Low-Rank O-Space Reconstruction

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Synopsis

Low-Rank O-Space presents a scheme to incorporate O-Space imaging with Low-Rank matrix recovery. The Low-Rank reconstruction based on iterative nonlinear conjugate gradient algorithm is applied to substitute the previous Kaczmarz and Compressed Sensing (CS) reconstructions to recover highly undersampled O-Space data. The simulations and experiments illustrate the proposed scheme can remove artifacts and noise in O-Space imaging at high reduction factors, compared to results recovered by Kaczmarz and CS. Moreover, the proposed method does not need to modify the conventional O-Space pulse sequences, and reconstruction results are better than those in radial imaging recovered by Kaczmarz, CS, or Low-Rank methods.

Audience

Researchers interested in parallel imaging, nonlinear gradient encoding, or low-rank matrix recovery

Purpose

Nonlinear spatial encoding magnetic fields (SEMs), such as those used in O-Space imaging [1,2], have been shown to improve image reconstructions under high acceleration factors. Low-Rank reconstruction [3-7], based on the development of Low-Rank matrix completion in Compressed Sensing (CS) theory [8], has been shown to provide excellent image recovery from reduced data sets when applied to appropriate sampling in k-space. In this paper, we present a scheme to incorporate O-Space imaging with Low-Rank matrix recovery. The simulations and phantom experiments illustrate that the proposed scheme can greatly remove artifacts and noise in O-Space imaging at high reduction factors, compared to Kaczmarz [1,2] and CS reconstructions [9,10].

Theory

Neglecting relaxation effects, the signal s_q from the q-th RF channel can be expressed as:

$$s_q = \int_{\omega} m(\boldsymbol{x}) C_q(\boldsymbol{x}) e^{-i\phi(\boldsymbol{x},t)} d\boldsymbol{x},$$

where $m(x)$ is the magnetization at location $\boldsymbol{x} = (x, y, z)$, $C_q(x)$ is the sensitivity of q-th coil, and the integral is over ω, which is the region of interest; $\phi(x,t)$ is the spatially dependent encoding phase. For the O-Space echo corresponding to the l-th center placement (CP) at (x_l, y_l), the spatially dependent encoding phase $\phi(x,t)$ of the signal equation becomes, which is

$$\phi_l(\boldsymbol{x},t) = k_x(t)x + k_y(t)y - 12k_{z2}(t)((x - x_l)^2 + (y - y_l)^2)$$

where

$$k_x(t) = \gamma \int_{t0} Gx(\tau) d\tau,$$
$$k_y(t) = \gamma \int_{t0} Gy(\tau) d\tau,$$

and

$$k_{z2}(t) = \gamma \int_{t0} Gz2(\tau)d\tau.$$

$G_x(t)$, $G_y(t)$ and $G_{z2}(t)$ are gradients waveforms on X, Y and Z2 directions; γ is the gyromagnetic ratio. To further improve image quality, we replace our standard Kaczmarz reconstruction or CS algorithm with Low-Rank matrix recovery. Similar to the previous work on CS reconstruction for O-Space imaging [9,10], we apply the Low-Rank reconstruction with O-Space imaging. Assuming a desired image in matrix form $\boldsymbol{S} \in C^{n\times m}$ in O-Space imaging, s is the vectorized version of the desired image by row concatenation, $s = vet(\boldsymbol{S})$, and this convex optimization may be written as:

$$s = argmin(\lambda_1 TV(\boldsymbol{S}) + \lambda_2 \parallel \boldsymbol{S} \parallel_* + \parallel \boldsymbol{f} - \boldsymbol{Es} \parallel_2^2)),$$

where $\boldsymbol{f}$ is the measured signal; $\|\cdot\|_2$ and $\|\cdot\|_*$ are ℓ_2 and nuclear norm; $vet(\cdot)$ is the vectorization function; $TV(\cdot)$ is total variation function; $\lambda 1$ and $\lambda 2$ are the relaxation convergence parameters and is typically set for strongly under-relaxed reconstructions for gradual convergence; E is the encoding matrix, with its inverse calculated by the Kaczmarz iterative algebraic reconstruction [9]. The iterative nonlinear conjugate gradient (NCG) method is applied to optimize the above problem.

Methods

The simulations used a geometric phantom with the 64×64 resolution to study the normalized mean-square-error (NMSE) of reconstruction results at reduction factors of 4, 8, 16 and 32. Experiments were performed on a SIEMENS MAGNETOM 3.0T Trio scanner (Erlangen, Germany). The Z2 SEM gradient inserts [9] were built by Resonance Research, Inc. (Billerica, MA), which of 38cm diameter can run a maximum current of 625 Ampere giving Z2 strength of 0.94 Gauss/cm^2. Another SIEMENS 8-channel head coil was used inside the gradient coil. Images were reconstructed with the 128×128 resolution.

Results

Figure 1 shows simulation results, including reference, radial and O-Space imaging at reduction factors of 8 and 16. Few improvements to reduce aliasing artifacts are observed if using the CS reconstruction in radial and O-Space imaging, but applying Low-Rank reconstruction clearly reduces undersampling artifacts, particularly for the O-Space encoded images. Figure 2 summarizes these results for a range of reduction factors and reconstruction methods applied to both radial and O-Space images. The proposed Low-Rank method improves NMSE over CS reconstruction, especially at high reduction factors, and the improvement is greater for O-Space encoded images. In Figure 3, experimental phantom results also show the proposed Low-Rank O-Space method reduces artifacts and recovers more detail (red arrows) than the either Kaczmarz or CS reconstruction of O-Space with pseudo-random disturbance. Moreover it is better than the best image attainable from radial encoding.

Discussion and Conclusion

In summary, the proposed method applies Low-Rank reconstruction to the problem of image reconstruction when imaging with nonlinear spatial encoding methods. The Low-Rank O-Space approaches can eliminate aliasing artifacts caused by undersampling in O-Space imaging. Moreover, the images are better than those achieved with radial data using either Kaczmarz, CS or Low-Rank reconstruction. It should also be noted that this method does not require modification of the O-Space acquisition strategy [9,10]. In the future, it may be beneficial to apply Low-Rank reconstruction to other nonlinear spatial encoding methods.

Acknowledgements

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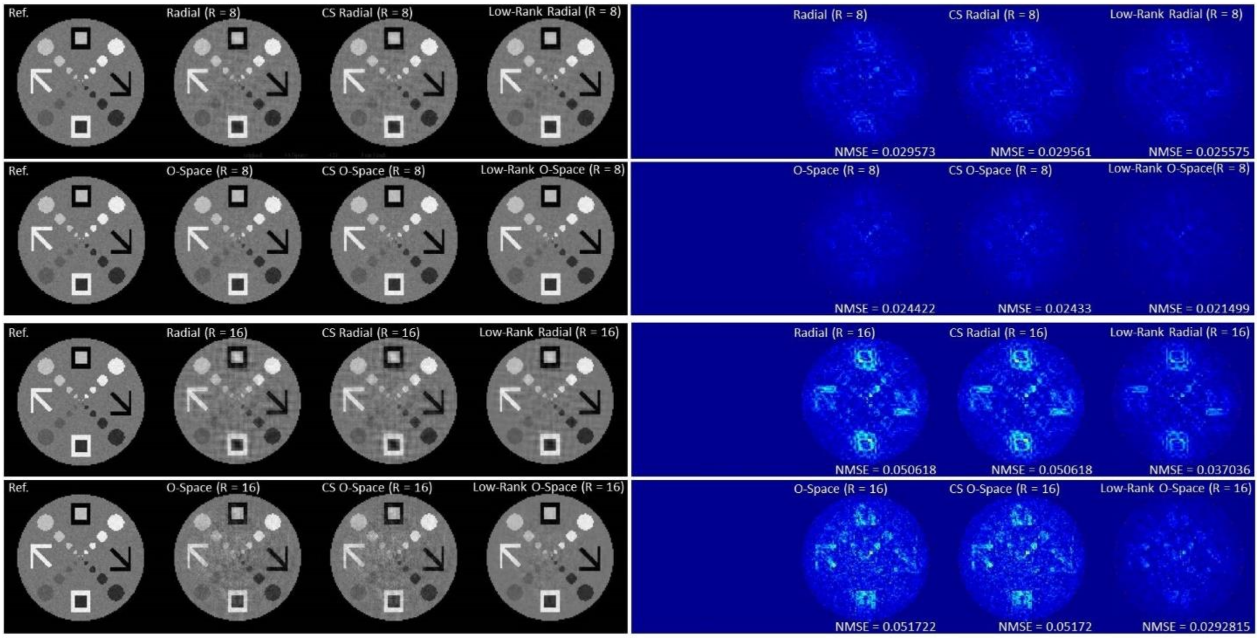

Figure 1: Simulations of geometric phantom with the 64×64 resolution including reference, radial imaging, O-Space imaging and their different images with the referent images at reduction factors of 8 and 16.

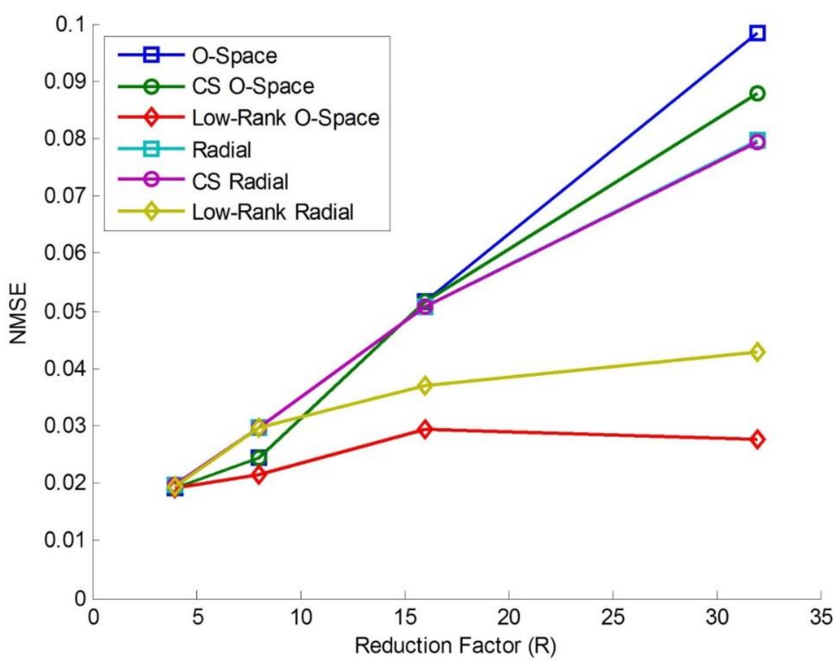

Figure 2: NMSE (Normalized Mean Square Error) of simulations (64×64) at different reduction factors including corresponding reconstruction methods of radial and O-Space imaging.

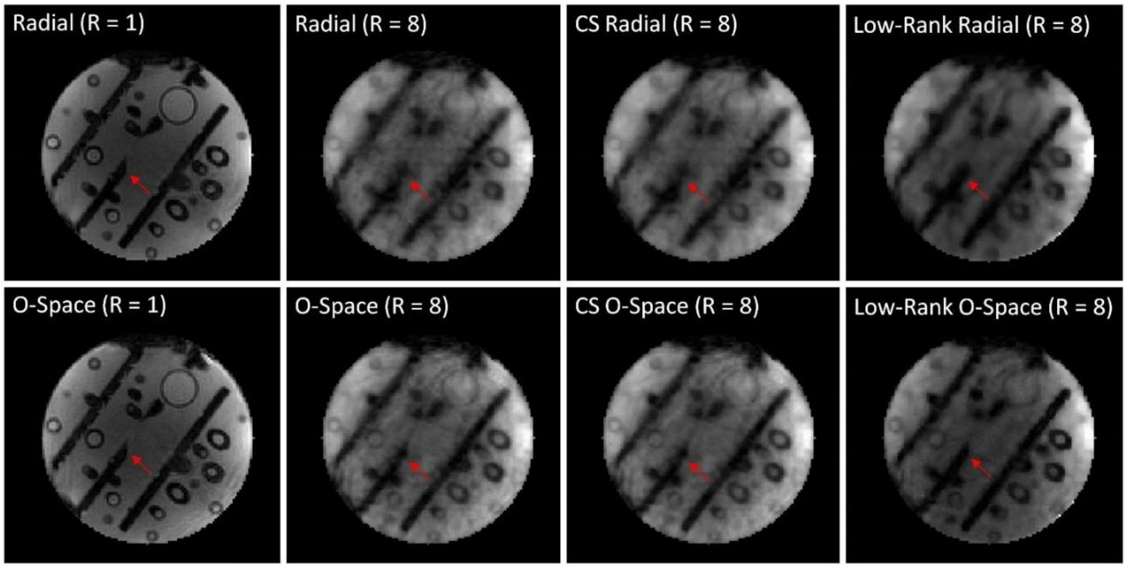

Figure 3. Experimental results with the 128×128 resolution including reference, radial imaging, and O-Space imaging at a reduction factor of 8.

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