

This is an Open Access document downloaded from ORCA, Cardiff University's institutional repository:<https://orca.cardiff.ac.uk/id/eprint/154793/>

This is the author's version of a work that was submitted to / accepted for publication.

Citation for final published version:

Wu, Wentao, Benner, Jingru and Luo, Zhiwen 2023. Developing analytical model for nighttime cooling of internal thermal mass. *Applied Thermal Engineering* 220 , 119798. 10.1016/j.applthermaleng.2022.119798

Publishers page: <http://dx.doi.org/10.1016/j.applthermaleng.2022.11...>

Please note:

Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher's version if you wish to cite this paper.

This version is being made available in accordance with publisher policies. See <http://orca.cf.ac.uk/policies.html> for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.



1 **Developing Analytical Model for Nighttime Cooling of Internal Thermal Mass**

2 *Wentao Wu^{1,2*}, Jingru Benner³, Zhiwen Luo^{4,5}*

3 *¹Department of Civil and Natural Resources Engineering, University of Canterbury,*
4 *Christchurch, 8041, New Zealand*

5 *²Department of Civil & Architectural Engineering, College of Engineering, Tennessee State*
6 *University, Nashville, TN 37209, USA*

7 *³College of engineering, Western New England University, Springfield, MA 01119, USA*

8 *⁴Welsh School of Architecture, Cardiff University, Cardiff, United Kingdom*

9 *⁵School of the Built Environment, University of Reading, Reading, United Kingdom*

10
11 **Corresponding author: Wentao Wu, wentao.wu@canterbury.ac.nz*

12 **Abstract**

13 Nighttime mechanical ventilation of internal building thermal mass has the potential to create
14 energy flexibility by shifting peak cooling demand. The research on nighttime cooling of internal
15 thermal mass is limited due to lacking simplified models by considering mass dimension and real-
16 world discrete climate data like outdoor air temperatures. This study develops an analytical model
17 for nighttime cooling of internal thermal mass with a constant air change rate and hourly varied
18 air temperatures. The analytical model is verified by both numerical, conduction transfer function
19 methods, and ANSI/ASHRAE Standard 140-Case 600. The analytical model is applied to quantify
20 the free cooling energy storage in 48 selected U.S. cities in different climate zones and in the 16
21 climate zones of California. Among the 48 cities, the maximum free cooling energy storage is
22 reported in Santa Fe, NM with a total free cooling energy storage of 19.1 kWh m⁻² a⁻¹ and a net
23 free cooling energy storage of 3.88 kWh m⁻² a⁻¹. Coastal regions in California are not suitable for
24 nighttime ventilation of internal thermal mass. The maximum total free cooling energy storage in
25 California achieves 27.5 kWh m⁻² a⁻¹, while the maximum net free cooling energy storage is 6.11
26 kWh m⁻² a⁻¹. The analytical model has a potential to be integrated into whole building energy
27 simulation software to improve the calculation of the effect of internal thermal mass.

28 **Key words:** energy flexibility; sensible heat storage; CFD; thermal inertia; night cooling; night
29 ventilation.

30 **1. Introduction**

31 The U.S. has committed to joining the global efforts to reach net-zero emissions by 2050 in the
32 26th Conference of the Parties (COP26). A key pathway toward carbon neutrality is to increase the
33 use of renewable energy sources. The intermittency and fluctuation of onsite renewable energy
34 sources may lead to imbalance between supply and demand. Demand-side energy flexibility with
35 intermediate energy storage is considered a promising solution to alleviate the supply and demand
36 mismatch [1]. Buildings can play an important role to create demand-side energy flexibility by
37 transforming massive structures into thermal mass to store thermal energy [2]. Thermal mass has
38 the capacity to absorb sensible heat during the daytime and release it during the nighttime. The
39 time lag of heat absorption and release shifts the peak thermal load in time and creates energy
40 flexibility [3]. The amount of energy flexibility created by thermal mass alone might be relatively
41 small due to its varied ability to shift heating and cooling demand in different climates [4].

42 The capacity of thermal mass to shift cooling demand depends on climates. In cold climates,

43 thermal mass on walls might increase building cooling demand in summer [5]. In warm and mild
44 climates, thermal mass on walls is reported to have low influence on energy demand reduction [6].
45 In hot climates, high thermal mass of insulated concrete wall increases cooling energy demand as
46 high ambient temperature causes more heat stored during daytime than that released during
47 nighttime [7]. An effective measure to increase the heat release from thermal mass is nighttime
48 cooling [8]. Cool outdoor air can be directly induced to flush the thermal mass and create a heat
49 sink to absorb space heat gains via convection and radiation in the following daytime. Nighttime
50 cooling of thermal mass has the potential to reduce the need for compressor-based cooling energy
51 demand [9]. The efficiency of nighttime cooling is most effective when night-time temperatures
52 are below 20°C, summer diurnal temperature swings are between 15-20°C, and maximum daytime
53 temperature is between 30 and 36°C [10]. A review [11] of related research undertaken between
54 1997 and 2017 reveals that most work has been done to investigate the energy demand reduction
55 of nighttime cooling of external thermal mass on walls. The external thermal mass is exposed to
56 varied outdoor climate conditions and the heat sink generated in the nighttime might be filled too
57 rapidly in the morning to shift the peak cooling load in the afternoon. One way to expand the
58 lifetime of the heat sink is to use internal thermal mass on the floor. The presence of internal
59 thermal mass can increase the building time constant by up to 42%, which is calculated as the time
60 needed to reach 63.2% of the temperature change between two steady states and measures the
61 ability of thermal mass to time-shift cooling load [12]. A question remains unanswered is: what is
62 the capacity of nighttime cooling of internal mass to reduce energy demand in different U.S.
63 climates?

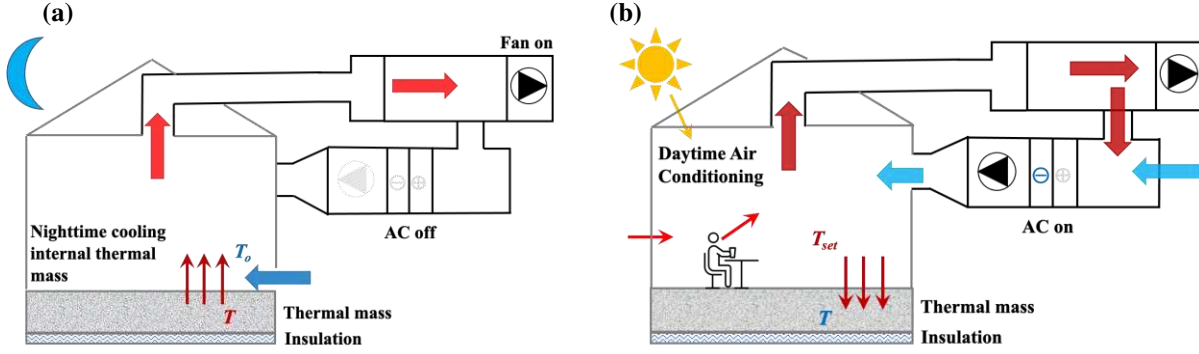
64 Limited research on nighttime cooling of internal thermal mass is attributed to the lack of
65 simplified but accurate models to quantify the cooling demand reduction under realistic climate
66 conditions [13,14]. In-situ measurements provide the most realistic quantification of the cooling
67 demand reduction of internal thermal mass. Experiments carried out in two real-scale buildings
68 showed that the use of thermal mass reduced the cooling energy demand by 67% to 75% during
69 14 days with only slight diurnal temperature variations [15]. However, measuring cooling energy
70 storage is challenging due to difficulties in placing sensors in thermal mass. In most experiments,
71 only air temperatures are measured and it is difficult to separate the influence of thermal mass from
72 other impact factors [16]. Whole building energy simulation software like EnergyPlus [17] and
73 REGCAP [18] are suitable to investigate the transient behavior of internal thermal mass and
74 nighttime ventilation. However, in EnergyPlus, internal thermal mass is either simplified as
75 interior construction materials with an effective area or lumped into the zone air with a capacitance
76 multiplier. The oversimplification might lead to overestimation of energy storage of internal
77 thermal mass that behaves superficially energy efficient. In REGCAP, the internal thermal mass
78 is described with an empirical relationship that scales with floor area. The empirical relationship
79 is based on the concept of the building time constant and does not consider the effect of thermal
80 properties. In most whole building energy simulation software, the thermal mass dimension is
81 completely ignored [19]. Reduced-order models are developed to consider details of internal
82 thermal mass and nighttime cooling. Thermal quadrupole approach [20] can be implemented to
83 calculate thermal energy storage in internal thermal mass. The approach is based on the analytical
84 solution of one-dimensional heat transfer in an infinitely large solid plane, and the boundary
85 conditions are limited to sinusoidal signals. Numerical methods such as finite element [21] or finite
86 difference [22] can be used to quantify the energy demand reduction with internal thermal mass
87 under realistic temperature boundary conditions. The drawback of numerical methods is the
88 requirement of a small time-step to ensure numerical stability, which increases the computational

89 time [23]. A case study of monthly temperature variation can take up to days to complete the
90 computation. Another approach that is essentially similar to numerical methods is the RC thermal
91 model [24]. A reduced-order RC (2R2C) model was adopted to study the performance of internal
92 thermal mass and results showed that ‘building as battery’ efficiency is between 40% and 80%
93 depending on the insulation level [25]. A limitation of the 2R2C model is the short prediction time
94 span of up to 12 h. An alternative reduced-order model is to directly derive analytical solutions to
95 the time-dependent heat conduction equation in the internal thermal mass [26]. The state-of-the-
96 art analytical models are limited to simplified boundary conditions. A typical example is to use a
97 sinusoidal function to approximate real-world diurnal outdoor air temperature variations [27]. The
98 sinusoidal approximation cannot reflect the actual complexity of time-varying temperature
99 boundary conditions [4], for example, the nighttime air temperatures are hourly-varied values and
100 the daytime air temperature is the constant setpoint value. To deal with the difference in the
101 nighttime and daytime temperature boundary conditions, Yang and Li [28] developed nighttime
102 and daytime analytical models separately by expressing the outdoor temperature as a sinusoidal
103 profile with a frequency of 24 h. The analytical model is developed for zero-dimensional thermal
104 mass during a period of 24 h. It is in urgent need to develop a simplified mathematical model for
105 nighttime cooling of internal thermal mass to consider mass details and real weather data. The
106 question is how to develop the model by considering mass dimension and real-world weather data,
107 for example, discrete (hourly varied) air temperatures?

108 In summary, much research has been done to investigate the nighttime cooling of external thermal
109 mass on walls. The research gap is to understand the potential of nighttime cooling of internal
110 thermal mass to reduce cooling demand in different climates. An urgent need is to develop a
111 simplified mathematical model for nighttime cooling of internal thermal mass by considering mass
112 dimension and real-world weather data, e.g., discrete air temperatures in a span of a year. The
113 detailed objectives of this study include: 1) developing a one-dimensional analytical model for
114 nighttime cooling of internal thermal mass with annually discrete air temperature boundary
115 conditions; the model will be validated against numerical simulations, traditional conduction
116 transfer function (CTF) method [29], and ANSI/ASHRAE Standard 140 -2017 [30]; 2) quantifying
117 cooling demand reduction with nighttime cooling of internal thermal mass in different climates.
118 This study will provide a new method to assess the efficiency of nighttime ventilation of internal
119 thermal mass to create energy flexibility [31] and enhancing the utilization of renewable energy
120 [32]. The validated analytical solution could also be used for model predictive control of thermal
121 mass [33] and potentially be integrated into whole building energy simulation software.

122 **2. Materials and methods**

123 The nighttime cooling in this study refers to a system that supplies unconditioned outdoor air into
124 the space to directly cool the concrete floor slab that is the internal thermal mass (Fig. 1a). During
125 the nighttime period, the supply air temperature is hourly varied. In the following daytime period,
126 the slab acts as a heat sink to absorb part of the space heat gains via convection and radiation (Fig.
127 1b); the air temperature distribution in the space is uniform and equal to a constant setpoint
128 temperature [28]. The daytime air conditioning and nighttime ventilation are provided by a
129 constant air volume system. The heat transfer in the slab is one-dimensional along the vertical
130 direction [34] and the bottom of the slab is well insulated. The slab is a single-layer material. The
131 heat transfer between the slab and the air through radiation and convection is described by a total
132 heat transfer coefficient.



133 **Fig. 1. (a) Nighttime cooling of internal thermal mass and (b) daytime air conditioning. T is the thermal mass**
 134 **temperature field; T_o is the nighttime hourly-varied supply air temperature; T_{set} is the daytime setpoint**
 135 **temperature of the conditioned space.**

136 The heat balance for a building with nighttime cooling of internal thermal mass can be described
 137 by the following equations presented by Yang and Li [28]:

$$Q_{cl} + Q_{wr} + Q_{win,cond} + Q_{win,sol} + Q_i + Q_{inf} + Q_{itm} = 0 \quad (1)$$

$$\rho_a c_a q_v (T_o - T_{room}) + Q_{itm} + Q_{wr} + Q_{win,cond} + Q_i + Q_{inf} = 0 \quad (2)$$

138 Equation (1) describes the heat balance during the daytime, where Q_{cl} is the cooling demand (W),
 139 Q_{wr} is the conduction cooling load through walls and roof (W), $Q_{win,cond}$ is the conduction cooling
 140 load through windows (W), $Q_{win,sol}$ is the cooling load due to solar heat gain through windows (W),
 141 Q_i is the cooling load due to internal heat gains (W), Q_{inf} is the infiltration heat gain (W), and Q_{itm}
 142 is the cooling load due to heat absorption or release from the internal thermal mass (W). The room
 143 air and the internal thermal mass exchange heat through convection and radiation. The total heat
 144 absorption or release from thermal mass is calculated as $h_i A_i (T_{surf} - T_{room})$, where h_i is the total heat
 145 transfer coefficient above the internal thermal mass ($\text{W m}^{-2} \text{K}^{-1}$), A_i is the total area of the internal
 146 thermal mass (m^2), T_{surf} is the surface temperature of the internal thermal mass (K), and T_{room} is
 147 the air temperature in the room (K). Equation (2) describes the heat balance during the nighttime
 148 and the first term is the supplied nighttime cooling, where ρ_a and c_a is the density (kg m^{-3}) and
 149 specific heat ($\text{J kg}^{-1} \text{K}^{-1}$) of air, respectively. The nighttime ventilation rate is $q_v (\text{m}^3 \text{s}^{-1})$.

150 The temperature field in the internal thermal mass is described by the following equation with
 151 Robin boundary conditions:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$-\lambda \frac{\partial T}{\partial y} \Big|_{y=L} = h_i [T(y=L, t) - T_r(t)] \quad (4)$$

$$\lambda \frac{\partial T}{\partial y} \Big|_{y=0} = 0 \quad (5)$$

152 Equation (3) assumes one-dimensional heat conduction in the internal thermal mass, where T is
 153 the slab temperature field (K), y is the vertical distance from the bottom of the internal thermal
 154 mass (m), t is time (s), ρ and c is the density (kg m^{-3}) and specific heat ($\text{J kg}^{-1} \text{K}^{-1}$) of the internal
 155 thermal mass, respectively. Equation (4) describes the heat exchange between the internal thermal

156 mass and the supply air temperature T_r (K), where, λ is the thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$) of the
 157 internal thermal mass, and L is the total slab thickness (m). T_r is a combination of unconditioned
 158 nighttime outdoor air temperatures and the daytime setpoint temperature of the conditioned space.
 159 T_r equals to the hourly varied temperature (T_o) of the outdoor air that is supplied to directly cool
 160 the slab in the nighttime period and is the setpoint temperature of the space in the daytime period
 161 (T_{set}) (Fig. 2). T_r can be described using the step function:

$$T_r(t) = [H(t) - H(t - t_1)] T_{r,1} + [H(t - t_1) - H(t - t_2)] T_{r,2} + \dots \quad (6)$$

$$+ [H(t - t_{M-2}) - H(t - t_{M-1})] T_{r,M-1} + H(t - t_{M-1}) T_{r,M}$$

$$H(t) = \begin{cases} 0 & t \leq 0 \\ 1 & t > 0 \end{cases} \quad (7)$$

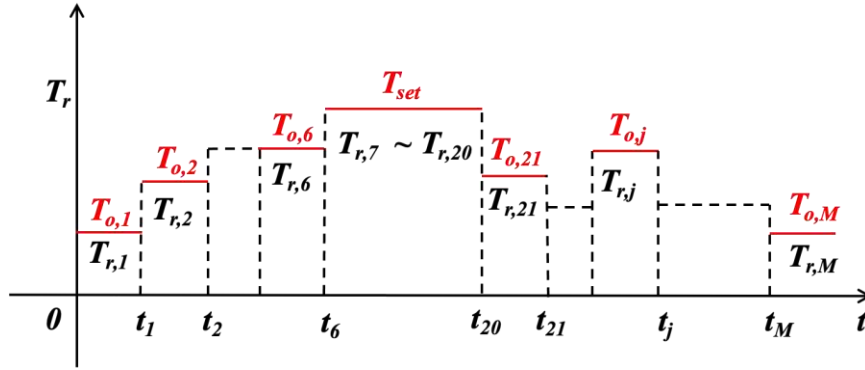


Fig. 2. Hourly varied air temperature.

164 Equation (5) describes a perfect insulation of the bottom of the internal thermal mass. The initial
 165 temperature of the internal thermal mass is set equal to the initial air temperature (T_{set}) and the
 166 effect of the initial temperature can be eliminated by periodic calculations.

167 2.1 Analytical model for nighttime cooling of internal thermal mass

168 The challenge to derive the analytical solution of equation (3) is the inhomogeneous and time-
 169 dependent boundary condition (6) that has discrete free-stream temperatures T_r . When separation
 170 of variables is applied, the first derivative of T_r will be introduced into the equation system. Since
 171 T_r is discrete and varies hourly in this study, its first derivative requires the use of Dirac delta
 172 function:

$$\frac{dT_r}{dt} = \delta(t) T_{r,1} + \sum_{m=1}^{M-1} \delta(t - t_m) [T_{r,m+1} - T_{r,m}] \quad (8)$$

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ 1 & t = 0 \end{cases} \quad (9)$$

173 where, M is the total number of days for nighttime cooling. The analytical solution can be sought
 174 by applying shifting function method [35], separation of variables, Fourier series expansion and
 175 variation of parameters (VOP) method [26] to the equations (4-9) normalized with:

$$\theta = \frac{T_r - T_{set}}{T_{set}}, \quad \tau = \frac{\alpha t}{L^2}, \quad \eta = \frac{y}{L}, \quad Bi = \frac{h_i L}{\lambda}, \quad \theta_r = \frac{T_r - T_{set}}{T_{set}} \quad (10)$$

176 where α is the thermal diffusivity ($\text{m}^2 \text{s}^{-2}$) and equals to $\lambda/\rho c$, Bi is the Biot number that measures
 177 the ratio of the thermal resistance in the internal thermal mass to the convection of nighttime
 178 cooling at the surface of the internal thermal mass. The detailed derivation is given in Appendix
 179 A. The final analytical model is established for nighttime mechanical ventilation of building
 180 thermal mass with discrete (hourly varied) temperature boundary conditions:

$$\theta = \sum_{n=1}^{\infty} \cos(\beta_n \eta) \left\{ \frac{S_n}{\beta_n^2} [\theta_{r,1} (e^{-\beta_n^2(\tau_M - \tau_1)} - e^{-\beta_n^2 \tau_M}) + \sum_{m=1}^{M-1} \theta_{r,m+1} (e^{-\beta_n^2(\tau_M - \tau_{m+1})} - e^{-\beta_n^2(\tau_M - \tau_m)})] - W_n [\theta_{r,1} e^{-\beta_n^2 \tau_M} + \sum_{m=1}^{M-1} (\theta_{r,m+1} - \theta_{r,m}) e^{-\beta_n^2(\tau_M - \tau_m)}] \right\} + \frac{\eta^2 - 1}{2} Bi \theta_r \quad (11)$$

181 2.1.1 Model verification with numerical simulation

182 The analytical model is verified and validated with numerical simulation, the traditional CTF
 183 method and ANSI/ASHRAE Standard 140-2017. This section verifies the time-dependent
 184 temperature variations in a concrete slab as internal thermal mass using the finite-volume method
 185 [36]. The verification process is targeted to represent a case as general as possible. The thermal
 186 mass is assumed to be on the floor and has a thickness (L) of 0.3 m. The material density is 2500
 187 kg m^{-3} . The thermal conductivity is $1.5 \text{ W m}^{-1} \text{ K}^{-1}$. The specific heat is $750 \text{ J kg}^{-1} \text{ K}^{-1}$. The nighttime
 188 ventilation rate is 1.5 h^{-1} . The hourly air temperatures are TMY3 dry bulb values between 19:00
 189 and 07:00 hours in the night on November 13th in Long Island of California. The daily air
 190 temperature is the room setpoint temperature of 24°C that is also the initial temperature of the
 191 internal thermal mass.

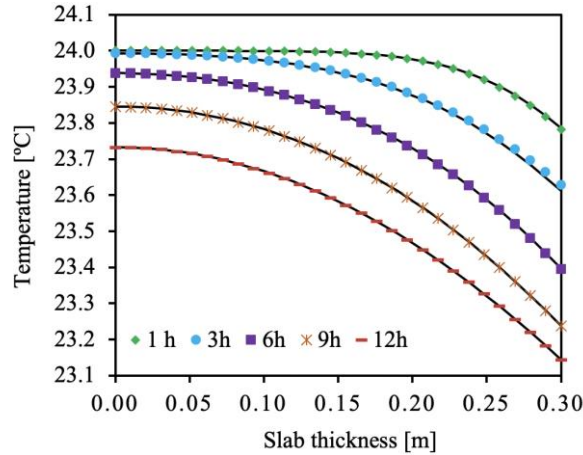


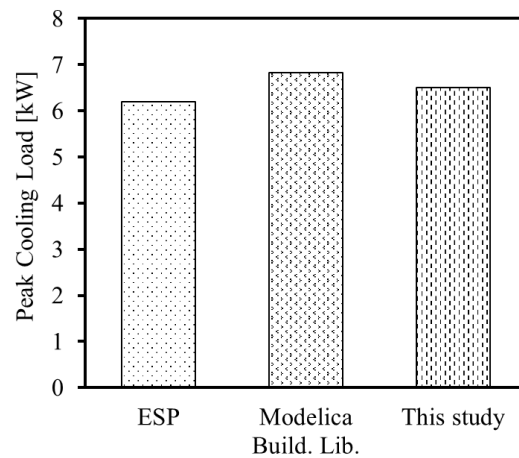
Fig 3. Temperature profiles at different hours: symbols and solid lines represents the results of the numerical simulation and the analytical model, respectively.

192 The finite-volume simulation was performed with the open source CFD code - OpenFOAM 7 and
 193 the laplacianFoam solver was used to solve the heat transfer in solids. The cell distance was chosen
 194 as $L/100$ uniformly based on both grid independence and sufficient resolution to compare with the
 195 analytical solution [26]. The time derivative was discretized with the first-order implicit scheme.
 196 The time step was chosen to satisfy the condition: $F_o (1 + \xi Bi) \leq 0.5$, where $F_o = \lambda \Delta t / (\rho c \Delta x^2)$, ξ is
 197 0 and 1 for internal and boundary nodes, respectively, Δt is time step and Δx is cell distance. The
 198 time step is determined to be 3 seconds. The Laplacian term was discretized by the central-
 199 differencing scheme. The boundary and initial conditions have the form of (4-5).

200 Fig. 3 compares the temperature profiles obtained from the finite-volume method and the
201 analytical model. The temperature profiles are shown along the thickness of the internal thermal
202 mass at different hours. The theoretical model predicts the same temperature profiles as the
203 numerical simulation. Both approaches show that the surface temperature (at 0.3 m) of the
204 thermal mass is reduced by 0.2°C after 1 hour and 0.9°C after 12 hours. The bottom temperature
205 of the slab starts to decrease after more than 3 hours and drops by 0.3°C after 12 hours.

206 2.1.2 ANSI/ASHRAE Standard 140 - Case 600 for model validation

207 ANSI/ASHRAE Standard 140 [30] is widely used in the building simulation community to
208 validate models. Standard 140 documents the simulated energy demand of a thermal zone using
209 different building energy simulation tools. In this study, Case 600 is presented for model
210 validation. Case 600 is a lightweight cubic zone (6 m × 8 m × 2.7 m) with two windows (3 m × 2
211 m each) on the south wall. All details including materials and climate can be found in the section
212 5.2 in Standard 140-2017. Construction material properties are summarized in Appendix B. The
213 U-value of the windows is 2.10 W m⁻² K⁻¹. The internal heat gains are 200 W during all 24 hours
214 per day for the full year. The internal heat gains are assumed 100% sensible and 60% radiative.
215 The infiltration is 0.5 ACH. TMY3 weather data for Denver is used for the case. The setpoint
216 temperature for cooling is 27°C. When nighttime cooling is applied to Case 600, the floor timer
217 is changed to the concrete slab in section 2.1.1. Figure 4 compares the annual hourly integrated
218 peak cooling loads predicted by the model in this study with those obtained from other two
219 building simulation tools. The peak cooling load is 6.50 kW predicted in this study is in very
220 close agreement with simulation results of other tools.



221

222

Fig 4. Case 600: peak cooling load at 13:00 on October 17th.

223 2.2. Nighttime cooling of internal thermal mass in different climates

224 The validated analytical model is used to study the capacity of nighttime cooling of internal
225 thermal mass to reduce cooling demand in different climates. The sources of the hourly weather
226 data are Typical Meteorological Year 3 (TMY3) and California Climate Zones 2 (CTZ2) data sets
227 that also provide climate data for EnergyPlus. Forty-eight TMY3 and sixteen CTZ2 climate data
228 are chosen to investigate the energy performance of nighttime mechanical ventilation of internal
229 thermal mass in cities located in forty-eight U.S. states and across the sixteen climate zones in
230 California, respectively. Filter criteria are applied to the climate data to ensure that it is beneficial
231 to apply nighttime cooling: 1) the nightly average relative humidity is in the range of 20-70% [10];

232 2) average nightly temperatures are lower than 18 °C and the average monthly diurnal temperature
 233 range are higher than 7°C [10]; 3) Nighttime ventilation is activated from 21:00 hours in the night
 234 to 07:00 hours in the following day. The months for nighttime ventilation are picked based on
 235 these three criteria. The hourly temperature data between 7:00 and 21:00 in TMY3 and CTZ2 is
 236 replaced with the setpoint temperature 24°C. Figure 2 illustrates a sample temperature profile for
 237 the first 21 hours of a 24-h period. The first 3-day data is duplicated in the beginning of the weather
 238 file to eliminate the influence of the assumption of uniform temperature distribution in the thermal
 239 mass.

240 The heat removed from the internal thermal mass during the period of 10-h nighttime ventilation
 241 is used to indicate the capacity to shift and reduce cooling demand and defined as the total free
 242 cooling energy:

$$Q_s = \rho c_p H T_{init} \left\{ \sum_{n=1}^{\infty} \frac{\sin \beta_n}{\beta_n} \Phi_n(\tau_J) - \sum_{n=1}^{\infty} \frac{\sin \beta_n}{\beta_n} \Phi_n(\tau_K) + \frac{1}{3} Bi [\theta_r(\tau_K) - \theta_r(\tau_J)] \right\} \quad (12)$$

243 where, J and K express each start and end hour of nighttime ventilation for all months. Total free
 244 cooling energy is the sum of all positive values of free cooling energy storage (Q_s). The total free
 245 cooling days are the number of nights with positive values of free cooling energy and also used as
 246 a capacity indicator. When mechanical ventilation is used, there will be fan energy consumption.
 247 Therefore, two more capacity indicators are defined by considering fan energy consumption. The
 248 net free cooling energy is the sum of all positive values of the difference between free cooling
 249 energy storage and fan energy consumption. The net free cooling days are the number of nights in
 250 which free cooling energy is larger than fan energy consumption.

251 2.2.1 Estimation of mechanical fan energy consumption

252 An accurate calculation of fan energy consumption requires the detailed information of the
 253 mechanical system including pressure drop and airflow rates that vary from building to building
 254 and from system to system. Such detailed information is often hard to obtain, and simplified
 255 methods are needed to estimate fan energy consumption in a more general fashion. An alternative
 256 to characterize fan energy performance is to use the metric of fan efficacy that is defined as the
 257 power required to transport a fixed volume of air ($W h m^{-3}$) [10]. Springer et al. [10] summarized
 258 the fan efficacy of a central fan system. The total area of the internal thermal mass is assumed to
 259 be 200 m². The total volume flow rate is 4800 m³ h⁻¹ to maintain a high ventilation rate of 8 ACH
 260 that is average value for commercial buildings like schools according to CIBSE Guide B2 [37].
 261 The total volume flow rate leads to a fan efficacy around 0.5 W h m⁻³.

262 2.2.2 Validation of total free cooling energy

263 The total free cooling energy calculated by equation (12) is validated by the time-dependent heat
 264 fluxes at the slab surface calculated using CTF method [38]. The CTF method is based on Z-
 265 transform to compute solutions of transient heat transfer. It relates the current heat flux to past and
 266 present values of interior and exterior boundary temperatures (air or surface temperatures) as well
 267 as heat flux history. The heat flux (q'') at the slab" surface at the present time (t) is given by

$$q_t'' = X_0 T_{es,t} + \sum_{m=1}^{M_{es}} X_m T_{es,t-m\Delta t} - Y_0 T_{is,t} - \sum_{m=1}^{M_{is}} Y_m T_{is,t-m\Delta t} + \sum_{m=1}^{M_q} Z_m q_{t-m\Delta t}'' \quad (13)$$

268 where, X_m ($\text{W m}^{-2} \text{K}^{-1}$), Y_m ($\text{W m}^{-2} \text{K}^{-1}$), Z_m are conduction transfer function coefficients for exterior
 269 temperatures (T_{es}), interior temperatures (T_{is}), and past heat fluxes, respectively. In this study, the
 270 exterior temperatures are the above-mentioned hourly air temperatures. Δt is the time step. A
 271 difficult and complex task is to determine the conduction transfer coefficients. Seem [39]
 272 developed a detailed procedure for calculating CTF coefficients for one-dimensional slabs. The
 273 procedure is coded with Python to generate CTF coefficients and calculate heat flux at the slab
 274 surface. The Python code is verified by calculating CTF coefficients and heat fluxes at the exterior
 275 and interior surface of a single-layer material presented in the thesis of Seem. When CTF method
 276 is applied, it is necessary to assume past values of exterior temperatures, interior temperatures, and
 277 heat fluxes for the initial calculation. Then the calculation is iterated on the periodical air
 278 temperatures until a steady periodic solution is reached. In this study, the heat fluxes during the
 279 third 24 hours are used to verify the analytical model.

280

Table 1 Heat flux (W m^{-2}) at the surface of the slab

hour	CTFM	Analytical	Relative difference
19	4.780	4.725	0.01151
20	4.973	4.936	0.007440
21	5.123	5.092	0.006051
22	5.349	5.320	0.005422
23	5.713	5.684	0.005076
0	6.010	5.981	0.004825
1	6.375	6.345	0.004706
2	6.064	6.041	0.003793
3	5.830	5.810	0.003431
4	5.534	5.516	0.003253
5	5.376	5.360	0.002976
6	5.220	5.204	0.003065
7	-1.005	-0.9578	0.04697
8	-0.9321	-0.9100	0.02371
9	-0.8932	-0.8793	0.01556
10	-0.8666	-0.8572	0.01085
11	-0.8470	-0.8406	0.007556
12	-0.8320	-0.8278	0.005048
13	-0.8202	-0.8177	0.003048
14	-0.8107	-0.8094	0.001603
15	-0.8029	-0.8025	0.0004982
16	-0.7963	-0.7966	0.0003767
17	-0.7906	-0.7915	0.001138
18	-0.7855	-0.7868	0.001655

281

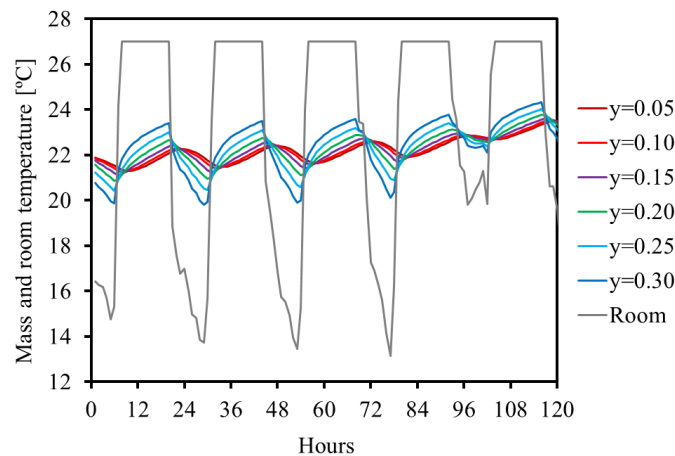
282 Table 1 compares the heat flux calculated with CTF method and the analytical method. In general,
 283 a very good agreement is found between the two methods. The relative difference in the heat flux
 284 ranges from 0.00003767 to 0.04697. The maxima occurs when the exterior air temperature is
 285 switched from TMY3 dry bulb temperature (14.8°C) at 7:00 to the room setpoint temperature
 286 (24°C) at 8:00. In other words, the maximum difference in heat flux is due to the maximum exterior
 287 temperature difference between the previous and current hour. The CTF coefficients are obtained
 288 by spatially discretizing the slab into 51 resistances and 50 capacitances. The limited number of

289 resistance-capacitance finite-difference discretization would inevitably result in accuracy issues.
290 The good agreement between the two methods indicate that the analytical model might be
291 instrumental to optimize the CTF coefficients when the exterior temperatures have large gradients.

292 3. Results

293 3.1. Analytical model to assess nighttime cooling of internal thermal mass in Case 600

294 The analytical model is applied to evaluate the potential of nighttime cooling of the internal
295 thermal mass in Case 600. Fig.5 shows the temperature profiles at different locations in the internal
296 thermal mass for 216 hours from June 2nd to June 11th in Denver. The line $y=0.30\text{m}$ represents the
297 temperature variations of the floor surface, whereas the line $y=0.05\text{m}$ represents a location near
298 the adiabatic boundary. The grey line represents the time-dependent temperature variations of the
299 room air. During the nighttime period (21:00-06:00), the room air temperature is calculated with
300 equation (2). During the daytime period (07:00-20:00), the room air temperature is assumed to be
301 the constant setpoint temperature (27°C). In the second full-day period in Fig.5, the room air
302 temperature differs by 9.8°C during the nighttime, whereas the temperature variation of thermal
303 mass surface is 2.4°C . The room air temperature cannot recover to the setpoint temperature 27°C
304 at 08:00, but it is in the vicinity of 25°C that will not cause thermal discomfort. The lowest room
305 air temperature is 13.7°C at 06:00 due to the lowest outdoor air temperature in the same hour. The
306 thermal mass surface temperature reaches to its lowest value 19.8°C at 06:00 but the thermal mass
307 temperature near the adiabatic surface reaches its lowest value 21.5°C in 4 hours later. The
308 maximum temperature difference between the daytime and nighttime period is 3.7°C on the floor
309 surface and 1.3°C at a depth of 0.15 m below the surface. The time-variation of the thermal mass
310 temperature at depths of 0.15 m below the surface is only 33% of that on the surface. Therefore, a
311 slab thickness of 0.15 m is optimum in terms of storing cooling energy during nighttime ventilation.
312 The analytical model shows a great performance to calculate the dynamic temperature variations
313 of the internal thermal mass under hourly varied and discrete air temperatures.



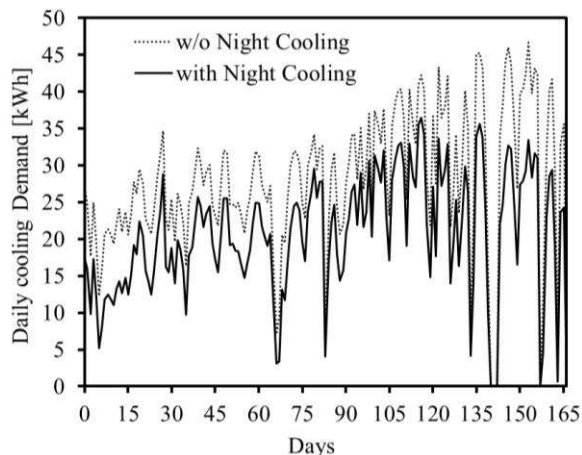
314

315

Fig. 5. Temperature variation at different locations of thermal mass in Case 600.

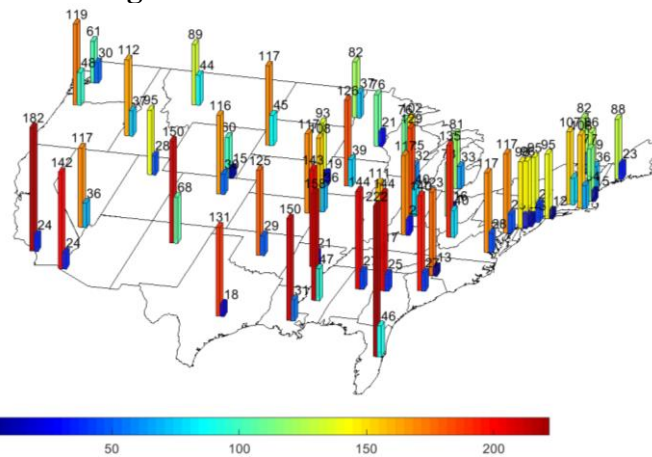
316 The daily cooling demand of Case 600 with and without nighttime cooling of internal thermal
317 mass is shown in Fig.6. For the entire season requiring cooling, nighttime cooling of internal
318 thermal mass can reduce daily cooling demand. The total cooling demand reduction is 1.2 MWh
319 and about 26% of the annual cooling demand. The average daily cooling demand reduction is 7.3

320 kWh with a standard deviation of 2.4 kWh. The maximum reduction is 13.9 kWh and about 30%
 321 of the daily cooling demand. The maximum peak demand reduction is 1.4 kWh. For a specific day
 322 like June 3rd, the hourly peak demand (3.2 kWh) occurs at 15:00. Nighttime cooling of internal
 323 thermal mass can reduce peak cooling demand by 706.2 Wh that covers the peak cooling load of
 324 640.5 W due to conduction heat gains through walls, roof and windows, internal and infiltration
 325 heat gains.



326
 327 **Fig. 6. Total cooling demand for Case 600 with and without nighttime cooling of internal thermal mass.**

328 **3.2. Capacity of nighttime cooling of internal thermal mass in different U.S. climates**



329
 330 **Fig 7. Total and net free cooling days in different U.S. cities.**

331 The analytical model is applied to evaluate the potential of nighttime mechanical ventilation of
 332 building internal thermal mass by calculating total and net free cooling days in different U.S. cities
 333 (Fig 7). In general, the number of total free cooling days decreases from southern states to northern
 334 states. This is associated to the change in the climate zones with latitude. In the southern U.S.
 335 region with hot-dry or hot humid climate, the average value of annual total and net free-cooling-
 336 days is 160 and 28, respectively. In the middle U.S. region with mixed humid climate, the average
 337 value of annual total and net free-cooling-days is 121 and 24, respectively. In the northern U.S.
 338 regions with cold climate, the average value of annual total and net free-cooling-days is 98 and 32,
 339 respectively.

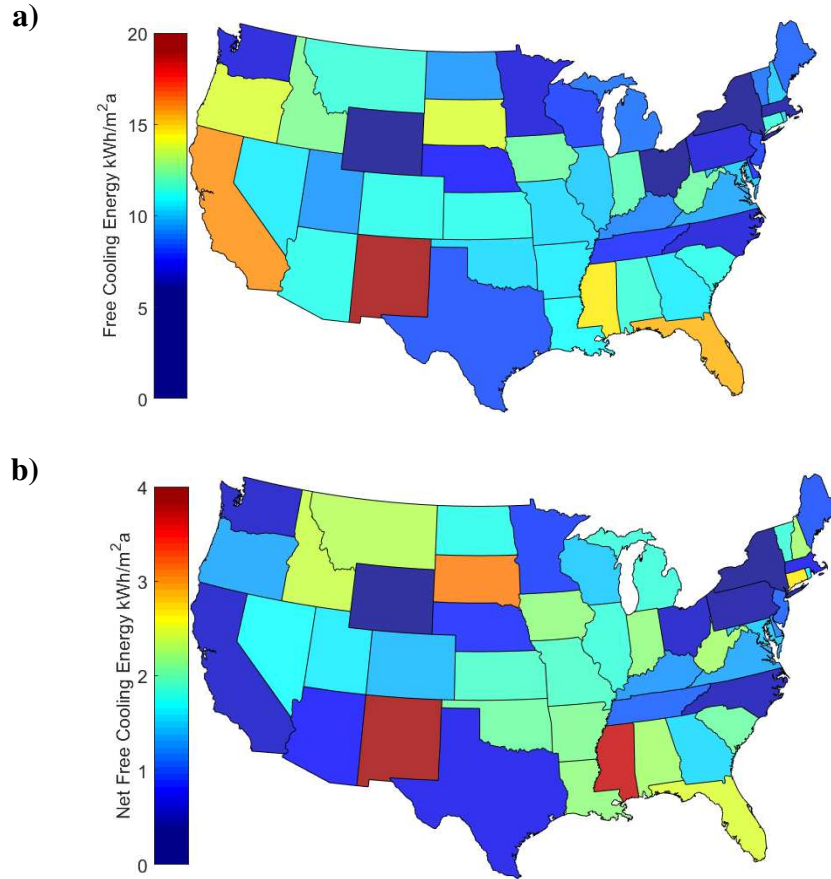
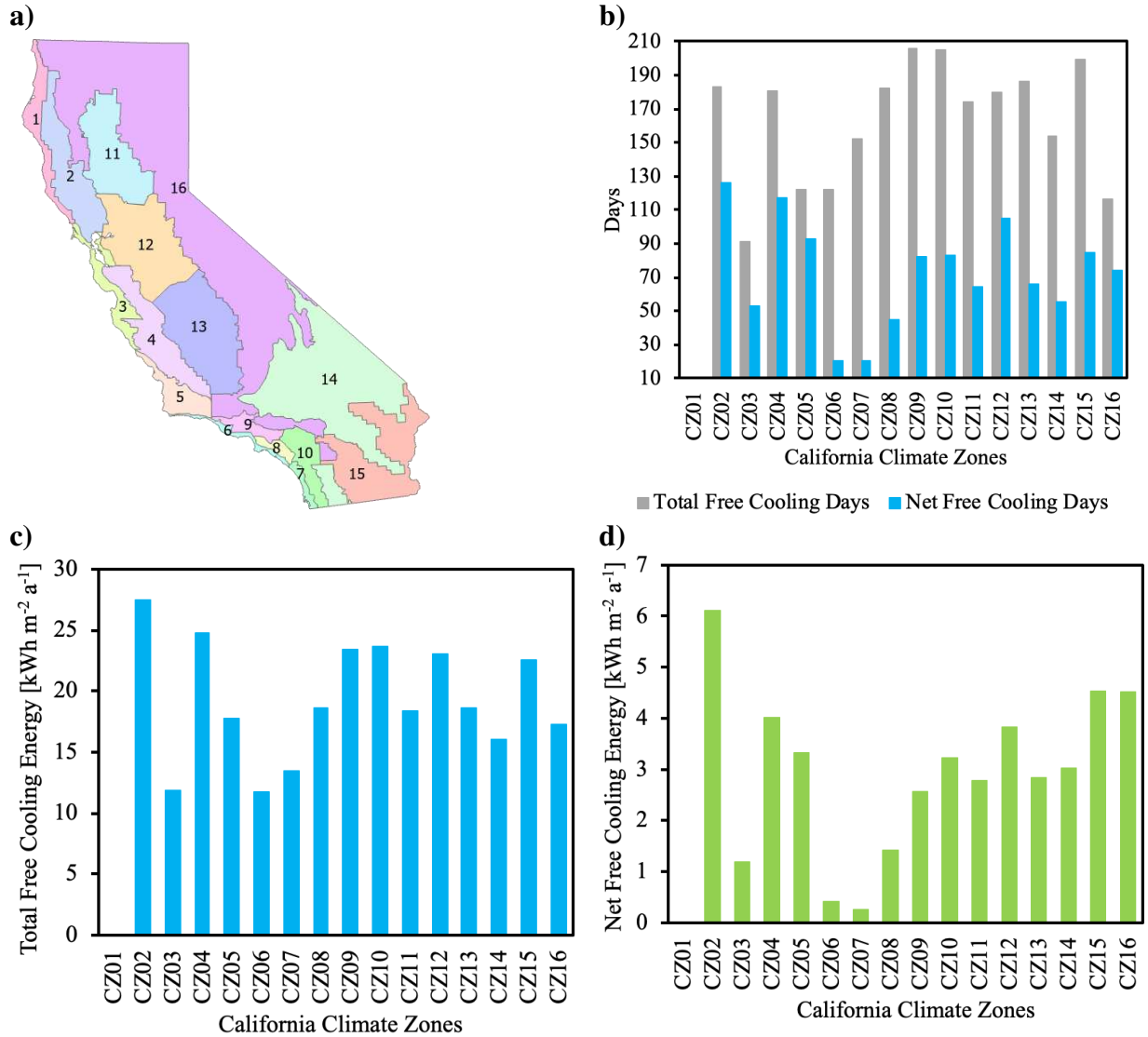


Fig. 8. a) Total and b) net free cooling energy storage in different U.S. cities.

340

341 For a region with hot-dry climate, the range of annual total free-cooling-days is from 117 to 182,
 342 while the range of annual net free-cooling-days is from 24 to 36. For example, the total free cooling
 343 days are 117 in Las Vegas, AZ, but the net energy saving is achieved for only 36 days. This is due
 344 to high average nighttime temperature of 20°C during the ventilation period. The total and net free
 345 cooling days are 117 and 24, respectively, in Long Beach, CA. For a region with hot-humid climate,
 346 the range of annual total free-cooling-days is from 131 to 222, while the range of annual net free-
 347 cooling-days is from 3 to 47. For example, the total free cooling days are 222 in Orlando, FL,
 348 while net energy saving is achieved in 47 of those days. The total and net free cooling days are
 349 179 and 3, respectively, in Honolulu, HI. The small number of net free cooling days in Honolulu
 350 is due to the high average nighttime temperature of 23°C during the ventilation period. For a region
 351 with marine climate, two cities are selected to demonstrate the potential of nighttime mechanical
 352 ventilation of internal thermal mass. In Seattle, WA, the total free cooling days are only 61, and
 353 net cooling storage is achieved in 50% of those nights. In Portland, OR, the total free cooling days
 354 increase to 119, but the net cooling storage is achieved in only 48 nights. For a region with mixed
 355 humid climate, total free cooling days range from 95 to 135, while most of the net free cooling
 356 days are below 30. Among the 15 tested cities in the mixed-humid climate zone, only Charleston,
 357 WV and Kansas City, MO have net cooling days of more than 30 days. In eastern coastal cities
 358 such as New York, the net free cooling days are only 12. For a region with cold or very cold
 359 climate, the total free cooling days range from 60 to 150, while the net free cooling days range
 360 from 15 to 68. Cheyenne, WY has least net free cooling days of 15 due to too cold nighttime

361 temperature. Santa Fe, NM has 68 net free cooling days and shows most promising to flush internal
 362 thermal mass with nighttime mechanical ventilation.



363 **Fig. 9. a) California climate zone. b) total and net free cooling days, c) total and d) net free cooling energy**
 364 **storage in different California climate zones.**

365 The analytical model is further applied to quantify the total and net free cooling energy storage in
 366 48 different U.S. cities (Fig. 8). The value of the total and free cooling energy storage in each city
 367 is used to contour the corresponding state. Compared to total free cooling days, the total free
 368 cooling energy storage does not show a decreasing trend from southern states to northern states.
 369 For all the three cities (Yuma, Long Beach and Las Vegas) with hot-dry climate, the total free
 370 cooling energy storage is above 10 kWh m⁻² per year, while the fan energy consumption is more
 371 than 80% of the total free cooling energy. For a region with hot-humid climate, the stored total
 372 free cooling energy ranges from 6.82 to 15.3 kWh m⁻² per year, while the net free cooling energy
 373 accounts for less than 26%. In Honolulu, HI, the net free cooling energy is nearly 0 kWh m⁻² per
 374 year. High total free cooling energy does not necessarily lead to high net free cooling energy. For
 375 example, the total free cooling energy in Orlando, FL, is 15.3 kWh m⁻² per year that is higher than

376 14.7 kWh m⁻² per year in Jackson, MS. On the contrary, the net free cooling energy in Orlando is
377 2.48 kWh m⁻² per year and lower than 3.74 kWh m⁻² per year in Jackson. In Portland and Seattle
378 with marine climate, the average total cooling energy is 10.4 kWh m⁻² per year, while the fan
379 energy consumption accounts for more than 90%. For a region with mixed humid climate, the
380 minimum total (5.86 kWh m⁻² a⁻¹) and net (0.39 kWh m⁻² a⁻¹) free cooling energy storage is
381 reported in New York. Birmingham, AL, has the maximum net free cooling energy storage of 2.24
382 kWh m⁻² per year, while the maximum total free cooling energy storage of 12.3 kWh m⁻² per year
383 is found Charleston, WV. For a region with cold or very cold climate, the total free cooling energy
384 ranges from 6.08 to 19.1 kWh m⁻² per year, while the net free cooling energy ranges from 0.34 to
385 3.88 kWh m⁻² per year. The maximum total and net free cooling energy are reported in Santa Fe,
386 NM. Comparing to Santa Fe, Rapid City, SD, has a lower total free cooling energy of 13.7 kWh
387 m⁻² per year, but the net free cooling energy of 3.11 kWh m⁻² per year is more similar.

388 Finally, the analytical model is applied to elaborate the free cooling energy storage in different
389 California climate zones (Fig. 9). In general, it is not suitable to apply nighttime ventilation of
390 internal thermal mass in coastal regions including climate zone CZ01, CZ03, CZ06 and CZ07. The
391 total free cooling days is determined as zero in CZ01. Despite an average total free cooling energy
392 of 12.4 kWh m⁻² per year in CZ03, CZ06 and CZ07, the average net free cooling energy of 0.62
393 kWh m⁻² per year indicates fan energy consumption accounts for 95%. In regions further into the
394 land, nighttime ventilation of internal thermal mass shows better performance. The total free
395 cooling days range from 116 to 206, while the net free cooling days range from 55 to 126. The
396 total free cooling energy storage is above 17.2 kWh m⁻² per year for these climate zones. Especially
397 in CZ02, the total free cooling energy storage achieves 27.5 kWh m⁻² per year, while the net free
398 cooling energy is 6.11 kWh m⁻² per year.

399 **4. Discussion**

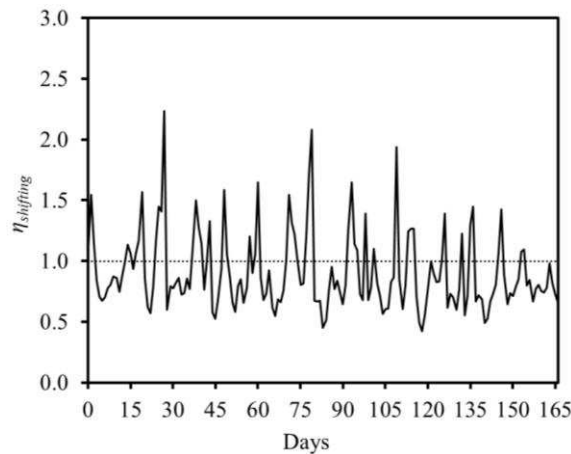
400 The research on nighttime cooling of internal thermal mass is limited due to lacking simplified
401 models by considering mass dimension and real-world discrete climate data like hourly outdoor
402 air temperatures. The analytical model in this study is developed to evaluate the performance of
403 nighttime cooling of internal thermal mass with discrete (hourly varied) air temperature boundary
404 conditions. It has a potential to be integrated into whole building energy simulation software, such
405 as EnergyPlus that sets internal thermal mass either as an effective area or a capacitance multiplier
406 for the zonal air. EnergyPlus predicts hourly zonal air temperature that can be coupled with the
407 analytical model. The temperature of the zonal air and the internal thermal mass can be updated
408 by iteration method. The fan energy consumption in this study is roughly estimated with fan
409 efficacy curve that affects the accuracy on quantifying net free cooling energy storage. Once the
410 analytical model is integrated into EnergyPlus, the fan energy can be simulated in details and the
411 estimation accuracy of the net free cooling energy can also be improved. The analytical model has
412 some advantages over existing simplified models and the methods dealing with internal thermal
413 mass in whole building energy simulation software. For example, the simplified model [28]
414 predicts the same temperature variation of room air and thermal mass due to the ignorance of
415 thermal mass dimension. The analytical model shows that the lag in temperature variation between
416 thermal mass and room air can be up to 4 hours (Fig.5). The ignorance of internal thermal mass
417 dimension like EnergyPlus multiplier method may lead to exaggerate the efficiency of internal
418 thermal mass for energy storage and such overestimation can be overcome by the integration of
419 the proposed analytical model in this study. The standalone analytical model can also be used to

420 quantify the free cooling energy storage with hourly temperature data without knowing the details
421 of building components, construction materials and etc. The limitation of the analytical model is
422 the requirement of constant ACH. The present form can be used only for constant-air-volume
423 system. Through the analysis of the total free cooling energy storage under different U.S. climates,
424 it will be very useful to further explore the performance of nighttime ventilation of internal thermal
425 mass with both hourly varied ventilation rates and air temperatures.

426 The analytical model is then applied to evaluate the performance of nighttime mechanical
427 ventilation of building internal thermal mass in different U.S. climate zones by total free cooling
428 days, net free cooling days, total free cooling energy storage and net free cooling energy. Total
429 free cooling days might be a good index to indicate energy flexibility. Total free cooling days are
430 above 60 days per year for all the 48 tested cities. It means that nighttime mechanical ventilation
431 of internal thermal mass can be designed to shift peak cooling demand for at least two months.
432 The dependency of total free cooling days on climate zones is not obvious (Fig. 7). It indicates that
433 analyzing only climate data is not adequate to evaluate the energy flexibility of nighttime cooling.
434 Therefore, total free cooling energy can be further used to quantify the energy flexibility. The total
435 free cooling energy is above 10 kWh m⁻² per year for 27 cities. Liu et al. [40] reported that the
436 energy demand of zero energy buildings should be less than 15 kWh m⁻² per year. Therefore,
437 design of nighttime mechanical ventilation and internal thermal mass is able to provide more than
438 67% useful energy demand for those type of buildings in the 27 cities. Nighttime cooling of
439 internal thermal mass is indeed promising in terms of energy flexibility for many U.S. climate
440 zones. Net free cooling days might be a good index to indicate energy savings. The difference in
441 net free cooling days is small between cities in various U.S. climate zones. Net free cooling days
442 range from 10 to 40 days for most cities. Santa Fe has the largest number of net free cooling days
443 of 68 days per year and Honolulu has the smallest number of net free cooling days of only 3 days
444 per year. Hence, applying nighttime mechanical ventilation of internal thermal mass is not an ideal
445 solution for energy saving purpose. Energy saving can be further quantified by net free cooling
446 energy. The maximum net free cooling energy in the 48 cities is 3.88 kWh m⁻² per year. If the
447 maximum energy demand of zero energy buildings is still used as reference, a maximum energy
448 saving of 26% can be achieved. A comparison between the total and net free cooling energy under
449 different U.S. climates enlightens that nighttime natural ventilation of internal thermal mass might
450 be a more suitable alternative for energy saving. Different locations in the same state might be in
451 different climate zones, for example, California has 16 climate divisions. Free cooling days and
452 energy storage in a single city of a state is not adequate to reveal the general pattern of the nighttime
453 mechanical ventilation of internal thermal mass under different U.S. climates. Therefore, the free
454 cooling days and energy storage are quantified for all the 16 climate zones in California. The more
455 detailed calculation offers new insights on nighttime mechanical ventilation of internal thermal
456 mass and the performance becomes better from coast regions further into the land. The maximum
457 total and net free cooling energy in California are about 1.5 times of those in Santa Fe.

458 The development of the analytical model is to implement the evaluation of capacity of the
459 nighttime cooling of the internal thermal mass to reduce cooling demand in different U.S. climates.
460 One main purpose to manage energy demand is to create energy flexibility for increasing the
461 integration of renewable energy. The energy flexibility created by nighttime cooling of internal
462 thermal mass is demonstrated using ANSI/ASHRAE Standard 140 – Case 600 in Denver climate.
463 The shifting efficiency ($\eta_{shifting}$) proposed by Dréau and Heiselberg [31] is adopted here to assess

464 the energy flexibility. Fig.10 shows the shifting efficiency of the nighttime cooling of internal
 465 thermal mass for Case 600. The efficiency lower than one means that not all stored cooling energy
 466 are used in the same day but shifted to the following day. The average efficiency during the cooling
 467 demand season is 0.9 with a standard deviation of 0.3. The range of the efficiency is 0.4 ~ 2.2 and
 468 a similar range is also reported by Dréau and Heiselberg [31]. It is observed that the maximum
 469 efficiency is 2.2 followed by 12 days with an average efficiency 0.7, which shows the capacity of
 470 the nighttime cooling of the internal thermal mass to create energy flexibility.



471

472 **Fig. 10. Shifting efficiency of nighttime cooling of internal thermal mass to create energy flexibility.**

473 **5. Conclusions**

474 The research on nighttime cooling of internal thermal mass is limited due to lacking simplified
 475 models by considering mass dimension and real-world discrete climate data like outdoor air
 476 temperatures. This study develops a one-dimensional analytical model for nighttime mechanical
 477 ventilation of internal thermal mass in buildings with a fixed air change rate, and constant air
 478 temperature in the daytime period but hourly varied air temperatures in the nighttime period.
 479 Temperature profiles for a general case obtained from the analytical model is confirmed by the
 480 results from time-dependent numerical simulations with the same boundary and initial conditions.
 481 The validated model is used to explore the free cooling energy storage of internal thermal mass
 482 during nighttime flush, and the main observations are:

- 483 1) Nighttime mechanical ventilation of internal thermal mass can be designed to shift peak
 484 cooling demand for at least two months for most locations. This is indicated by the total free
 485 cooling days, which are above 60 days per year for all the 48 cities tested. Among these U.S.
 486 cities in different climate zones, the maximum free cooling energy storage ($19.1 \text{ kWh m}^{-2} \text{ a}^{-1}$)
 487 is reported in Santa Fe, NM. Furthermore, the total free cooling energy is above 10 kWh m^{-2}
 488 per year for 27 selected cities, suggesting that they are suitable to apply nighttime cooling of
 489 internal thermal mass to shift peak cooling demand.
- 490 2) The energy savings by adopting nighttime mechanical ventilation of internal thermal mass is
 491 not exceptional when fan energy consumption is taken into account. Net free cooling days
 492 range from 10 to 40 days for different cities. Honolulu has the smallest number of net free
 493 cooling days with only 3 days per year. Santa Fe has the largest number of net free cooling
 494 days of 68 days per year. Santa Fe also has the maximum net free cooling energy of 3.88 kWh

495 $\text{m}^{-2} \text{a}^{-1}$. It is essential to reduce fan energy consumption to improve energy saving potential of
 496 internal thermal mass.

497 3) Free cooling energy quantified for the 16 climate zones in California shows that coastal regions
 498 are not suitable for nighttime mechanical ventilation of internal thermal mass. Large variation
 499 among different regions demonstrate that ventilation strategies need to be customized for
 500 different climate zones. The maximum total free cooling energy storage in California achieves
 501 $27.5 \text{ kWh m}^{-2} \text{ a}^{-1}$, while the maximum net free cooling energy is $6.11 \text{ kWh m}^{-2} \text{ a}^{-1}$.

502 The analytical model developed is convenient to examine the efficacy of nighttime mechanical
 503 ventilation of internal thermal mass to create energy flexibility. In addition to quantification of free
 504 cooling energy storage shown in this study, this model has the potential to be integrated into whole
 505 building energy simulation software to improve the efficiency and accuracy of evaluating the
 506 impact of internal thermal mass on indoor air temperature.

507 **Acknowledgement**

508 Dr. Wu would like to acknowledge the support of the National Science Foundation through Award
 509 Number: 2100781.

510 **Appendix A. Derivation of the analytical model**

511 Equation (4) can be normalized as:

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} \quad (\text{A1})$$

$$-\frac{\partial \theta}{\partial \eta} \Big|_{\eta=1} = Bi[\theta(\eta = 1, \tau) - \theta_r(\tau)] \quad (\text{A2})$$

$$\frac{\partial \theta}{\partial \eta} \Big|_{\eta=0} = 0 \quad (\text{A3})$$

$$\theta(\eta, 0) = \theta_r(0) = 0 \quad (\text{A4})$$

512 Shifting function method is applied to find the solution to equation (A1) with an inhomogeneous
 513 Robin boundary condition (A2). The solution takes the following form:

$$\theta = \Psi(\eta, \tau) + u(\tau)v(\eta) \quad (\text{A5})$$

514 where $v(\eta)$ is a shifting function and $u(\tau)$ contains time-dependent boundary values. The following
 515 equations are obtained by substituting equation (A5) into equation (A1-3):

$$\frac{\partial \Psi}{\partial \tau} + \frac{du}{d\tau} = \frac{\partial^2 \Psi}{\partial \eta^2} + u \frac{d^2 v}{d\eta^2}$$

$$\eta = 1: \quad -\frac{\partial \Psi}{\partial \eta} - u \frac{dv}{d\eta} = Bi(\Psi + uv - \theta_r) \quad (\text{A6})$$

$$\eta = 0: \quad \frac{\partial \Psi}{\partial \eta} + u \frac{dv}{d\eta} = 0$$

516 To make the Robin boundary condition homogeneous, u and v take the following form:

$$u = Bi\theta_r \quad (A7)$$

$$v = \frac{\eta^2 - 1}{2} \quad (A8)$$

517 The one-dimensional heat equation is recast as

$$\frac{\partial \Psi}{\partial \tau} = \frac{\partial^2 \Psi}{\partial \eta^2} + Bi\theta_r - \frac{1}{2}Bi(\eta^2 - 1)\frac{d\theta_r}{d\tau} \quad (A9)$$

518 Equation (A9) can be solved by using Separation of Variables and Fourier series expansion. The
519 separated function for space dimension satisfies homogeneous boundary conditions. The solution
520 takes the following form:

$$\Psi = \sum_{n=1}^{\infty} \cos(\beta_n \eta) \Phi_n(\tau) \quad (A10)$$

521 where the eigenvalues β_n are the roots of the transcendental equation

$$\beta_n \sin(\beta_n) = Bi \cos(\beta_n) \quad (A11)$$

522 Since the initial values of equation (A9) are zero, Φ can be determined by using the variation of
523 parameters (VOP) method [26] and the principle of orthogonality:

$$\Phi_n = \frac{1}{R_n} \int_0^1 \Psi \cos(\beta_n \eta) d\eta \quad (A12)$$

$$R_n = \int_0^1 \cos^2(\beta_n \eta) d\eta = \frac{\beta_n^2 + Bi^2 + Bi}{2(\beta_n^2 + Bi^2)}$$

524 The essence of VOP is to differentiate equation (A12) with respect to time and replace the partial
525 derivative with the right-hand side of equation (A9). An ordinary differential equation (ODE) for
526 Φ is obtained after carrying out the integration:

$$\frac{d\Phi_n}{d\tau} = -\beta_n^2 \Phi_n + S_n \theta_r - W_n \frac{d\theta_r}{d\tau} \quad (A13)$$

$$S_n = \frac{Bi \sin(\beta_n)}{R_n \beta_n}$$

$$W_n = \frac{Bi(\cos \beta_n - \sin \beta_n / \beta_n)}{R_n \beta_n^2} \quad (A15)$$

527 Employing the integral properties of Dirac delta function, the ODE can be expressed in the
528 following summation form:

$$\Phi_n = \frac{S_n}{\beta_n^2} \left\{ \theta_{r,1} (e^{-\beta_n^2(\tau_M - \tau_1)} - e^{-\beta_n^2 \tau_M}) + \sum_{m=1}^{M-1} \theta_{r,m+1} (e^{-\beta_n^2(\tau_M - \tau_{m+1})} - e^{-\beta_n^2(\tau_M - \tau_m)}) \right\} \quad (A16)$$

$$- W_n \left\{ \theta_{r,1} e^{-\beta_n^2 \tau_M} + \sum_{m=1}^{M-1} (\theta_{r,m+1} - \theta_{r,m}) e^{-\beta_n^2(\tau_M - \tau_m)} \right\}$$

529 Substituting Equation (A7-8) (A10) (A16) into Equation (A5), the final analytical model for
 530 nighttime cooling of internal thermal mass is obtained as equation (11).

531 **Appendix B. Simulation details of Case 600**

532 **Table B1 Thermal properties of construction materials**

Material (Inside to Outside)	λ W m⁻¹ K⁻¹	ρ kg m⁻³	specific heat J kg⁻¹ K⁻¹	Thickness m
Exterior Wall				
Plasterboard	0.16	950	840	0.012
Fiberglass quilt	0.04	12	840	0.066
Wood siding	0.14	530	900	0.009
Floor				
Timber	0.14	650	1200	0.025
Insulation	0.04	-	-	1.003
Roof				
Plasterboard	0.16	950	840	0.010
Fiberglass quilt	0.04	12	840	0.1118
Roof deck	0.14	530	900	0.019

533

534 **References**

535 [1] M. Saffari, C. Roe, D.P. Finn, Improving the building energy flexibility using PCM-
 536 enhanced envelopes, *Appl. Therm. Eng.* 217 (2022) 119092.
 537 <https://doi.org/https://doi.org/10.1016/j.applthermaleng.2022.119092>.

538 [2] P.H. Feng, B.C. Zhao, R.Z. Wang, Thermophysical heat storage for cooling, heating, and
 539 power generation: A review, *Appl. Therm. Eng.* 166 (2020) 114728.
 540 <https://doi.org/https://doi.org/10.1016/j.applthermaleng.2019.114728>.

541 [3] E. Alptekin, M.A. Ezan, Performance investigations on a sensible heat thermal energy
 542 storage tank with a solar collector under variable climatic conditions, *Appl. Therm. Eng.*
 543 164 (2020) 114423. <https://doi.org/https://doi.org/10.1016/j.applthermaleng.2019.114423>.

544 [4] S. Verbeke, A. Audenaert, Thermal inertia in buildings: A review of impacts across
 545 climate and building use, *Renew. Sustain. Energy Rev.* 82 (2018) 2300–2318.
 546 <https://doi.org/10.1016/j.rser.2017.08.083>.

547 [5] A. Reilly, O. Kinnane, The impact of thermal mass on building energy consumption,
 548 *Appl. Energy.* 198 (2017) 108–121. <https://doi.org/10.1016/j.apenergy.2017.04.024>.

549 [6] F. Stazi, E. Tomassoni, C. Bonfigli, C. Di Perna, Energy, comfort and environmental
 550 assessment of different building envelope techniques in a Mediterranean climate with a
 551 hot dry summer, *Appl. Energy.* 134 (2014) 176–196.
 552 <https://doi.org/10.1016/j.apenergy.2014.08.023>.

553 [7] L. Zhu, R. Hurt, D. Correia, R. Boehm, Detailed energy saving performance analyses on
 554 thermal mass walls demonstrated in a zero energy house, *Energy Build.* 41 (2009) 303–
 555 310. <https://doi.org/https://doi.org/10.1016/j.enbuild.2008.10.003>.

556 [8] J. Landsman, G. Brager, M. Doctor-Pingel, Performance, prediction, optimization, and
 557 user behavior of night ventilation, *Energy Build.* 166 (2018) 60–72.

- 558 <https://doi.org/https://doi.org/10.1016/j.enbuild.2018.01.026>.
- 559 [9] A. Gagliano, F. Patania, F. Nocera, C. Signorello, Assessment of the dynamic thermal
560 performance of massive buildings, *Energy Build.* 72 (2014) 361–370.
561 <https://doi.org/https://doi.org/10.1016/j.enbuild.2013.12.060>.
- 562 [10] D. Springer, B. Dakin, A. German, *Measure Guideline: Ventilation Cooling*, Davis,
563 California, 2012.
- 564 [11] E. Solgi, Z. Hamedani, R. Fernando, H. Skates, N.E. Orji, A literature review of night
565 ventilation strategies in buildings, *Energy Build.* 173 (2018) 337–352.
566 <https://doi.org/https://doi.org/10.1016/j.enbuild.2018.05.052>.
- 567 [12] H. Johra, P. Heiselberg, J. Le Dréau, Influence of envelope, structural thermal mass and
568 indoor content on the building heating energy flexibility, *Energy Build.* 183 (2019) 325–
569 339. <https://doi.org/10.1016/j.enbuild.2018.11.012>.
- 570 [13] Y. Liu, L. Yang, L. Hou, S. Li, J. Yang, Q. Wang, A porous building approach for
571 modelling flow and heat transfer around and inside an isolated building on night
572 ventilation and thermal mass, *Energy.* 141 (2017) 1914–1927.
573 <https://doi.org/10.1016/j.energy.2017.11.137>.
- 574 [14] H. Campaniço, P. Hollmuller, P.M.M. Soares, Assessing energy savings in cooling
575 demand of buildings using passive cooling systems based on ventilation, *Appl. Energy.*
576 134 (2014) 426–438. <https://doi.org/10.1016/j.apenergy.2014.08.053>.
- 577 [15] T. Kuczyński, A. Staszczuk, Experimental study of the influence of thermal mass on
578 thermal comfort and cooling energy demand in residential buildings, *Energy.* 195 (2020).
579 <https://doi.org/10.1016/j.energy.2020.116984>.
- 580 [16] R. Lindberg, A. Binamu, M. Teikari, Five-year data of measured weather, energy
581 consumption, and time-dependent temperature variations within different exterior wall
582 structures, *Energy Build.* 36 (2004) 495–501.
583 <https://doi.org/https://doi.org/10.1016/j.enbuild.2003.12.009>.
- 584 [17] E. Rodrigues, M.S. Fernandes, A.R. Gaspar, Á. Gomes, J.J. Costa, Thermal transmittance
585 effect on energy consumption of Mediterranean buildings with different thermal mass,
586 *Appl. Energy.* 252 (2019) 113437. <https://doi.org/10.1016/j.apenergy.2019.113437>.
- 587 [18] W.J.N. Turner, I.S. Walker, J. Roux, Peak load reductions: Electric load shifting with
588 mechanical pre-cooling of residential buildings with low thermal mass, *Energy.* 82 (2015)
589 1057–1067. <https://doi.org/10.1016/j.energy.2015.02.011>.
- 590 [19] T. Kalema, G. Jhannesson, P. Pylsy, P. Hagengran, Accuracy of energy analysis of
591 buildings: A comparison of a monthly energy balance method and simulation methods in
592 calculating the energy consumption and the effect of thermal mass, *J. Build. Phys.* 32
593 (2008) 101–130. <https://doi.org/10.1177/1744259108093920>.
- 594 [20] F. Leccese, G. Salvadori, F. Asdrubali, P. Gori, Passive thermal behaviour of buildings:
595 Performance of external multi-layered walls and influence of internal walls, *Appl. Energy.*
596 225 (2018) 1078–1089. <https://doi.org/10.1016/j.apenergy.2018.05.090>.

- 597 [21] B. Bozkaya, R. Li, T. Labeodan, R. Kramer, W. Zeiler, Development and evaluation of a
598 building integrated aquifer thermal storage model, *Appl. Therm. Eng.* 126 (2017) 620–
599 629. [https://doi.org/https://doi.org/10.1016/j.applthermaleng.2017.07.195](https://doi.org/10.1016/j.applthermaleng.2017.07.195).
- 600 [22] S.A. Al-Sanea, M.F. Zedan, S.N. Al-Hussain, Effect of thermal mass on performance of
601 insulated building walls and the concept of energy savings potential, *Appl. Energy*. 89
602 (2012) 430–442. <https://doi.org/10.1016/j.apenergy.2011.08.009>.
- 603 [23] W. Wu, T. Defraeye, Identifying heterogeneities in cooling and quality evolution for a
604 pallet of packed fresh fruit by using virtual cold chains, *Appl. Therm. Eng.* 133 (2018).
605 <https://doi.org/10.1016/j.applthermaleng.2017.11.049>.
- 606 [24] K. Shan, J. Wang, M. Hu, D. ce Gao, A model-based control strategy to recover cooling
607 energy from thermal mass in commercial buildings, *Energy*. 172 (2019) 958–967.
608 <https://doi.org/10.1016/j.energy.2019.02.045>.
- 609 [25] M.J.N. Oliveira Panão, N.M. Mateus, G. Carrilho da Graça, Measured and modeled
610 performance of internal mass as a thermal energy battery for energy flexible residential
611 buildings, *Appl. Energy*. 239 (2019) 252–267.
612 <https://doi.org/10.1016/j.apenergy.2019.01.200>.
- 613 [26] W. Wu, W. Zhang, J. Benner, A. Malkawi, Critical evaluation of analytical methods for
614 thermally activated building systems, *Renew. Sustain. Energy Rev.* 117 (2020) 109516.
615 <https://doi.org/https://doi.org/10.1016/j.rser.2019.109516>.
- 616 [27] A. Jeanjean, R. Olives, X. Py, Selection criteria of thermal mass materials for low-energy
617 building construction applied to conventional and alternative materials, *Energy Build.* 63
618 (2013) 36–48. <https://doi.org/https://doi.org/10.1016/j.enbuild.2013.03.047>.
- 619 [28] L. Yang, Y. Li, Cooling load reduction by using thermal mass and night ventilation,
620 *Energy Build.* 40 (2008) 2052–2058. <https://doi.org/10.1016/j.enbuild.2008.05.014>.
- 621 [29] B. Delcroix, M. Kummert, A. Daoud, M. Hiller, Improved Conduction Transfer Function
622 Coefficients Generation in TRNSYS Multizone Building Model, in: *Proceedings of*
623 *BS2013, Cambery, France, 2013*: pp. 2667–2674.
- 624 [30] N.J. Kruis, M. Kummert, R.T. Muehleisen, J.F. Pegues, J. Neymark, T.P. Mcdowell, M.J.
625 Witte, A. Roth, P. Strachan, M. Tosh, C.S. Barnaby, R.B. Burkhead, S.S. Hanson,
626 Standard method of test for the evaluation of building energy analysis computer programs,
627 *ASHRAE Stand.* 8400 (2004).
- 628 [31] J. Le Dréau, P. Heiselberg, Energy flexibility of residential buildings using short term heat
629 storage in the thermal mass, *Energy*. 111 (2016) 991–1002.
630 <https://doi.org/10.1016/j.energy.2016.05.076>.
- 631 [32] D.F. Dominković, P. Gianniou, M. Münster, A. Heller, C. Rode, Utilizing thermal
632 building mass for storage in district heating systems: Combined building level simulations
633 and system level optimization, *Energy*. 153 (2018) 949–966.
634 <https://doi.org/10.1016/j.energy.2018.04.093>.
- 635 [33] M. Hu, F. Xiao, J.B. Jørgensen, R. Li, Price-responsive model predictive control of floor
636 heating systems for demand response using building thermal mass, *Appl. Therm. Eng.* 153

- 637 (2019) 316–329. <https://doi.org/https://doi.org/10.1016/j.applthermaleng.2019.02.107>.
- 638 [34] W. Wu, Z. Tong, G. Zhang, A. Malkawi, X. Wang, J. Benner, An energy efficient
639 hydraulic system to cool manure and reduce ammonia emissions from livestock buildings,
640 J. Clean. Prod. 235 (2019) 920–929.
641 <https://doi.org/https://doi.org/10.1016/j.jclepro.2019.07.036>.
- 642 [35] T.W. Tu, S.Y. Lee, A New Analytic Solution for the Heat Conduction with Time-
643 Dependent Heat Transfer Coefficient, Int. J. Mech. Mechatronics Eng. 8 (2014) 1392–
644 1397.
- 645 [36] W. Wu, Z. Tong, G. Zhang, A. Malkawi, X. Wang, J. Benner, An energy efficient
646 hydraulic system to cool manure and reduce ammonia emissions from livestock buildings,
647 J. Clean. Prod. 235 (2019). <https://doi.org/10.1016/j.jclepro.2019.07.036>.
- 648 [37] CIBSE, CIBSE Guide B2 Ventilation and ductwork, CIBSE, 2016.
- 649 [38] F.C. McQuiston, J.D. Parker, Heating, Ventilating and Air Conditioning, Analysis and
650 Design, Sixth, John Wiley and Sons, New York, 1994.
- 651 [39] J.E. Seem, Modeling of Heat Transfer in Buildings, University of Wisconsin-Madison,
652 1987.
- 653 [40] Z. Liu, Y. Liu, B.J. He, W. Xu, G. Jin, X. Zhang, Application and suitability analysis of
654 the key technologies in nearly zero energy buildings in China, Renew. Sustain. Energy
655 Rev. 101 (2019) 329–345. <https://doi.org/10.1016/j.rser.2018.11.023>.
- 656