A new approach to the joint order batching and picker routing problem with alternative locations

Sajjad Hedayati[a,*], Mostafa Setak[a], Emrah Demir[b,c] , Tom Van Woensel[d]

[a] *Faculty of Industrial Engineering, K.N.Toosi University of Technology, Tehran, Iran;*

[b] *Panalpina Centre for Manufacturing and Logistics Research, Cardiff Business School, Cardiff University, UK;*

[c] *Khalifa University, Department of Engineering Systems and Management, Khalifa University, Abu Dhabi, United Arab Emirates;*

[d] *School of Industrial Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands;*

E-mail: hedayatis@email.kntu.ac.ir *[S. Hedayati];* setak@kntu.ac.ir *[M. Setak];*
demire@cardiff.ac.uk *[E. Demir];* t.v.woensel@tue.nl *[T. Van Woensel]*

Abstract

The clustered and generalized vehicle routing problem (CGVRP) extends the well-known vehicle routing problem by grouping the demand points into multiple distinct zones, and within each zone, further separation is made by forming clusters. The objective of the CGVRP is to determine the optimal routes for a fleet of vehicles dispatched from a depot, visiting all zones within each cluster. This requires making two simultaneous optimization decisions. Firstly, each zone must be visited by exactly one node, and secondly, all zones within a cluster must be visited by the same vehicle. In this paper, we introduce two mixed-integer linear programming formulations for the CGVRP, aimed at solving a joint order batching and picker routing problem with alternative locations (JOBPR-AL) in a warehouse environment featuring mixed-shelves configuration. Both formulations are tested on three scenarios of randomly generated small- and medium-sized instances. Additionally, we propose a general rule approach for calculating a cost matrix in a rectangular environment. The results demonstrate the effectiveness of the proposed mathematical formulations in efficiently solving problems with up to 120 nodes.

Keywords: Clustered generalized vehicle routing problem, mixed-integer programming, picker routing, order batching.

1. Introduction

Among many factors influencing the performance of fulfilling online orders in the business-to-customer e-commerce market, number of items and product types per each order are two main challenges in retrieving Stock Keeping Units (SKUs) operation. The former is small in the sense that each order consists of a few quantities per each item, and the latter is large in the variety of product types requested. For example, an order can consist of a mixture of items such as grocery products, automobile spare parts, digital devices, and more. Indeed, a crucial aspect of these challenges is order batching and picker routing problem because items associated with an order must be batched and picker needs to be routed to pick up different items of the same order (Boysen et al., 2019).

In the realm of collection items to order fulfillment, there is a significant overlap in terminologies used for the Vehicle Routing Problem (VRP), including cart capacity, optimal routes for pickers, and optimal batching. The primary goal of VRP is to optimize the routes taken by vehicles as they visit customers' locations while considering practical operational constraints such as capacity, distance, time windows, and specific requirements. Applying VRP principles could be a practical approach for handling routing and batching operations within a warehouse environment, as opposed to a broader geographical area. However, it is important to remember crucial factors such as accessibility to items on complex shelves, short distances from item to item, a high rate of order varies in different locations with different sizes and characters, and other relevant restrictions. Despite these considerations, there are still opportunities to map certain problems in the vehicle routing context, as we will demonstrate the joint order batching and picker routing problem (JOBPRP) in the following discussion. An insightful discussion on the role of the VRP in real world logistic operations is presented by Demir et al. (2022) and (Demir et al., 2019). As a solution to the JOBPRP, various technologies are implemented to quickly pick up and fulfill orders in tight time frames. Particularly, research on using new technologies such as drones to assist delivery services has been wiedly studied as this devices can improve delivery efficiency in coollaboartion with vehicles to perform operstions (Kundu et al., 2021). However, these efforts can be still time-consuming and costly, especially for e-retailers (Masae et al., 2020).

Flexible storage strategies, such as a mixed-shelves policy have been used to improve the order batching and picker routing problems as a long-term optimization policy. In the mixed-shelves environment, products are scattered on shelves in alternative locations throughout a warehouse (Weidinger and Boysen, 2018). This complex operational problem brings us the concept of generalized vehicle routing problem (GVRP), which is first presented by Ghiani and Improta (2000). The GVRP extends the VRP in which customers' locations are grouped into zones. Generally, the objective is to find minimum-cost routes such that a fleet of vehicles visits a particular location associated with a customer zone (Baldacci et al., 2010). This flexibility can be an attractive solution for mixed-shelves environments in which alternative locations are available for each item, but visiting only one of these locations is sufficient. Krushinsky et al. (2021) presented two scenarios in which in one of the customers have flexible location options. This scenario is based on the GVRP with time windows where there is more than one possible location for each customer to receive delivery services.

Another related problem to order batching and picker routing is the last mile delivery operations. That is, in each route assigned to a vehicle, the courier must consecutively visit a group of customer locations (Sevaux and Sörensen, 2008). This type of routing problem is known as clustered vehicle routing problem (CluVRP) in the literature, in which demand points are grouped into distinct clusters. The aim is to find a set of routes for a fleet of vehicles to visit customers in each of the clusters consecutively. The CluVRP can be categorized into two classes: soft-CluVRP and hard-CluVRP (Hintsch and Irnich, 2020). In the former, a vehicle can visit nodes from different clusters in its route, and the vehicle can leave a cluster and serve another cluster. However, all nodes in each cluster must be visited by exactly one vehicle. In the latter case, however, all demand points in each cluster must be visited consecutively by one vehicle. In other words, a vehicle cannot leave a cluster until all associated nodes have been visited.

By considering the GVRP and CluVRP concepts together, we reach to the clustered and generalized vehicle routing problem in which demand points can be grouped into distinct zones and then further the zones are grouped into clusters. This new setting, which is the focus of this study and called the clustered generalized vehicle routing problem (CGVRP), simultaneously helps decision makers to design a set of optimal routes for serving a set of demand points grouped into a zone. We note that it is required to select one node per zone as a serving point. Furthermore, after considering specific criteria such as the proximity of zones within a geographic area, the demand points are organized into clusters. Each demand point is then associated with a particular zone and cluster. Additionally, it is necessary for all zones within a cluster to be serviced by the same vehicle.

This paper considers the concept of CGVRP to define a new integrated approach for the JOBPRP with alternative locations, which is abbreviated as JOBPRP-AL. One may ask how the JOBPRP-AL can help decision makers improve operational processes in warehouses. A short answer is the benefits of accomplishing order batching and picker routing via one route in an optimal way, such as minimizing total traveling distance. In fact, the main motivation for this integrated approach can be stated as follows. Suppose a mixed-shelves setting where a wide range of product types are scattered in shelves throughout a warehouse, as an order might consist of a number of products, then an optimal route, for a picker with a capacitated cart, is a route by which all possible locations for each product type are grouped into a zone, and then all these zones are grouped into a cluster as a batch corresponding to the order. Hence, based on the capacity constraint of each cart, each picker might serve more than one order. It is assumed that each cart has enough capacity to serve at least one order completely and there is sufficient inventory to satisfy all orders.

Therefore, the CGVRP is considered to model and solve the JOBPRP-AL by mapping this problem as a variant of the VRP as presented in Table 1 and Figure 1.

Table 1. Mapping the JOBPRP-AL into the CGVRP context.

The element in the JOBPR-AL		The element in the CGVRP
Locations of items in an order	↔	Nodes
Orders	↔	Clusters (in the CluVRP)
Alternative locations for each item	↔	Zones (in the GVRP)
Carts	↔	Vehicles
The request for each item is placed in an order	↔	Demands
Carts depot/Packaging and/or sorting point	↔	Vehicles depot

The CGVRP concept is illustrated in Figure 1.

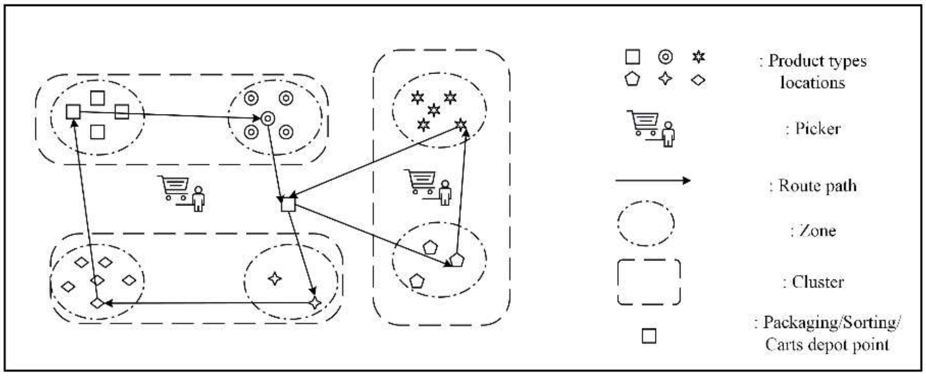


Figure 1. The CGVRP setting with a feasible solution.

As Figure 1 shows, the CGVRP is obtained by combining the concepts of the GVRP and CluVRP, and the JOBPR-AL is derived from the CGVRP in the mixed-shelves context. To provide further clarity, the JOBPR-AL is developed by adapting the terminologies of CGVRP to suit warehousing operations that involve mixed-shelves settings. The JOBPR-AL tackles the combined challenges of batch formation and routing within a mixed-shelves environment. The term '*alternative location*' refers to the different locations related to a particular type of product. As a result, to batch and pick items in a specific order, a picker with a cart starts its journey from a particular location such as carts depot and visits locations associated with the items in the order in such a way that the total travel distance is minimized. Therefore, the picker can complete at least an order via the route. If the cart has sufficient capacity to accommodate multiple orders, this scenario becomes feasible. Previous studies have not considered cart capacity, multiple-order processing capability, and integrated batching and picker routing simultaneously even in a traditional warehouse environment. Therefore, this study addresses these gaps by introducing the JOBPR-AL.

The contributions of this paper are threefold. First, as an integrated approach, we present an extended version of the CGVRP for modeling and solving the JOBPR-AL in warehouses with the mixed-shelves setting. Applying the JOBPR-AL in mixed-shelves warehouses can handle order picking efforts, which corresponds to all items associated with each order by a picker in a tour. Second, two mixed-integer linear programming (MILP) formulations are proposed for the JOBPR-AL. Third, a rule is presented to calculate the cost matrix in rectangular settings where to access to items on shelves, pickers must travel through aisles rather than Euclidean distance. This rule uses virtual nodes to calculate shortest distances between two points located in different aisles.

The remainder of the paper is organized as follows. Section 2 presents related works to this paper in two categories. Section 3 presents the definition of the problem and assumptions for the JOBPR-AL. Section 4 introduces mathematical models and presents a simple valid inequality to improve computational solution times for small- and medium-sized instances. Section 5 provides a detailed computational procedure to calculate a distance matrix in a rectangular environment with vertical and horizontal aisles. In Section 6, we create instances under three scenarios and solve them using the proposed IBM CPLEX solver (ILOG, 2023). In addition, Section 7 provides conclusions and future research directions.

2. Literature review

This section reviews the most related works to the JOBPR-AL in three categories. First, we review research studies related to the GVRP and its variants. Then, we investigate other publications associated with the CluVRP. An finally, we summarize the key studies related to the order picking problem in mixed-shelves environments.

2.1. The GVRP and its variants

Kara and Bektas (2003) presented an integer programming formulation for the GVRP and solved special cases of the problem, such as the generalized MTSP, GTSP, and CVRP. Bektaş et al. (2011) proposed mathematical formulations and B&C algorithms for the GVRP. Other mathematical models for the GVRP were presented by Pop et al. (2012). The authors also proposed new mathematical formulations for special cases, such as the GMTSP and GTSP. In addition, they studied the hard-CluVRP and introduced new formulations.

Regarding the solution approach, the literature in the GVRP is extensive (Hà et al., 2014). For exact algorithms, interested readers are referred to (Afsar et al., 2014), who presented a column generation approach for the GVRP. Reihaneh and Ghoniem (2017) proposed a B&C algorithm for the GVRP, and they used a dynamic programming approach to solve sub-problems. Prins et al. (2012) introduced an iterative local search for the GVRP. Pop et al. (2011)) presented heuristics algorithms for the same problem.

2.2. The CluVRP and its variants

Sevaux and Sörensen (2008) introduced the CluVRP. The basic problem is the hard-CluVRP, which does not allow vehicles to leave a cluster until they complete all delivery services. The soft-CluVRP was presented by Hintsch and Irnich (2020) in which vehicles can leave a cluster and then return to it to perform delivery services. Battarra et al. (2014) proposed an exact algorithm for the CluVRP, and they introduced a branch-price-and-cut and a B&C algorithm for the problem. Vidal et al. (2015) proposed hybrid metaheuristics for the CluVRP in which an iterative local search and a hybrid genetic algorithm were presented. In another study, Hintsch and Irnich (2018) proposed a large multiple neighborhood search for the CluVRP.

2.3. The picker routing problem in a mixed-shelves environment

Picker routing for retrieving or storage assignments is a relevant topic to the VRP. The applications of the TSP and VRP models and their extensions in warehousing operations are widely studied in the literature. A comprehensive review of routing problems for order picking was presented by Masae et al. (2020). The application of the TSP in a rectangular warehouse was studied by Ratliff and Rosenthal (1983). Later, (Scholz et al., 2016) addressed a single-picker routing problem in a block layout and proposed a MILP formulation for this problem. Briant et al. (2020) proposed a method for the joint order batching and picker routing problem and presented a column generation-based heuristic algorithm. Their work is based on the application of the TSP in both order batching and order picking in warehousing operations.

Weidinger and Boysen (2018) studied the scattered storage strategy. In another study, Weidinger et al. (2019) studied picker routing in a mixed-shelves setting for e-commerce retailers and presented a MILP formulation. Weidinger (2018) addressed picker routing in a rectangular mixed-shelves setting and presented procedures for optimizing warehouse operations.

This paper focuses on order picking strategies using routing concepts in which order batching and picking are handled using the CGVRP concept in a mixed-shelves environment. As far as we know, the use of the CGVRP has not been considered in a mixed-shelves setting in the literature. In this way, a sort-while-pick routing problem is considered in which multiple pickers, by using carts including some bins, pick up items associated with each order from alternative locations and sort them in different bins as a batch. Therefore, no subsequence sorting is needed after picking, and orders are ready to dispatch to destinations (Boysen et al., 2019).

In fact, using the CGVRP, parallel order-picking routes can be performed to pick items ordered by customers. This strategy in a setting with alternative locations for an item can be easily applied. On the other hand, we consider the CluVRP conditions to force the picker to complete picking all items of an order integrally for handling order batching. Regarding the use of the CluVRP in the setting of order batching and order selection problems, Aerts et al. (2021) applied the CluVRP to model the problem and proposed a two-level variable neighborhood search algorithm.

To the best of our knowledge, the CGVRP has not been considered in the literature. However, the TSP version of the problem for automated storage and retrieval systems (ASRS) vehicles is presented by Baniasadi et al. (2020), which is known as the Clustered GTSP. Foumani et al. (2018) studied this problem for optimizing ASRS robotic routes. They also presented a MILP formulation for the CGTSP and solved it using cross-entropy (CE). Similarly, the clustering strategy is considered to pick items in an order of magnitude. Baniasadi et al. (2020) also presented a transformation technique to the CGTSP. Table 2 shows related and selected works to this paper.

Table 2. Selected research related to JOBPR-AL.

Paper	JOBPR	Mixed-shelves	CluVRP	CGTSP	CGVRP
Weidinger (2018), Weidinger and Boysen (2018), Weidinger et al. (2019)		*			
Aerts et al. (2021)	*		*		
Baniasadi et al. (2020)	*			*	
This paper	*	*	*		*

3. Problem definition

This section introduces the JOBPR-AL and its features. Given a mixed-shelves environment where product types are scattered throughout a warehouse, each product type may be located in a number of different shelves in the warehouse. We assume that there is enough stock associated with each product type. In addition, there is a set of homogenous carts with capacity limitation. These carts are located at a particular location (i.e., packaging point). It is assumed that once routes are released to pickers, pickers can access the information instantly. Moreover, carts can carry more than one bin to handle simultaneous batching via pickers' routes, if needed. That is, bins, which are located in a cart, are considered as batches associated with different orders if the picker have to serve more than one order in a route. After receiving a number of orders, each of them is considered as a cluster. Then, the locations corresponding to each item in the order are grouped into a zone. If there is more than one item in the order, consequently more than one zone will be created. Hence, to create an optimal plan for a set of orders, the JOBPR-AL is applied to the related parameters. As a result, a set of optimal routes is produced, and each route is assigned to a picker with a cart. As mentioned above, each cart has enough capacity to handle at least one order, and each picker might perform more than one order. To process an order, pickers visit selected locations from each zone (i.e., a product type) in a cluster (i.e., an order). This action must be done sequentially for product types in an order. In other words, if there is more than one order assigned to a picker, the picker must batch and pick each order one-by-one along the route. The main attributes related to the JOBPR-AL are explained as follows.

As we assume that a company uses a mixed-shelves storage assignment strategy, items are scattered throughout a warehouse within shelves. This strategy is related to the GVRP concept of considering alternative locations for a product dispersed across shelves. Moreover, orders can be picked up by pickers with carts, and all carts are homogenous and located in a particular area called cart depot in the warehouse where a picker's route starts and ends. In addition, we assume that all products have similar shapes and volumes. For this reason, we focus on routing operations and assume that items are always available during the day. The problem is represented in Figure 2.

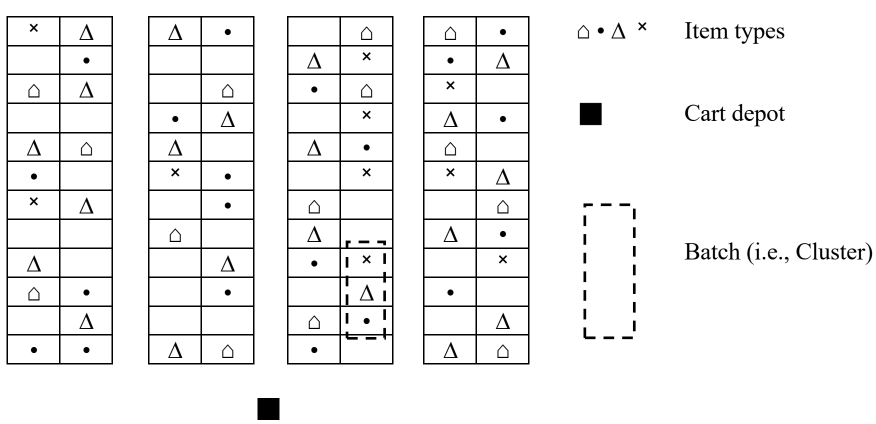


Figure 2. A basic illustration of the JOBPR-AL.

Figure 2 shows four shelves in which a picker must pick up an order, including items ●, Δ, and ×. These items are located throughout the warehouse as shown in Figure 2. Moreover, these three items are arranged in the nearest cluster to be picked up.

Due to the capacity restriction and customers' delivery time, orders may need to be divided into more than one route to pick up by more than one picker. In addition, pickers can pick up items related to more than one order in a route. For handling order batching in a route in which items related to an order must be picked up integrally, we consider the concept of the CluVRP for the GVRP modeling mixed-shelves settings. Therefore, the JOBPR-AL can be defined as a CGVRP.

4. Mathematical formulations

In this section, we present two MILP formulations. The first model is denoted as $F1$, and the second model is named $F2$. As we later show in section 6, while the former can handle situations in which ordered items in different orders overlap each other, the latter provides tighter lower bounds when there is no overlapping. In addition, model $F2$ needs less time to solve the problem optimally and provides a smaller gap in the final solution in most cases. Let the JOBPR-AL be defined on an undirected graph $G = (N, A)$ in which $N = \{0\} \cup \{1, \dots, k\}$ is associated with a set of nodes scattered in an area, which $\{0\}$ is the carts' depot, and $A = \{(i, j) \mid i \epsilon N, j \epsilon N, i \neq j\}$ is the set of arcs. A $(k+1) \times (k+1)$ matrix represents traveling distance between nodes i and j, which is denoted by e_{ij}. Note that in our setting carts are moved by pickers.

The nodes are grouped into $r+1$ zones in which the $L = \{0\} \cup \{L_1, \dots, L_r\}$ is the zone set. Each node i has a demand, $d_i > 0, i \epsilon \{1, \dots, k\}, d_0 = 0$ and all nodes grouped in zone L_h (e.g., $\forall w, l \epsilon L_h$) have the same demand, $d_w = d_l$. Zones are grouped into $m+1$ clusters, where $C = \{0\} \cup \{C_1, \dots, C_m\}$ is the cluster set. The size of the members (or zones) in cluster C_p is shown by $NC_p = |C_p|$. We also assume that there is a fleet of homogeneous capacitated carts with q number of carts, $V = \{V_1, \dots, V_q\}$, located at the depot (i.e., $i = \{0\}$).

The objective is to generate q routes with minimum cost. Each zone must be visited only once at one of the associated nodes of a zone. All zones in each cluster must also be visited sequentially by the same picker with a cart, subject to a set of side constraints. The necessary notations are listed in Table 3.

Table 3. Notations used in the models.

Notation	Description
Sets	
$N = \{0, 1 \dots, k\}$	Set of nodes
$N \setminus \{0\}$	Set of demand points
$A = \{(i,j) : i, j \epsilon N, i \neq j\}$	Set of arcs
$L = \{0\} \cup \{L_1, \dots, L_r\}$	Set of zones
$C = \{0\} \cup \{C_1, \dots, C_m\}$	Set of clusters
$V = \{V_1, \dots, V_q\}$,	Set of carts

$\{0\} \epsilon N$	The depot of carts
Parameters	
NC_p	The number of zones in cluster C_p
d_i	The demand of the node $i \; \epsilon N$
Q_f	The capacity of the cart $f \; \epsilon V$
Decision variables	
$x_{ijf} \epsilon \{0,1\}$	If the travel path *i* to *j* is passed by cart *f*, it is equal 1 and 0
$w_i \epsilon \{0,1\}$	If node *i* in a zone is visited by cart *f*, it is equal to *1* or *0* otherwise;
rc_{if}	The remaining capacity of the cart *f* after visiting node *i*.

Here, two formulations for the JOBPR-AL are proposed.

Model $F1$ is formulated as follows.

$$\min \sum_{i \in N} \sum_{j \in N} \sum_{f \in V} e_{ij} x_{ijf} \tag{1}$$

subject to:

$$\sum_{i \in L_h} w_i = 1 \qquad , \forall \, L_h \in \, L, \tag{2}$$

$$\sum_{j \in N} \sum_{f \in V} x_{ijf} = w_i \qquad , \forall \, i \in N, i \neq 0, i \neq j, \tag{3}$$

$$\sum_{j \in N} \sum_{f \in V} x_{jif} = w_i \qquad , \forall \, i \in N, i \neq 0, i \neq j, \tag{4}$$

$$\sum_{j \in N} \sum_{f \in V} x_{ijf} - \sum_{j \in N} \sum_{f \in V} x_{jif} = 0 \qquad , \forall \, i \in \, N, i \neq j, \tag{5}$$

$$\sum_{i \in C_p} \sum_{j \in C_p} \sum_{f \in V} x_{ijf} = NC_p - 1 \qquad , \forall \, C_p \in \, C, \tag{6}$$

$$\sum_{i \in N, i \neq 0} d_i . \sum_{j \in N} x_{ijf} \leq Q_f \qquad , \forall \, f \in \, V, \tag{7}$$

$$rc_{if} \leq Q_f + (d_i - Q_f) . x_{0if} \qquad , \forall \, i \in \, N, i \neq 0, \tag{8}$$

$$rc_{jf} \geq r_{if} + d_j - Q_f + Q_f . x_{ijf} + (Q_f - d_j - d_i) . x_{jif} \qquad , \forall \, i, j \in \, N \backslash \{0\}, i \neq j, ; \forall \, f \in \, V \tag{9}$$

$$\sum_{i \in N, i \neq 0} x_{i0f} \leq 1 \qquad , \forall \, f \in \, V, \tag{10}$$

$$\sum_{i \in N, i \neq 0} x_{0if} \leq 1 \qquad , \forall \, f \in \, V, \tag{11}$$

$$x_{ijf} \in \{0,1\} \qquad ,\forall (i,j) \in A, \forall f \in V, \qquad (12)$$

$$w_i \in \{0,1\} \qquad ,\forall i \in N, \qquad (13)$$

$$rc_{if} \in \mathbb{R}^+. \qquad (14)$$

The objective function (1) minimizes the total travel distance to pick and batch for pickers. Constraints (2)-(4) ensure that exactly one node is served in each zone to handle alternative locations for each item in an order. In fact, constraints (3)-(4) create intermediate variables, w_i, to use in constraints (2). Constraints (5) guarantee that for each zone the entering and leaving point must be the same. Constraint (6) says that zones grouped in each cluster must be visited successively. In other words, these constraints handle order batching via routes, forcing pickers to pick items associated with each order consecutively. Constraints (7) ensure that each route's total demand cannot exceed the cart's capacity. Constraints (8) and (9) are sub-tour elimination constraints in which constraint (8) is active for the remaining capacity of the cart f on node i, which must be visited after the carts' depot, and constraint (9) for others. Constraints (10) and (11) ensure that all tours must start at the depot with a minimum number of vehicles. Finally, constraints (12)-(14) are variables domain constraints.

Regarding model $F2$, we only present differences between *F1* and *F2* constraints which are formulated as follows.

$$\sum_{i\in L_h}\sum_{j\in L_h}\sum_{f\in V} x_{ijf} = 0 \qquad ,\forall L_h \in L, \qquad (15)$$

$$\sum_{i\in L_h}\sum_{j\in N}\sum_{f\in V} x_{ijf} = 1 \qquad ,\forall L_h \in L, \forall j \in N, i \neq j, \qquad (16)$$

$$\sum_{i\in L_h}\sum_{j\in N}\sum_{f\in V} x_{jif} = 1 \qquad ,\forall L_h \in L, \forall j \in N, i \neq j. \qquad (17)$$

The objective function (1) and constraints (5)-(14) function of the $F1$ are also applied for the $F2$, but constraints (2)-(4) in $F1$ are replaced by constraints (15)-(17) in $F2$. These new constraints (15)-(17) are used to ensure that only one node per zone will be visited by a picker and there is no path between nodes in a zone. We mentioned some key differences between models $F1$ and $F2$ earlier, but the performance, advantages, and disadvantages of both models are investigated in detail in section 6 by solving a wide range of instances.

We propose an inequality for the presented models. This inequality is based on the sub-tour elimination concept by which for $\forall (i,j) \epsilon A, j \neq i; \forall f \epsilon V$, it is impossible for each vehicle f to have a loop between vertices i and j. Only one path can be available between every two vertices. See Equation (18) and Figure 3.

$$x_{ijf} + x_{jif} \leq 1 \qquad ;\forall (i,j) \epsilon A, j \neq i; \forall f \epsilon V \qquad (18)$$

(a) (b) (c)

Figure 3. Forbidden path elimination between every two vertices:
(a) Forbidden tour (b) Feasible tour from i to j (c) Feasible tour from j to i.

As Figure 3 shows, case (a) cannot be possible between every two vertices, but at most one of cases (b) or (c) may be available in a feasible solution. Despite its simplicity, this valid constraint plays a crucial role in significantly reducing processing time.

5. Computing the distance matrix

This section presents a detailed warehouse layout and distance matrix computation procedure for the JOBPR-AL.

To ensure efficient picker routing in a warehouse environment with mixed shelves, it is crucial to establish a practical procedure for calculating the distance matrix. Additionally, three additional requirements must be taken into account for the general adoption of such a warehouse setting. First, the locations associated with each item must be categorized into specific zones. Each order should have at least one item with a minimum of one location to pick up. Second, all zones associated with each order should be clustered together within a single cluster. Therefore, each order is linked to a unique cluster. Third, with regards to the distance matrix, it is important to consider that accessing shelves in different aisles may require different rules compared to the last mile, where the distances between two nodes are calculated using the Euclidean formula. In this regard, we propose a specific procedure for warehouses consisting of three primary vertical aisles and one central depot. This rule can be extended to the same setting with more than three aisles. Here, we present a calculation procedure for generating a distance matrix in a warehouse with three vertical aisles and one carts depot or packaging point. Figure 4 shows a setting for presenting a procedure to calculate a distance matrix.

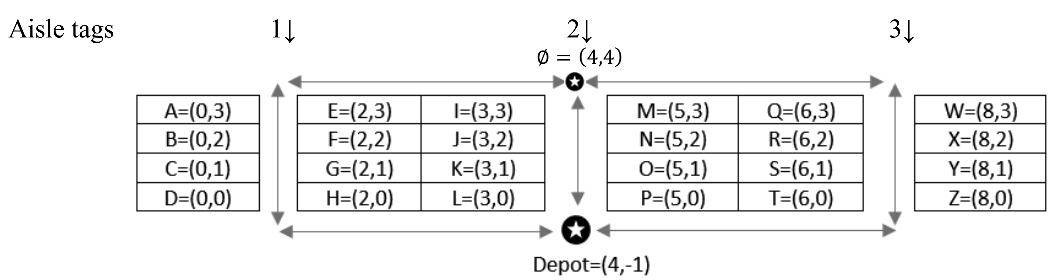


Figure 4. An example warehouse setting for calculating the distance matrix.

In this example, we have 24 labeled locations with associated coordinates. There are three aisles from left to the right with tags 1, 2 and 3, respectively.

If two locations are located in the same rack, such as *A*, *B, C*, and *D*, the distance is calculated as follows in equation (19):

$$if\ |x_i - x_j| = 0 \rightarrow distance(x, y) = |x_i - x_j| + |y_i - y_j| \tag{19}$$

If two locations are located in two different racks, but both of them are located in the same aisle, such as *K* and *N*, the distance is calculated as follows in equation (20):

$$if\ |x_i - x_j| = 2 \rightarrow distance(x, y) = |x_i - x_j| + |y_i - y_j| \tag{20}$$

If two locations are located in two different racks and aisles such as *G*, and *S*, the distance is calculated as follows in equation (21):

$$if\ |x_i - x_j| > 2 \rightarrow distance(x, y) = \min\left(2 + (|x_i - x_0| + |y_i - y_0|) + (|x_j - x_0| + |y_j - y_0|), 2 + (|x_i - x_\emptyset| + |y_i - y_\emptyset|) + (|x_j - x_\emptyset| + |y_j - y_\emptyset|) \right) \tag{21}$$

in which $\emptyset = (4{,}4)$ is a virtual node. In fact, for these locations located in different aisles, we need to calculate the minimum traveled distance based on feasible moving paths throughout the warehouse with respect to the original and virtual depots, respectively. Also, we need to add a "two" value because access to the shelves needs to move with one step. We call this value a correction value. The reason of incorporating this value is to account for virtual nodes when calculating distances. In fact, in this scenario where the aisles are separate, it is necessary to cross a virtual node denoted as $\emptyset$=(4,4) and the $carts\ depot = (-1{,}0)$ in order to access the shelves.

If two locations are located in two different racks and aisles but their racks are side by side, such as *N* and *R*, the distance is calculated as follows in Equation (22):

$$if\ |x_i - x_j| = 1 \rightarrow distance(x, y) = \min\left(2 + (|x_i - x_0| + |y_i - y_0|) + (|x_j - x_0| + |y_j - y_0|), 2 + (|x_i - x_\emptyset| + |y_i - y_\emptyset|) + (|x_j - x_\emptyset| + |y_j - y_\emptyset|) \right) \tag{22}$$

in the case of locations $\emptyset = (4{,}4)$ being a virtual node, it is necessary to calculate the minimum traveled distance considering the feasible moving paths throughout the warehouse with respect to the original and virtual depots. These locations, located in different aisles, require careful consideration. Additionally, a correction value of "two" needs to be added because accessing the shelves requires moving with one step. This correction value is necessary to account for the presence of virtual nodes in distance calculations. Specifically, in this scenario where the aisles are different, it is not possible to reach the shelves without passing through the virtual node $\emptyset = (4{,}4)$ and the $carts\ depot = (-1{,}0)$, which act as two separate points.

Therefore, the type of distance matrix used is the primary distinction between implementing CGVRP and adapting it to the JOBPR-AL environment.

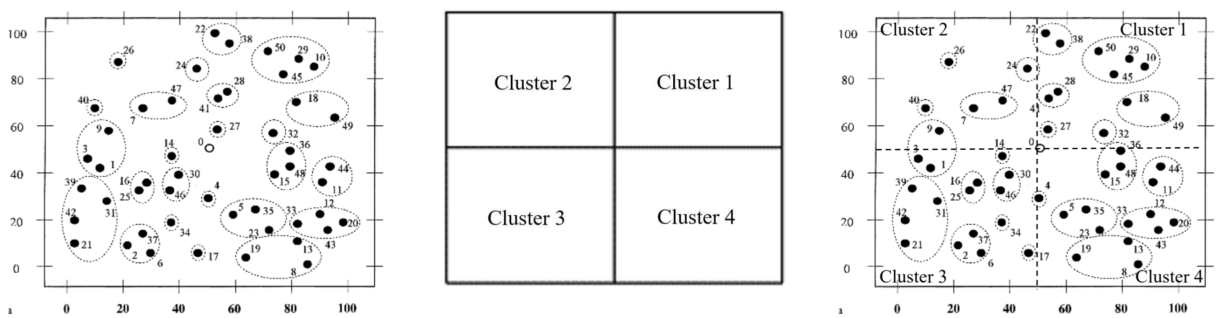
6. Numerical results

We use the CGVRP instances and solve them on a 64-bit operating system with 16 GB RAM and Core i7 configuration. All instances are solved by using CPLEX (ILOG, 2023) as a solver.

6.1. Model validation

To validate the proposed mathematical models, we adjusted the benchmark instances proposed by Ghiani and Improta (2000). First, we define four clusters in which different zones are grouped into specific clusters. For this, we divide the operation area into four segments and then consider each segment as a cluster as shown in Figure 5.

As Figure 5(a) shows, the operation area consist of 24+1 zones and each zone includes at least one node. This environment is virtually divided into 4+1 distinct clusters in Figure 5(b). The final operational area for the CGVRP model is shown in Figure 5(c). For detailed information on the zones and clusters, we refer to Table 4.



(a) (b) (c)

Figure 5. A representation of the clusters.

Table 4. Detailed information.

Notation	The members of clusters	The zones in a cluster	The members of zones
C_1	$C_1 = \{L_1, L_2, L_3, L_4, L_5, L_6\}$	$L_1, L_2, L_3, L_4, L_5, L_6$	$L_1 = \{22,38\}, L_2 = \{28,41\}, L_3 = \{27\}, L_4 = \{18,49\}, L_5 = \{32\}, L_6 = \{10,29,45,50\}$
C_2	$C_2 = \{L_7, L_8, L_9, L_{10}, L_{11}\}$	$L_7, L_8, L_9, L_{10}, L_{11}$	$L_7 = \{7,47\}, L_8 = \{26\}, L_9 = \{40\}, L_{10} = \{24\}, L_{11} = \{1,3,9\}$
C_3	$C_3 = \{L_{12}, L_{13}, L_{14}, L_{15}, L_{16}, L_{17}, L_{18}\}$	$L_{12}, L_{13}, L_{14}, L_{15}, L_{16}, L_{17}, L_{18}$	$L_{12} = \{21,42,31,39\}, L_{13} = \{14\}, L_{14} = \{34\}, L_{15} = \{17\}, L_{16} = \{16,25\}, L_{17} = \{30,46\}, L_{18} = \{2,6,37\}$
C_4	$C_4 = \{L_{19}, L_{20}, L_{21}, L_{22}, L_{23}, L_{24}\}$	$L_{19}, L_{20}, L_{21}, L_{22}, L_{23}, L_{24}$	$L_{19} = \{15,36,48\}, L_{20} = \{11,44\}, L_{21} = \{12,20,33,43\}, L_{22} = \{5,23,35\}, L_{23} = \{8,13,19\}, L_{24} = \{4\}$
{0}	{0}	{0}	{0}
Total	4+1 clusters	24+1 zones	50+1 nodes

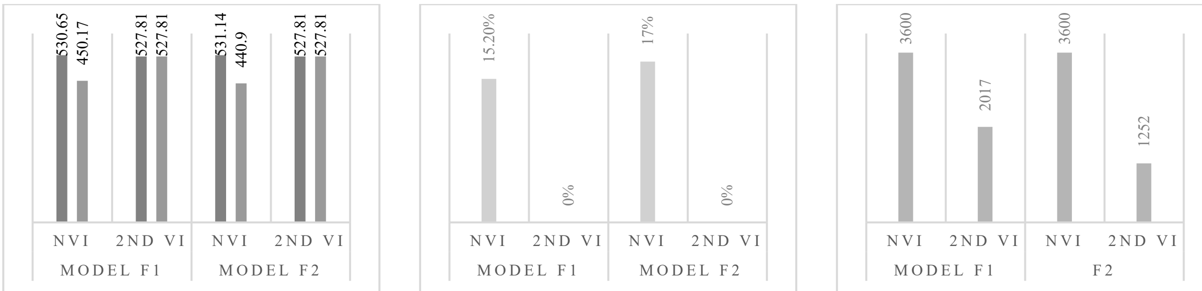
In the Euclidean matrix setting, the data used in this paper are based on the case proposed by Ghiani and Improta (2000). Additionally, we allocate a vehicle capacity of 16 units for the transportation process.

Before presenting the computational results of the JOBPR-AL, we compare our results with the results given by Ghiani and Improta (2000) and Kara and Bektas (2003). To this end, we exclude constraints (6) associated with grouping zones into clusters. In fact, by ignoring these constraints, we obtain the GVRP. Additionally, the capacity is set at 15 units for each theof vehicles. Our results are presented in Table 5 and Figure 6. The CPU execution time is restricted to a maximum of 3,600 seconds.

Table 5. Detailed results of the GVRP.

	Model $F1$		Model $F2$	
	NVI*	2nd VI**	NVI	2nd VI*
MIP Solution	530.65	**527.81**	531.14	**527.81**
Best Bound	450.17	**527.81**	440.90	**527.81**
CPU time (seconds)	3,600	**2017**	3,600	**1252**
Gap%	15.2%	**0%**	17%	**0%**

* NVI: Model without inequality constraints.
**2nd VI: Model with inequality constraints.



(a) (b) (c)

Figure 6. The results obtained from models F1 and F2 for the adjusted GVRP. (a), (b), and (c) display a comparison of the optimal solution and best bound, the gap between them, and the processing time.

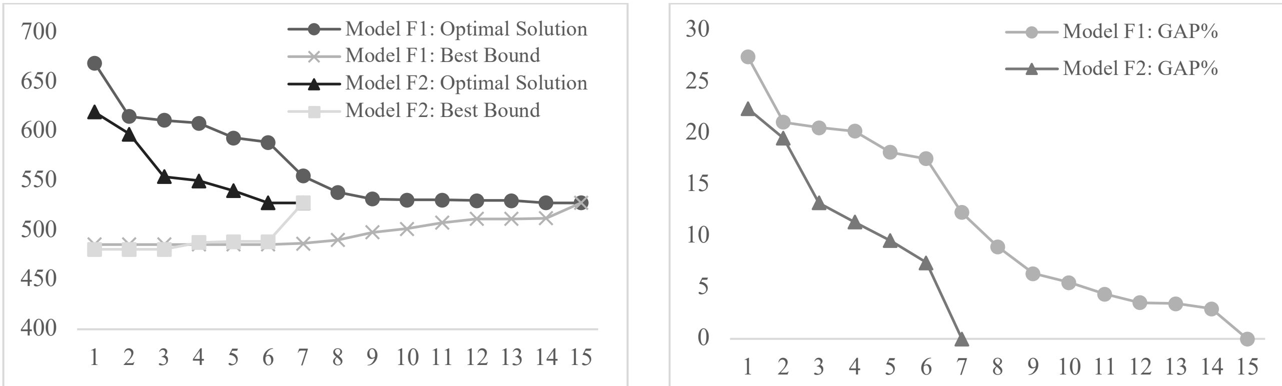
As Table 5 and Figure 6 highlight, our results obtained from models $F1$ and $F2$ are less than the best published objective function value presented by Ghiani and Improta (2000), which was 532.73. By applying a valid inequality, the $F1$ and $F2$ models obtain the optimal solution presented by Kara and Bektas (2003), which was 527.82. The slight difference 0.01 occurs because of the rounding in calculations. Therefore, two models $F1$ and $F2$ provides optimal solutions, and the effect of the valid inequality is significant. The inclusion of the valid inequality aids the solver in achieving an optimal MIP value within the time limit of 3,600 seconds. Moreover, the quality of the best bound for each model is equivalent to the MIP solution value, indicating a 0% gap.

In Table 6, the incumbent's progress associated with models $F1$ and $F2$ is compared together.

Table 6. The GVRP found incumbent associated with models $F1$ and $F2$.

$F1$ with inequality constraints			$F2$ with inequality constraints		
Optimal Solution	Best Bound	GAP%	Optimal Solution	Best Bound	GAP%
527.81	**527.81**	**0.00**	**527.81**	**527.81**	**0.00**
in 2,017 seconds.			in **1,252** seconds.		

As in Figure 7. shows, the performance of model $F2$ in reaching the optimal solution and its best bound progress is better than the model $F1$. It should be noted that for solving all the instances associated with the CGVRP and JOBPR-AL in this section, we have applied inequality constraints to both models $F1$ and $F2$.



(a) (b)

Figure 7. The MIP solutions corresponding to models F1 and F2 for the GVRP. (a) and (b) illustrate the variations observed in terms of the optimal solution, best bound, and gap values.

The comparative performance of model F2 in achieving optimal and near-optimal solutions surpasses that of model F1, as demonstrated in Tables 7-8 and Figure 8. We note that the objective value for the CGVRP is higher than that of the GVRP due to the introduction of constraint (6), which imposes an additional restriction on the model.

Table 7. Detailed results for the CGVRP.

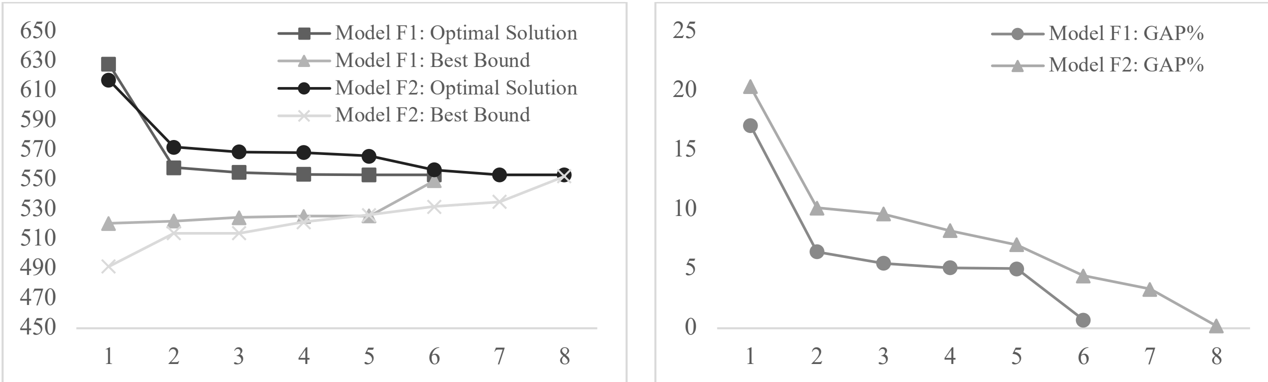
	Model $F1$		Model $F2$	
	NVI	2^{nd} VI	NVI	2^{nd} VI
MIP Solution	553.48	553.48	553.48	553.48
Best Bound	535.26	549.56*	535.66	**552.48***
Process time (seconds)	3,600	3,600	3,600	3,600
Gap%	3.29%	0.71%*	3.22%	**0.18%***

*The best results within each row are linked to their respective models.

Table 8. The MIP solutions associated with models F1 and F2 for the CGVRP.

Model $F1$ with the inequality constraints			Model $F2$ with the inequality constraints		
MIP Solution	Best Bound	GAP%	MIP Solution	Best Bound	GAP%
553.48	549.56	0.71	**553.48**	**552.48**	**0.18**

*The CPU running time limit = 3,600 seconds.



(a) (b)

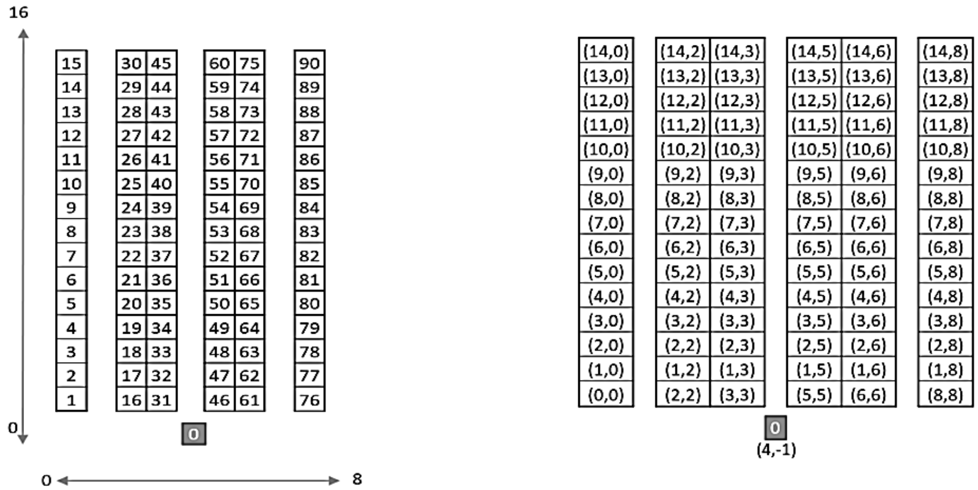
Figure 8. The MIP solutions associated with models $F1$ and $F2$ for the CGVRP. (a) and (b) represent the changes observed in reaching the optimal solution, best bound, and gap, respectively.

6.2. The joint order batching and picker routing instances

In this section, we generate instances for the JOBPR-AL by presenting three classes under three different scenarios. Based on the fact that there are no instances for the joint order batching and picker routing in a mixed-shelves environment in the literature, we present an instance generation method for our case as follows in Table 9 and Figures 9-10. We consider three scenarios. First, eight items are randomly scattered in each rack.

Table 9. Scenarios for the JOBPR-AL.

Mixed shelves setting	Scenarios		
	8 items	12 items	16 items
Number of vertical aisles	3	3	3
Number of racks	6	6	6
Number of shelves	90	90	90
Area's dimension	8*16	8*16	8*16
Number of orders	2	2	2
Number of items in each order	4	6	8
Number of depots	1	1	1
Number of carts (pickers)	2	2	2
Capacity of each cart (items)	4	6	8



(a) (b)

Figure 9. The mixed shelves instance setting.

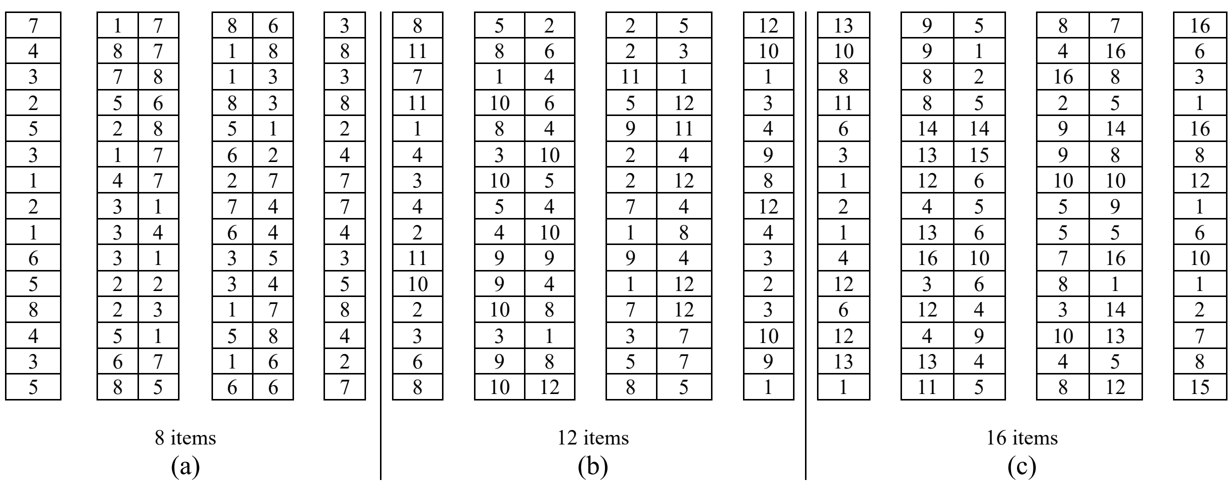


Figure 10. Three scattered scenarios in a mixed-shelves instance setting.

Figure 9 (a) shows the dimensions of the warehouse and the labels of shelves from 1 to 90. Also, as it depicts, the depot is located at the center bottom of the warehouse area. Figure 9 (b) shows the coordinates of each shelf. Based on the procedure explained in Section 5, the distance matrix is calculated using four conditions. Figure 10 shows three different scenarios in scattering items in a mixed-shelves environment. As it depicts, in case (a), we have eight items and randomly assign locations to each item. This procedure for both cases (b) and (c) is equal.

We apply both models $F1$ and $F2,$ to solve these cases. The computational results are shown in Tables 10-11 and Figure 11.

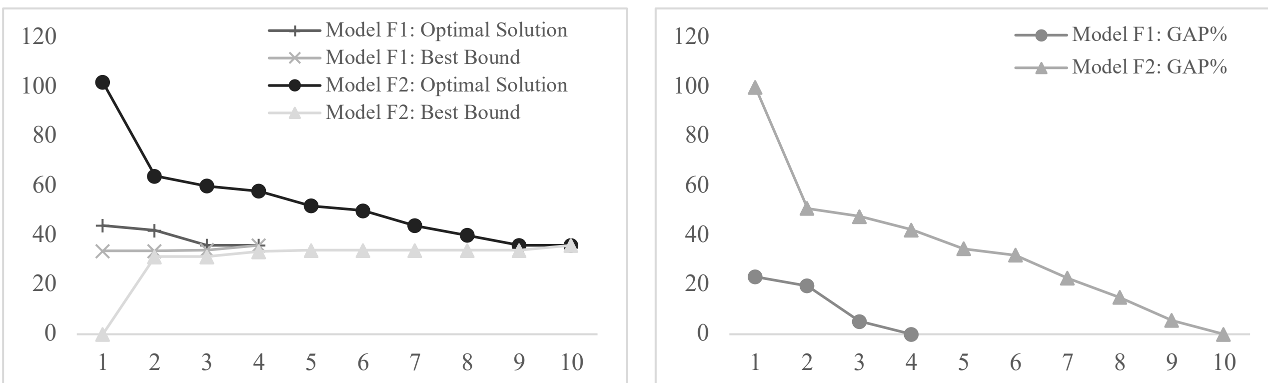
Table 10. The details of the orders for scenarios.

Mixed shelves setting	Scenarios		
	8 items	12 items	16 items
Order 1 (items)	1-4	1-6	1-8
Order 2(items)	5-8	7-12	9-16
Total number of items	8	12	16

Table 11. The MIP solutions associated with models $F1$ and $F2$ for the JOBPR-AL with 8 items.

Model $F1$ with inequality constraints			Model $F2$ with inequality constraints		
Optimal Solution	Best Bound	GAP%	Optimal Solution	Best Bound	GAP%
36.00*	**36.00**	**0.00**	**36.00***	**36.00**	**0.00**

*The running time of the CPU < 10 seconds.



(a) (b)

Figure 11. The MIP solutions associated with models $F1$ and $F2$ for the JOBPR-AL with 8 items. (a) and (b) represent the changes observed in reaching the optimal solution, best bound, and gap values.

As Table 11 shows, for the case with eight items, both models $F1$ and $F2$ can reach the optimal solution without any gap in less than 10 seconds. Figure 11(a) depicts the changes in obtaining optimal solutions and the best bound for each MIP solution. In addition, Figure 11(b) shows the changes in the gap during processing associated with the difference between the optimal solution and the best bound.

Table 12. The MIP solutions associated with models $F1$ and $F2$ for the JOBPR-AL with 12 items.

Model $F1$ with inequality constraints			Model $F2$ with inequality constraints		
Optimal Solution	Best Bound	GAP%	Optimal Solution	Best Bound	GAP%
62.00*	51.16	17.47	**62.00****	**62.00**	**0.00**

*The running time of the CPU =3,600 seconds. **The running time of the CPU =**2,100** seconds.

According to Table 12, the performance of model F2 in attaining the optimal solution and enhancing the best bound surpasses that of model F1. Model F2 exhibits a 17.47% gap after 3,600 seconds of processing time, but it can achieve a 0% gap within 2,100 seconds. The MIP solution for both models is equivalent to 62.00.

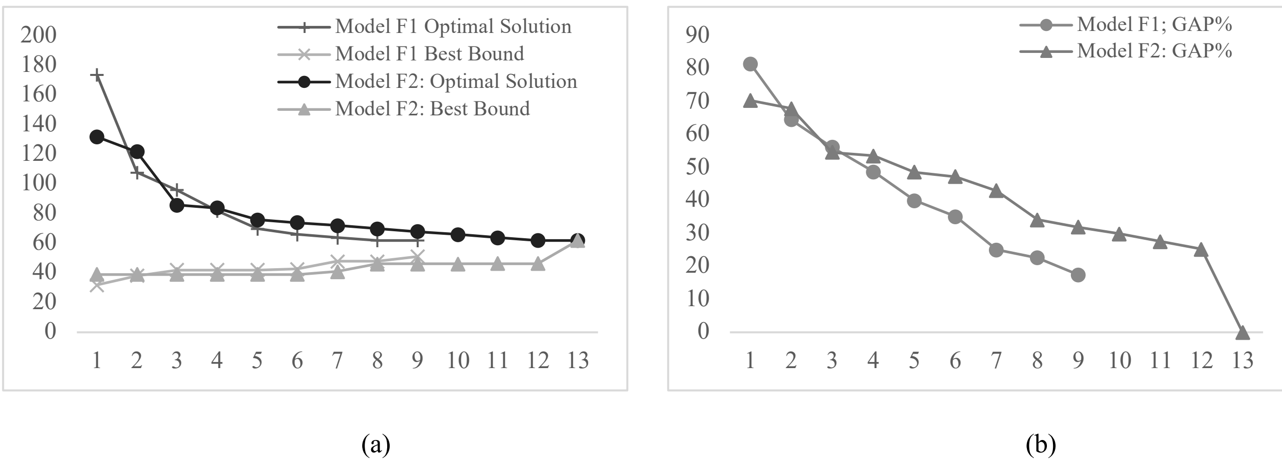


Figure 12. The identified MIP solutions associated with models $F1$ and $F2$ for the JOBPR-AL with 12 items. (a) and (b) represent the variations observed in the optimal solution, best bound, and gap values.

Figure 12(a) shows the changes in reaching MIP solution and the best bound in the models $F1$ and $F2$. However, Figure 12 (b) shows the decreasing gap path associated with the difference between the optimal solution and the best bound.

Table 13. The identified MIP solutions associated with models $F1$ and $F2$ in the context of the JOBPR-AL problem with 16 items.

Model $F1$ with the inequality constraints			Model $F2$ with the inequality constraints		
MIP Solution	Best Bound	GAP%	MIP Solution	Best Bound	GAP%
72.00*	60.83	15.51	**72.00***	**63.00**	**12.5**

*The running time of the CPU =3,600 seconds.

Table 13 highlights that model F2 outperforms model F1, as it exhibits 12.5% gap for a one-hour processing time. Notably, both models achieve an MIP solution of 72.00.

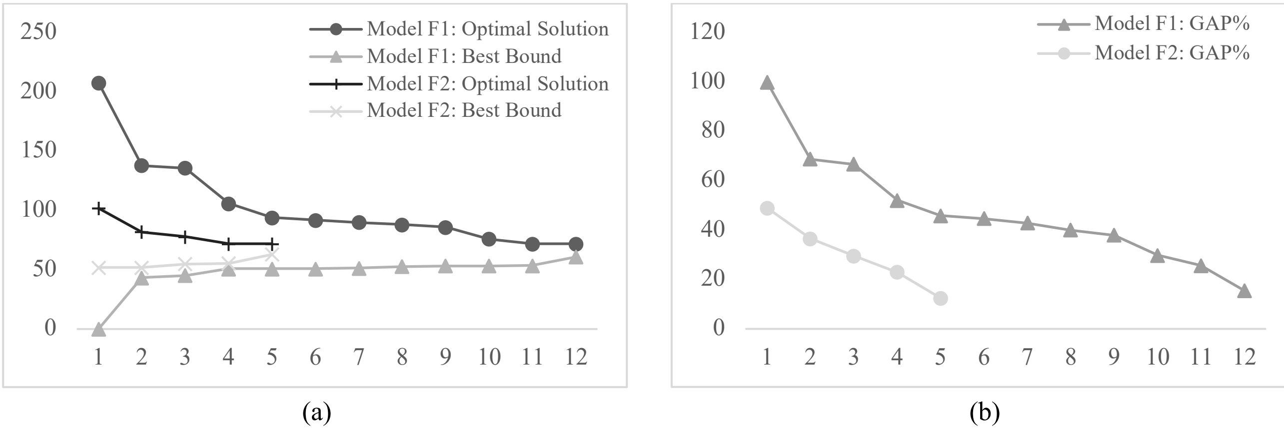


Figure 13. The relationship between the identified MIP solutions and the progression of the best bounds for models $F1$ and $F2$ in the context of the JOBPR-AL problem with 16 items. (a) and (b) depict the trends observed in reaching the optimal solution, best bound, and gap, respectively.

Figure 13(a) shows the trend in obtaining the optimal solution and best bound in each incumbent for models. On the other side, Figure 13(b) depicts the changes in the gap between the optimal solution and the best bound in each incumbent.

The results reveal two findings regarding the JOBPR-AL in a mixed shelves environment with three scenarios. First, in all cases, model $F2$ has better performance in improving best bound and reaching optimal solution without gap in one hour processing time. Second, the scenario with 16 items needs more processing time than cases with 12 and 8 items. Also, the case with 12 items needs more processing time than the case with 8 items. See Appendix A for graphical results related to the numerical results in this section.

6.3. Overlapping in ordered items

In section 6.2, we presented numerical results for a setting in which items in different orders are different. In fact, there is no overlap between items ordered by customers, so we face distinct clusters associated with each order. On the other hand, there is a situation in which a client may order items that overlap with other orders. Therefore, we consider four scenarios regarding this setting in which items in two different orders have details that overlap, as Table 14 shows. To do this, we duplicate the nodes corresponding to each location associated with each item, resulting in a node count that is twice as large as in the previous section. To do this, we copy nodes regarding each location associated with each item, so the number of nodes is twice that in the previous section. All parameters are the same as detailed in Table 9. In this new setting, if a location corresponding with an item is selected, this location is no longer available for other orders, and we add equation (23) to satisfy this requirement. For example, consider location i and its copy which is represented with i', equation (23) says that if one of these locations selected, another is no longer available. For this, we use model $F1$, as it can handle this issue.

$$w_i + w_{i'} \leq 1, \tag{23}$$

Table 14. The JOBPR-AL scenarios with overlapping.

Density of overlapping	Scenarios		
	8 items	12 items	16 items
25%	*		*
33%		*	
50%	*		*
66%		*	
75%	*		*
99%		*	
100%	*		*

The numerical results are shown in Table 15. As results show, when the density of overlapping is increased, the requisite processing time increases and the quality of solution dramatically decreases. Another point is that the number of items per order also has a significant effect on the solution process. That is, in scenario with 16 items and 8 items per order, the results are worse than the scenario with 12 items, and 8 items per order, and this scenario is worse than the scenario with 8 items and 4 items per order. Although the number of nodes is the same in all these scenarios, the number of clusters is different. Therefore, instances with more clusters and high density overlapping take more effort to solve optimally.

Table 15. Numerical results with overlapping by using model $F1$ with the inequality*.

Density of overlapping	Scenarios								
	8 items			12 items			16 items		
	Optimal Solution	Best Bound	GAP%	Optimal Solution	Best Bound	GAP%	Optimal Solution	Best Bound	GAP%
25%	40	40	0	-	-	-	88	51.91	41
33%	-	-	-	74	40.65	45.06	-	-	-
50%	46	37.33	18.84	-	-	-	112	42.66	61.9
66%	-	-	-	98	37.36	61.87	-	-	-
75%	52	36.77	29.27	-	-	-	114	42.14	63.03
99%	-	-	-	130	41.57	68.1	-	-	-
100%	66	37	43.78	-	-	-	No Solution		

*The running time of the CPU =3600 seconds.

7. Conclusions

We have studied CGVRP as an integrated approach for joint order batching and picker routing with alternative locations (JOBPR-AL). We have proposed two MILP formulations for the problem. In addition, we proposed valid inequality constraints for the proposed formulations in which eliminating forbidden paths between two nodes are considered. This inequality can improve total time to reach an optimal solution and best-bound values. We have adjusted benchmark instances found in the literature to validate our models and generated small and medium instances with three scenarios for the joint order batching and picker routing in a mixed-shelf setting.

Furthermore, the considered JOBPR-AL models, designed for the joint optimization of order batching and picker routing, effectively handle both order batching and order picking tasks. In settings with mixed-shelves, these models serve as valuable tools for operations with minimal cost. Among the models, model F2 consistently demonstrates superior performance in terms of reaching an optimal solution and achieving the best bound across all cases. In mixed-shelves settings, where larger items are dispersed in an area, the complexity of the model increases. Furthermore, two critical factors that have a notable influence on processing time and solution quality are the number of clusters or distinct items per order and the density of overlapping in the ordered items within each order. It is crucial to carefully consider these factors in order to achieve optimal results.

7.1. Managerial insights

E-commerce companies operate within a complex system where customers expect the convenience of placing orders from anywhere and at any time, with a desire for quick delivery. For companies that manage their own warehouses, optimizing the order picking process becomes crucial in meeting customer expectations, ensuring on-time delivery, and reducing operational time and costs. Such optimization efforts can lead to enhanced customer loyalty and long-term benefits for the company.

This paper propose models that address the joint optimization of order batching and picker routing problem. By enabling pickers in a warehouse to simultaneously execute order picking and order batching operations, separate batching processes can be seamlessly integrated with order picking activities. This approach effectively tackles three main challenges typically encountered in picking systems within mixed-shelves settings: order assignment to pickers, batch formation through picker routes, and selection of pick-up points for items within an order. The JOBPR-AL approach presented in this paper offers a comprehensive solution for order picking in mixed-shelves settings, simultaneously optimizing the aforementioned challenges. This approach can help companies to reduce process time and costs while improving the service level of their fulfillment operations. Moreover, this approach can also benefit the pickers themselves, as they can efficiently complete multiple orders while navigating through the warehouse aisles.

Implementing the JOBPR-AL approach in mixed-shelves or similar settings, where SKUs have alternative locations, can significantly reduce the need for separate batching or packaging operations. This is particularly valuable in the current e-commerce landscape, where there is a high variety of item types but relatively low demand for each individual item. By streamlining the order fulfillment process and increasing operational efficiency, companies can effectively reduce the effort required for order processing and enhance overall productivity.

7.2. Future research

In terms of future research directions, an interesting area to explore would be incorporating stochastic settings in both last mile delivery and joint order batching and picker routing problems. This would provide valuable insights into handling uncertainties and variability in real-world scenarios. Furthermore, integrating these models into broader warehousing and last mile operations would contribute to a more comprehensive understanding of supply chain management. This integration could uncover new opportunities for optimization and efficiency improvements.

Additionally, an open research area lies in the development of heuristic solution algorithms that can effectively tackle large-scale instances of these problems. Designing efficient algorithms capable of handling the computational complexity associated with real-world logistics operations would be highly valuable.

Data availability statement

The data underlying this article will be shared on reasonable request to the corresponding author.

Acknowledgements

We would like to extend our sincere gratitude to the Editors and the reviewers for their valuable and constructive comments on this study. Their input has made a substantial contribution to the enhancement and overall quality of our research.

References

Aerts, B., Cornelissens, T. & Sörensen, K. 2021. The joint order batching and picker routing problem: Modelled and solved as a clustered vehicle routing problem. *Computers & Operations Research,* 129**,** 105168.

Afsar, H. M., Prins, C. & Santos, A. C. 2014. Exact and heuristic algorithms for solving the generalized vehicle routing problem with flexible fleet size. *International Transactions in Operational Research,* 21**,** 153-175.

Baldacci, R., Bartolini, E. & Laporte, G. 2010. Some applications of the generalized vehicle routing problem. *Journal of the operational research society,* 61**,** 1072-1077.

Baniasadi, P., Foumani, M., Smith-Miles, K. & Ejov, V. 2020. A transformation technique for the clustered generalized traveling salesman problem with applications to logistics. *European Journal of Operational Research,* 285**,** 444-457.

Battarra, M., Erdoğan, G. & Vigo, D. 2014. Exact algorithms for the clustered vehicle routing problem. *Operations Research,* 62**,** 58-71.

Bektaş, T., Erdoğan, G. & Røpke, S. 2011. Formulations and branch-and-cut algorithms for the generalized vehicle routing problem. *Transportation Science,* 45**,** 299-316.

Boysen, N., De Koster, R. & Weidinger, F. 2019. Warehousing in the e-commerce era: A survey. *European Journal of Operational Research,* 277**,** 396-411.

Briant, O., Cambazard, H., Cattaruzza, D., Catusse, N., Ladier, A.-L. & Ogier, M. 2020. An efficient and general approach for the joint order batching and picker routing problem. *European Journal of Operational Research,* 285**,** 497-512.

Demir, E., Huckle, K., Syntetos, A., Lahy, A. & Wilson, M. 2019. Vehicle routing problem: Past and future. *Contemporary Operations and Logistics: Achieving Excellence in Turbulent Times*, 97-117.

Demir, E., Syntetos, A. & Van Woensel, T. 2022. Last mile logistics: Research trends and needs. *IMA Journal of Management Mathematics,* 33, 549-561.

Foumani, M., Moeini, A., Haythorpe, M. & Smith-Miles, K. 2018. A cross-entropy method for optimising robotic automated storage and retrieval systems. *International Journal of Production Research,* 56, 6450-6472.

Ghiani, G. & Improta, G. 2000. An efficient transformation of the generalized vehicle routing problem. *European Journal of Operational Research,* 122, 11-17.

Hà, M. H., Bostel, N., Langevin, A. & Rousseau, L.-M. 2014. An exact algorithm and a metaheuristic for the generalized vehicle routing problem with flexible fleet size. *Computers & Operations Research,* 43, 9-19.

Hintsch, T. & Irnich, S. 2018. Large multiple neighborhood search for the clustered vehicle-routing problem. *European Journal of Operational Research,* 270, 118-131.

Hintsch, T. & Irnich, S. 2020. Exact solution of the soft-clustered vehicle-routing problem. *European Journal of Operational Research,* 280, 164-178.

Ilog. 2023. *IBM ILOG CPLEX Optimization Studio* [Online]. Available: https://www.ibm.com/products/ilog-cplex-optimization-studio [Accessed].

Kara, I. & Bektas, T. Integer linear programming formulation of the generalized vehicle routing problem. EURO/INFORMS Joint International Meeting, Istanbul, July, 2003. Citeseer, 06-10.

Krushinsky, D., Guo, X. & Claassen, G. D. H. 2021. Location flexibility in parcel delivery operations: framework and empirical analysis. *IMA Journal of Management Mathematics,* 33, 563-581.

Kundu, A., Escobar, R. G. & Matis, T. I. 2021. An efficient routing heuristic for a drone-assisted delivery problem. *IMA Journal of Management Mathematics,* 33, 583-601.

Masae, M., Glock, C. H. & Grosse, E. H. 2020. Order picker routing in warehouses: A systematic literature review. *International Journal of Production Economics,* 224, 107564.

Pop, P. C., Kara, I. & Marc, A. H. 2012. New mathematical models of the generalized vehicle routing problem and extensions. *Applied Mathematical Modelling,* 36, 97-107.

Pop, P. C., Zelina, I., Lupşe, V., Sitar, C. P. & Chira, C. 2011. Heuristic algorithms for solving the generalized vehicle routing problem. *International Journal of Computers Communications & Control,* 6, 158-165.

Prins, C., Santos, A. C. & Afsar, H. M. 2012. Splitting procedure and a relaxed iterated local search for the generalized vehicle routing problem. *Procedings of the Latino-Iberoamericano de Investigación Operativa, Simpósio Brasileiro de Pesquisa Operacional, CLAIO-SBPO, Rio de Janeiro, Brazil*.

Ratliff, H. D. & Rosenthal, A. S. 1983. Order-picking in a rectangular warehouse: a solvable case of the traveling salesman problem. *Operations research,* 31, 507-521.

Reihaneh, M. & Ghoniem, A. 2017. A branch-cut-and-price algorithm for the generalized vehicle routing problem. *Journal of the operational research society*, 1-13.

Scholz, A., Henn, S., Stuhlmann, M. & Wäscher, G. 2016. A new mathematical programming formulation for the single-picker routing problem. *European Journal of Operational Research,* 253, 68-84.

Sevaux, M. & Sörensen, K. Hamiltonian paths in large clustered routing problems. Proceedings of the EU/MEeting 2008 workshop on Metaheuristics for Logistics and Vehicle Routing, EU/ME, 2008. Citeseer, 411-417.

Vidal, T., Battarra, M., Subramanian, A. & Erdogˇ An, G. 2015. Hybrid metaheuristics for the clustered vehicle routing problem. *Computers & Operations Research,* 58**,** 87-99.

Weidinger, F. 2018. Picker routing in rectangular mixed shelves warehouses. *Computers & Operations Research,* 95**,** 139-150.

Weidinger, F. & Boysen, N. 2018. Scattered storage: How to distribute stock keeping units all around a mixed-shelves warehouse. *Transportation Science,* 52**,** 1412-1427.

Weidinger, F., Boysen, N. & Schneider, M. 2019. Picker routing in the mixed-shelves warehouses of e-commerce retailers. *European Journal of Operational Research,* 274**,** 501-515.

Appendix A

This appendix presents the graphical results obtained from solving the proposed mathematical models for the JOBPR-AL. Figures A1-A3 display the optimal routes for the JOBPR-AL in cases with 8, 12, and 16 items, respectively.

It is important to note that the optimal location for each item are shown by *, as demonstrated in the all cases. For instance, picker 1 can pick up the items associated with order 1, i.e., batch 1, in the following sequence: 0 >> 33>> 34 >> 35 >> 37 >> 0. The labels on the cells in Figures A1-A3 correspond to the items' tags are shown by underlined numbers, while the locations' tags associated with each cell are presented by bold and italic numbers. For example, location 1 is reserved for item 5 as it is shown at the bottom left of the Figure A1. Further discussion and details can be found in Section 5.

15	7		1	***30***	***45***	7		8	***60***	***75***	6	3	***90***
14	4		8	***29***	***44***	7		1	***59***	***74***	8	8	***89***
13	3		7	***28***	***43***	8		1	***58***	***73***	3	3	***88***
12	2		5	***27***	***42***	6		8	***57***	***72***	3	8	***87***
11	5		2	***26***	***41***	8		5	***56***	***71***	1	2	***86***
10	3		1	***25***	***40***	7		6	***55***	***70***	2	4	***85***
9	1		4	***24***	***39***	7		2	***54***	***69***	7	7	***84***
8	2		3	***23***	***38***	1		7	***53***	***68***	4	7	***83***
7	1		3	***22***	***37****	4*		6	***52***	***67***	4	4	***82***
6	6		3	***21***	***36***	1		3	***51***	***66****	5*	3	***81***
5	5		2	***20***	***35****	2*		3	***50***	***65***	4	5	***80***
4	8		2	***19***	***34****	3*		1	***49***	***64****	7*	8	***79***
3	4		5	***18***	***33****	1*		5	***48***	***63****	8*	4	***78***
2	3		6	***17***	***32***	7		1	***47***	***62****	6*	2	***77***
1	5		8	***16***	***31***	5		6	***46***	***61***	6	7	***76***

0

Order 1(cluster 1)	1	2	3	4
Order 2(cluster 2)	5	6	7	8

Route 1*	***0→***	***33→***	***34→***	***35→***	***37→***	***0***
Route 2*	***0→***	***62→***	***63→***	***64→***	***66→***	***0***

Figure A1. An optimal route of the JOBPR-AL with 8 items.

15	8	5	30	45	2	2	60	75	5	12	90
14	11	8	29	44	6	2	59	74	3	10	89
13	7	1	28	43	4	11	58	73	1	1	88
12	11	10	27	42	6*	5*	57*	72	12	3	87
11	1	8	26	41	4	9	56	71*	11*	4	86
10	4	3	25	40	10	2	55	70	4	9*	85*
9	3	10	24	39	5	2*	54*	69	12*	8	84
8	4	5	23	38	4	7	53	68	4	12	83
7	2	4	22	37	10	1	52	67*	8*	4	82
6	11	9	21	36	9	9	51	66	4	3	81
5	10	9	20	35	4*	1*	50*	65	12	2	80
4	2	10	19	34	8	7	49	64	12	3	79
3	3	3	18	33	1	3*	48*	63	7	10*	78*
2	6	9	17	32	8	5	47	62*	7*	9	77
1	8	10	16	31	12	8	46	61	5	1	76

0

Order 1(cluster 1)	1	2	3	4	5	6
Order 2(cluster 2)	7	8	9	10	11	12

Route 1*	0→	48→	50→	54→	57→	42→	35→	0
Route 2*	0→	67→	69→	71→	85→	78→	62→	0

Figure A2. An optimal route of the JOBPR-AL with 12 items.

15	13	9	30	45	5	8	60	75	7	16	90
14	10	9	29	44	1	4	59	74	16	6	89
13	8	8	28	43	2	16	58	73	8	3	88
12	11	8	27	42	5	2	57	72	5	1	87
11	6	14	26	41	14	9	56	71	14	16	86
10	3	13	25	40	15	9	55	70	8	8	85
9	1	12	24	39	6	10	54	69	10	12	84
8	2	4	23	38	5	5	53	68	9*	1	83
7	1	13	22	37	6	5	52	67	5	6*	82*
6	4	16	21	36	10	7	51	66	16*	10*	81*
5	12	3	20	35	6	8	50	65	1	1*	80*
4	6	12	19	34	4	3*	49*	64	14*	2*	79*
3	12	4	18	33	9	10	48	63	13*	7*	78*
2	13	13	17	32	4	4*	47*	62	5*	8*	77*
1	1	11*	16*	31	5	8	46	61	12*	15*	76*

0

Order 1(cluster 1)	1	2	3	4	5	6	7	8
Order 2(cluster 2)	9	10	11	12	13	14	15	16

Route 1*	0→	47→	49→	78→	80→	82→	79→	77→	62→	0
Route 2*	0→	16→	64→	66→	81→	68→	63→	76→	61→	0

Figure A3. An optimal route of the JOBPR-AL with 16 items.