symmetry

Editorial

Special Issue Editorial "Symmetry of Hamiltonian Systems: Classical and Quantum Aspects"

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The Special Issue "Symmetry of Hamiltonian Systems: Classical and Quantum Aspects" is addressed to mathematical physicists wanting to find some fresh views on results and perspectives in symmetry analysis of a wide class of Hamiltonian systems featuring their many applications in modern classical and quantum theory. Its content establishes the importance of diverse symmetry approaches to analyzing Hamiltonian systems and the related geometric structures on manifolds, based on differential-geometric, Lie-algebraic, operator and strictly analytical tools of modern mathematical physics, effectively applied to studying various symmetry and geometry aspects of classical and quantum physical problems. We expect that this issue will lead to further insight on many interesting and current problems of modern classical and quantum field theory, based on exploring a variety of their symmetry properties motivated by deeply geometric structures naturally related with them.

The first two contributions by W. Jelonek [1] and I. Mykytyuk [2] bring interesting geometrical results devoted to investigating symmetry structures on smooth manifolds, including Köhler surfaces and by-Poissonian manifolds. In particular, W. Jelonek has demonstrated that neutral Koehler surfaces can be effectively described in terms of their positive twistor bundle, which made it possible to study such an important symmetry property as the existence of a vertical Killing vector field. Respectively, analyzing reductions of invariant bi-Poisson structures on manifolds subject to locally free actions on them, I. Mykytyuk stated that spaces of G-invariant functions on some subsets of a smooth connected submanifold and G-invariant functions on another canonically defined manifold are isomorphic as Poisson algebras with the corresponding bi-Poisson structures on them. Moreover, the second Poisson algebra of functions can be treated as the reduction of the first one with respect to a locally free action of a symmetry group.

Another very interesting and geometrically motivated trend is presented by the article by L. V. Bogdanov [3], devoted to studying a map from solutions of the dispersionless BKP (dBKP) equation to solutions of the Manakov–Santini (MS) system. It is shown that this map defines an Einstein–Weyl structure corresponding to the dBKP equation through the general Lorentzian Einstein–Weyl structure, corresponding to the MS system.

An important for classical and quantum mechanics problem of describing algebraic and differential invariants is thoroughly analyzed in an article by V. Lychagin [4]. In particular, he described in detail the fields of rational algebraic and rational differential invariants, as well as some their applications to the description of regular SO(3)-orbits of spherical harmonics.

The article by A.O. Smirnov [5] invites readers to consider an important aspect of modern classical field theory like nonlinear optics, plasma physics, etc., related to special completely integrable Hamiltonian systems on smooth functional manifolds and represented by means of evolution flows in partial derivatives. Its main topic is devoted to the spectral symmetry aspects of algebraic curves, whose detail analysis makes it possible to describe multi-phase solutions to the well-known derivative nonlinear Schrödinger

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type equations, presenting typical completely integrable Hamiltonian systems on suitably defined smooth functional manifolds.

An interesting role of special Hamiltonian systems in modern topological quantum field theory, related with so-called Frobenius manifold structures, is considered in the article by A.K. Prykarpatski and A.A. Balinsky [6]. They present a detailed Lie-algebraic construction of a new two-parametric hierarchy of commuting to each other Monge-type Hamiltonian vector fields jointly with a specially constructed reciprocal transformation, producing a Frobenius manifold potential function in terms of the solutions of these Monge-type Hamiltonian systems.

The article by L. Petrova [7] indicates the importance of considering some aspects of Hamiltonian systems in mathematical and physical from a differential-geometric point of view. A deep relationship with the Euler–Lagrange approach and related Hamiltonian systems is described by means of the differential-geometric tools.

The last review article of the issue, written by J. L. Cieśliński and D. Zhalukevich [8], is devoted to novel studying spectral properties of the so-called Lax-type integrable dynamical systems of mathematical physics and analyzing the conjecture that the related spectral parameter can be interpreted as the parameter of some one-parameter groups of transformation, provided that it cannot be removed by any gauge transformation. As a new result, it is argued that in a case when a non-parametric linear problem for a non-linear system is known like the Gauss–Weingarten equations as a linear problem for the Gauss–Codazzi equations in the geometry of submanifolds, by comparing symmetry groups, one can indicate the integrable cases.

Conflicts of Interest: The authors declare no conflict of interest.

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