New Formulation of Cardiff Behavioral Model Including DC Bias Voltage Dependence

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Abstract—A new mathematical formulation for the Cardiff nonlinear behavioral model is presented in this work which includes the DC bias voltages (drain and gate) into the model. It has been verified by modelling a GaN on SiC high electron mobility transistor (HEMT) at 3.5 GHz. For the case presented, interpolation of load-pull data has resulted in more than 90 % reduction in the density of the load-pull data required to generate a nonlinear behavioral model over a wide range of DC bias points.

Index Terms— Load-pull, Behavioral model, Power amplifier, Nonlinear device modelling

I. Introduction

Advancement in large-signal and waveform measurement systems[1], [2] has created an opportunity for direct utilization of measurement data (specifically load-pull measurement) into the computer-aided design (CAD) environment [3]. However, the generality requirement of advanced radiofrequency power amplifier (RFPA) designs demands load-pull data under various variables such as frequency, input power, and DC bias voltage which can be very time-consuming. Therefore, it is critical to adopt a strategy to reduce the measurement intensity and in doing so, reduce measurement time. One approach to reducing the density of the required load-pull data is to use an accurate and reliable nonlinear behavioral model to interpolate the data.

Current industry-leading nonlinear behavioral models are Cardiff University's Cardiff behavioral model [4] and Keysight's X-parameters [5]. Regarding the Cardiff model, to enhance its generality, variables such as frequency, transistor size, input power, etc. have been previously included into its mathematical formulation [6], [7]. In terms of DC bias, however, in the current formulation of Cardiff model [8] (and also X-parameters [9]) the DC bias voltages are treated as independent variables; hence, load-pull measurements at each DC bias voltage are required to generate a bias-dependent model. The CAD tools can be allowed perform simple interpolation.

Our previous work in [10] identifies the linear relationship between the Cardiff model coefficients and the gate voltage 'V_{gs}^{DC}' sweep and proposes a new Cardiff model formulation to include the V_{gs}^{DC} into the Cardiff model. In this paper, we present a new expansion for the Cardiff model to include both gate 'V_{gs}^{DC}' and drain 'V_{ds}^{DC}' bias variation while maintaining a rigorous, physics-based formulation underpinned by mixing theory. The new model is verified against experimental data collected on a GaN-on-SiC HEMT device, where a polynomial order of '3' for the 'V_{ds}^{DC}' variation was determined empirically.

II. Incorporation of the DC Bias Voltage into the Cardiff Model's Formulation

Mathematical development of the Cardiff behavioral model was based on the general mixing theory to account for the fact that when multiple harmonically related stimuli are injected into a multiport nonlinear system they interact ('mix') [11]. Polynomial expansion of the model has been developed around a limited operational domain about a large signal operating point (LSOP), for example at a fixed DC bias point, RF input drive level and frequency.

Equation (1) shows the general mathematical formulation of the Cardiff model, in the admittance domain, for fundamental load-pull only at a fixed input drive, DC bias condition and frequency.

$$I_{p,h} = (\angle V_{1,1})^h \sum_{r=0}^{1} \sum_{n=n_{min}}^{n_{max}} D_{p,h,m,n} |V_{2,1}|^m \left(\frac{\angle V_{2,1}}{\angle V_{1,1}}\right)^n$$

$$\begin{cases} n_{min} = -\left(w - h/2 - r\right) \\ n_{max} = h + \left(w - h/2 - r\right) \end{cases} \quad (1)$$

The 'p' is the port index and 'h' is the harmonic index, and 'm' and 'n' denote the coefficient related stimulus phasor ($V_{2,1}$) magnitude and phase exponents, respectively. The 'm' and 'n' are related as '$m = |n| + 2r$' where 'r' is the magnitude indexing term and its maximum value is usually restricted to '1'. The parameter 'w' represent the mixing order.

From a physics basis, it is logical to describe the device behavior in the admittance domain (current response as a function stimulus voltage). However, as RF systems measure incident 'A' and reflected 'B' travelling waves [12], it is common to define the device behavior in the travelling-wave domain (A, B domain):

$$B_{p,h} = (\angle A_{1,1})^h \sum_{r=0}^{1} \sum_{n=n_{min}}^{n_{max}} K_{p,h,m,n} |A_{2,1}|^m \left(\frac{\angle A_{2,1}}{\angle A_{1,1}}\right)^n \quad (2)$$

where for DC current ($h = 0$) the term '$B_{2,0} \triangleq I_{2,0}$'.

Note, the DC bias voltages are not included in the Cardiff model formulation in (1). Therefore, to generate a bias-dependent model, load-pull measurements at each DC bias condition are required.

To include both RF and DC bias voltages (v^{rf} and V^{DC}) into the Cardiff model formulation, a general mixing equation needs to be initially considered:

$$I = \sum_{w=0}^{w=order} \sum_{u=0}^{u=w} C_{u,(w-u)} \left(V_{gs}^{DC} + v_{gs}^{rf}\right)^u \left(V_{ds}^{DC} + v_{ds}^{rf}\right)^{(w-u)} \quad (3)$$

Equation (3) can be transformed to (4); separating out the RF and DC stimulus terms at both input and output of the device.

$$I = \sum_{w=0}^{w=order} \sum_{u=0}^{u=w} C_{u,(w-u)} \cdots$$
$$\left\{ \left[\sum_{x=0}^{x=u} C_x^{gs} \left(V_{gs}^{DC}\right)^x \left(v_{gs}^{rf}\right)^{u-x} \right] \left[\sum_{y=0}^{y=(w-u)} C_y^{ds} \left(V_{ds}^{DC}\right)^y \left(v_{ds}^{rf}\right)^{(w-u)-y} \right] \right\} \quad (4)$$

To be noted that now, two new sets of model coefficients 'C_x^{gs}' and 'C_y^{ds}' are added to model the mixing relationship of the RF and DC at the input and output of the device, respectively.

By further rearranging (4), it is possible to separate the RF and DC stimulus terms, separate mixing order can be used for RF and DC terms as:

$$I = \sum_{y=0}^{y=w} \left\{ \sum_{u=0}^{u_{max}} \sum_{v=u}^{v_{max}} G_{y,u,(v-u)} \left(V_{gs}^{DC}\right)^u \left(V_{ds}^{DC}\right)^{(v-u)} \right\} \left(v_{ds}^{rf}\right)^y \quad (5)$$

Since v_{gs}^{rf} is a constant it is no longer included in the formulation. Hence, '$G_{y,u,(v-u)}$' are the general coefficients including both DC and RF stimulus at a fixed input drive level ($v_{gs}^{rf} = constant$), and v_{max} and u_{max} are the polynomial fitting order for V_{ds}^{DC} and V_{gs}^{DC}, respectively.

The behavioural model formulation for the RF stimulus has been previously covered in several publications such as in [4], [7] and [13]. Therefore, by replacing the RF part of the (5) with the admittance domain Cardiff model formulation, a new Cardiff model formulation can be generated, as shown in (6), where both V_{gs}^{DC} and V_{ds}^{DC} are included into the model formulation.

$$I_{p,h} = (\angle V_{1,1})^h \sum_{r=0}^{1} \sum_{n=n_{min}}^{n_{max}} \cdots$$
$$\left\{ \sum_{u=0}^{u_{max}} \sum_{v=u}^{v_{max}} R_{p,h,m,n,u,(v-u)} \left(V_{gs}^{DC}\right)^u \left(V_{ds}^{DC}\right)^{(v-u)} \right\} \cdots \quad (6)$$
$$|V_{2,1}|^m \left(\frac{\angle V_{2,1}}{\angle V_{1,1}}\right)^n$$

Where '$R_{p,h,m,n,u,(v-u)}$' is the new model coefficient which is independent of the DC bias voltage. Equation (7) shows the new DC-inclusive Cardiff model, after transforming the RF part to the A, B domain which is more consistent with the experimental data. We recognise in this transformation the model coefficients '$L_{p,h,m,n,u,(v-u)}$' and complexity might be different from the admittance model's equivalent.

$$B_{p,h} = (\angle A_{1,1})^h \sum_{r=0}^{1} \sum_{n=n_{min}}^{n_{max}} \cdots$$
$$\left\{ \sum_{u=0}^{u=u_{max}} \sum_{v=u}^{v_{max}} L_{p,h,m,n,u,(v-u)} \left(V_{gs}^{DC}\right)^u \left(V_{ds}^{DC}\right)^{(v-u)} \right\} \cdots \quad (7)$$
$$|A_{2,1}|^m \left(\frac{\angle A_{2,1}}{\angle A_{1,1}}\right)^n$$

III. Experimental Verification

Load-pull measurements were conducted statically using CW excitation at a fundamental frequency of 3.5 GHz. Fig.1 shows the photograph of the real-time active load-pull measurement system used for this experiment, based on a PNA-X analyzer. Harmonics were terminated at 50 Ω.

The device under test (DUT) was a 4 W GaN-on-SiC HEMT from Ampleon. The drain voltage of the DUT was swept from 20 V to 50 V with a step of 3 V, while the gate bias was swept from -3.0 V to -2.0 V with 0.1 V step. For this experiment, 91 load points were measured configured to capture the optimum load points at all V_{ds}^{DC} voltages. Compression is 1 dB at 50 V and around 3 dB at 20 V.
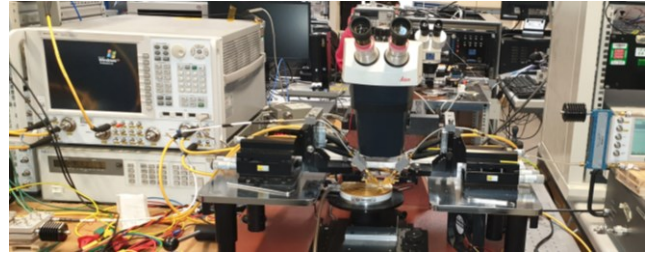

Fig. 1. Photograph of the active load-pull system.

Referring to (7), to use the model formulation, one needs to set the RF and DC mixing orders. In terms of RF mixing order (w), usually mixing order of 5 [14] is used to model the fundamental load-pull data. With regards to DC, the work in [10] shows a first-order polynomial can be used to accurately include the V_{gs}^{DC} into the model's formulation ($u_{max} = 1$). However, the relationship between the Cardiff model coefficients with V_{ds}^{DC} sweep has not been previously investigated. This will now be undertaken to identify 'v_{max}'.

A. Response of Cardiff model coefficients to V_{ds}^{DC} sweep.

To capture the relationship between the model coefficients and the V_{ds}^{DC} sweep, firstly, the model coefficients '$K_{p,h,m,n}$' were extracted from a 5th order conventional Cardiff model (2) at various V_{ds}^{DC} while V_{gs}^{DC}=-2.3 V. Secondly, both the imaginary and real parts of the coefficients were plotted against V_{ds}^{DC} as shown in Fig. 2 for $K_{2,0,m,n}$ and $K_{2,1,m,n}$.

Referencing to Fig. 2, both imaginary and real parts of the coefficients can be approximated reasonably well with a third-order polynomial function ($v_{max} = 3$).

B. Verification of including V_{ds}^{DC} into the Cardiff model.

As a first verification, the model for V_{ds}^{DC} only sweep ($u_{max} = 0$) at V_{gs}^{DC} =-2.3 V has been validated. The

measurement data at V_{ds}^{DC}=20, 29, 41, 50 V only has been used to extract the '$L_{p,h,m,n,u,(v-u)}$' coefficients.
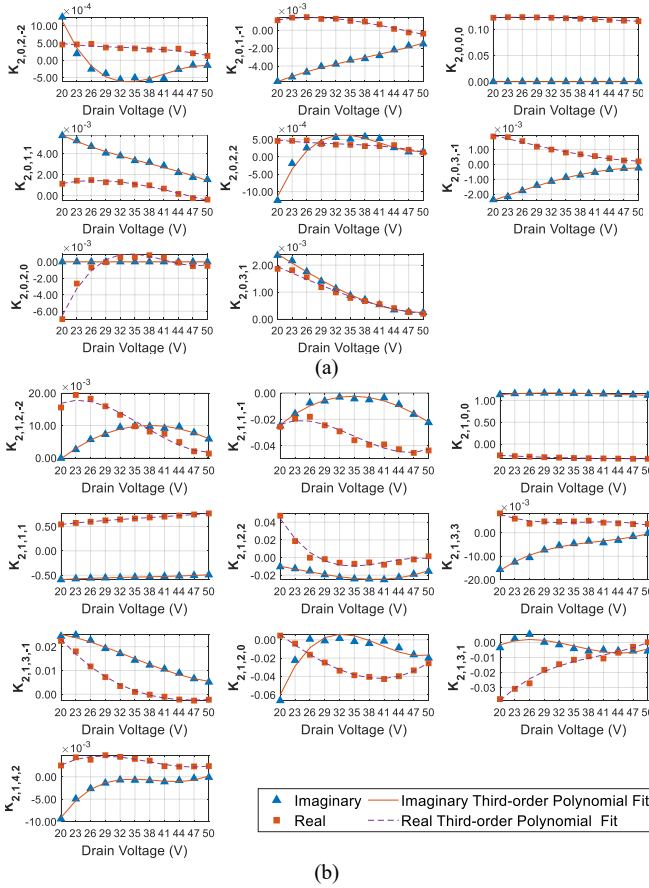

Fig. 2. Real and imaginary part of the model's coefficients (a) $K_{2,0,m,n}$ and (b) $K_{2,1,m,n}$ vs V_{ds}^{DC} (symbols). Third polynomial fitting (lines).

Fig. 3 shows the calculated normalized mean squared error NMSE (dB) (8) [15] value over the complete V_{ds}^{DC} set at V_{gs}^{DC} =-2.3 V. Using the new Cardiff model formulation, load-pull data can be accurately predicted over a range of V_{ds}^{DC} sweep with NMSE values less than -37 dB and -44 dB for $B_{2,0}$ and $B_{2,1}$, respectively.
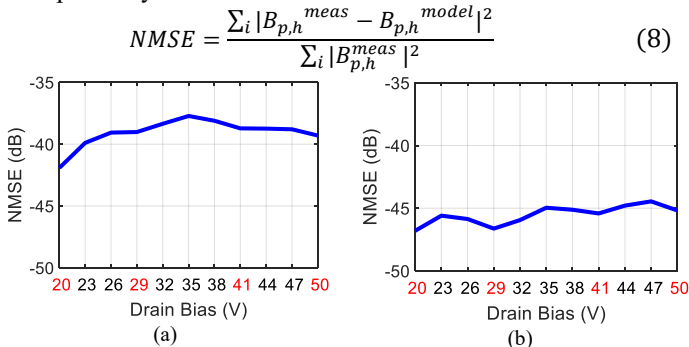
$$NMSE = \frac{\sum_i |B_{p,h}^{meas} - B_{p,h}^{model}|^2}{\sum_i |B_{p,h}^{meas}|^2} \qquad (8)$$

Fig. 3. Calculated NMSE (dB) for (a) $B_{2,0}$ and (b) $B_{2,1}$ at different V_{ds}^{DC} level. Only the V_{ds}^{DC} points highlighted in 'red' were used for model extraction.

C. Verification of Bias-dependent Cardiff Model

To verify the inclusion of both V_{gs}^{DC} and V_{ds}^{DC}, the model was extracted using a subset of bias points (V_{ds}^{DC}= 20, 29, 41, 50 V and V_{gs}^{DC}=-2.0, -2.5, -3.0 V) and validated on the full set of measurement data. Fig. 4 shows the NMSE for $B_{2,1}$.
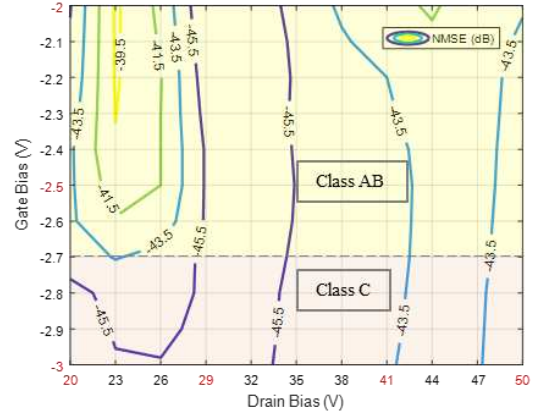

Fig. 4. NMSE (dB) of B_{21} at different DC bias conditions. The NMSE is better than -39.5 dB. The pinch-off voltage is at $V_{gs}^{DC} = -2.7\,V$. Only the bias points highlighted in "red" were used for model extraction.

The new Cardiff model formulation (7) is capable to accurately predict the load-pull data with an NMSE better than -39.5 dB across all the $B_{2,1}$ datasets, and -37 dB for $B_{2,0}$. Note, the pinch-off voltage is at V_{gs}^{DC}=-2.7 V and load-pull data have been successfully modelled from class AB to class C bias conditions. Fig. 5 compares measured and predicted power and efficiency contours at V_{ds}^{DC}=23 V and $V_{gs}^{DC} = -2.3\,V$ (DC bias condition with the lowest accuracy).
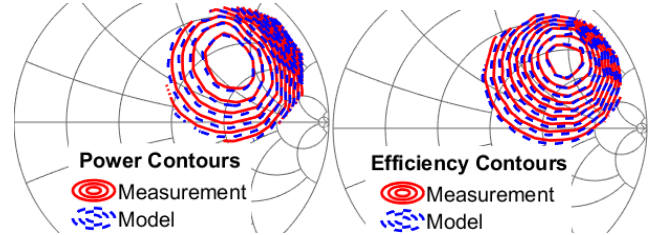

Fig. 5. Comparison of measured and interpolated load-pull power contours (0.5 dBm step from the maximum of 31.3 dBm) and efficiency contours (0.5 % step from a maximum of 61.8 %) at $V_{ds}^{DC} = 23\,V$ and $V_{gs}^{DC} = -2.3\,V$.

In this example, using the load-pull data of only 12 bias points (4 V_{ds}^{DC} points and 3 V_{gs}^{DC} points), the load-pull data of 121 bias points (11 V_{ds}^{DC} points and 11 V_{gs}^{DC} points) is predicted accurately. As the new formulation is capable to interpolate the load-pull data with respect to DC bias, compared to the conventional Cardiff model formulation (2), the total number of required load-pull data points have significantly reduced from 11011 points (91 load points at 121 DC bias points) to only 1092 points. That is, in this case, more than a 90% reduction in the load-pull measurement density.

IV. CONCLUSION

In this paper, the Cardiff model has been reformulated to incorporate both gate and drain bias variations. The model accuracy in the interpolation domain has been verified, using measurement data of a real device, and the NMSE is below -37 dB for all data points. The work shows that the dataset needed to accurately model this measurement space can be reduced by 90 % while still maintaining good accuracy.

V. ACKNOWLEDGEMENT

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