Bounded Rational Expectation: How It Can Affect the Effectiveness of Monetary Rules in the Open Economy

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Abstract

Since the channel for agents' expectations matters for the effectiveness of monetary policies, it is crucial for policy-makers to assess the degree to which economic agents are boundedly rational and understand how the bounded rationality affects the monetary rules in stabilising the economy. We investigate the empirical evidence for the bounded rationality in a small open economy model of the UK, and compare the results with those for the conventional rational expectations model. Overall, comparing the estimated models favours the bounded rationality framework. The results show that bounded rationality model helps to explain the hump-shaped dynamics of real exchange rate following monetary shocks, while the rational expectations model cannot. Also, we find that the exchange rate channel in the bounded rationality enlarges the effects of foreign mark-up shock, policymakers should send stronger signals over its target to the economics agents to combat the inflation. So the bounded rationality that can be found in the data still leaves scope for the forward guidance channel to work strongly enough to be exploited by policymakers.

Keywords: bounded rationality, monetary policy, small open economy, exchange rate channel
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1. Introduction

Managing expectations is a significant part of monetary policy. When central bank's policy interest rates have reached or been close to their lower bound, forward guidance as a monetary policy tool has been used by central banks such as the Bank of England, the European Central Bank, and the US Federal Reserve. Forward guidance is the practice of communicating the likely future plans for monetary policy instruments, for example, policy interest rates and price levels[1]. Monetary policies that rely on managing expectations are supposed to work very powerfully in standard New Keynesian models with rational agents, under the maintained assumption of credible commitment.

However, the last decade has seen an explosive increase in work criticised that the monetary policy analysis based on rational expectations cannot be considered realistic[2]. Bounded rationality has been increasingly suggested as a replacement for rational expectations in building macroeconomic models. Angeletos and Lian (2018) have explored how information frictions in the amount of knowledge agents have about the fundamentals and about one another's beliefs and actions cause the economy to respond to news about the future as if the agents were myopic. Farhi and Werning (2019) and García-Schmidt and Woodford (2019) argue that beliefs deviating from rational expectations behave like level-k thinking where agents are rational with respect to partial equilibrium effects, but do not quite understand general equilibrium effects. Thus, the expected future outcome is dampened. Gabaix (2020) achieves bounded rationality by assuming that the perceived law of motion of all the relevant economic variables exhibits less amplitude and less persistence than the true one.

Our aim in this paper is to investigate empirically, using a powerful estimation and testing strategy, how far a model of the UK open economy conforms to the fully rational and boundedly

[1] "Forward guidance is not new. Starting in the 1990s, central banks relied on qualitative descriptions of the main thrust of their interest rate policies to inform the public, which sometimes required deciphering 'code words' in official policy statements." Filardo and Hofmann (2014, p.38)

[2] Rational agents are assumed to know exactly the structural model, its parameters, and the stochastic shock distribution, when they form expectations about future outcomes.

rational expectations assumptions. From our estimates we evaluate the likely outcomes of different monetary policy strategies and understand how the bounded rationality affects the monetary rules relying on forward-looking agents in stabilising the economy.

This paper is also connected with three more groups of literature. First, there is a growing number of studies developing 'behavioural' macroeconomic models, which provide a response to the supposed shortcomings of mainstream macroeconomic models in the financial crisis. Driscoll and Holden (2014) summarise and discuss the alternative assumptions which depart from rational expectations, such as cognitive biases, rule-of-thumb consumption, hyperbolic discounting of savings and consumption. Most of the existing literature discusses the behavioural mechanism in the canonical three-equation New Keynesian model (Jump and Levine, 2019). In this paper we extend it to a small open economy setting with various real and nominal rigidities in order to analyse the role of the exchange rate channel under the assumption of bounded rationality.

Second, there is a strand of literature on the relation between expected interest rate differentials and the exchange rate in the open economy. According to uncovered interest parity (UIP), the exchange rate is equal to the undiscounted sum of all future expected interest rate differentials. This implies that the impact of an announcement of a future adjustment in interest rates on the current exchange rate is invariant to the timing of the adjustment and is larger the longer the horizon of implementation. However, Galí (2020) finds that expectations of interest differentials in the near future have larger impacts on the current exchange rate than expectations of interest rate differentials in the more distant future based on a partial equilibrium model. In the theoretical literature, Bacchetta and Van Wincoop (2019) provide a delayed portfolio adjustment model to explain Galí (2020)'s empirical finding. In their model, investors are assumed to face a quadratic adjustment cost of changing the international allocation of their portfolio and therefore the initial response of exchange rate to changes in expected interest rate differentials is muted. Instead, this paper attempts to examine whether the exchange rate reacts in a discounted manner to future expected interest rate differentials by testing the dynamic stochastic general equilibrium (DSGE) model's implied behaviour of exchange rate channel against the UK data.

Third, there is a long-lasting debate between inflation targeting and price level targeting. Under the rational expectations assumption, a survey by Hatcher and Minford (2016) suggests that price-level targeting outperforms inflation targeting, and forward guidance policy that aims to affect agents' expectation of future interest rate has a strong impact today and can provide near term stimulus at the zero lower bound. In the context of bounded rationality, Gabaix (2020) found that adopting price level targeting could be costly and suboptimal. And Benchimol and Bounader (2020) concluded that no definitive answer about which targeting policy to adopt can be drawn in a behavioural setting. We claim that the bounded rationality that can be found in the data still leaves scope for the forward guidance channel to work strongly enough to be exploited by policymakers.

The organisation of the paper is as follows. Section 2 discusses the model and generates some kinds of cognitive discounting in the Euler equation, the firms' price setting policy and exchange rate equation, which differ from the model with rational expectations. We evaluate both the boundedly rational model and the rational expectations model by Indirect Inference method. Moreover, the bounded rationality model is estimated and tested using UK data in Section 3. Policy implications are discussed in Section 4. Section 5 provides some concluding remarks. All technical derivations are delayed to Appendices.

2. The Model

We investigate bounded rationality in a New-Keynesian model of a small open economy (SOE), introducing extensions and modifications to typical models. Specifically, we depart from the assumption of perfect rationality in standard SOE New-Keynesian models (Gali and Monacelli (2005), Kollmann (2001, 2002) and Adolfson et al. (2015)), where rational agents possess complete knowledge of the structural model, its parameters, and the stochastic shock distribution, enabling them to form expectations about future outcomes accurately. In contrast, drawing inspiration from Gabaix (2020)'s 3-equation model, we adopt a bounded rationality approach, where agents exhibit myopic behaviour concerning deviations around the steady state in the distant future. This shift in expectation formation allows for a more realistic representation of agents' cognitive constraints.

Furthermore, while Gabaix (2020) focuses on a closed economy model, we extend their analysis to incorporate the dynamics of a small open economy, introducing additional complexities associated with exchange rate dynamic and firm's price setting. One of the key contributions of our bounded rationality SOE model lies in its generation of cognitive discounting in various equations, such as the Euler equation, the firms' price-setting policy, and the exchange rate equation. These cognitive discounting effects differ significantly from models with conventional rational expectations and from Gabaix (2020)'s closed economy model, offering new insights into the behaviour of agents in our bounded rationality framework.

Consider an open economy produced a continuum of tradable intermediate goods indexed by $j \in [0, 1]$ and a single non-tradable final good. It is assumed that intermediate goods markets are monopolistic competitions. Each intermediate good is either produced or imported by a single firm. A continuum of imported intermediate goods is indexed by $j \in [0, 1]$. Domestic labour and capital are used as inputs for producing the domestic intermediate goods, which are divided between home market use and exports. There is no price discrimination between these two markets. It is supposed that the representative household owns all shares of domestic firms and the capital stock, and also supplies labour. The markets for rental capital and for labour are competitive.

The assumption for the final-goods sector is perfect competition. The domestic produced and imported intermediate goods are used to produce the final good which is consumed and used for investment. The economy is small with respect to the rest of the world. In practice, home agents take the world nominal interest rate and price level as given.

2.1. Bounded Rationality Setup

Following Gabaix (2020), we assume that representative agents exhibit a form of myopia towards distant future events, as they do not fully understood the economy, especially events or economic policies that are far into the future. The bounded rational agent's perception of the state variables' flow motion evolves as

$$X_{t+1} = \overline{m} f(X_t, \epsilon_{t+1}) \tag{1}$$

where X_t is a state vector; ($\bar{m} \in [0, 1]$) represents the general myopia of the agent with regard to the economy's state; ϵ_{t+1} is a mean-0 innovations.

Agents' subjective conditional expectation about future economic variables equals the rational (model-consistent) conditional expectation multiplied by a geometric factor that is smaller than unity and decreasing in the forecast horizon. The relationship between boundedly rational agents and fully rational agents is shown as follow,

$$E_t^{BR}[X_{t+i}] = \bar{m}^i E_t X_{t+i} \tag{2}$$

where the $E_t^{BR}[X_{t+i}]$ is the subjective expectation under bounded rationality, and $E_t X_{t+i}$ is the rational expectation. The boundedly rational agent sees the events in the future with a dampened cognitive discount factor $\bar{m}^i$ at future horizon i. $\bar{m}^i$ is ith power of global cognitive discounting parameter $\bar{m}$, and it discounts future disturbances more as they are more distant in the future. When $\overline{m} = 1$, we recover the rational agent's law of motion.

The basic ingredients of the model are described in the following.

2.2. Household

We consider an infinitely lived representative household who is boundedly rational and maximises its lifetime utility

$$E_0^{BR}\sum_{t=0}^{\infty}\beta^t[\omega_c\epsilon_t^c\frac{C_t^{1-\gamma}-1}{1-\gamma}-(1-\omega_c)\epsilon_t^N\frac{N_t^{1+\phi}}{1+\phi}], \tag{3}$$

where C_t and N_t denote the household's aggregate consumption and the labour supply, respectively. $\frac{1}{\gamma}$ is the intertemporal elasticity of substitution in consumption. ω_c is a preference weight of the consumption in the household's utility function. ϕ is the elasticity of marginal dis-utility with respect to labour supply. ϵ_t^c and ϵ_t^N are exogenous preference shocks to the consumption and the labour supply, respectively.

Suppose the representative household owns all shares of domestic firms and accumulates physical capital. The law of motion of the physical capital stock is

$$K_{t+1} = K_t(1-\delta)+\epsilon_t^I[1-\Phi(\frac{I_t}{I_{t-1}})]I_t, \tag{4}$$

where K_t denotes capital stock and I_t represents the gross investment. The adjustment cost function $\Phi(.)$ is a positive function of changes in investment[3]. δ is the depreciation rate of capital. ϵ_t^I denotes a shock to the investment cost function.

The households finance its expenditures through labour incomes, capital incomes, financial wealth and dividends. More specifically, the households hold their financial wealth in the form of domestic bonds and foreign bonds, obtain factor payments by supplying labour and renting capital services to domestic-intermediate firms, receive dividend payments from the monopo-

[3]It is assumed that $\Phi(.)$ equals zero in steady state with a constant investment level, and the first derivative also equals zero around equilibrium. It implies that adjustment costs will only depend on the second-order derivatives, $\Phi''(.) = \kappa$. In addition, the steady state of the model does not depend on the adjustment cost parameter, κ, but the dynamics of the model are influenced by κ.

listically competitive intermediate-goods producers and importers, $\int_0^1 D_t^d(j)dj + \int_0^1 D_t^f(j)dj$[4]. The household uses some of its funds to pay lump-sum tax, and purchases consumption and investment, at a nominal price P_t.

The household faces the budget constraint in each period which (in nominal terms) is given by

$$P_t(C_t + I_t) + P_t^d\Psi(u_t)K_t + B_{t+1} + S_tB_{t+1}^f + T_t = W_tN_t + B_t(1 + i_{t-1}) + S_tB_t^f(1 + i_{t-1}^f)\Gamma(b_{t-1}^f, \epsilon_{t-1}^\Gamma)$$
$$+ R_t^K u_t K_t + \int_0^1 D_t^d(j)dj + \int_0^1 D_t^f(j)dj \tag{6}$$

where S_t denotes the nominal exchange rate; B_{t+1} and B_{t+1}^f denote net stocks of home and foreign currency bonds that mature in period t; i_{t+1} and i_{t+1}^f represent the nominal interest rates on the domestic and foreign bonds, respectively; W_t is the nominal wage rate; R_t^k is the nominal rental rate for service capital; T_t is a lump-sum tax.

$\Psi(u_t)K_t$ is the cost associated with variations in the degree of capital utilisation[5]. $(1 + i_{t-1}^f)\Gamma(b_{t-1}^f, \epsilon_{t-1}^\Gamma)$ is a risk-adjusted pre-tax gross interest rate. Following Gabaix and Maggiori (2015) and Dong et al. (2019), the term $\Gamma(b_t^f, \epsilon_t^\Gamma)$ is a premium on foreign bond holdings, which depends on the real net foreign asset to GDP ratio of the domestic economy[6]. The introduction of this risk-premium is needed in order to ensure a well-defined steady state in the model. ϵ_t^Γ is a time varying shock to the risk premium.

The household chooses a strategy $\{C_t, N_t, B_{t+1}, B_{t+1}^f, K_{t+1}, I_t, u_t\}_{t=0}^{t=\infty}$ to maximise its expected lifetime utility (3), subject to constraints (4) and (6) and to initial values B_0, B_0^f, K_0.

[4] K_t^s denotes the amount of effective capital that the household can rent to the firms and equals to

$$K_t^s = u_t K_t \tag{5}$$

where u_t is the utilisation rate of capital households choose.

[5] We impose two restrictions on the unit cost of capital utilisation function, $\Psi(u_t)$. First, we require that $u_t = 1$ in steady state. Second, we assume $\Psi(1) = 0$. Under the assumptions, the steady state of the model is independent of $\varphi = \Psi''(1)/\Psi'(1)$.

[6] The function $\Gamma(b_t^f, \epsilon_t^\Gamma)$ is assumed to be strictly decreasing in b_t^f and to satisfy $\Gamma(0,0) = 1$. Consequently, this function captures imperfect integration in the international financial markets.

We obtain a log-linearised discounted aggregate Euler equation

$$\hat{C}_t = \bar{m}E_t\hat{C}_{t+1} - \frac{1}{\gamma}\hat{r}_t + \frac{1}{\gamma}\epsilon_t^c \tag{7}$$

where $r_t \equiv i_t - E_t\pi_{t+1}$ is the real interest rate, π_t is inflation and the $\hat{}$ denotes deviations from steady state. Under bounded rationality with $\bar{m} < 1$, the future consumption appears to have less effect on current assumption.

The dynamics of the real exchange rate come from a no arbitrage condition in an imperfect international financial market is given by

$$\frac{1 + i_t}{(1 + i_t^f)\Gamma(b_t^f, \epsilon_t^\Gamma)} = \frac{E_t^{BR}S_{t+1}}{S_t}. \tag{8}$$

Linearising,

$$\hat{e}_t = \bar{m}E_t\hat{e}_{t+1} - \hat{r}_t + \hat{r}_t{}^f - \Gamma b_t^f + \epsilon_t^\Gamma \tag{9}$$

where $r_t{}^f$ is real foreign interest rate. e_t denotes the real exchange rate, which measures the relative price levels across countries ($e_t = S_t P_t^*/P_t$). P_t^* is the world general price level.

Equation (9) implies that the real exchange rate is related to the discounted sum of all future expected interest rate differentials [7].

The technical deviations of consumer's first order optimality conditions are relegated to Appendix A.

[7] Assume for simplicity that $lim_{T\to\infty}E_t(e_T)$ is well defined and bounded, and purchasing power parity holds in the long-run. In that case, (9) can be solved forward and rewritten

$$\hat{e}_t = \sum_{j=0}^{\infty} \bar{m}^j E_t(\hat{r}_{t+j}^f - \hat{r}_{t+j} - \Gamma b_{t+j}^f) + \epsilon_t^\Gamma$$

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2.3. The Firms

The firms are categorised into three types in this open economy: domestic intermediate goods producers, the foreign intermediate goods importers and the domestic final goods producers.

2.3.1. Final-goods sector / Aggregation sector

The final-goods sector is assumed to be perfect competitive market. Differentiated domestic and imported intermediate goods are inputs for a single non-tradable final good that is aggregated by CES production technology

$$Z_t = [(1-\omega_f)^{\frac{1}{v}} {y_t^d}^{\frac{v-1}{v}} + {\omega_f}^{\frac{1}{v}} {y_t^{im}}^{\frac{v-1}{v}}]^{\frac{v}{v-1}}, \tag{10}$$

where $0 < \omega_f < 1$ denotes a positive share of imported goods in the production of the final good, and $v > 0$ is the elasticity of substitution between domestic and imported goods. Z_t is final good output at date t and it is used for home consumption and investment, so that

$$Z_t = C_t + I_t. \tag{11}$$

y_t^d and y_t^{im} are quantity indices of domestic and imported intermediate goods, respectively:

$$y_t^d = (\int_0^1 y_t^d(j)^{\frac{v_t^d-1}{v_t^d}} dj)^{\frac{v_t^d}{v_t^d-1}}$$

and

$$y_t^{im} = (\int_0^1 y_t^{im}(j)^{\frac{v_t^{im}-1}{v_t^{im}}} dj)^{\frac{v_t^{im}}{v_t^{im}-1}},$$

where $v_t^d, v_t^{im} > 1$ are the time varying elasticity of substitution between domestic intermediate goods, and the elasticity of substitution between imported intermediate goods; both of them are related to the markup according to $\epsilon_t^p = v_t^d/(v_t^d - 1)$ and $\epsilon_t^{pf} = v_t^{im}/(v_t^{im} - 1)$; $y_t^d(j)$ and $y_t^{im}(j)$ are quantities of the domestic and imported type j intermediate goods.

Given the price level of the final good, P_t, the price index for domestic intermediate goods that are sold in the domestic market, P_t^d, and the price of imported goods, P_t^{im}, the competitive firm chooses y_t^d and y_t^{im} to maximise its profit. The maximisation problem is

$$\begin{aligned} &\underset{y_t^d, y_t^{im}}{\text{Max}} \quad P_t Z_t - P_t^d y_t^d - P_t^{im} y_t^{im}, \\ &\text{subject to} \quad Z_t = [(1-\omega_f)^{\frac{1}{v}} {y_t^d}^{\frac{v-1}{v}} + \omega_f{}^{\frac{1}{v}} {y_t^{im}}^{\frac{v-1}{v}}]^{\frac{v}{v-1}}. \end{aligned} \tag{12}$$

Profit maximisation implies the following demand functions for domestic and imported-composite goods:

$$y_t^d = (1-\omega_f)(\frac{P_t^d}{P_t})^{-v} Z_t; y_t^{im} = \omega_f(\frac{P_t^{im}}{P_t})^{-v} Z_t. \tag{13}$$

The demands for home produced goods and imported goods are negatively correlated with real prices of domestic goods and imported goods, respectively. Parameter v denotes the price elasticity of these demand functions.

The zero-profit condition implies that the price level of the final good (CPI), is linked to the producer price index (PPI) and importer-price index (IPI) through:

$$P_t = [(1-\omega_f){P_t^d}^{1-v} + \omega_f {P_t^{im}}^{1-v}]^{\frac{1}{1-v}}. \tag{14}$$

2.3.2. Intermediate-goods sector

Domestic-intermediate goods The technology of domestic intermediate goods firms that produce good j is

$$Y_t(j) = A_t K_t^s(j)^{\alpha} N_t(j)^{1-\alpha}, \tag{15}$$

where A_t is an exogenous productivity factor that is assumed to be identical across all firms. $Y_t(j)$ denotes the output of good j; $K_t^s(j)$ and $N_t(j)$ are the amounts of capital and labour input used by firm j. $1-\alpha$ is the output elasticity of labour.

The domestic-intermediate good is sold in the domestic market $y_t^d(j)$, $G_t(j)$, and exported, $y_t^{ex}(j)$:

$$Y_t(j) = y_t^d(j) + G_t(j) + y_t^{ex}(j). \tag{16}$$

Let R_t^K and W_t be the rental rate of capital and the nominal wage rate. The cost minimisation problem yields the following nominal marginal cost for intermediate firm:

$$MC_t = A_t^{-1}(R_t^K)^{\alpha} W_t^{1-\alpha} \alpha^{-\alpha}(1-\alpha)^{\alpha-1}. \tag{17}$$

Suppose there is no price discrimination between the domestic market and the export market, the export price is expressed as P_t^d/S_t. Following McCallum and Nelson (1999) and Kollmann (2001) it is assumed that the foreign demand for home produced intermediate goods (exports) is

$$y_t^{ex} = \left(\frac{P_t^d}{S_t P_t^\star}\right)^{-\eta} Z_t^F \tag{18}$$

where $\eta > 0$ denotes the price elasticity of the domestic country's aggregate exports. Z_t^F is the total world demand and modelled by AR(1) process.

Following the formalism proposed in Calvo (1983), firms can reset their price with probability $1-\theta_p$ in each period. Boundedly rational firms, which do not fully pay attention to future macroeconomic variables, choose the optimal price that maximises the current market value of the profits generated while that price remains effective. The first-order condition of the profit maximisation problem yields the following log-linearised discounting Phillips curve:

$$\hat{\pi}_t^d = \beta M^d E_t(\hat{\pi}_{t+1}^d) + \lambda(\hat{mc}_t + \epsilon_t^p) \tag{19}$$

where hat denotes log-deviation from steady-state (i.e., $\hat{X}_t = lnX_t - lnX$). $\hat{\pi}_t^d$, $\hat{mc}_t$ and ϵ_t^p denote inflation in the domestic sector, real marginal cost for producing domestic intermediate goods, and domestic price mark-up shock, respectively. $M^d = \bar{m}[\theta_p + \frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}}(1-\theta_p)]$, and $\lambda = \frac{1-\theta_p}{\theta_p}(1-\beta\theta_p)$.

Imported-intermediate goods The import sector is comprised of a continuum of firms that buy a homogeneous intermediate good at price $P_t^{\star}$ in the world market. The importer turns the differentiated imported goods into the imported-composite good, y_t^{im}, through CES technology. Followed by Calvo price setting, only $1-\theta_p^*$ proportion of importers are allowed to set their prices in each period, when they receive a random price change signal.

Each importing firm j reoptimise its price that maximises its weighted expected profits, given the nominal exchange rate, S_t, the price of the imported-composite good, P_t^{im}, and the world price level, $P_t^{\star}$. Solving this profit maximisation problem yields the log-linearised equation as follows.

$$\hat{\pi}_t^{im} = \beta M^* E_t(\hat{\pi}_{t+1}^{im}) + \lambda^*(\hat{S}_t + \hat{P}_t^* - \hat{P}_t^{im} + \epsilon_t^{pf}) \tag{20}$$

where $\hat{\pi}_t^{im}$ and ϵ_t^{pf} denote imported goods inflation and the time-varying markup shocks. $M^* = \bar{m}[\theta_p^* + \frac{1-\beta\theta_p^*}{1-\beta\theta_p^*\bar{m}}(1-\theta_p^*)]$, and $\lambda^* = \frac{1-\theta_p^*}{\theta_p^*}(1-\beta\theta_p^*)$.

2.4. Monetary Policy

The monetary authority sets policy according to

$$i_t = MAX(i_{ZLB}, i_t^T), \tag{21}$$

where i_{ZLB} is the interest rate at the effective lower bound and i_t^T follows a Taylor-type rule of the form

$$i_t^T = \phi_i i_{t-1}^T + \phi_\pi \tilde{\pi}_t + \phi_x \tilde{y}_t + \epsilon_t^M \tag{22}$$

where $\tilde{\pi}_t$ is inflation deviations from target in the home country, $\tilde{y}_t$ is the output gap, ϕ_π and ϕ_x are non-negative coefficients, chosen by the monetary authority.

2.5. Market Clearing Conditions

The evolution of net foreign assets at the aggregate level satisfies

$$S_t B_{t+1}^f = S_t(1 + i_{t-1}^f)\Gamma(b_{t-1}^f, \epsilon_{t-1}^{\Gamma})B_t^f + P_t^d EX_t - S_t P_t^* IM_t \tag{23}$$

where $IM_t = y_t^{im}$ and $EX_t = y_t^{ex}$; the net export is $NX_t = P_t^d EX_t - S_t P_t^* IM_t$.

2.6. Foreign Economy

Following the literature on small open economy modelling (Adolfson et. al, 2007; Kollmann, 2001), the variables for the rest of the world - foreign demand (z_t^F) and foreign interest rate (r_t^f) - are exogenously given by stochastic processes:

$$z_t^F = \mu_{z^f} + \rho_{z^F} z_{t-1}^F + \eta_{z^F,t} \tag{24}$$

$$r_t^f = \mu_{r^f} + \rho_{r^f} r_{t-1}^f + \eta_{r^f,t} \tag{25}$$

where $\eta_{z^F,t}$ and $\eta_{r^f,t}$ are innovations, ρ_{z^F} and ρ_{r^f} are autoregressive coefficients.

3. Testing the Bounded Rationality Model against the UK Data

In this section, we will estimate bounded rationality model, evaluate its ability to match UK data behaviour, and compare the results with those for the conventional rational expectations model.

3.1. Data

The UK has experienced several shifts in monetary regime in the post-Bretton Woods period. Since October 1992, it has moved to an ‘inflation targeting monetary regime’ and the pound has floated freely. To avoid this and earlier structural breaks, we use quarterly unfiltered data over the period 1993 Q1 to 2019 Q1 on key UK macroeconomic variables[8]. All real series are expressed in per capita terms after dividing by an working-age population index. The data is mainly obtained from Office for National Statistics (ONS).

3.2. Model Estimation and Evaluation by the Method of Indirect Inference

The bounded rationality model is estimated by the indirect inference method, which belongs to the family of the moment-matching estimation methodology. The main feature of the indirect inference method is the use of structural model specifications to explain results obtained using reduced-form models. The method of indirect inference was proposed by Smith (1993) and Smith (1993), and widely used in estimating structure models such as Le et al. (2011) and Yang et al. (2021).

The indirect inference estimation is based on the idea of using the data simulated from the estimated structural model to replicate estimates obtained using actual data in a given reduced-form analysis. Followed by Minford et al. (2009) and Le et al. (2011), the vector autoregressive model with exogenous variable (VARX) is used as the auxiliary model the approximation to the reduced form of the DSGE model. Real exchange rate, output, inflation and interest rate are the chosen variables, since they capture the causal relationships implied by the discounting in Euler equation, New Keynesian Phillips curves, and the exchange rate equation in the bounded rationality model. The VARX model include these four key variables, plus a deterministic time trend and the non-stationary technology shock that contains a common stochastic trend. The details about the choice of the auxiliary model and the estimates coefficients for the auxiliary model are shown in Appendix C and Appendix B.

In the indirect inference context, common choices of target moments to match include scores, impulse response functions, and estimates obtained in the auxiliary model. The VARX estimates are chosen as target moments to match in the bounded rationality model estimation.

[8] The ideal empirical results using UK data could be partially due to the high quality of the dataset and relatively sound financial market regulations.

We first simulate the bounded rationality model by bootstrapping its structure shocks. Secondly, VARX auxiliary model is estimated using simulated samples based on the structure model and the actual data. Then the Wald statistics is computed as follows,

$$WS = (\beta^a - \overline{\beta^s(\widehat{\theta_0})})'\Omega^{-1}(\beta^a - \overline{\beta^s(\widehat{\theta_0})}) \tag{26}$$

where β^a denotes the VARX estimates using the actual data; $\widehat{\theta_0}$ is the vector of parameters of the bounded rationality model[9]. $\Omega = cov(\beta^i(\widehat{\theta_0}) - \overline{\beta^s(\widehat{\theta_0})}) = \frac{1}{s}\sum_{i=1}^{s}(\beta^i(\widehat{\theta_0}) - \overline{\beta^s(\widehat{\theta_0})})(\beta^i(\widehat{\theta_0}) - \overline{\beta^s(\widehat{\theta_0})})'$ is the variance-covariance matrix of the distribution of simulated estimates β^i. In essence, it measures the gap between what the bounded rationality model says the data behaviour should be and what the observed data behaviour actually is.

In estimation of the bounded rationality model, a Simulated Annealing algorithm is used to find the minimum-value Wald statistic. This gives us the estimated set of parameters - the one that produces the simulations that are statistically the closest to actual data.

The estimation results of the bounded rationality model is given in Table 1. The cognitive discounting factor within 1993Q1 to 2019Q1 is estimated at around 0.8122, and its 95% confidence lower and upper bounds are 0.75 and 0.85, respectively. The estimate implies that the discounting parameters in the domestic producer's Philips Curve and the importer's Philips Curve are 0.75 and 0.76, respectively[10]. All parameters in the bounded rationality model are estimated except for depreciation rate (δ) and the quarterly discount factor (β), which are held fixed on theoretical grounds.

Furthermore, we use Indirect Inference test to evaluate both the bounded rationality and the rational expectations models by using some alternative auxiliary models[11].The results for model

[9] $\overline{\beta^s(\widehat{\theta_0})} = E(\beta^i(\widehat{\theta_0})) = \frac{1}{s}\sum_{i=1}^{s}\beta^i(\widehat{\theta_0})$ denotes the sample average of estimates of the coefficients in auxiliary model based on s sets of simulated data from the macroeconomic model, taking $\widehat{\theta_0}$ as given.

[10] The discounting parameters in the domestic producer's Philips Curve and the importer's Philips Curve are computed by $M^d = \bar{m}[\theta_p + \frac{1-\beta\theta_p}{1-\beta\theta_p\bar{m}}(1-\theta_p)]$, $M^* = \bar{m}[\theta_p^* + \frac{1-\beta\theta_p^*}{1-\beta\theta_p^*\bar{m}}(1-\theta_p^*)]$, respectively.

[11] In order to check the robustness of test results about the bounded rationality channel in the exchange rate dynamics, we reports the model evaluations based on alternative reduce-forms analyses.

evaluations are presented in Table 2. More specifically, the boundedly rational model is able to generate behaviour that mimics UK data of the output, the interest rate, the real exchange rate, and the inflation jointly within the 95% confidence interval. The overall p-value is 0.08 for the bounded rationality.

Also, we estimate separately the rational expectations model, where cognitive discounting parameter is set at one ($\bar{m} = 1$). The rational model can also replicate the data features of the real exchange rate and the output jointly. However, the addition of the interest rate to the VARX model weakens the rational expectation model's capability to match the data's moments; the model is borderline rejected at 5% significance. Moreover, the rational expectation model cannot accommodate the inflation and the interest rate in addition to the output and the real exchange rate in the auxiliary model, failing to pass the test even at 1% significance.

In summary, given the UK experience the idea of the bounded rationality appears plausible. The boundedly rational version of the agents' Euler equation, of the firms' price setting policy, and of the exchange rate channel are empirically supported by our test results. Intuitively, the impacts of the expected future real interest rate are discounted with the horizons of the policy implementations.

Baseline Parameters			
Symbol	Description	Estimation	95%Confidence Interval
Households			
β	a quarterly discount factor	0.9900	
γ	CRRA coefficient for consumption	1.4329	[1.25, 1.58]
ϕ	the inverse of Frisch labour supply elasticity	1.2490	[1.10, 1.59]
κ	the adjustment cost parameter	4.5474	[3.50, 5.50]
φ	the elasticity of capital utilisation with respect to the rental rate of capital	0.0655	[0.05, 0.08]
Firms			
α	output elasticity of capital	0.2000	[0.18, 0.25]
δ	a quarterly depreciation rate	0.0250	
ω_f	share of the imported good in the production of the final good	0.3634	[0.32, 0.46]
ν	the elasticity of substitution between domestic and imported goods	3.0367	[2.30, 3.66]
ν^{im}	the elasticity of substitution between imported goods	4.4399	[3.50, 5.00]
η	the price elasticity of the home country's aggregate exports	0.6813	[0.61, 0.74]
θ_p	index of price stickiness in home country	0.4864	[0.45, 0.49]
θ_p^*	index of price stickiness in the rest of the world	0.4076	[0.40, 0.48]
Financiers			
Γ	financiers' risk bearing capacity	0.3054	[0.25, 0.38]
Central Bank			
ϕ_i	monetary policy coefficient	0.8000	[0.75, 0.86]
ϕ_π	monetary policy coefficient	1.1000	[1.05, 1.21]
ϕ_x	monetary policy coefficient	0.1800	[0.13, 0.25]
Behavioural Parameters			
$\overline{m}$	cognitive discounting	0.8122	[0.75, 0.85]

Table 1: Indirect Inference Estimation for Bounded Rationality Model

Table 2: Indirect Inference Test Results for Bounded Rationality and Rational Expectation Models

	Model Test Results			
	Trans-Wald Statistic		P-Value	
VARX Auxiliary Model	BR	RE	BR	RE
Output, Real Exchange Rate	0.24	0.19	0.19***	0.20***
Output, Real Exchange Rate, Interest Rate	1.60	1.78	0.06**	0.04*
Output, Real Exchange Rate, Interest Rate, Inflation	1.34	4.10	0.08**	0.00

Notes: p-value with ***, ** and * indicate a rejection of the model at 1%, 5% and 10% significance level respectively. BR denotes Bounded Rationality model and RE denotes Rational Expectation model.

Table 3: Monte Carlo Results

Falseness(%)	*True*	*1*	*3*	*5*	*7*	*9*	*10*	*15*
Rejection Rate	5.02	9.53	62.60	95.08	99.20	100	100	100

Notes: Rejection rates for Indirect Inference test based on auxiliary models with output, real exchange rate, real interest rate and inflation.

3.3. Robustness Check by Monte Carlo Experiments

Model users such as policymakers want to know to what extent at least will the bounded rationality model to be relied on. We can find out by checking how powerful our test is. The Monte Carlo experiments have been constructed to quantify what percentage of the time will the test reject false models. Firstly, 10,000 repeated samples have been generated from the estimated bounded rationality model, treating it as the True model. Secondly, we construct False models by perturbing all the parameters alternately by + or − x% from the estimated parameters ('true values')[12]. Then, we generate 10,000 samples from each False model. Finally, we can carry out the 'size'of the test on each False model by checking how frequently the True data samples reject the False models as x% increases.

Monte Carlo experiments are reported in Table 3. The results imply that model users can rely on the estimated bounded rationality model to jointly match the output, real exchange rate, interest rate and inflation for the UK data within a bound of True to 9% False. In other words, we can be confident that the model could be up to 9% False.

[12] x is called the 'degree of falseness'. And each False model can be seen as the misspecified version of the True model.

Table 4: Stationarity of Shocks and Estimated AR(1) Coefficients

Shocks	Stationary Test (KPSS statistic)[a]		AR(1) coefficients	
	Rational	Bounded Rational	Rational	Bounded Rational
Demand Shocks				
Consumer Preferences	0.1652	0.2931	0.2405	0.8619
Investment Specific Technology	0.1293	0.1155	0.2787	0.4727
Export Demand	0.1642	0.1594	0.9095	0.9104
Import Demand	0.1947	0.1922	0.9659	0.9664
Government Demand	0.2949	0.2949	0.9905	0.9905
Supply Shocks				
Productivity	1.1797***	1.2032***	0.0507[b]	0.1168[b]
Domestic Price Markup	0.2196	0.2261	0.9095	0.9366
Import Price Markup	0.1661	0.1686	0.9659	0.9692
Labour Supply	0.1944	0.2335	0.9223	0.9359
Others shocks				
Monetary Policy	0.1059	0.1087	0.1793	0.1036
Currency Risk Premium	0.2915	0.1072	0.9535	0.8807
Foreign Demand	0.2576	0.2576	0.9686	0.9686
Foreign Interest Rate	0.0518	0.0518	0.1955	0.1955

Notes: a. For KPSS test, statistic with ***, ** and * indicate a rejection of the stationary process at 1%, 5% and 10% significance level respectively; b. It is the first difference AR(1) coefficient.

3.4. Shocks and Model Variances

The stochastic dynamics are driven by thirteen structural shocks. And we group the shocks into demand shocks, supply shocks and others. All shocks are assumed to follow a first-order autoregressive process. Table 4 reports the estimates and stationarity test of the shocks in both of the bounded rationality model and the rational expectation model .

The KPSS test results show that all the shocks series cannot reject the null hypothesis of the stationary process at the 5% significance level, except for the productivity shock. Thus, we model the productivity shock by first-difference AR(1) process with the form $\Delta lnA_t = \mu_A + \rho_A \Delta lnA_{t-1} + \eta_{A,t}$. And the other structural shocks are modelled as either stationary or trend-stationary in levels.

Table 4 shows that AR(1) coefficients for imported demand, government demand, import price markup, currency risk premium and foreign demand are over than 0.95, which implies that those shocks are high persistence.

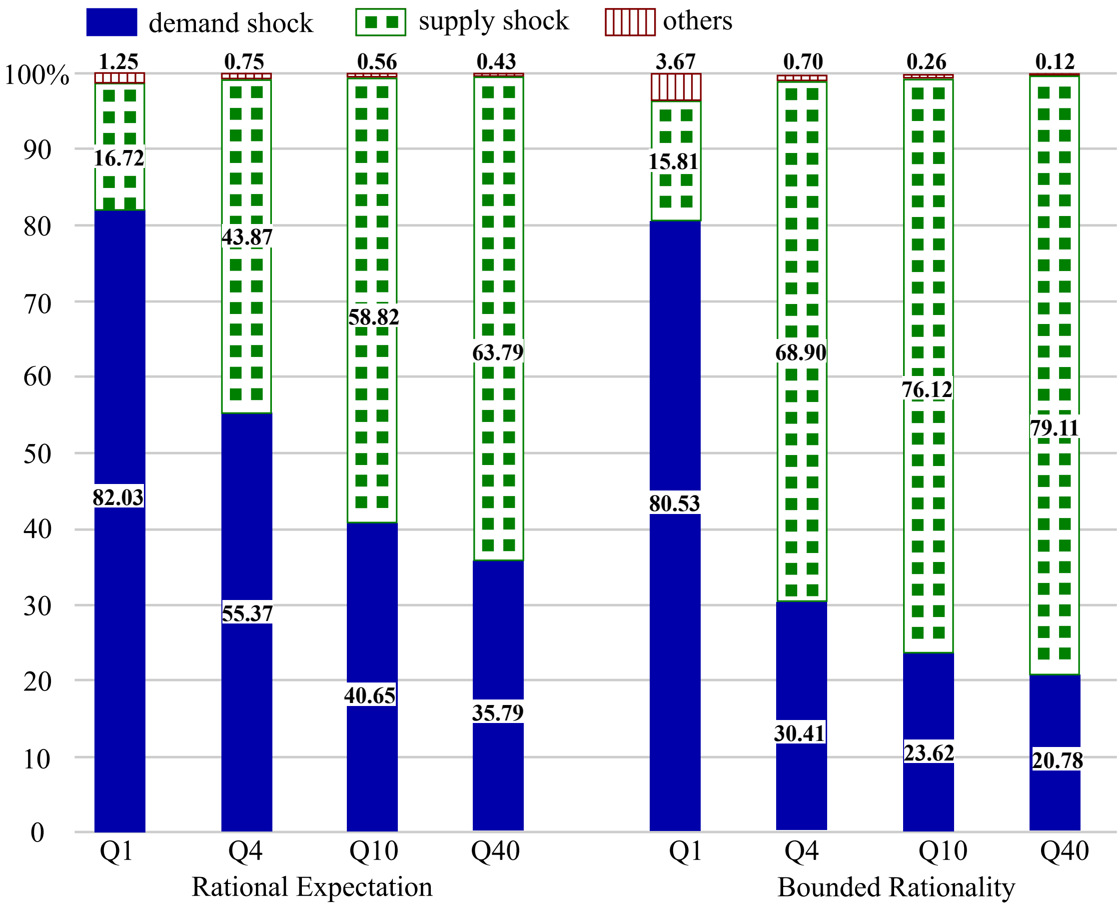


Figure 1: Variance Decomposition for Output

Figure 1 exhibits the forecast error variance decomposition of output at various horizons based on the estimated rational expectations (left) and boundedly rational (right) models. In the very short run (within one quarter), movements in real GDP are primarily driven by demand shocks, which account for around 80% of the error variance. Demand shocks continue to make up more than half of the error variance of output up to one year under the rational expectations, while supply shocks overtake the demand shocks and become the main driving force of output variation at one year horizon under the bounded rationality.

The impacts of demand shocks on output movement diminish over time, but they are more discounted under bounded rationality. Instead, supply shocks explain the most of the output variations in the medium to long run. Compared with rational expectations, supply shocks under bounded rationality, especially non-stationary productivity shock, are more persistent (see Table 4), and thus have a bigger effect on output variations.

4. Policy Discussion

4.1. Numerical Experiments on Forward Guidance in Open Economy

Next, we conduct some numerical experiments that allow us to understand how forward guidance in the boundedly rational model affects the economy differently from the rational expectation model. Suppose that at time 0, the home central bank credibly announces a one-period cut in the interest rate at 1%, to be implemented in period (quarter) T. Figure 2 displays the response of output, inflation and real exchange rate to the above experiment under three alternative time horizons for implementation: T=1,4,8. We used estimated models to run the experiments. Parameters are the same in both the boundedly rational model and the rational expectation model, except for the cognitive discounting parameter $\bar{m}$. For the rational expectation model, $\bar{m} = 1$, while $\bar{m} = 0.8122$ in the boundedly rational model.

In the right panel of Figure 2, the whole economy is fully rational. We see that the impact responses of output, inflation and real exchange rate are strictly increasing in T. The further away in the future is the forward guidance, the bigger is the effect of this policy on today's economy. In the left panel, both consumers and firms are bounded rationality, subjective expectations involve some cognitive discounting relative to rational expectations, in particular, when applied to intratemporal equations, New Keynesian Philips Curves, and the real exchange rate equation. We see that it is not necessarily the case anymore that the forward guidance about the distant future matters more for current outcomes with boundedly rational agents.

In the estimated boundedly rational model, the effect responses of output, inflation and real exchange rate associated with $T = 1$ are almost the same as those under rational expectations,

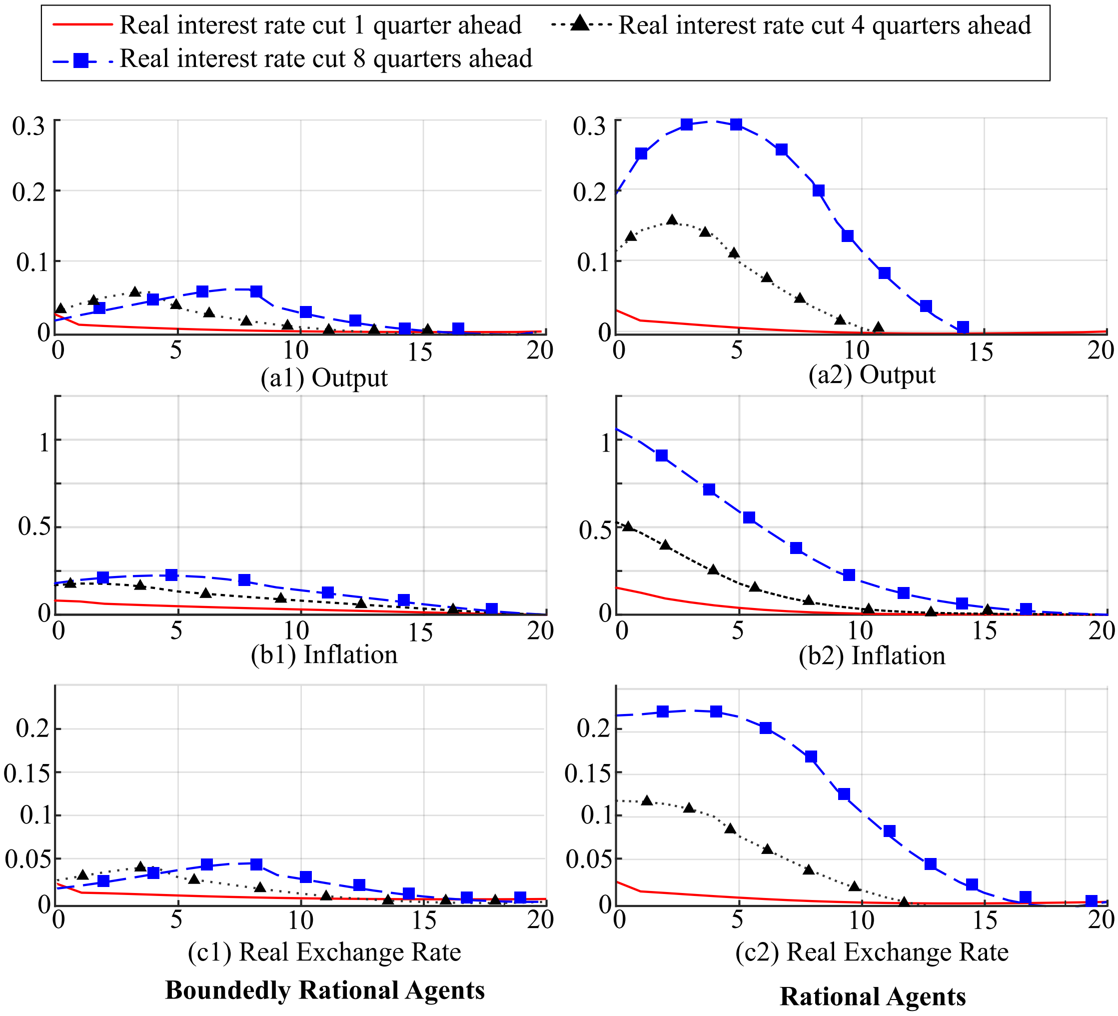


Figure 2: The Response Under Differing Expectations Assumptions of Output, Inflation and Real exchange rate to Forward Guidance About a One-Period Real Interest Rate Cut in T Quarters

but the effects of policy announcement refer to a further distant future on economy are much smaller, implying that there is a mitigating effect of distance into the future, with this mitigation the stronger the further out in the future the interest rate changes.

4.2. Further Discussion of the Monetary Policy Shock on Real Exchange Rate Dynamics

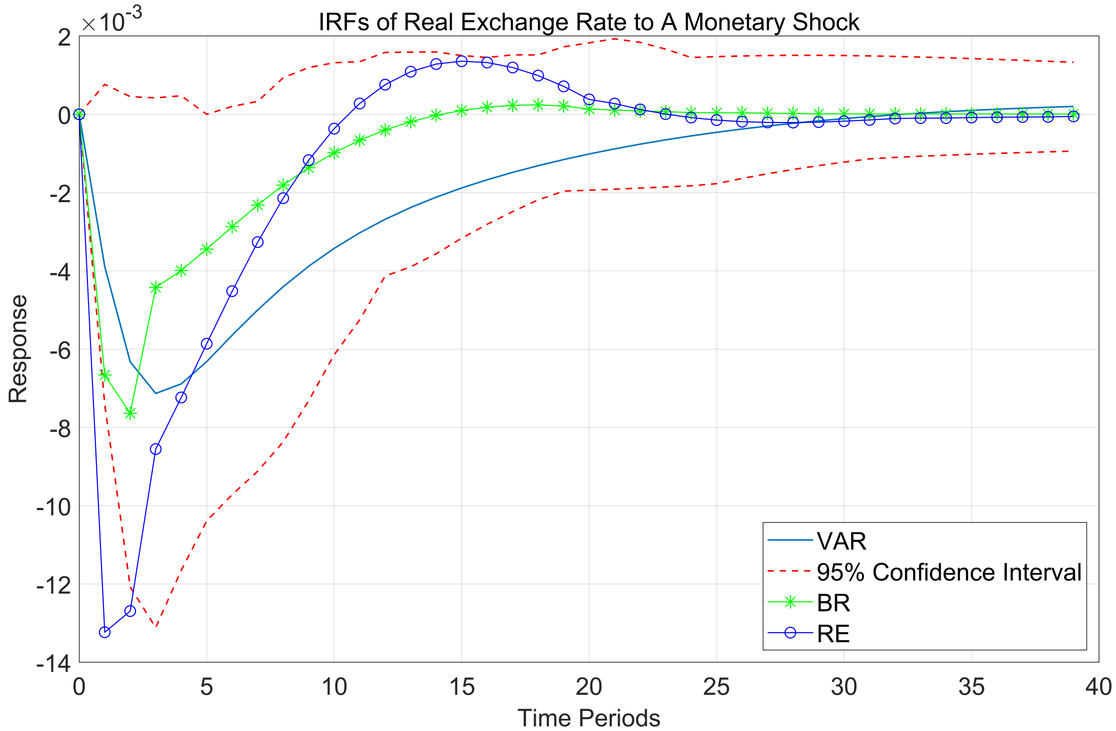
To further explore the impacts of the monetary policy shock on real exchange rate dynamics, we compare the impulse response functions (IRFs) of real exchange rate with the IRFs of the model under rational expectation as well as the IRFs under bounded rationality with the IRFs of the vector autoregressive (VAR) model.

We build a VAR model with four endogenous variables—output, real exchange rate, inflation and real interest rate. In the Figure 3, we see that associated with a one standard deviation increase in the interest rate, the responses are hump-shaped with the exception of the real exchange rate in the rational expectation model which jumps down (appreciates) and then returns to zeros from below. More specifically, if the agent is boundedly rational, we will see the hump-shaped response of the real exchange rate to monetary policy shocks. However, if the agent is fully rational, then we will see an immediate response of the real exchange rate. And the effect under bounded rationality is less dramatic than that under the rational expectation assumption.

The response of the real exchange rate under the bounded rationality model is well in line with that generated in VAR model[13]. As can be seen in Figure 3, the impulse response of the bounded rationality model remains enveloped within the VAR model's 95% confidence interval. But, an immediate response of the real exchange rate under the assumption of rational expecta-

[13]The hump-shaped dynamics of the foreign exchange rate following a monetary policy shock is a common pattern in this strand of literature, such as Kim and Roubini (2000) and Kim and Lim (2008). However, the convergence horizon and the degree of oscillation vary from country to country and are also affected by the specification of the VAR model, such as the composition of the VAR and the number of lags. We conduct the statistical information criteria to choose the lag length in VAR models. VAR(2) is preferred by the information criterion such as Schwartz Criterion and the Hannan-Quinn Criterion.

Figure 3: Impulse responses to a one standard deviation monetary policy shock



Notes: VAR model (solid line) and its confidence intervals (dash line), estimated bounded rationality model (line with star), and rational expectation model (line with circle).

tion is not within the 95% confidence level of VAR model[14]. The results align with the Indirect Inference test results: the bounded rationality model is better than rational expectation model in terms of matching exchange rate dynamics, when the output, real exchange rate, interest rate and inflation are jointly included in the reduce-form model.

4.3. Price Level Targeting vs Inflation Targeting

At the heart of the bounded rationality debate is the issue of how effective a monetary policy relying on forward-looking agents is. We examine this issue by evaluating the effectiveness of Price Level Targeting (PLT) in stabilising the economy compared with that of Inflation Targeting (IT). The effectiveness of PLT depends on agents understanding that an inflation deviation will trigger a long-lasting response of interest rates, designed to cause an equal and opposite deviation of cumulative future inflation; by contrast an IT rule will trigger a rise in interest rates today that will disappear once the inflation shock has died out. We will therefore find that PLT's relative effectiveness will fall as rational expectations become more bounded. What we want to establish with our empirical estimates of both models and the ranges within which each may differ from the truth, is how much boundedness matters for PLT policy effectiveness. If PLT continues to be relatively effective with the maximum possible boundedness, then we can conclude that the forward guidance channel while not as strong as in the pure rational expectations model still operates sufficiently in practice to be exploitable by policy.

To highlight the importance of this channel for monetary policy, we plot in Figure 4 how the economy responds to a 1% cost-push shock in both the bounded rationality model and the rational expectations model, when the central bank follows a PLT rule.

[14]After exploring 90% and 99% of confidence intervals of the VAR model, we consistently observe that the impulse responses of the bounded rationality model remain enveloped within the VAR model's confidence intervals, whereas those of the rational expectation model lie outside at the initial period. The conclusion regarding the contrast in impulse responses among different models remains robust across different confidence intervals.

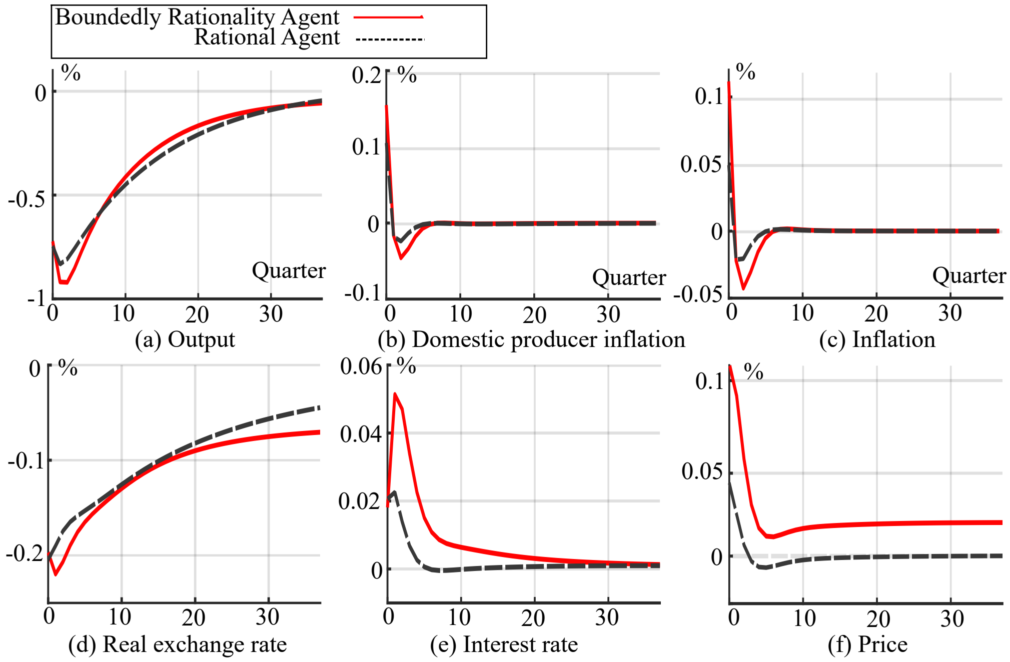


Figure 4: IRFs for a 1% cost-push shock to the Bounded Rationality Model (red solid) and Rational Expectations Model (black dash) under Price-level Targeting

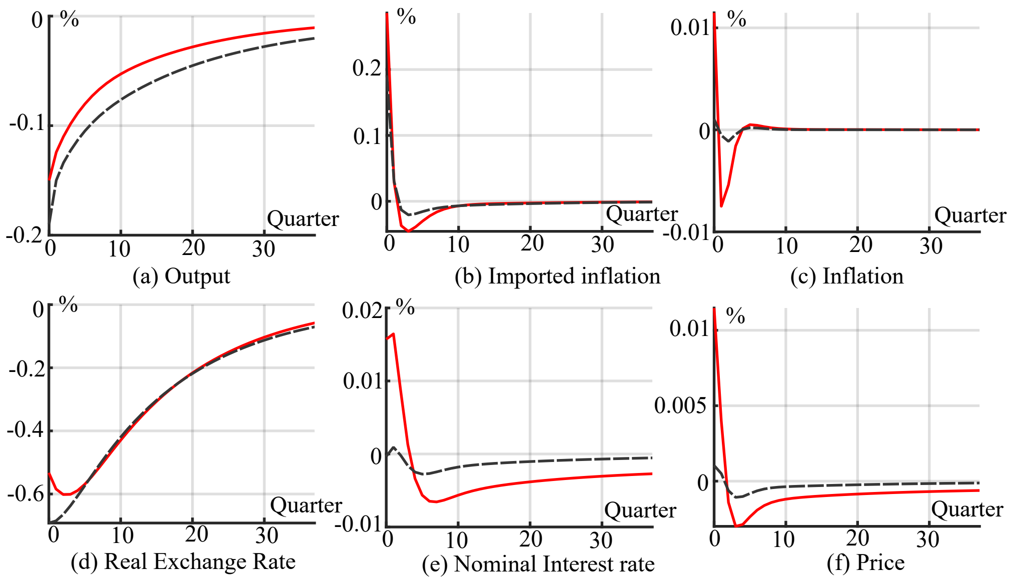


Figure 5: IRFs for a 1% imported markup shock to the Bounded Rationality Model (red solid) and Rational Expectations Model (black dash) under Price-level Targeting

Following a cost-push shock, the price level is above the target price path while output decreases. PLT calls for bringing about expected below-average inflation to stabilise the economy. Under rational expectations, this commitment to engineer a deflation in the future helps to reduce today's inflation volatility due to very forward looking agents.

In contrast, with the assumption of the bounded rational agents who are myopic to inflation, the effectiveness of PLT rule in stabilising the economy is reduced by the fall in forward lookingness both in the Phillips curve and exchange rate equation. Consequently, the price level and the inflation surge substantially and the output drops more. Moreover, in this case, we notice that the price level never returns to its steady state after a cost-push shock.

Figure 5 shows the impulse response of a markup shock in the import sector leads to an increase in imported inflation, which in turn pushes up the domestic price above the target price and causes higher inflation. Exchange rate appreciates less in the bounded rationality model than that in rational expectation model at short horizons, and thus that enlarges the effects of imported inflation shock on the inflation in the domestic economy. And output drops more in RE model due to the net export worsens more. Furthermore, the interest rate response is much more severe with bounded rational firms than with rational firms, so the central bank should raise the interest rate more aggressively to defeat the inflation.

To evaluate the stabilising power of PLT vs IT, it is assumed that the central bank seeks to minimise the quadratic loss function based on a Cole-Obstfeld preference(Cole and Obstfeld, 1991), which assumes the unitary elasticities of substitution:

$$L_1 = -\frac{(1-\omega_f)}{2}[\frac{v^d}{\lambda_\pi}\sigma_\pi^2 + (1+\phi)\sigma_y^2] \tag{27}$$

where σ_π^2 and σ_y^2 are the unconditional variances of the deviations of inflation from its targeted level and the output gap. $\lambda_\pi \equiv \frac{1-\theta_p}{\theta_p}(1-\beta\theta_p)$. The deviation of the welfare function can be found in Appendix D.

The form of IT rule has been specified in Equation 22, while the PLT rule is

$$i_t^P = \phi_i i_{t-1}^P + \phi_p(\tilde{p}_t) + \phi_x \tilde{y}_t + \epsilon_t^M \tag{28}$$

where $\tilde{p}_t = lnP_t - ln\bar{P}_t$, $\bar{P}_t$ denotes the predetermined long-run target price path, $\tilde{y}_t$ is the output gap and ϵ_t^M is monetary policy shock.

The optimal coefficients of PLT policy rule and Taylor rule $\{\phi_i, \phi_p, \phi_x\}$ have been derived optimally by computing the values that provides the smallest welfare loss (Equation 27).

In addition, we use the optimal policy rules derived from minimising loss function L_1 to calculate the conventional utility-based welfare loss function L_2, as the other criteria to evaluate monetary policy rules. We derive a second-order Taylor expansion to the utility losses of the domestic representative consumer resulting from deviations from the optimal policies. It can be algebraically written as

$$L_2 = -\frac{1}{2}[(1-\gamma)var(\hat{C}_t) - (\bar{p}^d(1-\alpha)\frac{\bar{Y}}{\bar{C}})(1+\phi)var(\hat{N}_t)]. \tag{29}$$

Specifically, the welfare of the representative household is adversely influenced by variability in consumption and employment.

Table 5 presents a comparative analysis of PLT and IT rules under both rational expectations and boundedly rational estimated models in terms of welfare loss functions and variances of the key variables. This table also reports coefficients for the corresponding optimal rules.

In the open economy model, the exchange rate is another key channel for the monetary transmission. First, changes in the exchange rate have a direct impact on inflation of imported goods through Equation 20. This strengthens the forward-looking channel of transmission and enhances the ability of PLT policy to reduce the volatility in inflation relative to IT rule. Second, Equation 42 implies that the exchange rate moves with the risk-adjusted uncovered interest parity.

Table 5: Compare behavioural model with rational model in different policies

		Rational Model		Bounded Rationality Model	
		IT	PLT	IT	PLT
Coefficients	ϕ_i	0.87	0.84	0.88	0.87
	ϕ_π or ϕ_p	2.80	2.65	2.90	3.50
	ϕ_x	0.17	0.22	0.16	0.10
Welfare Losses	L_1	1.28(1.29)	1.23(1.26)	1.27(1.29)	1.26(1.27)
	Incremental Benefit		3.9%(2.4%)		1.23%(1.6%)
	L_2	0.63(0.66)	0.59(0.61)	0.62(0.65)	0.61(0.64)
Avars	output gap	1.0148	1.0092	1.0094	1.0096
	inflation	0.9218	0.8445	0.9226	0.8957

Notes: To deal with the problem of model potential inaccuracy, we redo these policy simulations with maximally falsified parameters and see whether the policy rules still succeed in creating more stability. The corresponding welfare losses are shown in ().

Under the rational expectations model, results are in line with the literature surveyed by Hatcher and Minford (2016): PLT raises stability compared with IT both on the welfare loss function L_1 (by 3.9%) and on the utility-based welfare cost measure L_2 (by 6.3%). Even when the model is maximally false stability improves on both the measures (2.4% and 7.6% respectively).

When there is bounded rationality, PLT still raises stability compared with IT on inflation, but comes at the expense of higher output variability. The overall gain (1.23%) in stability due to PLT is not as great as under rational expectations but there is still a gain, even though the forward expectations channel is weakened. This remains the case when the model is maximally false - with PLT reducing L_1 by 1.6%. The advantage of PLT over IT in terms of inflation stabilisation weakens because of its large persistent reaction to inflation shocks. This echoes a frequent critique of PLT that it 'over-reacts' to inflation shocks, creating excessive future inflation variation in response to a current inflation shock. What our models show is that to offset this, a strong forward expectations channel is needed which checks current inflation.

Compared to the rational expectation models, the coefficients (ϕ_p) are larger in the bounded rationality models. Since agents are myopia to the future interest rate and inflation under the bounded rationality model, the transmissions of monetary policy to the output gap and inflation weaken. As a result, the policy-maker needs to react strongly in order to send the appropriate signal.

5. Concluding Remarks

After 2007-2009 financial crisis, models featuring bounded rationality have been widely studied. They have been used to explain the forward guidance puzzle or to deliver policy advices. This calls for a thorough investigation of whether the models without fully rational expectations are able to reflect factual economic developments.

To this end, we investigate the empirical evidence for the cognitive discounting form of bounded rationality in an open economy model of the UK, and compare the results with those for the fully rational expectations model. We take both models to the data and check how well they fit them on the features of the UK data. Overall, comparing the estimated models favours the bounded rationality framework.

We explore the implications of our results for monetary policy's abilities to exploit the expectations channel in stabilising the economy. The main conclusion from this analysis is that even in the presence of cognitive discounting and the potential errors in estimated parameters, price level targeting retains power to stabilise the economy, outperforming the Taylor inflation targeting rule, if the policy-maker reacts appropriately and sends strong signals over its target to the economics agents. So, the bounded rationality as found in the data still leaves scope for the forward channel to work strongly enough to be exploited by policymakers.

We remain cautious about the suitability of our current model in capturing the unique characteristics of the developing countries, such as foreign exchange rate interventions, fiscal policy constraints, foreign exchange reserves and switching regimes (Laxton and Pesenti, 2003). While we acknowledge the potential interest in testing the model for developing countries, undertaking such an extension is a significant endeavor that falls beyond the scope of this paper. Therefore, exploring these aspects would be more appropriately deferred to future research.

Appendix A: Model Deviations and The Log-linearised Model

A.1 Households' Lagrangian function is

$$\mathcal{L} = E_0^{BR}\sum_{t=0}^{\infty}\beta^t\{\omega_c\epsilon_t^c\frac{C_t^{1-\gamma}-1}{1-\gamma} - (1-\omega_c)\epsilon_t^N\frac{N_t^{1+\phi}}{1+\phi}$$
$$+ \lambda_{1,t}[W_tN_t + B_t(1+i_{t-1}) + S_tB_t^f(1+i_{t-1}^f)\Gamma(b_{t-1}^f, \epsilon_{t-1}^{\Gamma}) + R_t^K u_t K_t + \int_0^1 D_t^d(j)dj + \int_0^1 D_t^f(j)dj$$
$$- P_t(C_t+I_t) - P_t^d\Psi(u_t)K_t - B_{t+1} - S_tB_{t+1}^f - T_t]$$
$$+ \lambda_{2,t}[(1-\delta)K_t + \epsilon_t^I(1-\Phi(\frac{I_t}{I_{t-1}}))I_t - K_{t+1}]\} \tag{30}$$

Ruling out Ponzi schemes, the following equations are first-order conditions of this decision problems:

$$\omega_c\epsilon_t^c C_t^{-\gamma} - \lambda_{1,t}P_t = 0 \tag{31}$$

$$-(1-\omega_c)\epsilon_t^N N_t^{\phi} + \lambda_{1,t}W_t = 0 \tag{32}$$

$$-\beta^t\lambda_{1,t} + \beta^{t+1}E_t^{BR}\lambda_{1,t+1}(1+i_t) = 0 \tag{33}$$

$$-\beta^t S_t\lambda_{1,t} + \beta^{t+1}E_t^{BR}(\lambda_{1,t+1}S_{t+1})(1+i_t^f)\Gamma(b_t^f, \epsilon_t^{\Gamma}) = 0 \tag{34}$$

$$\lambda_{2,t} = \beta E_t^{BR}[\lambda_{1,t+1}(R_{t+1}^K u_{t+1} - P_{t+1}^d\Psi(u_{t+1})) + \lambda_{2,t+1}(1-\delta)] \tag{35}$$

$$\lambda_{1,t}P_t = \lambda_{2,t}\epsilon_t^I[1-\Phi(\frac{I_t}{I_{t-1}}) - \Phi'(\frac{I_t}{I_{t-1}})\frac{I_t}{I_{t-1}}] + \beta E_t^{BR}[\lambda_{2,t+1}\epsilon_{t+1}^I\Phi'(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2] \tag{36}$$

$$\frac{R_t^K}{P_t^d} = \Psi'(u_t) \tag{37}$$

where $\lambda_{1,t} = \frac{\lambda_t}{P_t}$ and $\lambda_{2,t}$ are the Lagrange multipliers associated with the budget and capital accumulation constraint respectively. Equation (35) and (36) can be expressed as

$$\lambda_{2,t} = \beta E_t^{BR}[\lambda_{t+1}\frac{P_{t+1}^d}{P_{t+1}}(\frac{R_{t+1}^K}{P_{t+1}^d}u_{t+1} - \Psi(u_{t+1})) + \lambda_{2,t+1}(1-\delta)] \tag{38}$$

$$\lambda_t = \lambda_{2,t}\epsilon_t^I[1 - \Phi(\frac{I_t}{I_{t-1}}) - \Phi'(\frac{I_t}{I_{t-1}})\frac{I_t}{I_{t-1}}] + \beta E_t^{BR}[\lambda_{2,t+1}\epsilon_{t+1}^I\Phi'(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2]. \tag{39}$$

Tobin'q is $Q_t = \frac{\lambda_{2,t}}{\lambda_t}$, which represents the shadow relative price of capital with regard to consumption goods. Thus, (38) and (39) can be rewritten as

$$Q_t = \beta E_t^{BR}\{\frac{\lambda_{t+1}}{\lambda_t}[\frac{P_{t+1}^d}{P_{t+1}}(\frac{R_{t+1}^K}{P_{t+1}^d}u_{t+1} - \Psi(u_{t+1})) + Q_{t+1}(1-\delta)]\} \tag{40}$$

$$1 = Q_t\epsilon_t^I[1 - \Phi(\frac{I_t}{I_{t-1}}) - \Phi'(\frac{I_t}{I_{t-1}})\frac{I_t}{I_{t-1}}] + \beta E_t^{BR}[\frac{\lambda_{t+1}}{\lambda_t}Q_{t+1}\epsilon_{t+1}^I\Phi'(\frac{I_{t+1}}{I_t})(\frac{I_{t+1}}{I_t})^2] \tag{41}$$

We can now derive the Euler Equation by combining (31) and (33),

$$\frac{\omega_c\epsilon_t^c C_t^{-\gamma}}{\omega_c E_t^{BR}(\epsilon_{t+1}^c C_{t+1}^{-\gamma})}\frac{E_t^{BR}P_{t+1}}{P_t} = \beta(1+i_t)$$

$$E_t^{BR}[\beta(1+i_t)\frac{P_t}{P_{t+1}}\frac{\epsilon_{t+1}^c}{\epsilon_t^c}(\frac{C_{t+1}}{C_t})^{-\gamma}] = 1$$

Equations (33) and (34) together imply:

$$\frac{1+i_t}{(1+i_t^f)\Gamma b_t^f} = \frac{E_t^{BR}S_{t+1}}{S_t} \tag{42}$$

Intra-temporal condition obtains by combining Equations (31) and (32),

$$\frac{(1-\omega_c)\epsilon_t^N N_t^\phi}{\omega_c\epsilon_t^c C_t^{-\gamma}} = \frac{W_t}{P_t}. \tag{43}$$

A.2 Firms

Following Calvo (1983), $1-\theta_p$ proportions of the firms can adjust their price ($\bar{p}_t^d(j)$) in each period by maximising the discounted present value of the profits. Instead of the assumption of rational expected firms, it is assumed that firms are boundedly rational that do not fully pay attention to future macroeconomic variables.

Formally, the behavioural firms solve the problem

$$\max_{\bar{P}_t^d(j)} \sum_{h=0}^{\infty}\theta_p^h E_t^{BR}\{\Lambda_{t,t+h}[Y_{t+h}(j)(\bar{P}_t^d(j) - MC_{t+h})]\} \tag{44}$$

subject to the sequence of demand constraints

$$Y_{t+h}(j) = (\frac{\bar{P^d_t}(j)}{P^d_{t+h}})^{-\frac{\epsilon^p_{t+h}}{\epsilon^p_{t+h}-1}} Y_{t+h} \tag{45}$$

where ϵ^p_{t+h} represents the time-varying markup shocks in the domestic goods market. E^{BR}_t denotes a subjective expectation of bounded rational agent. $Y_{t+h}(j)$ is output in period $t+h$ for a firm j that last adjust its price in period t, and MC_{t+h} is nominal marginal cost in period $t+h$.

Thus, $\bar{P^d_t}(j)$ must satisfy the first order condition

$$\sum_{h=0}^{\infty} \theta^h_p E^{BR}_t \{\Lambda_{t,t+h} Y_{t+h}(j)[\bar{P^d_t}(j) - \epsilon^p_{t+h} MC_{t+h}]\} = 0. \tag{46}$$

Using the fact that $\Lambda_{t,t+h} = \beta^h E^{BR}_t[(\frac{C_{t+h}}{C_t})^{-\gamma} \frac{P_t}{P_{t+h}}]$, the previous condition (46) can be rewritten as

$$\sum_{h=0}^{\infty} (\theta_p\beta)^h E^{BR}_t \{C^{-\gamma}_{t+h} Y_{t+h}(j) \frac{P^d_{t-1}}{P^d_{t+h}} (\frac{\bar{P^d_t}(j)}{P^d_{t-1}} - \epsilon^p_{t+h} \Pi^d_{t-1,t+h} mc_{t+h})\} = 0 \tag{47}$$

where $\Pi^d_{t-1,t+h} = \frac{P^d_{t+h}}{P^d_{t-1}}$, and $mc_{t+h} = \frac{MC_{t+h}}{P^d_{t+h}}$.

A first order Taylor expansion of first order condition of profit optimisation around the zero inflation steady state yields,

$$\hat{\bar{P^d_t}}(j) = (1-\beta\theta_p) \sum_{h=0}^{\infty} (\theta_p\beta)^h E^{BR}_t (\hat{mc}_{t+h} + \hat{P^d_{t+h}} + \hat{\epsilon^p_{t+h}}) \tag{48}$$

We recall the term structure of expectations for Gabaix(2016): $E^{BR}_t(\hat{P^d_{t+h}}) = m_\pi \bar{m}^h E_t(\hat{P^d_{t+h}})$ and $E^{BR}_t(\hat{mc_{t+h}}) = m_x \bar{m}^h E_t(\hat{mc}_{t+h})$, where $\bar{m}$ is the general cognitive discounting parameter, m_π is the inattention parameter to inflation disturbance, and m_x is inattention parameter to disturbance of macroeconomics variables. Then the Equation (48) becomes

$$\hat{\bar{P^d_t}}(j) = (1-\beta\theta_p) \sum_{h=0}^{\infty} (\theta_p\beta\bar{m})^h E_t[m_x(\hat{mc}_{t+h} + \hat{\epsilon^p_{t+h}}) + m_\pi \hat{P^d_{t+h}}] \tag{49}$$

Notice that all firms that are allowed to re-optimise will always set in the same price, thus $\bar{P^d_t}(j) = \bar{P}^d_t$. Under the assumed price-setting structure, the dynamics of the domestic price index are described by the equation

$$P_t^d = [\theta_p (P_{t-1}^d)^{\frac{1}{1-\epsilon_t^p}} + (1-\theta_p)(\bar{P_t^d})^{\frac{1}{1-\epsilon_t^p}}]^{1-\epsilon_t^p} \tag{50}$$

which can be log-linearised around the zero-inflation steady state to yield

$$\pi_t^d = (1-\theta_p)(\bar{P_t^d} - P_{t-1}^d) \tag{51}$$

Similarly, the maximisation problem for domestic importer is:

$$\max_{\bar{P_t^{im}}(j)} \sum_{h=0}^{\infty} \theta_p^{*h} E_t^{BR}\{\Lambda_{t,t+h}[y_{t+h}^{im}(j)(\bar{P_t^{im}}(j) - S_{t+h}P_{t+h}^*)]\} \tag{52}$$

subject to the sequence of demand constraints

$$y_{t+h}^{im}(j) = (\frac{\bar{P_t^{im}}(j)}{P_{t+h}^{im}})^{-\frac{\epsilon_{t+h}^{pf}}{\epsilon_{t+h}^{pf}-1}} y_{t+h}^{im} \tag{53}$$

where ϵ_{t+h}^{pf} is the time varying markup on the import goods. E_t^{BR} denotes a subjective expectation of bounded rational agent.

Thus, $\bar{P_t^d}(j)$ must satisfy the first order condition

$$\sum_{h=0}^{\infty} \theta_p^{*h} E_t^{BR}\{\Lambda_{t,t+h} y_{t+h}^{im}(j)[\bar{P_t^{im}}(j) - \epsilon_{t+h}^{pf} S_{t+h}P_{t+h}^*]\} = 0. \tag{54}$$

Using the fact that $\Lambda_{t,t+h} = \beta^h E_t^{BR}[(\frac{C_{t+h}}{C_t})^{-\gamma}\frac{P_t}{P_{t+h}}]$, we can rewrite the previous condition (46) as

$$\sum_{h=0}^{\infty} (\theta_p^* \beta)^h E_t^{BR}\{C_{t+h}^{-\gamma} y_{t+h}^{im}(j)(\frac{P_{t-1}^{im}}{P_{t+h}^{im}} \frac{\bar{P_t^{im}}(j)}{P_{t-1}^{im}} - \epsilon_{t+h}^{pf} \Pi_{t-1,t+h}^{im} mc_{t+h}^{im})\} = 0 \tag{55}$$

where $mc_{t+h}^{im} = \frac{S_{t+h}P_{t+h}^*}{P_{t+h}^{im}}$ is the real marginal cost of imported goods, and $\Pi_{t-1,t+h}^{im} = \frac{P_{t+h}^{im}}{P_{t-1}^{im}}$.

A first order Taylor expansion of first order condition of profit optimisation around the zero inflation steady state yields,

$$\hat{\bar{P_t^{im}}}(j) = (1-\beta\theta_p^*) \sum_{h=0}^{\infty} (\theta_p^* \beta)^h E_t^{BR}(\hat{mc}_{t+h}^{im} + \hat{\epsilon}_{t+h}^{pf} + \hat{P}_{t+h}^{im}) \tag{56}$$

Notice that all imported firms that are allowed to re-optimise will always set in the same price, thus $\bar{P_t^{im}}(j) = \bar{P_t^{im}}$. Under the assumed price-setting structure, the dynamics of the import price index are described by the equation

$$P_t^{im} = [\theta_p^*(P_{t-1}^{im})^{\frac{1}{1-\epsilon_t^{pf}}} + (1-\theta_p^*)(\bar{P_t^{im}})^{\frac{1}{1-\epsilon_t^{pf}}}]^{1-\epsilon_t^{pf}} \tag{57}$$

which can be log-linearised around the zero-inflation steady state to yield

$$\pi_t^{im} = (1-\theta_p^*)(\bar{P_t^{im}} - P_{t-1}^{im}) \tag{58}$$

A.3 Log-linearised Model

Log-linearised representations of structural models are expressed as

$$
\begin{aligned}
&lnC_t = \bar{m}lnE_tC_{t+1} - \frac{1}{\gamma}r_t + \frac{1}{\gamma}(ln\epsilon_t^c - ln\epsilon_{t+1}^c)\text{(Euler Equation)}\\
&lnY_t = \alpha(lnK_{t-1} + lnu_t) + (1-\alpha)lnN_t + lnA_t\text{(Production Equation)}\\
&lnN_t = ln\frac{1-\alpha}{\alpha} + lnK_{t-1} + lnu_t - lnw_t + lnr_t^K\text{(Demand of Labour)}\\
&lnQ_t = \bar{m}\beta(1-\delta)lnE_tQ_{t+1} + (1-\beta(1-\delta))\bar{m}(lnE_tr_{t+1}^K + lnp_{t+1}^d) - r_t\\
&\text{(Value of Capital-Tobin's q)}\\
&lnI_t = \frac{1}{1+\beta}lnI_{t-1} + \frac{\beta\bar{m}}{1+\beta}lnE_tI_{t+1} + \frac{\kappa}{1+\beta}lnQ_t + ln\epsilon_t^I\text{(Investment Equation)}\\
&lnK_t = (1-\delta)lnK_{t-1} + \delta lnI_t\text{(The accumulation of installed capital)}\\
&lnu_t = \frac{1}{\varphi}lnr_t^K\text{(Capital Utilisation)}\\
&lnr_t^K = \frac{\varphi\overline{Y}}{\overline{K}\overline{r^K}}[lnY_t - \frac{(1-\omega_f)\bar{p^d}^{-v}}{\overline{Y}}(\bar{I}lnI_t + \bar{C}lnC_t - (\overline{C}+\bar{I})vlnp_t^d) - \frac{\overline{G}}{\overline{Y}}lnG_t - \frac{\overline{EX}}{\overline{Y}}lnEX_t]\\
&\text{(Goods Market Condition)}\\
&lnw_t = \phi lnN_t + \gamma lnC_t - lnp_t^d + ln\varepsilon_t^N\text{(Labour Supply Equation)}\\
&\pi_t^d = \beta M^dE_t(\pi_{t+1}^d) + \lambda(lnmc_t + ln\epsilon_t^p)\text{(Domestic producer inflation Equation)}\\
&\pi_t^{im} = \beta M^*E_t(\pi_{t+1}^{im}) + \lambda^*(lnmc_t^{im} + ln\epsilon_t^{pf})\text{(Imported Inflation Equation)}\\
&lnmc_t = \alpha lnr_t^K + (1-\alpha)lnw_t - lnA_t - \alpha ln\alpha + (\alpha-1)ln(1-\alpha)\text{(Real Marginal Cost Equation)}\\
&lnmc_t^{im} = lne_t - lnp_t^{im}\\
&lnEX_t = -\eta(lnp_t^d - lne_t) + lnZ_t^F + ln\varepsilon_t^{EX}\text{(Export Equation)}\\
&lnIM_t = \frac{1}{\bar{C}+\bar{I}}(\bar{C}lnC_t + \bar{I}lnI_t) - vlnp_t^{im} + ln\varepsilon_t^{IM}\text{(Import Equation)}\\
&b_t^f = (1+\bar{r})\frac{1}{1+g}b_{t-1}^f + \frac{\overline{EX}}{\overline{Y}}lnEX_t - \frac{\overline{IM}\overline{e}}{\overline{Y}\overline{p^d}}(lnIM_t + lne_t - lnp_t^d)\text{(Evolution of Net Foreign Bonds)}\\
&lne_t = \bar{m}lnE_te_{t+1} + r_t{}^f - r_t - \Gamma b_t^f + ln\varepsilon_t^\Gamma\text{(UIP)}\\
&i_t = \phi_i i_{t-1} + \phi_\pi\tilde{\pi}_t + \phi_x\tilde{y}_t + \epsilon_t^M\text{(Taylor Rule)}\\
&r_t = i_t - E_t\pi_{t+1}\text{(Fisher Equation)}\\
&lnp_t^d = lnp_{t-1}^d + \pi_t^d - \pi_t\text{(Relative Producer Price)}\\
&lnp_t^{im} = lnp_{t-1}^{im} + \pi_t^{im} - \pi_t\text{(Relative Import Price)}\\
&\pi_t = (1-\omega_f)(\bar{p^d})^{1-v}\pi_t^d + \omega_f(\bar{p^{im}})^{1-v}\pi_t^{im}\text{(Inflation Equation)}\\
&lnG_t = \rho_G lnG_{t-1} + \eta_{G,t}\text{(Government Spending Equation)}\\
&lnZ_t{}^F = \mu_{Z^f} + \rho_{Z^F}lnZ_{t-1}^F + \eta_{Z^F,t}\text{(Rest of the World Demand Equation)}\\
&r_t^f = \mu_{r^f} + \rho_{r^f}r_{t-1}^f + \eta_{r^f,t}\text{(Rest of the World Real Interest Rate Equation).}
\end{aligned}
\tag{59}
$$

Appendix B: Some Empirical Results

The UK has experienced several shifts in monetary regimes since the post-Bretton Woods era. Given that alterations in monetary policy operating procedures are likely to lead to substantial changes in monetary policy behaviour and their effects on the economy, it can be challenging to capture these dynamics with a single model estimated over the entire floating exchange-rate regime period. Since October 1992, the UK has been transitioning to an ‘inflation-targeting monetary regime’, allowing the pound to float freely. To account for these changes and avoid potential structural breaks, we use quarterly data covering the inflation-targeting periods from 1993 Q1 to 2019 Q1 for key UK macroeconomic variables. This approach helps ensure a more accurate analysis and better captures the effects of monetary policy during the relevant periods. Table B1 provides the descriptive statistics of the UK data we use.

Table B.1:Descriptive Statistics of the UK data, 1993Q1-2019Q1

Series	Mean	Standard deviation	Range[Min,Max]
Consumption	8.96	0.1275	[8.66, 9.09]
Output	9.45	0.1082	[9.17, 9.55]
Hours	3.35	0.0168	[3.30, 3.37]
Investment	7.66	0.0888	[7.48, 7.81]
Capital	11.21	0.0657	[11.08, 11.30]
Real Wage	0.84	0.1496	[0.54, 1.01]
Export	8.05	0.2312	[7.48, 8.33]
Import	8.06	0.2744	[7.42, 8.46]
Government Spending	7.77	0.1116	[7.56, 7.88]
Net Foreign Asset/GDP	-0.67	0.7122	[-2.02, 0.26]
Real exchange rate (index)	112.61	0.0902	[94.15, 129.06]
Nominal interest rate	0.92%	0.0063	[0.07%, 1.91%]
Inflation rate	0.51%	0.0045	[-0.60%, 1.87%]

Notes: Except for rates and index series, real series are expressed in per capita terms by dividing them with the working-age population and then taking logarithms.

The reduced form of the structured model has been approximated by VARX(1). And the VARX(1) has been specified in the form of (60), which serves as the auxiliary model, being a parsimonious description of some key features of bounded rationality model derived in Section 2.

$$\begin{bmatrix} Y_t \\ e_t \\ r_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ e_{t-1} \\ r_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \beta_{15} & \beta_{16} & \beta_{17} \\ \beta_{25} & \beta_{26} & \beta_{27} \\ \beta_{35} & \beta_{36} & \beta_{37} \\ \beta_{45} & \beta_{46} & \beta_{47} \end{bmatrix} \begin{bmatrix} A_{t-1} \\ t \\ const \end{bmatrix} + \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \\ \eta_{3t} \\ \eta_{4t} \end{bmatrix} \tag{60}$$

The coefficient vector β^s used to construct the Direct Wald statistic includes OLS estimates of $\beta_{11}, \beta_{12}, \beta_{13}, \beta_{14}, \beta_{21}, \beta_{22}, \beta_{23}, \beta_{24}, \beta_{31}, \beta_{32}, \beta_{33}, \beta_{34}, \beta_{41}, \beta_{42}, \beta_{43}, \beta_{44}$ and the variances of the fitted stationary residuals η_{1t}, η_{2t}, η_{3t}, and η_{4t} based on each set of simulated data; the same coefficients make up β^a estimated on the observed data. The four variances of the residuals measure the volatility properties, and the coefficients represent the dynamic properties found in the model and data(see Table B.2).

Recall the Wald statistics is calculated by (26)

$$WS = (\beta^a - \overline{\beta^s(\widehat{\theta_0})})'\Omega^{-1}(\beta^a - \overline{\beta^s(\widehat{\theta_0})})$$

where β^a denotes the VARX estimates using the actual data; $\overline{\beta^s(\widehat{\theta_0})} = E(\beta^i(\widehat{\theta_0})) = \frac{1}{s}\sum_{i=1}^{s}\beta^i(\widehat{\theta_0})$ denotes the sample average of estimates of the coefficients in auxiliary model based on s sets of simulated data from the macroeconomic model, taking $\widehat{\theta_0}$ as given. Ω is the variance-covariance matrix of the distribution of simulated estimates β^i. In essence, it measures the gap between what the macroeconomic model says the data behaviour should be and what the observed data behaviour actually is.

Table B.2: Estimated Coefficients for the Auxiliary Model

VARX coeffs	Actual data estimates[a]	Simulated data average estimates[b]	
		Bounded rationality model	Rational model
β_{11}	0.9475	0.8279	0.7623
β_{21}	-0.0371	-0.0395	-0.0262
β_{31}	-0.0466	0.0049	-0.0218
β_{41}	0.0437	-0.0321	-0.0511
β_{12}	-0.0374	-0.0080	-0.0280
β_{22}	0.9737	0.8496	0.8454
β_{32}	-0.0422	-0.0411	-0.0192
β_{42}	0.0378	-0.0260	-0.0119
β_{13}	-0.8159	-0.9566	-0.6447
β_{23}	2.2723	0.1460	0.1226
β_{33}	0.6187	0.7016	-0.6270
β_{43}	0.2752	0.2147	-0.2087
β_{14}	-0.8374	-0.7212	0.0018
β_{24}	2.6979	-0.2772	-0.0150
β_{34}	1.0446	0.1798	0.2226
β_{44}	-0.1187	-0.2242	0.4294
$\sigma^2_{\eta_{1t}}$	0.000029	0.00015	0.0018
$\sigma^2_{\eta_{2t}}$	0.00055	0.00020	0.00092
$\sigma^2_{\eta_{3t}}$	0.000013	0.000063	0.000068
$\sigma^2_{\eta_{4t}}$	0.000014	0.000023	0.00059

Notes: a. actual data estimates are β^a in Wald statistics (26); b. simulated data average estimates are $\overline{\beta^s(\widehat{\theta_0})}$ in Wald statistics (26).

Appendix C.1: Granger Causality Test on the Auxiliary Model

The primary objective of this study is to investigate whether the bounded rationality model, incorporating discounting in the Euler equation, New Keynesian Philips curves, and the exchange rate equation, can establish implied causal relationships among key variables using UK data. Notably, the endogenous variables in these equations are output, real exchange rate, inflation, interest rate, and consumption. By employing Granger Causality tests, we identify the optimal combination of key variables for the auxiliary model. Specifically, the results shown in Table C suggest that output, real exchange rate, inflation, and interest rate effectively capture the causal relationships, while the inclusion of consumption does not yield significant results at the 5% significance level.

Table C: Granger Causality Test on the Auxiliary Model

Variable	Null Hypothesis	Chi-sq	df	P-value
Y	e, π,r cannot Granger-causes Y jointly	25.3845	3	0.0000***
e	Y, π,r cannot Granger-causes e jointly	8.5339	3	0.0362**
π	Y, e,r cannot Granger-causes π jointly	23.0783	3	0.0000***
r	Y, π,e cannot Granger-causes r jointly	170.6921	3	0.0000***
Adding Consumption				
C	Y, r, π,e cannot Granger-causes C jointly	8.7000	4	0.0690*
e	Y, r,C cannot Granger-causes e jointly	4.8064	3	0.1865
e	Y,π,C cannot Granger-causes e jointly	5.2554	3	0.1540

Notes: Y,e,π,r and C represent output, real exchange rate, inflation, real interest rate, consumption, respectively. p-value with ***, ** and * indicate a rejection of the model at 1%, 5% and 10% significance level respectively.

Appendix D: Welfare Loss Function

We now derive a welfare loss function 61 based on linear-quadratic approximation for a Cole-Obstfeld preference (Cole and Obstfeld, 1991), which assumes the unitary elasticities of substitution ($\sigma = \nu = \nu^{im} = 1$),

$$V = -\frac{(1-\omega_f)}{2}\sum_{t=0}^{\infty}\beta^t[\frac{\nu^d}{\lambda_\pi}\sigma_\pi^2 + (1+\phi)\sigma_y^2] \tag{61}$$

Taking the unconditional expectation of 61 with $\beta \to 1$ yields 62 which implies that the expected welfare losses of any policy that deviated from a strict inflation targeting can be written in terms of the variances of inflation (σ_π^2) and the output gap (σ_y^2).

$$L_1 = -\frac{(1-\omega_f)}{2}[\frac{\nu^d}{\lambda_\pi}\sigma_\pi^2 + (1+\phi)\sigma_y^2] \tag{62}$$

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