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Optimisation of livestock routing on farms

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ABSTRACT

A grass-based livestock farm will typically be partitioned into a set of fields which may not be contiguous. The livestock in question will be distributed among these fields plus a set of buildings. This distribution will change over time as a result of livestock being routed between different locations. This change in distribution is not a random process. It is instead planned by the farmer to satisfy a set of constraints while minimising workload. The set of constraints in question are designed to maximise the performance of the farm and, in many cases, will be large.

In this work, we refer to the above planning problem as the *Livestock Routing Problem* (LRP). We propose modelling the LRP as an integer program, which is a specific type of mathematical optimisation problem. Our model is general in nature whereby many farming activities can be incorporated. These activities include rotational grazing, silage production and livestock breeding.

In our analysis we consider many different instances of the LRP and attempt to solve these instances using an off-the-shelf integer program solver. In most cases, an optimal or close to optimal solution is found in reasonable time. These results demonstrate that the proposed methods could be used within a decision support system for livestock farmers and, in turn, reduce the workload associated with the routing of livestock.

1. Introduction

The practice of agriculture creates the majority of our world's food. In the United Kingdom (UK) agriculture activity accounts for 71% of total land use and has a total labour force of 472 thousand working on commercial farms (Department for Environment, Food and Rural Affairs, 2020). Agricultural activity can be divided into two main types, arable farming and livestock farming. Arable farming concerns the production of crops while livestock farming concerns the production of products such as meat and milk through the breeding of animals. The proportions of UK agricultural land dedicated to each of these types of farming is 72% and 27% respectively.

One of the main types of livestock farms in the UK is cattle farms. Grass is the predominant feed source on such farms and this is attributed to the suitability of the climate for grass growth (Higgins et al., 2019). The average grass- or pasture-based cattle farm in the UK is about 50 hectares in size and has a herd size of about 150 (Department for Environment, Food and Rural Affairs, 2020). In most farms, the cattle will spend most of their time directly grazing in fields where the grass grows. However, in the winter time when the conditions are poor and grass growth is limited, the cattle will typically spend time indoors eating fodder such as silage. Although the statistics presented above relate specifically to the UK, similar statistics are observed in many European countries.

A grass-based farm will typically be partitioned into a set of smaller fields. It is not uncommon for the fields in question to be many kilometres apart. The livestock will be distributed among these fields plus a set of buildings and this distribution will change over time. This distribution is not random; instead, it is planned by the farmer in order to satisfy a set of constraints while minimising workload. The set of constraints in question is designed to maximise the performance of the farm and, in many cases, this set will be large. Examples of possible elements in this set include the following. Ensuring cattle being fattened for sale are isolated from other cattle so that additional more expensive feed (e.g. pelleted feeds) can be made available to them alone. Ensuring sufficient land is set aside during the summer period to grow and harvest the fodder necessary for the winter period when growth levels are significantly less.

As evident from the examples above, satisfying many of the constraints involves the routing of cattle between different locations. Cattle are not domestic animals like cats and dogs, which can easily be manipulated. Therefore routing cattle between locations has a significant workload and generally requires a team of people. Given this workload,

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a farmer must plan his/her actions to ensure this activity is minimised while still satisfying the constraints in question. In this work, we refer to this planning problem as the *Livestock Routing Problem* (LRP). Note that this name is derived from the vehicle routing problem, which concerns the problem of routing vehicles as opposed to cattle. Given the large number of constraints and variables involved, it may be challenging for a farmer to manually solve the LRP optimally. To overcome this challenge, in this work we propose modelling the LRP as an integer program. This model is general in nature whereby many farming activities including rotational grazing, silage production and livestock breeding can be formulated. We propose to solve such instances using an existing integer program solver. To the authors' knowledge, this represents the first work to model and solve the problem in this way.

The main contributions of this paper are the following:

- We define the Livestock Routing Problem (LRP), which concerns the routing of livestock between different farm locations.
- We propose an integer program model of the LRP.
- We demonstrate that many farming activities can be formulated as instances of this model.
- We demonstrate that an off-the-shelf integer program solver can be used to determine useful solutions to problem instances of varying sizes in reasonable time.

The remainder of this paper is structured as follows. In Section 2 we review related works on optimising livestock farms and routing. In Section 3 we describe how the LRP can be modelled as an integer program. In doing so, we demonstrate how many common livestock farming activities correspond to instances of this model. In Section 4 we present an evaluation of the proposed approach for solving such instances. Finally, in Section 5 we conclude this work and discuss possible directions for future research.

2. Literature review

In this section, we first review related works on the application of optimisation methods to livestock farms. We then review related works on routing.

The use of data science and Internet of Things (IoT) technologies to help optimise the operation of livestock farms is an emerging field of research (Shalloo et al., 2021). Systems which aim to help perform this task are commonly referred to as decision support systems in the literature (Shalloo et al., 2018). In the following, we present a review of works most relevant to the current research. However, several more in-depth articles reviewing the entire field exist (Akhigbe et al., 2021; Benos et al., 2021; Hostiou et al., 2017; Michie et al., 2020; Shalloo et al., 2021; Wolfert et al., 2017).

The 'grass wedge' is a visualisation technique used to represent the relationship between the amount of grass and the number of livestock in individual fields (Macdonald et al., 2010). This technique aims to support decisions regarding the assignment of livestock to fields to maximise the use of grass. In a related work, Hanrahan et al. (2017) proposed a system for measuring grass growth in individual fields and used this information to assign livestock to fields. Higgins et al. (2019) proposed the use of data-driven approaches to soil nutrient management to maximise grass growth. French et al. (2015) proposed combining a system for measuring grass with virtual fencing, which is a technology that allows livestock movement to be restricted to a subset of a larger field using a virtual fence. Woodward et al. (1995) proposed a method for optimising rotational grazing of fields where fields are given time to recover between periods of being grazed by cattle. This problem is also referred to in the literature as pasture allocation. Several works have considered data-driven approaches to improving livestock breeding decisions (Mottram, 2016; Saint-Dizier & Chastant-Maillard, 2018). In some situations, individual livestock will need to be separated from others. This task is known as 'cow drafting' and several systems have been proposed to automate it (Shalloo et al., 2021). Bach and Cabrera (2017) proposed a data-driven system for personalised or precision feeding of livestock whereby each animal is assigned a different feeding policy based on their characteristics. In related work, Cabrera et al. (2020) proposed a system for assigning sets of cattle with the same feeding policy to fields or buildings. The above review demonstrates that several works have previously considered the problem of assigning livestock to different locations, such as fields and buildings, to achieve a given objective. However, none of these works have considered the problem of routing livestock between different locations, which must first be solved to achieve a given assignment. Modelling and optimising this routing problem represents the main contribution of the current work.

Although, we argue, modelling and optimising the routing of livestock is a novel problem, many related routing problems have been studied in other domains. In fact, routing is a ubiquitous activity performed in many contexts including, by packets in computer networks, people driving vehicles in street networks and autonomous drones on different planets. In the following we present a review of those works most relevant to the current research. A more in-depth review of works in this area can be found in Toth and Vigo (2002). The assignment problem is a classic optimisation problem which involves finding a matching in a weighted bipartite graph such that the sum of weights of the matching edges is minimum. A matching in a graph is a set of edges without common vertices. Many routing problems can be formulated as an assignment problem. For example, Fisher and Jaikumar (1981) formulated the problem of using a fleet of drivers to deliver products stored at a central depot to customers as an assignment problem. The Travelling Salesman Problem (TSP) is the most famous routing and combinatorial optimisation problem. Given a set of locations, the TSP is concerned with the problem of determining the shortest route that visits each location and then returns to the first location (Applegate et al., 2011). The Vehicle Routing Problem (VRP) is another routing problem. Given a set of locations and a set of vehicles, the VRP concerns determining the shortest set of vehicle routes such that each location is visited by one vehicle and all vehicles return to a start location when complete (Toth & Vigo, 2002). When there is only one vehicle, the VRP reduces to the TSP. There are also many generalisations of the VRP that involve adding additional constraints to the problem. These include adding capacity constraints so that vehicles have a maximum carrying capacity, and adding constraints so that each location can only be visited within a specific time window.

Another related routing problem is the orienteering problem. This problem concerns achieving the maximum total score obtained from visiting locations subject to a constraint on the total distance which can be travelled (Gunawan et al., 2016). Similar to the VRP, there are many generalisations of the orienteering problem. These include the team orienteering problem, which involves a team instead of a single agent, attempting to achieve a maximum total score. Parker et al. (2020) considered the problem of routing patients between different hospitals to maximise care. This problem is distinct from the LRP proposed in this work but contains some similarities. For example, in the former, individual hospitals have patient capacity constraints and therefore patients are routed between different hospitals to satisfy these constraints. In the latter, individual fields have livestock capacity constraints and therefore livestock are routed between different fields to satisfy these constraints.

3. Problem statement

As described in the introduction, the LRP concerns the problem of optimising livestock routing on farms. In this section, we describe how the LRP can be modelled as an integer program. Broadly speaking, this model aims to minimise the work required to successfully perform the routing in question. Minimising this work has many potential benefits such as reducing operational costs and improving the sustainability of farming. The remainder of this section is structured as follows. In

Table 1
Summary of model parameters and variables.

Parameter	Description
n	Number of livestock.
m	Number of locations (fields or buildings).
t	Number to time steps.
$w_{kk'}$	Number of units of works required to route a set of livestock from location k to location k'.
C^E	Set of equality constraints.
C^I	Set of inequality constraints.
Variable	**Description**
a_{ijk}	Equals 1 if and only if at timestep i, animal j is assigned to location k. Otherwise, this variable equals 0.
$z_{ikk'}$	Equals 1 if and only if one or more livestock are routed from location k to location k' between timesteps i and $i+1$.

Section 3.1 we formally define the model. In Section 3.2 we then demonstrate how common livestock farming activities can be formulated as instances of this model. Finally, in Section 3.3 we present a formal computational complexity analysis of the proposed model.

3.1. Model of livestock routing

In this section we present a formulation of the LRP as an integer program. A summary of the corresponding parameters and variables is provided in Table 1. For a given farm, let there be n cattle, m fields or buildings, and t timesteps over which modelling is performed. A timestep can be any duration such as a day or week and depends on the resolution one wishes to perform modelling. In the remainder of this article, we refer to fields and buildings as *locations*. We model the assignment of livestock to locations at each timestep as an integer valued tensor $A = (a_{ijk}) \in \{0,1\}^{t \times n \times m}$ subject to the constraint that $\sum_{k=1}^{m} a_{ijk} = 1$ for all values of i and j. The variable a_{ijk} equals 1 if and only if at timestep i, animal j is assigned to location k. Otherwise, this variable equals 0. The constraint $\sum_{k=1}^{m} a_{ijk} = 1$ for all values of i and j ensures that, at each timestep i, each animal j is assigned to a single location.

Now, let $W = (w_{ij}) \in \mathbb{R}^{m \times m}$ be the matrix where the value $w_{kk'}$ equals the number of units of work required to route a set of animals from location k to location k'. A unit of work may correspond to a person-hour and depends on how the modeller wishes to perform this quantification. We assume the diagonal elements of W equal zero because it requires zero work to route livestock between two locations which are the same location. Note that, the values $w_{kk'}$ may not be uniform and may instead vary as a function of the relationship between the locations in question. For example, the amount of work required to route an animal between two adjacent fields sharing an entrance will typically be less than that required to route an animal between two fields many miles apart. Furthermore, W may not be symmetric; that is, $w_{kk'}$ may not equal $w_{k'k}$. This would occur if the work required to round up and sort cattle varies between locations. Finally, we assume that each of the variables $w_{kk'}$ is a constant and does not vary as a function of the number of cattle being routed between k and k'. We believe this to be a reasonable approximation of reality. For example, it requires appropriately the same amount of work to route two animals between a pair of fields as it does to route a single animal between the same pair of fields.

Let C^E and C^I be sets of equality and inequality constraints respectively. These are defined with respect to the tensor A. For example, a possible constraint in the set C^E is the following. Consider the case where the farmer in question requires field k to be set aside during timestep i to grow silage. This requirement can be modelled using the equality constraint $\sum_{j=1}^{m} a_{ijk} = 0$. Note that, different instances of the LRP will be parameterised by different values of the above variables. For example, different farms may have a different number of livestock n, a different number of locations m, and different sets of constraints C^E and C^I.

The LRP concerns the problem of assigning livestock to locations such that all constraints are met and the cost of routing livestock between different locations is minimised. We formulate this problem as the following integer program (IP) where A is the single decision variable and all other variables are auxiliary or constant parameters.

$$\min_{A} \sum_{i=1}^{t-1} \sum_{k=1}^{m} \sum_{k'=1}^{m} z_{ikk'}\, w_{kk'} \tag{1a}$$

$$\text{s.t.} \sum_{k=1}^{m} a_{ijk} = 1,\ \forall\, i = 1,\dots,t,\ j = 1,\dots,n \tag{1b}$$

$$z_{ikk'} = \min(1, \sum_{j=1}^{n} a_{ijk}\, a_{i+1jk'}),\ \forall\, i = 1,\dots,t-1,\ k,k' = 1,\dots,m \tag{1c}$$

$$c = 0,\ \forall\, c \in C^E \tag{1d}$$

$$c \le 0,\ \forall\, c \in C^I \tag{1e}$$

$$a_{ijk} \in \{0,1\},\ \forall\, i = 1,\dots,t,\ j = 1,\dots,n,\ k = 1,\dots,m. \tag{1f}$$

Here, the tensor $Z = (z_{ikk'}) = \{0,1\}^{t-1 \times m \times m}$ defined in Eq. (1c) is an auxiliary variable. The variable $z_{ikk'}$ equals 1 if and only if one or more livestock are routed from location k to location k' between timesteps i and $i+1$. Otherwise, this variable equals 0. The objective function in Eq. (1a) can be interpreted as follows. The indices i, k and k' iterate over timesteps, locations and locations respectively. The variable $z_{ikk'}$ is multiplied by the variable $w_{kk'}$, which equals the amount of work required to route a set of livestock between the locations in question. Therefore, the summation in Eq. (1a) is a sum of the work required to route livestock between locations. As discussed above, the constraint $\sum_{k=1}^{m} a_{ijk} = 1$ for all values of i and j ensures that, at each timestep i, each animal j is assigned to a single location. The constraint sets C^E and C^I will vary between problem instances. An example constraint in the set C^E is described above. The integer valued constraint $a_{ijk} \in \{0,1\}$ for all values of i, j and k in Eq. (1f) ensures that, at any given time, an animal is either assigned or not assigned to a given location. That is, they cannot be half assigned to one location and half assigned to another location at the same time.

The above model of the LRP attempts to minimise the total sum of units of work. Consider the case where a unit of work corresponds to one person-hour. If we assume a single person works a given number of hours per day/week, the model may be used to determine the number of full-time jobs necessary to perform all the routing tasks in question. This information could, in turn, be used by managers and stakeholders to make higher-level strategic decisions.

3.2. Problem instances

The LRP model presented in the previous section is deliberately very general. Consequently, many common livestock farming activities can be formulated as instances of this model. In this section, we demonstrate this by considering five such activities and present the corresponding formulations.

3.2.1. Continuous and rotational grazing

In a grass-based system, there are two main livestock grazing policies implemented by farmers. In a rotational grazing policy, fields are left unoccupied between periods of grazing by livestock. The motivation for this is to permit the recovery and growth of fields after grazing. In a continuous grazing policy, livestock graze fields continuously and fields are not left unoccupied between periods of grazing. We now describe how continuous and rotational grazing policies can be formulated as instances of the proposed model.

To implement a continuous grazing policy, the number of livestock grazing in each field must be less than or equal to a corresponding feeding capacity at each timestep. This can be achieved by adding a corresponding inequality constraint for each field to the set C^I. Specifically, for a field with index k and continuous feeding capacity of x_k, the inequality constraint in question is defined as:

$$\sum_{j=1}^{n} a_{ijk} \leq x_k, \forall i = 1, \dots, t. \tag{2}$$

Integrating this constraint into the original optimisation problem defined in Eq. (1) gives the following optimisation problem:

$$\min_A \sum_{i=1}^{t-1} \sum_{k=1}^{m} \sum_{k'=1}^{m} z_{ikk'} w_{kk'} \tag{3a}$$

$$\text{s.t.} \sum_{k=1}^{m} a_{ijk} = 1, \forall i = 1, \dots, t, j = 1, \dots, n \tag{3b}$$

$$z_{ikk'} = \min(1, \sum_{j=1}^{n} a_{ijk} a_{i+1jk'}), \forall i = 1, \dots, t-1,\ k, k' = 1, \dots, m \tag{3c}$$

$$\sum_{j=1}^{n} a_{ijk} - x_k \leq 0, \forall i = 1, \dots, t,\ k = 1, \dots, m \tag{3d}$$

$$a_{ijk} \in \{0, 1\}, \forall i = 1, \dots, t,\ j = 1, \dots, n,\ k = 1, \dots, m. \tag{3e}$$

There are several ways that one could define a rotational grazing policy. In this work, we define such a policy as follows. Let $p, q \in \mathbb{Z}$, where $p \geq q$, be two user-defined model parameters. A rotational grazing policy ensures that, for each field, each contiguous sequence of $p + 1$ timesteps contains at most q timesteps where livestock are assigned to the field in question. To implement this policy we introduce an auxiliary variable $G = (g_{ik}) \in \{0, 1\}^{t \times m}$, defined as follows:

$$g_{ik} = \min(1, \sum_{j=1}^{n} a_{ijk}). \tag{4}$$

An individual value g_{ik} indicates whether or not at timestep i, one or more animals are assigned to field k. Given this, a rotational grazing policy is implemented by adding the following inequality constraint to the set C^I:

$$\sum_{i'=i}^{i+p} g_{i'k} \leq q, \forall i = 1, \dots, t-p,\ k = 1, \dots, m. \tag{5}$$

Integrating this constraint into the original optimisation problem defined in Eq. (1) gives the following optimisation problem:

$$\min_A \sum_{i=1}^{t-1} \sum_{k=1}^{m} \sum_{k'=1}^{m} z_{ikk'} w_{kk'} \tag{6a}$$

$$\text{s.t.} \sum_{k=1}^{m} a_{ijk} = 1, \forall i = 1, \dots, t, j = 1, \dots, n \tag{6b}$$

$$z_{ikk'} = \min(1, \sum_{j=1}^{n} a_{ijk} a_{i+1jk'}), \forall i = 1, \dots, t-1,\ k, k' = 1, \dots, m \tag{6c}$$

$$\sum_{j=1}^{n} a_{ijk} - x_k \leq 0, \forall i = 1, \dots, t,\ k = 1, \dots, m \tag{6d}$$

$$\sum_{i'=i}^{i+p} g_{i'k} - q \leq 0, \forall i = 1, \dots, t-p,\ k = 1, \dots, m \tag{6e}$$

$$g_{ik} = \min(1, \sum_{j=1}^{n} a_{ijk}), \forall i = 1, \dots, t,\ k = 1, \dots, m \tag{6f}$$

$$a_{ijk} \in \{0, 1\}, \forall i = 1, \dots, t,\ j = 1, \dots, n,\ k = 1, \dots, m. \tag{6g}$$

The above formulation also includes gazing capacity constraints in Eq. (6d). A farmer will set the gazing capacity values x_k in conjunction with the value q. For example, if higher gazing capacity values are set, then fields will need a greater time to recover and, in turn, a higher value of q will be required.

3.2.2. Silage production and winter housing

In many countries, it is not possible to perform rotational or continuous grazing all year due to seasonal variations in grass growth levels and weather conditions. For example, during the winter season in the UK and Ireland, grass growth is too small and weather conditions are too inclement for livestock to graze outdoors. To overcome this challenge farmers in such countries typically use a policy whereby they set aside fields during the summer to grow a feed known silage, which is then stored. Subsequently, during the winter season, farmers house livestock in buildings and feed them this silage. The policy can be implemented by adding two constraints to the original LRP.

The first constraint concerns ensuring that the fields to be set aside during the summer season are assigned zero livestock during this period. Let F be the set of fields to be set aside and let G be the set of timesteps during which they are to be set aside. The constraint in question can be achieved by adding the following equality constraints to the set C^E:

$$\sum_{j=1}^{n} a_{ijk} = 0, \forall i \in G,\ k \in F. \tag{7}$$

The second constraint concerns ensuring that, during the winter season, all livestock are assigned to buildings. Let H be the set of timesteps corresponding to the winter season and let B be the set of buildings to which livestock should be assigned during this period. Recalling that n is the number of livestock, the constraint in question can now be achieved by adding the following two equality constraints to the set C^E:

$$\sum_{k \in B} a_{ijk} = 1, \forall i \in H,\ j = 1, \dots, n \tag{8}$$

$$\sum_{k \in B} a_{ijk} = 0, \forall i \notin H,\ j = 1, \dots, n. \tag{9}$$

The first constraint ensures that, for all timesteps in H, all livestock are assigned to buildings in B. Meanwhile, the second constraint ensures that, for all timesteps not in H, zero livestock are assigned to buildings in B.

3.2.3. Livestock breeding

The breeding of new livestock is also an important activity on many farms. For example, on cattle farms a subset of cows will produce a new calf each year and the successful management of this process requires the farmer to implement several policies. We now describe three such policies and see how they can be formulated as instances of the proposed model.

A farmer will typically wish to implement a policy whereby one year old cows, known as heffors, are not assigned to the same location as a mature unneutered bull. The purpose of this is to ensure the heifers do not become pregnant which would negatively impact their growth and development. Let S and S' be the sets of bulls and heifers respectively and let G be the set of timesteps where these two sets must be separated. The policy in question can be implemented by adding the following equality constraint to the set C^E:

$$a_{ijk} a_{ij'k} = 0, \forall i \in G,\ j \in S,\ j' \in S',\ k = 1, \dots, m. \tag{10}$$

Note that a policy for weaning calves from cows, where calves are separated from corresponding parent cows, can also be implemented using a similar approach.

A farmer will, in most cases, be able to predict the calving period of each cow. Given this information, they will typically wish to implement a policy whereby, during this period, the cow in question will be assigned to a location suitable for calving. The purpose of this is to ensure that the farmer can easily monitor and care for the cow and calf. Let B be the set of suitable buildings and fields. Also, let H be a set of timesteps and cow pairs where $(i, j) \in H$ indicates that, at time i, cow j should be assigned to an element of B. The policy in question can be implemented by adding the following equality constraint to the set C^E:

$$\sum_{k \in B} a_{ijk} = 1, \forall (i, j) \in H. \tag{11}$$

After a cow has calved, a farmer will typically want to implement a policy whereby the cow and calf are assigned to the same location. The purpose of this is to ensure that the cow can care for the calf. Let P be the set of cow and calf pairs where $(p, p') \in P$ indicates that the cow p is the mother of the calf p'. The policy in question can now be implemented by adding the following equality constraint to the set C^E:

$$a_{ipk} = a_{ip'k}, \forall i = 1, \dots, t, (p, p') \in P. \tag{12}$$

3.2.4. Zero routing timesteps

A farmer may wish to implement a policy where no livestock routing is performed on a subset of timesteps. For example, on some days the farmer may wish to rest or not have sufficient assistance to perform the activity. Let G be the subset of timesteps on which the farmer does not wish to perform livestock routing. The policy in question can be implemented by adding the following equality constraint to the set C^E:

$$a_{ijk} = a_{i+1jk}, \forall i \in G, j = 1, \dots, n, k = 1, \dots, m. \tag{13}$$

For each timestep i in G, this constraint ensures that the assignments of livestock to locations at timesteps i and $i + 1$ are equal.

3.2.5. Livestock care

The objective function in Eq. (1a) of the original LRP formulation only models the work required to route livestock. Apart from this, a farmer may also wish to model the work required to care for livestock assigned to different locations. The work in question may not be uniform across locations and in such cases it is useful, if possible, to assign livestock to those locations where the corresponding workload is lower. For example, caring for livestock assigned to a field without a natural water source will require more work than a field with one. We now describe how the work required for caring can be modelled and how minimising this work can be formulated as an instance of the proposed model.

Let $R = (r_i) \in \mathbb{R}^m$ where an assignment $r_i = x$ indicates that it requires x units of work to care for an animal assigned to location i for a single timestep. A unit of work may be a person-hour and depends on how the modeller wishes to quantify work. Note that the values of r_i will not be uniform and will, instead, vary as a function of the location in question. To jointly minimise the work required to route and care for livestock, we replace the objective function in Eq. (1a) with the following, which contains an additional term for measuring the cost of caring:

$$\min_A \sum_{i=1}^{t-1} \sum_{k=1}^{m} \sum_{k'=1}^{m} z_{ikk'} \, w_{kk'} + \sum_{i=1}^{t} \sum_{j=1}^{n} \sum_{k=1}^{m} a_{ijk} \, r_k. \tag{14}$$

Note that, in the above, we described how several different farming activities can be independently formulated as instances of the proposed model. However, we can jointly formulate more than one of these activities as a single instance of the model. For example, we could jointly formulate the problems of continuous grazing and livestock care described above by defining an instance of the model which contains both the constraints defined in Eqs. (2) and (14) respectively.

3.3. Complexity analysis

As discussed in the previous section, the LRP formulation presented in Section 3.1 is very general in that many common livestock farming activities can be formulated as instances of this model. In this section we prove that the computational complexity of these instances can vary from polynomial to NP-hard.

First consider the problem instance that combines the activities of continuous grazing and livestock care described in Sections 3.2.1 and 3.2.5 respectively. Combining these activities gives the following integer program formulation, where the variables in question are defined in previous sections. Note that an optimal solution to the problem contains no routing of livestock between different locations.

$$\min_A \sum_{i=1}^{t-1} \sum_{k=1}^{m} \sum_{k'=1}^{m} z_{ikk'} \, w_{kk'} + \sum_{i=1}^{t} \sum_{j=1}^{n} \sum_{k=1}^{m} a_{ijk} \, r_k \tag{15a}$$

$$\text{s.t.} \sum_{k=1}^{m} a_{ijk} = 1, \forall i = 1, \dots, t, j = 1, \dots, n \tag{15b}$$

$$z_{ikk'} = \min(1, \sum_{j=1}^{n} a_{ijk} \, a_{i+1jk'}), \forall i = 1, \dots, t-1, k, k' = 1, \dots, m \tag{15c}$$

$$\sum_{j=1}^{n} a_{ijk} - x_k \le 0, \forall i = 1, \dots, t, k = 1, \dots, m \tag{15d}$$

$$a_{ijk} \in \{0, 1\}, \forall i = 1, \dots, t, j = 1, \dots, n, k = 1, \dots, m. \tag{15e}$$

Theorem 1. *The integer program defined in Eq. (15) can be solved in $O(n^3)$ time where n equals the number of livestock.*

Proof. Let U be the set of n livestock and let V be a set that contains x_k copies of each location k where x_k is the livestock feeding capacity of location k. The set V therefore contains a total of $\sum x_k$ elements. Let T be a $|U| \times |V|$ matrix where T_{uv} equals the units of work required to care for an animal assigned to location u for a single timestep. The integer program in Eq. (15) corresponds to an instance of the assignment problem which involves matching elements of U to elements of V and can be solved in $O(n^3)$ time using the Hungarian algorithm (Edmonds & Karp, 1972). □

We now consider the problem instance which involves the activity of rotational grazing described in Section 3.2.1. The corresponding integer program formulation is defined in Eq. (6) where the variables in question are defined in the above section.

Theorem 2. *The integer program defined in Eq. (6) is NP-hard.*

Proof. Consider the case where the number of timesteps t equals the number of fields m, the feeding capacity x_k of each field equals 1, the number of livestock n equals 1 and the rotational grazing parameters p and q equal t and $t - 1$ respectively. That is, there are an equal number of fields and timesteps, and each field can only be grazed for a single timestep. This case corresponds to an instance of the Travelling Salesman Problem (TSP) which is NP-hard (Applegate et al., 2011). □

Finally, consider the problem instance that combines the activities of continuous grazing and livestock breeding described in Sections 3.2.1 and 3.2.3 respectively. For livestock breeding, we specifically consider the problem instance where a set H of livestock pairs must be assigned to different locations. Combining these activities gives the following integer program formulation, where the variables in question are defined in the above sections. Note that, the constraint in Eq. (16d) ensures that each pair $(j, j') \in H$ of livestock is not assigned to the same location at any timestep.

$$\min_A \sum_{i=1}^{t-1} \sum_{k=1}^{m} \sum_{k'=1}^{m} z_{ikk'} \, w_{kk'} \tag{16a}$$

$$\text{s.t.} \sum_{k=1}^{m} a_{ijk} = 1,\ \forall\, i = 1,\dots,t,\ j = 1,\dots,n \tag{16b}$$

$$z_{ikk'} = \min\,(1, \sum_{j=1}^{n} a_{ijk}\, a_{i+1jk'}),\ \forall\, i = 1,\dots,t-1,\ k,k' = 1,\dots,m \tag{16c}$$

$$\sum_{k=1}^{m} a_{ijk}\, a_{ij'k} = 0,\ \forall\, i = 1,\dots,t,\ \forall (j,j') \in H \tag{16d}$$

$$\sum_{j=1}^{n} a_{ijk} - x_k \le 0,\ \forall\, i = 1,\dots,t,\ k = 1,\dots,m \tag{16e}$$

$$a_{ijk} \in \{0,1\},\ \forall\, i = 1,\dots,t,\ j = 1,\dots,n,\ k = 1,\dots,m. \tag{16f}$$

Theorem 3. *The integer program defined in Eq. (16) is NP-hard.*

Proof. Consider the case where the number of timesteps t equals 1 and the feeding capacity x_k of each field equals the number of livestock n. We construct a graph where each livestock corresponds to a vertex and an edge exists between two vertices if and only if the corresponding pair of livestock is an element of H. That is, an edge exists between two vertices if and only if the corresponding pair of livestock must be assigned to different locations. Finding a solution to this case corresponds to finding an m-colouring of the above graph where m is the number of locations. This problem is NP-hard (Lewis, 2021). □

4. Results and analysis

In this section we present an evaluation of the proposed integer programming model for solving the LRP. As demonstrated in Section 3.2, the proposed model is general in nature whereby many common farming activities can be formulated as instances of the model. In this section we consider three specific problem instances where each combines two or more of the activities presented in Section 3.2. All integer program formulations were implemented in Python using the Gurobi Optimiser version 9.1.2, which is a commercial integer program solver. This optimiser uses a branch and cut algorithm for solving individual problem instances (Mitchell, 2002). All experiments were run on a PC with 16 GB of RAM and an Intel CPU containing eight cores each running at 2.80 GHz. The time limit of the optimiser was set to 600 s per run. If an optimal solution is not found before this limit is reached, the optimiser returns the current best solution found. The optimiser always returns the relative optimality gap which equals the relative difference between the best observed solution and current best bound on the optimal solution. This value is expressed as a percentage, where a globally optimal solution has a 0% relative optimality gap.

4.1. Continuous grazing, silage production & winter housing

The first model we consider combines the activities of continuous grazing with silage production and winter housing. These are described in Sections 3.2.1 and 3.2.2 respectively. Combining these activities gives the following integer program formulation.

$$\min_{A} \sum_{i=1}^{t-1} \sum_{k=1}^{m} \sum_{k'=1}^{m} z_{ikk'}\, w_{kk'} \tag{17a}$$

$$\text{s.t.} \sum_{k=1}^{m} a_{ijk} = 1,\ \forall\, i = 1,\dots,t,\ j = 1,\dots,n \tag{17b}$$

$$z_{ikk'} = \min\,(1, \sum_{j=1}^{n} a_{ijk}\, a_{i+1jk'}),\ \forall\, i = 1,\dots,t-1,\ k,k' = 1,\dots,m \tag{17c}$$

$$\sum_{j=1}^{n} a_{ijk} - x_k \le 0,\ \forall\, i = 1,\dots,t,\ k = 1,\dots,m \tag{17d}$$

Table 2
An example matrix $W = (w_{ij}) \in \mathbb{R}^{m\times m}$ is displayed where $m = 4$. An assignment $w_{ij} = x$ indicates that it requires x units of work to route a single cow from location i to location j.

Location	1	2	3	4
1	0	1	4	6
2	2	0	4	7
3	4	4	0	6
4	6	6	5	0

$$\sum_{j=1}^{n} a_{ijk} = 0,\ \forall\, i \in G,\ k \in F \tag{17e}$$

$$\sum_{k\in B} a_{ijk} = 1,\ \forall\, i \in H,\ j = 1,\dots,n \tag{17f}$$

$$\sum_{k\in B} a_{ijk} = 0,\ \forall\, i \notin H,\ j = 1,\dots,n \tag{17g}$$

$$a_{ijk} \in \{0,1\},\ \forall\, i = 1,\dots,t,\ j = 1,\dots,n,\ k = 1,\dots,m. \tag{17h}$$

Note that, the constraint in Eq. (17d) relates to the continuous grazing aspect of the problem. The constraints in Eqs. (17e), (17f) and (17g) relate to the silage production and winter housing aspects of the problem.

We first illustrate the above problem using a small toy problem instance. Let the number of timesteps t be 4, the number of livestock n be 6 and the number of locations m be 4. Let the feeding capacities x^1, x^2, x^3 and x^4 be 3, 3, 3 and 6 respectively and let the matrix $W \in \mathbb{R}^{m\times m}$ of routing work values equal that displayed in Table 2. Let the set F of fields set aside for silage be $\{1\}$ and let the set G of timesteps that these fields are to be set aside equal $\{2\}$. Finally, let the set B of buildings used for housing cattle during the winter season equal $\{4\}$ and let the set H of timesteps corresponding to this season equal $\{4\}$.

The solution obtained to this problem is illustrated in Fig. 1. Specifically, Figs. 1(a), 1(b), 1(c) and 1(d) display slices of the solution tensor $A = (a_{ijk})$ for i equal to 1, 2, 3 and 4 respectively (the assignment of livestock to locations at each timestep). For example, at timestep 1, livestock 1, 2 and 3 are assigned to field 2, while livestock 4, 5 and 6 are assigned to field 3. We can see that this solution satisfies all problem constraints. For example, the continuous feeding capacity of each field is not exceeded, field 1 is set aside for silage at timestep 2, and all cattle are assigned to building 4 at timestep 4.

To evaluate the scalability of the proposed method for solving the above problem we considered problem instances of increasing size. For a given number of locations m and livestock n we defined a corresponding random problem instance using the following approach. We defined the number of timesteps t to equal m. We defined each of the feeding capacities x_k to be 10 except for a single random building, which was defined to have a feeding capacity equal to n. All livestock were assigned to this building during the winter season, which was defined to be the final timestep t. A randomly selected field was set aside for silage during the summer season, which was defined to be the first timestep 1. Finally, the matrix $W \in \mathbb{R}^{m\times m}$ of routing work values was defined to be a random zero-diagonal matrix where each non-zero element is a random integer in the range 3 to 10 inclusive.

For each pair of m and n values, we generated and solved ten corresponding random problem instances using the above approach. Table 3 displays the number of instances solved optimally, the mean running time of those instances solved optimally measured in seconds, and the mean relative optimality gap of those instances not solved optimally. Recall that if an optimal solution is not found before a running time of 600 s is reached, the optimiser will return the current best solution plus the corresponding relative optimality gap. As an example, for the set of problem instances with m and n equal to 10 and 70 respectively, 9 of the 10 instances were solved optimally, the mean running time of those instances solved optimally was 296 s and the

Table 3
For each value of m and n, this table displays the number of instances solved optimally, the mean running time of those instances solved optimally measured in seconds and the mean relative optimality gap of those instances not solved optimally. A dash ('–') represents that the problem instances are infeasible. The problem instances in question correspond to the problem in Eq. (17).

m, n	10	20	30	40	50	60	70	80	90	100
3	10,1,0	–	–	–	–	–	–	–	–	–
4	10,1,0	10,1,0	–	–	–	–	–	–	–	–
5	10,1,0	10,1,0	10,1,0	–	–	–	–	–	–	–
6	10,1,0	10,1,0	10,1,0	10,6,0	–	–	–	–	–	–
7	10,1,0	10,1,0	10,2,0	10,8,0	10,7,0	–	–	–	–	–
8	10,1,0	10,2,0	10,4,0	10,7,0	10,10,0	10,17,0	–	–	–	–
9	10,1,0	10,5,0	10,13,0	10,23,0	10,42,0	10,54,0	9,123,2	–	–	–
10	10,1,0	10,6,0	10,9,0	10,51,0	10,44,0	10,211,0	9,296,3	9,140,3	–	–

(a)

Livestock \ Location	1	2	3	4
1	0	1	0	0
2	0	1	0	0
3	0	1	0	0
4	0	0	1	0
5	0	0	1	0
6	0	0	1	0

(b)

Livestock \ Location	1	2	3	4
1	0	1	0	0
2	0	1	0	0
3	0	1	0	0
4	0	0	1	0
5	0	0	1	0
6	0	0	1	0

(c)

Livestock \ Location	1	2	3	4
1	0	1	0	0
2	0	1	0	0
3	0	1	0	0
4	0	0	1	0
5	0	0	1	0
6	0	0	1	0

(d)

Livestock \ Location	1	2	3	4
1	0	0	0	1
2	0	0	0	1
3	0	0	0	1
4	0	0	0	1
5	0	0	0	1
6	0	0	0	1

Fig. 1. Slices of the solution tensor $A = (a_{ijk})$ for i equal to 1, 2, 3 and 4 are displayed in (a), (b), (c) and (d) respectively. Each slice represents the assignment of livestock to locations at the timestep in question.

mean relative optimality gap of those instances not solved optimally was 3%. A dash ('–') in Table 3 represents the fact that the problem instances in question are infeasible; that is, no corresponding feasible solution exists. We can see from the statistics in the above table that the optimiser generally finds a solution with a small relative optimality gap within the 600 s time limit.

Infeasible problem instances may potentially be made feasible by adjusting some of the corresponding variables or relaxing some of the corresponding constraints. For example, a problem instance may be infeasible because the farm in question has too many livestock and is overstocked. In this case, the problem instance may be made feasible by reducing the number of livestock. Note that, some problem instances correspond to the case where the farm in question is understocked. In such cases, the number of livestock might also be increased while still maintaining feasibility.

4.2. Rotational grazing & livestock care

The second problem instance we consider combines the activities of rotational grazing and livestock care described in Sections 3.2.1 and 3.2.5 respectively. Combining these activities gives the following integer program formulation.

$$\min_{A} \sum_{i=1}^{t-1} \sum_{k=1}^{m} \sum_{k'=1}^{m} z_{ikk'} \, w_{kk'} + \sum_{i=1}^{t} \sum_{j=1}^{n} \sum_{k=1}^{m} a_{ijk} \, r_k \tag{18a}$$

$$\text{s.t.} \sum_{k=1}^{m} a_{ijk} = 1, \; \forall \, i = 1, \dots, t, j = 1, \dots, n \tag{18b}$$

$$z_{ikk'} = \min(1, \sum_{j=1}^{n} a_{ijk} \, a_{i+1jk'}), \; \forall \, i = 1, \dots, t-1, \; k, k' = 1, \dots, m \tag{18c}$$

$$\sum_{j=1}^{n} a_{ijk} - x_k \le 0, \; \forall \, i = 1, \dots, t, \; k = 1, \dots, m \tag{18d}$$

$$\sum_{i'=i}^{i+p} g_{i'k} - q \le 0, \; \forall \, i = 1, \dots, t-p, \; k = 1, \dots, m \tag{18e}$$

$$g_{ik} = \min(1, \sum_{j=1}^{n} a_{ijk}), \; \forall \, i = 1, \dots, t, \; k = 1, \dots, m \tag{18f}$$

$$a_{ijk} \in \{0, 1\}, \; \forall \, i = 1, \dots, t, \; j = 1, \dots, n, \; k = 1, \dots, m. \tag{18g}$$

Note that, the constraints in Eqs. (18d), (18e) and (18f) relate to the rotational grazing aspect of the problem. The second term in the objective in Eq. (18a) relates to the livestock care aspect of the problem.

We first illustrate the solution to a small toy instance of this problem. Let the number of timesteps t be 4, the number of livestock n be 5 and the number of fields m be 4. Let the grazing capacity x_k of each field be 3 and the matrix $W \in \mathbb{R}^{m \times m}$ of routing work values equal that displayed in Table 2. Let the livestock caring work values r_1, r_2, r_3 and r_4 be 1, 1, 1 and 2 respectively. Finally, let the rotational grazing parameters p and q both be 2. That is, for each field, each contiguous sequence of 3 timesteps contains at most 2 timesteps where livestock are assigned to the field in question. The solution obtained to this problem is illustrated in Fig. 2. We can see that this solution satisfies all problem constraints. For example, field 1 has livestock assigned at timesteps 2 and 3 but not at timesteps 1 and 4 thus satisfying the rotational grazing constraint. Furthermore, given the high livestock caring work corresponding to field 4, no livestock are assigned to this field at any timestep.

In our experiments, for a given number of locations m and livestock n, we defined a corresponding random problem instance using the following approach. We defined the number of timesteps t to equal m. We defined each of the feeding capacities x_k to be 10. The matrix $W \in \mathbb{R}^{m \times m}$ of routing work values was defined to be a random zero-diagonal matrix where each non-zero element was a random integer in the range 3 to 10 inclusive. Each livestock caring work value r^k was defined to be a random real number in the interval $[1, 2]$. Finally, the rotational grazing parameters p and q were both defined to be 2.

For each pair of m and n values, we generated and solved ten corresponding random problem instances using the above approach. Table 4 displays the results of applying the optimiser to these instances. We can see from the statistics in this table that the optimiser generally finds a solution with a small relative optimality gap within the 600 s time limit. However, for larger problem instances, the mean relative optimality

(a)

Livestock \ Location	1	2	3	4
1	0	1	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	1	0
5	0	0	1	0

(b)

Livestock \ Location	1	2	3	4
1	0	1	0	0
2	0	1	0	0
3	1	0	0	0
4	1	0	0	0
5	1	0	0	0

(c)

Livestock \ Location	1	2	3	4
1	0	0	1	0
2	0	0	1	0
3	1	0	0	0
4	1	0	0	0
5	1	0	0	0

(d)

Livestock \ Location	1	2	3	4
1	0	0	1	0
2	0	0	1	0
3	0	1	0	0
4	0	1	0	0
5	0	1	0	0

Fig. 2. Slices of the solution tensor $A = (a_{ijk})$ for i equal to 1, 2, 3 and 4 are displayed in (a), (b), (c) and (d) respectively. Each slice represents the assignment of livestock to locations at the timestep in question.

Table 4
For each value of m and n, this table displays the number of instances solved optimally, the mean running time of those instances solved optimally measured in seconds and the mean relative optimality gap of those instances not solved optimally. A dash ('–') represents that the problem instances are infeasible. The problem instances in question correspond to the problem in Eq. (18).

m, n	10	20	30	40	50	60	70	80	90	100
3	10,1,0	10,1,0	–	–	–	–	–	–	–	–
4	10,1,0	10,1,0	–	–	–	–	–	–	–	–
5	10,1,0	10,3,0	10,63,0	–	–	–	–	–	–	–
6	10,1,0	10,47,0	0,0,2	0,0,2	–	–	–	–	–	–
7	10,2,0	10,96,0	0,0,1	0,0,1	–	–	–	–	–	–
8	10,1,0	10,215,0	0,0,1	0,0,2	0,0,4	–	–	–	–	–
9	10,4,0	0,0,1	0,0,3	0,0,7	0,0,7	0,0,8	–	–	–	–
10	9,0	0,0,1	0,0,4	0,0,7	0,0,8	0,0,8	–	–	–	–

gap increases significantly to 8%. This can be partially attributed to the fact that, as discussed in Section 3.3, optimising rotational grazing is NP-hard.

4.3. Continuous grazing & livestock breeding

The final problem instance we consider combines the activities of continuous grazing and livestock breeding described in Sections 3.2.1 and 3.2.3 respectively. Combining these activities gives the following integer program formulation where the variables in question are defined in the above sections.

$$\min_{A} \sum_{i=1}^{t-1} \sum_{k=1}^{m} \sum_{k'=1}^{m} z_{ikk'}\, w_{kk'} \tag{19a}$$

$$\text{s.t.} \sum_{k=1}^{m} a_{ijk} = 1,\ \forall\, i = 1, \dots, t,\ j = 1, \dots, n \tag{19b}$$

$$z_{ikk'} = \min\,(1, \sum_{j=1}^{n} a_{ijk}\, a_{i+1jk'}),\ \forall\, i = 1, \dots, t-1,\ k, k' = 1, \dots, m \tag{19c}$$

$$\sum_{j=1}^{n} a_{ijk} - x_k \le 0,\ \forall\, i = 1, \dots, t,\ k = 1, \dots, m \tag{19d}$$

$$\sum_{k \in B} a_{ijk} = 1,\ \forall\, (i, j) \in H \tag{19e}$$

(a)

Livestock \ Location	1	2	3	4
1	0	1	0	0
2	0	0	0	1
3	0	1	0	0
4	0	0	1	0
5	0	0	1	0
6	1	0	0	0

(b)

Livestock \ Location	1	2	3	4
1	0	1	0	0
2	0	0	0	1
3	0	1	0	0
4	0	0	1	0
5	0	0	1	0
6	1	0	0	0

(c)

Livestock \ Location	1	2	3	4
1	0	1	0	0
2	0	0	1	0
3	0	1	0	0
4	0	0	0	1
5	0	0	1	0
6	1	0	0	0

(d)

Livestock \ Location	1	2	3	4
1	0	1	0	0
2	0	0	1	0
3	0	1	0	0
4	0	0	0	1
5	0	0	1	0
6	1	0	0	0

Fig. 3. Slices of the solution tensor $A = (a_{ijk})$ for i equal to 1, 2, 3 and 4 are displayed in (a), (b), (c) and (d) respectively. Each slice represents the assignment of livestock to locations at the timestep in question.

$$a_{ijk} \in \{0, 1\},\ \forall\, i = 1, \dots, t,\ j = 1, \dots, n,\ k = 1, \dots, m. \tag{19f}$$

Note that, the constraints in Eqs. (19d) and (19e) relate to the continuous grazing and livestock breeding aspects of the problem respectively.

We first illustrate the solution to a small toy instance of this problem. Let the number of timesteps t be 4, the number of livestock n be 6 and the number of locations m be 4. Let the feeding capacities x^1, x^2, x^3 and x^4 be 2, 2, 2 and 1 respectively and let the matrix $W \in \mathbb{R}^{m\times m}$ of routing work values equal that displayed in Table 2. Finally, let B and H be the sets $\{4\}$ and $\{(2,2),(3,4)\}$ respectively. That is, the set of locations suitable for cows calving is $\{4\}$ where this single location has a feeding capacity of 1. Furthermore, cow 2 is predicted to calf during timestep 2 and cow 4 is predicted to calf during timestep 3. During their respective calving timestep, the cow in question should be assigned to location 4.

The solution obtained to this problem is illustrated in Fig. 3. We can see that this solution satisfies all problem constraints. For example, at timestep 2 cow 2 is assigned to location 4. Similarly, at timestep 3 cow 4 is assigned to location 4.

For a given number of locations m and livestock n we define a corresponding random problem instance using the following approach. We defined the number of timesteps t to equal m. We defined each of the feeding capacities x_k to be 10 except for x^m which was defined to have a feeding capacity equal to 1. The matrix $W \in \mathbb{R}^{m\times m}$ of routing work values was defined to be a random zero-diagonal matrix where each non-zero element was a random integer in the range 3 to 10 inclusive. Finally, we defined the sets B and H to equal $\{m\}$ and $\{(i,i) : i = 1, \dots, t\}$ respectively.

For each pair of m and n values, we generated and solved ten corresponding random problem instances using the above approach. Table 5 displays the results of applying the optimiser to these instances. We can see from the statistics in this table that the optimiser generally finds a solution with a small relative optimality gap within the 600 s time limit.

Table 5
For each value of m and n, this table displays the number of instances solved optimally, the mean running time of those instances solved optimally measured in seconds and the mean relative optimality gap of those instances not solved optimally. A dash ('–') represents that the problem instances are infeasible. The problem instances in question correspond to the problem in Eq. (18).

m, n	10	20	30	40	50	60	70	80	90	100
3	10,1,0	10,1,0	–	–	–	–	–	–	–	–
4	10,1,0	10,1,0	10,1,0	–	–	–	–	–	–	–
5	10,1,0	10,1,0	10,1,0	10,1,0	–	–	–	–	–	–
6	10,1,0	10,1,0	10,1,0	10,2,0	10,4,0	–	–	–	–	–
7	10,1,0	10,1,0	10,1,0	10,5,0	10,18,0	10,21,0	–	–	–	–
8	10,1,0	10,2,0	10,4,0	10,7,0	10,22,0	10,36,0	10,98,0	–	–	–
9	10,1,0	10,3,0	10,6,0	10,10,0	10,31,0	10,49,0	10,115,0	8,553,1	–	–
10	10,1,0	10,4,0	10,6,0	10,14,0	10,38,0	10,56,0	10,130,0	0,0,3	0,0,5	–

5. Conclusions

This work has identified a planning problem in the livestock farming domain, which we refer to as the Livestock Routing Problem (LRP). To the authors' knowledge, this problem has not previously been considered by the research community. In our analysis we demonstrate that many common farming activities can be modelled as instances of LRP and in turn can be optimised with respect to the amount of work required for successful completion.

The proposed LRP model has many potential uses and applications. This model could potentially act as a decision support system for livestock farmers and, in turn, reduce the workload associated with the routing of livestock. Given the fact that the average age of a UK farmer is 60 and increasing, the proposed research could help to ensure the sustainability of the sector (Department for Environment, Food and Rural Affairs, 2020). Such a decision support system could also be used to evaluate the effects of prospective managerial decisions. For example, the system could be used to evaluate if increasing the number of livestock on a given farm by a given amount significantly increases workload. This type of analysis could, in turn, be used to identify potential opportunities for economies of scale. In many countries, farming activities will be constrained by corresponding government rules and policies. To give a concrete example of this, consider the Sustainable Farming Scheme proposed by the Welsh Government (Welsh Government, 2022). Farmers participating in this scheme will receive additional payments if they perform their farming activities more sustainably. This includes, for example, reducing the risk of livestock catching and spreading disease by ensuring all new livestock in a given farm are isolated for at least six days before mixing with existing livestock. Ensuring that such rules and policies are satisfied could be achieved by modelling them as an instance of the LRP. Apart from the obvious benefit to farmers by ensuring that they satisfy the necessary rules and policies, this modelling could also be of benefit to policymakers. For example, a policymaker could evaluate the potential consequences of a new scheme with respect to the farmer's workload.

A famous expression in the field of statistics states that all models are wrong, but some are useful. As a first work to consider and model the LRP, we do not claim the proposed model to be all-encompassing. However, for the reasons discussed above, we do believe it is useful. There exist many possible directions of future research to improve the accuracy and in turn usefulness of the model. We now describe some of these directions.

As seen, we have modelled the LRP as an integer program that assumes all model parameters are known. However, in reality, many of these parameters may have some associated uncertainty. For example, when modelling continuous and rotational grazing we assume the grazing or feeding capacity of each field is known and constant. In reality, this will not be the case and, instead, this capacity will vary over time as a function of weather conditions. This limitation could potentially be overcome by explicitly modelling the uncertainty in the model parameters using a stochastic integer program. Another potentially useful future research direction would be to combine the proposed model with advances in Internet of Things (IoT) technologies for agriculture such as virtual fencing or partitioning and automated grass measurement devices (Murphy et al., 2021).

CRediT authorship contribution statement

Padraig Corcoran: Conceptualization, Methodology, Software, Validation, Formal analysis, Writing – original draft, Writing – review & editing. **Rhyd Lewis:** Methodology, Validation, Formal analysis, Writing – original draft, Writing – review & editing.

Data availability

No data was used for the research described in the article.

References

Akhigbe, B. I., Munir, K., Akinade, O., Akanbi, L., & Oyedele, L. O. (2021). Iot technologies for livestock management: A review of present status, opportunities, and future trends. *Big Data and Cognitive Computing*, *5*, 10.

Applegate, D. L., Bixby, R. E., Chvátal, V., & Cook, W. J. (2011). *The traveling salesman problem*. Princeton University Press.

Bach, A., & Cabrera, V. (2017). Robotic milking: Feeding strategies and economic returns. *Journal of Dairy Science*, *100*, 7720–7728.

Benos, L., Tagarakis, A. C., Dolias, G., Berruto, R., Kateris, D., & Bochtis, D. (2021). Machine learning in agriculture: A comprehensive updated review. *Sensors*, *21*, 3758.

Cabrera, V. E., Barrientos-Blanco, J. A., Delgado, H., & Fadul-Pacheco, L. (2020). Symposium review: Real-time continuous decision making using big data on dairy farms. *Journal of Dairy Science*, *103*, 3856–3866.

Department for Environment, Food and Rural Affairs (2020). *Agriculture in the united kingdom 2020*.

Edmonds, J., & Karp, R. M. (1972). Theoretical improvements in algorithmic efficiency for network flow problems. *Journal of the ACM*, *19*, 248–264.

Fisher, M. L., & Jaikumar, R. (1981). A generalized assignment heuristic for vehicle routing. *Networks*, *11*, 109–124.

French, P., O'Brien, B., & Shalloo, L. (2015). Development and adoption of new technologies to increase the efficiency and sustainability of pasture-based systems. *Animal Production Science*, *55*, 931–935.

Gunawan, A., Lau, H. C., & Vansteenwegen, P. (2016). Orienteering problem: A survey of recent variants, solution approaches and applications. *European Journal of Operational Research*, *255*, 315–332.

Hanrahan, L., Geoghegan, A., O'Donovan, M., Griffith, V., Ruelle, E., Wallace, M., & Shalloo, L. (2017). Pasturebase ireland: A grassland decision support system and national database. *Computers and Electronics in Agriculture*, *136*, 193–201.

Higgins, S., Schellberg, J., & Bailey, J. (2019). Improving productivity and increasing the efficiency of soil nutrient management on grassland farms in the uk and ireland using precision agriculture technology. *European Journal of Agronomy*, *106*, 67–74.

Hostiou, N., Fagon, J., Chauvat, S., Turlot, A., Kling-Eveillard, F., Boivin, X., & Allain, C. (2017). Impact of precision livestock farming on work and human-animal interactions on dairy farms. a review. *Biotechnologie, Agronomie, Société et Environnement/Biotechnology, Agronomy, Society and Environment*, *21*, 268–275.

Lewis, R. (2021). *Guide to graph colouring: Algorithms and applications* (2nd ed.). Springer.

Macdonald, K., Glassey, C., & Rawnsley, R. (2010). The emergence, development and effectiveness of decision rules for pasture based dairy systems. In *Australasian dairy science symposium: Meeting the challenges for pasture-based dairying* (pp. 199–209).

Michie, C., Andonovic, I., Davison, C., Hamilton, A., Tachtatzis, C., Jonsson, N., Duthie, C. A., Bowen, J., & Gilroy, M. (2020). The internet of things enhancing animal welfare and farm operational efficiency. *Journal of Dairy Research*, *87*, 20–27.

Mitchell, J. E. (2002). Branch-and-cut algorithms for combinatorial optimization problems. In *Handbook of applied optimization, Vol. 1* (pp. 65–77).

Mottram, T. (2016). Animal board invited review: precision livestock farming for dairy cows with a focus on oestrus detection. *Animal*, *10*, 1575–1584.

Murphy, D. J., Murphy, M. D., O'Brien, B., & O'Donovan, M. (2021). A review of precision technologies for optimising pasture measurement on Irish grassland. *Agriculture*, *11*, 600.

Parker, F., Sawczuk, H., Ganjkhanloo, F., Ahmadi, F., & Ghobadi, K. (2020). Optimal resource and demand redistribution for healthcare systems under stress from COVID-19. arXiv preprint arXiv:2011.03528.

Saint-Dizier, M., & Chastant-Maillard, S. (2018). Potential of connected devices to optimize cattle reproduction. *Theriogenology*, *112*, 53–62.

Shalloo, L., Byrne, T., Leso, L., Ruelle, E., Starsmore, K., Geoghegan, A., Werner, J., & O'leary, N. (2021). A review of precision technologies in pasture-based dairying systems. *Irish Journal of Agricultural and Food Research*.

Shalloo, L., O'Donovan, M., Leso, L., Werner, J., Ruelle, E., Geoghegan, A., Delaby, L., & O'leary, N. (2018). Grass-based dairy systems, data and precision technologies. *Animal*, *12*, s262–s271.

Toth, P., & Vigo, D. (2002). *The vehicle routing problem*. Society for Industrial and Applied Mathematics.

Welsh Government (2022). *Sustainable farming scheme outline proposals for 2025*.

Wolfert, S., Ge, L., Verdouw, C., & Bogaardt, M. J. (2017). Big data in smart farming–a review. *Agricultural Systems*, *153*, 69–80.

Woodward, S. J., Wake, G. C., & McCall, D. G. (1995). Optimal grazing of a multi-paddock system using a discrete time model. *Agricultural Systems*, *48*, 119–139.