Integrated demand forecasting and planning model for repairable spare part: an empirical investigation

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Abstract

Efficient resource management methods are essential for spare parts used in the maintenance and repair of equipment. Forecasting plays a critical role in planning, especially under demand uncertainty. Existing works regarding spare parts with intermittent demand focus on the mere forecasting model while integrating the planning and forecasting models are not sufficiently investigated. We examine the interaction between two models to optimise planning and forecasting decisions and prevent sub-optimality. This paper presents two mathematical models, including a planning model that determines stock level, spare part order assignment to suppliers, equipment repair assignment, and the number of intervals over the planning horizon. The second model is the forecasting model by Support Vector Machine (SVM). Considering uncertainty, demand estimation is performed by piecewise linearization considering the optimal number of intervals in the planning model used in forecasting. An interactive procedure is developed to optimise models. We use an empirical investigation from an oil company providing the spare part supply chain data. The analyses show that demand estimation by piecewise method and integrating the decisions optimises the cost, improves the forecasting accuracy, and planning performance. Moreover, we offer several insights to practitioners that shed light on spare part planning and forecasting decisions.

Keywords: Forecasting, Inventory management, Planning, Spare part, Support Vector Machine

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1. Introduction

Most organisations consider profitability a vital objective, so the decisions should be organised to facilitate achieving the goals and competing in the global market. Unexpected downtime can be disastrous for continuous production and the company's bottom line, i.e., the net profit (Comparesoft 2021). Given the equipment maintenance, developing the operations above standard causes safety risks, business risks, and operational obstacles; therefore, these risks may affect the continuous operations of ageing assets (Keystone Energy Tools 2021). Maintenance and repair are critical operations in industries that guarantee production continuation, supported by robust planning integrated with a well-structured forecasting approach. Repair operations significantly impact downtime costs causing the lost sale, which can never be regained (World oil 2021); in other words, a few hours of downtime can prominently be critical in a strategic industry (Jin and Liao 2009). A spare part supply chain should provide a high service level since the shortage lead to prominent inventory costs; however, overstocking may cause huge costs (Cantini et al. 2022).

We focus on the high-value and low-demand spare parts that are the most important resources used in equipment repair operations, which may be crippled if their required resources are not sufficiently supplied (Christiansen 2021). Optimising the resources has gained increasing attention worldwide, so there are many tools to manage the activities for this purpose. Circular Economy (CE) is one of the concepts for optimising resources which shows the growing popularity regarding the increasing attention to supply, production, and distribution (Gonzalez et al. 2019). Spare parts have characteristics that distinguish them from other inventories, such as criticality, speciality, demand pattern, and price (value). Moreover, in the US military, the repairable spare parts were estimated at $10 billion in 1976 (Nahmias 1981). The government accountability office reports that the items are worth about $4.3 billion just in one of the army's depots, reported in 2021, which points out the importance of using an effective forecasting method to optimise planning. The spare part's demand pattern has two aspects: quantity and forecastability; the irregular pattern makes it difficult to control forecasting (Huiskonen 2001).

Integrating the supply and repair decisions can improve planning and increase supply chain responsiveness. Existing research is based on Multi-Echelon Technique for Recoverable Item Control (METRIC) model that discusses repairable spare part inventory management with uncertain demand during the lead-time by a probability distribution with constant parameters; however, using data for estimation is both theoretically and practically important, mainly when uncertain intermittent demand exists. Goltsos et al. (2019) declare that forecasting is vital in any supply chain, especially when dealing with a closed-loop network, the uncertainty in return rate (quantity and time) and quality becomes important. We implement the renewal process using piecewise linearization to approximate the demand in each time interval over the planning horizon, enhancing planning effectiveness by improving the demand estimation. The estimated demand is used both in planning and forecasting. Indeed, this method determines the number of intervals so that the demand in each interval becomes near-linear, which is simpler to deal with. We use "interval" instead of time interval throughout this article for simplicity.

Several kinds of research regarding the intermittent demand forecasting models, such as (Balugani et al. 2019; Prestwich et al. 2014; A.A. Syntetos, Babai, and Altay 2012), consider forecasting by a single model while integrated planning and forecasting models can improve the results. Cyplik et al. (2009) discuss integrated planning and forecasting decisions, but demand estimation, supply, and repair decisions are not sufficiently investigated. According to Goltsos et al. (2022), a 1% reduction in forecast errors can lead to a 10–15% decrease in inventory costs, which illustrates the criticality of considering the integration between forecasting and planning and prevents overstocking and shortages. Moreover, as Tapia-Ubeda et al. (2020) declared, the integration of various decisions is vital to enhancing the supply chain performance and cost-effectiveness

We are looking to answer the following questions: 1) what are the optimal forecasting model parameters for the spare parts with intermittent demand? 2) what is the optimal number of intervals (periods)? 3) what failed equipment should be assigned to each repair centre? 4) what and how many spare parts to order? 5) where the repaired equipment should be held (warehouses or installation bases)? 6) what is the optimal stock level of spare parts?

This paper presents two models: the repairable spare part planning that determines the stock level of warehouses, repair, order assignment, and the optimal number of the planning intervals, and an SVM-based forecasting model that uses the estimated demand considering the optimal number of intervals from the planning model. An interactive procedure optimises the models. An important factor in forecasting accuracy is the number of periods (intervals) that affects the demand estimation. The estimated demand is the basis of forecasting in future periods. The rest of the article is organised into the following sections: a literature review is presented in section 2. Then, the problem and model are described in section 3. The case study, computations, and results are provided in section 4. The conclusion is expressed in section 5.

2. Literature review

The related works regarding spare part supply chain planning and forecasting are discussed below. The literature review findings are provided as the research gaps at the end of this section.

2.1. Planning models

Spare part characteristics distinguish it from other similar products in the aspect of demand, value, and other properties. As one of the most critical activities in industrial sections, inventory management involves supplying, ordering, purchasing, and inventory control. All the activities should be organised in a way that helps meet the demand while minimising total costs (Haj shirmohammadi 2014). If the inventory level is too high, the inventory costs will be high while the shortage decreases. A low inventory level decreases the inventory costs while shortages increase.

Sherbrooke (1968) developed a METRIC model, as the pioneer, to optimise the stock level aiming to minimise costs. Yangi and Saski (1991) considered the same model with a continuous review policy that applies the repair capacity in repair centres to evaluate the performance. Subsequently, Axsäter (2003) examined a single-echelon, single-item model examining unilateral transhipment between warehouses to reduce shortages. Jain and Raghavan (2009) extended the queuing model for inventory planning in a multi-tier supply chain. Manufacturers, warehouses, and vendors are considered in this network, and the M/M/∞ queuing model evaluates the performance. Considering the basic model, (Hertzler 2010) investigated the effect of the replacement for the low-demand, high-value spare parts. Using the decision tree model and the Markov chain, they also analysed the effect of unidirectional replacement in a multi-period supply chain.

Van Jaarsveld et al. (2015) used an integer programming model for a multi-location, single-echelon, and multi-item repairable spare parts network. Continuous inventory review and base stock replenishment policy are considered in this problem. Lateral transhipment is allowed to reduce the shortage probability. Tavakkoli-Moghaddam et al. (2018) developed a two-tier closed-loop spare part supply chain, including distributor, repair centres, and operating bases considering multi-modal repairs, formulated for location-allocation to minimise costs and determine each spare part's repair service mode. Ruiz-Torres et al. (2019) published a paper on optimising supplier selection for a spare part network. Supplier capacity is deterministic, but the returned spare part is uncertain.

Bitton and Cohen (2019) focused on the repairable spare parts network in the aviation industry. In this study, two groups of spare parts are analysed to obtain optimal decisions regarding inventory management and repair. No-Go spare parts ground the aircraft when they fail, but Go parts' failure allows the aircraft to be operational for a predetermined period. This research uses Erlang-A (M/M/c) and Erlang-B (M/M/c/c) models for the second and first groups of parts, respectively. Mohtashami et al. (2020) considered a green closed-loop network to minimise negative environmental effects. The long waiting time in queues is due to the limited capacity of the facilities such as distribution, recycling, and repair centres. The queuing model is defined as G/M/S, aiming to minimise the waiting time to reduce negative environmental effects. Qin et al. (2021) presented two models with profit- and cost-centric objective functions for a two-echelon repairable spare part service network. The models consider the performance assessment in repair centres under the uncertain failure rate.

The reviewed researches investigated the performance assessment in planning decisions by queuing models to obtain optimal decisions, while supply decisions are rarely examined besides the other decisions. Also, other research does not consider repair constraints such as repair expertise (skills) and capacity, which are commonly used in real-world problems. Moreover, there is no order assignment decision to suppliers regarding the supply decisions integrating with repair and inventory management. Most of the works focused on the basic METRIC model from the aspect of uncertainty, while this method may not fit the demand pattern in many cases.

2.2. Forecasting models

Demand plays a crucial role in spare part supply chain planning since the stock level depends significantly on demand and value. Since uncertainty in demand is a critical factor in forecasting, we first review the literature regarding the spare part demand uncertainty; then, the forecasting context is examined.

Machine learning (ML) is the branch of artificial intelligence (AI) that allows applications to obtain results without programming for every purpose. Historical data are used as the input for prediction. Machine learning is divided into different types according to the algorithms used in learning: 1) unsupervised learning, 2) supervised learning, 3) semi-supervised learning, and 4) reinforcement learning. The algorithm is selected based on the data used in the prediction (Burns 2021).

Demands of spare parts have specific characteristics that make forecasting complicated. These characteristics are such as 1) type of demand: I) Intermittent (irregular demand occurrence with low demand quantity variation); II) Lumpy (irregular demand occurrence with high demand quantity variation); III) Erratic (regular demand occurrence with high demand quantity variation); IV) Smooth (regular demand occurrence with low demand quantity variation) 2) Dependence on descriptive factors: There are several factors related to maintenance and repair, and working condition that affects the failure rate, i.e., the demand (SILVER 1981).

Croston (1972) proposed the first forecasting model for intermittent demand that divides the demand intervals and quantity. He used exponential smoothing in forecasting, which outperforms routine exponential smoothing. Also, Aris A. Syntetos and Boylan (2005) developed the Syntetos-Boylan approximation (SBA), and (Babai et al. 2019) presented a modified SBA (MSBA). Lindsey and Pavur (2014) uses the Bayesian model to forecast intermittent demand to minimise costs that obtain spare parts' inventory level when demand is uncertain. Hua and Zhang (2006) developed an approach that assesses the effectiveness of time series and descriptive variables on demand forecasting. They also adapted the Support Vector Machine model to forecast demand. Using the descriptive variables with the proposed model improves the forecast accuracy.

Amin-Naseri and Tabar (2008) examined recurrent neural network (NN) for forecasting spare parts with lumpy demand. They compared the proposed approach with Boylan, Synetos, and Croston's methods. The results confirmed the superiority of the NN-based AI. (Amirkolaii et al. 2017) studied the forecasting approach for irregular demands of spare parts. They analysed the neural network for single- and multi-feature demand. The results show more accuracy for spare parts with higher features, especially those with intermittent demands. (N. Pawar and B. Tiple 2019) compared the accuracy of the ML when new features (variables and predictors) are added to the model. The results show that adding new features enhances accuracy. Yang et al. (2021) studied the spare part classification using convolutional neural networks. They considered various criteria to categorise the spare parts. A transfer learning-based is used according to stock level, characteristics, cost, and lead-time criteria. Dodin et al. (2021) developed a machine learning forecasting framework for intermittent and non-intermittent demands of aircraft spare parts in the aftermarket to pursue the recent literature. The proposed framework prominently improves the accuracy and run time.

The literature review shows that the relation of demand estimation in planning models with forecasting is not investigated, especially in nonparametric methods that are data-based. Integrating forecasting and planning models can efficiently bridge this research gap, especially for the repairable spare parts supply chain.

2.3. Research gaps

The findings of the reviewed literature are listed below:

- Demand is a critical factor used in forecasting, affecting planning performance. Many research considers a probability distribution for demand estimation, but existing methods may not fit the demand pattern appropriately, especially for spare parts with intermittent demand, which highly affects demand forecasting.
- Parameters in forecasting models such as SVM are often separately optimised while reflecting planning decisions is scarcely discussed for optimising decisions considering the demand estimation by piecewise linearization.
- The integrated supply and repair decisions are not comprehensively considered in other models; for example, the supply capacity, defect, and delivery time constraints are significant decisions affecting the planning performance. Also, repair constraints such as capacity, time, and expertise are critical constraints that are not considered in other research. Integrating the above decisions contributes to the literature and prevents sub-optimal solutions.

3. Model

This section discusses the problem description and model formulation, including forecasting and planning models.

3.1. Problem description

The present research considers a repairable spare part supply network (RSPSN) model regarding supply and repair decisions. The RSPSN is illustrated in Figure 1. The flows begin where the equipment is installed. The failed equipment is replaced with an operational one that is intact, and the defective one is moved to inspection centres where the experts are gathered to examine the entries from technical aspects to determine whether the repair is economical.

The repair assignments to repair centres are analysed considering the repair time, expertise, technical, and capacity aspects. The company assigns the repair operations to either inner or outer-company repair centres based on recent factors. Inner-company repair centres depend on the company's resources, such as labour, material (spare parts used in repairing), and the budget, while the outer-company repair centres serve the company and receive the repair cost based on the predefined contract. Central warehouses provide the spare parts to repair centres used in repairing the equipment. The order assignment to suppliers is performed according to price, delivery time, and defect rate.

In case of failure, the spare parts are stocked in central and local warehouses to meet the demands of installation bases and central warehouses. The central warehouses also supply the local warehouses, and the demands of installation bases are supplied from the local warehouses. Additionally, the central warehouses supply the repair centres for the spare parts used in repair operations. The optimal decisions regarding the planning model are such as order assignment to suppliers, including what to order and the amount of order, assignment of the failed equipment to repair centres considering the repair capacity, expertise, and time, the stock level of central and local warehouses, repair equipment from repair centres to central and local warehouses and installation bases, the number of time intervals in the planning horizon. The demand, i.e., the failure rate, may follow a known distribution in the present planning horizon, but this distribution cannot be reliable for long-term forecasting since demand's mean and variance may change over the planning horizon. Hence, integrating the planning and forecasting models can improve the performance of models.

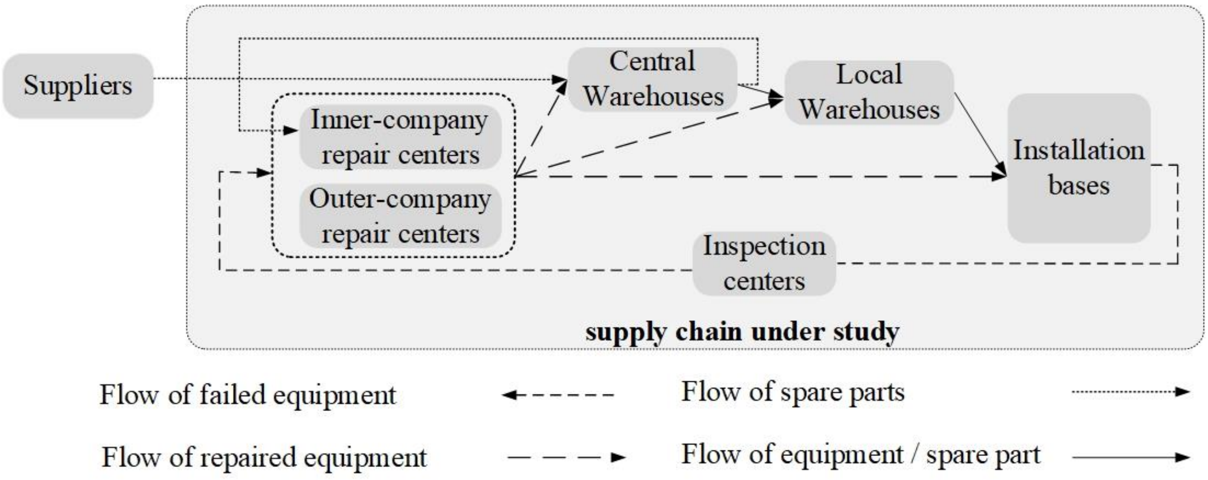


Figure 1. RSPSN representation

Figure 1 Alt Text. This figure shows the network structure, type and direction of flows. The figure shows the network that includes suppliers, repair centres, warehouses, inspection centres, and installation bases. The equipment moves from installation bases to repair centres; then, they move to warehouses and installation bases when they are repaired. Also, the spare parts move from suppliers to warehouses and warehouses to repair centres.

3.2. Notations

- **Indices and sets**

$s \in S$ where $s_1, s_2 \subseteq s$	Equipment / Spare parts
$j, j' \in J$ where $w, r, i, c, s' \subseteq j, j'$	All nodes
$w, w' \in W; w_1 \subseteq w; w_2 \subseteq w$	Warehouses; Central warehouses; Local warehouses
$r \in R;\ r_1 \in r; r_2 \in r$	Repair centres; Inner-company; Outer-company repair centres
$i \in I$	Inspection centre
$c \in C$	Installation bases
$s' \in S'$	Suppliers
$l \in N$	Number of time intervals

- **Parameters**

d_{sc}	Spare part *s* demand (failure rate) at installation base *c*
$tc_{sjj'}$	Transportation cost from node *j* to j'
rt_{sr}	Spare part *s* repair time in repair centre *r*
$scap_{ss'}$	Capacity of supplier s' for spare part *s*
$pr_{ss'}$	Spare part *s* procurement cost ordered from supplier s'
$pu_{s_1s_2}$	Probability of demand for spare part s_1 in equipment $s_2 \in s$ in repair centre, otherwise 0
h_{sj}	Spare part *s* holding cost in facility *j* (warehouse *w* or repair centre *r*)
$def_{ss'}$	Defect rate of spare part *s* from supplier s'
$del_{ss'}$	Spare part *s* delivery time from supplier s'
$mdef_s$	Maximum mean defect of spare part *s*
$mdel_s$	Minimum acceptable mean delivery time of spare part *s*
I_{sw}^0	Initial inventory of spare part *s* in warehouse *w*
$\tau_{sw_1w_2}$	Traveling time of spare part *s* from central warehouse w_1 to local warehouse w_2
π'_s	Shortage cost of spare part *s*
$\mu_{ss'w_1}$	Spare part *s* supply time from supplier s' to central warehouse w_1

$\tau_{sw_1} = \sum_{s'} \mu_{ss'w_1}$	Travel time of spare part *s* from supplier s' to central warehouse w_1
G_{si}	Probability of repairability of equipment *s* in inspection centre *i*
rc_{sr}	The repair cost of spare part *s* in repair centre *r*
cp_{rs}	1, if repair centre *r* has the capability of repairing spare part *s*, otherwise 0
cap_r	Repair centre *r* capacity (Man-Hour)

- **Decision variables**

x'_{scil}	Amount of equipment *s* from installation base *c* to inspection centre *i* in interval *l*
y'_{sirl}	Amount of equipment *s* from inspection centre *i* to repair centre *r* in interval *l*
$x^{(1)}{}_{ss'w_1l}$	Amount of spare part *s* from supplier s' to central warehouse w_1 in interval *l*
$x^{(2)}{}_{srw_1l}$	Amount of equipment *s* from repair centre *r* to central warehouse w_1 in interval *l*
$y^{(1)}{}_{sw_1w_2l}$	Amount of equipment *s* from central warehouse w_1 to local warehouse w_2 in interval *l*
$y^{(2)}{}_{srw_2l}$	Amount of equipment *s* from repair centre *r* to local warehouse w_2 in interval *l*
$z^{(1)}{}_{sw_2cl}$	Amount of equipment *s* from local warehouse w_2 to installation base *c* in interval *l*
$z^{(2)}{}_{srcl}$	Amount of equipment *s* from repair centre *r* to installation base *c* in interval *l*
$wa_{sw_{1l}}$	Spare part *s* waiting time in the central warehouse w_1 in interval *l*
ws_{sw_1rl}	Amount of spare part *s* from central warehouse w_1 to repair centre *r* in interval *l*
I^+_{swl}	Average on-hand inventory of spare part *s* in warehouse *w* in interval *l*
I^-_{swl}	Average shortage of spare part *s* in warehouse *w* in interval *l*
st_{sjl}	Stock level of spare part *s* facility *j* (warehouse *w* or repair centre *r*) in interval *l*
λ_{swl}	Demand rate of spare part *s* in warehouse *w* in interval *l*

3.3. Formulation

3.3.1. Planning model

The planning model is formulated in this section. The objective function and constraints are presented below. We assume in this model, I) central warehouses do not confront shortages, II) demand of equipment (LRUs[1]) depends on spare parts (SRUs[2]), III) each SRU only lies in one LRU, IV) base stock (*S-1, S*) replenishment policy is used for all equipment and spare parts.

- **Objective function and constraints**

The objective function minimises total costs in each time interval *l* shown in Eqs. (3-1).

$Min\ Z_l$ Eq. (3-1)

$= \sum_s \sum_{s'} \sum_{w_1} tc_{ss'w_1} x^{(1)}{}_{ss'w_1l}$ transportation costs from suppliers to central warehouses

$+ \sum_s \sum_r \sum_{w_1} tc_{srw_1} x^{(2)}{}_{srw_1l}$ transportation cost from repair centres to warehouses and installation bases

[1] Line-Replaceable Unit
[2] Shop-Replaceable Unit

$$+\sum_{s}\sum_{r}\sum_{w_2} tc_{srw_2}{y^{(2)}}_{srw_2l} + \sum_{s}\sum_{r}\sum_{c} tc_{src}z^{(2)}_{srcl}$$

$$+\sum_{s}\sum_{w_1}\sum_{w_2} tc_{sw_1w_2}{y^{(1)}}_{sw_1w_2l}$$
transportation cost between the warehouses

$$+\sum_{s}\sum_{w_2}\sum_{c} tc_{sw_2c}\, z^{(1)}_{sw_2cl}$$
travelling cost from local warehouses to installation bases

$$+\sum_{s}\sum_{c}\sum_{i} tc_{sci}\, x'_{scil}$$
transportation cost from installation bases to inspection centres

$$+\sum_{s}\sum_{i}\sum_{r} tc_{sir}y'_{sirl}$$
transportation cost from installation bases to inspection and repair centres

$$+\sum_{s}\sum_{w_1}\sum_{r_1} tc_{sw_1r_1}\, ws_{sw_1r_1l}$$
travelling cost of spare parts used in repairing equipment, from central warehouses to repair centres

$$+\sum_{s}\sum_{w_1}\sum_{s'} pr_{ss'}{x^{(1)}}_{ss'w_1l}$$
Procurement cost, including ordering and purchase cost

$$+\sum_{s}\sum_{i}\sum_{r} rc_{sr}y'_{sirl}$$
repair cost

$$+\sum_{s}\sum_{w} h_{sw}I^{+}_{swl}$$
Holding cost in warehouses

$$+\sum_{s}\sum_{r} h_{sr}st_{srl}$$
Holding cost in repair centers

$$+\sum_{s}\sum_{w} {\pi'}_{s}I^{-}_{swl}$$
shortage costs

The renewal process is used to formulate the uncertainty in demand over the planning horizon that considers IID[3] variables (Jin and Tian 2012). First, we consider $N(t)$ as the number of random events in [0, *t*], then an integral equation is introduced as $u = a + u * F$ for an unknown function $u\colon [0,\infty) \to \mathbb{R}$ that is the renewal equation of *u*. $M_{sc}\ (t)$ is the number of arrivals in [0, *t*]. f_s and F_s are respectively the density and cumulative functions that are interpreted as the first arrival and time-to-failure over interval [0, *s*]. K_j and $Z_{sc}(t)$ respectively are the time of happening *j^th^* event the overall failures, shown in Eq. (3-2).

$$Z_{sc}(t) = M_{sc}(t) + \sum_{j=1}^{N_{sc}(t)} M_{sc}(t - K_{scj}) \qquad \text{Eq. (3-2)}$$

To simplify the time-varying demands, stepwise linearization is used. Accordingly, the length of each interval is *L* in the planning horizon with length *t*. Consider *t* as the length of interval *l* where $l \in \{1, \dots, N\}$. is calculated in Eq. (3-3).

$$n = \frac{t}{L_l} \qquad \text{Eq. (3-3)}$$

[3] Independent and identically distributed

It is assumed that the time-to-failure follows the exponential distribution with a rate of α. The average number of arrivals in [0, *t*] is presented in Eq. (3-4) (David and Nagaraja 2004).

$$E(Z_{sc}(t)) = \alpha_s t + \alpha_s t \frac{N(t)}{2} \qquad \text{Eq. (3-4)}$$

Considering the arrival rate as λ, $N(t) = \lambda t$. The average demand in interval [0, t_l] is calculated by Eq. (3-5). Then, it is rewritten for each spare part and installation base.

$$E(Z(t)) = \alpha t + \frac{\alpha \lambda t^2}{2}$$

$$\bar{d}_{scl} = \frac{E(Z_{sc}(t_l) - Z_{sc}(t_{l-1}))}{t_l - t_{l-1}} = \alpha + \frac{\alpha\lambda(t_l + t_{l-1})}{2}, l = 1, \dots, n \ ; \quad n = \frac{t}{L_l} \qquad \text{Eq. (3-5)}$$

The amount of failed equipment to inspection centres is calculated by Eq. (3-6), in which the right-hand side is reproduced from Eq. (3-5). Eq. (3-7) shows the balance equation at installation bases

$$\sum_i x'_{scil} = L_k . \bar{d}_{scl} \qquad \forall s, c, l \qquad \text{Eq. (3-6)}$$

$$\sum_{w_2} z^{(1)}{}_{sw_2cl} + \sum_r z^{(2)}{}_{srcl} = L_k . \bar{d}_{scl} \qquad \forall s, c, l \qquad \text{Eq. (3-7)}$$

Eqs. (3-8) – (3-14) are the METRIC model constraints such as expected on-hand inventory, shortage, and waiting time. The model developed by Sherbrooke (1968) for repairable items is later extended with adding multi-item, multi-location, capacity limitation, and other properties. Eq. (3-8) presents the average on-hand inventory. Eqs. (3-9) and (3-10) are the demand of local from central warehouses and the average shortage. The little law in Eq. (3-11) computed the expected waiting time given the average shortage. The average replenishment time in local warehouses is presented in Eq. (3-12). Finally, the average on-hand and shortage inventory of local warehouses are shown in Eqs. (3-13) and (3-14).

$$I^+_{sw_1l} = \sum_{j=1}^{st_{sw_1l}} j.P(X = j) = \sum_{j=1}^{st_{sw_1l}} j \frac{e^{-\lambda_{sw_1}\tau_{sw_1}} (\lambda_{sw_1l}\tau_{sw_1})^{st_{sw_1l} - j}}{(st_{sw_1l} - j)!} \qquad \text{Eq. (3-8)}$$

$$\lambda_{sw_1l} = \sum_{w_2} y^{(1)}_{sw_1w_2l} \qquad \text{Eq. (3-9)}$$

$$\forall s, w_1, l$$

$$st_{sw_1l} \geq I^+_{sw_1l} - I^-_{sw_1l} + \lambda_{sw_1l}\tau_{sw_1} \qquad \text{Eq. (3-10)}$$

$$wa_{sw_1l} = \frac{I^-_{sw_1l}}{\lambda_{sl}}, \lambda_{sl} \neq 0 \qquad \text{Eq. (3-11)}$$

$$\bar{\tau}_{sw_2l} = \sum_{w_1} (\tau_{sw_1w_2} + wa_{sw_1l}) \qquad \text{Eq. (3-12)}$$

$$\lambda_{sw_2l} = \sum_{w_1} y^{(1)}_{sw_1w_2l} \qquad \forall s, w_2, l$$

$$I^+_{sw_2l} = \sum_{j_s=1}^{st_{sw_2l}} j_s \times \frac{e^{-\lambda_{sw_2l}\bar{\tau}_{sw_2l}} (\lambda_{sw_2l}\bar{\tau}_{sw_2l})^{st_{sw_2l} - j_s}}{(st_{sw_2l} - j_s)!} \qquad \text{Eq. (3-13)}$$

$$I^-_{sw_2l} = I^+_{sw_2l} - (st_{sw_1l} - \lambda_{sw_2l}\bar{\tau}_{sw_2l}) \qquad \text{Eq. (3-14)}$$

Inner-company repair centres use tools, budget, materials, and energy, while outer-company repair centres operate independently. The minimum spare parts required in inner-company repair centres that the central warehouses may supply are calculated by Eq. (3-15). Eqs. (3-16) and (3-17) are the capacity and expertise constraints.

$$\sum_{w_1} ws_{sw_1r_1l} \geq pu_{ss_1} \times \sum_i y'_{s_1ir_1l} + st_{s_1r_1l} \qquad \forall s, r, l \qquad \text{Eq. (3-15)}$$

$$\sum_i rt_{sr} \times y'_{sirl} \leq cap_r \qquad \forall s, r \qquad \text{Eq. (3-16)}$$

$$\sum_i y'_{sirl} \leq M \times cp_{rs} \qquad \forall s, r, l \qquad \text{Eq. (3-17)}$$

The supply constraints related to capacity, maximum defect, and delivery time are shown in Eqs. (3-18) - (3-20). The balance equation in local and central warehouses are respectively presented in Eq. (3-21) and (3-22).

$$\sum_{w_1} x^{(1)}{}_{ss'w_1l} \leq scap_{ss'} \qquad \forall s, s', l \qquad \text{Eq. (3-18)}$$

$$\sum_{s'} \sum_{w_1} def_{ss'} x^{(1)}{}_{ss'w_1l} \leq mdef_s \sum_{w_1} \sum_{s'} x^{(1)}{}_{ss'w_1l} \qquad \forall s, l \qquad \text{Eq. (3-19)}$$

$$\sum_{w_1} \sum_{s'} del_{ss'} x^{(1)}{}_{ss'w_1l} \leq mdel_s \sum_{w_1} \sum_{s'} x^{(1)}{}_{ss'w_1l} \qquad \forall s, l \qquad \text{Eq. (3-20)}$$

$$I^0_{sw_2} + \sum_{w_1} y^{(1)}{}_{sw_1w_2l} + \sum_r y^{(2)}{}_{srw_2l} = st_{sw_2l} + \sum_c z^{(1)}{}_{sw_2cl} \qquad \forall s, w_2, l \qquad \text{Eq. (3-21)}$$

$$I^0_{sw_1} + \sum_{s'} x^{(1)}{}_{ss'w_1l} + \sum_r x^{(2)}{}_{srw_1l} = st_{sw_1l} + \sum_{w_2} y^{(1)}{}_{sw_1w_2l} + \sum_r ws_{sw_1rl} \qquad \forall s, w_1, l \qquad \text{Eq. (3-22)}$$

The amount of repairable equipment is computed in Eq. (3-23) based on the probability of repairability. Eq. (3-24) shows the flow balance in repair centres. Finally, domains of variables are presented.

$$\sum_r y'_{sirl} = \sum_c G_{si} \times x'_{scil} \qquad \forall s, i, l \qquad \text{Eq. (3-23)}$$

$$\sum_i y'_{sirl} = \sum_{w_1} x^{(2)}{}_{srw_1l} + \sum_{w_2} y^{(2)}{}_{srw_2l} + \sum_c z^{(2)}{}_{srcl} \qquad \forall s, r, l \qquad \text{Eq. (3-24)}$$

$$x'_{scil},\ y'_{sirl},\ x^{(1)}_{ss'w_1l}, x^{(2)}_{srw_1l},\ y^{(1)}_{sw_1w_2l},\ y^{(2)}_{srw_2l},\ z^{(1)}_{sw_2cl},\ z^{(2)}_{srcl},\ st_{swl} \in \mathbb{Z}^+;\quad I^+_{swl},\ I^-_{swl} \in \mathbb{R}^+$$

3.3.2. Forecasting model

Spare part intermittent demand forecasting methods are divided into three categories: time-series, contextual forecasting, and comparative studies (Pinçe e al. 2021). Contextual forecasting refers to external factors such as equipment life, maintenance scheduling, and working conditions. Comparative studies provide performance benchmarks for spare part forecasting models based on the available data sets. Spare part demand forecasting by time series includes parametric and nonparametric approaches that we focus on the latter one. Parametric approaches consider the demand over the lead-time as a predefined parameter with a known probability distribution, while nonparametric approaches extract the distribution from the data.

The demand may follow a known distribution in the present planning horizon, but the specified distribution cannot be reliable for future forecasting since demand's mean and variance may change over the planning horizon; therefore, we use a nonparametric method to determine the future demand.

Machine learning techniques use learning algorithms to determine demand patterns. Support Vector Machine (SVM) is a supervised algorithm that provides accurate results compared with other models. This method does not depend on the model and requires a smaller number of parameters. Also, it is not dependent on the model type, such as linear and stationary process, and is guaranteed to obtain the optimal solution (N. I. Sapankevych and R. Sankar 2009).

This method has its cons, e.g., it is time-consuming, and its robustness is not sufficiently investigated. Jiang, et al. (2021) present a new version of SVM considering outliers and errors that outperform the basic model considering computation time and non-smooth demand forecasting accuracy; however, the validation period is constant. $\langle w, x\rangle$ is defined as the dot product where *w, x,* and b are the coefficient, attribute vector, and intercept. It is allowed that data fluctuates in the interval $[-\varepsilon, +\varepsilon]$, the outlier distance is defined as the penalty $\delta_{sj}^{l,i'}$ where $i, j \in I$ counts the data samples used in the training process. The nonlinear form of attribute is generally denoted by $\emptyset(x)$. The forecasting error formulated by Mean Absolute Scaled Error (MASE) is used to measure accuracy. An important factor in forecasting accuracy is the number of intervals utilised in estimating demand. We determine the number of intervals by the planning model, expressed in section 3.3.1. The indices may be omitted for simplicity while explaining.

First, it is assumed that the demand for each spare part is initially considered to be time-independent. The initial demand obtained by $d_s^l = \sum_c d_{sc}^l$ initialising the forecasting model. $d_s^{l,i'} = \sum_l^{l+i'-1} \sum_c d_{sc}^l$ is the demand of each spare part in interval with length i' from period *l*. A so-called kernel function is implemented to estimate the random variable's conditional expectation. Kernel function computes the dot product of two vectors, which means how much of the force vector is applied in the direction of the other vector. The forecast variable is denoted by $F_s^{l,i'}(\omega_{i'})$ where $\omega = \{\Psi, \varepsilon, \xi\}$. $F_s^{l,i'}(\omega_{i'})$ is the forecast for i' periods later from *l*. The initial parameters in the kernel function are $d \in N$ and $r \geq 0$. The learning process includes the initial training, validation, and testing processes defined for each spare part. Given the planning horizon with length *T*, it is divided into three parts 1 to T_1 for initial training, $T_1 + 1$ to $T_1 + V$ for validation, and $T_1 + V + 1$ to *T* for the testing process. Considering the validation process, it is decomposed into the main model in Eqs. (3-25)-(3-27), and subproblem in Eqs. (3-28) - (3-32). The strong dual counterpart is shown in Eqs. (3-33)-(3-35) used to simplify the existing complexities. The subproblem determines the optimal parameters of the regression line while the main model generates the hyperparameters by calling the dual model.

$$\omega_{i'}^* = \arg\min_{\omega_{i'}} \sum_{l=T_1+1}^{T_1+V} \sum_{s=1}^{S} \frac{\frac{\left|d_s^{l,i'} - F_s^{l,i'}(\omega_{i'})\right|}{S.V}}{\frac{\sum_{q=T_1+2}^{T_1+V} \left|d_s^{q,i'} - d_s^{q,i'-1}\right|}{V-1}} \qquad \forall i' \qquad \text{Eq. (3-25)}$$

$$F_s^{l,i'}(\omega_{i'}) = \sum_{j=1}^{l-i'} b_s^{l,i'}(\omega_{i'}) + \left(u_{j,s}^{l,i'}(\omega_{i'}) - u'^{\,l,i'}_{j,s}(\omega_{i'})\right) \varphi_{i'}(x_s^j, x_s^l) \qquad \forall\, i', s, l \qquad \text{Eq. (3-26)}$$

$$\varphi_{i'}(x_s^j, x_s^l) = \begin{cases} (x_s^j)^T x_s^t + r, & linear\ form \quad \forall s, l, i' \\ \exp\left(-\gamma_{i'} \left\|x_s^j - x_s^l\right\|^2\right), & radial\ basis\ form \quad \forall s, l, j, i' \\ [\gamma_{i'} \left(x_s^j\right)^T x_s^t + r]^d, \gamma_{i'} > 0 & polynomial\ form \quad \forall s, l, j, i' \\ \tanh\left(\gamma_{i'} (x_s^j)^T x_s^t + r\right), & sigmoid\ form \quad \forall s, l, j, i' \end{cases} \qquad \text{Eq. (3-27)}$$

The subproblem model formulation determines the optimal parameters of the regression line, which are the SVM variables. The support vectors are formulated in Eqs. (3-29) and (3-30). δ and δ' are the slack variables that capture the errors out of $[-\varepsilon, +\varepsilon]$. The objective function in Eq. (3-28) includes two terms; the first term is flatness, defined as the distance or volume surrounded by the hyperplanes. The balance between two terms of the objective function is tunned with a decision variable denoted by Ψ in the second term.

$$\left\{w_s^{l,i'}, b_s^{l,i'}, \delta_s^{l,i'}, \delta_s'^{\,l,i'}\right\}^* = \arg \min_{w_s^{l,i'}, b_s^{l,i'}, \delta_s^{l,i'}, \delta_s'^{\,l,i'}} \frac{1}{2}\left(w_s^{l,i'}\right)^T w_s^{l,i'} + \Psi_{i'} \sum_{j=1}^{l-i'} (\delta_{sj}^{l,i'} + \delta_{sj}'^{\,l,i'}) \quad \forall s, i', l \qquad \text{Eq. (3-28)}$$

$$d_s^{j,i'} - \left(w_s^{l,i'}\right)^T \emptyset(x_s^j) - b_s^{l,i'} \le \varepsilon_{i'} + \delta_{sj}^{l,i'} \qquad \text{Eq. (3-29)}$$

$$\left(w_s^{l,i'}\right)^T \emptyset(x_s^j) + b_s^{l,i'} - d_s^{j,i'} \le \varepsilon_{i'} + \delta_{sj}'^{\,l,i'} \quad \forall j, s, l, i' \qquad \text{Eq. (3-30)}$$

$$\delta_{sj}^{l,i'}, \delta_{sj}'^{\,l,i'} \ge 0 \qquad \text{Eq. (3-31)}$$

$$\Psi_{i'}, \varepsilon_{i'} \ge 0 \quad \forall i' \qquad \text{Eq. (3-32)}$$

The dual model presented below that determines $u_{j,s}^{l,i'}$ and $u_{j,s}'^{\,l,i'}$. This model simplifies the complexity made by large dimensional $w_s^{l,i'}$. Eq. (3-33) shows the objective function and Eqs. (3-34) and (3-35) are the constraints. Lagrangian relaxation is used to approximate the dual model.

$$\left\{u_s^{l,i'}, u_s'^{\,l,i'}\right\} = \arg \max_{u_s^{l,i'}, u_s'^{\,l,i'}} -\varepsilon_{i'} \sum_{j=1}^{l-i'} \left(u_{j,s}^{l,i'} + u_{j,s}'^{\,l,i'}\right) + \sum_{j=1}^{l-i'} \left(u_{j,s}^{l,i'} - u_{j,s}'^{\,l,i'}\right) d_s^{j,i'} - \frac{1}{2} \sum_{i=1}^{l-i'} \sum_{j=1}^{l-i'} \left(u_{i,s}^{l,i'} - u_{i,s}'^{\,l,i'}\right) \left(u_{j,s}^{l,i'} - u_{j,s}'^{\,l,i'}\right) \varphi_l\left(x_s^i, x_s^j\right) \quad \forall s, i', l \qquad \text{Eq. (3-33)}$$

$$\sum_{i=1}^{l-i'} \left(u_{i,s}^{l,i'} - u_{i,s}'^{\,l,i'}\right) = 0 \quad \forall s, i', l \qquad \text{Eq. (3-34)}$$

$$0 \le u_{i,s}^{l,i'}, u_{i,s}'^{\,l,i'}, u_{j,s}^{l,i'}, u_{j,s}'^{\,l,i'} \le \Psi_{i'} \quad \forall i, j, s, i', l \qquad \text{Eq. (3-35)}$$

4. Computations and discussions

This section presents the empirical investigation, interactive procedure, and results and sensitivity analyses.

4.1. Case study and solution method

Twenty-five months of data for the spare parts with intermittent demand is provided from the oil industry to validate the models. The National Iranian South Oilfields Company (NISOC), a prominent Iranian oil company, is selected with three central and six local warehouses. Additionally, one outer company and two inner-company repair centres are considered. Table 1 presents the demand data which initialised the planning model. Holding cost per unit is 20%, and the shortage cost equals the production loss value. Four categories of spare parts are considered, including 200 spare parts with intermittent demand fall into one of these categories. A maximum five number of intervals (periods) are considered. This table shows that all the squared coefficient of variations (CVs) for demand are less than 0.5.

Table 1. Spare parts category demand statistics in 25 months

	Spare parts category			
	1	**2**	**3**	**4**
Mean	7.12	8.15	7.97	7.4
Std. Dev.	4.29	4.52	4.64	5.14
C.V.	0.36	0.31	0.34	0.48
Minimum	0	0	0	0
Maximum	15	25	19	24

The models are solved using the case study data. The initialisation process consists of 12 months, validation involves seven months, and testing covers six months. Demand and the maximum number of intervals (in addition to other crucial parameters) are initially set to solve the planning model, then the number of intervals is obtained. The range of Ψ, ξ, and ε are respectively $[10^{-2}, 10^{2}]$, $[10^{-1}, 10^{0}]$, and $[10^{-2}, 10^{-1}]$. The kernel parameters such as r and d are zero and three.

4.2. Numerical results

Since the models are simplified, GAMS software is capable of solving both of the models. A PC with 16GB RAM and Intel(R) Core (TM) i5-9400F CPU @ 2.90GHz is used to run the models. The forecasting model uses the output of the first model (number of intervals) to forecast the demand in successive periods. The interactive procedure optimises two models and runs the termination condition when the optimal number of intervals no longer changes, as shown in Figure 2. The optimal number of time intervals is $n^* = 4$ with the optimal cost of 1.56×10^{7}. The number of intervals over the planning horizon is used for demand estimation and forecasting. Table 2 demonstrates the costs for different time intervals. Also, Table 3 outlines MASE in different periods. The optimal stock levels in warehouses are presented in Table A-1 in the appendix.

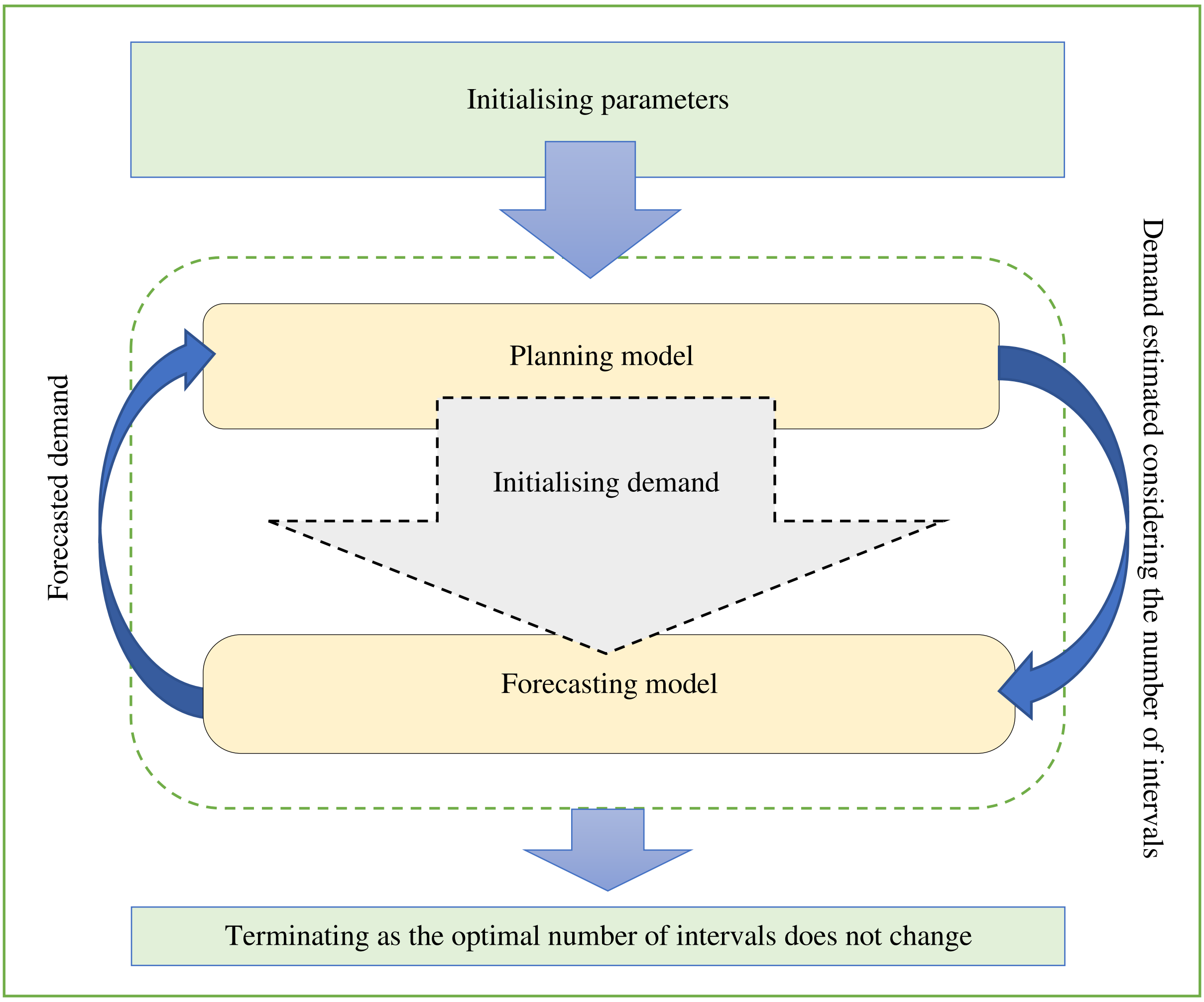


Figure 2. Interactive procedure for optimising planning and forecasting models

Figure 2 Alt Text. The iterative optimisation procedure is illustrated in this figure. First, the parameters are initialised in the planning model. After solving the planning model, it gets the number of time intervals from the planning model (to estimate demand in each interval). Then, the demand forecasted by the forecasting model initialises the planning model. The procedure continues until no significant changes appear in the number of intervals (and consequently no change in forecasted demand).

Table 2: Costs over the time intervals

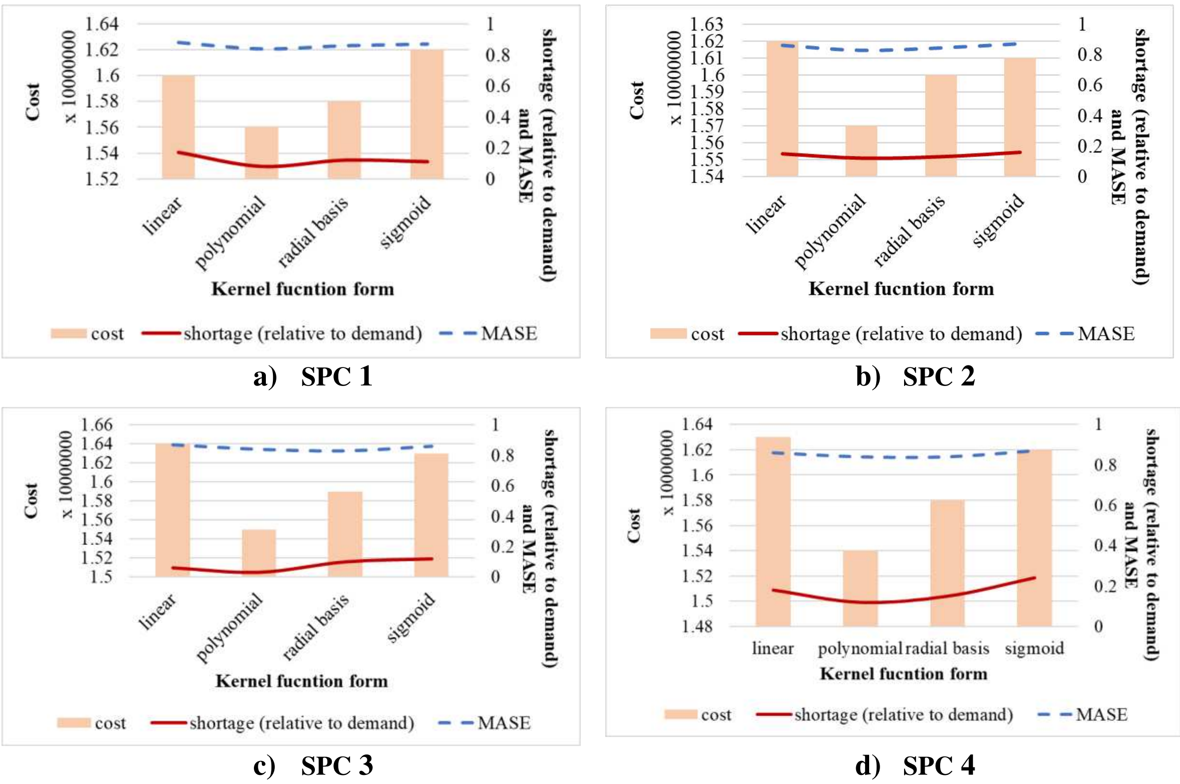
Time interval	Cost
1	1.63×10^7
2	1.63×10^7
3	1.6×10^7
4	1.56×10^7
5	1.59×10^7

Table 3. MASE in different time intervals and iterations

Iterations	Time interval				
	1	**2**	**3**	**4**	**5**
10	0.88	0.87	0.868	0.866	0.877
20	0.874	0.869	0.868	0.862	0.858
30	0.865	0.864	0.86	0.858	0.855
50	0.861	0.86	0.858	0.855	0.854
70	0.857	0.85	0.85	0.846	0.84
100	0.855	0.848	0.847	0.844	0.851

4.3. The effect of changing forecasting model parameters

Considering the forecasting criteria, Figure 3 illustrates the effect of implementing the kernel function on MASE, shortage, and cost. The kernel function is defined in linear, polynomial, radial basis, and sigmoid forms that affect data fitness and accuracy accordingly. MASE is decreasing by the cost, resulting in more accurate forecasting leading to lower shortages and costs. In other words, the emphasis on collecting more data can enhance forecasting since the nonparametric estimation highly depends on data sets. Also, it can be helpful in parametric estimation that uses probability distribution for the estimation. It is noteworthy that organising a well-structured forecasting system improves inventory management by modifying the ordering process; therefore, it can cut shortages since the data accuracy will positively affect the planning. Another finding is regarding the kernel function estimation, which plays an essential role in fitting the regression line for the data. Here, the polynomial form outperforms the others, but higher forecasting accuracy does not necessarily guarantee a good inventory management performance; however, it may require considering all the parameters in practice. SVM implements various hyperparameters used in tuning, but simultaneous tuning helps prevent overfitting, although it is recommended that each spare part be tuned separately to adopt the specifications of each item.



a) SPC 1 b) SPC 2

c) SPC 3 d) SPC 4

Figure 3. MASE deviation, cost, and shortage (relative to demand) for each spare parts category (SPC)

Figure 3-a Alt Text. Considering SPC 1, the Polynomial form of kernel function has the minimum cost in comparison with other forms, followed by radial basis. Also, the minimum MASE and shortages are obtained in polynomial form. The sigmoid form has closer results to polynomial. **Figure 3-b Alt Text.** The polynomial form has the minimum cost, shortage, and MASE for SPC 2, but other forms of kernel functions prominently have higher value. Moreover, MASE changes significantly when we move the end of the diagram. **Figure 3-c Alt Text.** Regarding SPC 3, the radial basis form has the minimum MASE, but the polynomial has the minimum cost. Also, linear and sigmoid forms have similar results to linear forms for cost, while the sigmoid form has higher MASE and shortage compared to other forms of the kernel function. **Figure 3-d Alt Text.** SPC 4 shows that minimum cost occurs in the polynomial form where it has the minimum MASE, but cost, MASE, and shortage sharply react to other forms of kernel functions that lead to higher values.

The relation of total spare parts' stock level of warehouses and different forms of kernel function is also shown below in Figure 4. Inventory is a critical factor that significantly affects the supply chain's performance since it imposes prominent costs for holding excessive stock. Forecasting can improve inventory planning to avoid shortages and high costs. In the SVM model, the kernel function helps map the present attributes to new dimensions to investigate more features in the forecasting model. The polynomial function best fits the data because the minimum stock level confirms previous findings. Overfitting can be observed as we proceed to the right side of the diagram resulting from tightening the parameter.

Several measures are used to evaluate intermittent demand forecasting models categorised as absolute, mean, percentage, square, scaled, and relative errors. Also, there are hybrid measures, e.g. mean absolute; some are symmetric or geometric (Hyndman and Koehler 2006). They recommend the absolute measures to be independent among the various time series. Also, (Aris A. Syntetos and Boylan 2005) proposed scaled mean error (SME) widely used in bias analysis (Petropoulos et al. 2016). Another independent measure is alternative mean absolute percentage error (AMAPE), which is a good alternative for dealing with division-by-zero problems. We consider Mean absolute scaled error MASE, AMAPE, and SME as measures to compare the accuracy of the present forecasting model with others.

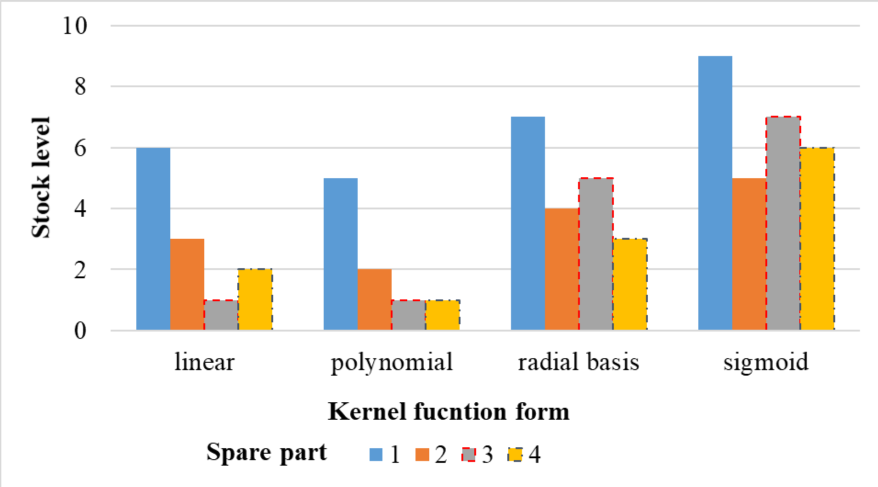


Figure 4. Stock levels vs kernel functions

Figure 4 Alt Text. The stock level changes are shown in different forms as the kernel functions change. The minimum stock level is obtained by using the polynomial kernel function. Other forms of kernel functions cause overstocking that is due to more errors in forecasting

The results are presented in Table 4. The effect of running other models such as SBA, MSBA, Croston, and SVM are analysed considering the various intervals, shown in Figure 5. It can be seen that the cost reduces by using the SVM integrated with the planning model, which is the result of improvement in the forecasting model's accuracy. It can be seen that when considering the integration of forecasting and planning decisions, the results improve compared with other methods. Nevertheless, the results are comparable as we go through different time intervals. The comparison shows that the trade-off between the number of intervals and forecasting accuracy gives insights into determining the optimal length of each interval (by dividing the length of the period by the number of time intervals). As we move to four intervals, the errors reduce, but it starts to rise when the number of intervals increases. The best-performing result is highlighted in table 4. In practice, the number of planning periods can be considered as the number of intervals in the theoretical context, but the decisions in this regard should be modified for each spare part due to different demand patterns.

Table 4. Forecasting accuracy comparison by different measures

Time interval		SBA	MSBA	Croston	SVM	**SVM integrated with planning**
1	MASE	0.889	0.880	0.890	0.869	0.855
	SME	0.087	0.007	0.161	-0.141	-0.152
	AMAPE	1.112	0.752	0.833	1.050	1.090
2	MASE	0.818	0.810	0.869	0.865	0.848
	SME	0.080	0.006	0.148	-0.144	-0.158
	AMAPE	1.023	0.692	0.766	0.966	0.940
3	MASE	0.791	0.783	0.792	0.759	0.847
	SME	0.077	0.006	0.143	-0.145	-0.166
	AMAPE	0.990	0.669	0.741	0.935	0.926
4	MASE	0.765	0.757	0.765	0.734	0.844
	SME	0.075	0.006	0.138	-0.149	-0.182
	AMAPE	0.956	0.647	0.716	0.903	0.807
5	MASE	0.782	0.774	0.783	0.751	0.851
	SME	0.077	0.006	0.142	-0.154	-0.178
	AMAPE	0.979	0.662	0.733	0.924	0.906

The number of intervals affects demand estimation, inventory costs, and other related factors. Total cost reduces to a minimum value as the number of intervals increases. However, the cost starts to rise when it passes four intervals. There are two practical and theoretical concepts. Practically, this could be justified from the aspect of risk-sharing (flexibility in spare part stock level and order) caused by increasing the number of intervals. There is a break-even point between the costs and the number of intervals. The demand estimation theoretically improves as the optimal number of intervals increases since the demand curve closes to a near-linearised form. Figure 5 shows that minimum cost is obtained when there are four intervals for all the forecasting methods, so using the optimal number of intervals can be interpreted as an improvement in parameter tuning which is achieved by integrating planning and forecasting decisions. The integration causes simultaneous consideration of the decisions and their effects on each other, so multiple attributes in the decision-making process help the results be more reliable since various aspects are considered.

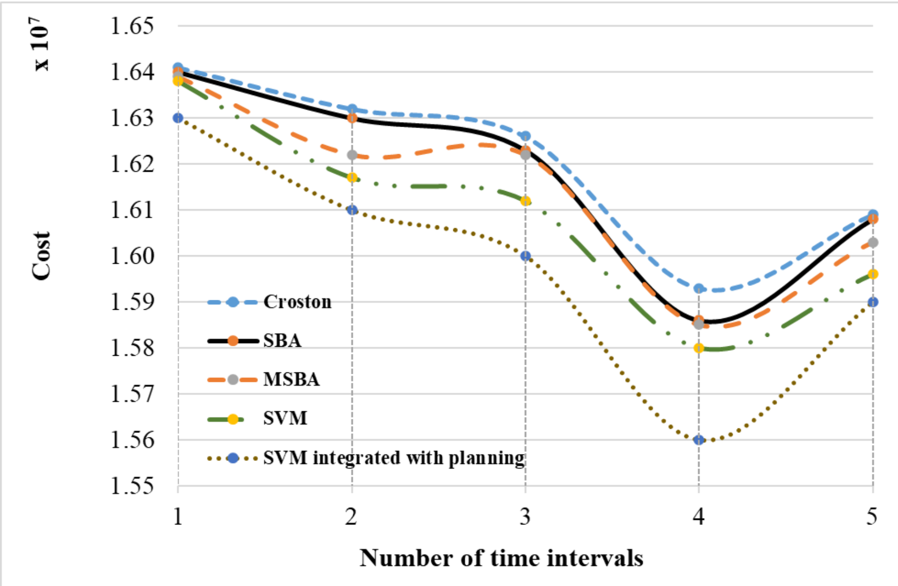


Figure 5. The cost fluctuation for different models as the number of intervals changes

Figure 5 Alt Text. The cost is compared when using different intermittent demand forecasting models, including Croston, SBA, MSBA, SVM, and SVM integrated with planning. The minimum cost is obtained at interval four, where the SVM is integrated with the planning model. Also, SBA and MSBA show comparable results, while Croston has the highest cost.

4.4. The impact of changing planning model parameters

Considering the changes in the number of intervals, stock level fluctuations are provided in Figure 6. It illustrates that the optimal number of the time intervals does not necessarily relate to stock level; however, it indirectly affects the stock levels. As presented earlier, the optimal number of time intervals has the minimum cost. We can observe that stock levels of spare parts 1 and 3 will proceed to the minimum value as the number of intervals is closing to four, while this analysis is suitable for spare parts 1 and 4 in local warehouses. These differences show that each warehouse and spare part may require an adaptive policy for planning since the demand makes the difference in planning. These analyses result in two concepts for planning decisions: centralisation and decentralisation of decisions. The first concept suggests considering the same decision or similar decisions of planning, but the second concept considers separate decision-making for each item, i.e., spare parts. The centralised decision-making provides savings in cost and resources, but the decisions may not be flexible and adaptive to each spare part. Conversely, decentralised decision-making leads to more adaptive results since the decisions can be made according to demand patterns and other attributes of the spare part; however, it is not as cost-effective as the centralised one.

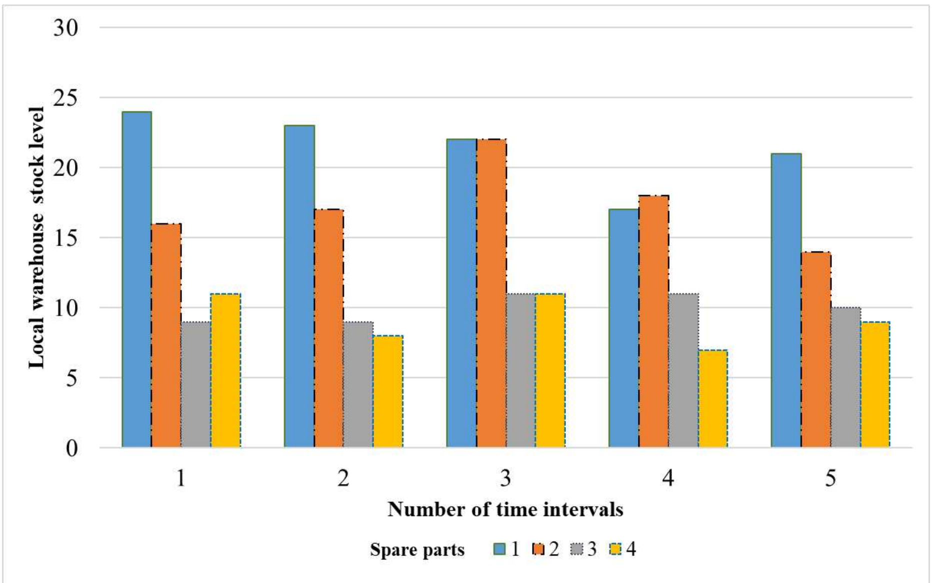


Figure 6. Local warehouse stock levels when the number of time intervals changes

Figure 6 Alt Text. Stock levels of local warehouses are illustrated when the number of intervals changes. Four categories of spare parts are compared. It can be seen that categories 1 and 4 have minimum stock levels when the optimal number of intervals is four, but categories two and three have optimal stock levels when the number of intervals is respectively five and one.

Figure 7 illustrates the effect of change in shortage cost on total cost and MASE. As the shortage cost increases, the total cost also increases. Since the objective function minimises the total cost, increasing the shortage cost necessitates more accurate forecasting to optimise the planning decisions. The forecasting model tunes the planning data, i.e., the future demand, by minimising MASE, reducing the forecasting error as the shortage cost increases. In practice, the shortage cost can be interpreted as lost sale cost, damage cost to machines, or imposed labour cost. The spare parts with higher shortage costs need to be planned more accurately due to their crucial role in the operation.

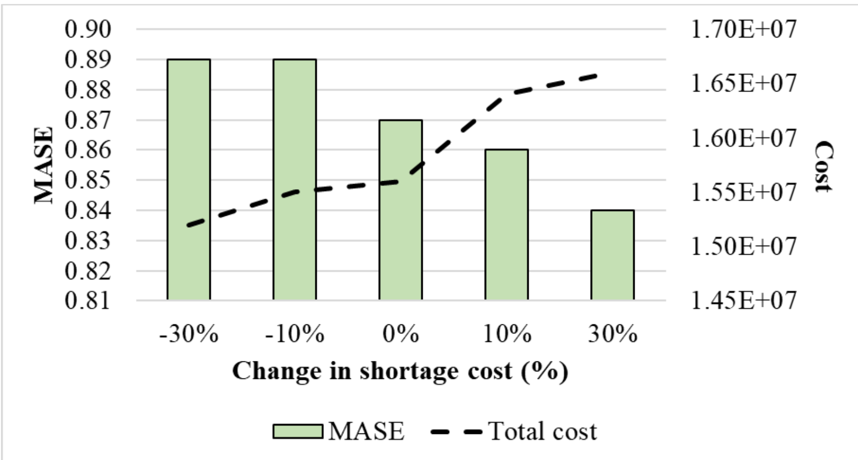


Figure 7. The effect of shortage cost fluctuation on MASE and cost

Figure 7 Alt Text. MASE and cost are shown when the shortage cost change. MASE decreases, and total costs increase as shortage cost increases. The more the shortage cost is, the less the forecasting error will be due to improvement in MASE.

4.5. Managerial insights

- The planning performance highly depends on the accuracy of demand data, i.e., failure rate. The less the forecasting error is, the more the planning performance will be. For example, the repair response time depends on the supply planning of spare parts used in the repairing operations, which can be improved by optimising forecasting.

- Integrating planning and forecasting decisions and considering the uncertainty improves the robustness and optimises the cost. Also, filtering the data and considering more attributes reduces the noise and overfitting.
- Nonparametric approaches derive the demand distribution from data, so it is recommended that spare parts with similar demand patterns be tuned separately to adopt the specifications of each category for demand estimation. These differences show that each warehouse and spare part may require its adaptive policy for planning since the demand makes the difference in planning; however, it may differ from one industry to another.
- Considering the category of spare parts with a similar demand pattern improves the forecasting efficiency due to demand aggregation since it gives minor variation and more accurate forecasts.
- Owing to the characteristics of repairable spare parts with intermittent demand, considering the supply and repair planning with forecasting optimises the stock level of spare parts, affecting the inventory costs. The servicing performance is enhanced due to improvement in planning since separate optimisation leads to sub-optimal solutions.
- Estimating the demand by piecewise linearization gives better results because the dynamic number of intervals adapts to the demand pattern of each spare part category. Also, the planning decisions can be determined by analysing the trade-off between the forecasting results and planning decisions.

5. Conclusion

In this study, we examine integrated forecasting and planning models for repairable spare part supply chain, including repair and supply decisions for low-demand spare parts following the intermittent demand pattern, which necessitates a well-defined forecasting mechanism to improve the performance of planning decisions. This research considers two mathematical models; the first model includes planning decisions, while the second focuses on demand forecasting. An interactive procedure optimises two models based on the iterative procedure. The planning model determines optimal variables such as the number of time intervals, order assignment to suppliers, stock level of warehouses, and order assignment to repair centres. The forecasting model uses the first model's estimated demand using the piecewise linearization technique and forecasts the demand. The procedure iterates until the number of time intervals does not change significantly. An empirical study of an oil company is presented to validate the models solved by GAMS that implement data related to 200 spare parts obtained from an oil company.

The contributions of this research include 1) Demand estimation using the piecewise linearization technique by determining the number of intervals, 2) using an interactive procedure for optimising the forecasting and planning models, 3) Integrating the repair and supply decisions, 4) Modifying SVM-based forecasting model by considering the integration of planning and forecasting decisions. The findings are two parts: the findings related to planning model analyses and the findings of demand forecasting: I) Determining the number of intervals is theoretically and practically significant. Estimating demand by piecewise linearization is advantageous since it simplifies dealing with uncertainty. Additionally, it can be used as a criterion for specifying the number of planning periods. The proposed methodologies can be used for planning (such as determining the stock level, order quantity, and assignment to operational bases). The number of intervals determines the length of each interval, e.g., a two-year planning period with four intervals results in six-month-length periods so that the demand will be broken down according to this logic to reduce the forecasting error and provide robust planning decisions. Moreover, we noticed that all the forecasting methods examined in this study, except SVM integrated with planning, are very competitive in the lower number of intervals. This is a critical result in cases where one interval exists; in other words, the mentioned methods (SBA, MSBA, Croston, and SVM) can be used interchangeably in long-term decision-making.

II) Decreasing the forecasting error causes the planning to be more accurate since the piecewise linearization technique improves the forecasting accuracy and outperforms the basic planning model used in the METRIC. This method enables the central and decentral decision-making for each item and warehouse to manage the resources and costs. In comparing the variations of the errors, we observe that some methods outperform the others in a specific number of intervals that can be used for parameter tuning.

III) Filtering the historical data, aggregating, and considering more attributes reduces the noise and overfitting; moreover, using the kernel function modifies the possible errors. It enhances forecasting accuracy; however, the characteristics of each spare part should be considered according to the centralised or decentralised concept.

IV) The forecasting's result significantly impacts the planning decisions such as stock levels, order assignment, and repair assignment since they correlate with demand data, i.e., failure rate. In this case, the trade-off between the forecasting error measures such as MASE and planning performance could contribute to defining multiple scenarios for adapting to repair, supply, and inventory management decisions. We presented a model for planning and forecasting a spare part supply chain that can support the repair, supply, and other related processes in an information system.

The piecewise linearization technique is helpful for spare parts with intermittent demand, which other researchers can develop to deal with lumpy, erratic, and smooth demand. The forecasting method used in this paper is proper for intermittent demand, so the forecasting methods for other types of demand can be developed regarding the machine learning approaches. Although, the training process can be computationally complicated, and attributes may need to be selected empirically. Future research can investigate adding each spare part category's feature to modify the present forecasting model, e.g., examining the aggregation level is valuable for analysing the properties of each category. Also, using clustering methods can be helpful for the aggregation of the spare parts demand. Researchers may also examine other decisions such as lateral and emergency transhipment and pooling in the planning model, which are significant decisions connecting with forecasting models. Moreover, considering the various scenarios and implementing a decision tree provides flexible results, which can be an exciting throughput for the managers; therefore, they can easily opt for the desired choices. Another extension of this work is to consider the network design as a higher decision level of planning that allows examining the effect of the network structure on other decisions; additionally, it may result in new methodologies in integrated forecasting, network design, and planning which multi-level models can be used in this case.

Disclosure statement

The authors reported no conflict of interest.

Data availability statement

The data used in this research are available from the corresponding author upon reasonable request.

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Appendix:

Table A-1: Costs and stock levels in warehouses at different time intervals

Time interval	Warehouses	Spare parts				Cost
		1	2	3	4	

1	**Central**	1	0	0	0	1.63×10^7
		1	1	1	1	
		3	1	2	3	
	Local	0	0	0	1	
		0	0	1	0	
		0	2	0	0	
		2	0	2	2	
		1	0	1	0	
		0	0	0	0	
2	**Central**	2	0	0	0	1.61×10^7
		1	0	1	2	
		1	1	2	2	
	Local	0	0	0	0	
		0	0	1	0	
		0	4	0	0	
		0	0	0	0	
		0	0	0	0	
		0	0	0	0	
3	**Central**	2	0	0	0	1.6×10^7
		0	0	1	2	
		0	1	2	2	
	Local	1	0	0	0	
		0	0	1	0	
		0	1	2	0	
		0	0	0	0	
		0	0	1	0	
		0	0	0	0	
4	**Central**	2	0	0	0	1.56×10^7
		0	0	1	2	
		0	0	2	2	
	Local	3	0	0	0	
		0	1	1	0	
		1	0	0	0	
		0	0	0	1	
		0	1	0	0	
		1	0	0	0	
5	**Central**	0	0	0	0	1.59×10^7
		1	1	1	1	
		1	1	2	0	
	Local	0	0	0	1	
		0	3	1	0	
		0	2	0	0	
		2	0	2	2	
		1	0	1	0	
		0	0	0	0	