FISCAL POLICY IN UNCERTAIN TIMES

By

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Abstract

This thesis examines the effectiveness and design of fiscal policy in terms of rare disasters-induced uncertainty. Rare disasters, acting as adverse shocks, generally depress the economy significantly. Simultaneously, the uncertainty caused by rare disasters may further exacerbate the negative effects of rare disasters. The fiscal stimulus package, the main tool for the government, can be adopted to combat rare disasters and recover the economy. However, a high degree of uncertainty induced by such a disaster might affect the efficacy of fiscal policy through the real allocation of resources. Especially how households respond to increases in government spending might become uncertain as their demand for private and public consumption shifts during rare disasters. This thesis takes the COVID-19 pandemic as an example to estimate the disaster-induced uncertainty and its effect on the effectiveness and optimisation of fiscal policy. A reformulated Smets-Wouters type new Keynesian model that includes government spending in households' utility is employed to find a larger uncertainty in household demand for private and public consumption that reduces the size of fiscal multipliers. Consequently, the fiscal policy package becomes less effective. Intuitively, the pandemic generates uncertainty in the degree of negative wealth effects that makes households cautious, resulting in lower private consumption and the fiscal multiplier. In such disaster-induced uncertainty, a greater fiscal stimulus is needed to maintain the same optimal welfare level as normal times. In particular, when the nominal interest rate is binding at zero, the optimal fiscal stimulus is even larger. From the welfare perspective, the disaster-induced uncertainty itself reduces households' welfare. As a result, the fiscal stimulus package has the scope to combat rare disasters and help the economy recover, but policymakers need to take the disaster-induced uncertainty into account and significantly increase the size of the package.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

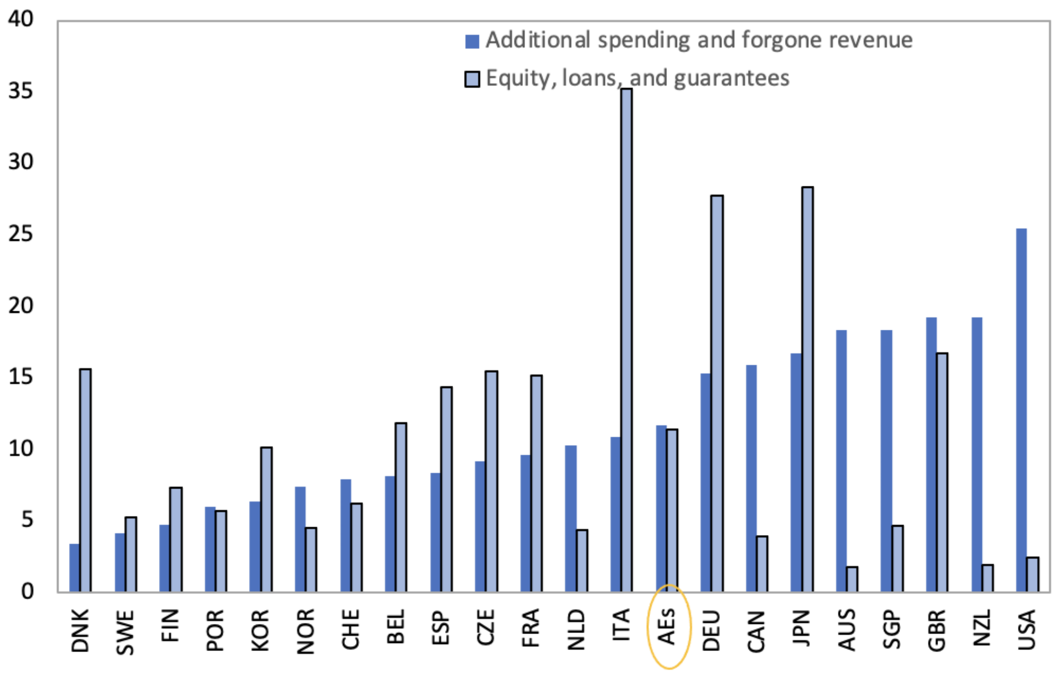
Rare disasters, such as pandemics, wars, and natural disasters, refer to significant in scale and largely unforeseen, adverse shocks, which have a recessionary effect on the economy. Rare disasters generally manifest themselves as a sudden disruption of economic activity resulting in a large drop in aggregate output. Simultaneously, the uncertainty caused by rare disasters may further exacerbate the negative effects. Recently, there has been a significant increase in the number of rare disasters. For example, during the period between 2010 and 2019, the amount of weather-related disasters, consequences of abnormal increases in global temperatures was five times more than it was between 1970 and 1979.[1] As a result, economic losses caused by rare disasters are increasing. And frequent disasters are likely to cause uncertainty to remain at an increasingly high level.

Rare disasters-induced uncertainty therefore pose a difficult challenge to policy makers burdened with the task of macroeconomic stabilisation. For policymakers and governments, a main tool to stabilise the economy is to use fiscal policy as suggested by Keynes (1937). Governments could directly compensate the drop in aggregate demand with a fiscal policy package. Indeed, to deal with the consequences of the COVID-19 pandemic, governments worldwide have adopted massive fiscal stimulus packages, which is substantially greater than that in the

[1]See the recent report by the World Meteorological Organisation (2021).

2008 financial crisis (See Figure 1.1).[2]

Figure 1.1: Fiscal stimulus packages during COVID-19 in Advanced Economies



Sources: Database of Country Fiscal Measures in Response to the COVID-19 Pandemic; and IMF staff estimates.

Figure 1.1 indicates that half of the advanced economies conducted fiscal stimulus exceeding 10% of GDP, which is substantially greater than that in the 2008 financial crisis.

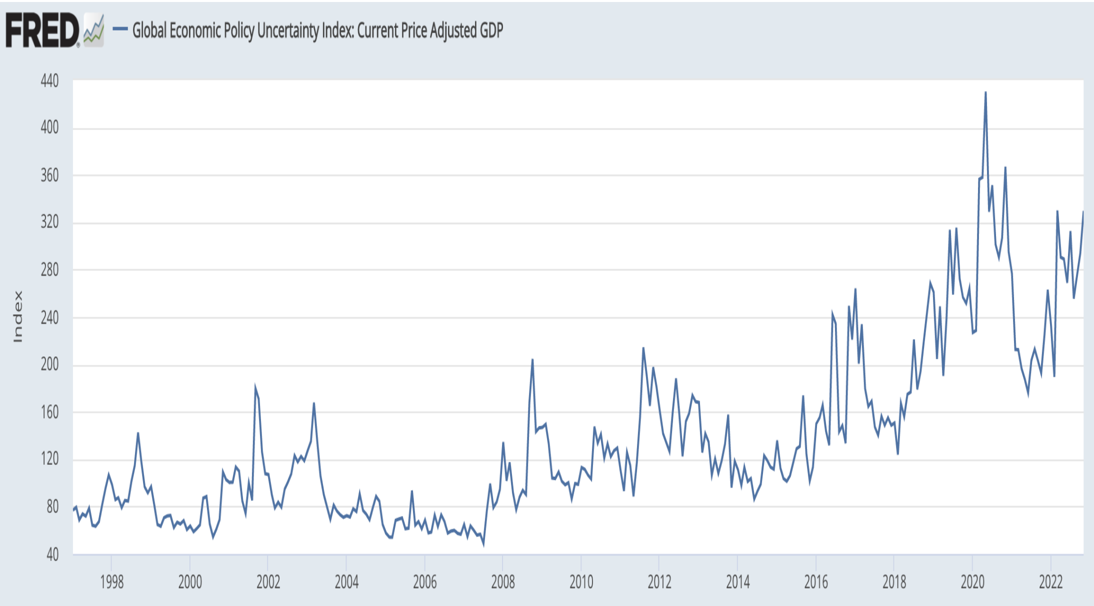
This practice raises the question of whether adopting such a massive fiscal policy package is necessary and adequate to stimulate the economy. Many economists doubted that the cost of such a package might exceed its benefits. The main issue is that such a massive package may be dramatically inflationary and thus cause a high cost to the economy. But how inflation changes in rare disasters might differ

[2]GDP weights country group averages in US dollars adjusted by purchasing power parity. Data labels use International Organization for Standardization country codes. AEs denote advanced economies.

from normal times because rare disasters induce a high degree of uncertainty in the economy. Aggregate demand might not increase as expected. Thus, inflation might not rise dramatically.

A key factor related to fiscal policy during rare disasters is the increase in disaster-induced uncertainty. The COVID-19 pandemic, for example, caused a substantial increase in uncertainty as the virus has been mutating and recurring on a global scale. Its depth and duration have become uncertain (see Figure 1.2).[3] Figure 1.2 shows a huge increase in uncertainty during the pandemic. Although it declined after that, its level remained at a higher value than before the pandemic. Moreover, the disaster-induced uncertainty is substantially larger

Figure 1.2: Global Economic Policy Uncertainty



Sources: Baker, Scott R.; Bloom, Nick; Davis, Stephen J.

than that induced by economic recessions, as rare disasters are more sudden and infrequent than recessions. Taking disaster-induced uncertainty into account, it comes up with new questions about the effects of fiscal policy and the optimal

[3]Sources: https://fred.stlouisfed.org/series/GEPUCURRENT

fiscal policy.

Therefore, It is crucial to investigate the fiscal policy in times of disaster-induced uncertainty. This thesis focuses on it and provides possible policy implications. First, chapter 2 investigates the fiscal multipliers in the face of disaster-induced uncertainty. Second, Chapter 3 examines the optimal fiscal policy with that kind of uncertainty. Finally, Chapter 4 quantifies the welfare effect of disaster-induced uncertainty.

This thesis focuses on the uncertainty in households' demand for private and public consumption. Due to the high uncertainty in the economy and fiscal stimulus package, the distribution of total resources might be affected. How households respond to the package might become uncertain because the uncertain nature of rare disasters gives rise to uncertainty about households' decisions on consumption. Moreover, the government is unable to observe the actual responses of households.

To be specific, households' consumption consists of private and public consumption, where the public consumption is supplied by government services and goods [4]. Government spending can directly affect the utility of households through public consumption. Furthermore, there is a substitutable relationship between private and public consumption. Households' responses to a fiscal stimulus depend on the demand for both private and public consumption, indicating a shift in household preferences between the two kinds of consumption. In the face of rare disasters, households might suffer health risks and work disruptions. Thus, rare disasters induce a shift in the distribution of preferences, thereby fluctuating households' decisions on consumption. In other words, their demand for private and public consumption might change and even be uncertain because of the disaster-induced uncertainty. Hence, I refer to the volatility in demand for private and public consumption as the disaster-induced preference uncertainty.

[4] It is also called the effective consumption defined by Bailey (1971). Furthermore, this kind of consumption has become a standard specification adopted by recent literature

One of the expected outcomes of the fiscal policy package is to stimulate aggregate demand. However, households' preference for consumption shifts and fluctuates, a stand-in for fluctuations in fiscal policy and rare disasters. That is to say, since the demand for both public and private consumption is uncertain, the effect of fiscal stimulus on households might not be the same as expected.

Chapter 2 first introduces the households' demand for private and public consumption, which is measured by the marginal rate of substitution between private consumption and government spending (says γ_{gt}). That is to say, γ_{gt} captures how much private consumption changes in response to a change in government spending. And then, I follow Leeper et.al (2017) to estimate the values of γ_{gt} in a reformulated Smets-Wouters type new Keynesian model. The estimation results show that γ_{gt} fluctuates over time, especially a dramatic fluctuation during COVID-19. It also suggests that the demand for private and public consumption changes over time and is deeply affected by rare disasters. To further estimate its volatility, I assume that the time series γ_{gt} follows a shock process with stochastic volatility and use Born and Pfeifer (2021) estimation method to obtain the magnitude of γ_{gt} uncertainty. The result confirms a dramatically high degree of volatility during COVID-19.

Second, I analyse the effects of disaster-induced uncertainty on the efficacy of fiscal policy through the values of fiscal multipliers in a simple RBC model. The result shows that the uncertainty does reduce the fiscal multipliers. Intuitively, the increased government spending has a negative wealth effect on households because the high government spending will be financed by future tax increases. As rare disasters induce the uncertainty in economic activities and fiscal policy, the degree of the negative wealth effect becomes uncertain. Thus, it triggers uncertainty in the demand of households for both private consumption and government spending. Therefore, households become cautious about their consumption and increase their precautionary savings, causing a lower optimal level of private consumption.

Numerically, based on the estimation of γ_{gt}, a Smets-Wouters type new Keynesian model, including sticky prices and wages, and consumption habitat, is used to examine further consequences of disaster-induced uncertainty (I focus on the uncertainty during COVID-19). The result shows that pandemic-induced uncertainty has a recessionary effect on the economy, where output and consumption considerably fall. At the same time, the recessionary effect substantially dampens the positive effect of increased government spending. Similarly, the impact on fiscal multipliers is consistent with the analytical result that the increasing uncertainty reduces the values of fiscal multipliers. Specifically, in normal times, the size of the fiscal multiplier is positive but below 1. With disaster-induced uncertainty, the size of it considerably falls to 0.03. Consequently, the pandemic-induced uncertainty reduces the efficacy of fiscal stimulus to a larger extent.

As known fiscal policy packages become less effective, Chapter 3 analyses the optimal fiscal policy during rare disasters. As the disaster-induced uncertainty reduces the fiscal multipliers, this uncertainty may require a more significant optimal fiscal policy to maintain the same optimal level of welfare in normal times. Thus, I consider two scenarios to examine the impact of disaster-induced uncertainty on optimal government spending in normal times and rare disasters, respectively. In detail, in normal times, the monetary policy is assumed to be passive, in which the nominal interest rate is set to a constant. In rare disasters, the economy is hit by a considerably negative demand shock. Thus, the nominal interest rate is generally binding at zero. Under two scenarios, the fiscal policymaker takes discretionary decisions depending on a welfare-based objective function obtained from a second-order approximation of the representative household's utility function.

The results indicate that the optimal government spending is larger with disaster-induced uncertainty both in normal times and at the zero lower bound (ZLB). And the size of optimal fiscal stimulus at the ZLB is more significant than that in normal times because of the recessionary effect of the rare disaster itself. The expected level of optimal government spending is no longer ideal in the face of uncertainty. Consequently, it is desirable to have a larger scale of fiscal stimulus. Additionally, the more persistent rare disasters, the greater the size of optimal government spending required.

Chapter 4 further studies the welfare effect of disaster-induced uncertainty. Unlike Chapter 2 and 3, Chapter 4 does not focus on how disaster-induced uncertainty affects the effect of a fiscal stimulus. Rather, it focuses on how the uncertainty itself affects the optimal utility of households. In other words, it investigates whether households prefer to stay in a stochastic state.

First, I analyse that the preference shock for private and public consumption with mean preserving spread (MPS) property does affect households' welfare in a simple RBC model. The shock is assumed to follow a log-normal distribution with a non-zero mean. Second, to measure the welfare effect of uncertainty, I use perturbation methods (Grohe and Uribe, 2004, Heiberger and Maußner, 2020) in a standard RBC model by taking a second-order approximation of the policy function and market clearing conditions around the model's non-stochastic steady state. The welfare cost is equivalent to the amount of private consumption that is compensated for in the presence of uncertainty to remain the same as in the absence of uncertainty.

The main result of this chapter is that an increase in disaster-induced uncertainty (the preference uncertainty between private and public consumption) reduces household welfare. Households prefer to stay in a steady state rather than a stochastic state. In the face of uncertainty, households smooth their consumption. In addition, due to the negative wealth effect, they are willing to supply more labours and then obtain more income but face a higher tax, thus decreasing their consumption. Consequently, they decrease more consumptions and suffer from welfare loss.

Finally, I investigate how other preference parameters impact the welfare effect of disaster-induced uncertainty. First, when households become more risk-averse, the uncertainty further deepens the negative effect on welfare. Second, if the labour supply has a high degree of Frisch elasticity, households suffer a greater negative wealth effect because of the uncertainty. Moreover, I examine the welfare effect of uncertainty in the absence of the wealth effect. Supposing that labour and consumption are non-separate in the utility function, the welfare cost becomes less but is still positive. And households still do not prefer uncertainty. As a result, disaster-induced uncertainty has a negative welfare effect, which policymakers should take into account.

Chapter 5 concludes the main results and provides some possible policy implications.

Chapter 2

The fiscal multiplier in rare disaster-induced uncertainty

2.1 Introduction

The aim of this chapter is to understand how the size of fiscal multipliers is affected by rare disasters. I focus especially on the effects of increasing uncertainty on the value of the fiscal multiplier. For my analysis, I employ Dynamic Stochastic General Equilibrium (DSGE) models. My analysis is predicated on the idea that during a rare disaster, there may be a greater preference for government spending. I assume that total consumption consists of both private and public consumption. As a consequence, government spending – since it is part of consumption – can directly affect households' utility.[1] I model the marginal rate of substitution between private consumption and government spending (γ_{gt}) a random preference shock with stochastic volatility. This assumption is meant to capture preference shifts between private consumption and government spending as well as an increase in uncertainty during rare disasters. Following the approach put forward by Leeper et al. (2017), I first estimate γ_{gt} for the US economy at a quarterly frequency. These authors use a reformulated Smets-Wouters type new Keynesian model that allows government spending to be in the utility function

[1] To the best of my knowledge, this specification is first employed in Bailey (1971). Recent examples include Bouakez and Rebei (2007), F'eve et al. (2013), Ganelli and Tervala (2009), Leeper et al. (2017) and Sims and Wolff (2018).

and Bayesian methods to estimate γ_{gt}. I restrict γ_{gt} to be between zero and one and provide estimates for it for the period between 1979 to 2022. Then, I use the time series for γgt and the procedure employed in Born and Pfeifer (2021) to estimate a shock process for γ_{gt} with stochastic volatility.

I find that γ_{gt} was low at around 0.1 for most of the sample periods until the start of the pandemic. At the start of the pandemic, γ_{gt} increased substantially to 0.7. Estimation results also confirm the insight that there was a significant increase in the stochastic volatility of γ_{gt} during the pandemic.

Then, I use the models to understand how heightened uncertainty affects the size of the fiscal multiplier. I first carry out the analysis within a RBC economy. And I derive a simple analytical expression and establish a clear link between uncertainty and the fiscal multiplier, showing that uncertainty reduces the fiscal multiplier. In the limiting case, where uncertainty is infinitely large, the fiscal multiplier approaches zero. Then, I use a Smets-Wouters type model to quantify the empirical impact of uncertainty on the fiscal multiplier using a Smets and Wouters type new Keynesian (NK) model by calibrating the model with the estimated shock process. The results from this experiment confirm my analytical conclusion. In normal times, the fiscal multiplier is a little below one. With uncertainty, the multiplier is a lot lower at around 0.03.

The intuition behind these results is as follows. As is well-known, in DSGE models, an increase in government spending causes a negative wealth effect.[2] This is because an increase in government spending will be financed by future tax increases. The more persistent the increase in government spending is, the larger the negative wealth effect. In a rare disaster, as the depth and duration of the crisis are uncertain, this makes the degree of the negative wealth effect uncertain. It is this uncertainty in the degree of negative wealth effect that

[2]The negative wealth effect refers to Galí et al. (2007): *An increase in (non-productive) government purchases, financed by current or future lump-sum taxes, has a negative wealth effect which is reflected in lower consumption. It also induces a rise in the quantity of labour supplied at any given wage.*

makes households cautious, resulting in lower private consumption and the fiscal multiplier. Lower private consumption and higher government spending mean that during a rare disaster, households effectively substitute private consumption with government spending.

Micro-level evidence provided by Parker et al. (2022) reinforces these insights suggested by the models. Parker et al. (2022) use the Consumer Expenditure Interview Survey and measure the average response of consumer spending to EIP handouts and find that people spent less of their EIPs compared to similar previous policy episodes such as the 2008 Tax rebate. Parker et al. (2022) note that the depth and duration of the pandemic were uncertain, especially during the first round of EIPs was being disbursed and cite this uncertainty as a key factor for low spending of the EIPs. Kubota et al. (2021) examine the COVID-19 stimulus payments in Japan. The Japanese government implemented a programme similar to EIPs, where all individuals received a payment of around 950$. Kubota et al. (2021) reach a similar conclusion to that of Parker et al. (2022).

This chapter is related to the paper by Auerbach and Gorodnichenko (2012). These authors find that fiscal multipliers are state-dependent and are more effective during recessions. Estimation results suggest that the level and uncertainty of γ_{gt} are not sensitive to whether the economy is subjected to a recession (or an expansion). It appears that recessions do not induce a significant preference shift towards government spending as much as rare disasters do. This chapter is also related to the growing literature on uncertainty. Since the seminal paper by Bloom (2009), there has been extensive research on understanding the macroeconomic implications of uncertainty (see Bloom (2009) for a survey). Fernández-Villaverde et al. (2015) and Basu and Bundick (2017) present a thorough analysis of the effects of increasing uncertainty on the macroeconomy using new Keynesian models. These authors emphasise the presence of a precautionary savings channel in the model. This channel suggests that higher uncertainty leads households to save more. While this is also true our model, public consumption increases in my model.

The rest of the paper is organised as follows. Section 2.2 presents a theory and evidence for γ_g. Specifically, in this section, I estimate a shock process for γ_{gt} with stochastic volatility. Section 2.3 presents a simple RBC model and illustrates analytically that increasing uncertainty lowers the fiscal multiplier. This section also quantifies the effect of uncertainty on the fiscal multiplier using a Smets and Wouters (2007) type new Keynesian model. The model features sticky prices, sticky wages and habit persistence in consumption. Section 2.4 concludes the paper.

2.2 A Theory and Evidence for γ_g

In this section, I first define γ_g and then provide empirical evidence for it.

2.2.1 A Theory for γ_g

There is a continuum of households, indexed by $h \in [0,1]$. Households have identical preferences and derive utility not only from private consumption and leisure but also from government goods and services. I first consider the following general utility function for the representative household:

$$E_t \sum_{t=1}^{\infty} \beta^t \left(U(C_t^p, G_t, N_t) \right) \tag{2.1}$$

The intertemporal budget constraint is given by

$$C_t^p + B_{t+1} = N_t w_t - T_t + (1 + r_t) B_t \tag{2.2}$$

where E_t is the expectations operator, β is the discount factor, C_t^p is household's private consumption, $L_t = 1 - N_t$ is leisure and N_t is hours worked. G_t denotes government spending. G_t in the utility function can be thought of as a traditional public good such as education, health, economic affairs, public order and environment. B_t denotes bonds accumulated by the household until period

t. w_t is the real wage rate, T_t denotes the lump-sum tax paid by households, and r_t denotes the nominal interest rate.

The marginal rate of substitution between private consumption and government spending ($\gamma_g = (\frac{\partial U}{\partial G})$ / $(\frac{\partial U}{\partial C^p})$) is given by

$$\gamma_g = -\frac{dC^p}{dG} \tag{2.3}$$

where γ_g is the marginal rate of substitution between private consumption and government spending and captures how much private consumption changes in response to a change in government spending. The derivative on the RHS of this equation is assumed to be negative and between zero and -1. This assumption implies that households may choose to substitute some of their consumption with government spending. Given these assumptions, γ_g will be positive and between zero and 1. Government spending provides utility to the extent that it substitutes for private consumption. When the derivate is -1 and, therefore, $\gamma_g = 1$, households decrease their consumption one-to-one with government spending.

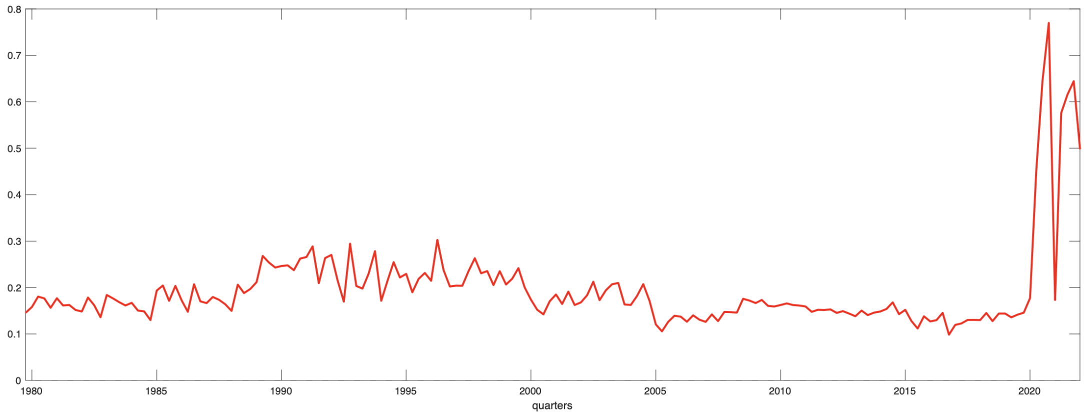
A key assumption here is that I assume that γ_{gt} is a time-varying and stochastic variable. This assumption is meant to capture changes in demand shifts for public goods due to pandemics, wars and natural disasters. Then, I further assume that the value of γ_{gt} is revealed to households at the beginning of period t.

2.2.2 Empirical evidence for γ_{gt}

This section presents estimates of γ_{gt} using data from 1955 to 2022. Then, I use the estimates to gauge the degree of stochastic volatility in γ_{gt}. To do so, we follow Leeper et al (2017). These authors estimate the value of γ_{gt} using a Smets and Wouters type model and Bayesian techniques. They use data for eight observables at a quarterly frequency from 1955Q1 to 2014Q2. These observables are log differences of aggregate consumption, investment, real wages, real government consumption, the GDP deflator, log hours worked, the federal funds rate and the real market value of government debt. Starting from 1979Q1, they estimate a 25-year (100 quarters) rolling average of γ_{gt} until 2014.

I update the dataset until 2022Q2 and, therefore, the dataset includes data from the COVID-19 pandemic. And I re-estimate the model by Leeper et al. and provide estimates of γ_{gt} until 2022Q2. The quarterly estimates of γ_{gt} from 1979Q1 to 2022Q2 are reported in Figure 2.1.

Figure 2.1: Estimated values of γ_{gt}



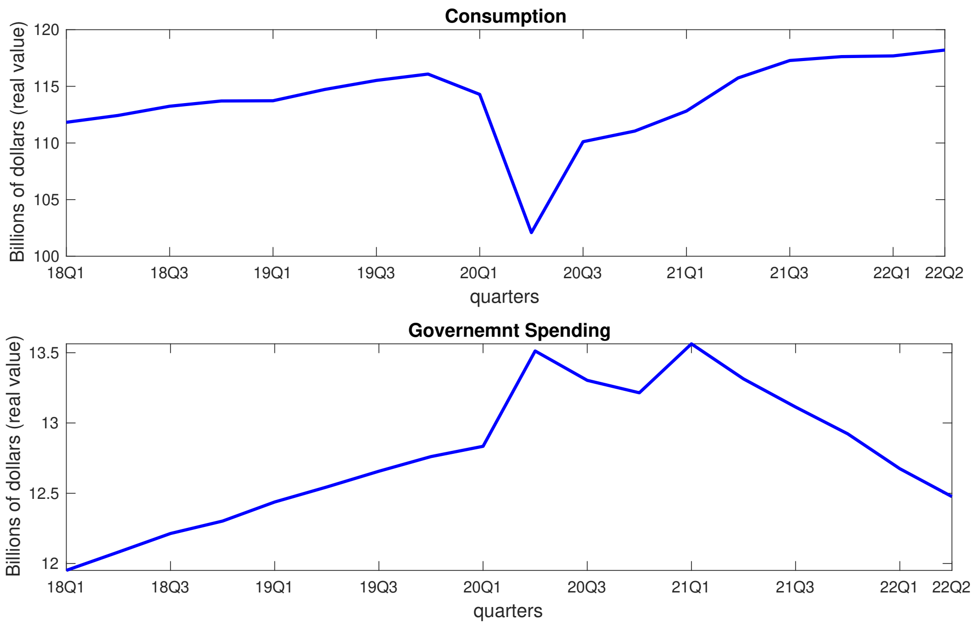
Note: The confidence bounds for γ_{gt} is assumed to be from 0 to 1. $\gamma_g t$ is estimated as a parameter in the New Keynesian model following Leeper et al (2017). The time variation in γ_{gt} is sequentially estimated by using a 25-year rolling window with quarter steps.

Figure 2.1 shows that γ_{gt} was low and relatively stable from 1979 to 1990. I see from the figure that during this period, the mean value of γ_{gt} was around 0.15. During the 1990s, γ_{gt} was higher at around 0.25 and more volatile. It then gradually reverted back to its value during the 1980s until the pandemic.

A striking feature of the figure is that there is a significant increase in the volatility of γ_{gt} after COVID-19. With the outbreak of COVID-19, in the second quarter of 2020, γ_{gt} increased substantially to around 0.7. It returned back to its steady-state value and then increase again to around 0.6. The high volatility in γ_{gt} seems to resemble the pattern of government expenditures during the pandemic. Real private consumption and government expenditures from 2018 and 2022 are plotted in Figure 2.2.[3]

[3]The total consumption includes non-durable goods and services consumptions. And gov-

Figure 2.2: Private Consumption and Government Spending During the Pandemic



As is evident from Figure 2.3, there was a significant decline in private consumption and a significant increase in government expenditures during this period. I also see that there was a "M-shaped" adjustment in government expenditures.

Next, using the time series for γ_{gt}, I estimate the stochastic volatility in γ_{gt}. To do so, I assume that γ_{gt} follows the following shock process with stochastic volatility:

$$\gamma_{gt} = \left(1 - \rho_{\gamma_g}\right) \gamma_g + \rho_{\gamma_g} \gamma_{gt-1} + \sigma_t \epsilon_{\gamma_{gt}} \tag{2.4}$$

$$\sigma_t = (1 - \rho_\sigma) \overline{\sigma} + \rho_\sigma \sigma_{t-1} + \sigma_\sigma \epsilon_{\sigma t} \tag{2.5}$$

where ρ_{γ_g} and ρ_σ are the persistence parameters. $\epsilon_{\gamma_g t}$ and $\epsilon_{\sigma t}$ are first-moment and second-moment shocks that characterise the innovation to the level and

ernment spending consists of Federal government expenditures and gross investment. (Nominal values are converted to real values using the GDP deflator.

volatility of the degree of the substitutability of consumption goods. Both shocks are i.i.d. and follow a unit normal distribution.

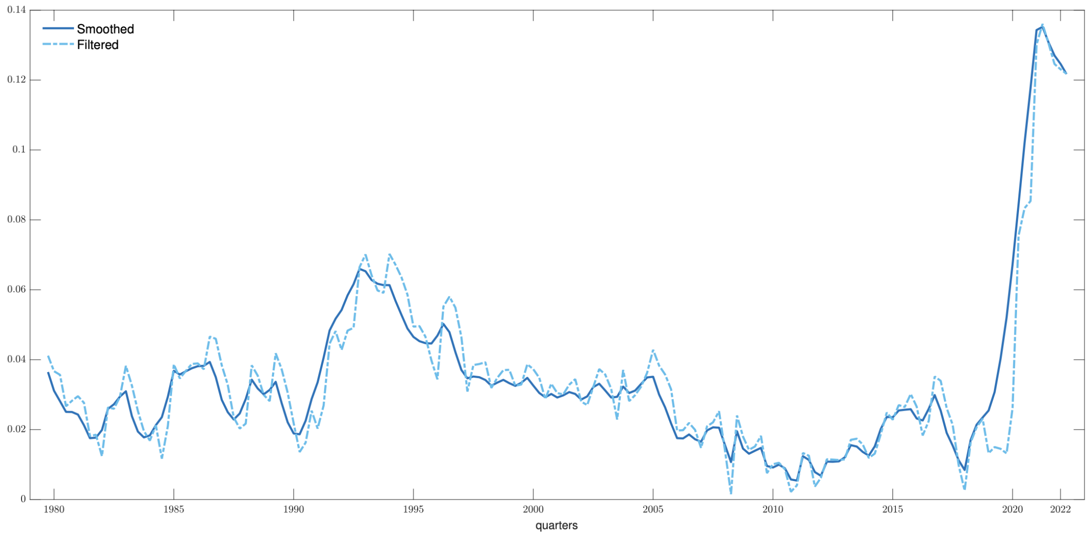
I estimate the shock processes in Equations 2.4 and 2.5 using data from 1955 to 2022. Table 2.1 reports parameter estimates and Figure 2.3 plots the implied stochastic volatility in γ_{gt}.

Table 2.1: Prior and Posterior Distributions of the Shock Processes

Parameter	Prior distribution			Posterior distribution		
	Distribution	Mean	Std. Dev.	Mean	5 Percent	95 Percent
$\rho_{\sigma^{\gamma_g}}$	Beta*	0.90	0.100	0.961	0.887	0.999
ρ_{γ_g}	Beta*	0.90	0.100	0.998	0.997	0.999
$\sigma_{\sigma_{\gamma_g}}$	Gamma	0.50	0.100	0.010	0.008	0.013
$\overline{\sigma}^{\gamma_g}$	Uniform	0.05	0.014	0.043	0.012	0.070

Note: Beta* indicates that the parameter divided by 0.999 follows a beta distribution. Posterior means and 90 percent credible intervals in brackets.

Figure 2.3: volatility of γ_{gt}



A key finding from Figure 2.3 is that there was a substantial increase in uncertainty in γ_{gt} during the pandemic. It is also worth pointing out that uncertainty was higher during the 1990s, relative to the 1980s, but not significantly so. It is also interesting to note that uncertainty in γ_{gt} did not change much during recessions such as the Great Recession in 2008.

Table 2.1 shows that the first-moment and the second-moment shocks are highly persistent. The standard deviation of the level shock is 4.3%, while that of the second-moment shock is 1%.

2.3 The fiscal multiplier and uncertainty

In this section, I derive consumption and fiscal multipliers in the model and examine how an increase in uncertainty affects the multipliers. To do so, I first employ a simple RBC model to illustrate the main ideas in a simple way and then a Smets and Wouters (2007) type new Keynesian model to quantify the effects in the fiscal multipliers.

2.3.1 The fiscal multiplier and uncertainty in the RBC model

I assume that the utility function takes the following form:

$$U(C_t^p, G_t, N_t) = ln(C_t^p + \gamma_{gt} G_t) - \frac{N_t^{1+\eta}}{1+\eta} \tag{2.6}$$

The household's budget constraint is the same as before. We repeat here for convenience:

$$C_t^p + B_{t+1} = N_t w_t - T_t + (1 + r_t) B_t \tag{2.7}$$

The government's budget constraint is given by

$$G_t + R_t B_t = B_{t+1} - B_t + T_t \tag{2.8}$$

The rest of the model is standard RBC. There is a continuum of identical and price-taker prices. Both product and labour markets are perfectly competitive. The production function is given:

$$Y_t = A N_t \tag{2.9}$$

where Y_t is output and A denotes constant productivity. The wage rate is equal to labour's marginal product:

$$w_t = A \tag{2.10}$$

The resource constraint is given by

$$Y_t = C_t^p + G_t \tag{2.11}$$

Finally, we assume the following process for γ_{gt}:

$$\gamma_{gt} = \bar{\gamma}_g + \epsilon_t \tag{2.12}$$

where $\bar{\gamma}_g$ is the steady-state value of γ_{gt} and ϵ_t is a white noise stochastic process with zero mean and variance $\sigma^2_{\gamma_g}$. The uncertainty in γ_{gt} is captured by an increase in variance $\sigma^2_{\gamma_g}$.[4]

Following the steps in Woodford (2011) and making use of Equations (2.9), (2.10) and (2.11) to obtain the following equation (the value of A is assumed as 1.):

$$\frac{1}{Y_t - (1-\gamma_{gt})G_t} = Y_t^{\eta} \tag{2.13}$$

Substituting Equation (2.12) into Equation (2.13) for γ_{gt} and taking the first derivate of the resulting equation with respect to government spending gives fiscal multiplier in the economy[5]

$$E_t(\frac{dY_t}{dG_t}) = \frac{1-\bar{\gamma}_g}{1+\eta E_t Y_t^{\eta-1}\left\{[E_tY_t - (1-\bar{\gamma}_g)E_tG_t]^2 + \sigma^2_{\gamma_g}(E_tG_t)^2\right\}} \tag{2.14}$$

This equation shows that the fiscal multiplier depends on $\bar{\gamma}_g$ and the variance of γ_g. It is straightforward to see that since $0 \leq \bar{\gamma}_g < 1$, the multiplier is positive. Our main conclusion is evident from this equation: an increase in uncertainty, which is captured by $\sigma^2_{\gamma_g}$, reduces the value of the multiplier.[6]

[4]The detailed shock process for γ_{gt} is shown in Equation 2.4 and 2.5, which is used in numerical discussion.

[5]The Appendix provides detailed derivations of fiscal and consumption multipliers.

[6]This conclusion holds even when $\bar{\gamma}_g$ is negative.

To understand this result, we calculate the consumption multiplier. The consumption multiplier is given by

$$E_t(\frac{dC_t^p}{dG_t}) = -\frac{A_1(\sigma^2_{\gamma_g}(E_tG_t)^2 + A_2) + \bar{\gamma}_g}{1 + A_1(\sigma^2_{\gamma_g}(E_tG_t)^2 + A_2)} \tag{2.15}$$

where $A_1 = \eta(E_tC_t^p + E_tG_t)^{\eta-1} > 0$ and $A_2 = (E_tC_t^p + \bar{\gamma}_g E_tG_t)^2 > 0$. The equation for the consumption multiplier is more cumbersome than before. To establish the link between the consumption multiplier and uncertainty, we take the first order condition of Equation 2.15 with respect to $\sigma^2_{\gamma_g}$. The resulting FOC is

$$\frac{d(E_t(\frac{dC_t^p}{dG_t}))}{d\sigma_{\gamma_g^2}} = -\frac{A_1(1-\bar{\gamma}_g)(E_tG_t)^2}{(1 + A_1(\sigma^2_{\gamma_g}(E_tG_t)^2 + A_2))^2} \tag{2.16}$$

Since A_1 is positive and the steady-state value of $\bar{\gamma}_g$ is between zero and one, Equation 2.16 indicates that this first-order condition is negative. This result implies that in response to an increase in government spending, in the presence of uncertainty, there is a reduction in private consumption. Reduced private consumption and the increase in government spending mean that the share of public consumption in the consumption basket increase with uncertainty.

What is the intuition behind these results? As is known, in the RBC model, government spending causes a negative wealth effect. It is useful to consider the standard RBC case in which $\bar{\gamma}_g$ is zero. As Equation 2.14 shows, the multiplier depends crucially on η, which measures the elasticity of the marginal disutility of work with respect to output increases. When $\eta = 0$, meaning that the disutility of work is low, households respond to fiscal expansion by increasing their labour supply. Increased labour supply completely offsets the negative wealth effect of higher government spending, resulting in a fiscal multiplier of one.

The multiplier can be even greater than one depending on the value of $\bar{\gamma}_g$. As emphasised by Feve et al. (2013), Ganelli and Tervala (2009) and Leeper et al. (2017), if $\bar{\gamma}_g < 0$, meaning that private consumption and government spending are complements, then the multiplier will be greater than one. The multiplier decreases if they are substitutes.

Therefore, in the baseline model with γ_{gt}, the value of the fiscal multiplier depends on the strength of the consumption-leisure substitution channel as well as whether private consumption and government spending are strategic complements or substitutes.

Uncertainty in γ_{gt} weakens the consumption-leisure substitution channel. This uncertainty makes households more cautious. Consequently, they choose lower consumption and higher leisure. Therefore, with higher uncertainty, the consumption-leisure substitution channel becomes less effective in offsetting the negative wealth effect, resulting in lower fiscal multipliers. Therefore, fiscal policy becomes less effective.

In my setup, the strength of negative wealth effects depends inversely on γ_{gt}: the lower the γ_{gt}, the greater the negative wealth effect. To see this, I combine the households' budget constraints with those of the government. By doing so, I obtain

$$C_t + (1 - N_t)W_t = W_t - (1 - \gamma_{gt})G_t \tag{2.17}$$

where $C_t = C_t^p + \gamma_{gt}G_t$. As this equation shows, an increase in government spending has a negative wealth effect . In my set-up, the degree of the negative wealth effect depends on the value of γ_{gt}. When $\bar{\gamma}_g = 1$, there is no wealth effect. This is because households substitute private consumption with government spending and still get the same utility. When $\bar{\gamma}_g = 0$, the negative wealth effect is the largest.

Uncertainty in γ_{gt} has consequences for the size of the negative wealth effect. Uncertainty in γ_{gt} makes the size of the negative wealth effect uncertain. This uncertainty leads households to be more cautious. The variable households can control is their consumption. Therefore, the optimal level of private consumption occurs at a lower level.

2.3.2 Quantifying the effect of uncertainty on the fiscal multiplier

In this section, I quantify the effect of uncertainty on the fiscal multiplier by employing the NK model. The model features sticky prices, wages, and habit persistence in consumption. As the baseline model is standard, the model's exposition is kept brief.

2.3.3 The New Keynesian model

The utility function and the corresponding budget constraint are given by

$$E_0 \sum_{t=1}^{\infty} \beta^t \left(\frac{(C_t - \gamma_b C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right) \tag{2.18}$$

subjected to the intertemporal budget constraint:

$$C_t^p + \frac{B_{t+1}}{P_t} = N_t w_t - T_t + R_t \frac{B_t}{P_t} + \mathit{\Pi}_t \tag{2.19}$$

Where, $C_t = C_t^p + \gamma_{gt} G_t$, β is the discount factor, γ_b is the persistence of habit formation, σ is the intertemporal elasticity of substitution in consumption, η is the inverse of Frisch elasticity of labour supply. N_t denotes hours worked, B_t is the one-period government bond that households hold, w_t is the real wage, T_t is the lump-sum tax paid by households, R_t denotes the nominal gross interest rate, and $\mathit{\Pi}_t$ denotes the profit that households obtain from the firms.[7]

The first-order conditions of the household optimisation problem are

$$\lambda_t = (C_t - \gamma_b C_{t-1})^{-\sigma} - \beta \gamma_b \left(E_t C_{t+1} - \gamma_b C_t \right)^{-\sigma} \tag{2.20}$$

$$\lambda_t = \beta E_t [\lambda_{t+1} \frac{R_t}{\pi_{t+1}}] \tag{2.21}$$

Where λ_t is the lagrange multiplier.

[7]Equation 2.3 still holds focusing on period t.

Wage setting

Households supply differentiated labour input $N(j)$. Different labour inputs are combined into a composite labour input by a union. The aggregation labour input is done according to the following equation:

$$N_t = \left(\int_0^1 N_t(j)^{\frac{\epsilon_n - 1}{\epsilon_n}} dj \right)^{\frac{\epsilon_n}{\epsilon_n - 1}} \tag{2.22}$$

where j is an index for differentiated labour inputs, ϵ_n is the elasticity of substitution between different labour inputs. The demand for type j labour input is given by

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_n} N_t \tag{2.23}$$

The demand for type j labour input depends on her own wage $(W_t(j))$, aggregate wage (W_t) and aggregate labour demand. Consequently, the aggregate wage rate is

$$W_t = \left(\int_0^1 W_t(j)^{1-\epsilon_n} dj \right)^{\frac{1}{1-\epsilon_n}} \tag{2.24}$$

As with firms, households set their wages according to the Calvo process. In each period, only a fraction of $1 - \theta_w$ of households can adjust the nominal wage. As households will update to an identical reset wage (W_t^*), it is given by

$$(W_t^*)^{1+\epsilon_n \eta} = \frac{\epsilon_n}{\epsilon_n - 1} \frac{F_t^1}{F_t^2} \tag{2.25}$$

Where:

$$F_t^1 = E_t \sum_{s=0}^{\infty} (\beta \theta_w)^s \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{-\epsilon_n (1+\eta)} W_{t+s}^{\epsilon_n (1+\eta)} N_{t+s}^{1+\eta} \tag{2.26}$$

$$F_t^2 = E_t \sum_{s=0}^{\infty} (\beta \theta_w)^s \lambda_{t+s} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{1-\epsilon_n} W_{t+s}^{\epsilon_n} N_{t+s} \tag{2.27}$$

Then, I express wages in terms of their real value where $w_t^* = \frac{W_t^*}{P_t}$ is the real reset wage and $w_t = \frac{W_t}{P_t}$ is the real wage. Therefore, the real reset wage is:

$$(w_t^*)^{1+\epsilon_n \eta} = \frac{\epsilon_n}{\epsilon_n - 1} \frac{f_t^1}{f_t^2} \tag{2.28}$$

where f_t^1 and f_t^2 are defined as $\frac{F_t^1}{P_t^{\epsilon_n(1+\eta)}}$ and $\frac{F_t^2}{P_t^{\epsilon_n-1}}$ respectively, and they are expressed recursively:

$$f_t^1 = (w_t)^{\epsilon_n(1+\eta)} (N_t)^{1+\eta} + \beta\theta_w E_t (\pi_{t+1})^{\epsilon_n(1+\eta)} f_{t+1}^1 \tag{2.29}$$

$$f_t^2 = (w_t)^{\epsilon_n} \lambda_t N_t + \beta\theta_w E_t (\pi_{t+1})^{\epsilon_n-1} f_{t+1}^2 \tag{2.30}$$

The average (real) wage is given by

$$w_t^{1-\epsilon_n} = \theta_w \pi_t^{\epsilon_n-1} w_{t-1}^{1-\epsilon_n} + (1-\theta_w) w_t^{*1-\epsilon_n} \tag{2.31}$$

Firms

There is a continuum of firms (indexed by i). Firms have monopoly power over a specific good and produce differentiated goods. Firms operate a constant technology that transforms labour into output subject to constant productivity: $Y_{it} = AN_{it}$ where $A = 1$. The differentiated goods are then combined to produce the final consumption good according to the standard Dixit-Stiglitz production function.

$$Y_t = (\int_0^1 Y_{it}^{\frac{\epsilon_p-1}{\epsilon_p}} di)^{\frac{\epsilon_p}{\epsilon_p-1}} \tag{2.32}$$

where ϵ_p is the elasticity of substitution between different intermediate goods. The corresponding price index is

$$P_t = (\int_0^1 P_{it}^{1-\epsilon_p} di)^{\frac{1}{1-\epsilon_p}} \tag{2.33}$$

Where P_t is the general price level. With these assumptions, the demand for firm i's output is given by

$$Y_{it} = (\frac{P_{it}}{P_t})^{-\epsilon_p} Y_t \tag{2.34}$$

The marginal cost is given by

$$mc_t = w_t \tag{2.35}$$

I assume that prices are sticky and firms set their prices according to the Calvo process. The reset price P_t^* is given by

$$\frac{P_t^*}{P_t} = \frac{\epsilon_p}{\epsilon_p - 1}\frac{Z_t^1}{Z_t^2} \tag{2.36}$$

$$Z_t^1 = E_t \sum_{j=0}^{\infty} (\theta_p \beta)^j \lambda_{t+j} mc_{t+j} \Pi_{tt+j}^{\epsilon_p} Y_{t+j} \tag{2.37}$$

$$Z_t^2 = E_t \sum_{j=0}^{\infty} (\theta_p \beta)^j \lambda_{t+j} \Pi_{tt+j}^{\epsilon_p - 1} Y_{t+j} \tag{2.38}$$

Where $1 - \theta_p$ is the hazard rate. Π_{tt+j} is the cumulative gross inflation rate over j periods. Z_t^1 denotes the present discounted value of future marginal cost and Z_t^2 denotes the present discounted value of marginal revenue. Z_t^1 and Z_t^2 can be rewritten as follows:

$$Z_t^1 = \lambda_t mc_t Y_t + \beta \theta_p E_t[\pi_{t+1}^{\epsilon_p} Z_{t+1}^1] \tag{2.39}$$

$$Z_t^2 = \lambda_t Y_t + \beta \theta_p E_t[\pi_{t+1}^{\epsilon_p - 1} Z_{t+1}^2] \tag{2.40}$$

The price index evolves according to the following equation.

$$P_t^{1-\epsilon_p} = \theta_p P_{t-1}^{1-\epsilon_p} + (1 - \theta_p) P_t^{*1-\epsilon_p} \tag{2.41}$$

The interest rate is set according to the Smets-Wouters type Taylor rule (in logs):

$$R_t - R = \rho R_{t-1} + (1 - \rho)(r_\pi \pi_t + \phi_y Y_t) + \phi_\Delta y(Y_t - Y_{t-1})) \tag{2.42}$$

The nominal gross interest rate (R_t) responds to inflation, output and the growth of output $\frac{Y_t}{Y_{t-1}}$. $\phi_\pi > 1$ and $\phi_y > 0$ are the coefficients in front of the targeting variables. R is the steady-state gross nominal interest rate.

Then, I need to define the government's budget constraint to complete the model. I assume that it is the same as before (see Equations 2.8−2.44). Following Galí et al. (2007), the fiscal instrument rule is given by

$$t_t = \phi_g g_t + \phi_b b_{t-1} \tag{2.43}$$

where $t_t \equiv \frac{T_t - T_{ss}}{Y_{ss}}$, $g_t \equiv \frac{G_t - G_{ss}}{Y_{ss}}$ and $b_{t-1} \equiv \frac{B_{t-1}\frac{R_{t-1}}{\pi_t} - B_{ss}\frac{R_{ss}}{\pi_{ss}}}{Y_{ss}}$. ϕ_g and ϕ_b are positive parameters. The government spending $g_t \equiv \frac{G_t - G_{ss}}{Y_{ss}}$ is assumed to be exogenous and follow AR(1) process as follows:

$$g_t = \rho_g g_{t-1} + \epsilon_{gt} \tag{2.44}$$

where ρ_g is the persistence parameter, and ϵ_{gt} is the government spending shock process with i.i.d. and follows a unit normal distribution.

2.3.4 Choice of Parameters

I calibrate the model at quarterly frequency using values common in the business cycle literature. The values are listed in Table 2.2. I set the discount factor β to 0.994. The relative risk aversion parameter σ is assumed to be 1. The consumption habit persistence γ_b is set to 0.8. Following Smets and Wouters (2007), both elasticities of substitution ϵ_p and ϵ_w are set to 10. The Frisch elasticity of labour supply $\frac{1}{\eta}$ is set to 0.5. The Calvo price stickiness θ_p is 0.6, implying a hazard rate of 0.4. The Calvo wage stickiness θ_w is 0.75.

The values chosen for the Taylor rule coefficients follows Smets and Wouters (2007): $\phi_\pi = 2.03$, $\phi_y = 0.08$, $\phi_\Delta y = 0.22$ and $\rho = 0.81$. Following Galí et al. (2007), in the fiscal instrument rule, the coefficient on debt ϕ_b is set to 0.33, while that on government ϕ_g is assumed to be 0.1. The steady-state ratio of government spending g_y ($\frac{G}{Y}$) is 0.2. Following Kara and Sin (2018), we set the steady-state ratio of debt-to-GDP b_y to 0.4 .

The steady-state value of γ_{gt} is set to $\bar{\gamma}_g = 0.2$, which is the estimated mean of this parameter from 1955Q1 to 2022Q1. Following Christiano et al. (2011), the persistence ρ_g of the government spending shock process is set to 0.8.

Table 2.2: Calibration

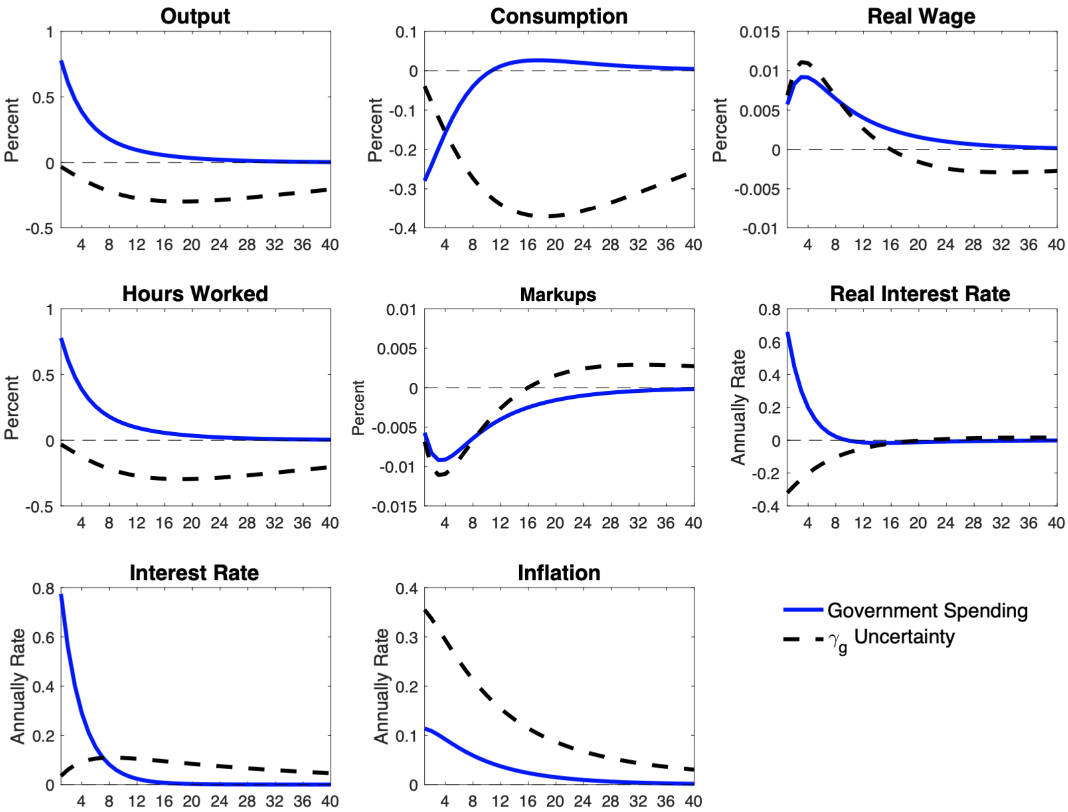
Structure Parameters		
β	0.994	Discount factor
σ	1	Risk aversion
γ_b	0.8	Consumption habit persistence
$\frac{1}{\eta}$	0.5	Frisch elasticity
θ_p	0.6	Calvo probability (Price)
θ_w	0.75	Calvo probability (Wage)
ϵ_p	10	Elasticity of subs. (goods)
ϵ_w	10	Elasticity of subs. (labour)
$\bar{\gamma_g}$	0.2	households' response coefficient
Policy parameters		
ϕ_π	2.03	Inflation response coefficient
ϕ_y	0.08	Output response coefficient
$\phi_\Delta y$	0.22	Output growth response coefficient
ρ	0.81	Interest rate smoothing
ϕ_b	0.33	Fiscal rule coefficient on debt
ϕ_g	0.1	Fiscal rule coefficient on government spending
g_y	0.2	Steady-state share of government spending
b_y	0.4	Steady-state share of bonds
Shock parameters		
ρ_g	0.8	Persistence of government spending

2.3.5 Dynamic responses of uncertainty shocks

In this section, to understand the macroeconomic effects of uncertainty shocks (second-moment shocks), I solve the model by taking a third-order Taylor approximation of equilibrium conditions around the steady state. Then, I use the estimated shock processes (see Table 2.1) to examine the effects of rare disaster-induced shocks on the uncertainty about γ_{gt} on the value of the fiscal multiplier. In Figure 2.4 and 2.5, I report the impulse response functions (IRFs) of the key variables in the NK model to an increase in government spending with and without uncertainty. I assume that the size of the government spending shock is 1% of the GDP. The estimated size of the γ_{gt} uncertainty is 0.12, which is shown in

Figure 2.3. Table 2.3 reports the resulting fiscal multipliers. [8] As a point of reference, Figure 2.4 also plots the IRFs to an uncertainty shock (without government spending).

Figure 2.4: Impulse Response Functions



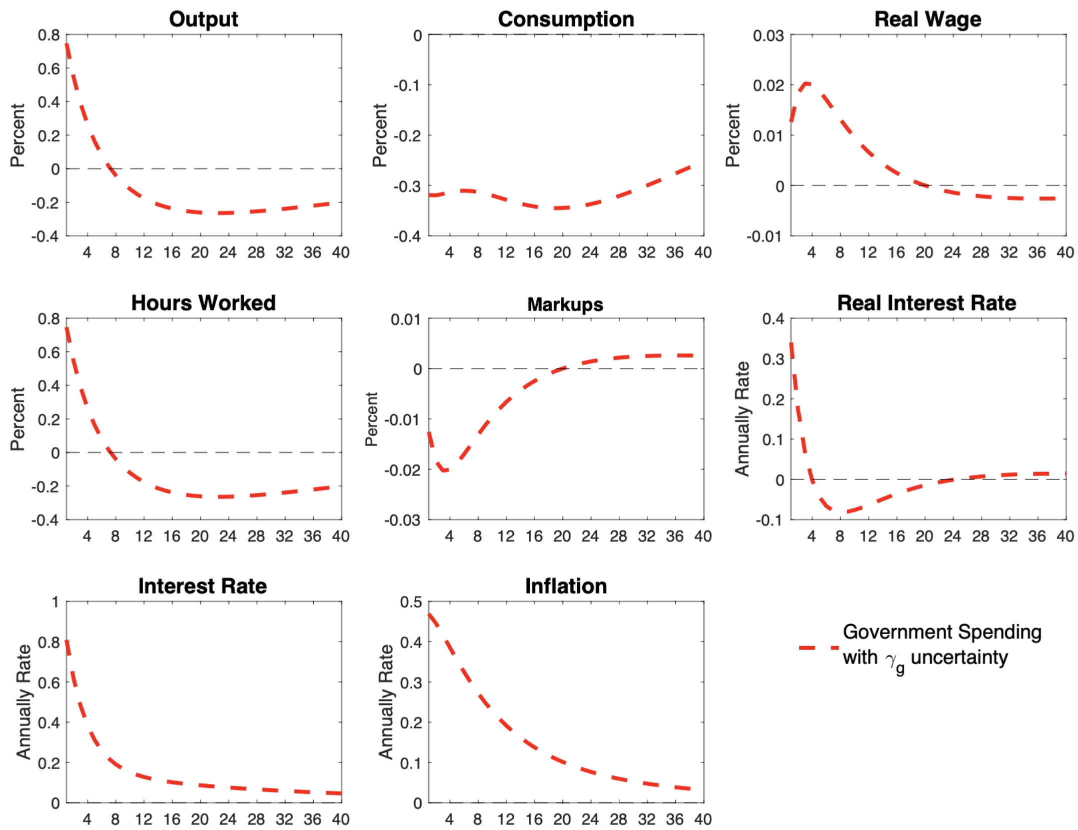
Note: The blue line captures the effects of the government spending shock, and the dashed black line shows the effects of γ_{gt} uncertainty shock.

I start the discussion with the case without uncertainty. As Figure 2.4 shows, in response to the increase in government spending, consumption and output rise. As Table 2.3 shows, in this case, the 10-quarters cumulative fiscal multiplier is 0.8. The fiscal multiplier is less than but close to one. If I look at the effect of fiscal stimulus on inflation, I see that the impact is positive but small. Inflation increases in response to the fiscal stimulus and gradually returns to the initial

[8]The formula for the cumulative multiplier, following Kara and Sin (2018), is defined as $\frac{E_t \sum_{t=0}^{\infty} dY_t}{E_t \sum_{t=0}^{\infty} dG_t}$.

steady state after four years of the shock.

Figure 2.5: Impulse Response Functions



Note: The dashed red line shows the effects of γ_{gt} uncertainty shock and government spending shock simultaneously.

I now examine the effectiveness of fiscal policy in times of increasing uncertainty. Before discussing the effects of fiscal stimulus in such an environment, it is helpful to explore the impact of increasing uncertainty on the economy. The dashed black line in Figure 2.4 shows the IRFs in response to the estimated uncertainty shock process. Figure 2.5 shows that both output and consumption fall significantly with increasing uncertainty. Both variables adjust in a hump−shaped manner and are persistent. They peak around three years after the shock. Given the recessionary effect of the shock, both hours worked and real wages fall.

If I look at the inflation response, the inflation is higher with increasing un-

Table 2.3: Fiscal Multiplier

Model Features	Without γ_{gt} Uncertainty	With γ_{gt} Uncertainty
Impact	0.77	0.62
4 quarters	0.76	0.45
8 quarters	0.78	0.17
10 quarters	0.79	0.03

certainty. However, it turns out that the effect is negligible. Inflation is only 0.4% percentage points higher. The positive inflation response is because, since prices are sticky, firms set larger price markups when resetting their prices. They do so to protect their prices against higher uncertainty during the period when the price will remain fixed.

Now, I turn to my main question: what is the impact of higher uncertainty on the fiscal multiplier? The dashed red line shows the effects of increasing government spending in the presence of uncertainty. In response to the rise in government spending, output increases. But the increase is smaller than in the case without uncertainty. While the impact multiplier is around 0.62, the 10-quarter cumulative fiscal multiplier is almost zero. This is because excluding the period immediately after the shock, the rise in government spending almost doesn't impact consumption. With higher uncertainty, after the initial increase in consumption, consumption falls and remains persistently negative for a long time. Therefore, as uncertainty holds consumption back, after the initial increase, output falls and remains negative during the reporting periods.

Finally, I look at the effect of government spending on inflation. Figure 2.4 and 2.5 show that inflation mirrors the behaviour of output. I see from the figures that inflation increases in response to government spending. However, the increase is not large.

2.3.6 The role of each friction

My model has three key frictions: price stickiness, wage stickiness and habit persistence in consumption. Now, I examine the contribution of each friction to my results. Table 2.4 reports fiscal multipliers when each friction is turned off at a time.[9] For comparison, Row (1) shows the results from the benchmark model with all three frictions, which we have discussed.

Table 2.4: Cumulative Fiscal Multiplier with Frictions

Model Features	Cumulative Multiplier	
	Without	With uncertainty
Price and Wage Rigidity with consumption habit	0.79	0.03
Price and Wage Rigidity	0.62	-0.49
Price Rigidity with habit formation	0.52	0.45
Wage Rigidity with habit formation	0.78	0.025

Row (2) shows multipliers without habit persistence. When there is no uncertainty, removing habit persistence does not significantly affect the value of the fiscal multiplier (0.79 vs 0.62). However, in the presence of uncertainty, habit persistence plays a more critical role. Consumption habits mitigate the effects of uncertainty on consumption and without habits, there is a significant fall in the multiplier. Without habit persistence, the multiplier is -0.5, while with habit persistence, it is 0.03.

Row (3) shows the value of the multiplier with sticky prices, habit persistence and flexible wages. In this case, the impact of uncertainty on the multiplier is limited. Since wages are flexible, wages fall, mitigating the effect of increasing uncertainty on the economy.

Row (4) shows the value of the multiplier with sticky wages, habit persistence and flexible prices. When prices are flexible, uncertainty significantly affects the fiscal multiplier than when wages are flexible (Row 3). This result indicates that sticky wages are a crucial friction, intensifying the negative impact of uncertainty

[9]The formula for fiscal multipliers is the same as Table 2.3.

on the fiscal multiplier.

2.4 Summary and conclusions

I have developed my approach to understanding how rare disasters-induced uncertainty affects the fiscal multiplier. I have focussed on the preference uncertainty induced by rare disasters. And I have examined how such uncertainty affects the effectiveness of fiscal policy and the value of the fiscal multiplier. In my framework, total consumption consists of private and public consumption. Government spending directly affects the utility of the representative household. I have assumed that the parameter governing the marginal rate of substitution between private consumption and government spending (γ_{gt}) is a time-varying stochastic process with stochastic volatility. This can be thought of as a random preference shift capturing the changes in the demand for government spending.

I carried out my analysis in two steps. First, we estimate γ_{gt} using the procedure put forward by Leeper et al. (2017) and show that there was a substantial increase in both the level and uncertainty of this parameter. Second, using DSGE models, I show that such uncertainty lowers the value of the multiplier. I first illustrate the general point within the basic RBC model. Then I show analytically that there is a negative relationship between increasing uncertainty and the size of the fiscal multiplier. Next, I quantify the effect of growing uncertainty on the fiscal multiplier using a Smets and Wouters (2007) style new Keynesian DSGE. A shock process for the households' response coefficient with stochastic volatility is estimated and I use it to calibrate the model. Without uncertainty, in my model, the fiscal multiplier is a little less than one. With uncertainty, the value of the multiplier reduces significantly to around 0.03.

A policy implication of my findings is that in times of large economic uncertainty, a government aiming to close a negative output gap should design a fiscal stimulus package larger than what would be required in regular times.

2.5 Appendix

2.5.1 Output Multiplier

Differentiating Equation (2.13) with respect to government spending yields:

$$\left(-\frac{1}{[Y_t-(1-\gamma_{gt})G_t]^2}\right)dY_t+\left((-\frac{1}{[Y_t-(1-\gamma_{gt})G_t]^2})(\gamma_{gt}-1)\right)dG_t=\left(\eta Y_t^{\eta-1}\right)dY_t \tag{2.45}$$

Simplifying equation (2.45) gives:

$$\frac{dY_t}{dG_t}=\frac{1-\gamma_{gt}}{1+\eta Y_t^{\eta-1}[Y_t-(1-\gamma_{gt})G_t]^2} \tag{2.46}$$

Finally, we combine this equation with Equation (2.12) to obtain the fiscal multiplier in the main text, where $E_t(\gamma_g t)=\gamma_g$ and $E_t(\gamma_g)^2=\gamma_g^2+\sigma_{\gamma_g}^2$:

$$\begin{aligned}\frac{E_t dY_t}{E_t dG_t}&=\frac{1-E_t(\gamma_{gt})}{1+\eta E_t Y_t^{\eta-1}[E_tY_t-(1-E_t(\gamma_{gt}))E_tG_t]^2}\\&=\frac{1-\gamma_g}{1+\eta E_tY_t^{\eta-1}[(E_tY_t)^2+(1+E_t(\gamma_{gt})^2-2E_t(\gamma_{gt}))(E_tG_t)^2-2E_tY_t(1-E_t(\gamma_{gt}))E_tG_t]}\\&=\frac{1-\gamma_g}{1+\eta E_tY_t^{\eta-1}[(E_tY_t)^2+(1+\gamma_g^2+\sigma_{\gamma_g}^2-2\gamma_g))(E_tG_t)^2-2E_tY_t(1-\gamma_g))E_tG_t]}\\&=\frac{1-\gamma_g}{1+\eta E_tY_t^{\eta-1}\left\{[E_tY_t-(1-\gamma_g)E_tG_t]^2+\sigma_{\gamma_g}^2E_tG_t^2\right\}}\end{aligned}$$

Simplifying the last expression gives the equation in the main text.

2.5.2 The Private Consumption Multiplier

Using the equation for the resource constraint, Equation (2.13) can be re-written as:

$$\frac{1}{C_t^p+\gamma_{gt}G_t}=(C_t^p+G_t)^\eta \tag{2.47}$$

Differentiating Equation (2.47) gives

$$\left(-\frac{1}{[C_t^p+\gamma_{gt}G_t]^2}\right)dC_t^p+\left(\gamma_{gt}(-\frac{1}{[C_t^p+\gamma_{gt}G_t]^2})\right)dG_t=(\eta(C_t^p+G_t)^{\eta-1})dC_t^p+(\eta(C_t^p+G_t)^{\eta-1})dG_t \tag{2.48}$$

Rearranging this equation gives

$$\frac{dC_t^p}{dG_t} = -\frac{\eta(C_t^p + G_t)^{\eta-1}(C_t^p + \gamma_{gt}G_t)^2 + \gamma_{gt}}{1 + \eta(C_t^p + G_t)^{\eta-1}(C_t^p + \gamma_{gt}G_t)^2} \tag{2.49}$$

Finally, we combine this equation with Equation (2.12) to obtain the fiscal multiplier in the main text, where $E_t(\gamma_g t) = \gamma_g$ and $E_t(\gamma_g)^2 = \gamma_g^2 + \sigma_{\gamma_g}^2$:

$$\begin{aligned}
\frac{E_t dC_t^p}{E_t dG_t} &= -\frac{\eta(E_tC_t^p + E_tG_t)^{\eta-1}(E_tC_t^p + E_t(\gamma_{gt})E_tG_t)^2 + E_t(\gamma_{gt})}{1 + \eta(E_tC_t^p + E_tG_t)^{\eta-1}(E_tC_t^p + E_t(\gamma_{gt})E_tG_t)^2} \\
&= -\frac{\eta(E_tC_t^p + E_tG_t)^{\eta-1}[(E_tC_t^p)^2 + E_t(\gamma_{gt})^2(E_tG_t)^2 + 2E_tC_t^pE_t(\gamma_{gt})E_tG_t] + E_t(\gamma_{gt})}{1 + \eta(E_tC_t^p + E_tG_t)^{\eta-1}[(E_tC_t^p)^2 + E_t(\gamma_{gt})^2(E_tG_t)^2 + 2E_tC_t^pE_t(\gamma_{gt})E_tG_t]} \\
&= -\frac{\eta(E_tC_t^p + E_tG_t)^{\eta-1}[(E_tC_t^p)^2 + (\sigma_{\gamma_g}^2 + \gamma_g^2)(E_tG_t)^2 + 2\gamma_gE_tC_t^pE_tG_t] + \gamma_g}{1 + \eta(E_tC_t^p + E_tG_t)^{\eta-1}[(E_tC_t^p)^2 + (\sigma_{\gamma_g}^2 + \gamma_g^2)(E_tG_t)^2 + 2\gamma_gE_tC_t^pE_tG_t]} \\
&== -\frac{\eta(E_tC_t^p + E_tG_t)^{\eta-1}[(E_tC_t^p + \gamma_gE_tG_t)^2 + \sigma_{\gamma_g}^2(E_tG_t)^2] + \gamma_g}{1 + \eta(E_tC_t^p + E_tG_t)^{\eta-1}[(E_tC_t^p + \gamma_gE_tG_t)^2 + \sigma_{\gamma_g}^2(E_tG_t)^2]}
\end{aligned}$$

Simplifying the last expression gives the equation in the main text.

Chapter 3

Optimal fiscal policy in rare disasters-induced uncertainty

3.1 Introduction

Rare disasters, such as the pandemic, wars, and natural disasters, have a recessionary effect on economy. Especially, the uncertainty induced by rare disasters further exacerbates the effect. To be specific, the outbreak of COVID-19 pandemic has induced a higher uncertainty in households' preference for private and public consumption, resulting in a decline in output and consumption, which is supported by the empirical evidence in Chapter 2.[1]

Fiscal policy has become a significant tool for government to stimulate economy in rare disasters. For example, to combat COVID-19 pandemic and subsequently support the recovery, a large-scale fiscal stimulus packages were conducted by many countries. Japan government has adopted the Emergency Economic Package which corresponds to 20.9 % of GDP. Italy, German and US governments have also undertaken a large package, amounting to more than 10% of their GDP. These packages significantly exceeded the economic stimulus packages of each country in the 2008 financial crisis. This raises the question of why governments around the world have chosen to adopt such massive fiscal stimulus

[1]The result of Chapter 2 shows that an increase in uncertainty leads to output declines of around 0.45% and consumption declines of around 0.58%. And the uncertainty in preference for private consumption and government spending is shown in Figure 2.3 in Chapter 2.

in response to the pandemic. One possible reason is that the disaster-induced uncertainty reduces the effectiveness of the fiscal stimulus, which is supported by the lower fiscal multiplier obtained from Chapter 2. During rare disasters, the shift in preference of households between private consumption and government spending becomes uncertain, which is referred as a preference shock and is allowed for stochastic volatility. This uncertainty results in households not responding to the increased government spending as governments would expect them to in normal times. At the same time, the government is unable to observe the actual responses of households to the increased government spending. As a result, fiscal policy becomes less effective. And the general fiscal stimulus is not sufficient to recovery economy in the face of the disaster-induced uncertainty. A massive fiscal policy package might be desirable in rare disasters.

However, it remains an open question of whether such massive fiscal policy packages are optimal in the presence of higher uncertainty in a rare disaster. This chapter aims to provide an answer to this question. I especially focus on the effects of disaster-induced preference uncertainty on the size of fiscal policy packages. To do so, I use DSGE models and derive a welfare-based objective function for the government by taking the second-order approximation of the representative household's utility function. I show that the welfare function depends on the variability in inflation, in the output gap and in the government spending. Additionally, I investigate the optimal fiscal policy in normal times and at the zero lower bound (ZLB) respectively.

I proceed the analysis into two steps. First, I analyse the desirability of fiscal policy in presence of the preference uncertainty between private and public consumption. The result suggests that this uncertainty plays an important role in designing the optimal fiscal policy. When the preference shock is uncertain, households become more cautious as they perceive that they may face higher costs after the disaster, and they tend to boost their precautionary savings. As a result, their consumption falls and then the demand for aggregate output decreases.

Therefore, the size of optimal government spending needs to be increased to maintain the same level of welfare. More generally, the optimal size of fiscal stimulus package increases with that kind of uncertainty.

Second, of course, during rare disasters, the zero lower bound on the nominal interest rate may be binding. In this case, I assume that the economy is hit by a disaster shock which directly reduces the demand for aggregate output. In response to it, the central bank lowers the nominal interest rate to the zero lower bound. Thus, the zero lower bound arises endogenously. At the same time, the disaster shock induces a high degree of uncertainty in the households' response to government spending. The result shows that in the presence of the zero lower bound, the disaster-induced uncertainty gives rise to a even larger need for fiscal policy in rare disasters. The reason for this is that, in addition to the preference uncertainty, the disaster shock also hits the economy and thus lowers aggregate output. It is the further reduction in output that increases the need for fiscal policy. Additionally, the size of optimal government spending also increases if the disaster shock is highly persistent. Consequently, when the ZLB is binding, the fiscal stimulus should even be larger in face of the uncertainty in households' preference for private consumption and government spending.

A similar stream of literature looks at the optimal fiscal policy but with different assumptions. Woodford (2011) analytically shows that when the zero lower bound is binding, government spending should be larger in the New Keynesian framework. He discusses the ZLB under the assumption of financial crisis rather than that of rare disasters. This chapter extends his analysis to a rare disaster case. To investigate the desirability of optimal fiscal policy, some papers assume government spending is separate from private consumption in the utility of households (e.g. Nakata, 2016; Bilbiie et al., 2014; Schmidt, 2013). They do not consider the possible relationship between private consumption and government spending. This chapter complements their analysis, focusing on the role of substitution between private consumption and government spending.

In the above respect, this chapter is also related to the growing literature on households' preference for private and public consumption. Sims and Wolff (2018) consider the non-separate private consumption government spending in the utility to analyse the welfare effect of fiscal policy. Their results shows that it is optimal for government to conduct a fiscal stimulus at the ZLB. However, they model the degree to which government spending substitutes private consumption as a constant parameter. However, according to the estimation form Chapter 2, the households' preference for private and public consumption fluctuates over time. And a high degree of volatility occurs during the pandemic. Thus, unlike their paper, I focus on the preference uncertainty between private consumption and government spending.

In addition, there is a number of papers taking uncertainty into account to investigate the optimal fiscal policy in interaction with monetary policy. Nakata (2016) and Schmidt (2013) finds that a greater fiscal stimulus is desirable to optimise welfare with respect to the discount factor uncertainty. Recently, Billi and Walsh (2021) suggest that an active fiscal policy interacted with a passive monetary policy can enhance welfare at the ZLB if the debt-financed fiscal stimulus is unbacked by future fiscal adjustments.

The rest of the chapter is organized as follows. Section 3.2 introduces household preference on private and public goods. Section 3.3 characterises the New Keynesian model with sticky price and its corresponding log-linearised form. Section 3.4 describes the calibrates parameters. Section 3.5 figures out the optimal government spending with preference uncertainty when the monetary policy is passive. Section 3.6 considers a disaster shock and the ZLB to evaluate fiscal policy in response to the preference uncertainty. Section 3.7 summarises.

3.2 Households' Preference on private and public consumption

Bailey (1971) defines that households' effective consumption is the sum of their private and public consumption. The public one is provided by government goods and services, which is shown as a fraction of government spending. Therefore, the effective consumption is given by:

$$C_t = C_t^p + \gamma_{gt} G_t \tag{3.1}$$

where C_t^p is the private consumption, G_t is government spending. Additionally, γ_{gt} measures the marginal rate of substitution between private consumption and government spending, which also indicates the household preference on private and public goods. This also refers to a direct relationship between government spending and households' consumption, which has become a standard specification adopted by recent literature.[2] Thus, government spending not only enhances households' utility, but also impacts their private consumption.

Then, households' utility function becomes $U(C_t^P, G_t, N_t)$, which is twice partially differentiable with continuous second derivatives, and strictly quasi-concave. To be specific, I assume the first partial derivatives $U_{C_t^P}$ and U_{G_t} with respect to C_t^P and G_t are positive. Thus, the utility rises with increasing private consumption and government spending. Inversely, the first partial derivative with respect to labour (working hours) is negative, implying that the utility decreases as working hours increase. The positive U_{G_t} indicates that government spending could partly enhance the utility and households obtain private benefits from public goods and services.

According to Equation (3.1), γ_{gt} is equivalence to the following equation:

$$\gamma_{gt} = \frac{U_{G_t}}{U_{C_t^P}} \tag{3.2}$$

[2] See, for example, Boehm, 2020; Sims and Wolff, 2018; Dawood and Francois, 2018; Leeper et.al, 2017 and Fève, Matheron and Sahuc, 2013.

where the RHS is the expression of the marginal rate of substitution between private consumption and government spending. Thus, γ_{gt} implies how many units of C_t^p households are willing to forego to obtain an additional unit of G_t. And the range for γ_{gt} is assumed to be between 0 and 1. If γ_{gt} is close to 0, it indicates that households find there is few private benefits from public goods and services, and then the government spending becomes wasteful. In contract, if γ_{gt} is close to one, it implies that households choose to only consume public goods and services. That is to say, public consumption completely substitutes private one, and government spending fully crowds out private consumption. The value of γ_{gt} is estimated in Chapter 2 which uses US data for eight observables at a quarterly frequency from 1955Q1 to 2022Q2.[3] The estimation result shows that γ_{gt} has fluctuated around 0.2 until COVID-19 pandemic. During the pandemic, the values of γ_{gt} increases dramatically but decrease immediately. It suggests that the demand of households for private and public consumption is greatly affected by economic environment, such as the occurrence of a rare disaster.

Moreover, the volatility of γ_{gt} is sharply increasing during the pandemic, which is also estimated in Chapter 2. The estimation result shows the pandemic induces a high volatility in households' preference for private and public consumption, which triggers a greater incentive for households to reduce their private consumption and increase the precautionary savings. At the same time, government conducts a large scale of fiscal stimulus to cope with the pandemic. Households might tend to rely more on public consumption in line with the high value of γ_{gt} in the pandemic.

[3]These observables are log differences of aggregate consumption, investment, real wages, real government consumption, the GDP deflator, log hours worked, the federal funds rate and the real market value of government debt. Starting from 1979Q1, I estimate a 25-year rolling average of γ_{gt} until 2022.

3.3 A simple New Keynesian model with government spending

In this chapter, to analyse how uncertainty in γ_{gt} impacts the optimal fiscal policy, I employ a closed-economy New Keynesian model with sticky price setting, flexible wages, and without capital accumulation (Woodford, 2011). Additionally, government spendings are financed through lump-sum taxes and the issuance of debt.

3.3.1 Households

The representative household chooses consumptions C_t^p and working hours N_t to maximise the utility, which is subject to his budget constraint, including the wages, profits from firms, tax and government bonds. The maximisation problem is:

$$\max_{C_t^p, N_t} \sum_{j=0}^{\infty} \beta^j \left\{ \frac{\left(C_t^p + \gamma_g G_t\right)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right\} \tag{3.3}$$

subject to

$$C_t^p + \frac{B_{t+1}}{P_t} = N_t \frac{W_t}{P_t} - T_t + (1 + i_t) \frac{B_t}{P_t} + \Pi_t \tag{3.4}$$

where β is the discount factor, σ is the inverse inter-temporal elasticity of substitution in consumption, η is the inverse of (substitution between labour and wage) Frisch elasticity of labour, P_t is the price level of consumption, B_t is the government bonds, i_t is the nominal interest rate, W_t is the nominal wage rate, T_t is the lump-sum tax, and Π_t is the profit from firms. To analytically discuss the role of γ_g, I firstly set the preference of the household on private and public consumption as a constant $\bar{\gamma}_g$ (The steady state value of γ_{gt}). Then, its uncertainty is discussed in section 5 and 6.

3.3.2 Firms

In production side of the economy, it consists of intermediate and final goods producers. The final-good producer is under the perfect competition. A representative final-good firm use intermediate goods to produce homogeneous goods. For intermediate-good producers, there is a continuum of firms imperfectly competing. And they hire labour to produce differentiated goods.

Final goods producers

With a standard constant return to scale (CES) technology, the final good is a constant elasticity of substitution aggregate of intermediate goods. The production function is given:

$$Y_t = (\int_0^1 Y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di)^{\frac{\epsilon}{\epsilon-1}} \tag{3.5}$$

where $\frac{\epsilon}{\epsilon-1}$ measures the price mark-up of intermediate goods.

Under the perfect competition, the final goods producers take the final and intermediate goods' prices as given and choose intermediate inputs to maximize their profits. After maximizing their profit, their optimal demand for intermediate inputs is derived as follows:

$$Y_{i,t} = (\frac{P_{i,t}}{P_t})^{-\epsilon} Y_t \tag{3.6}$$

where $P_{i,t}$ and P_t are the price levels of intermediate and final goods respectively. Combining final goods production function and the demand for intermediate inputs, I get a relationship between the final and intermediate goods' prices as follows:

$$P_t = (\int_0^1 P_{i,t}^{1-\epsilon} di)^{\frac{1}{1-\epsilon}} \tag{3.7}$$

Intermediate goods producers

Intermediate goods producers employ households and have access to a fixed technology (A) to produce the intermediate goods. Their production function is given as:

$$Y_{i,t} = N_{i,t}^d \tag{3.8}$$

where $N_{i,t}^d$ is the input labour, and the fixed technology A is assume as 1.

In order to pursue a maximized profit, the intermediate good producers firstly choose the amount of inputs $N_{i,t}^d$ to minimize their cost. Through this process, the real marginal cost is represented as follows:[4]

$$mc_t = w_t \tag{3.9}$$

where $w_t = \frac{W_t}{P_t}$ is the real wage rate.

Secondly, these imperfectly competitive producers can choose intermediate goods' prices to maximize their profits. I assume that the setting of the intermediate goods' price follows Calvo (1983) mechanism. In detail, all intermediate good producers face a hazard rate $1-\theta$ at each period, in which they can freely adjust their prices. In addition, to maximize the profit, I assume that there is an opportunity for the intermediate good producers to adjust its price at period t, but their prices will remain stuck in future. Then, the expected discount nominal profit for the producer is shown as:

$$\max_{P_{i,t}} E_t \sum_{j=0}^{\infty} (\theta\beta)^j \frac{(C_{i,t+j})^{-\sigma}}{(C_t)^{-\sigma}} (\frac{P_{i,t}}{P_{t+j}} Y_{i,t+j} - \frac{W_{t+j}}{P_{t+j}} \frac{Y_{i,t+j}}{A_{t+j}}) \tag{3.10}$$

Subjected to the demand for intermediate goods

$$Y_{i,t} = (\frac{P_{i,t}}{P_t})^{-\theta} Y_t \tag{3.11}$$

where $\beta^j \frac{(C_{i,t+j})^{-\sigma}}{(C_t)^{-\sigma}}$ denotes the stochastic discounted factor, and $C_t^{-\sigma}$ is the derivative of households' utility with respect to their consumption. When solving the maximizing problem, the optimal price for the intermediate firm is derived from the first order condition, which is simplify as follows: (considering a symmetric equilibrium, the optimal price for the intermediate goods is equal to the optimal one for the final goods)

$$\pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{Z_t^1}{Z_t^2} \tag{3.12}$$

$$Z_t^1 = E_t \sum_{j=0}^{\infty} (\theta\beta)^j C_{t+j}^{-\sigma} mc_{t+j} \Pi_{t,t+j}^{\epsilon} Y_{t+j} \tag{3.13}$$

[4] With notation A, the real marginal cost is $mc_t = \frac{w_t}{A}$.

$$Z_t^2 = E_t \sum_{j=0}^{\infty} (\theta\beta)^j C_{t+j}^{-\sigma} \Pi_{t,t+j}^{\epsilon-1} Y_{t+j} \tag{3.14}$$

where π_t^* is defined as $\frac{P_{i,t}^*}{P_t}$, and $\Pi_{t,t+j}$ is defined as $\frac{P_{t+j}^*}{P_t}$ which indicates the cumulative gross inflation rate over j periods. Z_t^1 denotes the present discounted value of future marginal cost, and its discounting part consists of future expected output and inflation levels, which can be interpreted as the marginal change in production for a unit change in the optimal reset price (Ascari and Ropele, 2009). Z_t^2 denotes the present discounted value of marginal revenue. In addition, Z_t^1 and Z_t^2 are also rewritten as follows:

$$Z_t^1 = C_t^{-\sigma} mc_t Y_t + \beta\theta E_t[\pi_{t+1}^{\epsilon} Z_{t+1}^1] \tag{3.15}$$

$$Z_t^2 = C_t^{-\sigma} Y_t + \beta\theta E_t[\pi_{t+1}^{\epsilon-1} Z_{t+1}^2] \tag{3.16}$$

Based on Calvo (1983) pricing mechanism, the price index evolves:

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta) P_t^{*1-\epsilon} \tag{3.17}$$

3.3.3 Government

The Monetary Policy

The monetary authority follows a simple Taylor rule which is given:

$$\frac{i_t}{i} = (\frac{\pi_t}{\pi_{target}})^{\phi_\pi} (\frac{Y_t}{Y_{t-1}})^{\phi_y} \tag{3.18}$$

where the nominal interest rate i_t responses to inflation and the growth of output $\frac{Y_t}{Y_{t-1}}$, $\phi_\pi > 1$ and $\phi_y > 0$. The inflation target is assumed as zero ($\pi_{target} = 1$).

The Fiscal Policy

Government spending is financed through lump-sum tax and bonds. Thus, shifts in government spending paths are thought to imply changes in tax and bond paths to maintain government solvency over time. The government budget constraint is

$$G_t + B_{t-1} \frac{i_{t-1}}{\pi_t} = B_t + T_t \tag{3.19}$$

Following Gali et al. (2007), the fiscal instrument rule is given by

$$t_t = \phi_g g_t + \phi_b b_{t-1} \tag{3.20}$$

where $t_t \equiv \frac{T_t - T_{ss}}{Y_{ss}}$, $g_t \equiv \frac{G_t - G_{ss}}{Y_{ss}}$ and $b_t \equiv \frac{B_{t-1}\frac{i_{t-1}}{\pi_t} - B_{ss}\frac{i_{ss}}{\pi_{ss}}}{Y_{ss}}$. ϕ_g and ϕ_b are positive parameters. The government spending $g_t \equiv \frac{G_t - G_{ss}}{Y_{ss}}$.

3.3.4 Market Clearing and Equilibrium

Under the market clearing condition, I derive the aggregate demand for the final goods as follows:

$$Y_t^d = \frac{N_t}{v_t^p} \tag{3.21}$$

where v_t^p is define as $\int_0^1 (\frac{P_{it}}{P_t})^{-\epsilon}$. According to the price index of Calvo's pricing, it is derived as follows:

$$v_t^p = \theta \pi_t^{\epsilon} v_{t-1}^p + (1-\theta)\pi_t^{*-\epsilon} \tag{3.22}$$

Moreover, the resource constraint in the market clearing is:

$$Y_t = C_t^P + G_t \tag{3.23}$$

3.3.5 Log-linearised Model

The model is log-linearised from the steady state with no trend growth, constant government spending and zero inflation, and with a subsidy that exactly offsets the steady-state distortions arising from price mark-ups. Additionally, the technology is assumed to be constant ($A = 1$).[5] A detailed derivations of log-linearised equations are presented in Appendix. The following are competitive equilibrium conditions after linearisation:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}(i_t - E_t \hat{\pi}_{t+1} - \bar{r}) \tag{3.24}$$

[5] The value of A is directly applied in the left sections.

$$\hat{c_t} = \frac{1-g_y}{1-(\bar{\gamma}_g - 1)g_y}\hat{c_{pt}} + \frac{\bar{\gamma}_g}{1-(\bar{\gamma}_g - 1)g_y}\hat{g}_t \tag{3.25}$$

$$\varphi \hat{n}_t = -\sigma \hat{c}_t + \hat{w}_t \tag{3.26}$$

$$\hat{w}_t = \hat{mc_t} \tag{3.27}$$

$$\hat{\pi}_t = \frac{(1-\theta\beta)(1-\theta)}{\theta}\hat{mc_t} + \beta E_t \hat{\pi_{t+1}} \tag{3.28}$$

$$\hat{y}_t = \hat{n}_t \tag{3.29}$$

$$\hat{y}_t = (1-g_y)\hat{c}_{pt} + \hat{g}_t \tag{3.30}$$

where Equation 3.24 is the linearised Euler equation; $\bar{r}$ is the steady state interest rate which is given by $-\log\beta$. And Equation 3.25 is linearised effective consumption, where $g_y = \frac{\bar{g}}{\bar{y}}$ denotes steady-state government spending share of output. $\hat{g}_t$ is defined as $\frac{g_t - \bar{g}}{\bar{y}}$ which is the deviation of government spending from its steady state as a share of steady-state output. Equation 3.26 and 3.27 are the linearised labour supply and demand condition respectively. And Equation 3.28 is the linearised Phillips curve, which comes from price-setting conditions that follows Calvo (1983) mechanism. Equation 3.29 is the linearised production function. Since the linearised equilibrium conditions are derived from zero inflation steady state and the absence of technology shock, production solely depends on working hours $\hat{n}_t$. Equation 3.30 is the linearised resource constraint.

In conjunction with above equations (Equation 3.24- 3.30), the New Keynesian Phillips curve is given by

$$\hat{\pi}_t = \lambda_1 \kappa(\hat{y}_t - \Gamma \hat{g}_t) + \beta E_t \hat{\pi}_{t+1} \tag{3.31}$$

where the output gap is equivalence to $\hat{y}_t - \Gamma\hat{g}_t$, The parameters λ_1, κ and Γ are given by:

$$\lambda_1 \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta} \tag{3.32}$$

$$\kappa \equiv \eta + \frac{\sigma}{1+(\bar{\gamma}_g - 1)g_y} \tag{3.33}$$

$$\Gamma \equiv \frac{(1-\bar{\gamma_g})\sigma}{\sigma + \eta(1 + (\bar{\gamma_g} - 1)g_y)} \tag{3.34}$$

The New Keynesian IS curve derived from the Euler equation (Equation (3.24)) can be rewritten as:

$$\hat{y}_t - (1-\bar{\gamma_g})\hat{g}_t = E_t\hat{y}_{t+1} - (1-\bar{\gamma_g})E_t\hat{g}_{t+1} - \frac{1 + (\bar{\gamma_g} - 1)g_y}{\sigma}(i_t - E_t\hat{\pi}_{t+1} - \bar{r}) \tag{3.35}$$

If $\bar{\gamma_g}$ and g_y are set to 0, the Phillips and IS curve are identical to those of Woodford (2011).

3.4 Parameter values

Table 3.1: Calibration

Structure Parameters		
β	0.994	Discount factor
σ	1	Risk aversion
$\frac{1}{\eta}$	0.5	Frisch elasticity
θ	0.6	Calvo probability (Price)
ϵ	10	Elasticity of subs. (goods)
$\bar{\gamma_g}$	0.2	households' response coefficient
Policy parameters		
ϕ_π	1.5	Inflation response
ϕ_y	0.5/4	Output response
ϕ_b	0.33	Fiscal rule coefficient on debt
ϕ_g	0.1	Fiscal rule coefficient on government spending
g_y	0.2	Steady-state share of government spending
b_y	0.4	Steady-state share of bonds

Table 3.1 shows the baseline values used in remaining sections. As the model is similar to that in Chapter 2, the parameter values are referred to the same sources and motives, which is list in Table 2.1, but the Taylor rule follows a standard setting. The discount factor β is set to 0.994. The relative risk aversion parameter σ is assumed to be 1. Following Smets and Wouters (2007), the elasticity of

substitution ϵ is set to 10. The Frisch elasticity of labour supply $\frac{1}{\eta}$ is set to 0.5. The Calvo price stickiness θ is 0.6, implying a hazard rate of 0.4.

The values chosen from the literature: $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. Following Galí et al. (2007), in the fiscal instrument rule, the coefficient on debt ϕ_b is set to 0.33, while that on government ϕ_g is assumed to be 0.1. The steady-state ratio of government g_y ($\frac{G}{Y}$) is 0.2. Following Kara and Sin (2018), I set the steady-state ratio of debt-to-GDP to 0.4. The steady-state value of γ_{gt} is set to $\bar{\gamma_g}$= 0.2, which is the estimated mean of this parameter from 1955Q1 to 2022Q2.

3.5 Optimal Fiscal with Passive Monetary Policy

In this section, I evaluate the optimal fiscal policy in the simple scenario which assumes a passive monetary policy. This implies that the nominal interest rate is constant and the central bank does not respond to changes in the economy. Thus, I define the nominal interest rate as i_C which is constant and greater than its steady state $\bar{r}$ ($i_C - \bar{r} > 0$). And I only consider changes in variables at the period t. Therefore, the Phillips and IS curve become:

$$\hat{\pi}_t = \lambda_1 \kappa(\hat{y}_t - \Gamma \hat{g}_t) \tag{3.36}$$

$$\hat{y}_t - (1 - \bar{\gamma_g})\hat{g}_t = -\frac{1 + (\bar{\gamma_g} - 1)g_y}{\sigma}(i_C - \bar{r}) \tag{3.37}$$

3.5.1 Welfare Analysis

The optimal fiscal policy is evaluated from a welfare perspective. For this purpose, I derive the welfare loss based on a second-order approximation to the utility of the representative household as a consequence of fluctuations around an efficient steady state with zero inflation. Additionally, the fiscal policy is assumed to be discretionary. Then, I follow Woodford (2011) to express the periods social welfare loss as a fraction of steady-state conditions:

$$L = \left[\lambda_y(\hat{y}_t - \Gamma\hat{g}_t)^2 + \lambda_g\hat{g}_t^2 + \frac{\epsilon}{\lambda_1}\hat{\pi}_t^2\right] \tag{3.38}$$

where the welfare loss function depends on output gap , inflation and government spending. Thus, government spending can affect welfare in both direct and indirect ways. These parameters $\Gamma, \lambda_1, \lambda_y, \lambda_g$ and B are all positive, which is proved in appendix. And λ_y, λ_g and B are given by:

$$\lambda_y \equiv \eta + \frac{\sigma}{B} \tag{3.39}$$

$$\lambda_g \equiv \frac{\sigma(1-\bar{\gamma}_g)^2}{B} - \lambda_y\Gamma^2 \tag{3.40}$$

$$B \equiv 1 - (1-\bar{\gamma}_g)g_y \tag{3.41}$$

Governments choose the size of government spending to minimize the welfare loss subject to two constraints (Equation 3.36 - 3.37). Thus, the first order condition with respect to g_t is given as:

$$2(\lambda_y + \epsilon\lambda_1\kappa^2)(1-\bar{\gamma}_g-\Gamma)\left[(1-\bar{\gamma}_g-\Gamma)\hat{g}_t + \frac{B}{\sigma}(\bar{r}-i_C)\right] + 2\lambda_g\hat{g}_t = 0 \tag{3.42}$$

Then, the optimal government spending $\hat{g}_t^*$ becomes:

$$\hat{g}_t^* = \frac{\eta(\eta B+\sigma)(\epsilon\lambda_1(\eta B+\sigma)+B)}{\eta^2(1-\bar{\gamma}_g)(\epsilon\lambda_1(\eta B+\sigma)+B)+\sigma\eta(1-\bar{\gamma}_g)} \cdot \left(\frac{-\bar{r}+i_C}{\sigma}\right) > 0 \tag{3.43}$$

Additionally, the optimal output $\hat{y}_t^*$ is given by:

$$\hat{y}_t^* = \frac{\eta\sigma\epsilon\lambda_1(\eta B+\sigma)}{\eta^2(1-\bar{\gamma}_g)(\epsilon\lambda_1(\eta B+\sigma)+B)+\sigma\eta(1-\bar{\gamma}_g)} \cdot \left(\frac{-\bar{r}+i_C}{\sigma}\right) > 0 \tag{3.44}$$

where the signs of Equation 3.43 and 3.44 are proved in Appendix.

As the optimal government spending (Equation 3.43) is positive, it shows that a fiscal stimulus is desirable and improves welfare when monetary policy is passive. Moreover, $\bar{\gamma}_g$ occurs in both Equation 3.43 and 3.44. Hence, the value of $\bar{\gamma}_g$ can have an effect on optimal government spending. When the households' preference on private consumption and government spending (γ_{gt}) fluctuates over the time, the optimal value of government spending and output also changes. When there is an uncertainty in γ_{gt}, the size of optimal government spending might be affected.

3.5.2 Uncertainty about the household demand between private consumption and government spending

In this section, I assume that the household demand between private and public consumption is time-varying captured by an innovation shock (a preference shock), which is given by $\gamma_{gt} = \bar{\gamma}_g + \epsilon_t$. And ϵ_t follows a normal distribution with zero mean and standard deviation σ_{γ_g} ($\epsilon_t \sim N(0, \sigma^2_{\gamma_g})$). The volatility or uncertainty in γ_{gt} is measured by an increase in the variance $\sigma^2_{\gamma_g}$.

With these assumptions, the expected optimal level of government spending $E_t\hat{g}_t^{*un}$ is given by

$$E_t\hat{g}_t^{*un} = \frac{A_1 + (\eta g_y)^2(\epsilon\lambda_1\eta + 1)\sigma^2_{\gamma_g}}{A_2 - \eta^2 g_y(\epsilon\lambda_1\eta + 1)\sigma^2_{\gamma_g}} \cdot \left(\frac{-\bar{r} + i_C}{\sigma}\right) \tag{3.45}$$

where A_1 and A_2 are positive, and satisfy:

$$A_1 \equiv \eta(\eta B + \sigma)(\epsilon\lambda_1(\eta B + \sigma) + B) \tag{3.46}$$

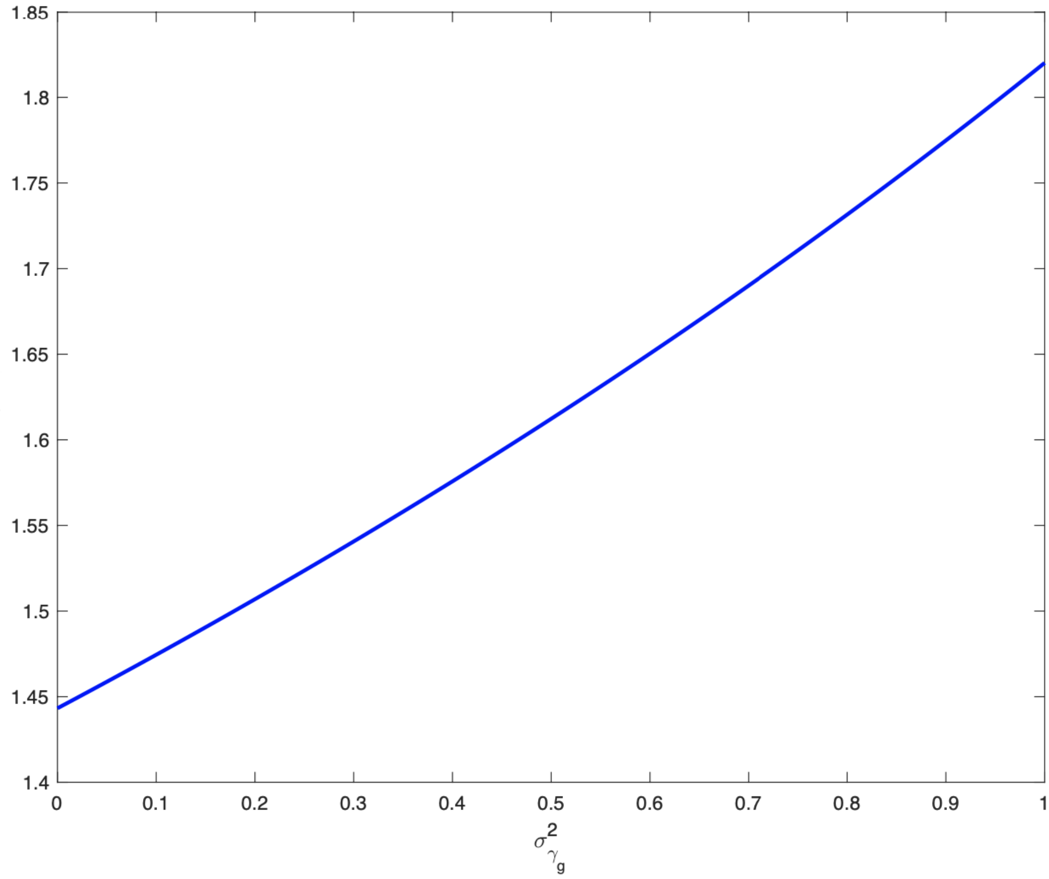
$$A_2 \equiv \eta^2(1 - \bar{\gamma}_g)(\epsilon\lambda_1(\eta B + \sigma) + B) + \sigma\eta(1 - \bar{\gamma}_g) \tag{3.47}$$

According to Equation 3.45, the magnitude of $E_t\hat{g}_t^{*un}$ is affected by the variance $\sigma^2_{\gamma_g}$. Moreover, comparing Equation 3.43 and 3.45, the original size of optimal fiscal stimulus is no longer optimal in face of uncertainty. To further understand how uncertainty in γ_{gt} affects the optimal government spending, I take the first derivative with respect to the variance $\sigma^2_{\gamma_g}$. Formally,

$$\frac{\partial E_t\hat{g}_t^{*un}}{\partial \sigma^2_{\gamma_g}} = \frac{\eta^2 g_y(\epsilon\lambda_1\eta + 1)(g_y A_2 + A_1)}{\left(A_2 - \eta^2 g_y(\epsilon\lambda_1\eta + 1)\sigma^2_{\gamma_g}\right)^2} \left(\frac{-\bar{r} + i_C}{\sigma}\right) > 0 \tag{3.48}$$

As the values of A_1 and A_2 are positive, the first derivative (Equation 3.48) is also positive. Thus, it implies that the size of optimal government spending increases with the variance of γ_{gt}. The increasing variance indicates a rise in uncertainty. Thus, an increase in γ_{gt} uncertainty requires a larger size of optimal government spending. More specifically, in regards to the preference uncertainty, government needs to boost the quantity of fiscal package to optimise households' welfare.

Figure 3.1: The optimal fiscal policy with increasing γ_{gt} uncertainty



Note: The vertical axis denotes the value of $g(\sigma^2_{\gamma_g})$. It changes with different values of variance $\sigma^2_{\gamma_g}$

To further investigate the effect of increasing γ_{gt} uncertainty on the optimal government spending, I rearrange Equation 3.45 as:

$$g(\sigma^2_{\gamma_g}) \equiv \frac{\hat{g}_t^{*un}}{i_C - \bar{r}} = \frac{A_1 + (\eta g_y)^2(\epsilon\lambda_1\eta + 1)\sigma^2_{\gamma_g}}{\sigma\left(A_2 - \eta^2 g_y(\epsilon\lambda_1\eta + 1)\sigma^2_{\gamma_g}\right)}$$

Since the value of $(i_C - \bar{r})$ is constant and positive, the effect of $\sigma^2_{\gamma_g}$ on $\hat{g}_t^{*un}$ is equivalent to that on $g'(\sigma^2_{\gamma_g})$. Then, with calibrated parameters values from Table 3.1, Figure 3.1 plots how the optimal value of $g(\sigma^2_{\gamma_g})$ changes with increasing γ_{gt} uncertainty $(\sigma^2_{\gamma_g})$. For a given passive monetary policy and γ_{gt} uncertainty by $(i_C - \bar{r}, g(\sigma^2_{\gamma_g}))$, the optimal size of the increase in government spending can be obtained from Figure 3.1 by observing the optimal ratio for that value of $g(\sigma^2_{\gamma_g})$, and then multiplying by the value of $i_C - \bar{r}$. Figure 3.1 shows that an increase in γ_{gt} variance leads to a rise in the value of $g(\sigma^2_{\gamma_g})$. Thus, a higher uncertainty in γ_{gt} requires a large optimal size of government spending to enhance welfare.

3.6 Optimal Fiscal Policy at the Zero Lower Bound

As the uncertainty in γ_{gt} can impact the optimal fiscal policy under a passive monetary policy, I further investigate how this uncertainty affects optimal fiscal policy in a more realistic scenario. Supposing that a disaster shock (e.g. wars, the pandemic, earthquakes) occurs, it has a recessionary impact on the economy and the nominal interest rate is pegged at zero. In order to solve it, the government wants to design a optimal fiscal stimulus package to recover the economy.

First, the rare disaster shock is assumed to be a large and negative aggregate demand shock (μ_t) and follow an AR(1) process with autoregressive coefficient ρ and standard deviation σ_μ. And this exogenous process of the shock is assumed to be unaffected by either monetary or fiscal policy choices. As the Phillips and IS curves are purely forward-looking, they can be expressed in terms of μ_t and ρ, then become:

$$\hat{\pi}_t = \frac{\lambda\kappa}{1 - \beta\rho}(\hat{y}_t - \Gamma\hat{g}_t) \tag{3.49}$$

$$(1-\rho)\left(\hat{y}_t-(1-\bar{\gamma}_g)\hat{g}_t\right)=\frac{1+(\bar{\gamma}_g-1)g_y}{\sigma}(\bar{r}-i_t+\rho\hat{\pi}_t-\mu_t) \tag{3.50}$$

where I assume $E_tX_{t+1}=\rho X$, X denotes the variables $\hat{y}_t, \hat{g}_t$ and $\hat{\pi}_t$.

Second, the monetary policy follows the simple Taylor rule (Equation 3.18), but the zero lower bound is most likely to become a binding constraint on the monetary policy. Therefore, the nominal interest rate becomes:

$$\hat{i}_t=\max\left[\bar{r}+\phi_\pi\hat{\pi}_t+\phi_y\hat{y}_t,0\right] \tag{3.51}$$

In conjunction with the Phillips and IS curves (Equation 3.49 and 3.50), it shows the equilibrium output given the level of government spending, which is given as:

$$\hat{y}_t=\vartheta_r(\bar{r}-i_t-\mu_t)+\vartheta_g\hat{g}_t \tag{3.52}$$

where ϑ_r, ϑ_g and B are greater than 0 [6] and given by:

$$\vartheta_r \equiv \frac{(1-\beta\rho)B}{\sigma(1-\rho)(1-\beta\rho)-\lambda\kappa\rho B}>0 \tag{3.53}$$

$$\vartheta_g \equiv \frac{(\sigma+\eta B)(1-\rho)(1-\beta\rho)-\lambda\kappa\rho B}{\sigma(1-\rho)(1-\beta\rho)-\lambda\kappa\rho B}\Gamma>0 \tag{3.54}$$

When the rare disaster shock μ_t is large enough and the nominal interest rate i_t is pegged at 0, the value of $\vartheta_r(\bar{r}-i_t-\mu_t)$ becomes negative. Thus, in Equation 3.52, government spending should increase to make output $\hat{y}_t$ positive. In other words, to boost aggregate demand, the effective way is to increase government spending.

3.6.1 Welfare at the ZLB

To further study the desirability of government spending at the ZLB, I examine the size of the optimal fiscal stimulus. Following the same steps in section 5, the

[6] The detail about how to derive these equations is shown in Appendix

welfare loss function is derived by a second-order approximation which is similar to Equation 3.38.

$$L = \frac{1}{1-\beta\mu}\left[\lambda_y(\hat{y}_t - \Gamma\hat{g}_t)^2 + \lambda_g\hat{g}_t^2 + \frac{\epsilon}{\lambda_1}\hat{\pi}_t^2\right] \tag{3.55}$$

where the weights are related to the model parameters according to:

$$\Gamma \equiv \frac{(1-\bar{\gamma_g})\sigma}{\sigma + \eta B} \tag{3.56}$$

$$\lambda_1 \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta} \tag{3.57}$$

$$\lambda_y \equiv \eta + \frac{\sigma}{B} \tag{3.58}$$

$$\lambda_g \equiv \frac{\sigma(1-\bar{\gamma_g})^2}{B} - \lambda_y\Gamma^2 \tag{3.59}$$

$$B \equiv 1 - (1-\bar{\gamma_g})g_y \tag{3.60}$$

The optimal government spending is obtained by minimising the welfare lose function which is subject to the Phillips and IS curves (Equation 3.49 and 3.50). Thus, the first derivative with respect to $\hat{g}_{zlb}$ is taken as follows:

$$(\lambda_y + \frac{\epsilon\lambda_1\kappa^2}{(1-\beta\rho)^2})(\vartheta_g - \Gamma)\left[(\vartheta_g - \Gamma)\hat{g}_{zlb} + \vartheta_r(\bar{r} - \mu_t)\right] + \lambda_g\hat{g}_{zlb} = 0 \tag{3.61}$$

Therefore, a linear solution for the optimal government spending $\hat{g}^*_{zlb}$:

$$\hat{g}^*_{zlb} = \frac{\left(\lambda_y + \frac{\epsilon\lambda_1\kappa^2}{(1-\beta\rho)^2}\right)(\vartheta_g - \Gamma)\vartheta_r}{\left(\lambda_y + \frac{\epsilon\lambda_1\kappa^2}{(1-\beta\rho)^2}\right)(\vartheta_g - \Gamma)^2 + \lambda_g}(\mu_t - \bar{r}) \tag{3.62}$$

It can be simplified as following:

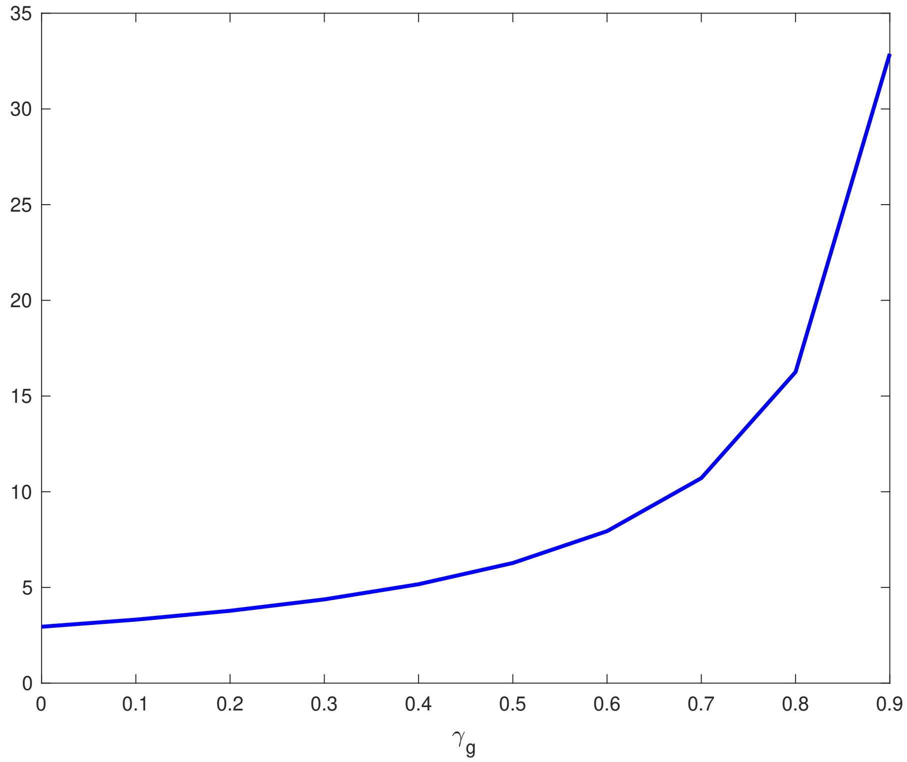
$$\hat{g}^*_{zlb} = \frac{(1-\beta\rho)^2(1-\rho)(\sigma+\eta B)\left((1+\xi\eta)B + \xi\sigma\right)}{(1-\beta\rho)^2(1-\rho)^2\sigma\eta^2(1-\bar{\gamma_g})\left((1+\xi\eta)B+\xi\sigma\right) + A_3}(\mu_t - \bar{r}) > 0 \tag{3.63}$$

where A_3 and ξ are defined as:

$A_3 \equiv (1-\bar{\gamma_g})\eta\left[\sigma(1-\beta\rho)(1-\rho) - \lambda_1\rho(\sigma + B\eta)\right]^2$, and $\xi \equiv \frac{\epsilon\lambda_1}{(1-\beta\rho)^2}$

In face of a rare disaster shock, the optimal government spending at the ZLB is positive shown in the Equation 3.63. It indicates a fiscal stimulus package is desirable to improve households' welfare during rare disasters. Consequently, a massive fiscal stimulus package is an effective fiscal policy to revive economy and improve households' welfare in response to rare disasters.

Figure 3.2: The optimal fiscal policy with different values of γ_{gt}



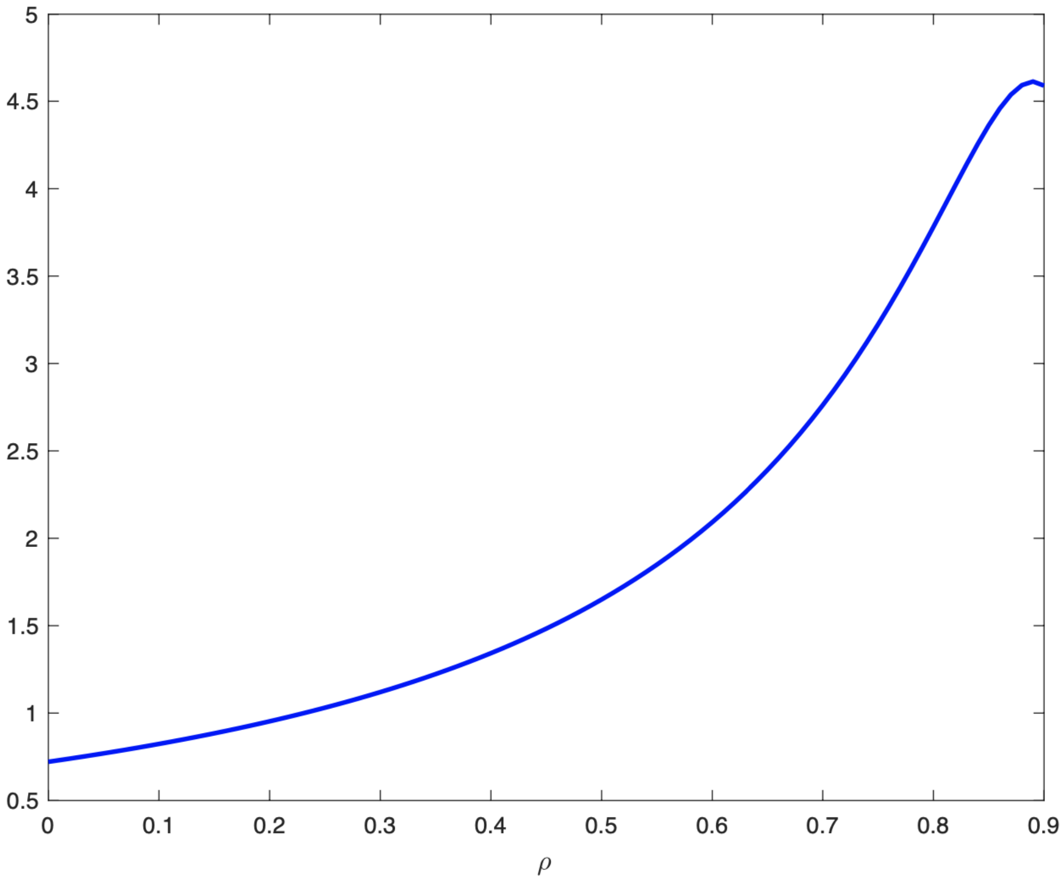
Note: The vertical axis denotes the value of $g_{zlb}(\sigma^2_{\gamma_g})$. It changes with different values of γ_g.

Additionally, Equation 3.63 indicates the persistence of the rare disaster shock ρ and the value of $\bar{\gamma}_g$ can impact the size of optimal government spending. To examine the impacts of these two parameters, I rearrange Equation 3.63 which is similar to generating Figure 3.1. The rewritten equation is obtained as follows:

$$g_{zlb}(\sigma^2_{\gamma_g}) \equiv \frac{\hat{g}_t^{*un}}{\mu_t - \bar{r}} = \frac{(1-\beta\rho)^2(1-\rho)(\sigma+\eta B)\left((1+\xi\eta)B+\xi\sigma\right)}{(1-\beta\rho)^2(1-\rho)^2\sigma\eta^2(1-\gamma_g)\left((1+\xi\eta)B+\xi\sigma\right)+A_3}$$

Then, I use $g_{zlb}(\sigma^2_{\gamma_g})$ to show how preference on private and public consumption $\bar{\gamma}_g$ and the persistence of rare disasters ρ affect the optimal government spending respectively. By applying the parameter values, the effects are shown in Figure 3.2 and 3.3. For a given rare disaster shock and constant preference on

Figure 3.3: The optimal fiscal policy with different values of ρ



Note: The vertical axis denotes the value of $g_{zlb}(\sigma^2_{\gamma_g})$. It changes with the persistence of rare disasters ρ.

private and public consumption by $(\mu_t, \rho, \bar{\gamma}_g)$, the optimal size of the increase in government spending can be obtained from Figure 3.2 by observing the optimal ratio for that value of $g_{zlb}(\sigma^2_{\gamma_g})$, and then multiplying by the value of $\mu_t - \bar{r}$.

Figure 3.2 plots the optimal government spending which rises as the value of $bar\gamma_g$ goes up.[7] When households rely more on public consumption, a greater

[7]When the value of $\bar{\gamma}_g$ is 1, $g_{zlb}(\sigma^2_{\gamma_g})$ goes to infinity.

scale of government spending is required to optimal their welfare during rare disasters.

Furthermore, Figure 3.3 shows that the size of optimal fiscal policy also rises with the increasing persistence of rare disaster ρ, suggesting that a larger optimal size of government spending is needed to withstand rare disaster shocks. Besides, when the value of persistence is greater than 0.4, the increase in slope becomes significantly larger. That is to say, a quite substantial optimal government spending is desirable when the rare disaster shock is highly persistent.

3.6.2 Households' preference uncertainty in private consumption and government spending

A dramatically high uncertainty is also considered during rare disasters. Similar to section 5, I assume the same process for γ_{gt} as previous section. $\gamma_{gt} = \bar{\gamma}_g + \epsilon_t$, and ϵ_t denotes the shock with zero mean and standard deviation σ_{γ_g}. Thus, the uncertainty in γ_{gt} is captured by the increase in variance $\sigma^2_{\gamma_g}$.

Therefore, the optimal government spending $\hat{g}^{*un}_{zlb}$ is obtained as follows:

$$\hat{g}^{*un}_{zlb} = \frac{A_4 + B_1\sigma^2_{\gamma_g}}{A_5 + A_3 - (B_2 + B_3)\sigma^2_{\gamma_g}} (\mu_t - \bar{r}) \tag{3.64}$$

where $A_4 \equiv (1-\beta\rho)^2(1-\rho)(\sigma+\eta B)\left((1+\xi\eta)B + \xi\sigma\right) > 0$

$A_5 \equiv (1-\beta\rho)^2(1-\rho)^2\sigma\eta^2(1-\bar{\gamma_g})\left((1+\xi\eta)B + \xi\sigma\right) > 0$

$A_3 \equiv (1-\bar{\gamma_g})\eta\left[\sigma(1-\beta\rho)(1-\rho) - \lambda_1\rho(\sigma+B\eta)\right]^2 > 0$

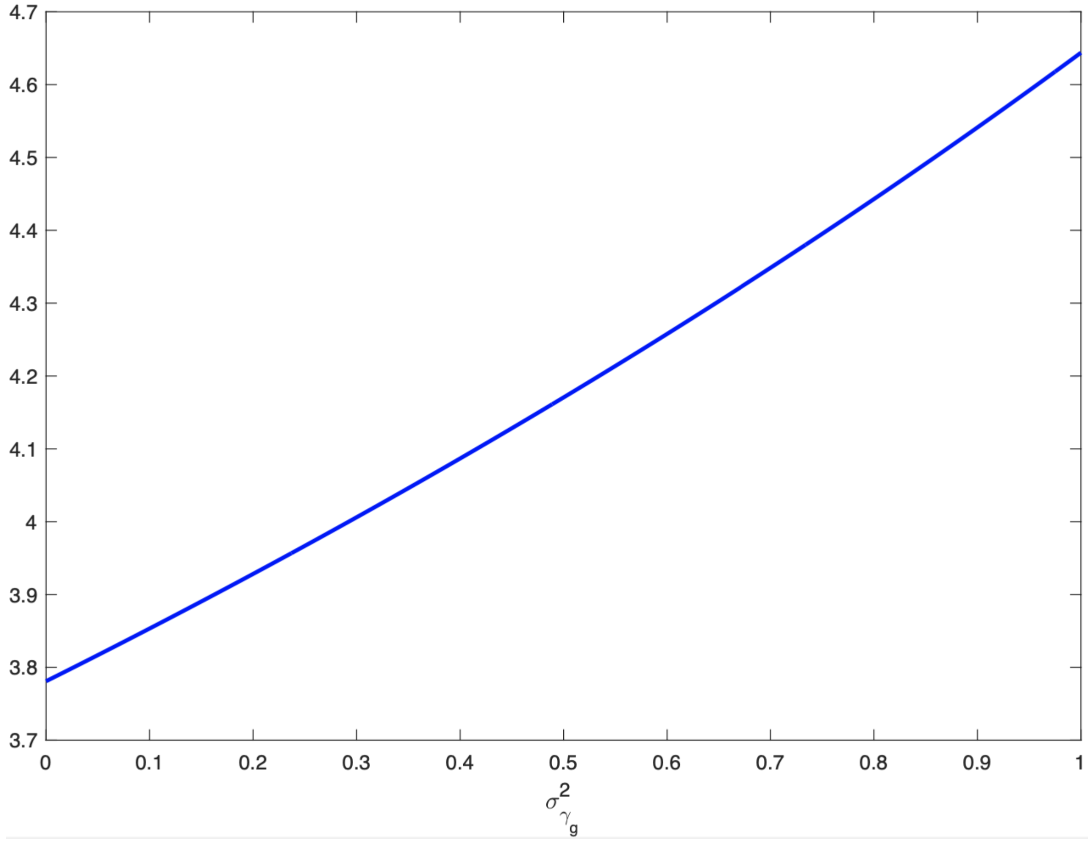
$B_1 \equiv (1-\beta\rho)^2(1-\rho)(1+\xi\eta)g_y^2$; $B_2 \equiv (1-\beta\rho)^2(1-\rho)^2\eta^2\sigma(1+\xi\eta)g_y > 0$

$B_3 \equiv 2\eta\lambda_1\rho gs\left[\lambda_1\rho(\sigma+1-g_y) - \sigma\eta(1-\beta\rho)(1-\rho)\right] > 0$

Similar to the result of section 5, the variance $\sigma^2_{\gamma_g}$ can impact the size of optimal government spending. To figure out how the uncertainty affects the size, the first order condition is derived with respect to $\sigma^2_{\gamma_g}$ which is given by:

$$\frac{\partial \hat{g}^{*un}_{zlb}}{\partial \sigma^2_{\gamma_g}} = \frac{(A_4 + A_5)B_1 + A_4(B_2 + B_3)}{\left(A_5 + A_3 - (B_2 + B_3)\sigma^2_{\gamma_g}\right)^2}(\mu_t - \bar{r}) > 0 \tag{3.65}$$

Figure 3.4: The optimal fiscal policy with increasing γ_{gt} uncertainty at the ZLB

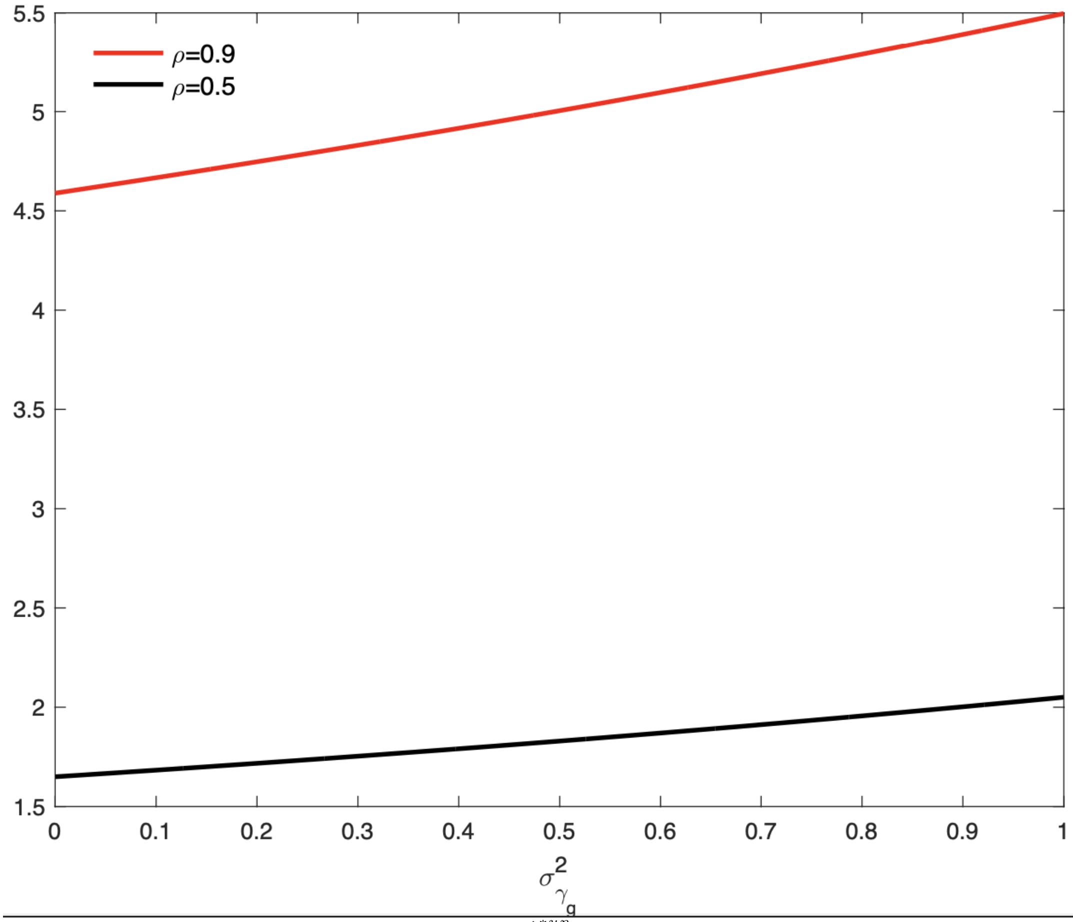


Note: The vertical axis denotes the value of $\frac{\hat{g}^{*un}_{zlb}}{\mu_t - \bar{r}}$. It changes with the variance $\sigma^2_{\gamma_g}$.

As all parameters are greater than 0, [8] the first order condition becomes positive. It illustrates that an increase in γ_{gt} uncertainty requires a larger amount of government spending to optimise households' welfare at the ZLB. Due to the disaster-induced uncertainty in households' preference, they tends to increase their precautionary savings and thus reducing their private consumption. At the same time, it further leads to changes in the demand of households for private and

[8] The prove is shown in Appendix

Figure 3.5: The slope with increasing γ_{gt} uncertainty at the ZLB



Note: The vertical axis denotes the value of $\frac{\hat{g}^{*un}_{zlb}}{\mu_t - \bar{r}}$. It changes with the variance $\sigma^2_{\gamma_g}$ with different persistence of rare disaster shock ρ.

public consumption, suggesting that they rely more on public one. Consequently, a larger scale of fiscal stimulus is needed to the optimal welfare of households.

Similarly, A upward sloping line in Figure 3.4 indicates the optimal size of fiscal stimulus increases with raising uncertainty in γ_{gt} ($\rho = 0.8$). For a given rare disaster shock and preference uncertainty on private and public consumption by $(\mu_t, \rho, \sigma^2_{\gamma_g})$, the optimal size of the increase in government spending can be obtained from Figure 3.4 by observing the optimal ratio for that value of $\frac{\hat{g}^{*un}_{zlb}}{\mu_t - \bar{r}}$, and then multiplying by the value of $\mu_t - \bar{r}$. That is to say, in terms of enhancing welfare, it is optimal to use discretionary government spending to partially offset the decline in output caused by the preference uncertainty. Compared with the scenario in section 5, with the persistence of the rare disaster shock, the optimal value of government spending is significantly greater. This means that a more aggressive fiscal policy is feasible to deal with a rare disaster.

As the persistence of the rare disaster shock impacts the size of optimal government spending, Figure 3.5 is generated with low and high persistences. Both of them suggest that the increasing uncertainty in γ_{gt} triggers a higher values of optimal government spending. Comparing with the low persistence, the increase of optimal government spending is larger with the high persistence.

3.7 Conclusion

This chapter has studied how disaster-induced uncertainty impacts the efficacy of a fiscal stimulus in normal times and at the Zero Lower bound, where rare disasters induce uncertainty in households' preference for private and public consumption. Several interesting results emerge:

First, the analytical results show that a fiscal stimulus is effective when government spending directly affects households' utility in normal times (the monetary policy is passive) and at the ZLB. It is optimal to have a fiscal stimulus to enhance the welfare of households. Second, with the disaster-induced uncertainty, the results indicate that it reduces the efficacy of fiscal stimulus. Thus, optimal government spending should be massive. More specifically, the previous optimal government spending is no longer optimal. Consequently, from the perspective of welfare, it is desirable to conduct a large scale of fiscal stimulus in the face of disaster-induced uncertainty.

The results suggest that the disaster-induced uncertainty in households' preference for private and public consumption is a significant factor that policymakers should take into account.

3.8 Appendix

3.8.1 Appendix: Log-linear model

This appendix shows a log-linearised model which is used to derive welfare loss function in main part. For variable X_t, $\hat{X}_t \equiv \ln X_t - \ln X$ indicates the log-deviation of X_t from its steady-state value X.

Households

Firstly, the effective consumption is log-linearised as:

$$\hat{c}_t = \frac{1 - g_y}{1 + (\gamma_g - 1)g_y}\hat{c}_t^p + \frac{\gamma_g}{1 + (\gamma_g - 1)g_y}\hat{g}_t \tag{3.66}$$

From households optimal problem, the first order conditions are derived as:

$$\lambda_t = C_t^{-\sigma} \tag{3.67}$$

$$N_t^\eta = \lambda_t w_t \tag{3.68}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{i_t}{\pi_{t+1}} \tag{3.69}$$

where λ_t is the Lagrange multiplier

Euler equation and labour supply are log-linearised as:

$$\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}(i_t - E_t \hat{\pi}_{t+1} - \bar{r}) \tag{3.70}$$

$$\eta \hat{n}_t = -\sigma \hat{c}_t + \hat{w}_t \tag{3.71}$$

where $\bar{r} \equiv -\log \beta$.

Firms

The firms production function can be log-linearised as:

$$\hat{y}_t = \hat{n}_t \tag{3.72}$$

As assuming zero-inflation target and no technology shock, the profit maximization problem gives the following log-linearised equations:

$$\hat{w}_t = \hat{mc}_t \tag{3.73}$$

With the Calvo pricing setting, the marginal cost and inflation equation:

$$\hat{\pi}_t = \frac{(1-\theta\beta)(1-\theta)}{\theta}\hat{mc}_t + \beta E_t \hat{\pi_{t+1}} \tag{3.74}$$

In the equilibrium condition, resource constraint is log-linearised as

$$\hat{y}_t = (1-g_y)\hat{c}_t^p + \hat{g}_t \tag{3.75}$$

Substituting labour supply Equation (3.71) into inflation Equation (3.74) gives:

$$\begin{aligned}
\hat{\pi}_t &= \frac{(1-\theta\beta)(1-\theta)}{\theta}\left(\eta\hat{n}_t + \sigma\hat{c}_t\right) + \beta E_t \hat{\pi_{t+1}} \\
&= \frac{(1-\theta\beta)(1-\theta)}{\theta}\left(\eta\hat{y}_t + \sigma\left(\frac{1-g_y}{1+(\gamma_g-1)g_y}\hat{c}_t^p + \frac{\gamma_g}{1+(\gamma_g-1)g_y}\hat{g}_t\right)\right) + \beta E_t \hat{\pi_{t+1}} \\
&= \frac{(1-\theta\beta)(1-\theta)}{\theta}\left(\eta\hat{y}_t + \sigma\left(\frac{1-g_y}{1+(\gamma_g-1)g_y}\left(\frac{\hat{y}_t-\hat{g}_t}{1-g_y}\right) + \frac{\gamma_g}{1+(\gamma_g-1)g_y}\hat{g}_t\right)\right) + \beta E_t \hat{\pi_{t+1}} \\
&= \frac{(1-\theta\beta)(1-\theta)}{\theta}\left(\eta + \frac{\sigma}{1+(\gamma_g-1)g_y}\right)\left[\hat{y}_t - \frac{(1-\gamma_g)\sigma}{\sigma+\eta(1+(\gamma_g-1)g_y)}\hat{g}_t\right] + \beta E_t \hat{\pi}_{t+1}
\end{aligned}$$

The Phillips curve can therefore be written as:

$$\hat{\pi}_t = \lambda_1\kappa(\hat{y}_t - \Gamma\hat{g}_t) + \beta E_t \hat{\pi}_{t+1} \tag{3.76}$$

where $\lambda_1 \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta}$, $\kappa \equiv \eta + \frac{\sigma}{1+(\gamma_g-1)g_y}$ and $\Gamma \equiv \frac{(1-\gamma_g)\sigma}{\sigma+\eta(1+(\gamma_g-1)g_y)}$

The Euler equation form consumption can be written as:

$$\frac{1}{1+(\gamma_g-1)g_y}\hat{y}_t - \frac{1-\gamma_g}{1+(\gamma_g-1)g_y}\hat{g}_t = \frac{1}{1+(\gamma_g-1)g_y}E_t\hat{y}_{t+1} - \frac{1-\gamma_g}{1+(\gamma_g-1)g_y}E_t\hat{g}_{t+1} - \frac{1}{\sigma}(i_t - E_t\hat{\pi}_{t+1} - \bar{r})$$

Rearranging it, the log-linearised Euler equation is shown in the following form:

$$\hat{y}_t - (1-\gamma_g)\hat{g}_t = E_t\hat{y}_{t+1} - (1-\gamma_g)E_t\hat{g}_{t+1} - \frac{1+(\gamma_g-1)g_y}{\sigma}(i_t - E_t\hat{\pi}_{t+1} - \bar{r}) \quad (3.77)$$

3.8.2 The Utility-based Loss Function

The time t utility function:

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} + V(G_t)$$

where $C_t = C_t^p + \gamma_g G_t$ and $Y_t = C_t^p + G_t$

Rearranging the utility function, the new form is:

$$U_t = F(Y_t, G_t) - \frac{N_t^{1+\eta}}{1+\eta} + V(G_t)$$

where $F(Y_t, G_t) = \frac{(Y_t-(1-\gamma_g)G_t)^{1-\sigma}}{1-\sigma}$

$$\begin{aligned} F(Y_t, G_t) \approx & F_Y Y\left(\frac{Y_t-Y}{Y}\right) - \frac{1}{2}F_{YY}Y^2\left(\frac{Y_t-Y}{Y}\right)^2 + F_{YG}Y^2\left(\frac{Y_t-Y}{Y}\right)\left(\frac{G_t-G}{Y}\right) \\ & + F_G Y\left(\frac{G_t-G}{Y}\right) + \frac{1}{2}F_{GG}Y^2\left(\frac{G_t-G}{Y}\right)^2 + t.i.p \end{aligned}$$

where t.i.p. stands for terms independent of policy.

Using the second order approximation for the percentage changes in consumption implies that:

$$
\begin{aligned}
F(Y_t, G_t) \approx & F_Y Y \left(\hat{y}_t + \frac{1}{2}\hat{y}_t^2 \right) - \frac{1}{2} F_{YY} Y^2 \hat{y}_t^2 + F_{YG} Y^2 \hat{y}_t \hat{g}_t \\
& + F_G Y \left(\hat{g}_t + \frac{1}{2}\hat{g}_t^2 \right) + \frac{1}{2} F_{GG} Y^2 \hat{g}_t^2 + t.i.p \\
\approx & F_Y Y \left[\hat{y}_t + \frac{1}{2} \left(1 + \frac{F_{YY}}{F_Y} Y \right) \hat{y}_t^2 + \frac{F_{YG}}{F_Y} Y^2 \hat{y}_t \hat{g}_t \right. \\
& \left. + \frac{F_G}{F_Y} \left(\hat{g}_t + \frac{1}{2}(1 + \frac{F_{GG}}{F_G} Y) \hat{g}_t^2 \right) \right] + t.i.p
\end{aligned}
$$

where F_X and $F_X X$ denote the first and second derivatives with respect to variable X $(X = Y, G)$.

$$
\begin{aligned}
F_Y &\equiv (Y_t - (1-\gamma_g) G_t)^{-\sigma} \\
F_{YY} &\equiv -\sigma \left(Y_t - (1-\gamma_g) G_t \right)^{-\sigma-1} \\
F_G &\equiv -(1-\gamma_g) \left(Y_t - (1-\gamma_g) G_t \right)^{-\sigma} \\
F_{GG} &\equiv -\sigma (1-\gamma_g)^2 \left(Y_t - (1-\gamma_g) G_t \right)^{-\sigma-1} \\
F_{YG} &\equiv \sigma (1-\gamma_g) \left(Y_t - (1-\gamma_g) G_t \right)^{-\sigma-1}
\end{aligned}
$$

The sub-utility function for labour supply is:

$$
\begin{aligned}
\frac{N_t^{1+\eta}}{1+\eta} &\approx N^{(1+\eta)} \left(\frac{N_t - N}{N} \right) + \frac{1}{2} \eta N^{(1+\eta)} \left(\frac{N_t - N}{N} \right)^2 + t.i.p \\
&\approx N^{(1+\eta)} \left[\hat{n}_t + \frac{1}{2}(1+\eta) \hat{n}_t^2 \right] + t.i.p
\end{aligned}
$$

The production function can be taken second-order approximation as the following form:

$$
\hat{y}_t + \frac{1}{2}\hat{y}_t^2 \approx \hat{n}_t + \frac{1}{2}\hat{n}_t^2 - \hat{v_{p}}_t^{\,2} + t.i.p
$$

where $\hat{v_{p}}_t$ denotes the second-order approximation for price dispersion.

The second-order approximation for labour supply can be rewritten as:

$$
\frac{N_t^{1+\eta}}{1+\eta} \approx N^{(1+\eta)} \left[\hat{y}_t + \frac{1}{2}(1+\eta) \hat{y}_t^2 + \hat{v_{p}}_t^{\,2} \right] + t.i.p
$$

The sub-utility function $V(G)$ is rewritten as:

$$\begin{aligned} V(G) &\approx V_G Y(\hat{g}_t + \frac{1}{2}\hat{g}_t^2) + \frac{1}{2}V_{GG}Y^2\hat{g}_t^2 + t.i.p \\ &\approx V_G Y\left[\hat{g}_t + \frac{1}{2}(1 + \frac{V_{GG}}{V_G}Y)\hat{g}_t^2\right] + t.i.p \end{aligned}$$

where V_G and V_{GG} are the first and second derivatives with respect to G.

Consequently, the second-order approximation for utility function is:

$$\begin{aligned} \hat{U}_t \approx & F_Y Y\left[\hat{y}_t + \frac{1}{2}\left(1 + \frac{F_{YY}}{F_Y}Y\right)\hat{y}_t^2 + \frac{F_{YG}}{F_Y}Y^2\hat{y}_t\hat{g}_t + \frac{F_G}{F_Y}\left(\hat{g}_t + \frac{1}{2}(1 + \frac{F_{GG}}{F_G}Y)\hat{g}_t^2\right)\right] \\ & - N^{(1+\eta)}\left[\hat{y}_t + \frac{1}{2}(1+\eta)\hat{y}_t^2 + \hat{v^p}_t^2\right] + V_G Y\left[\hat{g}_t + \frac{1}{2}(1 + \frac{V_{GG}}{V_G}Y)\hat{g}_t^2\right] + t.i.p \end{aligned}$$

According to the labour supply, its steady state condition is:

$$N^\eta = wC^{-\sigma} = AF_Y$$

Additionally, the steady state resource constraint is:

$$Y = C^p + G = AN$$

As a result, the following equation is derived by combining above two equations:

$$N^{1+\eta} = YF_Y$$

Government spending should be conducted if and only if the marginal utility of it is the same as that of additional private consumption. Then, government spending is used to maximize $F(Yt, Gt) + V(Gt)$ by taking the quantity of aggregate output Y_t as given. In order to maximise households utility, government spending should satisfy following first order condition.

$$F_G(G_t, Y_t) + V_G = 0$$

In the equilibrium condition, the price dispersion equals 1. The can be simplified as:

$$\begin{aligned} \frac{\hat{U}_t}{F_Y Y} \approx & \hat{y}_t + \frac{1}{2}\left(1 + \frac{F_{YY}}{F_Y}Y\right)\hat{y}_t^2 + \frac{1}{2}\frac{F_{GG}}{F_Y}Y\hat{g}_t^2 \\ & - \left[\hat{y}_t + \frac{1}{2}(1+\eta)\hat{y}_t^2 + \hat{v^p}_t^2\right] + \frac{1}{2}\frac{V_{GG}}{F_Y}\hat{g}_t^2 + t.i.p \end{aligned}$$

Since $\frac{1}{2}\frac{V_{GG}}{F_Y}\hat{g}_t^2$ is irrelevant to γ_g, I assume that $\frac{V_{GG}}{F_Y}$ equals zero. This is also a case that is considered in Woodford (2011) in his Figure 4, namely case A.

Using the steady-state conditions, $\frac{F_{YY}}{F_Y}Y$ and $\frac{F_{GG}}{F_Y}Y$ can be simplified as:

$$\frac{F_{YY}}{F_Y}Y \equiv -\sigma(1-(1-\gamma_g)g_y)^{-1}$$
$$\frac{F_{GG}}{F_Y}Y \equiv -\sigma(1-\gamma_g)^2(1-(1-\gamma_g)g_y)^{-1}$$

where g_y is the steady-state ratio of government spending to output .

The above equations can be simplified as:

$$\begin{aligned}\frac{\hat{U}_t}{F_Y Y} \approx& \frac{1}{2}\left[1-\frac{\sigma}{1-(1-\gamma_g)g_y}\right]\hat{y}_t^2 - \frac{\sigma(1-\gamma_g)^2}{2(1-(1-\gamma_g)g_y)}\hat{g}_t^2 - \left[\frac{1}{2}(1+\eta)\hat{y}_t^2 + \hat{v^p}_t^2\right] + t.i.p \\ \approx& -\frac{1}{2}\left[(\eta+\frac{\sigma}{1-(1-\gamma_g)g_y})\hat{y}_t^2 + \frac{\sigma(1-\gamma_g)^2}{(1-(1-\gamma_g)g_y)}\hat{g}_t^2 - 2\hat{v^p}_t^2\right] + t.i.p\end{aligned}$$

Following the chapter 6 in Woodford (2006), the price dispersion $\hat{v}_t^p$ can be expressed by $\hat{\pi}_t$. As a result, the discounted loss function is defined as:

$$L = -\frac{1}{2}F_Y Y\sum_{t=0}^{\infty}\beta^t\hat{U}_t = \sum_{t=0}^{\infty}\beta^t\left[(\eta+\frac{\sigma}{1-(1-\gamma_g)g_y})\hat{y}_t^2 + \frac{\sigma(1-\gamma_g)^2}{(1-(1-\gamma_g)g_y)}\hat{g}_t^2 + \frac{\epsilon}{\lambda_1}\hat{\pi}_t^2\right]$$

The output gap is expressed as $\hat{y}_t - \Gamma\hat{g}_t$ where $\Gamma \equiv \frac{(1-\gamma_g)\sigma}{\sigma+\eta(1+(\gamma_g-1)g_y)}$. Substituting output gap to the loss function, it is shown as following:

$$L = \sum_{t=0}^{\infty}\beta^t\left[\lambda_y(\hat{y}_t-\Gamma\hat{g}_t)^2 + \lambda_g\hat{g}_t^2 + \frac{\epsilon}{\lambda_1}\hat{\pi}_t^2\right]$$

where $\Gamma \equiv \frac{(1-\gamma_g)\sigma}{\sigma+\eta B}$, $\lambda_1 \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta}$, $\lambda_y \equiv \eta + \frac{\sigma}{B}$, $\lambda_g \equiv \frac{\sigma(1-\gamma_g)^2}{B} - \lambda_y\Gamma^2$ and $B \equiv 1-(1-\gamma_g)g_y$ (All parameters $\Gamma, \lambda_1\lambda_g\lambda_y$ are positive).

3.8.3 Optimal fiscal policy

Passive monetary policy

Assumptions: 1. The nominal interest rate is constant ($i_t = i_C$). 2. The expected variables for $t+1$ period are zero.

Consequently, the Phillips curve and Euler equation is rewritten as:

$$\hat{\pi}_t = \lambda_1 \kappa (\hat{y}_t - \Gamma \hat{g}_t) \tag{3.78}$$

$$\hat{y}_t = (1-\gamma_g)\hat{g}_t + \frac{B}{\sigma}(\bar{r} - i_C) \tag{3.79}$$

The discretionary welfare loss minimization problem is defined as:

$$\max \frac{1}{1-\beta}\left[\lambda_y(\hat{y}_t - \Gamma\hat{g}_t)^2 + \lambda_g \hat{g}_t^2 + \frac{\epsilon}{\lambda_1}\hat{\pi}_t^2\right]$$

subject to Equation (3.78) and (3.79)

Substituting Equation (3.78) and (3.79) to loss function, I get following equation:

$$\begin{aligned}
L_t &\equiv (\lambda_y + \epsilon\lambda_1\kappa^2)(\hat{y}_t - \Gamma\hat{g}_t)^2 + \lambda_g\hat{g}_t^2 \\
&\equiv (\lambda_y + \epsilon\lambda_1\kappa^2)\left[(1-\gamma_g)\hat{g}_t + \frac{B}{\sigma}(\bar{r} - \mu_t) - \Gamma\hat{g}_t\right]^2 + \lambda_g\hat{g}_t^2 \\
&\equiv (\lambda_y + \epsilon\lambda_1\kappa^2)\left[(1-\gamma_g-\Gamma)\hat{g}_t + \frac{B}{\sigma}(\bar{r} - \mu_t)\right]^2 + \lambda_g\hat{g}_t^2
\end{aligned}$$

Then, I take first order condition with respect to $\hat{g}_t$ to maximise welfare. Consequently, the first order condition is

$$\frac{\partial L_t}{\partial \hat{g}_t} = 2(\lambda_y + \epsilon\lambda_1\kappa^2)(1-\gamma_g-\Gamma)\left[(1-\gamma_g-\Gamma)\hat{g}_t + \frac{B}{\sigma}(\bar{r} - \mu_t)\right] + 2\lambda_g\hat{g}_t$$

As the optimal condition satisfies that first-order derivative equals zero, the optimal $\hat{g}_t$ is:

$$\hat{g}_t^* = \frac{B(\lambda_y + \epsilon\lambda_1\kappa^2)(1-\gamma_g-\Gamma)}{(\lambda_y + \epsilon\lambda_1\kappa^2)(1-\gamma_g-\Gamma)^2 + \lambda_g} \cdot \left(\frac{-\bar{r} + \mu_t}{\sigma}\right) > 0$$

where $-\bar{r} + \mu_t > 0$, $B \equiv 1 - (1-\gamma_g)g_y$, $\Gamma \equiv \frac{(1-\gamma_g)\sigma}{\sigma + \eta B}$, $\lambda_1 \equiv \frac{(1-\theta\beta)(1-\theta)}{\theta}$, $\lambda_y \equiv \eta + \frac{\sigma}{B}$ and $\lambda_g \equiv (1-\gamma_g)^2 \frac{\sigma\eta}{\sigma + \eta B}$.

The optimal government spending can be simplified as:

$$\hat{g}_t^* = \frac{\eta(\eta B + \sigma)(\epsilon\lambda_1(\eta B + \sigma) + B)}{\eta^2(1-\gamma_g)(\epsilon\lambda_1(\eta B + \sigma) + B) + \sigma\eta(1-\gamma_g)} \cdot \left(\frac{-\bar{r} + \mu_t}{\sigma}\right)$$

Therefore, the optimal solution of $\hat{y_t}^*$ in terms of government spending $\hat{g}_t^*$ is given by:

$$
\begin{aligned}
\hat{y_t}^* &= (1-\gamma_g)\hat{g}_t^* + \frac{B}{\sigma}(-\bar{r}+\mu_t) \\
&= \left(\frac{(1-\gamma_g)\eta(\eta B+\sigma)(\epsilon\lambda_1(\eta B+\sigma)+B)}{\eta^2(1-\gamma_g)(\epsilon\lambda_1(\eta B+\sigma)+B)+\sigma\eta(1-\gamma_g)} - B \right) \cdot \left(\frac{-\bar{r}+\mu_t}{\sigma} \right) \\
&= \frac{\eta\sigma\epsilon\lambda_1(\eta B+\sigma)}{\eta^2(1-\gamma_g)(\epsilon\lambda(\eta B+\sigma)+B)+\sigma\eta(1-\gamma_g)} \cdot \left(\frac{-\bar{r}+\mu_t}{\sigma} \right) > 0
\end{aligned}
$$

Zero lower bound

Assumptions: 1. The nominal interest rate is pegged at zero ($i_t = 0$). 2. There is a disaster shock that can be regarded as a negative demand shock μ_t with persistence ρ.

Therefore, the Phillips curve and Euler equation is rewritten as:

$$\hat{\pi}_t = \frac{\lambda\kappa}{1-\beta\rho}(\hat{y}_t - \Gamma\hat{g}_t) \tag{3.80}$$

$$(1-\rho)\left(\hat{y}_t - (1-\gamma_g)\hat{g}_t\right) = \frac{1+(\gamma_g-1)g_y}{\sigma}(\bar{r} - i_t + \rho\pi_t - \mu_t) \tag{3.81}$$

The Euler equation can be simplified as:

$$\hat{y}_t = \frac{1-\gamma_g}{1-\rho}\hat{y}_t + \frac{1+(\gamma_g-1)g_y}{\sigma(1-\rho)}(\bar{r}+\rho\hat{\pi}_t - \mu_t) \tag{3.82}$$

Combine Equation 3.80 and 3.82, output can be expressed by government spending, disaster shock and the steady-state interest rate.

$$(1-\frac{\rho\lambda\kappa(1+(\gamma_g-1)g_y)}{\sigma(1-\rho)(1-\beta\rho)})\hat{y}_t = (\frac{1-\gamma_g}{1-\rho} - \frac{\Gamma\rho\lambda\kappa(1+(\gamma_g-1)g_y)}{\sigma(1-\rho)(1-\beta\rho)})\hat{g}_t + \frac{1+(\gamma_g-1)g_y}{\sigma(1-\rho)}(\bar{r}-\mu_t) \tag{3.83}$$

Reducing Equation (3.83), the output becomes:

$$
\begin{aligned}
\hat{y}_t =& \frac{(1-\beta\rho)(1+(\gamma_g-1)g_y)}{\sigma(1-\rho)(1-\beta\rho)-\lambda\kappa\rho(1+(\gamma_g-1)g_y)}(\bar{r}-\mu_t) \\
&+ \frac{(\sigma+\eta(1+(\gamma_g-1)g_y))(1-\rho)(1-\beta\rho)-\lambda\kappa\rho(1+(\gamma_g-1)g_y)}{\sigma(1-\rho)(1-\beta\rho)-\lambda\kappa\rho(1+(\gamma_g-1)g_y)}\Gamma\hat{g}_t
\end{aligned}
$$

where ϑ_r and ϑ_g are defined as follows:

$$\vartheta_r \equiv \frac{(1-\beta\rho)B}{\sigma(1-\rho)(1-\beta\rho)-\lambda\kappa\rho B} > 0 \tag{3.84}$$

$$\vartheta_g \equiv \frac{(\sigma+\eta B)(1-\rho)(1-\beta\rho)-\lambda\kappa\rho B}{\sigma(1-\rho)(1-\beta\rho)-\lambda\kappa\rho B}\Gamma > 0 \tag{3.85}$$

The discretionary welfare loss minimization problem is similar to the previous section:

$$L = \frac{1}{1-\beta\mu}\left[\lambda_y(\hat{y}_t - \Gamma\hat{g}_t)^2 + \lambda_g\hat{g}_t^2 + \frac{\epsilon}{\lambda_1}\hat{\pi}_t^2\right]$$

subject to Equation (3.80) and (3.83).

Substituting Equation (3.80) and (3.83) to the loss function, I get following equation:

$$\begin{aligned}
(1-\beta\rho)L_t &\equiv (\lambda_y + \frac{\epsilon\lambda_1\kappa^2}{(1-\beta\rho)^2})(\hat{y}_t - \Gamma\hat{g}_t)^2 + \lambda_g\hat{g}_t^2 \\
&\equiv (\lambda_y + \frac{\epsilon\lambda_1\kappa^2}{(1-\beta\rho)^2})\left[\vartheta_g\hat{g}_t + \vartheta_r(\bar{r}-\mu_t) - \Gamma\hat{g}_t\right]^2 + \lambda_g\hat{g}_t^2 \\
&\equiv (\lambda_y + \frac{\epsilon\lambda_1\kappa^2}{(1-\beta\rho)^2})\left[(\vartheta_g - \Gamma)\hat{g}_t + \vartheta_r(\bar{r}-\mu_t)\right]^2 + \lambda_g\hat{g}_t^2
\end{aligned}$$

Then, the first-order condition with respect to $\hat{g}_{zlb}$ is derived to maximise welfare. Consequently, the first-order condition is

$$\frac{\partial L_t}{\partial \hat{g}_{zlb}} = 2(\lambda_y + \frac{\epsilon\lambda_1\kappa^2}{(1-\beta\rho})^2)(\vartheta_g - \Gamma)\left[(\vartheta_g - \Gamma)\hat{g}_t + \vartheta_r(\bar{r}-\mu_t)\right] + 2\lambda_g\hat{g}_t$$

As the optimal condition satisfies that first-order derivative equals zero, the optimal $\hat{g}^*_{zlb}$ is:

$$\hat{g}^*_{zlb} = \frac{(1-\beta\rho)^2(1-\rho)(\sigma+\eta B)\left((1+\xi\eta)B+\xi\sigma\right)}{(1-\beta\rho)^2(1-\rho)^2\sigma\eta^2(1-\gamma_g)\left((1+\xi\eta)B+\xi\sigma\right)+A_3}(\mu_t - \bar{r}) > 0$$

Chapter 4

Welfare cost in rare disasters-induced uncertainty

4.1 Introduction

The effect of uncertainty on the macroeconomy and on household welfare is an important question in macroeconomics. Lucas (1987) argues that consumption uncertainty leads to welfare costs, but these costs are small and negligible. The possible reason for Lucas's conclusion is that the estimation of consumption uncertainty is under normal times. A welfare cost might be greater during rare disaster. Barro (2009) considers the disaster risk to estimate the assets pricing uncertainty to reflect the consumption uncertainty and examine its welfare effects. Barro's result indicates that the welfare cost is equivalent to about 20 percent deduction in GDP under rare disaster, which is quite larger than that of Lucas.

During rare disaster, there is a new factor that affects consumption uncertainty and further reduces welfare. According to Chapter 2, rare disasters, like COVID-19 pandemic, causes the government to adopt a fiscal stimulus to combat it. But the greater uncertainty induced by the pandemic affects the allocation of resources and households' response to the fiscal stimulus. That is to say, the pandemic induces the uncertainty in the demand of households for private consumption and government spending, which is also the source of consumption uncertainty.

To be specific, government spending can directly impact households' utility through consumption. Thus, household utility depends on the effective consumption which consists of private (C_t^p) and public ($\gamma_{gt} G_t$) ones. ($C_t = C_t^p + \gamma_{gt} G_t$). Public consumption is provided by government goods and services, which is shown as a fraction γ_{gt} of government spending (G_t). And how households choose public consumption depends on the shifts in their preference between private consumption and government spending, which fluctuates over time. The shift is captured by the variable γ_{gt}, which is assumed to a preference shock (say γ_{gt}) and fluctuates stochastically. Especially, rare disasters (e.g. COVID-19 pandemic) induce a high degree of volatility in this preference. It is natural to ask whether such greater uncertainty has a significantly negative impact on welfare. .

This chapter focuses on the welfare effect of the uncertainty in households' responses to government spending. And I use a standard Real Business Cycle (RBC) model to analyse the welfare cost driven by the preference shock between private consumption and government spending.

In order to measure the welfare effect of uncertainty, I follow Cho et al. (2015) to assume that γ_{gt} is a shock with mean preserving spread (MPS) property and follows a log-normal distribution. Thus, the log of γ_{gt} is governed by a normal distribution i.e. $\ln \gamma_{gt} \sim N(\ln \bar{\gamma}_g - \frac{\sigma^2}{2}, \sigma^2)$. The variance σ^2 is used to measure the degree of uncertainty in γ_{gt}. Thus, the welfare cost of uncertainty is obtained by taking the difference in utility based on different values of σ^2. In addition, due to the mean preserving spread property, the mean of σ^2 does not vary with its variance. Thus, the mean of $\ln \gamma_{gt}$ should be $\ln \bar{\gamma}_g - \frac{\sigma^2}{2}$, which becomes a function of its variance.

The model is solved using the extended perturbation methods of Heiberger and Maußner (2020), accounting for the effect of the mean preserving spread property on welfare cost. As the mean of a MPS shock is a function of its variance, the method takes this property into account and thus accurately measures the welfare costs of uncertainty in the model. And this chapter computes both conditional and unconditional welfare cost.

The main result of this chapter is that an increase in disaster-induced uncertainty in the demand of households for private consumption and government spending is welfare reducing. Households prefer to stay in a steady state rather than a stochastic state. According to Cho et al. (2015), the welfare effect consists of fluctuations and mean effects. To be specific, in response to the uncertainty, households smooth their consumption which refers to the fluctuations effect. The result shows that a 0.85% increase in conditional welfare cost in the presence of a significant high degree of γ_{gt} uncertainty. Additionally, due to the mean effect, households supply more labours and then obtain more income. But they faces a higher tax because of the negative wealth effect and thus reduce their consumption. Consequently, they decrease consumption more and suffer from a welfare loss, which leads to a greater welfare cost (2.7%).

In addition, I investigate how other preference parameters impact the welfare effect of γ_{gt} uncertainty. First, when households become more risk averse, the γ_{gt} uncertainty further deepen the negative effect on the welfare. Second, if the labour supply has a high degree of Frisch elasticity, households suffer greater negative wealth effect because of γ_{gt} uncertainty. Finally, I examine the welfare effect of γ_{gt} uncertainty in the absence of the wealth effect. In order to eliminate the wealth effect, I assume that the labour and consumption are non-separate in utility function. In this specification, the welfare cost becomes less but is still positive. Households do not like γ_{gt} uncertainty. And when they become more risk averse, the welfare cost increases.

There is already extensive research focusing estimating welfare cost of business cycle with more complex settings. More estimations, especially recent ones, have demonstrated that there is a non-negligible welfare cost. Krusell et al (2009) find considerably high welfare cost from perspective of household heterogeneous preference. Unlike this chapter, it does not consider the possible effects of public consumption on households. Cho et al. (2015) argue a positive welfare when taking TPF uncertainty into account instead of consumption one. Lester et. al (2014) complements the analysis of Cho et al. (2015) with other shocks such as investment-specific productivity shock. However, they do not consider the case of rare disasters, as investigated by Barro (2009). Barro suggests that disaster-induced uncertainty in assets pricing dramatically reduce welfare. This chapter complements their analysis, focusing on the disaster-induced uncertainty in households demand for private consumption and government spending, as the source of consumption uncertainty.

The rest of the chapter is organized as follows. Section 4.2 uses a simply model without capital to analyse how γ_{gt} affects household welfare. Section 4.3 introduces the mean preserving shock to the preference on private and public goods. Section 4.4 presents a canonical RBC model and its calibration. Section 4.5 examines the welfare cost of uncertainty in γ_{gt}. Section 4.6 investigates the robustness (different risk aversion and labour elasticity). Section 4.7 summarises.

4.2 A RBC Model without Capital

In order to analytically examine the effect of preference shock on private and public consumption, I assume a simple RBC model without capital. Thus, analytical policy functions can be obtained from this simple model, which further yields a utility function defined by the preference shock and other parameters. Based on this utility function, it is clear to know that how an increasing uncertainty in the preference for private consumption and government spending impacts welfare. Moreover, γ_{gt} is the only source of aggregate fluctuations in the model.

The economy has a representative household and firm. They take fiscal policies as given when making private optimal decisions. The government uses lump-sum taxes as revenue for the provision of public services and transfers.

The representative household maximises his utility function defined by the effective consumption ($C_t = C_t^P + \gamma_{gt}G_t$) and the working hours (N_t), subject to a budget which is constraint by his net income. Therefore, the problem of optimal households utility is expressed as follows:

$$\max_{C_t^p N_t} \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^p + \gamma_{gt}G_t)^{1-\sigma}}{1-\sigma} - \frac{(N_t)^{1+\eta}}{1+\eta} \right) \tag{4.1}$$

where C_t^p is the private consumption, G_t is the government spending, β is the discount factor, σ is the elasticity of intertemporal substitution (EIS) and the relative risk aversion parameter, and η is the inverse of Frisch elasticity of labour supply.

Additionally, γ_{gt} demotes the marginal substitution between private consumption and government spending. It shows that the preference of households on public and private goods, and thus their demand for government spending and private consumption. To be specific, they expect to forego their private consumption by the amount of γ_{gt} when there is one unit increase in government spending. The value of γ_{gt} fluctuates over time. Besides, the volatility of γ_{gt} is mainly impacted by the change in economic environment e.g. natural disasters, wars or pandemics. For example, the COVID-19 pandemic has induced a dramatic high volatility in γ_{gt}.

The household budget constraint is derived as the following equation:

$$C_t^p \leqslant w_t N_t - T_t \tag{4.2}$$

where w_t is the real wage rate and T_t is the lump-sum tax.

A representative firm hires the household to produce goods, and a constant returns scale production function is assumed as a linear one. Thus, the output (Y_t) is

$$Y_t = N_t \tag{4.3}$$

Moreover, I assume that the government spending is financed through lump-sum tax, a constant rate tax based on the household's income (Barro, 1990). And government spending can provide positive utility to the representative household (Lucas and Stokey, 1983).

$$G_t = \Gamma Y_t \tag{4.4}$$

where Γ is the tax rate.

The resource constraint is given by:

$$C_t^p + G_t = Y_t \tag{4.5}$$

There are no frictions in the model. The competitive equilibrium can be characterized as the solution to a household optimisation problem that the representative household maximises his utility constrained by the resource constraint. As there is no endogenous state variable, the optimisation problem can be rewritten as a one-period problem. To be specific, in conjunction with Equations 4.1 to 4.5, the optimization problem for the representative household in one-period can be rewritten as:

$$\begin{aligned} \max_{C_t^p N_t} \frac{\left(C_t^p + \gamma_{gt} G_t\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_t\right)^{1+\eta}}{1+\eta} \\ s.t \\ C_t^p + G_t \leqslant N_t \\ G_t = \Gamma Y_t \end{aligned}$$

The policy function for the optimal choices of C_t^p and N_t with respect to γ_{gt} are:

$$C_t^p = (1-\Gamma)\left(1-\Gamma+\gamma_{gt}\Gamma\right)^{\frac{1-\sigma}{\sigma+\eta}} \tag{4.6}$$

$$N_t = \left(1-\Gamma+\gamma_{gt}\Gamma\right)^{\frac{1-\sigma}{\sigma+\eta}} \tag{4.7}$$

After substituting the policy functions into the objective function and defining $\hat{U}(\gamma_{gt})$ as the optimally indirect utility function, some algebraic manipulations yield:

$$\hat{U}(\gamma_{gt}) = \frac{\sigma+\eta}{(1-\sigma)(1+\eta)}\left(1-\Gamma+\gamma_{gt}\Gamma\right)^{\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}} \tag{4.8}$$

The first and second derivatives with respect to γ_{gt} are:

$$\hat{U}_t' = \Gamma\left(1-\Gamma+\gamma_{gt}\Gamma\right)^{(\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}-1)} > 0 \tag{4.9}$$

$$\hat{U}_t'' = \Gamma^2\left(\frac{1-2\sigma-\sigma\eta}{\eta+\sigma}\right)\left(1-\Gamma+\gamma_{gt}\Gamma\right)^{(\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}-2)} \tag{4.10}$$

If the value of $2\sigma+\sigma\eta$ is greater than 1, the second derivative $\hat{U}_t''$ is negative. The household's utility is concave in the preference shock (γ_{gt}). Then, by Jensen's inequality, $\hat{U}(E(\gamma_{gt})) > E(\hat{U}(\gamma_{gt}))$. This result is considered as a fluctuations effect of uncertainty according to Cho et al. (2015). Risk averse households prefer to smooth their consumptions. Thus, the expected utility is smaller than the utility obtained from expected outcome. If this condition is satisfied, it means that a mean-preserving spread on γ_{gt} will reduce welfare. In other words, with expectation, the household prefers less volatility in γ_{gt}.

However, if the value of $2\sigma+\sigma\eta$ is smaller than 1, the utility function becomes convex in γ_{gt}. Thus, the welfare is increasing with the volatility in γ_{gt}, which causes from the mean effect. The household is more willing to work for compensating the effect of a high volatility in γ_{gt}. Thus, the mean of consumption and output might increase with uncertainty. However, more working means high income. Then, it leads to a higher tax and thereby a higher government spending. Therefore, the household might reduce his private consumption due to the negative wealth effect. Moreover, his expected relative values between public and private consumptions are higher with volatility. Hence, the household tends to decrease his private consumption. Even the utility is convex, uncertainty might have no or negative effect on welfare. In the model, the mean effect might be dominated by the negative welfare effect. If setting the value of risk aversion σ and the inverse of Frisch elasticity of labour η to 0.5 and 0 respectively, an increase in uncertainty reduces the welfare.[1] Therefore, both the mean and fluctuation effects lead to a reduction in household welfare.[2]

4.2.1 A Shock with Mean-preserving spread (MPS) property

Supposing the γ_{gt} shock has mean-preserving (MPS) property and its uncertainty affects households utility, I follow Lucas (1987) to assume that $\ln \gamma_{gt}$ is governed by a AR(1) process. And γ_{gt} follows a log-normal distribution. Thus, $\ln \gamma_{gt}$ follows a normal distribution as follows:[3]

$$\ln \gamma_{gt} = \rho \ln \gamma_{gt-1} + \epsilon_t, \epsilon_t \sim N\left((1-\rho)\ln \bar{\gamma}_g - \frac{\tau^2}{2(1+\rho)}, \tau^2\right) \tag{4.11}$$

where ρ is the persistence of γ_{gt} shock.

As a result of the persistence, following Cho et al. (2015), I assume that γ_{gt} follows a MA process. Therefore, γ_{gt} can be expressed as follows:

$$\gamma_{gt} = \exp\left\{\sum_{j=0}^{\infty} \rho^j \epsilon_{t-j}\right\}$$

The first two moments of γ_{gt} can be obtained as:

$$E(\gamma_{gt}) = \exp\left\{\sum_{j=0}^{\infty}\left[\rho^j E(\epsilon_t) + \frac{\rho^{2j} Var(\epsilon_t)}{2}\right]\right\}$$
$$E(\gamma_{gt}^2) = \exp\left\{\sum_{j=0}^{\infty}\left[2\rho^j E(\epsilon_t) + 2\rho^{2j} Var(\epsilon_t)\right]\right\}$$

If simplifying the first moment, the mean of γ_{gt} is equal to $\bar{\gamma}_g$. Moreover, in conjunction with the first and second moments, the variance of γ_{gt} is expressed as:

[1] The figure of welfare cost is shown in Appendix
[2] The settings for σ and η follows Lester et al. (2014)
[3] The detailed process is shown in Appendix

$$
\begin{aligned}
Var(\gamma_{gt}) &= E(\gamma_{gt}^2) - [E(\gamma_{gt})]^2 \\
&= \sum_{j=0}^{\infty} 2\exp(\ln\bar{\gamma}_g)[\exp(\rho^{2j}\tau^2) - 1]
\end{aligned}
$$

Therefore, the mean and variance of γ_{gt} are obtained as:

$$
\begin{aligned}
E(\gamma_{gt}) &= \bar{\gamma}_g \\
Var(\gamma_{gt}) &= \bar{\gamma}_g{}^2 \left(\exp\left(\frac{\tau^2}{1-\rho^2}\right) - 1\right)
\end{aligned}
$$

In this distribution, to capture the MPS property, the mean of γ_{gt} is constant. A change in τ^2 indicates a mean preserving spread of the shock γ_{gt}.

As γ_{gt} follows the log-normal distribution, it allows the uncertainty (a changes in γ_{gt} variance) to affect the household expected utility. To be specific, under expectation, the optimal period utility function (Equation 4.8) can be rewritten as:

$$
E_t\left(\hat{U}(\gamma_{gt})\right) = \frac{\sigma+\eta}{(1-\sigma)(1+\eta)} E_t\left(1 - \Gamma + \gamma_{gt}\Gamma\right)^{\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}} \tag{4.12}
$$

To illustrate how uncertainty affects the expected utility via an analytical solution, I present a simple example where the value of τ is assumed to be 1 and the value of ρ to be 0. Thus, the expected function becomes:

$$
E_t\left(\hat{U}(\gamma_{gt})\right) = \frac{\sigma+\eta}{(1-\sigma)(1+\eta)} E_t\left(\gamma_{gt}\right)^{\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}} \tag{4.13}
$$

where $\ln\gamma_{gt}$ follows normal distribution with the mean $\ln\bar{\gamma}_g - \frac{\tau^2}{2}$ and the variance τ^2, the distribution of $\ln\left((\gamma_{gt})^{\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}}\right)$ becomes,[4]:

$$
\ln\left((\gamma_{gt})^{\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}}\right) \sim iidN\left(\left(\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}\right)(\ln\bar{\gamma}_g - \frac{\tau^2}{2}), \left(\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}\right)^2 \tau^2\right)
$$

[4]In Equation (4.13 $\ln\left((\gamma_{gt})^{\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}}\right) = \frac{(1-\sigma)(1+\eta)}{\sigma+\eta}\ln\gamma_{gt}$

Therefore, the expected value of it is given by:

$$E_t\left(\ln\left((\gamma_{gt})^{\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}}\right)\right) = e^{\left(\left(\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}\right)(\ln\bar{\gamma}_g - \frac{\tau^2}{2}) + \left(\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}\right)^2 \frac{\tau^2}{2}\right)} \tag{4.14}$$

As a result, the expected period utility becomes:

$$E_t\left(\hat{U}(\gamma_{gt})\right) = \frac{\sigma+\eta}{(1-\sigma)(1+\eta)} e^{\left(\left(\frac{(1-\sigma)(1+\eta)}{\sigma+\eta}\right)(\ln\bar{\gamma}_g + \frac{(1-2\sigma-\sigma\eta)}{2(\sigma+\eta)}\tau^2)\right)} \tag{4.15}$$

where this expression shows that the expected utility depends on the variance τ^2, as well on the other parameters. If the value of $2\sigma + \sigma\eta$ is greater than 1, the expected utility decreases in τ^2. Thus, the fluctuations effect plays a role. To discuss exhaustively the impacts of γ_{gt} uncertainty on the household's welfare, the standard model RBC is taken into account in the next section.

4.3 A canonical RBC model with Capital

In this section, I assume a standard RBC model which includes capital (K_t). Thus, the firm uses capital and labour as inputs. Additionally, I assume total factor productivity A is constant which equals 1. The production function is obtained as following equation:

$$Y_t = N_t^{1-\alpha} K_t^{\alpha} \tag{4.16}$$

where α denotes capital's share of income which is between 0 and 1.

Moreover, a typical law of motion of the capital is assumed as follows:

$$K_{t+1} = (1-\delta)K_t + i_t \tag{4.17}$$

where δ is the depreciation rate of capital, and i_t is the investment.

The optimization problem of the representative household in a standard RBC model consists of the following equations:

$$\max_{C_t N_t K_{t+1}\}} \frac{(C_t^p + \gamma_{gt} G_t)^{1-\sigma}}{1-\sigma} - \frac{(N_t)^{1+\eta}}{1+\eta}$$

$$s.t.$$

$$C_t^p + i_t + G_t \leqslant N_t^{1-\alpha} K_t^{\alpha}$$

$$K_{t+1} = (1-\delta)K_t + i_t$$

$$G_t = \Gamma Y_t$$

Furthermore, the optimal problem can be recursively rewritten as a Bellman equation:

$$V(\gamma_{gt}, K_t) = \max_{C_t^p N_t K_{t+1}} U(C_t^p, N_t) + \beta V(\gamma_{gt+1}, K_{t+1})$$

$$s.t$$

$$C_t^p + K_{t+1} - (1-\delta)K_t \leqslant (1-\Gamma)N^{1-\alpha} K_t^{\alpha}$$

The conditions characterizing an optimal interior solution to this problem are given by:

$$-U_N(C_t^p, N_t) = U_{C^p}(C_t^p, N_t)(1-\Gamma)(1-\alpha)(\frac{K_t}{N_t})^{\alpha} \tag{4.18}$$

$$U_{C^p}(C_t^p, N_t) = \beta E_t \left(U_{C^p}(C_{t+1}^p, N_{t+1}) \left((1-\Gamma)\alpha(\frac{K_{t+1}}{N_{t+1}})^{\alpha-1} + 1 - \delta \right) \right) \tag{4.19}$$

The preference shock γ_{gt} follows a log-normal distribution which is described in Equation 4.11.

4.4 Welfare Measure

The aim of this chapter is to report how γ_{gt} preference uncertainty affects the households' optimal utility. To examine this effect, I apply perturbation methods to solve the model following Schmitt-Grohe and Uribe (2004). As the γ_{gt}

shock has the MPS property, its mean changes with its variance. The general method perturbs ignores the effects of the changing mean. Therefore, I use the extended perturbation solution introduced by Heiberger and Maußner (2020), which considers the effect of MPS property.

In order to measure the welfare effect of γ_{gt} uncertainty, I take a second-order approximation of the policy function and market clearing conditions around the model's non-stochastic steady state. According to Schmitt-Grohé and Uribe (2004a), the welfare is measured as the part of private consumption which the representative household is willing to forgo in the presence of uncertainty to be equally well-off as in the absence of uncertainty.

In addition, I compute the conditional and unconditional welfare cost. Following Heiberger and Maußner (2020), the conditional welfare cost is the variation between two economies which stay in the same condition at the beginning, but one economy suffer from the γ_{gt} uncertainty that drives the business cycle. The unconditional welfare is measured based on the expectation of the value function. Thus, it includes both the mean and fluctuation effects of γ_{gt} uncertainty.

4.4.1 Conditional Welfare

Suppose C_{it} and N_{it} are the optimal consumption and labour in two different equilibrium time paths $i \in n, u$. Thus, the expected present discounted value of life-time utility can be expressed as:

$$
\begin{aligned}
V_{it}(\gamma_{gt}, K_t) &= E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\left(C^p_{it+j} + \gamma_{git+j} G_{it+j}\right)^{1-\sigma}}{1-\sigma} - \frac{(N_{it+j})^{1+\eta}}{1+\eta} \right) \\
&= \left(\frac{1}{1-\beta} \right) \left(\frac{(C^p_i + \gamma_{gi} G_i)^{1-\sigma}}{1-\sigma} - \frac{(N_i)^{1+\eta}}{1+\eta} \right)
\end{aligned}
$$

In order to express in a clear form, two auxiliary value functions are defined as:

$$V_i(\gamma_g, K) = V_i^C(\gamma_g, K) + V_i^N(\gamma_g, K)$$

$$V_i^C(\gamma_g, K) = \frac{1}{1-\beta}\left(\frac{(C_i^p + \gamma_{gi}G_i)^{1-\sigma}}{1-\sigma}\right)$$

$$V_i^N(\gamma_g, K) = -\frac{1}{1-\beta}\left(\frac{(N_i)^{1+\eta}}{1+\eta}\right)$$

According to Schmitt-Grohe and Uribe (2004), the welfare cost equals the portion of consumption that the representative household is willing to give up in equilibrium 'n' in order to obtain the same welfare as in the alternative equilibrium 'u'. Therefore, the relationship between two equilibriums is defined as:

$$V_u(\gamma_g, K) = E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{((1+\lambda_c)C_{nt+j}^p + \gamma_{gut+j}G_{nt+j})^{1-\sigma}}{1-\sigma} - \frac{(N_{nt+j})^{1+\eta}}{1+\eta}\right) \tag{4.20}$$

$$= \left(\frac{1}{1-\beta}\right)\left(\frac{((1+\lambda_c)C_n^p + \gamma_{gu}G_n)^{1-\sigma}}{1-\sigma} - \frac{(N_n)^{1+\eta}}{1+\eta}\right)$$

where 'n' denotes the equilibrium without γ_{gt} uncertainty, and 'u' is with uncertainty. And λ_c measures the private consumption that households need to forgo or increase in equilibrium 'n' to be equally well-off as in the alternative equilibrium 'u'. That is to say, λ_c indicates the conditional welfare loss or gain in terms of γ_{gt} uncertainty.

Simplifying Equation 4.20, the welfare cost λ_c is derived in the following equation:

$$\lambda_c = \frac{1}{C^{pn}}\left\{[(1-\beta)(1-\sigma)(V_u(\gamma_g, K) + V_n(\gamma_g, K))]^{\frac{1}{1-\sigma}} - \gamma_{gn}G^n\right\} - 1 \tag{4.21}$$

where variables with upper subscript 'n' denotes under the equilibrium 'n' without uncertainty, and the subscript 'c' indicates that this measure is conditioned on the initial point (γ_{gt}, K_t). If the value of λ_c is negative, it shows that an increase in γ_{gt} variance reduces households utility. In other words, households dislike the uncertainty in γ_{gt}.

4.4.2 Unconditional Welfare

The expression of unconditional welfare is similar to the conditional one. Value functions are evaluated at the unconditional expectation. Thus, the unconditional welfare gain or loss λ_u is obtained as follows:

$$\lambda_u = \frac{1}{E_t C_t^{pn}} \left\{ \left[(1-\beta)(1-\sigma)\left(E_t V_u(\gamma_{gt}, K_t) + E_t V_{nt}^N(\gamma_{gt}, K_t)\right)\right]^{\frac{1}{1-\sigma}} - E_t \gamma_{gt}^n G_t^n \right\} - 1 \tag{4.22}$$

For the unconditional welfare cost, the economy with γ_{gt} uncertainty includes an increase in the expected mean of γ_{gt}. Thus, the welfare cost is different from the conditional one. With a higher mean of γ_{gt}, households weigh more on public goods than private one. Thus, the mean of endogenous variables varies with γ_{gt} volatility. These changes show the optimal response of labour supply and capital accumulation to γ_{gt} uncertainty. And it also includes the tax of these changes. Comparing with the conditional welfare cost, the unconditional one can be regarded as a long term consequence of γ_{gt} volatility.

4.4.3 Parameters Calibration and Result

Parameters Calibration

To measure the value of welfare cost, I first calibrate parameters' values and then use the perturbation method from Heiberger and Maußner (2020) to get both conditional and unconditional welfare costs. The calibrated parameters are listed in Table 4.1. In the benchmark model, I set the discount factor β to 0.994, implying a steady-state annual real interest rate of 2.4%, in line with the value from Fernández-Villaverde et al. (2015). The household's relative risk aversion is set to 2. And the inverse of Frisch elasticity of labour supply is set to 3. According to the estimation of Chapter 2, the mean value of $\bar{\gamma_g}$ is 0.2. Additionally, the tax rate Γ is 0.2, which is also considered as the government's spending ratio for output. In the γ_{gt} shock process, the persistence of $\ln \gamma_{gt}$ is assumed as 0.95. And its innovation variance is discussed in the following part.

Table 4.1: Calibration Parameters

Structure Parameters		
β	0.994	Discount factor
σ	2	Relative risk aversion
η	2	the inverse of Frisch elasticity of labour supply
$\bar{\gamma_g}$	0.2	Coefficient of government spending in household utility
Γ	0.2	The tax rate
ρ_z	0.95	Persistence of the preference shock (γ_{gt})

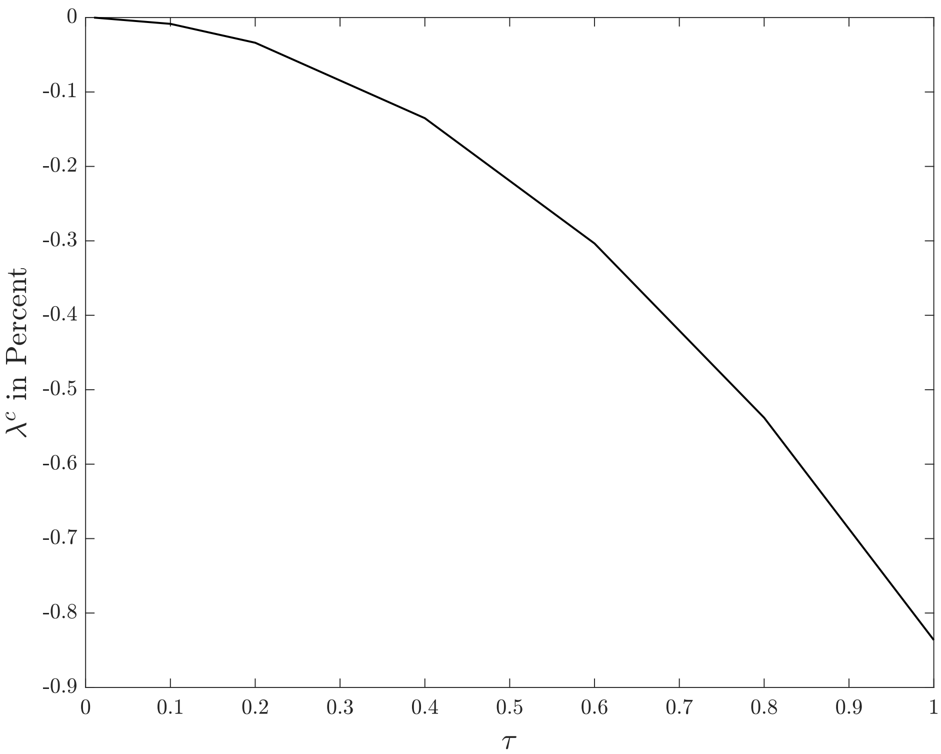
Results

With the calibration of parameters, Figure 4.1 and 4.2[5] show the changes of conditional and unconditional welfare cost with the increasing innovation variance of γ_{gt}.[6] The values of welfare cost are expressed by percentage, which can be interpreted as a proportion of private consumption. And the standard deviation of γ_{gt} (τ) is set from 0.01 to 1. As rare disasters, such as, pandemics, wars and earthquakes, generally induce a dramatically high uncertainty in economy, I set the highest value of the standard deviation as 1.

In both figures (Figure 4.1 and 4.2), the values of welfare cost are negative under the conditional and unconditional measures, which suggests a welfare loss with an increase in γ_{gt} variance. That is to say, the uncertainty in γ_{gt} is welfare reducing. And households prefer staying in a stationary state rather than a

[5]How Figure 4.1-4.8 are constructed is explained in Appendix.
[6]The increase in variance τ^2 indicates an uncertainty in γ_{gt}.

Figure 4.1: Conditional Welfare Measure with increasing uncertainty in γ_{gt}

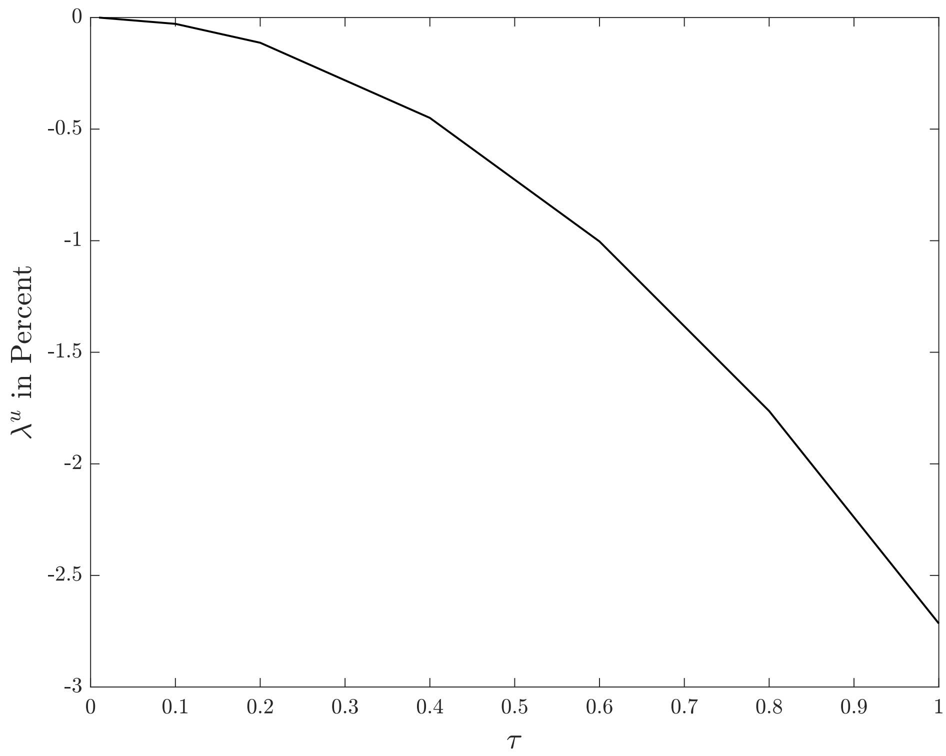


Note: The horizontal axis denotes the standard deviation τ, and the vertical axis denotes the percentage changes of welfare

stochastic one. When γ_{gt} is uncertain, the representative household tends to decrease his consumption on private goods as they value more on pubic goods and services. At the same time, the household becomes more cautious and tends to increase his precautionary savings, which further reduces consumption. Besides, the volatility in γ_{gt} pushes the household to work more refer to the mean effect, but leads to a higher income tax. Thus, the household reduces his private consumption, which is due to the negative wealth effect. Combining the fluctuation and negative wealth effects, the household private consumption falls more.

Moreover, comparing Figure 4.1 and 4.2, the unconditional welfare cost is larger than the conditional one, because the expected mean rises with an in-

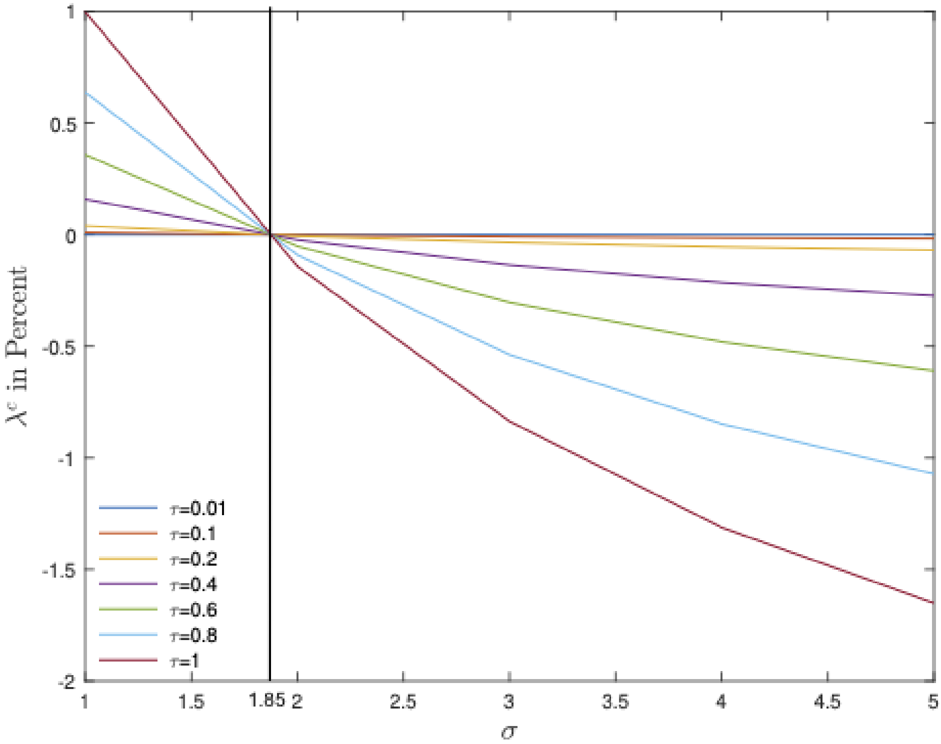
Figure 4.2: Unconditional Welfare Measure with increasing uncertainty in γ_{gt}



Note: The horizontal axis denotes the standard deviation τ, and the vertical axis denotes the percentage changes of welfare

crease in γ_{gt} variance.[7] The household with a higher mean of γ_{gt} weighs more on public goods and services rather than private ones. Thus, he tends to replace private consumption with more public consumption. Consequently, the welfare cost is higher under the unconditional measure. In the long term, the preference uncertainty in private consumption and government spending can severely reduce the welfare of households.

Figure 4.3: Conditional Welfare Measure with different degree of relative risk aversion



Note: The horizontal axis denotes the household relative risk aversion σ, and the vertical axis denotes the percentage changes of welfare. The value of η is set to 2.

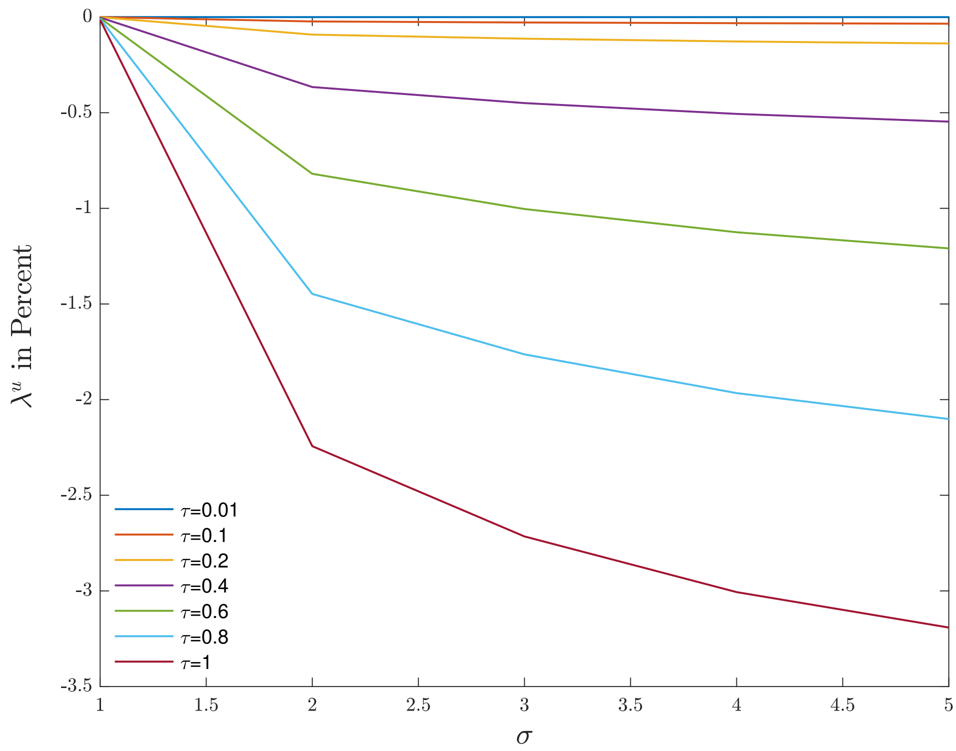
4.4.4 Sensitivity Analysis

Sensitivity to parameters

As the uncertainty in γ_{gt} is considered as a preference shock for the household, the effect of γ_{gt} might be impacted by other household's preference parameters, such as the relative risk aversion and the elasticity of labour supply. As the first section shows the sign of the second-order derivative of the utility function with respect to γ_{gt} ($\frac{\partial^2 U_t}{\gamma_{gt}}$) depends on the sign of $1 - 2\sigma - \sigma\eta$, where σ is the relative risk aversion and η is the inverse of Frisch elasticity of labour supply. These preference parameters might impact the welfare effect of γ_{gt} uncertainty. Therefore,

[7] In the unconditional welfare, its cost includes the effects of changes in expected mean of γ_{gt}.

Figure 4.4: Unconditional Welfare Measure with different degree of relative risk aversion



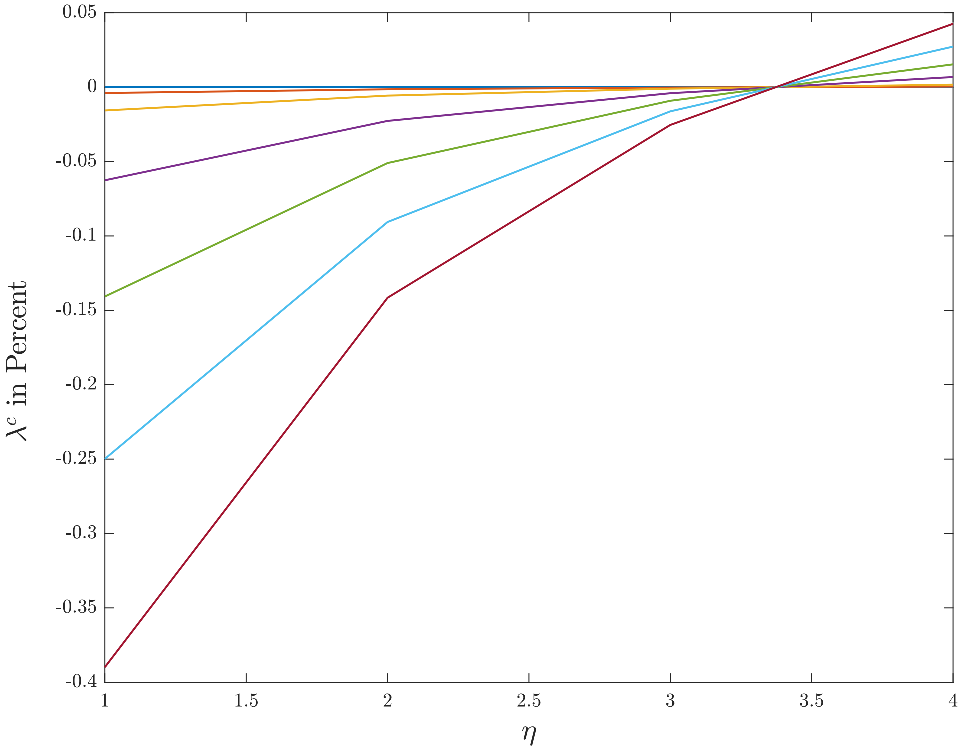
Note: The horizontal axis denotes the household relative risk aversion σ, and the vertical axis denotes the percentage changes of welfare. The value of η is set to 2.

I measures the welfare cost with different values of these two parameters.

Figure 4.3 and 4.4 illustrate how the conditional and unconditional welfares change with different values of σ (from 1 to 5).[8] And each figure shows the welfare cost under different sizes of variance in γ_{gt}. In both figures, the increasing σ reduces the household welfare under different degrees of variance in γ_{gt}. If σ is fixed at a higher value, the welfare also decreases with the increasing volatility in γ_{gt}. But in Figure 4.3, when the value of relative risk aversion is low (below 1.85), the welfare is positive and increases with γ_{gt} uncertainty which indicates a welfare improving. When $\sigma = 1.85$, there is no welfare cost with an increase in standard deviation τ. It indicates the household works more to compensate

[8]The value of η is set to 2

Figure 4.5: Conditional Welfare Measure with different degree of η



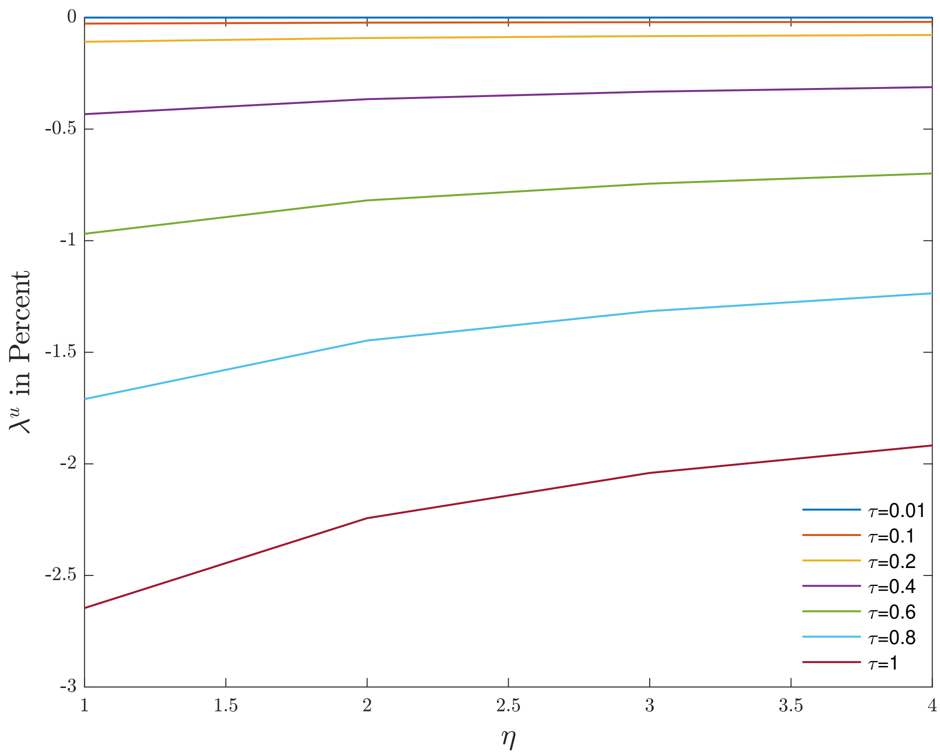
Note: The horizontal axis denotes the inverse of Frisch elasticity of labour η, and the vertical axis denotes the percentage changes of welfare. The value of σ is set to 2

the effect of uncertainty in γ_{gt} so that he has zero welfare cost. With lower risk aversion, the household makes full use of the fluctuation in γ_{gt}. That is to say, the household prefers the uncertainty in γ_{gt} with low risk aversion.

However, with a higher value of σ, the welfare dramatically reduces with increasing uncertainty in γ_{gt}, which shows a larger welfare cost. To be specific, the household who is more risk aversion dislikes the volatility in γ_{gt}. He is more willing to keep in a stable situation rather than in a stochastic one. In response to the γ_{gt} uncertainty, the household need to forego more their consumption to return to the steady state.

Moreover, comparing Figure 4.3 and 4.4, the welfare cost under the unconditional measure is also greater than that under the conditional measure as the

Figure 4.6: Unconditional Welfare Measure with different degree of η



Note: The horizontal axis denotes the inverse of Frisch elasticity of labour η, and the vertical axis denotes the percentage changes of welfare. The value of σ is set to 2

expected mean of the increasing volatility in γ_{gt} is considered in the unconditional welfare measure.

Figure 4.5 and 4.6 focus on another preference parameter η, the inverse of Frisch elasticity of labour supply. The welfare cost decreases with the a higher value of η, but the values of λ remain negative until the value of η is more than 3.4. Under the unconditional measure, the welfare reducing becomes less with the raising η, but the values of welfare are always negative. For both figures, given the value of η, the welfare cost rises with the volatility in γ_{gt} increasing.

Intuitively, a higher value of η illustrate that the household is less willing to work. Thus, there is less negative wealth effect on the household even with the γ_{gt} volatility. When the household does not supply more labour to production, their income does not increase. Thus, there is no more rise in tax. The uncertainty in γ_{gt} cannot further reduce the household private consumption more.

Non-separate utility between consumption and labour

The benchmark model includes the separate labour and consumption in the household's utility. In face of the preference uncertainty in private consumption and government spending, the household smooths his consumption and labour separately. The negative wealth effect plays a main role on the changes of consumption and labour. Therefore, in this part, I examine the welfare effect of γ_{gt} uncertainty, excluding the wealth effect. In order to eliminate the wealth effect, I set the consumption and labour as non-separate (e.g. Bilbiie et al. 2018). Thus, the household's utility function becomes:

$$U(C_t^P, N_t) = \frac{1}{1-\sigma}\left((C_t^p + \gamma_{gt}G_t)^{\theta}(1-N_t)^{1-\theta}\right)^{1-\sigma} \tag{4.23}$$

where θ denotes the relative proportion of the effective consumption in terms of utility and its value is between 0 and 1.

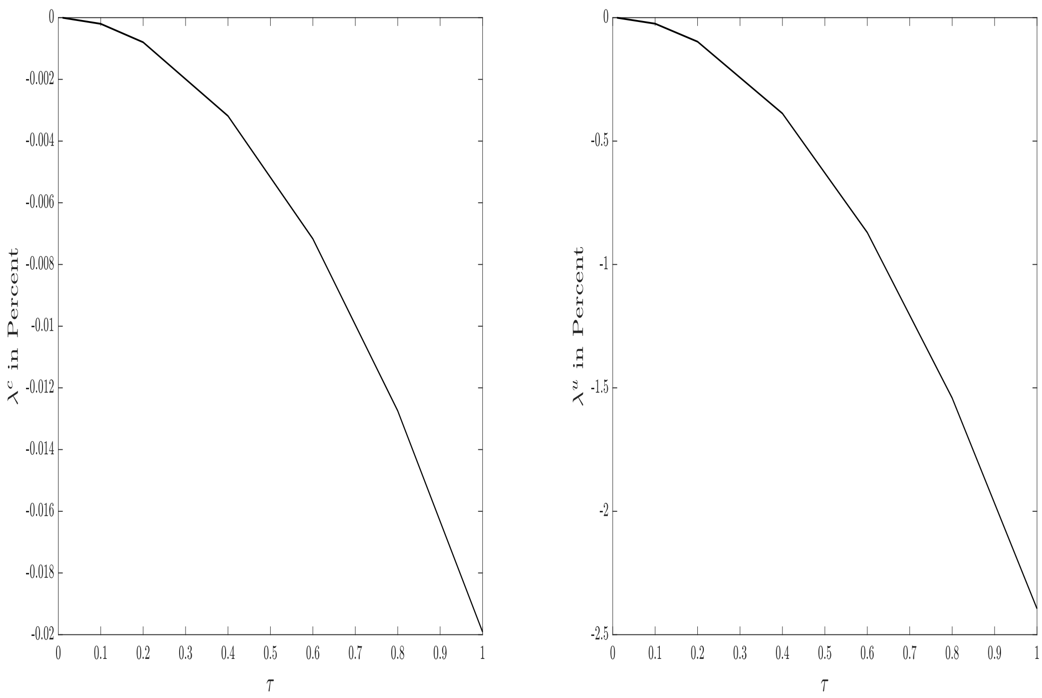
The production side is the same as in benchmark specification with variable capital utilization. The first-order conditions characterizing an optimal solution to the problem are:

$$-U_N(C_t^p, N_t) = U_{C^p}(C_t^p, N_t)(1-\Gamma)(1-\alpha)(\frac{K_t}{N_t})^{\alpha} \tag{4.24}$$

$$U_{C^p}(C_t^p, N_t) = \beta E_t\left(U_{C^p}(C_{t+1}^p, N_{t+1})\left((1-\Gamma)\alpha(\frac{K_{t+1}}{N_{t+1}})^{\alpha-1} + 1 - \delta\right)\right) \tag{4.25}$$

Under the competitive equilibrium with the non-separate utility function, the conditional and unconditional welfare costs are presented in Figure 4.7. Figure (a) and (b) show that the value of welfare falls as the standard deviation of γ_{gt} rises. With the non-separate utility function, the uncertainty in γ_{gt} is still

Figure 4.7: Welfare Measures with increasing uncertainty



(a) The Conditional Welfare Measure (b) The Unconditional Welfare Measure

Note: The horizontal axis denotes the standard deviation τ, and the vertical axis denotes the percentage changes of welfare.

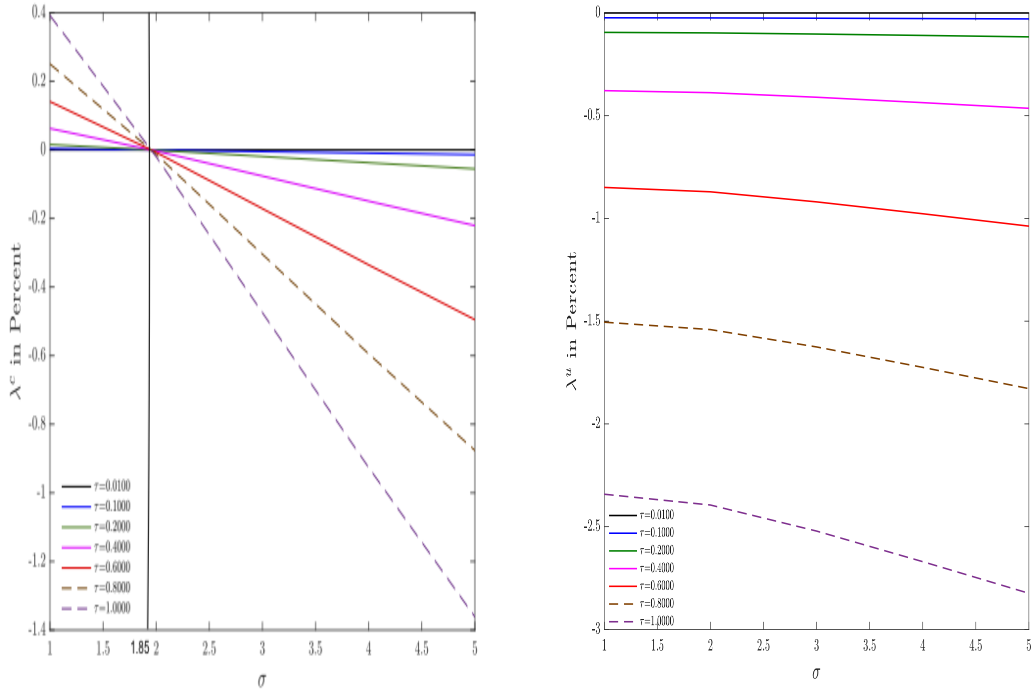
welfare reducing. The household still prefers a stabilised state, and thus smooths his private consumption to offset the uncertainty.

Additionally, for both conditional and unconditional measures, the decrement of welfare cost is less than that of separate utility function.[9] Particularly, under the conditional measure, the magnitude of welfare cost is considerably less than that from the separate utility function. This indicates that the negative wealth effect does play an important role on welfare effect of γ_{gt} uncertainty.

To further investigate how the household with different degrees of relative risk aversion reacts to the γ_{gt} uncertainty, Figure 4.8 is generated, which shows the welfare effect of γ_{gt} uncertainty with different values of σ. Figure 4.8 (a) and (b)

[9]Compare Figure 4.3 and 4.4 with Figure 4.7.

Figure 4.8: Welfare Measures with different degree of relative risk aversion



(a) The Conditional Welfare Measure (b) The Unconditional Welfare Measure

Note: The horizontal axis denotes the household relative risk aversion σ, and the vertical axis denotes the percentage changes of welfare.

show that give the size of standard deviation τ , the value of welfare falls with a higher σ. More risk-averse household responds to γ_{gt} uncertainty by reducing his private consumption more. Thus, the welfare cost is higher. Oppositely, when the value of σ is less than 1.85, the value of welfare increases. This means that the household who is less risk averse prefers the uncertainty and benefits from it.

In the figure 4.8 (b), the unconditional welfare cost also rises with the increasing relative risk aversion. And the magnitudes of welfare cost are greater than the conditional ones. Besides, the less risk-averse household also does not prefer the γ_{gt} uncertainty as the expected mean of γ_{gt} is considered in the unconditional welfare measure.

4.5 Conclusion

This chapter has studied how the preference uncertainty in private and public consumption itself impacts the welfare of households. To be specific, the preference uncertainty is induced by rare disasters and indicates the volatility in households' demand for private consumption and government spending.

The analytical result from a simple RBC model shows that an increase in preference uncertainty might reduce households' welfare. Households prefer a steady state instead of a stochastic state. According to Cho et al. (2015), the source of the welfare effect is from fluctuations effect and mean effect. The result also demonstrates the fluctuations effect that households prefer a steady state and smooth their consumption in the face of uncertainty. However, the mean effect in my case might not be welfare-enhancing. Although households work more to get higher incomes in response to uncertainty, they will face a higher tax. The negative wealth effect leads households to consume less. Consequently, the preference uncertainty in private and public consumption is welfare reducing.

To further investigate the welfare effect of the uncertainty, the numerical result is obtained from a canonical RBC model by computing the welfare cost (the compensating variation of private consumption). The result shows that the welfare cost rises with the increasing preference uncertainty in private and public consumption. This indicates the preference uncertainty is welfare reducing. Consequently, households dislike staying in an uncertain environment.

The robustness of results has been shown by different levels of preference parameters and specifications. Nevertheless, this would be a fruitful area for further work. One possible extension is to include heterogeneity of households or firms. As individual preferences are likely to vary, the welfare effects of uncertainty might become different.

4.6 Appendix

4.6.1 γ_{gt} shock process with mean-preserving property

To show how to generate the shock process for γ_{gt}, I assume a randow variable X_t that follows a log-normal distribution.

$$\ln(X_t) \sim N(\mu, \tau_x^2) \tag{4.26}$$

The mean and variance of X_t are derived as follows:

$$E(X_t) = exp(\mu + \frac{\tau_x^2}{2}) \tag{4.27}$$

$$Var(X_t) = exp(2\mu + \tau_x^2)[exp(\tau_x^2) - 1] \tag{4.28}$$

Equation 4.27 indicates that the mean of X_t changes with the value of τ_x^2. However, γ_{gt} shock has the mean-preserving proprty. Thus, the mean of $gamma_{gt}$ should be constant. I change the distribution of $\ln(X_t)$ as:

$$\ln(X_t) \sim N(\mu - \frac{\tau_x^2}{2}, \tau_x^2) \tag{4.29}$$

In this case, the mean and variane of X_t become:

$$E(X_t) = exp(\mu) \tag{4.30}$$

$$Var(X_t) = exp(2\mu)[exp(\tau_x^2) - 1] \tag{4.31}$$

Therefore, the mean of X_t is constant and changes in τ_x^2 means the mean preserving spread of X_t.

As γ_{gt} shock follows a log-normal distribution and has persistence and mean-preserving property, $\ln \gamma_{gt}$ is expressed as follows:

$$\ln(\gamma_{gt-1}) = \rho \ln(\gamma_{gt-1}) + \epsilon_t, \epsilon_t \sim N((1-\rho)\ln \gamma_{gt} - \frac{\tau^2}{2(1+\rho)}, \tau^2) \tag{4.32}$$

As the persistence of γ_{gt} shock, I assume it follows a MA process. Thus, γ_{gt} can be rewritten as follows:

$$\gamma_{gt} = \exp\left\{\sum_{j=0}^{\infty} \rho^j \epsilon_{t-j}\right\}$$

the first and second moments of γ_{gt} become:

$$\begin{aligned}
E(\gamma_{gt}) &= \exp\left\{\sum_{j=0}^{\infty}\left[\rho^j E(\epsilon_t) + \frac{\rho^{2j}Var(\epsilon_t)}{2}\right]\right\} \\
&= \exp\left\{\sum_{j=0}^{\infty}\rho^j E(\epsilon_t) + \sum_{j=0}^{\infty}\rho^{2j}\frac{Var(\epsilon_t)}{2}\right\} \\
&= \exp\left\{\frac{E(\epsilon_t)}{1-\rho} + \frac{Var(\epsilon_t)}{2(1-\rho^2)}\right\} \\
&= \exp\left\{\frac{E(\epsilon_t)}{1-\rho} + \frac{Var(\epsilon_t)}{2(1-\rho)(1+\rho)}\right\}
\end{aligned}$$

$$\begin{aligned}
E(\gamma_{gt}^2) &= \exp\left\{\sum_{j=0}^{\infty}\left[2\rho^j E(\epsilon_t) + 2\rho^{2j}Var(\epsilon_t)\right]\right\} \\
&= \exp\left\{\sum_{j=0}^{\infty}\rho^j 2E(\epsilon_t) + \sum_{j=0}^{\infty}\rho^{2j}2Var(\epsilon_t)\right\} \\
&= \exp\left\{\frac{2E(\epsilon_t)}{1-\rho} + \frac{2Var(\epsilon_t)}{1-\rho^2}\right\} \\
&= \exp\left\{\frac{2E(\epsilon_t)}{1-\rho} + \frac{2Var(\epsilon_t)}{(1-\rho)(1+\rho)}\right\}
\end{aligned}$$

As $\epsilon_t \sim N((1-\rho)\ln\bar{\gamma}_g - \frac{\tau^2}{2(1+\rho)}, \tau^2)$, the mean of γ_{gt} becomes:

$$\begin{aligned}
E(\gamma_{gt}) &= \exp\left\{\frac{(1-\rho)\ln\bar{\gamma}_g - \frac{\tau^2}{2(1+\rho)}}{1-\rho} + \frac{\tau^2}{2(1-\rho)(1+\rho)}\right\} \\
&= \exp\left\{\ln\gamma_{gt} - \frac{\tau^2}{2(1+\rho)(1-\rho)} + \frac{\tau^2}{2(1-\rho)(1+\rho)}\right\} \\
&= \exp\{\ln\bar{\gamma}_g\} \\
&= \bar{\gamma}_g
\end{aligned}$$

The variance of γ_{gt} becomes:

$$
\begin{aligned}
Var(\gamma_{gt}) &= E(\gamma_{gt}^2) - [E(\gamma_{gt})]^2 \\
&= \exp\left\{\frac{2E(\epsilon_t)}{1-\rho} + \frac{2Var(\epsilon_t)}{(1-\rho)(1+\rho)}\right\} - \left(\exp\left\{\frac{E(\epsilon_t)}{1-\rho} + \frac{Var(\epsilon_t)}{2(1-\rho)(1+\rho)}\right\}\right)^2 \\
&= \exp\left\{\frac{2(1-\rho)\ln\bar{\gamma_g} - \frac{\tau^2}{(1+\rho)}}{1-\rho} + \frac{2\tau^2}{(1-\rho)(1+\rho)}\right\} \\
&- \left(\exp\left\{\frac{(1-\rho)\ln\bar{\gamma_g} - \frac{\tau^2}{2(1+\rho)}}{1-\rho} + \frac{\tau^2}{2(1-\rho)(1+\rho)}\right\}\right)^2 \\
&= \exp\left\{2\ln\bar{\gamma_g} + \frac{\tau^2}{(1-\rho)(1+\rho)}\right\} - \exp\left\{\ln\bar{\gamma_g}\right\}^2 \\
&= \bar{\gamma_g}^2\left(\exp\left(\frac{\tau^2}{1-\rho^2}\right) - 1\right)
\end{aligned}
$$

4.6.2 Figures

Figures for the welfare cost

The welfare is measured by computing the difference of private consumption between with and without γ_{gt} uncertainty. In detail, first, I use the second-order approximation of the household utility $V(\gamma_g, K)$ at the stationary solution following Heiberger and Maußner (2020). Then, to compare an economy without uncertainty and, thus, without the γ_{gt} shock to one with no risk (τ=0). Thus, I assume that both economies stay in the stationary condition and that the one without shocks stays there for ever. The economy with γ_{gt} uncertainty with a changing amount of risk (increases in τ^2).

Therefore, the welfare cost equals the portion of consumption that the representative household is willing to give up in equilibrium 'n' in order to obtain the same welfare as in the alternative equilibrium 'u'. Therefore, the welfare is defined as Equation 4.22:

$$\lambda = \frac{1}{C^{pn}} \left\{ \left[(1-\beta)(1-\sigma) \left(V_u(\gamma_g, K) + V_n(\gamma_g, K) \right) \right]^{\frac{1}{1-\sigma}} - \gamma_{gn} G^n \right\} - 1 \quad (4.33)$$

where variables with subscript 'n' denotes under the equilibrium 'n' with zero variance, and variables with subscript 'u' denotes under the equilibrium 'u' with the increasing variance.

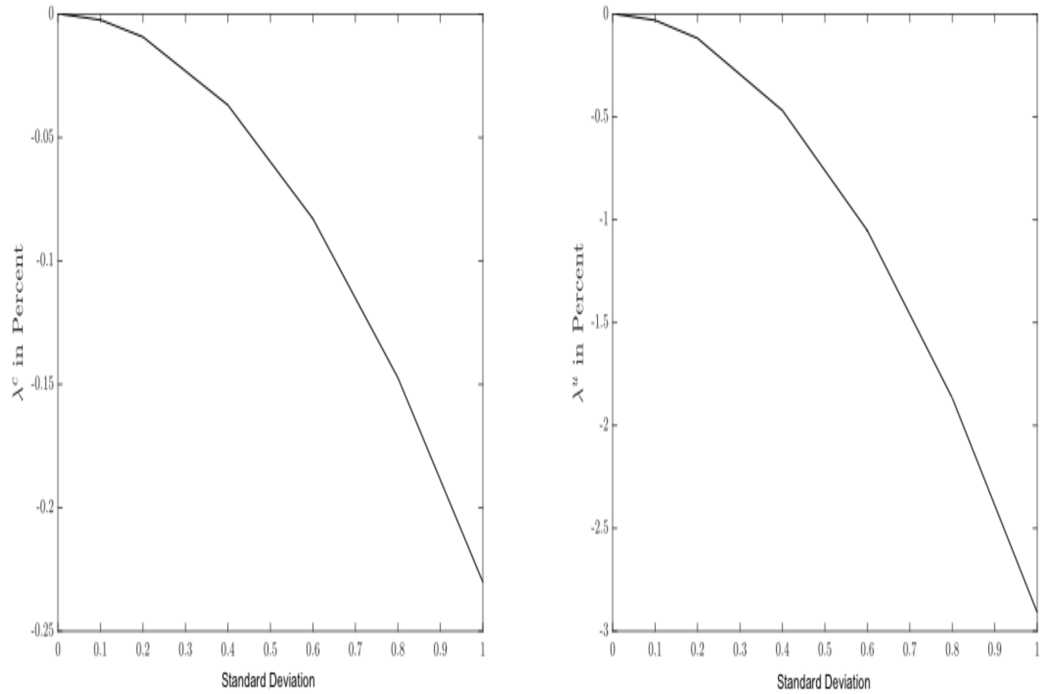
With increasing value of τ^2, the value of the household utility $V_u(\gamma_g, K)$ changes. Each time $V_u(\gamma_g, K)$ changes, I find a value of *lambda* such that $V_n(\gamma_g, K)$ is again equal to $V_u(\gamma_g, K)$. Figure 4.1, 4.2, 4.7 and 4.9 show λ with increasing standard deviation of γ_{gt}.

Additionally, With changes in the value of parameter σ and η, the value of the household utility $V_u(\gamma_g, K)$ changes. Therefore, given the value of standard deviation τ, I calculate the value of *lambda* such that $V_n(\gamma_g, K)$ is again equal to $V_u(\gamma_g, K)$ when parameters changes. The results are shown in Figure 4.3-4.6 and 4.8.

Figures with σ and η

Figure 4.9 shows the case that σ is 0.5 and η is 0.

Figure 4.9: Welfare Measures with $\eta = 0$ and σ =0.5



(a) The Conditional Welfare Measure (b) The Unconditional Welfare Measure

Note: The horizontal axis denotes the standard deviation τ, and the vertical axis denotes the percentage changes of welfare.

Chapter 5

Conclusion

This thesis has studied disaster-induced uncertainty and its impacts on the economy, especially on the efficacy of fiscal policy. The recent COVID-19 pandemic in 2020 has induced a large scale of uncertainty that adversely affects the economy, such as the fall in output and consumption. In the face of such kind of uncertainty, many countries conducted a large-scale fiscal stimulus to revive the economy. Consequently, it has prompted a reconsideration of how desirable such massive fiscal stimulus is. In particular, it is worth examining whether fiscal stimulus is effective when disaster-induced uncertainty is substantial. This thesis provides a new scope for understanding disaster-induced uncertainty and how it impacts the effectiveness of the fiscal policy.

First, I focus on the uncertainty in demand of households for private and public consumption because the uncertain nature of rare disasters gives rise to uncertainty about households' decisions on consumption. More specifically, rare disasters cause households to suffer health risks and work disruptions and thereby induce a shift in the distribution of their preferences for private and public consumption, indicating a fluctuation in their decisions on consumption. In other words, households' demand for these two consumption might change and even be uncertain because of the disaster-induced uncertainty. This demand is estimated in Chapter 2, which is measured by the marginal rate of substitution between private consumption and government spending. The estimation result indicates that this demand fluctuates over time, referred to as a preference shock with stochastic volatility. Especially during the COVID-19 pandemic, there is high volatility in this preference shock.

Second, government spending directly affects households' utility through public consumption supplied by government services and goods. Thus, the demand for private and public consumption directly affects the effect of the fiscal policy package. Hence, I have analytically and numerically examined fiscal multipliers with disaster-induced uncertainty. The result shows that this uncertainty reduces the value of fiscal multipliers. Intuitively, the increased government spending has a negative wealth effect on households. As rare disasters induce uncertainty in economic activities and fiscal policy, the consumption-leisure substitution channel becomes weaker. Thus, it triggers uncertainty in the demand of households for both private consumption and government spending. Therefore, households become cautious about their consumption and increase their precautionary savings, causing a lower optimal level of private consumption.

Due to the lower fiscal multiplier, Chapter 3 analytically investigates the optimal fiscal policy in rare disasters. To design optimal policy more realistic, the nominal interest rate binding at zero is considered in the model. At the ZLB, the optimal government spending is more significant to maintain the same optimal level of welfare. That is to say. It is desirable to adopt a large-scale fiscal stimulus package in response to rare disasters.

Moreover, from a welfare perspective, Chapter 4 has further studied the volatility in households' demand for private and public consumption itself affects households. The result indicates that the increasing preference uncertainty in private and public consumption reduces welfare and increases the compensating variation to make households indifferent between with and without uncertainty. Households dislike an uncertain environment. Intuitively, households prefer a steady state and smooth their consumption in the face of uncertainty. Furthermore, households tend to work more to get higher incomes to combat uncertainty.

However, they will face a higher tax. The negative wealth effect leads households to consume less. Consequently, the preference uncertainty is welfare-reducing, which is supported by the robust analysis.

This thesis first analyses the disaster-induced uncertainty in households' demand for private and public consumption and investigates its economic impacts. Based on the results, there are several possible policy suggestions. First, the disaster-induced uncertainty in households' demand for private and public consumption is a significant factor that policymakers should consider. The reasons are that rare disasters might happen more frequently, and households dislike an uncertain environment. Second, the massive fiscal policy package adopted by many countries is desirable to combat the COVID-19 pandemic. Because of the high disaster-induced uncertainty, the size of fiscal stimulus can be even more significant.

However, several challenges and extensions are still worth investigating. First, the households' demand for private and public consumption might vary among individuals. Thus, the welfare effects of uncertainty might become different. One possible extension is to include heterogeneity of households. Second, this thesis only focuses on the public consumption in the fiscal stimulus package, but there are liquidity supports such as equity, loans and guarantees. Combining the above aspects could be a fruitful avenue for future research.

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