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Modified age-based replacement

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ABSTRACT

The maintenance policy age-based replacement (ABR) is widely specified in OEM instructions. The practical application of ABR raises concerns about ensuring consistent adherence to prescribed replacement schedules for extended periods. ABR lacks periodicity, resulting in scheduling asynchrony with designated time slots, while alternative policies such as block replacement (BR) provide periodicity at the expense of efficiency. Additionally, scepticism about ABR is based on its simplicity and restrictive assumptions, which include ideal replacements and the one-component system assumption. The task of estimating component lifetime distributions and defining critical parameters such as cost of failure presents significant challenges. We study "modified age-based replacement" (MABR) in response to the limitation of aperiodicity, so that preventive replacements exhibit quasi-periodic behavior. We quantify the cost-inefficiency of MABR compared to ABR, thus informing the practical implications of introducing periodicity into the ABR policy and highlighting the need to incorporate real-world constraints, such as time slots for maintenance actions. The findings indicate that MABR and a special case are reasonably efficient provided the slot-interval is not too large. This is a useful insight for practical application of ABR type policies for scheduling preventive maintenance.

1. Introduction

Age-based replacement is a maintenance policy with a long history, and it is often the first model to be described in textbooks on maintenance modeling and management (e.g. [1–3]). Nonetheless, evidence for the use of the policy in practice is lacking [4,5]. There are compeling reasons for this. Despite periodic (time-based) preventive maintenance (PM) being the norm in industry [6], age-based replacement (ABR) does not schedule PM in such a way that it is periodic in time. We define periodicity in time as generating time instances or slots is ($i = 1, 2, \ldots$), for some given s [7–10]. A discrete-time setting (e.g. [11,12]) has similar slots although these would typically correspond to cycles of operation or loading. In aside, Confining PM to slots can also simplify large maintenance optimization problems (e.g. [13]).

ABR is not "periodic" because replacement is scheduled based on age, so that the scheduled PMs of a system will become asynchronous with respect to slots is ($i = 1, 2, \ldots$) as soon as the first failure (and corrective replacement) has occurred. This is a bar to implementation of the policy in practice. Block replacement (BR) [1] and modified block replacement [14,15] on the other hand are periodic policies. A maintenance action on a system under the regime of a BR policy is carried out preventively at regular time intervals iT ($i = 1, 2, \ldots$), regardless of the age of the item. As with ABR, BR is such that corrective actions are performed when the system fails. However, unlike ABR, BR does not reset the time-origin of the equipment's preventive replacement cycle. On the other hand, an item may be replaced preventively soon after a corrective replacement [16]. For this reason, BR is cost-inefficient compared to ABR.

There are other reasons why practitioners might be sceptical about using ABR. The model is perhaps too simplistic, or put another way, the assumptions of the model are perhaps too restrictive. Let us examine the particular shortcomings of the assumptions that underlie ABR. First is the renewal assumption, and by implication the choice of the decision criterion, the steady-state cost-rate, for determining the best value of the

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Abbreviations: ABR, age-based replacement; BR, block replacement; DFR, decreasing failure rate; IFR, increasing failure rate; MABR, modified age-based replacement; OEM, original equipment manufacturer; PM, preventive maintenance.

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critical age for preventive replacement. However, rejection of this criterion implies rejection of most maintenance models. Alternative criteria are also problematic. A cost analysis over a finite planning horizon (e.g. [17]) raises issues of where and how to terminate the horizon. Real options theory may be appropriate for large capital items [18], but its use for planning for routine maintenance of critical parts is questionable. Renewal also implies that maintenance actions are perfect, but the reality may be far from this. This assumption has been relaxed in many models [19–22], but application of models in practice is often neglected [23].

Next is the one-component system assumption [24]. There has been much work on multi-component system extensions wherein interactions complicate the simple sequence of periodic replacements, for example, opportunistic replacement [8,25,26], and grouping [27–30]. Nonetheless, an underlying periodic schedule based on the one-component system assumption is useful.

Third is the assumption about the lifetime of a component. To determine the optimal age-based replacement, it is crucial to know the lifetime distribution, but there are numerous challenges in accurately defining this distribution [31]. Misalignment between vendor guidelines and actual usage and maintenance practices is a primary reason for this, often due to a lack of comprehensive knowledge. Furthermore, data collection has long been a concern in the reliability and maintenance community [32,33], with records frequently having: inaccuracies and omissions; heavy right-censoring; and general data scarcity. These make it difficult to obtain accurate estimates of the parameters required for models, so that uncertainty related to the parameters of a given lifetime distribution is quite common. This raises two important concerns. The first is the choice of a lifetime distribution to reduce the effort to estimate the parameters, and the second is to understand the influence of the parameter variation on the optimum replacement age. Regarding the first concern, many authors advocate the use of distribution with few parameters [34]. The two-parameter Weibull offers this. In fact, the Weibull distribution is especially noted for its ability to describe various aging profiles in life distributions, such as those with increasing, decreasing, or constant failure rates [35,36]. Related to the second concern, De Jonge et al. [31] and also Fouladirad et al. [37] offer a significant contribution in addressing uncertainties in lifetime distribution parameters for determining the optimal replacement age. These studies underscore how minor variations in estimated parameters can significantly impact optimal maintenance policies, highlighting the need for robust strategies adaptable to parameter uncertainty. Their main contribution lies in raising awareness of this sensitivity, emphasizing the importance of resilient maintenance policies, providing support for decision-making tools that consider parameter variations, and ultimately driving cost reduction and increased operational efficiency by minimizing downtime, reducing maintenance costs, and enhancing system reliability.

Another challenge is when the lifetimes of the components are highly variable, leading to the characterization of markedly different failure processes, which may be due to heterogeneity within the component population [25]. This topic is extensively explored in Finkelstein [38], who argues that heterogeneous populations are encountered much more frequently than typically considered in failure rate modeling. The disparity between reality and modeling can lead to misunderstandings and errors in reliability analysis. Consequently, the application of mixture models to address lifetime modeling for heterogeneous populations stands out as a significant and pivotal area of research. Typically, employing a mixture of lifetime functions proves to be an effective approach for dealing with the various patterns of failures. The work of Aalen [39] provides some very interesting insights into mixture analysis. Although it focuses on survival analysis when examining the lifetimes of patients facing specific diseases, such as cancer, the work offers reflections that can be highly valuable in terms of system reliability analysis, especially concerning specific peculiarities that may justify markedly different behavior patterns regarding observed failures. Determining the optimal age for replacement is a challenge, especially given the significant variability in time to failure among different subpopulations. The condition of IFR (Increasing Failure Rate) that is typically required for a cost-effective policy can become problematic when dealing with a mixture of two IFR distributions, as it can result in a DFR (Decreasing Failure Rate) distribution. Scarf et al. [40] study the challenges of devising effective maintenance plans for such scenarios and suggest a policy that integrates age-based replacement with inspections, particularly when an intermediate state before failure (defect) can be identified during an inspection.

Fourth is assumption about the cost, and hence the consequences of failure. It can be challenging to specify the cost of failure. When used as a fixed value, regardless of the time to return to operational status, it essentially represents an average cost of failures related to various failure scenarios, each with potentially varying downtime [41–43]. Alternatively, the failure cost can be decomposed into constant components, reflecting personnel and material costs (components) added to the loss of value of operation, which varies depending on the specific time required to return to operational status [44]. Some authors advocate using a penalty cost that encompasses all costs incurred due to failures beyond the maintenance activity itself (personnel and material). The concept of penalty implies that failure consequences can be completely translated into financial losses, expressed as costs. In a broader sense, it is interesting to revisit some early ideas on the use of a loss function associated with the failure scenario and the effort to prevent failure as a more general alternative to better define maintenance intervals [45,46]. Zelen [46] in particular discusses findings from a seminar held by the Mathematics Research Center, United States Army, at the University of Wisconsin, Madison. The seminar was attended by leading researchers of the day who established a new field of study in applied mathematics, combining maintenance and reliability models that are crucial for proper management of the functioning of equipment. During the seminar, there were several mentions of cost, which, without loss to the models used, can be replaced by any other quantity that better reflects the need to avoid failure. This possibility is useful when estimating losses in one dimension is more challenging than in another. For example, instead of cost, one can use the respective values of corrective-action execution-time (related to a failure) and preventive-action time. Regarding the difficulty of estimating costs, Scarf et al. [47] offer a valuable framework for maintenance decision-making, emphasizing the crucial balance between cost and reliability. The innovative concept of the implied cost of failure is introduced, providing a practical approach to assess the consistency between cost and reliability measures. Additionally, their approach to determining the age limit in age-based replacement policies is especially useful, particularly in situations, where specifying failure costs directly may be challenging. A broader perspective is offered by De Almeida et al. [48], especially when there are significantly different consequence dimensions for a failure, and translating them into financial losses does not accurately represent the characteristics of a problem. For systems that provide essential services or have severe safety implications due to failure, this multicriteria approach is particularly useful. Finally, in practice replacement costs may be time-varying [49]. Then, for example, advancing or postponing maintenance to periods when there is lower demand for maintenance resources may be beneficial (see [50]).

Next are assumptions about instantaneous changes of state. Again, there are models that have relaxed this restriction, in connection with replacement (e.g. [44,51–53]) and with failure (e.g. [54,55]), but evidence for the use of these models is lacking [4,56]. The notion of immediate replacement, that is, no postponement or delay, either on failure or at a PM is similar, and again there are models available that account for delays e.g. defaulting [8,57] and postponement [58–60].

All these models, and much more (see e.g. [61]), form the body of “theory” that is maintenance modeling. From a critical appraisal of this corpus, we might conclude that researchers have sought to fix the restrictions of the simple ABR model, so that it is more applicable.

However, perhaps these fixes (see for example [62]) may not be sufficient to ensure applicability. Perhaps aperiodicity is the key issue. There is evidence that industry prefers a fixed schedule of actions, so that resources and the costs associated with the provision of resources are (mostly) known and constant [63,64]. This raises a question that to our knowledge has not been studied before in the literature: what are the implications for cost-efficiency of imposing periodicity on the age-based replacement model? To answer this question, we study a policy in which a component is replaced on failure or at the first slot after the system attains age T, that is, at age ks such that $(k-1)s < T < ks$, whichever occurs sooner, so that preventive replacements are always quasi-periodic. In particular, we compare the minimum cost-rate of this modified policy with those of age-based replacement and block replacement.

Notice that the modified policy is such that, if the maintainer has a fleet of systems, then failed systems are subject to immediate corrective replacement and, at the slots is ($i = 1, 2, \ldots$), those systems that are at least age T are preventively replaced. This policy, which we call "modified age-based replacement" (MABR), is cost-inefficient relative to ABR. Our aim is to quantify the extent of this inefficiency. We are not the first to study this policy. Bajestani and Banjevic [65] propose it, and use the term "calendar-based" age replacement, in the context of geographically dispersed items. However, their purpose is to present a formal analysis of the policy. Our purpose is different: to measure the cost-inefficiency of the policy relative to ABR and relative to BR. Furthermore, we think the modified policy is more ubiquitous in practice but nonetheless inefficiently applied. Therefore, we think it is important to study the merits of the MABR policy relative to ABR and BR, in order to underline the practical relevance of MABR. This is the aim of our paper.

The slots themselves may correspond to scheduled shutdowns of a wider plant within which the system(s) of interest is (are) embedded. Then, preventive replacements would not coincide with scheduled missions, which might themselves induce preventive replacements at random times [66]. Nonetheless, relative to system age and at steady state, preventive replacements will appear to occur at random times so that the MABR policy falls within the class of "non-precise" replacement policies. Other non-precise policy models can be found in Zhao et al. [67], Zhao and Nakagawa [68], and Schouten et al. [50].

The policies we compare are described in the next section. There we also discuss how they are practically specified. Then, in Section 3, we compare the policies numerically, showing graphically their relative cost-inefficiencies. The paper ends with a brief discussion.

2. The maintenance policies

2.1. Age-based replacement

The classic age-based replacement (ABR) policy [69] supposes the repetition of an install-and-use cycle of a component in a socket ad infinitum, with replacement (installation) either correctively on failure or preventively at age T, whichever ends the renewal (install-and-replace) cycle. For a one-component system with component reliability function $R(t)$, the long-run expected cost (or the cost-rate for short) is given by

$$C_{\infty}^{\text{ABR}} = \frac{c_{\text{F}}(1 - R(T)) + c_{\text{P}}R(T)}{\int_0^T R(t)\text{d}t}, \tag{1}$$

where c_{F} is the cost of corrective (failure) replacement and c_{P} is the cost of preventive replacement. A cost-minimising (optimum) T, T^*_{ABR}, exists provided $c_{\text{F}} > c_{\text{P}}$ and the distribution of the lifetime of a (notional) component has a strictly increasing hazard-rate function [1]. In practice c_{F}, c_{P} and the parameters of $R(t)$, must be specified, and in so doing the assumptions of the model must be justified.

2.2. Modified age-based replacement

The modified age-based replacement (MABR) policy is defined as follows: replace the component on failure (corrective replacement) or at age ks (preventive replacement) such that $(k-1)s < T < ks$, whichever occurs sooner. Here, s is the time interval between slots, or slot-interval for short. Thus, once the system attains age T, preventive replacement occurs at the next slot, provided it does not fail beforehand. In this way, preventive replacement can occur only at a slot, and since slots are periodic preventive replacements are quasi-periodic. In practice, preventive replacement is managed by querying the age of each system (in the fleet) at every slot and then replacing those aged at least T. Corrective replacement is the same as for the classic policy, ABR: a failed system is replaced immediately. To determine the value of critical age for replacement that minimizes the cost-rate we use simulation (Fig. 1). Details of the coding are described in Section 3.

It is much simpler to optimize ABR using Eq. (1) than to optimize MABR using the simulator (Fig. 2). Therefore, we also consider a sub-optimal variation of MABR with $T = T^*_{\text{ABR}}$. This sub-optimal variation of MABR is useful for practice because it will allow a practitioner to specify the policy easily. Therefore, because we are interested to compare the cost-efficiency of practical policies, we include it in our comparison. We denote the optimal MABR policy by MABR* and the sub-optimal MABR policy by MABR$_{T\text{-ABR}}$. The key point is this: MABR$_{T\text{-ABR}}$ is a sensible policy to use in practice because: (i) it will be close to globally optimal (the globally optimal policy is ABR*); (ii) preventive replacements are quasi-periodic; (iii) T^*_{ABR} is easy to find by minimising (1); (iv) corrective replacements are carried out immediately (the maintainer does not wait for a slot).

Further, on the subject of comparisons with sub-optimal policies, we also consider block replacement, which we describe next.

2.3. Block replacement

Block replacement (BR) is a model that makes the same assumptions as ABR, but the replacement policy is different and simpler: a component is replaced correctively on failure and preventively replaced periodically at times kT, $(k = 1, 2, \ldots)$ [1]. The cost-rate is given by

$$C_{\infty}^{\text{BR}}(T) = [c_{\text{F}}M(T) + c_{\text{P}}]/T,$$

where $M(T)$ is the expected number of failures in $(0, T]$, which is itself a function of $R(t)$ that must be evaluated numerically. A useful approximation exists when $R(t) = \exp\{-(t/\eta)^{\beta}\}$ [70] provided $c = c_{\text{F}}/c_{\text{P}} \gg 1$:

$$T^*_{\text{BR}} \simeq \eta(1+c)^{-1/\eta}\left\{c\beta - c - 1 - \sqrt{(c\beta - c - 1)^2 - 2}\right\}^{1/\eta} \tag{2}$$

Since we use simulation (Fig. 1) to optimize MABR, we do likewise for BR (Fig. 2) rather than use the approximation (2), which is less exact. Results of this comparison are not reported.

3. Numerical comparison of policies

We compare the policies generally, by allowing the model parameters to vary within the framework of the Weibull lifetime distribution, $R(t) = \exp\{-(t/\eta)^{\beta}\}$. In this comparison, the units of time and cost are arbitrary. Therefore, without loss of generality we set $c_{\text{P}} = 1$, so that the cost of preventive replacement is the unit of cost, and $\eta = 10$ so that the unit of time (e.g. one year) is a fraction (1/10th) of the characteristic life of the system (e.g. 10 years). Then, the free parameters are β, which characterizes lifetime uncertainty, s, the slot-interval, and c_{F}, the consequences (cost) of an individual system failure. Necessarily, $\beta > 1$ and $c_{\text{F}} > 1$.

Optimization was coded in Python. For ABR, we employed the SLSQP Sequential Least Squares Programming method, utilizing the 'scipy.

```
Initiate MABR's simulator
Determine the values of input parameters ($\eta$, $\beta$, $C_P$, $C_F$, $s$) and decision variable ($T$)
Initialize auxiliary variables ($\mathrm{Cos}t$, $Life$, $C_\infty$) as null
While (Criterion of termination not satisfied) do:
    Generate $z$ based on the component's life distribution function
    $x \leftarrow Life + z$
    $T \leftarrow T + Life$
    Define $k$ based on the following relation: $(k-1)\cdot s \le T \le k\cdot s$
    If $x < k\cdot s$ then:
        $\mathrm{Cos}t \leftarrow \mathrm{Cos}t + c_F$
        $Vida \leftarrow x$
    Else:
        $Cost \leftarrow Cost + c_P$
        $Vida \leftarrow k\cdot s$
    $C_\infty \leftarrow \mathrm{Cos}t/Life$
Return $C_\infty$
End MABR's simulator
```

Fig. 1. Pseudocode for simulation of modified age-based replacement.

```
Initiate Simulator
Initialize auxiliary variables ($\mathrm{Cos}t$, $Life$, $C_\infty$) as null
While (Criterion of termination not satisfied) do:
    Generate $x$ based on the component's life distribution function
    If $x < T$ then:
        $\mathrm{Cos}t \leftarrow \mathrm{Cos}t + c_F$
        $Life \leftarrow Life + x$
        While True do:
            Generate $z$ based on the component's life distribution function
            If $z < (T - x)$ do:
                $Cost \leftarrow Cost + c_F$
                $Life \leftarrow Life + z$
                $x \leftarrow x + z$
            Else:
                $Cost \leftarrow Cost + c_P$
                $Life \leftarrow Life + (T - x)$
            End-while
    Else:
        $\mathrm{Cos}t \leftarrow Cost + c_P$
        $Life \leftarrow Life + T$
    $C_\infty \leftarrow \mathrm{Cos}t/Life$
Return $C_\infty$
End BR's simulator
```

Fig. 2. Pseudocode for simulation of block replacement.

optimize.minimize' function from the scipy library. The parameter boundaries were defined by the relationship $(\eta/10, 10\eta)$, and the initial guess was set as $\eta/4$. The only constraint enforced was dictated by the ABR maintenance policy, where $T > 0$. For the BR maintenance policy, we utilized the 'scipy.differential_evolution' function from the scipy library. The required parameters for this function, including boundaries and constraints, were the same as those for the ABR maintenance policy. Lastly, to optimize the values of the decision variable returned by the MABR maintenance policy, we conducted an exhaustive search. This involved iterating through the range of T with a step size of 0.05, considering the boundaries (1, 15).

Fig. 3 shows the cost-rate behavior (left) and the value of the critical age for replacement (right) for the competing policies, for a range of values of the slot-interval, s. The examined results were obtained with a fixed increment of 0.05 between each s value. As required, the results for ABR* and BR* remain constant regardless of changing s.

ABR is the global cost-minimizing policy. Therefore, Fig. 4 makes same policy comparison, but for a range of values of β and c_F, and displaying the cost-inefficiency of the inefficient policies relative to ABR, $(C^*_{\mathrm{Policy}} - C^*_{\mathrm{ABR}})/C^*_{\mathrm{ABR}}$.

Thus, Fig. 3 presents the minimum cost-rates of MABR and $\mathrm{MABR}_{\mathrm{T\text{-}ABR}}$ within the notional boundaries of the ABR (efficient) and BR

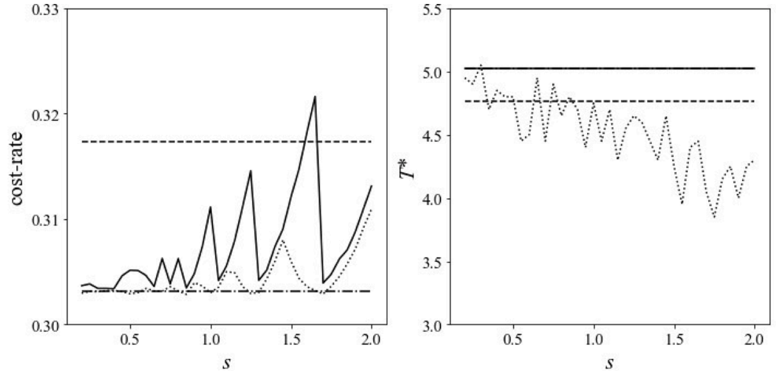

Fig. 3. Minimum cost-rate and optimum value of the critical age for replacement (T^*) for ABR* (— ■ —), BR* (— —), MABR* (■ ■ ■) and MABR$_{T\text{-ABR}}$ (—) as a function of the slot-interval, s, in the base case: $\beta = 3$, $c_F = 5$.

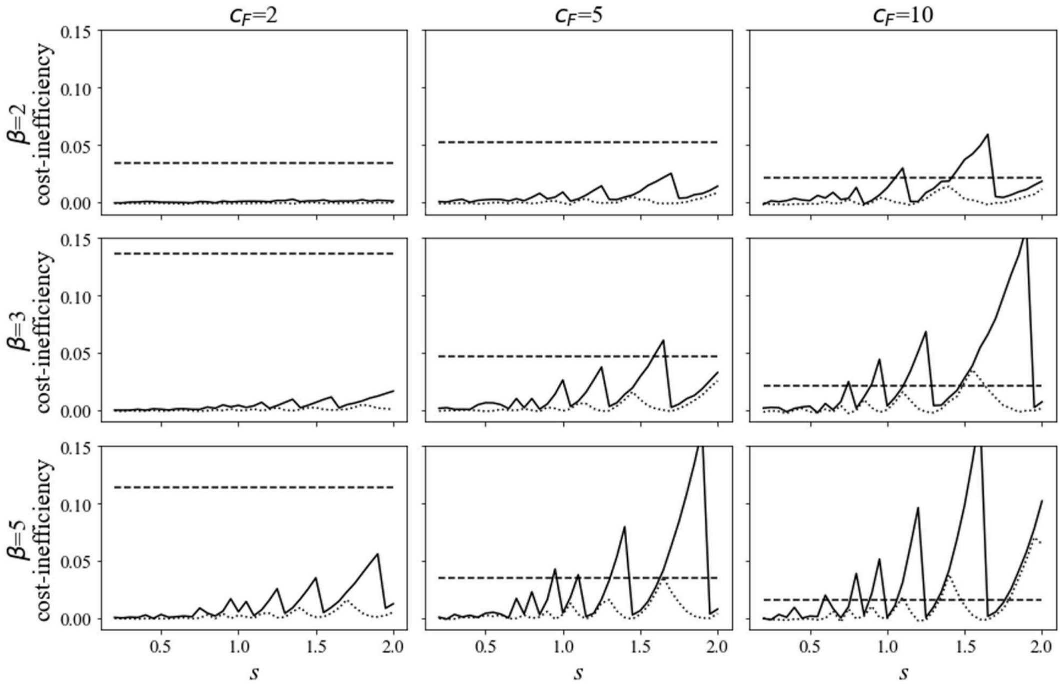

Fig. 4. Cost-inefficiency of BR* (— —), MABR* (■ ■ ■) and MABR$_{T\text{-ABR}}$ (—) relative to ABR* as a function of s for various values of β and c_F.

(inefficient). Notionally, we shall think of ABR as the benchmark for efficiency and BR as the benchmark for inefficiency, and if the minimum cost-rate of a proposed policy (MABR, MABR$_{T\text{-ABR}}$) is close to that of ABR, we deem it to be efficient. In this framework, we can see that MABR and MABR$_{T\text{-ABR}}$ are efficient provided s is not too large, while the critical age of replacement for MABR is quite sensitive to the value of s. Fig. 4 demonstrates that the efficiency of MABR and MABR$_{T\text{-ABR}}$ is not universal. A larger cost of failure and/or more certain component lifetime reduces the preference for MABR and MABR$_{T\text{-ABR}}$ over BR. This efficiency of ABR is known because ABR has no restrictions on when replacement will be executed. However, as previously mentioned, the comparator policy (MABR) provides a perspective closer to reality, where there is indeed a moment when maintenance actions are confined to the time slot defined by the maintenance planning.

Importantly, for small values of s, the cost-rates of MABR* and MABR$_{T\text{-ABR}}$ are only marginally greater than ABR*, but much less than that of BR*. These values of s correspond to those that might be encountered in practice. Thus, here $\eta = 10$ and $s = 1$ correspond to a fleet of components with a ten-year expected life, say, and subject to annual preventive maintenance across the fleet.

Returning to Fig. 3, we can see that as the value of s increases, a notable trend in the decision variable for the MABR* policy is its consistent decrease relative to T^* for the ABR* policy. The reason behind this phenomenon is that the MABR policy aims to balance the estimated equipment lifetimes based on the determination of the decision variable. Consequently, as we configure and increase the time slots, more time will elapse between the time obtained from the decision variable T and the actual system rejuvenation action at some ks. Thus, MABR and MABR$_{T\text{-ABR}}$ become less efficient as s increases, with this inefficiency more pronounced as lifetimes become more predictable (Fig. 4). The same effect can be observed with an increase in the corrective replacement cost. These phenomena were expected. The key point we seek to understand is how far from the ideal (ABR* policy) we are when adopting the MABR* and MABR$_{T\text{-ABR}}$ policies, using BR* as the benchmark for inefficiency.

3.1. *Benchmarking inefficiency using parameter uncertainty*

The Weibull distribution is widely employed for modeling lifetimes. However, experts in Weibull analysis commonly agree that estimating the scale parameter (η) is challenging. Consequently, in Weibull-based reliability analysis, it is often encountered that the shape parameter is known, whereas the scale parameter introduces uncertainty [71]. This suggests an alternative way to benchmark inefficiency, wherein we quantify the inefficiency of a policy that uses a parameter value that is different from its true value. Thus, Fig. 5 shows the cost-inefficiency of ABR* as the used value of the scale parameter deviates from its true value. Thus, the true value is $\eta* = 10$ (as above) but the value used to determine $T*$ is allowed to vary from 8 to 12 (with increments of 0.1). Then, we observe that a cost-inefficiency of up to 5% would not be unusual in practical circumstances in which η is unknown. Noticeably, this is the same as the cost-inefficiency of BR* in the base case (Fig. 4, middle panel). Thus, we can broadly conclude that using an inefficient policy, such as MABR or MABR$_{T\text{-ABR}}$, has a lower consequence cost-wise than uncertainty about the lifetime of components.

3.2. *Defaulting*

In a practical regime in which preventive maintenance is carried out only periodically, defaulting may also be common. Thus, a preventive replacement may be postponed until a subsequent slot. Resource constraints have been proposed as the mostly likely reason for postponement [72]. Thus, Fig. 6 illustrates the same relationships as those depicted in Fig. 3, but with the possibility of at most one postponement of a preventive replacement (probability 0.4) to the subsequent slot. Note, the model of default applies equally to all policies in Fig. 6.

Two points emerge. Firstly, it is noteworthy that the patterns observed in Fig. 3 persist in Fig. 6, whether it be the periodicity of the cost rate in relation to the slot-interval or the relationships between maintenance policies. Secondly, an increase in the cost-rate is observed with the increase of the slot-interval. This outcome is logical, as defaults will lead to a progressively longer expected delay in the execution of maintenance actions as the slot-interval increases.

4. Conclusions

This study examines the age-based replacement (ABR) policy, widely cited in OEM instructions but fraught with practical challenges due to its lack of periodicity, leading to scheduling asynchrony with designated time slots for preventive maintenance. The block replacement (BR) policy offers periodicity but at a cost inefficiency. Despite the simplicity and restrictive assumptions of ABR, such as perfect replacements and the one-component system assumption, it is regarded theoretically as an important maintenance strategy. Nonetheless, there is little evidence that it is used in practice. We argue that “modified age-based replacement” (MABR), the policy we discuss in this paper, is more practical because preventive maintenance can occur only at times that follow a fixed, periodic schedule. Therefore, we are interested to know the efficiency of the MABR policy relative to ABR, which is aperiodic, and to BR, which is periodic but cost-inefficient. Our findings show the practical implications of favoring periodicity, and thus emphasizing the need to consider real-world constraints like time slots for maintenance actions.

Using ABR and BR as benchmark policies for the range of cost-efficiency a maintainer might expect to achieve, with ABR being efficient and BR notionally inefficient, we show that MABR (a policy like ABR but which executes preventive replacement at the next available slot after a component has reached its critical age for replacement) is efficient provided slots are reasonably frequent and the cost consequence of a failure is not too large and component lifetime is unpredictable. Furthermore, a sub-optimal MABR policy, which uses the critical age for replacement of the ABR policy, and which we term MABR$_{T\text{-ABR}}$, is also efficient under the same circumstances. Obviously, MABR$_{T\text{-ABR}}$ is not as efficient as MABR which is not as efficient as ABR. However, MABR$_{T\text{-ABR}}$ would be simpler to manage in practice than MABR. The disbenefits of the managerially simpler policies are thus quantified.

An alternative benchmark, based on parameter uncertainty, is also used to qualify the inefficiency of the studied policies. We show that a cost-inefficiency of up to 5% would not be unusual in practical circumstances in which the lifetime of components is unknown. This is broadly commensurate with the cost-inefficiency of block replacement and greater than the cost-inefficiencies of modified age-base replacement (MABR) and the sub-optimal version (MABR$_{T\text{-ABR}}$), except when the slot-interval or the cost of failure or the lifetime shape-parameter is atypically large.

It might be argued that a cost-related comparison of the two classic models, ABR and BR, that to an extent motivate our study of MABR, is not practically relevant because ABR and BR are used in different scenarios: ABR for high-cost, highly critical systems, where failure may result in a very large loss; BR where potential losses are not large and periodic preventive maintenance is convenient. Nonetheless, the MABR policy offers the better of both in that it is convenient and near to cost-optimal. Therefore, we think that MABR is practical in any application for which BR is appropriate, and could be considered practical in many applications in which ABR is appropriate provided the slot-interval is not too large.

Acknowledging limitations, our research primarily focuses on a simple model of a single-component system with specific parameters, which may not fully represent the complexity of maintenance scenarios. The quasi-periodic nature of MABR may have different implications in various contexts, warranting further investigation. Additionally, the study assumes certain factors, such as constant costs and the simple way to consider maintenance defaults. In practice, these factors may vary and influence maintenance policy performance differently.

Future research could explore diverse equipment types and industries, providing insights into the applicability of MABR. Further investigations might address the challenge of accurately estimating component lifetime distributions, considering data quality and availability. Developing decision support tools based on MABR principles could aid maintenance planners in making more informed decisions. Finally, addressing the last point in the previous paragraph, the policies we study use a simple model of failure with a homogeneous population of components with an increasing failure rate. It would be interesting to

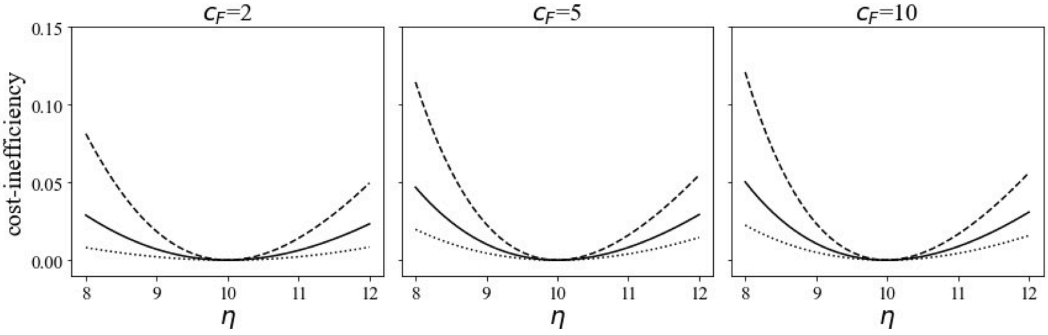

Fig. 5. Parameter uncertainty benchmark: cost-inefficiency of ABR policy when uncertain about η, for $\beta = 2$ (■ ■ ■), $\beta = 3$ (—) and $\beta = 5$ (—). $c_F = 5$.

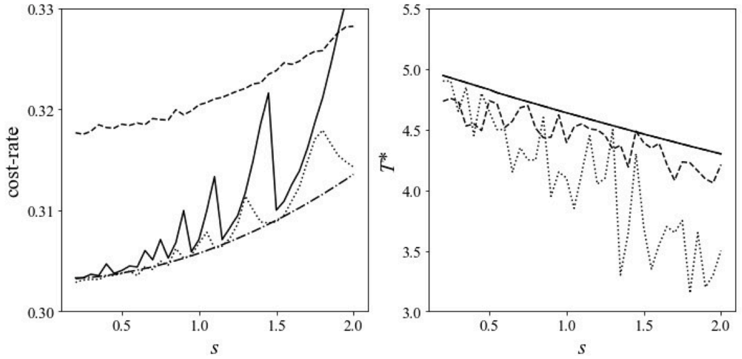

Fig. 6. The case of defaulting: minimum cost-rate and optimum value of the decision variable (T^*) for ABR* (— ■ —), BR* (— —), MABR* (■ ■ ■) and MABR$_{T\text{-ABR}}$ (—) as a function of s in the base case: $\beta = 3, c_F = 5$.

study MABR and the notion of slots for preventive maintenance beyond this failure model framework, that is, when time to failure is modeled as a mixture or modeled with a non-monotonic failure rate.

CRediT authorship contribution statement

Phil Scarf: Writing – review & editing, Writing – original draft, Validation, Project administration, Methodology, Formal analysis, Conceptualization. **Naif Mohammed Alotaibi:** Writing – review & editing, Resources, Investigation, Funding acquisition, Conceptualization. **Cristiano A.V. Cavalcante:** Writing – original draft, Supervision, Investigation, Funding acquisition, Conceptualization. **Yan R. Melo:** Visualization, Validation, Software, Investigation, Formal analysis. **Augusto J.S. Rodrigues:** Writing – original draft, Visualization, Validation, Software, Investigation, Formal analysis, Data curation.

Declaration of competing interest

As corresponding author I can confirm there has been no conflict of interest in preparation of the manuscript “Modified age-based replacement”

Data availability

Data are generated with code that can be requested

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