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A novel framework to classify opinion dynamics of mobile agents under the bounded confidence model

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Abstract

The formation and evolution of public opinion have been widely studied to understand how consensus forms due to atomic interactions between individuals. While many studies have paid attention to modelling influence and interaction, most of the literature assumes static agents, ignoring the frequent changes in physical locations expected in real-life. This feature naturally allows humans to interact with diverse people and avoid disagreement, which heavily impacts the co-evolution of opinions, communities or isolation in human societies.
Our previous work proposed an extension of the bounded confidence model inspired by the theories of homophily and cognitive dissonance, which concern humans' natural behaviours of attraction and disagreement. Although this demonstrated a marked difference to a static opinion model and purely random mobility, the limited experiments gave little insight into the causes or the resulting structures of consensus.
This paper addresses these shortcomings through a thorough investigation of the impact of mobility modelled by different mechanisms. Through extensive simulation, we observe a transition from multiple stable opinion clusters to complete consensus and a shift from a geographically-based organisation to isolated structure-less agents. Lastly, we propose a novel classification of the different outcomes of self-organisation in opinion models, highlighting the patterns of emerging behaviours across the spectrum of interaction range and influence parameters.

Keywords

Opinion, communities, mobility, self-organisation, agent-based modelling, homophily, cognitive dissonance

Introduction

Public opinion is one of the main factors that drives the formation of communities among humans. These opinions are formed through the interactions that we undergo with peers within our immediate proximity. Extensive work has been undertaken to develop this field of opinion modelling, as found across several surveys by Castellano et al. (2009); Xia et al. (2011); Abid et al. (2018).

In sociology, empirical evidence highlights geographical proximity as an indicator of increased interactions between peers (Latané 1981; Lambiotte et al. 2008). The theory of propinquity supports this, stating that physical proximity and frequency of regular encounters raises the chance of friendship or romantic relationships (Festinger et al. 1950).

Furthermore, Latané et al. (1995) studied proximity impact with empirical data and concluded that the average number of interactions people find noteworthy or *memorable* is proportional to the inverse of the distance at which individuals live. Other researchers have tried to increase the chances of meeting other attendees in a conference to expand their social network (Chin et al. 2012). They used proximity and homophily in order to recommend a new contact. Cho et al. (2011) have shown that social relationships can explain 10-30% of the human movement while periodic or pattern movement explains more than 50%. Also, Monge and Kirste (1980) noted the fact that proximity is dynamic and the distance fluctuates with time due to peoples movement over time.

In fact, similarities and differences in opinion or culture have been highlighted as drivers that cause people to change their location (Castles 2002; Motyl et al. 2014). At a more general level, studies have shown how the preference of people to be co-located with neighbours holding similar ideologies to co-location can lead to segregation, as they move out of a certain community or neighbourhood to a more similar one (Schelling 1971).

When it comes to modelling assumptions, opinion models have often been criticised for neglecting a number of realistic features of social interaction, perhaps most significantly at a local level, the possibility for agents to actually move in physical space, in relation to others (Sobkowicz (2009); Schweitzer and Hołyst (2000); Castellano et al. (2009); Xia et al. (2011)). Given the substantial social psychological research (Latané 1981; Lambiotte et al. 2008; Latané et al. 1995) on the relationship between impact and distance (i.e., the proximity-influence relationship), it is surprising that there is limited research on opinion evolution in settings where personal mobility influenced by psychology is included as an explicit feature. Indeed, most research in this area is conducted on static settings where the locations of agents are not dynamic (Castellano et al. (2009)). Therefore,

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studying the impact of proximity in continuous space instead of discrete space can raise interesting findings.

One of the most well-studied opinion models is the Bounded Confidence (BC) models proposed in Krause (1997); Deffuant et al. (2000); Hegselmann et al. (2002) using a threshold for influence. Hegselmann et al. (2002) proposes a model with a simultaneous update rather than the pairwise sequential interactions implemented in Deffuant et al. (2000). The need to calculate opinion averages of large groups of agents makes computer simulations of the Hegselmann et al. (2002) model rather lengthy as compared to Deffuant et al. (2000) model (Castellano et al. (2009)).

In this paper we use the Deffuant-Weisbuch BC model (Deffuant et al. 2000), here denoted as *DW*. Our focus is not the opinion model itself but the self-organisation emerging from the impact of different mobility mechanisms. This helps understand how communities emerge and identifies the main drivers that could form or break communities. Therefore, for simplicity and faster computation we applied the DW model, as if agents are having a face-to face meeting or a one-on-one interaction.

Despite the wide range of different approaches in the literature, very few take into account the fundamental principle of human mobility (Sobkowicz (2009); Schweitzer and Hołyst (2000); Castellano et al. (2009); Xia et al. (2011); Gracia-Lázaro et al. (2009)). Alraddadi et al. (2020) proposed the addition of a new mobility mechanism into the bounded confidence model, which included attractive and repulsive forces between agents based on their agreement/disagreement, inspired by psychological theories. This model can be considered a generalisation of the filter bubble problem (Nguyen et al. 2014), where information is usually shared within a group and without external influences. This makes it hard for opinions to change and can support misinformation spreading. This was supported by results which showed a greater likelihood that clusters of distinct opinions would survive.

Related work

Opinion models that include mobility explicitly have typically applied uniformly random changes in location without considering any direction of movement (Centola et al. (2007); Gracia-Lázaro et al. (2009); Kozma and Barrat (2008); Qiang et al. (2008); Schelling (1971)). Furthermore, they commonly are based on analogies to moving particles, typically ignoring the psychological triggers that lead to movement (Sousa et al. (2008); Zhang et al. (2018); Ree (2011); Martins (2008b); Galam et al. (1998)).

Our previous work (Alraddadi et al. (2020)) demonstrated that including random mobility triggered by differences of opinion (i.e. where agents move to a random nearby location whenever they have an interaction in which they disagree with their peer) produces very similar results to the DW global interaction method (i.e. convergence to a very limited number of opinions across the population). We suggested that the similarity exhibited by a purely random model could explain why incorporating mobile agents has not been widely studied in opinion models, because results don't produce any significant impact on opinion dynamics. The focus of Alraddadi et al. (2020) was to study the overall impact of a more complex model of mobility that reflected psychological theories of homophily (agents move closer to those with similar opinions) and cognitive dissonance (agents move away from those they disagree with). It is worth noting that while cognitive dissonance generally represents the psychological distress felt through misalignment of beliefs and actions, we are here focusing on its direct reflection on the social network of individuals, when actors holding strong ties act in a manner that is difficult to reconcile with their respective ideological positions. This form of dissonance is known in the literature as *vicarious dissonance* (Norton et al. 2003) and motivates individuals to restore consonance by, for example, changing the ties and relationships that they hold with others.

However, the range of experiments considered was somewhat limited, failing to consider the individual mechanisms that caused these differences, and whether changes in location or opinion are driving convergence. To address these shortcomings, in this paper we decompose the mobility model into distinct components in order to study their individual impact on the agent's organisation, allowing us to use extensive simulation to propose a novel classification that identifies a small number of different outcomes that may occur. We also propose additional metrics that explicitly consider agents convergence in location in addition to convergence in opinion. This detailed study of mobility components and wide parameter spectrum can help identify the drivers for certain patterns of behaviours.

Mobility is defined in many different ways across the literature. The simplest approaches locate agents at discrete locations on a lattice, with mobility either changing the locations of individuals (Schelling (1971); Schweitzer and Hołyst (2000); Galam et al. (1998); Sousa et al. (2008); Pfau et al. (2013); Zhang et al. (2018); Gracia-Lázaro et al. (2009); Hamann (2018); Ree (2011); Qiang et al. (2008)) or swapping places occupied by pairs of agents (Martins (2008a)). Other works refer to mobility in terms of allowing an agent to interact with a far away agent, even though neither agent will actually change their location (Guo et al. (2015)). A distinctive approach is presented in Feliciani et al. (2017); Martínez et al. (2015), which applies mobility on a toroidal grid. However in Feliciani et al. (2017) opinion and location don't co-evolve, as the agents first move to organise themselves, after which a period of opinion dynamics begins.

Table 1 summaries the major opinion models that include any type of explicit *mobility* or change in their *network structure*, allowing us to draw parallels between different approaches and highlight omissions. In these dynamic networks, agents change their links with others depending on the policy or reaction upon the model. To extend the depth of mobility models we have included a small number of dynamic networks for comparison. Finally, although the model does not study opinion dynamics, we include the work of Schelling (1971) due to the use of disagreement/cultural difference as a trigger to movement.

In the table, we categorise the details of the models, including the *mobility trigger*, which describes the reason or stimulus that causes an agent to take action. Some models perform movement after disagreement is encountered while others randomly move at each time step. When an action is taken, the *mobility dynamics* describes the mechanism that determines the location for the next movement. Following

Table 1. Opinion models with incorporated mobility - N/A denotes feature not considered.

Reference	Opinion model	Environment	Interaction	Trigger	Dynamic	Inspiration
Alraddadi et al.	BC	Free space	Neighbour	Agreement and disagreement	Move closer or away from a peer	Homophily (McPherson et al.) and Cognitive Dissonance (Festinger)
Centola et al.	(Axelrod)	Lattice	Neighbour	Disagreement	Augment neighborhood	N/A
Galam et al.	Voting (Galam)	Lattice	Neighbour	Random	Move (to unoccupied)	Reaction-diffusion automata (Chopard and Droz)
Gargiulo and Huet	BC	Network	Local and external	Disagreement	Re-linking	Cognitive Dissonance (Festinger)
Gracia-Lázaro et al.	(Axelrod)	Lattice	Neighbour	Disagreement	Moving (to unoccupied)	Intolerence (Schelling)
Guo et al.	Majority rule	Network (small world) (Newman and Watts)	Local and global	N/A	N/A	Levy flights (Gonzalez et al.)
Hamann	(Galam) and (Hamann)	Free space	Neighbour	Random	Move	Swarms
Holme and Newman	Voter (Clifford and Sudbury; Holley and Liggett)	Network (random)	Neighbour	Random	Re-linking	N/A
Kozma and Barrat	BC	Network (random)	Neighbour	Disagreement	Re-linking	N/A
Martins	Voter (Clifford and Sudbury; Holley and Liggett)	Lattice and network (small world)	Neighbour	Random	Swap	N/A
Pfau et al.	(Axelrod)	Lattice	Distance/link strength	Disagreement	Move (to unoccupied)	(Castles)
Qiang et al.	BC	Lattice and network (scale free)	Neighbour	Disagreement	Move (to unoccupied)	N/A
Ree	BC	Lattice	Neighbour	Random	Move	N/A
Schelling	N/A	Lattice	Neighbour	Disagreement	Move (to unoccupied)	Discrimination
Schweitzer and Hołyst	Social Impact Theory (Nowak et al.; Lewenstein et al.)	2D spatial structure	Social distance	Agreement and disagreement	Move	Brownian particles, Langevin equations
Sousa et al.	(Sznajd-Weron and Sznajd)	Lattice (various)	Neighbour	Random	Move (to unoccupied)/swap	Lattice gas (Ausloos et al.)
Zhang et al.	BC	Lattice	Neighbour	Random	Move (to unoccupied)	N/A

this, we describe the *inspiration* behind the dynamics, which are more frequently based on physical phenomena rather than psychological concepts.

A majority of works apply randomness in their decision making, either in their *mobility trigger* or their *mobility dynamics*. There are also some models that are triggered by disagreement. However, the agent's choice of location does not reflect the disagreement but this is simply randomly chosen, with the exception of Gargiulo and Huet (2010); Pfau et al. (2013). This has inspired the inclusion of repelling forces in our proposed model, *triggered* based on disagreement to move directly away from the peer, following the principle of *cognitive dissonance* (Festinger (1957)). Similarly, our model includes an attractive force which captures the concept of *homophily* (McPherson et al. (2001)). Therefore, we find it important to explore the area and investigate the dynamics of the mobility forces that would explain the emergence of opinions and communities. Specifically, the model captures the following ideas: 1. It explores the impact of physical but directed mobility (i.e.relocation) on opinion evolution instead of random mobility. 2. It enables analysis of the impact of distance (instead of explicit or fixed links) on interactions and community formation, which captures features not considered in previous studies. 3. It helps to identify different scenarios of self-organisation that may emerge.

This paper will discuss the results of the models and then present a classification summary of the distinct outcomes. The results section is broken down to initially discuss the impact of mobility compared to the static model. From that section, we chose random mobility as a benchmark for the rest of the paper. Following that, we discuss the repulsive and attractive components of directed mobility models with an analysis of quantitative results. Then, the outcomes of the directed mobility models are classified into common outcomes from the perspective of each metric individually. Finally, the results across the metrics are synthesised and combined to produce an overall classification diagram that describes the limited number of possible outcomes from the model in a number of scenarios.

Contribution

The related work section highlights the different features of the most noteworthy mobility mechanisms published in the literature, including our previous contribution (Alraddadi et al. 2020). Of particular note is the focus on either random mobility or the use of disagreement as a trigger for movement, with very little consideration on homophilous/attractive movement when agents are in agreement. This triggers the research question for this paper: *How do the individual components of mobility impact the co-evolution of opinion in terms of structure and self-organisation?* For example, is emergent behaviour purely a result of disagreement, or do some outcomes result from agreement? To address this, we use (Alraddadi et al. (2020)) as a starting point, which models mobility by taking into account human psychological behaviour, where individuals have control over their social structures. However, in this paper, we go further by decomposing the mobility model into distinct attractive and repulsive components, allowing in-depth analysis of the self-organised outcomes that may emerge. In contrast to many works, we do not impose a fixed number of communities, or force agents to belong to a community, but instead measure directly the emergent behaviour. Note that many papers consider a dynamic network model (e.g. based on lattices with rewiring), however we note that this forces an explicit binary structure on the potential for interactions that may hide more realistic behaviour. For example, Latané et al. (1995) shows that geographical distance is a factor for the people we choose to interact with frequently. This motivates our use of a more general 2D spatial setting.

To perform this study, we consider the Hybrid Model (HM) proposed in Alraddadi et al. (2020) (where it was termed *homophilous*), which combines both the attract and repel mobility components. The original study found that when restricting the interaction range, HM naturally acts as a driver to stimulate more opinion clusters. In this paper, we study the attract and repel mobility components in isolation, while exploring the parameter space of influence and interaction range.

We analyse the agent's organisation at both the micro and macro level and return quantitative results, where macro-levels describe the status of the entire population and micro-levels demonstrate an individual's inner circle or local area. To study forms of organisation, we consider the presence of isolated agents whom are without a group membership and isolated either geographically or in opinion. In this paper, these isolated agent's are referred to as loners. Loners are used as an indicator for the overall (dis)organisation in the system.

For rigour and consistent results, we propose a metric that can capture stability in movement, whereas previous works typically only consider convergence of opinion. These are a novel addition to the field that allows new insight into the structures that emerge among agents. This will highlight the conditions and mechanisms that stimulate complete consensus or allow diversity to persist. In particular, attractive mobility stimulates multiple opinion clusters under a large influence spectrum (ϵ). This highlights that opinion clusters emerge with larger influence spectrum.

The final contribution is a novel classification of six types of behaviours that describe the emergence of self-organisation. This highlights the formation of communities, isolated agents and tolerance between communities within proximity. This classification provides a clear understanding of the transition in behaviour or in particular, self-organisation. These various findings highlight the significance of mobility in naturalistic settings and emphasise the importance of applying mobility under the inspiration of psychological theories to resemble more human reactions. This classification provides a basis for assessing mobility models and sets a benchmark that can be carried forward and followed against other models.

The model

Model framework

In this paper we propose a model for the co-evolution of opinion and location. We consider a population of agents, $A = \{a_1, \ldots, a_n\}$, where each agent a_i is defined by a location $xy_i = (x_i, y_i)$ and opinion $op_i \in [0, 1]$.

Following the DW opinion model (Deffuant et al. 2000), a pair of agents a_i, a_j will interact if and only if their respective opinions (op_i, op_j) are within an opinion threshold ϵ, where μ is a global parameter controlling the effect of a peer's opinion (termed convergence rate in the original model).

In the DW model an agent can interact with any other agent in the population regardless of its location. In our case, the selection of an agent depends on the distance between them, thus only allowing interactions between agents that are close not only in opinion but also location. Let $d(i, j)$ denote the Euclidean distance between agents a_i and a_j, and let $N(xy_i, r_s) = \{a_j \in A - \{a_i\} : d(i, j) \leq r_s\}$ be the set of agents that are at most distance r_s from agent a_i. For each *time step* we select an *inviting agent* a_i at random from the population A, and select a peer at random from $N(xy_i, r_s) \neq \emptyset$. However, if no neighbors are found then nothing changes to the properties of a_i.

At the end of an interaction, the agent's details are updated, both the opinions of a_i and a_j and the location of a_i (instigator). Movement is applied to a_i with probability p. A range of p has been studied in (Alraddadi et al. 2020) and it mainly has impact on the convergence. For the purpose of this paper we set $p = 1$ for mobility and $p = 0$ for static agents where there is no movement nor change in location. A parameter λ indicates the range of the step movement between a distanced pair of agents. It is applied to control the scale of movement of a_i towards a_j, with $\lambda = 0$ leading to no movement and $\lambda = 1$ denoting that a_i moves directly to the same exact location as a_j. In this paper the experiments are fixed to $\lambda = 0.6$ (based on previous experiments from Alraddadi et al. (2020)).

Incorporating mobility

The mobility models proposed reflect on how the agents may potentially respond in physical space towards a peer. Firstly, we demonstrate a *pure* random mobility model. This concept has been previously considered where agents have continuous change of their sites or links in (Sousa et al. 2008; Zhang et al. 2018; Ree 2011; Hamann 2018; Martins 2008b; Galam et al. 1998). Secondly, we propose two mobility models that incorporate two distinct dynamic processes inspired by human behaviour: *attractive* and *repulsive* mobility. The former movement is inspired by the theory of homophily McPherson et al. (2001), where agents are *attracted* to move closer to peers that share similar opinions. Repulsive is inspired by cognitive dissonance theory (Festinger (1957)), where agents are *repelled* by those that hold significantly different opinions. Homophily is one of the most common theories that is used in the modelling of mechanisms for interaction (e.g.Axelrod (1997); Deffuant et al. (2000); Gargiulo and Gandica (2016); Holme and Newman (2006)). Cognitive dissonance has been used to translate change in an agent's location or links (Gargiulo and Huet 2010; Gracia-Lázaro et al. 2009; Schelling 1971) (shown in Table 1). Cognitive dissonance represents a repulsive force that motivates a move away from the source agent(s). We consider a simple directed movement to reflect a co-evolution that combines both opinion dynamics and structure in a geographical space.

In this paper, we will further compare the original model (HM) in Alraddadi et al. (2020) to both the attract and repel mobility components individually. In Kossinets and Watts (2009) the authors state that mobility is constrained geographically by the distance an individual can travel within a day. Therefore, we conduct further experiments to study the impact of the distance moved. In addition, we investigate a full range of $\epsilon \in [0, 1]$ rather than stopping at $\epsilon = 0.5$. However, our results show consistency with (Fortunato 2004) in terms of when we observe differences in behaviour, hence we only show results for $0.1 \leq \epsilon \leq 0.5$. Below we present the alternative mobility models used in our experimentation.

Pure Random Mobility Pure Random Mobility (PRM) is applied after every interaction, with the inviting agent a_i moving to a random location in their local area regardless of the relative opinions of the pair (Algorithm 1).

Algorithm 1 Pure Random Mobility (PRM)

```
function PRM(a_i, a_j, ε, λ)                    ▷ Move
    r ← r_s λ √U(0,1)
    θ ← 2πU(0,1)
    return (x_i + r cos θ, y_i + r sin θ)
end function
```

Hybrid Mobility The Hybrid Model (HM) combines both the attract and repel mobility, incorporating forces aligned to both homophily and cognitive dissonance. Agent a_i moves closer to their peer a_j if they are close in opinion, and further away in the opposite direction if they differ (Algorithm 2).

Attractive Mobility Under Attractive Mobility (AM), after an inviting agent a_i interacts with a random neighbour a_j, it will move closer if their opinions are similar (Algorithm 3).

Algorithm 2 Hybrid Mobility - HM

```
function HM(a_i, a_j, ε, λ)
    if |op_i − op_j| ≤ ε then
        xy'_i ← xy_i + λ(xy_j − xy_i)           ▷ a_i attracted to similar peer
    else
        xy'_i ← xy_i − λ(xy_j − xy_i)           ▷ a_i repelled from different peer
    end if
    return xy'_i
end function
```

Algorithm 3 Attractive Mobility (AM)

```
function AM(a_i, a_j, ε, λ)
    if |op_i − op_j| ≤ ε then
        return xy_i + λ(xy_j − xy_i)            ▷ Move closer
    else
        return xy_i                             ▷ Don't move
    end if
end function
```

Repulsive Mobility Repulsive Mobility (RM) model is triggered by disagreement, and as a consequence the inviting agent a_i will physically move away in exactly the opposite direction (Algorithm 4).

Algorithm 4 Repulsive Mobility (RM)

```
function RM(a_i, a_j, ε, λ)
    if |op_i − op_j| ≤ ε then
        return xy_i                             ▷ Don't move
    else
        return xy_i − λ(xy_j − xy_i)            ▷ Move away
    end if
end function
```

Method

Algorithm 5 presents the overall framework for simulation, which is dependent on the input parameters described in Table 2. *Social interaction* is the process of selecting an agent to communicate with (here we use proximity as a factor similarly to Latané et al. (1995)). *Social influence* is the process in which individuals change their opinion, revise their beliefs, or change their behaviour as a consequence of an interaction (Moussaïd et al. (2013)). We consider two subsequent phases of social influence with *opinion influence* describing the change in opinion and *mobility Influence* reflecting the agent's response to this opinion change by choosing their preferred location. The type of movement is determined by the MOVE() function, where MOVE can be either the PRM, HM, AM and RM algorithms specified above. Finally, the agents properties are *Updated* for both their opinion and location accordingly.

We consider three evaluation metrics to assess self-organisation among agents. *Convergence* focuses on stability, measuring how many iterations pass before changes in opinion or location become trivially small. *Clustering* captures similar groups that emerge in opinion and/or location at a macro-level in the population. Finally, *tolerance* considers

Table 2. Input parameters.

Parameter	Description	Value
$L \times L$	Region size	10×10
n	Number of agents	100
$limit$	Number of time steps per run	70,000
ϵ	Opinion threshold	$[0.1, 0.2 \ldots, 1]$
μ	Convergence rate	0.5
r_s	Interactive radius	$[2, 3, 5, 10]$
p	Probability of movement	$[0, 1]$
δ_{op}	Opinion change threshold	0.01
δ_{mov}	Movement distance change threshold	1
N_F	Number of time steps without opinion change	10000

Algorithm 5 Simulation framework

Require: Input $(n, limit, r_s, \epsilon, p, \mu, \lambda, \delta_{op}, \delta_{mov}, N_F, MOVE)$
  for $i \leftarrow 1$ to n **do** ▷ Randomly create population A
    $op_i \leftarrow U(0, 1)$
    $xy_i \leftarrow (U(0, L), U(0, L))$
  end for
  for $limit$ time steps **do**
    $a_i \in A$ ▷ Social interaction
    $a_j \in N(xy_i, r_s)$
    ▷ Opinion influence (Deffuant et al. 2000)
    if $|op_i - op_j| < \epsilon$ **then**
      $op'_i \leftarrow op_i + \mu(op_j - op_i)$
      $op'_j \leftarrow op_j + \mu(op_i - op_j)$
    else
      $op'_i \leftarrow op_i$
      $op'_j \leftarrow op_j$
    end if
    if $U(0, 1) < p$ **then**
      $xy'_i \leftarrow$ MOVE$(a_i, a_j, \epsilon, \lambda)$ ▷ Mobility influence
    else
      $xy'_i \leftarrow xy_i$ ▷ Don't move
    end if
    $op_i \leftarrow op'_i$; $op_j \leftarrow op'_j$ ▷ Update
    $xy_i \leftarrow xy'_i$
  end for

the diversity of opinion at a micro-level within the local area of each agent.

Convergence The **opinion convergence time** of a simulation run is defined as the lowest value of t such that no agent changes their opinion by more than δ_{op} between iteration t and $t + N_F$ (with N_F and δ_{op} set as input parameters). Similarly, the **movement convergence time** is the lowest value of t such that no agent changes location by a distance greater than δ_{mov} between iteration t and $t + N_F$ (with δ_{mov} an input parameter). For both measures, a convergence time of $limit$ denotes that the system did not stabilise.

Clusters The Density-based Spatial Clustering of Applications with Noise (DBSCAN) algorithm (Ester et al. 1996) is used to identify clusters of agents that are similar in opinion and/or location once the system has converged. Only non-trivial clusters containing at least 5 agents are considered, with all other agents being termed *loners* (with the number of these denoted by N_{loners}).

Clusters based on **opinion** are identified by considering the distance between two agents a_i and a_j to be $|op_i - op_j|$, and setting the DBSCAN distance threshold to be δ_{op}, being the maximum distance between a pair of agents for which they are classified into a the same cluster. To identify **communities** that are close in both opinion and location, the distance between agents a_i and a_j is defined as:

$$\begin{array}{ll} d(xy_i, xy_j) & \text{if } |op_i - op_j| < \delta_{op} \\ \infty & \text{otherwise} \end{array}$$

The threshold δ_{mov} is then used as an input to the DBSCAN algorithm to identify clusters that are both geographically close and similar in opinion.

Tolerance Agents within δ_{mov} are considered to be in the same local area, and δ_{mov} is specifically used to evaluate the geographical structure between the agents. For an agent a_i, the set of agents sharing the same local area is given by $N(xy_i, \delta_{mov})$, and we define N' to denote the subset of these agents that hold a different opinion, where:

$$N'(xy_i, \delta_{mov}) = \{j \in N(xy_i, \delta_{mov}) : |op_j - op_i| > \delta_{op}\}$$

The **tolerance** of an agent a_i, denoted by $tol(a_i)$, is defined as the proportion of their local peers that hold a different opinion (at the end of a simulation), where:

$$tol(a_i) = \begin{cases} \dfrac{|N'(xy_i, \delta_{mov})|}{|N(xy_i, \delta_{mov})|} & \text{if } |N(xy_i, \delta_{mov})| > 0 \\ 0 & \text{otherwise} \end{cases}$$

The tolerance of a population A is denoted $tol(A)$ and defined as the mean tolerance of all agents $a_i \in A$:

$$tol(A) = \frac{1}{|A|} \sum_{a_i \in A} tol(a_i)$$

Results

Experiments are presented for a population of $n = 100$ agents located in a 10×10 bounded 2D space. The initial position and opinion of each agent are set uniformly randomly to $xy_i = (U(0, 10), U(0, 10))$ and $op_i = U(0, 1)$ respectively. When an agent's mobility would take it beyond the confines of the region, it bounces back to remain within boundaries.

A general property (Deffuant et al. 2000) of the DW model is that when ϵ is large the system reaches *complete consensus* with a single dominant opinion. Results from the literature shows that $\epsilon \geq 0.5$ will always result in complete consensus (Fortunato 2004) regardless of the topology. However, the critical threshold of where complete consensus is found is different between the models depending on the different additional rules. Qiang et al. (2008) demonstrated that, with mobility under scale free networks, complete consensus is obtained with a smaller ϵ threshold than in a lattice. Kozma and Barrat (2008) showed that a larger ϵ is necessary under an adaptive network compared to a static network, stating that rewiring favors formation of different clusters. Gargiulo and Huet (2010) states that under a smaller ϵ complete consensus can be found in a dynamic network, explaining that the group dynamics has the effect of removing those individuals that usually occur between $0.26 < \epsilon < 0.5$. However, with lower opinion thresholds, more opinion clusters are likely to emerge.

To address this wide variation, in this paper we include a thorough investigation of the parameter space of opinion threshold (ϵ) and interactive radius (r_s). Note that when the interactive radius is equal to one ($r_s = 1$) then the average number of neighbors typically consists of only two agents, therefore we restrict our attention to $r_s \geq 2$, where neighbourhoods of at least 10 agents are typical. Each simulation run has a maximum of $limit = 70,000$ iterations and all results presented are averaged over 20 independent simulation runs with different random seeds. Other simulation parameters are listed in Table 2.

Static agents or random mobile agents

Most of the literature either employs static agents or only considers random mobility, often without a stimulus to trigger movement. Therefore, we first compare static and randomly mobile agents (given by Algorithm 5, where MOVE=PRM) before introducing models of directed mobility. For the static case, we apply the DW model (by setting $p = 0$ in Algorithm 5 to prevent movement) but vary the interactive radius r_s to restrict the locality of interactions. The red box in Figure 1a and 1c denotes the mimicked DW model under large interaction range $r_s = L$ (shown in the red box).

Figure 1a shows that the number of opinion clusters increases when interactions are restricted through distance ($r_s < 3$) and/or opinion ($\epsilon < 0.4$). This is similar to (Kozma and Barrat 2008) and (Castellano et al. 2009), which stated that consensus forms around a larger number of opinions under the restricted interaction of a static model. However, their results show that communities do not form in large numbers, with most agents being loners (Figure 1c), showing that similar opinions are spread out across the region. In contrast, a single opinion dominates for $\epsilon > 0.3$, irrespective of r_s.

Under PRM (Figure 1b), changing r_s has little effect for each ϵ, with the number of opinion clusters similar to the static DW model (Figure 1a, red box). This is in line with Zhang et al. (2018) which incorporated mobility and found opinion consensus for a small interaction range or low probability of movement. Similarly, Sousa et al. (2008) found that random mobility removes small factions of opposing opinions so that all agents reach full consensus.

Other evaluation metrics behave likewise, with values similar under both PRM and the DW model, although restricting interactions does delay opinion convergence. Furthermore, due to the PRM constant mobility convergence in movement is never found. However, both PRM and the DW model display high tolerance when multiple opinions are present (for low ϵ).

To conclude, in the static DW model, where all peers may interact, the spread of an opinion is only dependent on ϵ. In contrast, restricting the interaction range leads to more opinions at a macro-level (Figure 1a), although restricting interaction did not impact random mobility models (Figure 1b) as there is still a high probability that any pair of agents will meet at some point. As a result we find that random mobility leads to the *same number of opinion clusters* as in the static DW model with a global interactive method. Adding random mobility tends to *eliminate the effect* of restricting the static DW model to local interactions and increases the numbers of opinion clusters, thus highlighting it's *insensitivity to* r_s.

Directed mobility

In this section we consider a more sophisticated mobility that models agents reaction to each other's opinion, with the nature of these interactions guiding the direction of movement. Since PRM behaves as the static DW model under a global interactive method, we use PRM as the baseline to compare with the directed mobility models.

Convergence Results show that the speed of convergence in either opinion or in movement is dependant on the opportunity for an agent to interact with their peers (controlled by ϵ and r_s). For each of the mobility models (AM, RM, HM), increasing ϵ reduces the time taken to *converge in opinion*, with convergence found within 2,000 time steps in all cases.

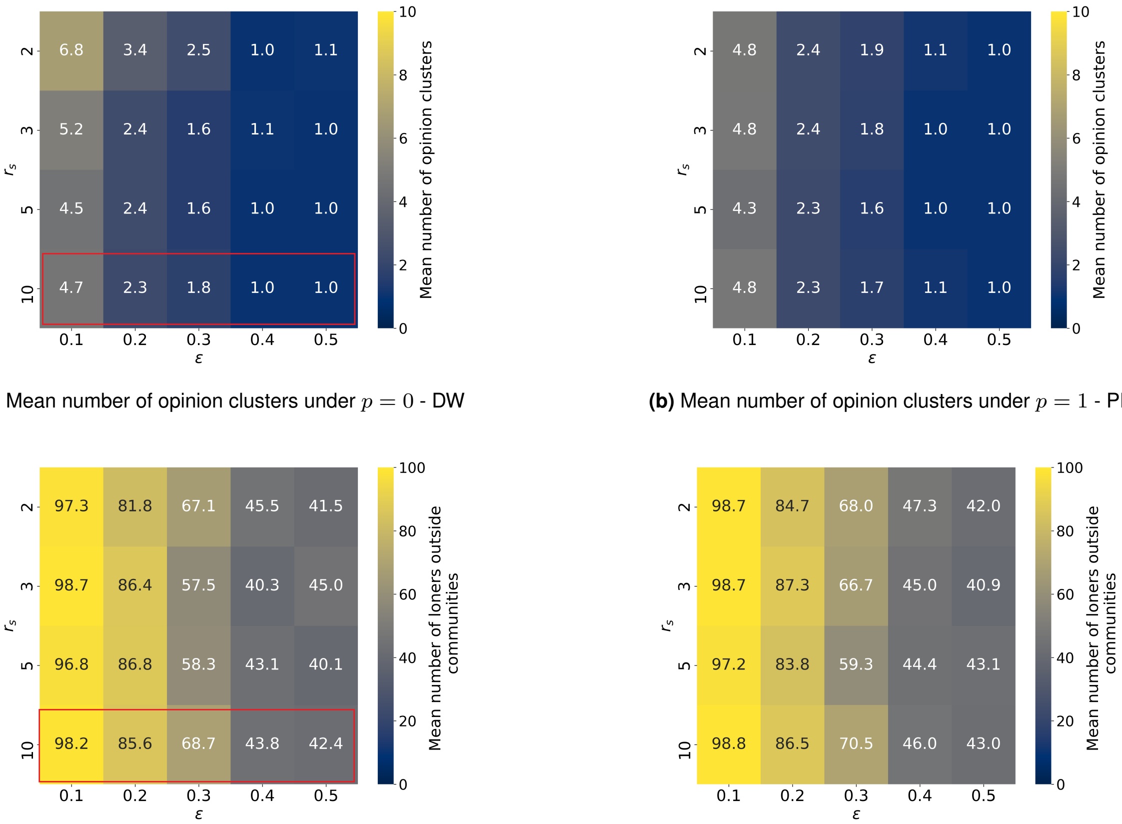
Figure 2 shows convergence in movement for the different mobility models, particularly highlighting that convergence is slow or non-existent under RM and HM for the combination of small ϵ with high r_s. Low ϵ means that agents are less likely to agree and successfully interact with their local peers, while high r_s leads to a lack of local structure as agents move.

Overall, convergence in opinion is typically faster than convergence in movement, suggesting that opinions are usually settled before agents change their location in search for a "content" neighbourhood. For repulsive mobility, large r_s leads to difficulty in finding such a location. In line with Zhang et al. (2018), convergence in opinion gets faster as the interaction range, r_s, increases.

Opinion diversity at a macro-level and micro-level In this section we assess the extent to which different opinions can co-exist under each mobility scheme both at a macro and a micro level within a local area. Figure 3 shows that restricting interactions ($r_s < 5$) stimulates a larger number of opinion clusters than PRM for all other mobility models, with little variation in particular between AM and HM. Diverse opinion clusters persist even when ϵ increases since lower values of r_s restrict the opportunities for agents to interact with diverse peers.

With RM under higher r_s, agents that are repelled expose a new neighbourhood of peers to their opinion, hence this mechanism tends to produce a single opinion cluster. Their common repel mechanism means that both RM and HM have similar tolerance (Figure 3), as neither model is stable when agents with different opinions are in the same neighbourhood. A higher tolerance only occurs for small ϵ, which allows multiple opinions to persist.

AM shows higher tolerance for levels as high as $\epsilon = 0.3$, where agents are less likely to influence their peers opinion, but lack a mechanism to move away.



(a) Mean number of opinion clusters under $p = 0$ - DW

(b) Mean number of opinion clusters under $p = 1$ - PRM

(c) Mean number of loners outside communities under $p = 0$ - DW

(d) Mean number of loners outside communities under $p = 1$ - PRM

Figure 1. Opinion clusters and loners outside communities under static ($p = 0$) and PRM ($p = 1$). Each cell represents an average of 20 simulation runs.

When $r_s \geq 5$, all directed mobility models (AM, RM, HM) result in a similar number of opinion clusters as the maximum number of $\frac{1}{2\ \epsilon}$ clusters that can be produced by DW (Deffuant et al. (2000)). This is because there is a high chance that a pair of agents will be within the interaction range r_s. A similar effect is found in Guo et al. (2015), where agents move around a lattice structure augmented by a small number of global links, and increased mobility results in the formation of a single opinion.

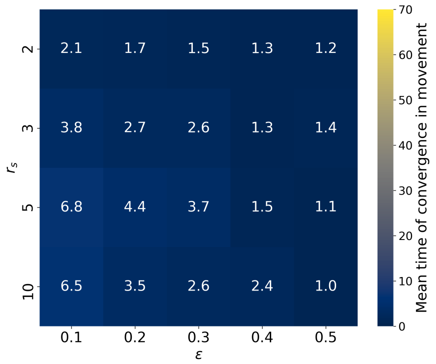
In conclusion, in comparison to purely random movement, we find that directed mobility (AM, RM, HM) results in a larger number of opinion clusters when the interactive radius is restricted, even for large values of ϵ. This result highlights the importance of attraction and repulsion in the mobility model, a feature that is often overlooked in other studies. For example, in a network based approach, Kozma and Barrat (2008) found that fewer clusters emerge with higher frequency of rewiring, when considering only purely random re-linking. We note in particular that random mobility behaves in a similar manner to the DW model.

Community formation In this section we investigate how mobility affects the number of communities that evolve in terms of agent opinions and geographical locations as well as the levels of emerging loners. Other research has investigated the DW model and found that loners are frequently found. Lorenz (2007) reports that outliers exist for structural reasons, and identifies that Deffuant et al. (2000); Weisbuch et al. (2002) chose to ignore clusters with one agent. Lorenz (2007) further highlights that Hegselmann et al. (2002) (group-based interactions) in comparison to the DW model (peer-based interactions) have no minor clusters. Gargiulo and Huet (2010); Kozma and Barrat (2008) demonstrates the DW model while also implementing group-based interactions while permitting rewiring and these outliers disappear, similar to the static Hegselmann et al. (2002) model. Therefore, in this paper we propose a quantifiable measurement of the occurrence of isolated agents to be considered along with the formation of communities.

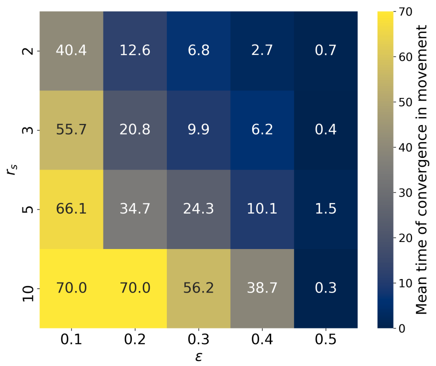
Figure 4 shows that despite the lack of an attractive component, RM with restricted interactions ($\epsilon \leq 0.3$ and $r_s < 5$) encourages the formation of geographic clusters with few loners (mean 17%). For both AM and HM across r_s, self-organisation is easier and able to produce multiple communities with minimum number of loners due the nature of attractive mobility. However, when AM is highly restricted ($\epsilon < 0.2$ and $r_s < 3$), the number of loners increases dramatically, as agents lack opportunity explore new areas with the potential for successful interactions.

For AM and HM, large ϵ and $r_s > 3$ lead to a single opinion, as the attract component minimises the number of loners and encourages agents to gather into a single centered cluster. In contrast, when $r_s \leq 3$, many more communities are detected across the different ϵ levels.

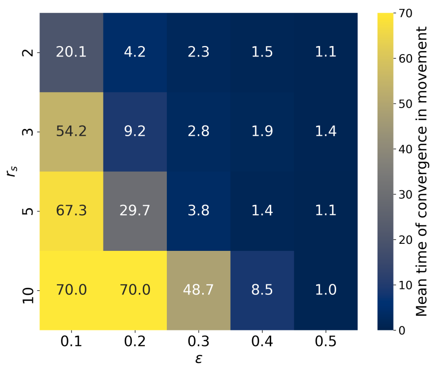
In conclusion, HM has both the repel and attract components and as a result it produces the most uniform



(a) Mean time of convergence in movement under AM for a combination of r_s and ϵ values.



(b) Mean time of convergence in movement under RM for a combination of r_s and ϵ values.



(c) Mean time of convergence in movement under HM for a combination of r_s and ϵ values.

Figure 2. Movement convergence time under $p = 1$, values of 70,000 denote convergence is not found. Each cell represents an average of 20 simulation runs scaled down for representation.

clusters geographically with minimum noise. However, due to the repulsive impact, when there are multiple opinions and the interaction range is up to its full potential, a noisy geographical structure results, because of the constant repelling jumps.

Identifying behaviour change under 2D metric spectrum

In this section we classify the behaviour and outcomes of each evaluation metric as ϵ and r_s vary under the directed mobility models (AM, RM, HM) compared to random mobility (PRM).

For each metric we show an abstract classification diagram illustrating “quadrants” of similar outcomes, and provide an example of a single run for specific ϵ, r_s which exhibits this behaviour.

Opinion clusters Three different outcomes are evident in our experiments, represented graphically in Figure 5:

(i) **A single opinion cluster** emerges that is shared by the entire population.
(ii) **Multiple opinion clusters** are formed across the population, at a similar rate to DW.
(ii) **Exceeds static model** where more opinion clusters are formed than in the static DW model. This corresponds to more than the maximum number of $\frac{1}{2\,\epsilon}$ clusters that can be produced by DW (Deffuant et al. (2000)).

In quadrant QIII (low ϵ, low r_s), all mobility models (AM, RM, HM) produce more heterogeneity in opinion clusters than the static DW. Moreover, for the higher ϵ in quadrant QII, attract forces (AM, HM) are needed to enable multiple opinions to persist, with RM leading to complete consensus.

In quadrants QI and QIV (i.e. larger r_s) all mobility models (AM, RM, HM) result in a similar number of opinion clusters as with the PRM model.

Communities To demonstrate the organisation of the agents in geographical space, we first classify in Figure 6 the potential outcomes of:

(i) **Multiple communities** of agents that are close in both opinion and distance.
(ii) **A single community** of agents holding the same opinion.
(iii) **Undefined** when no coherent communities are formed (with at least 5 members).

A second geographical classification (Figure 7) is based on the presence of loners:

(i) **No structure** where many agents are isolated outside of communities ($N_{Loners} > 30$).
(ii) **Organised** where most of the agents are part of a community, located in the same neighborhood and holding the same opinion ($N_{Loners} \leq 30$).

Mobility models with attract components (AM, HM) display organisation for almost all values of ϵ, r_s. Repulsion only (RM) leads to organisation when interactions are most restricted (QIII), with some exceptional cases for high r_s (likely due to the high volatility that comes with larger movement).

When considered together, Figures 6 and 7 demonstrate the importance of attraction in building structured communities with little noise. AM and HM only differ in the small regions that lack structure. For example, the behaviour of AM in QIII means that agents are not attracted towards like

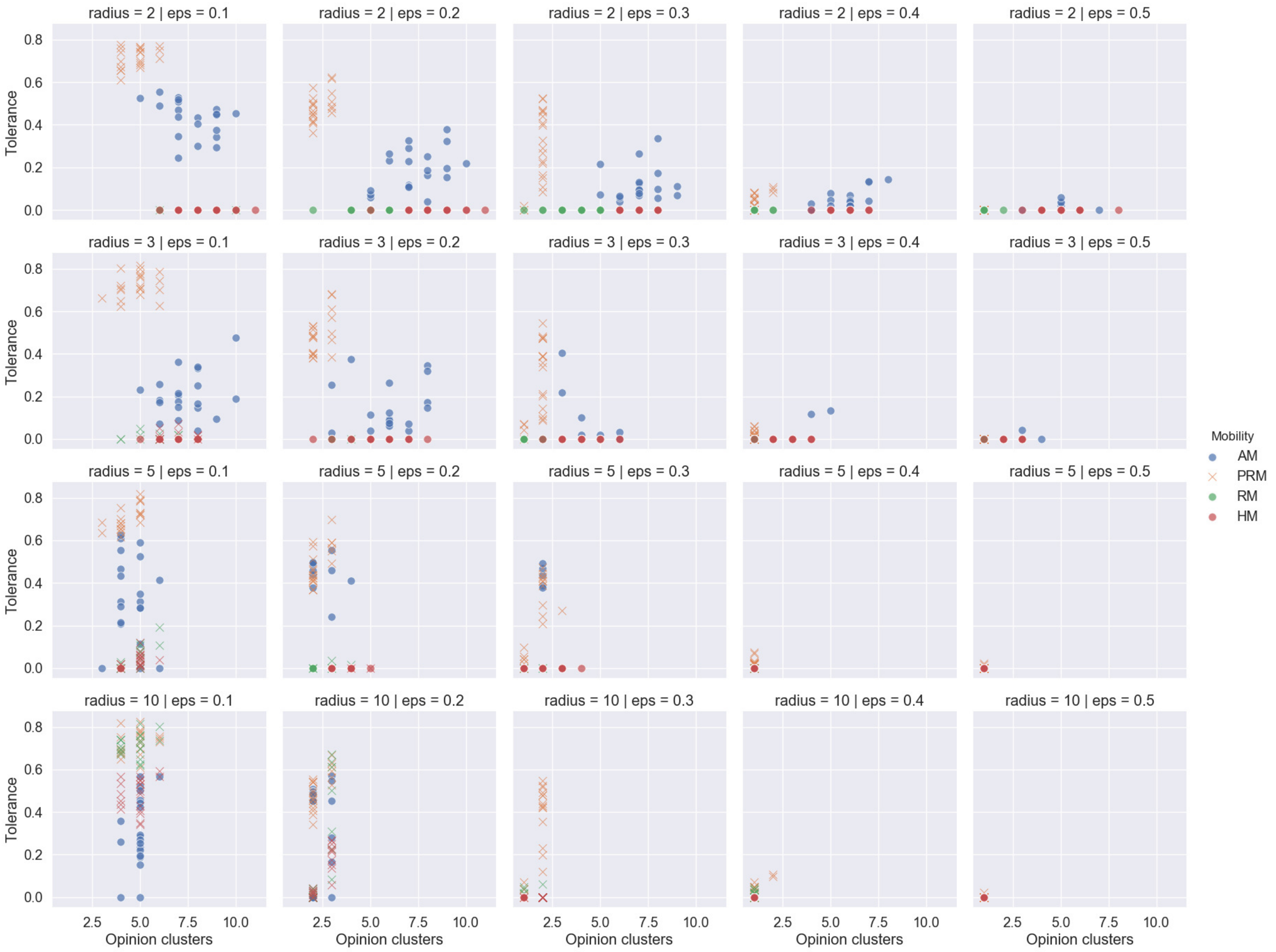


Figure 3. Opinion clusters and tolerance. 20 runs shown for each experiment, $\circ$ denotes the simulation converges in movement, and $\times$ no convergence.

minded agents beyond the immediate region, but also are not influenced by local opinions. In contrast, HM lacks structure in QIV, where small ϵ leads to frequent disagreement, but large r_s results in unstructured movement.

Tolerance in the neighbourhood Figure 8 classifies the two outcomes of tolerance:

(i) **Mixed opinion** where geographical neighbourhoods hold a range of opinions ($tol(A) \geq 0.1$).
(ii) **Homogeneous opinion** where neighbourhoods largely hold the same opinion ($tol(A) < 0.1$).

Models with a repel component (RM and HM) behave similarly in all four quadrants, as agents move away from dissimilar opinions. Although *Mixed opinion* neighborhoods are present in QIV, these are not stable and do not converge in movement (Figure 2b and 2c).

In contrast, AM always converges in movement, but for low ϵ, agents can not convince peers to adapt their opinion, leading to high tolerance and mixed opinions.

Classification of Self-organisation

In this section we synthesise and combine all the results and develop a novel classification diagram that identifies the different self-organised outcomes listed below. As before, we express these in terms of behaviours for each quadrant formed by ϵ and r_s (Figure 9) and provide indicative examples of each outcome in Figure 10.

Outcomes

For a full picture, we outline each outcome and discuss the causes, with Figure 10 showing a representative example from the simulations.

Multiple uniform clusters. Multiple stable opinion clusters emerge that are segregated (more or less evenly) in the geographical space (Figure 10a), with agents located at identical coordinates. This behaviour emerges as a result of the attract component (AM or HM).

Multiple clumped clusters. Multiple opinion clusters emerge that are segregated in the geographical space (Figure 10b), although with agents not exactly co-located. This outcome only occurs with the repel component (low ϵ and r_s), where limited movement and interaction prevents global consensus but allows small pockets to form. The lack of an attractive component prevents agents from co-locating precisely.

Multiple mix clusters. Multiple opinion clusters emerge that overlap within the same neighbourhood but don't interact (Figure 10c). Multiple mixed clusters are only found with the *attractive* mobility via low ϵ, regardless of r_s.

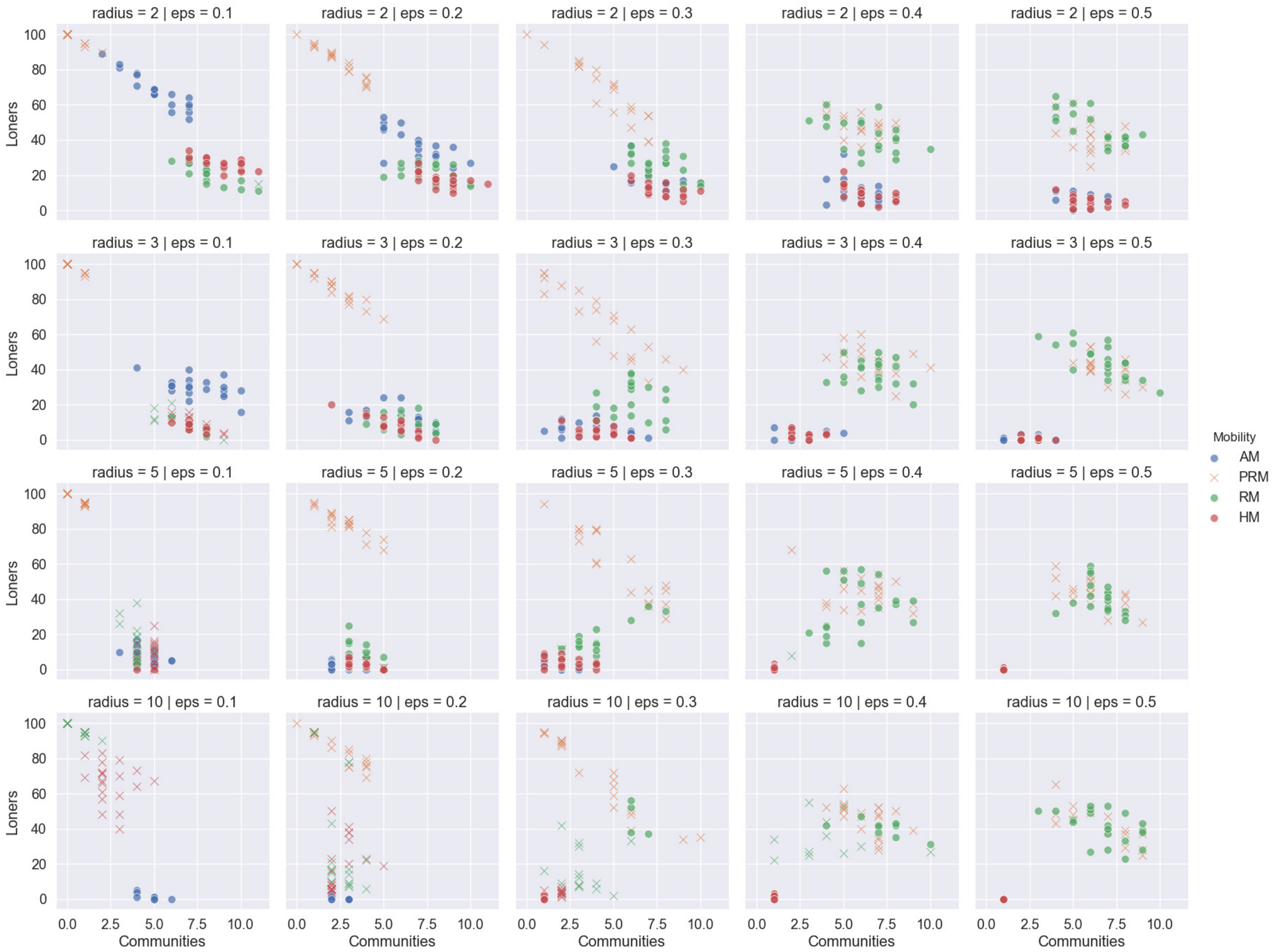


Figure 4. Communities and loners. 20 runs shown for each experiment, $\circ$ denotes the simulation converges in movement, and $\times$ no convergence.

Clusters with the same opinions converge to a single location while unaffected by nearby opinions that are different, often resulting in higher tolerance.

Single uniform cluster. A single opinion cluster is formed at a single location, which dominates the region Figure 10d. This requires large ϵ and r_s together with an attractive component (AM or HM).

Single scattered clusters. A single opinion cluster dominates, however the agents lack structure and are scattered across the region (Figure 10e). This behaviour is present in the RM model under large ϵ for all r_s. Note that this scattered distribution of a single opinion exhibits a similar behaviour to the static DW model.

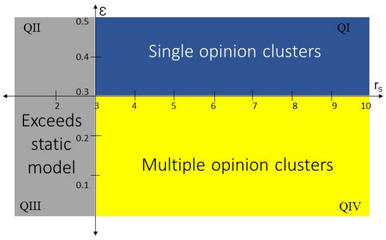
Multiple opinion unstable organisation. This describes the case when multiple opinion clusters are formed (low ϵ), but there is no self-organisation or communities formed (Figure 10f). This is observed when r_s is large with repel component, causing no convergence in movement. The structure is close to random, giving a similar outcome to the static DW model.

Conclusions

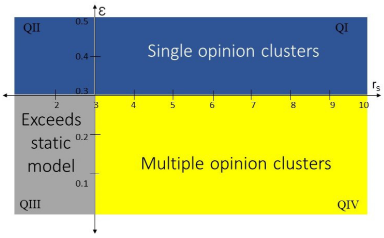
Flache et al. (2017) stated that Axelrod (1997) posed the question of 'why not all differences eventually disappear if social influence reducing differences between people is such a pervasive force in social interaction'? Considering more structured forms of mobility may be one of the explanations for what we see in our every day lives. We have proposed a framework for the co-evolution of opinion and location based on the well-studied bounded confidence model. In contrast to many previous implementations of mobility in opinion dynamics, we consider individual attractive and repulsive components, inspired by psychological theory. Thorough simulation is used to classify the potential outcomes in terms of local clustering and opinions emergence.

Our results show that purely random mobility produces similar outcomes to the corresponding DW model, with random dynamics supporting consensus, in line with the literature. For example, Sousa et al. (2008) show that random mobility makes complete consensus easier to obtain, while in (Qiang et al. 2008) random re-linking and mobility leads to a decrease in the critical value of ϵ for consensus. The similarity with the static case may explain why opinion formation with mobility has received relatively little attention in the literature. In contrast, a more structured movement triggered by agreement/disagreement is able to produce a wider range of distinct outcomes.

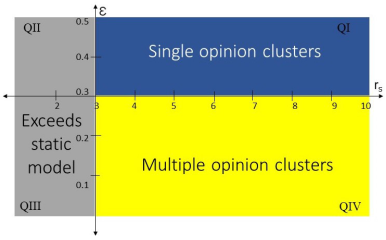
Our *attractive mobility* model is inspired by homophily, which typically leads to heterogeneous opinions at both population and local levels. In contrast, a *repulsive model* based on the principle of cognitive dissonance, frequently



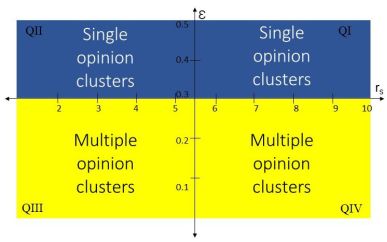
(a) Under AM, intersect at $r_s = 3$ and $\epsilon = 0.3$



(b) Under RM, intersect at $r_s = 3$ and $\epsilon = 0.3$

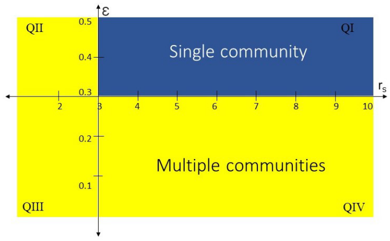


(c) Under HM, intersect at $r_s = 3$ and $\epsilon = 0.3$

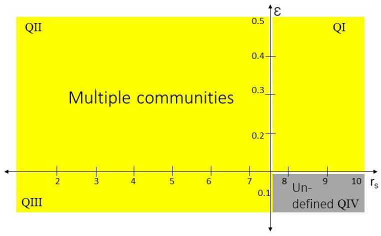


(d) Under PRM, intersect at $\epsilon = 0.3$ and insensitive to r_s

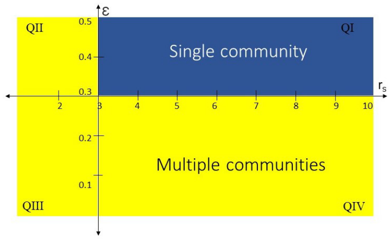
Figure 5. Opinion cluster classification for different mobility models, each colour represents a behaviour



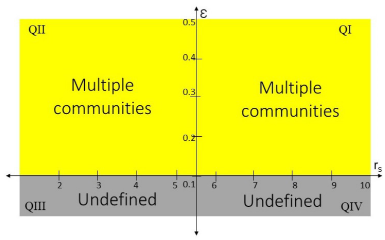
(a) Under AM, intersect at $r_s = 3$ and $\epsilon = 0.3$



(b) Under RM, intersect at $r_s = 7.5$ and $\epsilon = 0.1$



(c) Under HM, intersect at $r_s = 3$ and $\epsilon = 0.3$



(d) Under PRM, intersect at $\epsilon = 0.1$ and insensitive of r_s

Figure 6. Community classification for different mobility models, each colour represents a behaviour

results in homogeneous opinions at both population and local level. Finally, a *hybrid model* combining both approaches is able to produce scenarios with diverse opinions globally while producing consensus locally. Our results show how the outcomes of all these three models are heavily influenced by the opportunity for peers to interact and influence each other. This is controlled by measures of closeness in both opinion and location. Typically, more opinions are sustained as opportunities to interact are reduced.

In conclusion, our model provides an effective approach to the abstraction and synthesis of the complicated behaviour of real life agents by capturing some characteristics of opinion evolution in a free space dynamic environment. This provides further insights on the generic mechanisms observed in opinion formation.

Future enhancements of the model could implement heterogeneous distributions of the exogenous parameters (ϵ and r_s) across the population, reflecting different levels of influence across agents. This may also lead to a less uniform response in terms of mobility, such as agents that are more tolerant would be expected to remain close to dissenting peers.

Acknowledgements

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Website: http://www.sunrise-setting.co.uk

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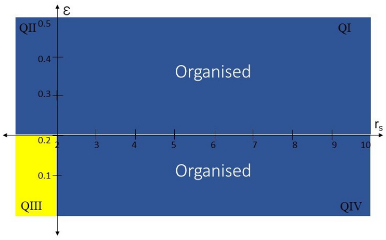
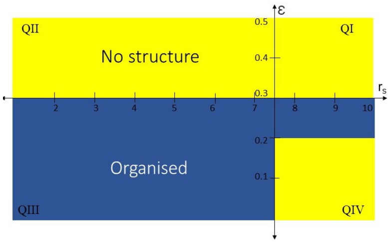
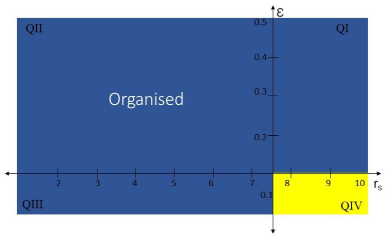


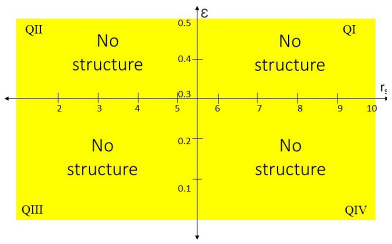
(a) Under AM, intersect at $r_s = 2$ and $\epsilon = 0.2$



(b) Under RM, intersect at $r_s = 7.5$ and $\epsilon = 0.3$

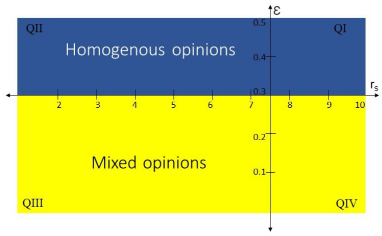


(c) Under HM, intersect at $r_s = 7.5$ and $\epsilon = 0.1$

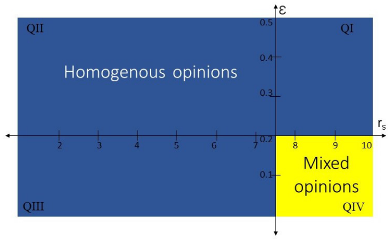


(d) Under PRM, insensitive to both r_s and ϵ

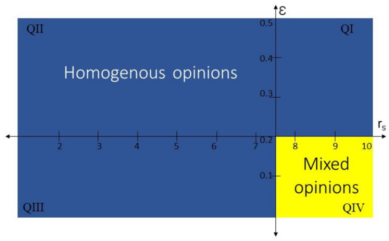
Figure 7. Loners classification for different mobility models, each colour represents a behaviour



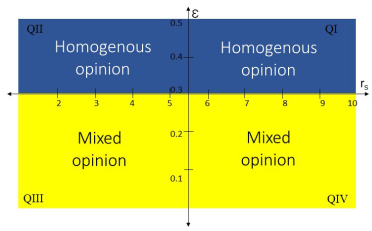
(a) Under AM, intersect at $r_s = 7.5$ and $\epsilon = 0.3$



(b) Under RM, intersect at $r_s = 7.5$ and $\epsilon = 0.2$



(c) Under HM, intersect at $r_s = 7.5$ and $\epsilon = 0.2$



(d) Under PRM, intersect at $\epsilon = 0.3$ and insensitive to r_s

Figure 8. Tolerance classification for different mobility models, each colour represents a behaviour

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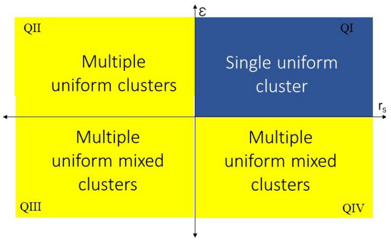
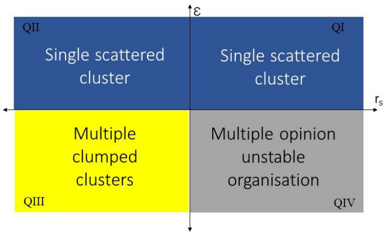
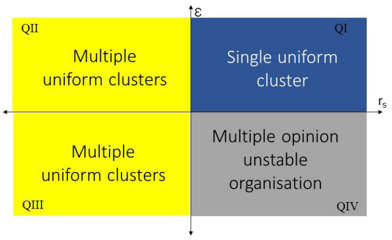


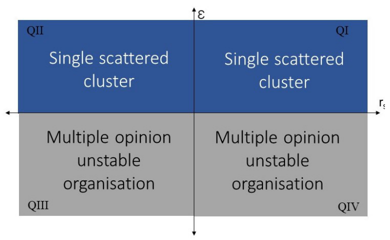
(a) Under AM model for a combination of r_s and ϵ values.



(b) Under RM model for a combination of r_s and ϵ values.



(c) Under HM model for a combination of r_s and ϵ values.



(d) Under PRM model for a combination of r_s and ϵ values.

Figure 9. Summary of outcomes for different mobility models, each colour represents a behaviour.

Simulation 20(4).

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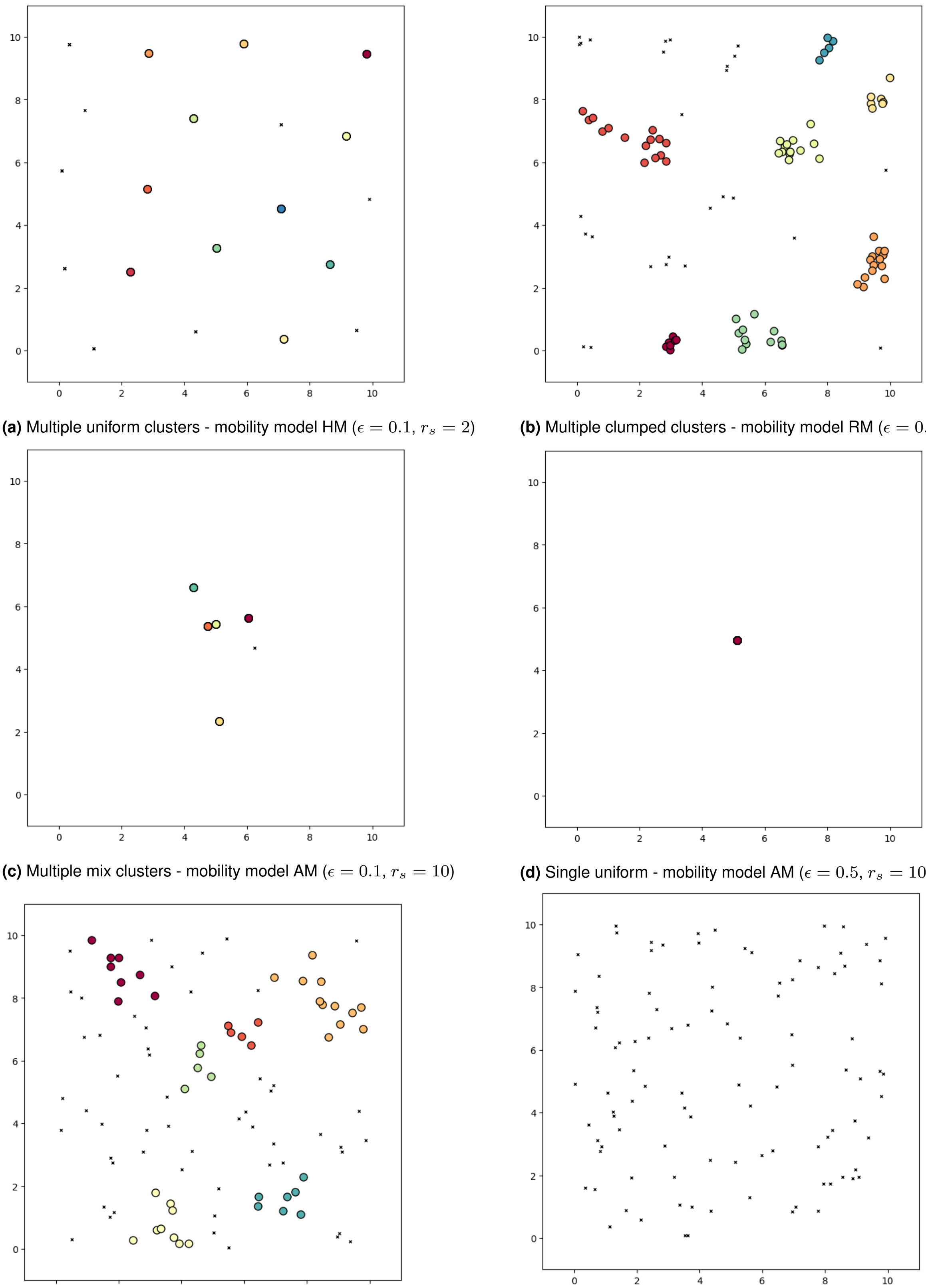

(a) Multiple uniform clusters - mobility model HM ($\epsilon = 0.1$, $r_s = 2$)

(b) Multiple clumped clusters - mobility model RM ($\epsilon = 0.1$, $r_s = 2$)

(c) Multiple mix clusters - mobility model AM ($\epsilon = 0.1$, $r_s = 10$)

(d) Single uniform - mobility model AM ($\epsilon = 0.5$, $r_s = 10$)

(e) Single scattered clusters - mobility model RM ($\epsilon = 0.5$, $r_s = 2$)

(f) Multiple opinion unstable organisation - mobility model RM ($\epsilon = 0.1$, $r_s = 10$)

Figure 10. Examples of potential outcomes of a single run. Agent's distribution is on a 10×10 region. Coloured $\circ$ denote communities where agents hold the same opinion and $\times$ are loners.

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