Supplementary Information for

Topology-optimized thermal metamaterials traversing full-parameter anisotropic space

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Supplementary Note 1: Full-parameter anisotropic space

As shown in Supplementary Figure 1(a), for an arbitrary point $M(\kappa_x^i,\ \kappa_y^i)$, we can consider the ETC tensor of a mixture structure is $\kappa_M = \begin{bmatrix} \kappa_x^i & 0 \\ 0 & \kappa_y^i \end{bmatrix}$. The ETC tensor will vary when the mixture structure is rotated. Assuming that the rotation angle of the mixture structure is θ, the updated ETC tensor κ_{M_R} can be calculated by

$$\begin{aligned} \kappa_{M_R} &= \begin{bmatrix} \kappa_{xx}^i & \kappa_{xy}^i \\ \kappa_{yx}^i & \kappa_{yy}^i \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \kappa_x^i & 0 \\ 0 & \kappa_y^i \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T \\ &= \begin{bmatrix} \kappa_x^i \cos^2\theta + \kappa_y^i \sin^2\theta & (\kappa_x^i - \kappa_y^i)\sin\theta\cos\theta \\ (\kappa_x^i - \kappa_y^i)\sin\theta\cos\theta & \kappa_y^i \cos^2\theta + \kappa_x^i \sin^2\theta \end{bmatrix} \end{aligned} \tag{1}$$

As shown in Supplementary Figure 1(b), we use $(\kappa_{xx}, \kappa_{yy}, \kappa_{xy})$ to describe the ETC tensor. Then, we analyze the trajectory of the point $M(\kappa_x^i,\ \kappa_y^i,\ 0)$ when the mixture structure is rotated.

From Equation (1), it can be found that the sum of components on the main diagonal of the ETC tensor κ_{M_R} is a constant $\kappa_x^i + \kappa_y^i$. Therefore, as θ changes from 0 to π, the trajectory of the point $M(\kappa_x^i,\ \kappa_y^i, 0)$ is located on the plane $\begin{cases} \kappa_{xx} + \kappa_{yy} = \kappa_x^i + \kappa_y^i \\ \kappa_{xy} = \kappa_{xy} \end{cases}$, which is perpendicular to the line $\begin{cases} \kappa_{xx} = \kappa_{yy} \\ \kappa_{xy} = 0 \end{cases}$. Next, we build a new coordinate system with the κ'_{xy} axis and $\frac{\kappa_{xx} - \kappa_{yy}}{\sqrt{2}}$ axis, as seen in Supplementary Figure 1(b). The distance from the point $M_R(\kappa_{xx}^i, \kappa_{yy}^i, \kappa_{xy}^i)$ to the κ'_{xy} axis is

$$d_{\kappa'_{xy}} = \frac{\kappa_{xx}^i - \kappa_{yy}^i}{\sqrt{2}} = \frac{(\kappa_x^i - \kappa_y^i)(\cos^2\theta - \sin^2\theta)}{\sqrt{2}} \tag{2}$$

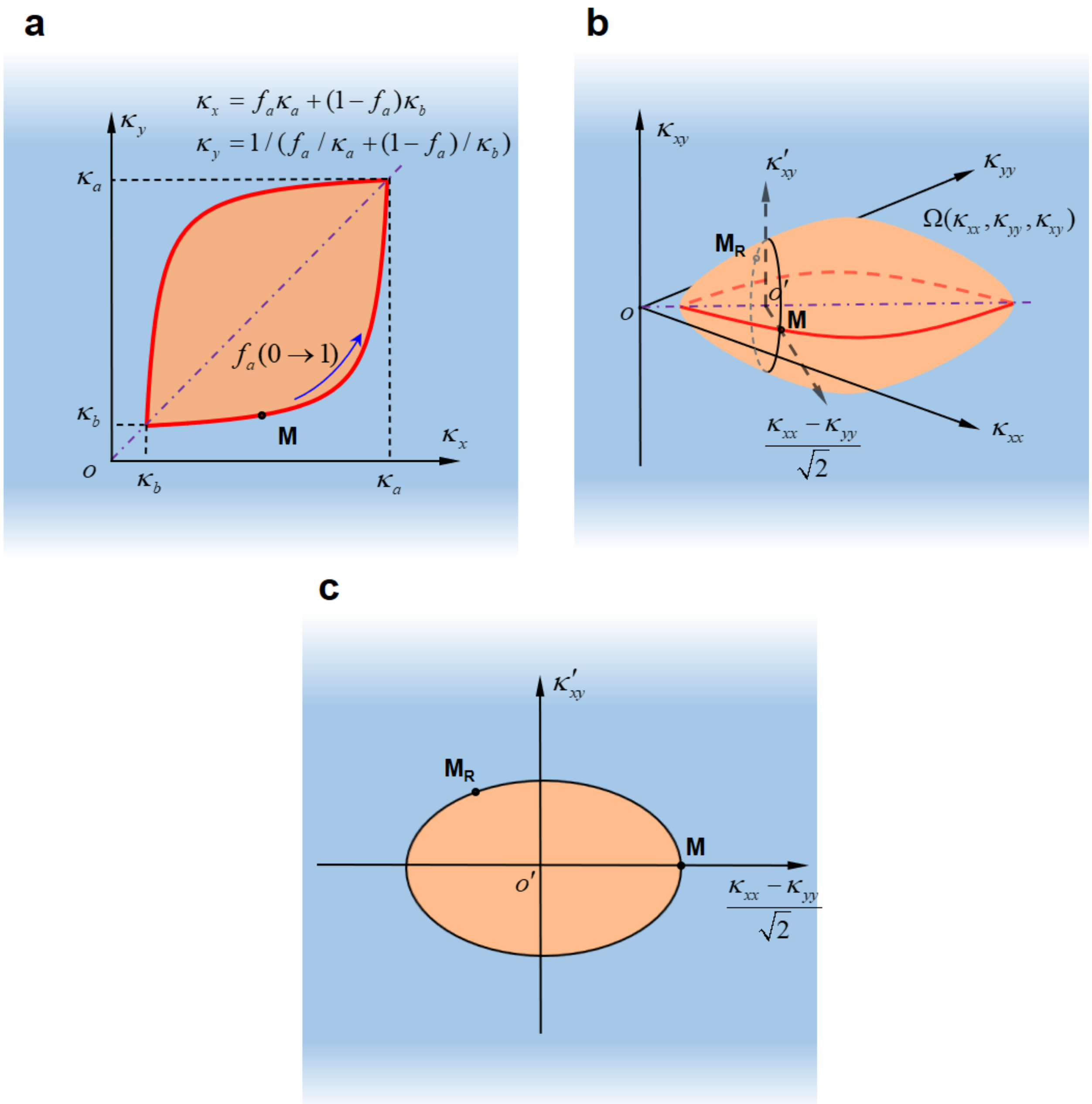
and the distance from the point $M_R(\kappa_{xx}^i, \kappa_{yy}^i, \kappa_{xy}^i)$ to the $\frac{\kappa_{xx} - \kappa_{yy}}{\sqrt{2}}$ axis is:

$$d_{\frac{\kappa_{xx}-\kappa_{yy}}{\sqrt{2}}} = \kappa_{xy}^i = (\kappa_x^i - \kappa_y^i)\sin\theta\cos\theta \tag{3}$$

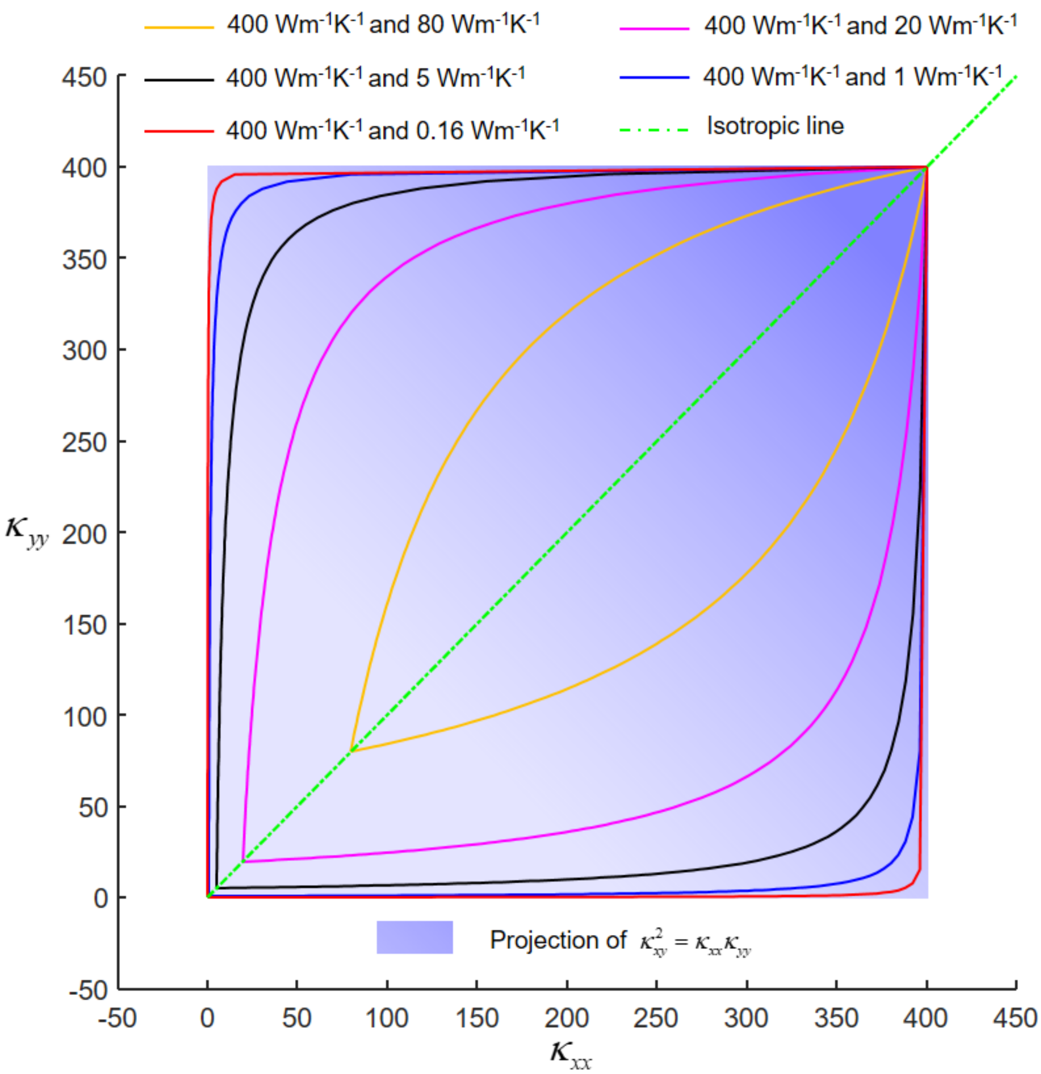
It is noted that $d_{\kappa'_{xy}}$ and $d_{\frac{\kappa_{xx}-\kappa_{yy}}{\sqrt{2}}}$ have the following relationship

$$\frac{d^2_{\kappa'_{xy}}}{\left(\frac{\kappa_x^i - \kappa_y^i}{\sqrt{2}}\right)^2} + \frac{d^2_{\frac{\kappa_{xx}-\kappa_{yy}}{\sqrt{2}}}}{\left(\frac{\kappa_x^i - \kappa_y^i}{2}\right)^2} = 1 \tag{4}$$

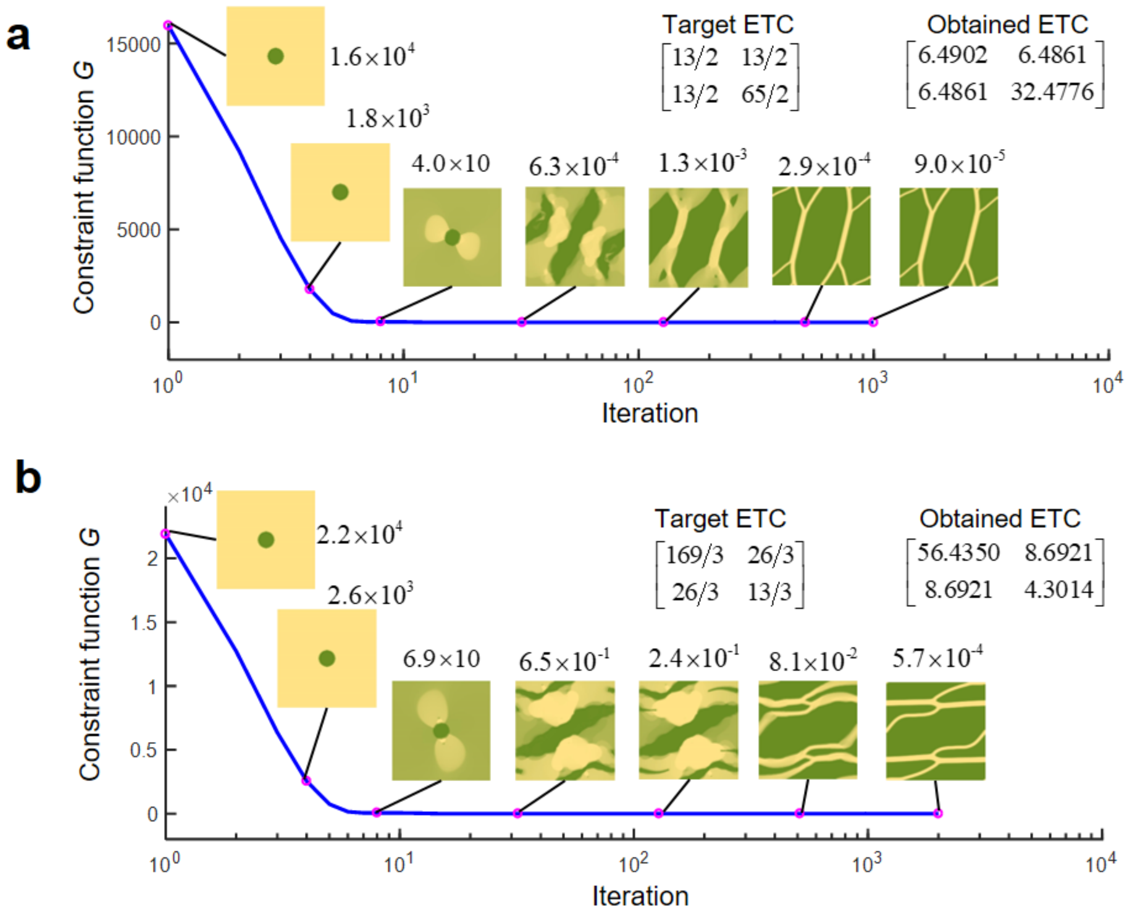
Therefore, the trajectory of point $M(\kappa_x^i,\ \kappa_y^i, 0)$ as θ changes from 0 to π is an ellipse with the major axis a=$\frac{\kappa_x^i - \kappa_y^i}{\sqrt{2}}$ and minor axis b=$\frac{\kappa_x^i - \kappa_y^i}{2}$, as shown in Supplementary Figure 1(c).



Supplementary Figure 1. Full-parameter anisotropic space of thermal conductivity. (**a**) ETC with κ_{xy}=0. The orange region represents the possible ETC of a mixture structure with two given materials, and the red curves are the Wiener bounds of ETC. κ_a and κ_b are the thermal conductivities of two given materials respectively, and the f_a and f_b are the corresponding volume fractions with f_a+f_b=1. (**b**) Schematic of full-parameter anisotropic space Ω(M$_a$, M$_b$). The dashed arrows denote the new coordinate system. (**c**) The front view of the new coordinate system in (**c**).



Supplementary Figure 2. κ_{xy}**-direction projection of** $\kappa_{xx}\kappa_{yy}=\kappa_{xy}\kappa_{yx}$ **and full-parameter anisotropic space achieved by TFC strategy under different two materials.**



Supplementary Figure 3. Topology optimization process for different target ETC tensors with several intermediate designs. (**a**) Target ETC $\boldsymbol{\kappa}_2 = \begin{bmatrix} 13/2 & 13/2 \\ 13/2 & 65/2 \end{bmatrix}$. (**b**) Target ETC $\boldsymbol{\kappa}_3 = \begin{bmatrix} 169/3 & 26/3 \\ 26/3 & 13/3 \end{bmatrix}$. The different colors denote different materials (yellow and green respectively represent copper and PDMS) in the mixture structures. The horizontal and vertical coordinates represent the number of iterations and the value of constraint function G, respectively.

Supplementary Note 2: Details for designing thermal metamaterials by regionalized scattering cancellation method

Based on Fig. 2(d) of main article, we expand Equations (2) and (3) to show the heat flux relationship when the heat flux crosses interface $l^{(j)}$(j=1) as

$$\left(\mathbf{q}^{(in,out)}\right)_{\perp l^{(j)}}=\left|\left(\mathbf{q}^{(in,out)}\right)_{l^{(j)}}\right|\cdot\left(\sin\alpha^{(in,out)}\right)_{l^{(j)}} \tag{5}$$

$$\left(\mathbf{q}^{(in,out)}\right)_{l^{(j)}}=\begin{bmatrix}\left(q_x^{(in,out)}\right)_{l^{(j)}}\\ \left(q_y^{(in,out)}\right)_{l^{(j)}}\end{bmatrix}=-\boldsymbol{\kappa}^{(i,i+1)}\cdot\nabla\left(\mathbf{T}^{(i,i+1)}\right)_{l^{(j)}}=-\begin{bmatrix}\kappa_{xx}^{(i,i+1)} & \kappa_{xy}^{(i,i+1)}\\ \kappa_{yx}^{(i,i+1)} & \kappa_{yy}^{(i,i+1)}\end{bmatrix}\begin{bmatrix}\left(\partial T^{(i,i+1)}/\partial x\right)_{l^{(j)}}\\ \left(\partial T^{(i,i+1)}/\partial y\right)_{l^{(j)}}\end{bmatrix} \tag{6}$$

$$\sqrt{\left(\kappa_{xx}^{(i)}\left(\partial T^{(i)}/\partial x\right)_{l^{(j)}}+\kappa_{xy}^{(i)}\left(\partial T^{(i)}/\partial y\right)_{l^{(j)}}\right)^2+\left(\kappa_{yx}^{(i)}\left(\partial T^{(i)}/\partial x\right)_{l^{(j)}}+\kappa_{yy}^{(i)}\left(\partial T^{(i)}/\partial y\right)_{l^{(j)}}\right)^2}\cdot\sin\left(\alpha^{(in)}\right)_{l^{(j)}}=$$
$$\sqrt{\left(\kappa_{xx}^{(i+1)}\left(\partial T^{(i+1)}/\partial x\right)_{l^{(j)}}+\kappa_{xy}^{(i+1)}\left(\partial T^{(i+1)}/\partial y\right)_{l^{(j)}}\right)^2+\left(\kappa_{yx}^{(i+1)}\left(\partial T^{(i+1)}/\partial x\right)_{l^{(j)}}+\kappa_{yy}^{(i+1)}\left(\partial T^{(i+1)}/\partial y\right)_{l^{(j)}}\right)^2}\cdot\sin\left(\alpha^{(out)}\right)_{l^{(j)}}$$

(7)

where the meanings of all the symbols keep unchanged as those in main article, and $\left(\sin\alpha^{(in,out)}\right)_{l^{(j)}}$ is the included angle of $\left(\mathbf{q}^{(in,out)}\right)_{l^{(i)}}$ with $l^{(i)}$, which can be calculated by

$$\left(\alpha^{(in)}\right)_{l^{(j)}}=\left(\theta\right)_{l^{(j)}}-\arccos\left(\frac{\kappa_{xx}^{(i)}\left(\partial T^{(i)}/\partial x\right)_{l^{(j)}}+\kappa_{xy}^{(i)}\left(\partial T^{(i)}/\partial y\right)_{l^{(j)}}}{\sqrt{\left(\kappa_{xx}^{(i)}\left(\partial T^{(i)}/\partial x\right)_{l^{(j)}}+\kappa_{xy}^{(i)}\left(\partial T^{(i)}/\partial y\right)_{l^{(j)}}\right)^2+\left(\kappa_{yx}^{(i)}\left(\partial T^{(i)}/\partial x\right)_{l^{(j)}}+\kappa_{yy}^{(i)}\left(\partial T^{(i)}/\partial y\right)_{l^{(j)}}\right)^2}}\right)$$
$$\left(\alpha^{(out)}\right)_{l^{(j)}}=\left(\theta\right)_{l^{(j)}}+\arccos\left(\frac{\kappa_{xx}^{(i+1)}\left(\partial T^{(i+1)}/\partial x\right)_{l^{(j)}}+\kappa_{xy}^{(i+1)}\left(\partial T^{(i+1)}/\partial y\right)_{l^{(j)}}}{\sqrt{\left(\kappa_{xx}^{(i+1)}\left(\partial T^{(i+1)}/\partial x\right)_{l^{(j)}}+\kappa_{xy}^{(i+1)}\left(\partial T^{(i+1)}/\partial y\right)_{l^{(j)}}\right)^2+\left(\kappa_{yx}^{(i+1)}\left(\partial T^{(i+1)}/\partial x\right)_{l^{(j)}}+\kappa_{yy}^{(i+1)}\left(\partial T^{(i+1)}/\partial y\right)_{l^{(j)}}\right)^2}}\right)$$

(8)

Supplementary Note 3: Design of thermal connector

As shown in Fig. 4(a) of main article, we divide the area into four areas by three interfaces, and the shape and functionality of thermal connector are determined by geometry parameters β_1 and β_2 (the angle the angle between the line BC and the horizontal line). The temperature gradient components in different areas are given in Supplementary Table 1. Based on Equation (2), the heat flux across $l^{(1)}$, $l^{(2)}$ and $l^{(3)}$ satisfies the following relationship

$$\begin{bmatrix} \left(q_x^{(in)}\right)_{l^{(1)}} \\ \left(q_y^{(in)}\right)_{l^{(1)}} \end{bmatrix} = \begin{bmatrix} \kappa_b \phi \\ 0 \end{bmatrix}, \left(\alpha^{(in)}\right)_{l^{(1)}} = (\theta)_{l^{(1)}} - \arccos\left(\frac{\left(q_x^{(in)}\right)_{l^{(1)}}}{\sqrt{\left(\left(q_x^{(in)}\right)_{l^{(1)}}\right)^2 + \left(\left(q_y^{(in)}\right)_{l^{(1)}}\right)^2}} \right)$$
$$\begin{bmatrix} \left(q_x^{(out)}\right)_{l^{(1)}} \\ \left(q_y^{(out)}\right)_{l^{(1)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(2)} \phi \\ \kappa_{yx}^{(2)} \phi \end{bmatrix}, \left(\alpha^{(out)}\right)_{l^{(1)}} = (\theta)_{l^{(1)}} + \arccos\left(\frac{\left(q_x^{(out)}\right)_{l^{(1)}}}{\sqrt{\left(\left(q_x^{(out)}\right)_{l^{(1)}}\right)^2 + \left(\left(q_y^{(out)}\right)_{l^{(1)}}\right)^2}} \right) \tag{9}$$

$$\begin{bmatrix} \left(q_x^{(in)}\right)_{l^{(2)}} \\ \left(q_y^{(in)}\right)_{l^{(2)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(2)} \phi \\ \kappa_{yx}^{(2)} \phi \end{bmatrix}, \left(\alpha^{(in)}\right)_{l^{(2)}} = (\theta)_{l^{(2)}} - \arccos\left(\frac{\left(q_x^{(in)}\right)_{l^{(2)}}}{\sqrt{\left(\left(q_x^{(in)}\right)_{l^{(2)}}\right)^2 + \left(\left(q_y^{(in)}\right)_{l^{(2)}}\right)^2}} \right), \cot\beta_1 = \frac{\left(q_x^{(in)}\right)_{l^{(2)}}}{\left(q_y^{(in)}\right)_{l^{(2)}}}$$
$$\begin{bmatrix} \left(q_x^{(out)}\right)_{l^{(2)}} \\ \left(q_y^{(out)}\right)_{l^{(2)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(3)} \phi \\ \kappa_{yx}^{(3)} \phi \end{bmatrix}, \left(\alpha^{(out)}\right)_{l^{(2)}} = (\theta)_{l^{(2)}} + \arccos\left(\frac{\left(q_x^{(out)}\right)_{l^{(3)}}}{\sqrt{\left(\left(q_x^{(out)}\right)_{l^{(2)}}\right)^2 + \left(\left(q_y^{(out)}\right)_{l^{(2)}}\right)^2}} \right), \cot\beta_2 = \frac{\left(q_x^{(out)}\right)_{l^{(2)}}}{\left(q_y^{(out)}\right)_{l^{(2)}}} \tag{10}$$

$$\begin{bmatrix} \left(q_x^{(in)}\right)_{l^{(3)}} \\ \left(q_y^{(in)}\right)_{l^{(3)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(3)} \phi \\ \kappa_{yx}^{(3)} \phi \end{bmatrix}, \left(\alpha^{(in)}\right)_{l^{(3)}} = (\theta)_{l^{(3)}} - \arccos\left(\frac{\left(q_x^{(in)}\right)_{l^{(3)}}}{\sqrt{\left(\left(q_x^{(in)}\right)_{l^{(3)}}\right)^2 + \left(\left(q_y^{(in)}\right)_{l^{(3)}}\right)^2}} \right)$$
$$\begin{bmatrix} \left(q_x^{(out)}\right)_{l^{(3)}} \\ \left(q_y^{(out)}\right)_{l^{(3)}} \end{bmatrix} = \begin{bmatrix} \kappa_b \phi \\ 0 \end{bmatrix}, \left(\alpha^{(out)}\right)_{l^{(3)}} = (\theta)_{l^{(3)}} + \arccos\left(\frac{\left(q_x^{(out)}\right)_{l^{(3)}}}{\sqrt{\left(\left(q_x^{(out)}\right)_{l^{(3)}}\right)^2 + \left(\left(q_y^{(out)}\right)_{l^{(3)}}\right)^2}} \right) \tag{11}$$

where κ_b is thermal conductivity of area 1. Then, we substitute Equations (9) to (11) into Equation (7), and obtain

$$\kappa_b \cdot \sin(\theta)_{l^{(1)}} = \sqrt{\left(\kappa_{xx}^{(2)}\right)^2 + \left(\kappa_{yx}^{(2)}\right)^2} \cdot \sin\left((\theta)_{l^{(1)}} + \arccos\left(\frac{\kappa_{xx}^{(2)}}{\sqrt{\left(\kappa_{xx}^{(2)}\right)^2 + \left(\kappa_{yx}^{(2)}\right)^2}} \right) \right) \tag{12}$$

$$\sqrt{\left(\kappa_{xx}^{(2)}\right)^2 + \left(\kappa_{yx}^{(2)}\right)^2} \cdot \sin\left((\theta)_{l^{(2)}} - \arccos\left(\frac{\kappa_{xx}^{(2)}}{\sqrt{\left(\kappa_{xx}^{(2)}\right)^2 + \left(\kappa_{yx}^{(2)}\right)^2}} \right) \right) =$$
$$\sqrt{\left(\kappa_{xx}^{(3)}\right)^2 + \left(\kappa_{yx}^{(3)}\right)^2} \cdot \sin\left((\theta)_{l^{(2)}} + \arccos\left(\frac{\kappa_{xx}^{(3)}}{\sqrt{\left(\kappa_{xx}^{(3)}\right)^2 + \left(\kappa_{yx}^{(3)}\right)^2}} \right) \right) \tag{13}$$

$$\kappa_b \cdot \sin(\theta)_{l^{(3)}} = \sqrt{\left(\kappa_{xx}^{(3)}\right)^2 + \left(\kappa_{yx}^{(3)}\right)^2} \cdot \sin\left((\theta)_{l^{(3)}} - \arccos\left(\frac{\kappa_{xx}^{(3)}}{\sqrt{\left(\kappa_{xx}^{(3)}\right)^2 + \left(\kappa_{yx}^{(3)}\right)^2}}\right)\right) \tag{14}$$

Combining Equations (12) to (14), we obtain the general solution as follows:

$$\kappa_b \cdot \sin(\theta)_{l^{(1)}} = \sqrt{\left(\kappa_{xx}^{(2)}\right)^2 + \left(\kappa_{yx}^{(2)}\right)^2} \cdot \sin\left((\theta)_{l^{(1)}} + \arccos\left(\frac{\kappa_{xx}^{(2)}}{\sqrt{\left(\kappa_{xx}^{(2)}\right)^2 + \left(\kappa_{yx}^{(2)}\right)^2}}\right)\right) \tag{15}$$

$$\begin{aligned} &\sqrt{\left(\kappa_{xx}^{(2)}\right)^2 + \left(\kappa_{yx}^{(2)}\right)^2} \cdot \sin\left((\theta)_{l^{(2)}} - \arccos\left(\frac{\kappa_{xx}^{(2)}}{\sqrt{\left(\kappa_{xx}^{(2)}\right)^2 + \left(\kappa_{yx}^{(2)}\right)^2}}\right)\right) = \\ &\sqrt{\left(\kappa_{xx}^{(3)}\right)^2 + \left(\kappa_{yx}^{(3)}\right)^2} \cdot \sin\left((\theta)_{l^{(2)}} + \arccos\left(\frac{\kappa_{xx}^{(3)}}{\sqrt{\left(\kappa_{xx}^{(3)}\right)^2 + \left(\kappa_{yx}^{(3)}\right)^2}}\right)\right) \end{aligned} \tag{16}$$

$$\kappa_b \cdot \sin(\theta)_{l^{(3)}} = \sqrt{\left(\kappa_{xx}^{(3)}\right)^2 + \left(\kappa_{yx}^{(3)}\right)^2} \cdot \sin\left((\theta)_{l^{(3)}} - \arccos\left(\frac{\kappa_{xx}^{(3)}}{\sqrt{\left(\kappa_{xx}^{(3)}\right)^2 + \left(\kappa_{yx}^{(3)}\right)^2}}\right)\right) \tag{17}$$

$$\cot\beta_1 = \frac{\left(\kappa_{xx}^{(2)}\right)_{l^{(2)}}}{\left(\kappa_{yx}^{(2)}\right)_{l^{(2)}}}, \cot\beta_2 = \frac{\left(\kappa_{xx}^{(3)}\right)_{l^{(2)}}}{\left(\kappa_{yx}^{(3)}\right)_{l^{(2)}}} \tag{18}$$

When $(\theta)_{l^{(1)}} = 90^\circ$, $\cot\beta_1 = 1$, $\cot\beta_2 = 1$ and $(\theta)_{l^{(2)}} = 90^\circ$, Equations (15) to (17) can be simplified to

$$\begin{aligned} &\kappa_b = \kappa_{xx}^{(2)} = \kappa_{xy}^{(2)} \\ &\kappa_{xx}^{(2)} = \kappa_{xx}^{(3)} \\ &\kappa_b = \kappa_{xx}^{(3)} = \kappa_{xy}^{(3)} \end{aligned} \tag{19}$$

Supplementary Note 4: Design of thermal reflector

As shown in Supplementary Figure 4(a), we divide the design area into four areas by three interfaces, and the shape and functionality of thermal reflector are determined by geometry parameters β_1, β_2, β_3 and β_4. The temperature gradient components in different areas are given in Supplementary Table 1. Based on Equation (2), the heat flux across $l^{(1)}$, $l^{(2)}$ and $l^{(3)}$ satisfies the following relationship

$$\begin{bmatrix} \left(q_x^{(in)}\right)_{l^{(1)}} \\ \left(q_y^{(in)}\right)_{l^{(1)}} \end{bmatrix} = \begin{bmatrix} \kappa_b \phi \\ 0 \end{bmatrix}, \left(\alpha^{(in)}\right)_{l^{(1)}} = (\theta)_{l^{(1)}} - \arccos\left(\frac{\left(q_x^{(in)}\right)_{l^{(1)}}}{\sqrt{\left(\left(q_x^{(in)}\right)_{l^{(1)}}\right)^2 + \left(\left(q_y^{(in)}\right)_{l^{(1)}}\right)^2}} \right)$$

$$\begin{bmatrix} \left(q_x^{(out)}\right)_{l^{(1)}} \\ \left(q_y^{(out)}\right)_{l^{(1)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(2)} \phi \\ \kappa_{yx}^{(2)} \phi \end{bmatrix}, \left(\alpha^{(out)}\right)_{l^{(1)}} = (\theta)_{l^{(1)}} + \arccos\left(\frac{\left(q_x^{(out)}\right)_{l^{(1)}}}{\sqrt{\left(\left(q_x^{(out)}\right)_{l^{(1)}}\right)^2 + \left(\left(q_y^{(out)}\right)_{l^{(1)}}\right)^2}} \right) \tag{20}$$

$$\begin{bmatrix} \left(q_x^{(in)}\right)_{l^{(2)}} \\ \left(q_y^{(in)}\right)_{l^{(2)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(2)} \phi \\ \kappa_{yx}^{(2)} \phi \end{bmatrix}, \left(\alpha^{(in)}\right)_{l^{(2)}} = (\theta)_{l^{(2)}} - \arccos\left(\frac{\left(q_x^{(in)}\right)_{l^{(2)}}}{\sqrt{\left(\left(q_x^{(in)}\right)_{l^{(2)}}\right)^2 + \left(\left(q_y^{(in)}\right)_{l^{(2)}}\right)^2}} \right), \tan\beta_3 = \frac{\left(q_x^{(in)}\right)_{l^{(2)}}}{\left(q_y^{(in)}\right)_{l^{(2)}}}$$

$$\begin{bmatrix} \left(q_x^{(out)}\right)_{l^{(2)}} \\ \left(q_y^{(out)}\right)_{l^{(2)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(3)} \phi \tan\beta_1 \cot\beta_2 - \kappa_{xy}^{(3)} \phi \tan\beta_1 \\ \kappa_{yx}^{(3)} \phi \tan\beta_1 \cot\beta_2 - \kappa_{yy}^{(3)} \phi \tan\beta_1 \end{bmatrix}, \left(\alpha^{(out)}\right)_{l^{(2)}} = (\theta)_{l^{(2)}} + \arccos\left(\frac{\left(q_x^{(out)}\right)_{l^{(2)}}}{\sqrt{\left(\left(q_x^{(out)}\right)_{l^{(2)}}\right)^2 + \left(\left(q_y^{(out)}\right)_{l^{(2)}}\right)^2}} \right), \cot\beta_4 = \frac{\left(q_x^{(out)}\right)_{l^{(2)}}}{\left(q_y^{(out)}\right)_{l^{(2)}}}$$

(21)

$$\begin{bmatrix} \left(q_x^{(in)}\right)_{l^{(3)}} \\ \left(q_y^{(in)}\right)_{l^{(3)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(3)} \phi \tan\beta_1 \cot\beta_2 - \kappa_{xy}^{(3)} \phi \tan\beta_1 \\ \kappa_{yx}^{(3)} \phi \tan\beta_1 \cot\beta_2 - \kappa_{yy}^{(3)} \phi \tan\beta_1 \end{bmatrix}, \left(\alpha^{(in)}\right)_{l^{(3)}} = (\theta)_{l^{(3)}} - \arccos\left(\frac{\left(q_x^{(in)}\right)_{l^{(3)}}}{\sqrt{\left(\left(q_x^{(in)}\right)_{l^{(3)}}\right)^2 + \left(\left(q_y^{(in)}\right)_{l^{(3)}}\right)^2}} \right)$$

$$\begin{bmatrix} \left(q_x^{(out)}\right)_{l^{(3)}} \\ \left(q_y^{(out)}\right)_{l^{(3)}} \end{bmatrix} = \begin{bmatrix} \kappa_b \phi \cot\beta_2 \\ -\kappa_b \phi \end{bmatrix}, \left(\alpha^{(out)}\right)_{l^{(3)}} = (\theta)_{l^{(3)}} + \arccos\left(\frac{\left(q_x^{(out)}\right)_{l^{(3)}}}{\sqrt{\left(\left(q_x^{(out)}\right)_{l^{(3)}}\right)^2 + \left(\left(q_y^{(out)}\right)_{l^{(3)}}\right)^2}} \right)$$

(22)

where κ_b is thermal conductivity of area 1. Then, we substitute Equations (20) to (22) into Equation (7), and obtain

$$\kappa_b \cdot \sin(\theta)_{l^{(1)}} = \sqrt{\left(\kappa_{xx}^{(2)}\right)^2 + \left(\kappa_{yx}^{(2)}\right)^2} \cdot \sin\left((\theta)_{l^{(1)}} + \arccos\left(\frac{\kappa_{xx}^{(2)}}{\sqrt{\left(\kappa_{xx}^{(2)}\right)^2 + \left(\kappa_{yx}^{(2)}\right)^2}} \right) \right) \tag{23}$$

$$\sqrt{\left(\kappa_{xx}^{(2)}\right)^2+\left(\kappa_{yx}^{(2)}\right)^2}\cdot\sin\left(\left(\theta\right)_{l^{(2)}}-\arccos\left(\frac{\kappa_{xx}^{(2)}}{\sqrt{\left(\kappa_{xx}^{(2)}\right)^2+\left(\kappa_{yx}^{(2)}\right)^2}}\right)\right)=$$

$$\sqrt{\left(\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1\right)^2+\left(\kappa_{yx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{yy}^{(3)}\tan\beta_1\right)^2}\cdot$$

$$\sin\left(\left(\theta\right)_{l^{(2)}}+\arccos\left(\frac{\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1}{\sqrt{\left(\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1\right)^2+\left(\kappa_{yx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{yy}^{(3)}\tan\beta_1\right)^2}}\right)\right) \tag{24}$$

$$\sqrt{\left(\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1\right)^2+\left(\kappa_{yx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{yy}^{(3)}\tan\beta_1\right)^2}\cdot$$

$$\sin\left(\left(\theta\right)_{l^{(3)}}-\arccos\left(\frac{\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1}{\sqrt{\left(\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1\right)^2+\left(\kappa_{yx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{yy}^{(3)}\tan\beta_1\right)^2}}\right)\right)=$$

$$\sqrt{\left(\kappa_b\cot\beta_2\right)^2+\left(-\kappa_b\right)^2}\cdot\sin\left(\left(\theta\right)_{l^{(3)}}+\arccos\left(\frac{\kappa_b\cot\beta_2}{\sqrt{\left(\kappa_b\cot\beta_2\right)^2+\left(-\kappa_b\right)^2}}\right)\right) \tag{25}$$

Combining Equations (23) to (25), we obtain a general solution as follows:

$$\kappa_b\cdot\sin\left(\theta\right)_{l^{(1)}}=\sqrt{\left(\kappa_{xx}^{(2)}\right)^2+\left(\kappa_{yx}^{(2)}\right)^2}\cdot\sin\left(\left(\theta\right)_{l^{(1)}}+\arccos\left(\frac{\kappa_{xx}^{(2)}}{\sqrt{\left(\kappa_{xx}^{(2)}\right)^2+\left(\kappa_{yx}^{(2)}\right)^2}}\right)\right) \tag{26}$$

$$\sqrt{\left(\kappa_{xx}^{(2)}\right)^2+\left(\kappa_{yx}^{(2)}\right)^2}\cdot\sin\left(\left(\theta\right)_{l^{(2)}}-\arccos\left(\frac{\kappa_{xx}^{(2)}}{\sqrt{\left(\kappa_{xx}^{(2)}\right)^2+\left(\kappa_{yx}^{(2)}\right)^2}}\right)\right)=$$

$$\sqrt{\left(\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1\right)^2+\left(\kappa_{yx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{yy}^{(3)}\tan\beta_1\right)^2}\cdot \tag{27}$$

$$\sin\left(\left(\theta\right)_{l^{(2)}}+\arccos\left(\frac{\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1}{\sqrt{\left(\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1\right)^2+\left(\kappa_{yx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{yy}^{(3)}\tan\beta_1\right)^2}}\right)\right)$$

$$\sqrt{\left(\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1\right)^2+\left(\kappa_{yx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{yy}^{(3)}\tan\beta_1\right)^2}\cdot$$

$$\sin\left((\theta)_{l^{(3)}}-\arccos\left(\frac{\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1}{\sqrt{\left(\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1\right)^2+\left(\kappa_{yx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{yy}^{(3)}\tan\beta_1\right)^2}}\right)\right)=$$

$$\sqrt{\left(\kappa_b\cot\beta_2\right)^2+\left(-\kappa_b\right)^2}\cdot\sin\left((\theta)_{l^{(3)}}+\arccos\left(\frac{\kappa_b\cot\beta_2}{\sqrt{\left(\kappa_b\cot\beta_2\right)^2+\left(-\kappa_b\right)^2}}\right)\right) \tag{28}$$

$$\tan\beta_3=\frac{\kappa_{xx}^{(2)}}{\kappa_{yx}^{(2)}},\cot\beta_4=\frac{\kappa_{xx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{xy}^{(3)}\tan\beta_1}{\kappa_{yx}^{(3)}\tan\beta_1\cot\beta_2-\kappa_{yy}^{(3)}\tan\beta_1} \tag{29}$$

When $\cot\beta_1=3, \beta_2=90^\circ, \beta_3=45^\circ, \beta_4=45^\circ$, $(\theta)_{l^{(1)}}=90^\circ$ and $(\theta)_{l^{(2)}}=90^\circ+\beta_1$, Equations (26) to (29) can be simplified to

$$\begin{aligned}&\kappa_b=\kappa_{xx}^{(2)}=-\kappa_{xy}^{(2)}\\&2\kappa_b-\kappa_{yx}^{(2)}=-\kappa_{xy}^{(3)}\\&-\kappa_{xy}^{(3)}/3=\kappa_{yy}^{(3)}/3=\kappa_b\end{aligned} \tag{30}$$

Supplementary Note 5: Design of thermal concentrator

As shown in Supplementary Figure 4(b), we divide a quarter of the design area into three areas by two interfaces, and the shape and functionality of thermal concentrator are determined by geometry parameters $|\mathrm{CB}|, |\mathrm{DC}|, |\mathrm{OD}|, (\theta)_{l^{(1)}}$ and $(\theta)_{l^{(2)}}$. The temperature gradient components in different areas are given in Supplementary Figure 1. Based on Equation (2), the heat flux across $l^{(1)}$ and $l^{(2)}$ satisfies the following relationship

$$\begin{bmatrix}\left(q_x^{(in)}\right)_{l^{(1)}}\\\left(q_y^{(in)}\right)_{l^{(1)}}\end{bmatrix}=\begin{bmatrix}\kappa_b\phi\\0\end{bmatrix},\left(\alpha^{(in)}\right)_{l^{(1)}}=(\theta)_{l^{(1)}}-\arccos\left(\frac{\left(q_x^{(in)}\right)_{l^{(1)}}}{\sqrt{\left(\left(q_x^{(in)}\right)_{l^{(1)}}\right)^2+\left(\left(q_y^{(in)}\right)_{l^{(1)}}\right)^2}}\right)$$

$$\begin{bmatrix}\left(q_x^{(out)}\right)_{l^{(1)}}\\\left(q_y^{(out)}\right)_{l^{(1)}}\end{bmatrix}=\begin{bmatrix}\kappa_{xx}^{(2)}\phi\frac{|\mathrm{CB}|}{|\mathrm{DB}|}+\kappa_{xy}^{(2)}\phi\frac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot(\theta)_{l^{(1)}}\\\kappa_{yx}^{(2)}\phi\frac{|\mathrm{CB}|}{|\mathrm{DB}|}+\kappa_{yy}^{(2)}\phi\frac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot(\theta)_{l^{(1)}}\end{bmatrix},\left(\alpha^{(out)}\right)_{l^{(1)}}=(\theta)_{l^{(1)}}+\arccos\left(\frac{\left(q_x^{(out)}\right)_{l^{(1)}}}{\sqrt{\left(\left(q_x^{(out)}\right)_{l^{(1)}}\right)^2+\left(\left(q_y^{(out)}\right)_{l^{(1)}}\right)^2}}\right)$$

$$\tag{31}$$

$$\begin{bmatrix} \left(q_x^{(in)}\right)_{l^{(2)}} \\ \left(q_y^{(in)}\right)_{l^{(2)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(2)}\phi\dfrac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{xy}^{(2)}\phi\dfrac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(2)}} \\ \kappa_{yx}^{(2)}\phi\dfrac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{yy}^{(2)}\phi\dfrac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(2)}} \end{bmatrix}, \left(\alpha^{(in)}\right)_{l^{(2)}} = \left(\theta\right)_{l^{(2)}} - \arccos\left(\frac{\left(q_x^{(in)}\right)_{l^{(2)}}}{\sqrt{\left(\left(q_x^{(in)}\right)_{l^{(2)}}\right)^2 + \left(\left(q_y^{(in)}\right)_{l^{(2)}}\right)^2}}\right)$$

$$\begin{bmatrix} \left(q_x^{(out)}\right)_{l^{(2)}} \\ \left(q_y^{(out)}\right)_{l^{(2)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(3)}\phi\dfrac{|\mathrm{OC}|}{|\mathrm{OD}|} \\ \kappa_{yx}^{(3)}\phi\dfrac{|\mathrm{OC}|}{|\mathrm{OD}|} \end{bmatrix}, \left(\alpha^{(out)}\right)_{l^{(2)}} = \left(\theta\right)_{l^{(2)}} + \arccos\left(\frac{\left(q_x^{(out)}\right)_{l^{(2)}}}{\sqrt{\left(\left(q_x^{(out)}\right)_{l^{(2)}}\right)^2 + \left(\left(q_y^{(out)}\right)_{l^{(2)}}\right)^2}}\right)$$

(32)

where κ_b is thermal conductivity of area 1. Then, we substitute Equations (31) and (32) into Equation (7), and obtain

$$\kappa_b \cdot \sin\left(\theta\right)_{l^{(1)}} = \sqrt{\left(\kappa_{xx}^{(2)}\frac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{xy}^{(2)}\frac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(1)}}\right)^2 + \left(\kappa_{yx}^{(2)}\frac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{yy}^{(2)}\frac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(1)}}\right)^2} \cdot$$
$$\sin\left(\left(\theta\right)_{l^{(1)}} + \arccos\left(\frac{\kappa_{xx}^{(2)}\dfrac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{xy}^{(2)}\dfrac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(1)}}}{\sqrt{\left(\kappa_{xx}^{(2)}\dfrac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{xy}^{(2)}\dfrac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(1)}}\right)^2 + \left(\kappa_{yx}^{(2)}\dfrac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{yy}^{(2)}\dfrac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(1)}}\right)^2}}\right)\right)$$

(33)

$$\sqrt{\left(\kappa_{xx}^{(2)}\frac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{xy}^{(2)}\frac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2 + \left(\kappa_{yx}^{(2)}\frac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{yy}^{(2)}\frac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2} \cdot$$
$$\sin\left(\left(\theta\right)_{l^{(2)}} - \arccos\left(\frac{\kappa_{xx}^{(2)}\dfrac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{xy}^{(2)}\dfrac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(2)}}}{\sqrt{\left(\kappa_{xx}^{(2)}\dfrac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{xy}^{(2)}\dfrac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2 + \left(\kappa_{yx}^{(2)}\dfrac{|\mathrm{CB}|}{|\mathrm{DB}|} + \kappa_{yy}^{(2)}\dfrac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2}}\right)\right) =$$

$$\sqrt{\left(\kappa_{xx}^{(3)}\frac{|\mathrm{OC}|}{|\mathrm{OD}|}\right)^2 + \left(\kappa_{yx}^{(3)}\frac{|\mathrm{OC}|}{|\mathrm{OD}|}\right)^2} \cdot \sin\left(\left(\theta\right)_{l^{(2)}} + \arccos\left(\frac{\kappa_{xx}^{(3)}\dfrac{|\mathrm{OC}|}{|\mathrm{OD}|}}{\sqrt{\left(\kappa_{xx}^{(3)}\dfrac{|\mathrm{OC}|}{|\mathrm{OD}|}\right)^2 + \left(\kappa_{yx}^{(3)}\dfrac{|\mathrm{OC}|}{|\mathrm{OD}|}\right)^2}}\right)\right)$$

(34)

Combining Equations (33) and (34), we obtain a general solution as follows:

$$\begin{aligned}
&\kappa_{yx}^{(2)}\frac{|\mathrm{CB}|}{|\mathrm{DB}|}+\kappa_{yy}^{(2)}\frac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot(\theta)_{l^{(1)}}=0\\
&\kappa_{xx}^{(2)}\frac{|\mathrm{CB}|}{|\mathrm{DB}|}+\kappa_{xy}^{(2)}\frac{|\mathrm{DC}|}{|\mathrm{DB}|}\cot(\theta)_{l^{(1)}}=\kappa_b\\
&\kappa_{yx}^{(3)}\frac{|\mathrm{OC}|}{|\mathrm{OD}|}=0\\
&\kappa_{xx}^{(3)}\frac{|\mathrm{OC}|}{|\mathrm{OD}|}=\kappa_b
\end{aligned} \tag{35}$$

When $|\mathrm{DC}|=2|\mathrm{OD}|=2|\mathrm{CB}|=25\,\mathrm{mm}, (\theta)_{l^{(1)}}=45^\circ$ and $\cot(\theta)_{l^{(2)}}=1/4$, Equation (35) can be simplified to

$$\begin{aligned}
&\kappa_{yx}^{(2)}=-2\kappa_{yy}^{(2)}\\
&\kappa_{xx}^{(2)}+2\kappa_{xy}^{(2)}=3\kappa_b\\
&3\kappa_{yx}^{(3)}=0\\
&3\kappa_{xx}^{(3)}=\kappa_b
\end{aligned} \tag{36}$$

Supplementary Note 6: Design of thermal cloak

As shown in Supplementary Figure 4(c), we divide a quarter of the design area into three areas by two interfaces, and the shape and functionality of thermal cloak are determined by geometry parameters $(\theta)_{l^{(1)}}$, $(\theta)_{l^{(2)}}$, $|\mathrm{CB}|$ and $|\mathrm{OB}|$. The temperature gradient components in different areas are given in Supplementary Table 1, and ξ is a small positive number. Based on Equation (2), the heat flux across $l^{(1)}$ and $l^{(2)}$ satisfies the following relationship

$$\begin{aligned}
&\begin{bmatrix}\left(q_x^{(in)}\right)_{l^{(1)}}\\ \left(q_y^{(in)}\right)_{l^{(1)}}\end{bmatrix}=\begin{bmatrix}\kappa_b\phi\\ 0\end{bmatrix},\left(\alpha^{(in)}\right)_{l^{(1)}}=(\theta)_{l^{(1)}}-\arccos\left(\frac{\left(q_x^{(in)}\right)_{l^{(1)}}}{\sqrt{\left(\left(q_x^{(in)}\right)_{l^{(1)}}\right)^2+\left(\left(q_y^{(in)}\right)_{l^{(1)}}\right)^2}}\right)\\
&\begin{bmatrix}\left(q_x^{(out)}\right)_{l^{(1)}}\\ \left(q_y^{(out)}\right)_{l^{(1)}}\end{bmatrix}=\begin{bmatrix}\kappa_{xx}^{(2)}\phi\frac{|\mathrm{OB}|}{|\mathrm{CB}|}-\kappa_{xy}^{(2)}\phi\frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot(\theta)_{l^{(2)}}\\ \kappa_{yx}^{(2)}\phi\frac{|\mathrm{OB}|}{|\mathrm{CB}|}-\kappa_{yy}^{(2)}\phi\frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot(\theta)_{l^{(2)}}\end{bmatrix},\left(\alpha^{(out)}\right)_{l^{(1)}}=(\theta)_{l^{(1)}}+\arccos\left(\frac{\left(q_x^{(out)}\right)_{l^{(1)}}}{\sqrt{\left(\left(q_x^{(out)}\right)_{l^{(1)}}\right)^2+\left(\left(q_y^{(out)}\right)_{l^{(1)}}\right)^2}}\right)
\end{aligned}$$

(37)

$$\begin{bmatrix} \left(q_x^{(in)}\right)_{l^{(2)}} \\ \left(q_y^{(in)}\right)_{l^{(2)}} \end{bmatrix} = \begin{bmatrix} \kappa_{xx}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{xy}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}} \\ \kappa_{yx}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{yy}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}} \end{bmatrix}, \alpha^{(in)} = \left(\theta\right)_{l^{(2)}} - \arccos\left(\frac{\left(q_x^{(in)}\right)_{l^{(2)}}}{\sqrt{\left(\left(q_x^{(in)}\right)_{l^{(2)}}\right)^2 + \left(\left(q_y^{(in)}\right)_{l^{(2)}}\right)^2}} \right)$$

$$\begin{bmatrix} \left(q_x^{(out)}\right)_{l^{(2)}} \\ \left(q_y^{(out)}\right)_{l^{(2)}} \end{bmatrix} = \begin{bmatrix} \kappa_o \xi \\ 0 \end{bmatrix}, \alpha^{(out)} = \left(\theta\right)_{l^{(2)}} + \arccos\left(\frac{\left(q_x^{(out)}\right)_{l^{(2)}}}{\sqrt{\left(\left(q_x^{(out)}\right)_{l^{(2)}}\right)^2 + \left(\left(q_y^{(out)}\right)_{l^{(2)}}\right)^2}} \right)$$

(38)

where κ_b is thermal conductivity of area 1. Then, we substitute Equations (37) and (38) into Equation (7), and obtain

$$\kappa_b \cdot \sin\left(\theta\right)_{l^{(1)}} = \sqrt{\left(\kappa_{xx}^{(2)} \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{xy}^{(2)} \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2 + \left(\kappa_{yx}^{(2)} \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{yy}^{(2)} \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2} \cdot$$

$$\sin\left(\left(\theta\right)_{l^{(1)}} + \arccos\left(\frac{\kappa_{xx}^{(2)} \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{xy}^{(2)} \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}}{\sqrt{\left(\kappa_{xx}^{(2)} \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{xy}^{(2)} \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2 + \left(\kappa_{yx}^{(2)} \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{yy}^{(2)} \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2}} \right) \right)$$

(39)

$$\kappa_o \xi \cdot \sin\left(\theta\right)_{l^{(2)}} = \sqrt{\left(\kappa_{xx}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{xy}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2 + \left(\kappa_{yx}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{yy}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2} \cdot$$

$$\sin\left(\left(\theta\right)_{l^{(2)}} - \arccos\left(\frac{\kappa_{xx}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{xy}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}}{\sqrt{\left(\kappa_{xx}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{xy}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2 + \left(\kappa_{yx}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|} - \kappa_{yy}^{(2)}\phi \frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}\right)^2}} \right) \right)$$

(40)

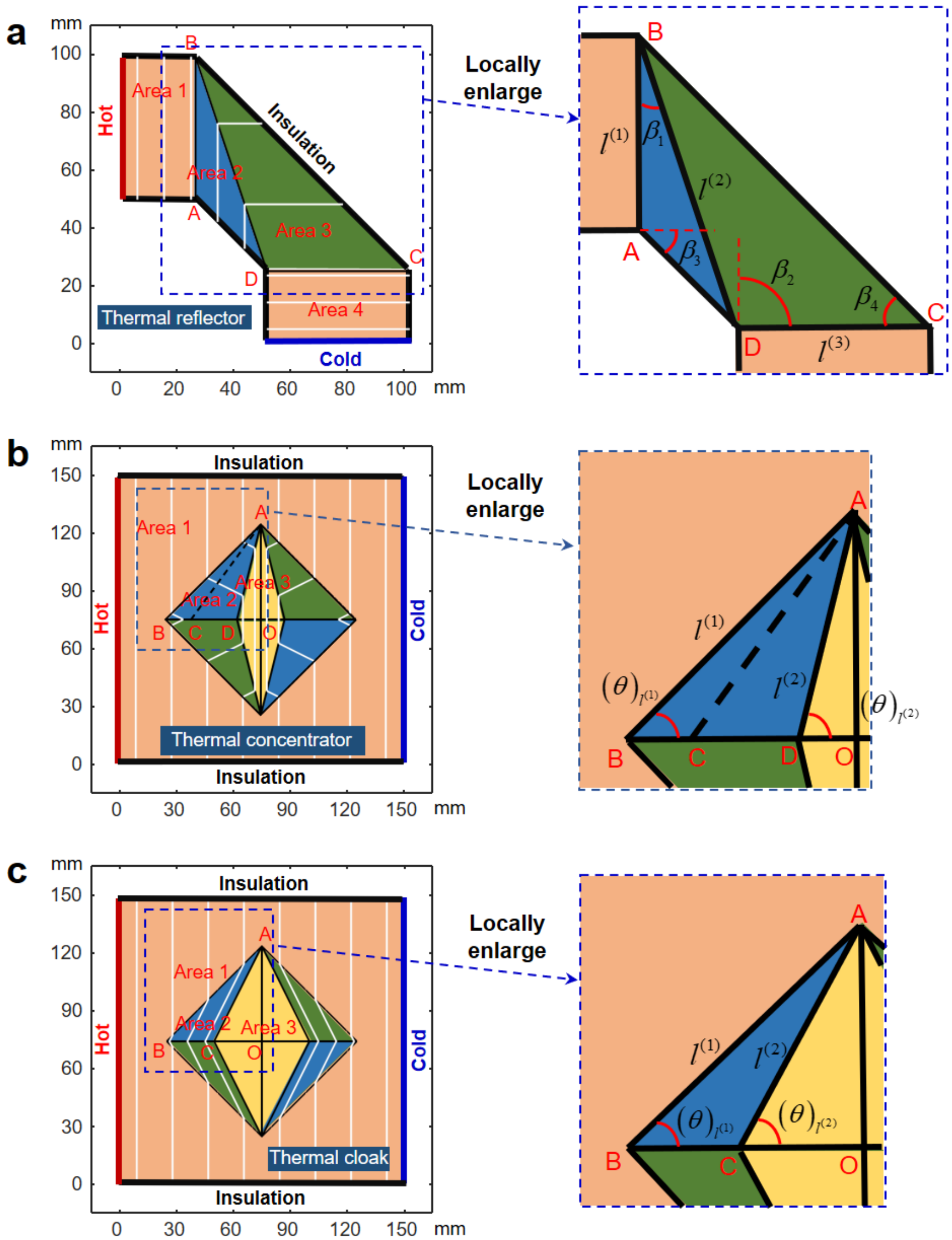
Combining Equations (39) and (40), we obtain a general solution as follows:

$$
\begin{aligned}
&\kappa_{yx}^{(2)}\frac{|\mathrm{OB}|}{|\mathrm{CB}|}-\kappa_{yy}^{(2)}\frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}=0\\
&\kappa_{xx}^{(2)}\frac{|\mathrm{OB}|}{|\mathrm{CB}|}-\kappa_{xy}^{(2)}\frac{|\mathrm{OB}|}{|\mathrm{CB}|}\cot\left(\theta\right)_{l^{(2)}}=\kappa_b\\
&\kappa_o=\kappa_b\phi/\xi
\end{aligned}
\tag{41}
$$

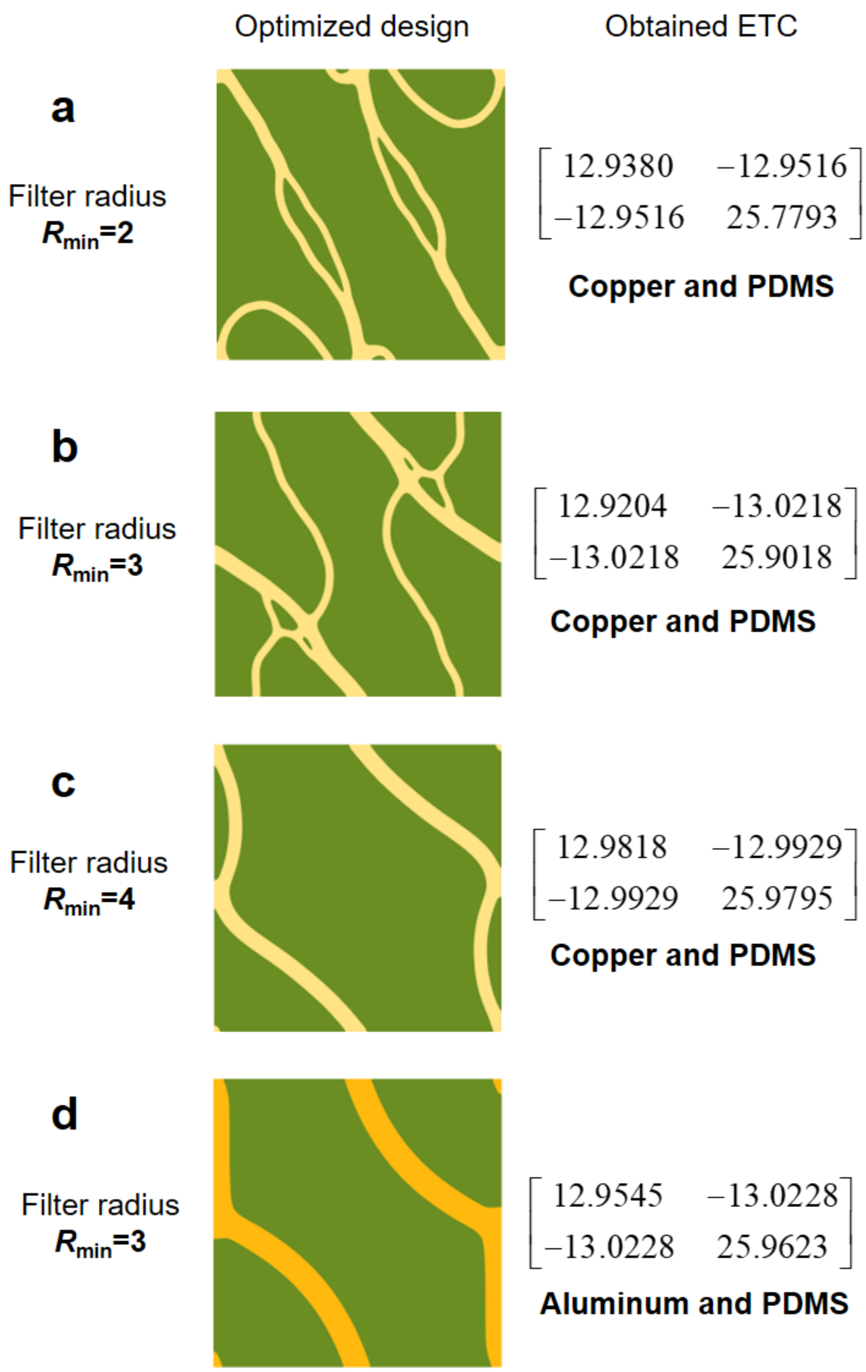
When $|\mathrm{OB}|=2|\mathrm{CB}|=50\ mm$, $\left(\theta\right)_{l^{(1)}}=45^\circ$ and $\cot\left(\theta\right)_{l^{(2)}}=1/2$, Equation (41) can be simplified to

$$
\begin{aligned}
&2\kappa_{yx}^{(2)}-\kappa_{yy}^{(2)}=0\\
&2\kappa_{xx}^{(2)}-\kappa_{xy}^{(2)}=\kappa_b\\
&\kappa_o=\kappa_b\phi/\xi
\end{aligned}
\tag{42}
$$

Based on Equations (19), (30), (36) and (42), we provide a general solution in Supplementary Table 1 for designing four thermal metadevices with the consideration of the reciprocity and nonnegativity of thermal conductivity tensor. The corresponding numerical simulations are displayed in Figs. 3(a) to 3(c) of main article.



Supplementary Figure 4. Design of thermal metamaterials. (**a**) Thermal reflector. (**b**) Thermal concentrator. (**c**) Thermal cloak. Different colors represent different areas. The orange area represents the background, the yellow area represents thermal functionality area, and the blue and green areas represent thermal metamaterials. The white curves are isotherms and parallel in each area.



Supplementary Figure 5. Optimized design for $\kappa_1 = \begin{bmatrix} 13 & -13 \\ -13 & 26 \end{bmatrix}$ with different parameters. (a)-(c) Optimized design with different filter radius (a) R_{min}=2, (b) R_{min}=3 and (c) R_{min}=4. (**d**) Optimized design with aluminum (237 $Wm^{-1}K^{-1}$) and PDMS. The different colors denote different materials in the mixture structures.

Supplementary Table

Supplementary Table 1. Details for designing thermal metamaterials

Thermal metamaterials	Area	Components of temperature gradient	Thermal conductivity tensor (a feasible solution)
Thermal connector	1	$\begin{cases} \dfrac{\partial T^{(1)}}{\partial x} = -\phi \\ \dfrac{\partial T^{(1)}}{\partial y} = 0 \end{cases}$	$\begin{bmatrix} 13 & 0 \\ 0 & 13 \end{bmatrix}$
	2	$\begin{cases} \dfrac{\partial T^{(2)}}{\partial x} = -\phi \\ \dfrac{\partial T^{(2)}}{\partial y} = 0 \end{cases}$	$\begin{bmatrix} 13 & 13 \\ 13 & 26 \end{bmatrix}$
	3	$\begin{cases} \dfrac{\partial T^{(3)}}{\partial x} = -\phi \\ \dfrac{\partial T^{(3)}}{\partial y} = 0 \end{cases}$	$\begin{bmatrix} 13 & 13 \\ 13 & 26 \end{bmatrix}$
	4	$\begin{cases} \dfrac{\partial T^{(4)}}{\partial x} = -\phi \\ \dfrac{\partial T^{(4)}}{\partial y} = 0 \end{cases}$	$\begin{bmatrix} 13 & 0 \\ 0 & 13 \end{bmatrix}$
Thermal reflector	1	$\begin{cases} \dfrac{\partial T^{(1)}}{\partial x} = -\phi \\ \dfrac{\partial T^{(1)}}{\partial y} = 0 \end{cases}$	$\begin{bmatrix} 13 & 0 \\ 0 & 13 \end{bmatrix}$
	2	$\begin{cases} \dfrac{\partial T^{(2)}}{\partial x} = -\phi \\ \dfrac{\partial T^{(2)}}{\partial y} = 0 \end{cases}$	$\begin{bmatrix} 13 & -13 \\ -13 & 26 \end{bmatrix}$
	3	$\begin{cases} \dfrac{\partial T^{(3)}}{\partial x} = -\phi \tan\beta_1 \cot\beta_2 \\ \dfrac{\partial T^{(3)}}{\partial y} = \phi \tan\beta_1 \end{cases}$	$\begin{bmatrix} 130/3 & -39 \\ -39 & 39 \end{bmatrix}$
	4	$\begin{cases} \dfrac{\partial T^{(4)}}{\partial x} = -\phi \cot\beta_2 \\ \dfrac{\partial T^{(4)}}{\partial y} = \phi \end{cases}$	$\begin{bmatrix} 13 & 0 \\ 0 & 13 \end{bmatrix}$

Thermal concentrator	1	$\begin{cases} \dfrac{\partial T^{(1)}}{\partial x} = -\phi \\ \dfrac{\partial T^{(1)}}{\partial y} = 0 \end{cases}$	$\begin{bmatrix} 13 & 0 \\ 0 & 13 \end{bmatrix}$
	2	$\begin{cases} \dfrac{\partial T^{(2)}}{\partial x} = -\phi \dfrac{\lvert \mathrm{CB} \rvert}{\lvert \mathrm{DB} \rvert} \\ \dfrac{\partial T^{(2)}}{\partial y} = -\phi \dfrac{\lvert \mathrm{DC} \rvert}{\lvert \mathrm{DB} \rvert} \cot(\theta)_{l^{(1)}} \end{cases}$	$\begin{bmatrix} 169/3 & -26/3 \\ -26/3 & 13/3 \end{bmatrix}$
	3	$\begin{cases} \dfrac{\partial T^{(3)}}{\partial x} = -\phi \dfrac{\lvert \mathrm{OC} \rvert}{\lvert \mathrm{OD} \rvert} \\ \dfrac{\partial T^{(3)}}{\partial y} = 0 \end{cases}$	$\begin{bmatrix} 13/3 & 0 \\ 0 & 39 \end{bmatrix}$
Thermal cloak	1	$\begin{cases} \dfrac{\partial T^{(1)}}{\partial x} = -\phi \\ \dfrac{\partial T^{(1)}}{\partial y} = 0 \end{cases}$	$\begin{bmatrix} 13 & 0 \\ 0 & 13 \end{bmatrix}$
	2	$\begin{cases} \dfrac{\partial T^{(2)}}{\partial x} = -\phi \dfrac{\lvert \mathrm{OB} \rvert}{\lvert \mathrm{CB} \rvert} \\ \dfrac{\partial T^{(2)}}{\partial y} = \phi \dfrac{\lvert \mathrm{OB} \rvert}{\lvert \mathrm{CB} \rvert} \cot(\theta)_{l^{(2)}} \end{cases}$	$\begin{bmatrix} 13 & 13 \\ 13 & 26 \end{bmatrix}$
	3	$\begin{cases} \dfrac{\partial T^{(3)}}{\partial x} = -\xi \\ \dfrac{\partial T^{(3)}}{\partial y} = 0 \end{cases}$	$\begin{bmatrix} 230 & 0 \\ 0 & 230 \end{bmatrix}$