the number of times out of 24 that each model yields the lower RRMSE.

Table 5. Aggregated RMSE for $1, 2, \ldots, 6$-step ahead forecasts for 8 IIPs for 3 countries

	SSA-STS	Basic SSA	ARIMA	ETS	NN
UK					
Food	2.46	2.45	3.31	3.44	3.34
Basic Metals	5.36*	6.41	6.59	6.42	9.37
Chemicals	3.22	3.67	3.95	3.74	4.30
Fabricated Metal	4.24*	4.76	5.03	5.30	4.82
Utilities	4.44	5.04	4.86	4.76	5.24
Machinery	5.85	9.03	6.81	6.75	7.38
Electrical Equipment	4.67*	5.22	5.29	4.94	6.06
Vehicles	5.41	6.16	6.77	5.56	7.51
RRMSE		**0.86**	**0.83**	**0.87**	**0.76**
Germany					
Food	2.50	2.54	2.92	3.09	2.73
Basic Metals	3.16	3.66	3.18	3.25	3.57
Chemicals	3.01*	3.32	3.95	3.63	3.63
Fabricated Metal	3.48*	4.68	4.46	4.90	7.05
Utilities	3.15	3.10	2.99	4.47	3.88
Machinery	3.85*	4.49	4.70	5.47	6.54
Electrical Equipment	2.11	2.46	2.03	2.29	3.56
Vehicles	7.56	8.36	7.20	7.33	11.26
RRMSE		**0.89**	**0.92**	**0.84**	**0.72**
France					
Food	1.98	2.14	2.74	3.17	2.33
Basic Metals	3.46	3.52	3.65	3.55	3.66
Chemicals	4.44	4.60	4.45	4.65	5.53
Fabricated Metal	4.34	4.71	5.36	5.18	4.96
Utilities	5.22	5.38	4.94	6.14	5.76
Machinery	5.44*	5.83	6.13	6.18	7.06
Electrical Equipment	3.47*	3.99	4.27	4.33	4.56
Vehicles	6.75*	7.84	7.73	7.83	8.85
RRMSE		**0.93**	**0.89**	**0.85**	**0.83**
RRMSE (overall)		0.89	0.88	0.85	0.77
Score (overall)	19	1	4	0	0

Note:* indicates that forecasting errors for a method is significantly smaller than errors for all other methods by the one-sided KSPA test based on a p-value of 0.1.

In terms of producing smaller RMSE, the results in Tables 5 and 6 provide strong evidence that the SSA-STS model is generally superior to Basic SSA, ARIMA, ETS and NN models for all the three countries. Overall, the results

Table 6. Aggregated RMSE for $7, 8, \ldots, 12$-step ahead forecasts for 8 IIPs for 3 countries

	SSA-STS	Basic SSA	ARIMA	ETS	NN
UK					
Food	2.67	2.73	4.24	3.82	4.72
Basic Metals	6.97*	7.78	10.10	9.69	13.03
Chemicals	3.23*	3.88	5.21	4.41	5.27
Fabricated Metal	4.52	6.14	5.39	5.23	5.37
Utilities	5.13	5.11	5.16	5.47	6.11
Machinery	6.30	11.53	8.26	8.97	11.53
Electrical Equipment	5.03*	6.66	7.36	5.89	7.56
Vehicles	6.84*	7.88	9.44	7.22	9.64
RRMSE		**0.83**	**0.74**	**0.81**	**0.66**
Germany					
Food	2.53	2.60	3.32	3.34	3.02
Basic Metals	3.32	3.84	3.13	3.39	4.43
Chemicals	3.60	3.67	5.19	3.99	4.58
Fabricated Metal	3.98*	4.59	4.99	5.49	11.10
Utilities	3.353.57	3.27	4.47	4.37	
Machinery	4.12*	4.71	4.62	5.76	8.57
Electrical Equipment	3.38*	3.73	3.46	4.08	4.92
Vehicles	8.28	9.18	8.59	8.26	16.36
RRMSE		**0.91**	**0.90**	**0.83**	**0.65**
France					
Food	2.18*	2.14	3.00	3.28	2.40
Basic Metals	3.53*	3.96	3.99	3.92	4.49
Chemicals	4.27*	4.47	5.10	5.22	6.45
Fabricated Metal	4.21*	4.78	5.83	5.24	5.91
Utilities	5.20	5.56	4.92	5.28	5.04
Machinery	5.57	5.94	5.76	6.09	7.83
Electrical Eqipment	3.75*	5.01	4.86	4.84	5.08
Vehicles	7.13*	8.00	8.76	8.60	11.90
RRMSE		**0.91**	**0.85**	**0.84**	**0.77**
RRMSE (overall)		0.88	0.83	0.83	0.69
Score (overall)	18	2	3	1	0

Note:* indicates that forecasting errors for a method is significantly smaller than errors for all other methods by the one-sided KSPA test based on a p-value of 0.1.

show that for 1 to 6 step ahead forecasts, SSA-STS outperformed Basic SSA, ARIMA, ETS and NN by 11%, 12%, 15% and 23%, whereas for 7 to 12 step ahead the difference was 12%, 17%, 17% and 31%, respectively. Hence, for longer horizons, SSA-STS performance exceeds all other methods by a larger margin. Although the Score (overall) statistics corroborates these results, by indicating that the SSA-STS model produces lower RMSE for 19 and 18 cases out of the 24 (in aggregate for 1–6 and 7–12 step ahead forecasts respectively), one would expect the statistics to be switched if they were to support the findings that SSA-STS performs better for longer horizon forecasts.

Although, at face value the predictive accuracy advantage of SSA-STS is clear, we perform the one-sided KSPA test to ensure the results are statistically significant. Based on the 0.1 p-value, we see in Tables 5 and 6 that the SSA-STS method consistently had statistically significant lower errors for many IIPs over both or some horizons. As in the simulation study, for the real time series SSA-STS proved to be particularly superior when forecasting over longer horizons for time series with complex trends, in particular, for many of the French IIPs.

5.2.1 Depictions for UK IIPs

We show graphically the UK IIPs 12-month forecasts created at each cutting point in different colours for each forecasting scheme in Figure 2. Hence, the amount of deviation, or spread of the colourful lines from the reconstructed series (in black) confirms the adequacy of the forecasting methods deduced from the pooled RMSE results.

For instance, the Utilities IIP shows most of the forecasts coinciding for all the forecasting methods, as expected due to its simple trend and consistent periodic movement.

Whereas, for more complex IIPs, such as Chemicals and Machinery and Equipment, we observe the advantage of SSA-STS quite clearly. Since its retrospective forecasts concur, indicating that this method would be a reliable way of accurately forecasting these series, when compared to the benchmark methods. Interestingly, for the series of Machinery and Equipment the trend is especially complex and there seems to be some type of structural break around 2014, hence the benchmark methods fail to account for this. The steep red lines which are increasing when the true reconstruction declines (in black) depicts this problem precisely.

6. CONCLUSIONS

Building on the work in [13, 14, 11], we develop a greatly flexible SSA-STS forecasting algorithm for the application to series with complex trends and the presence of seasonality, such as commonly seen in IIP data. Its performance is investigated against that of popular methods, including Basic SSA, ARIMA, ETS and NN, on real and simulated data. The real data used consists of IIPs of eight major sectors in Germany, France and the UK. In the simulation study a series of similar form is considered, but allowing for variation in trend complexity. The superiority of SSA-STS is evident on both sets of data, with some minor exceptions. Those being in cases where there is no trend complexity, which eradicates the SSA-STS algorithm's advantage. A good example of such a series is the Utility IIPs, that have more systematic behavior than the majority of the other series, due to very

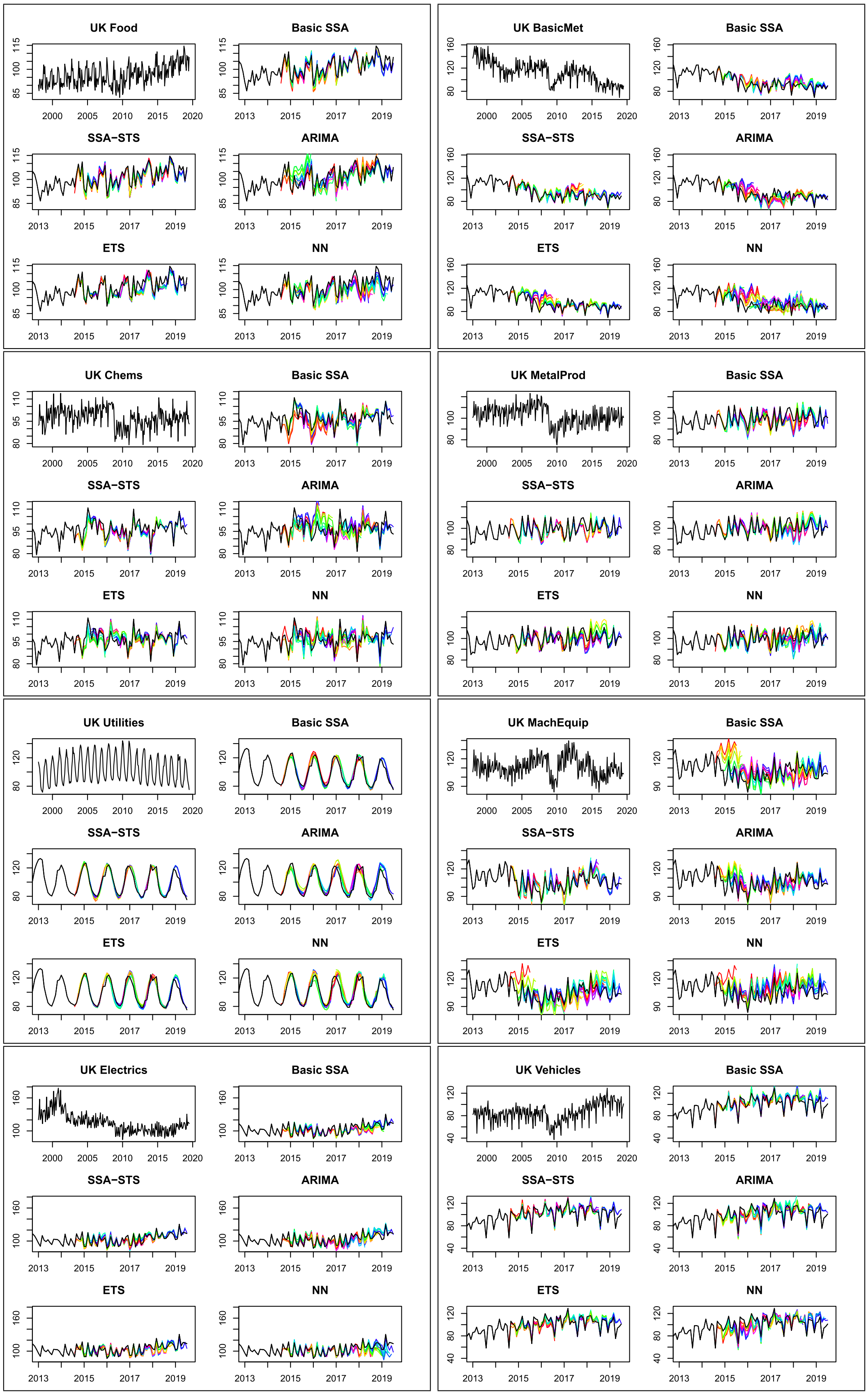

Figure 2. 8 UK IIPs and retrospective 12-month ahead forecasts by 5 methods.

slight variations in the periodic annually and the absence of a trend and structural breaks. Hence, for such series, the other methods may create similar or preferable results than the SSA-STS algorithm.

Since the dominance of SSA-STS stems from the separate evaluation of the trend and seasonal sub-cycles, this entails a larger set of parameter options. Hence, an important constituent for the success of the SSA-STS algorithm is the correct selection of parameters L, r, L_s, r_s, L_t, r_t, as for many SSA-type methods this choice can determine the performance.

At a more general level, we can conclude that the SSA-STS forecasting algorithm outperforms other methods in instances where a trend of complex shape is present. This is indicated during the development stages of the algorithm, as forecasting the trend and periodic separately would suggest that forecasting each in isolation enhances the final result, especially in the case of a complex trend. This is substantiated by the results from the retrospective forecasts discussed. Since the majority of industrial production indicators have complex trend, the SSA-STS algorithm is a favorable forecasting method for such data. However, it can also be successfully applied for forecasting many other types of real data, and in particular recommended for forecasting series with trend complexities and structural breaks.

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