The Role of Heterogeneity in Price Rigidities for Delayed Nominal Exchange Rate Overshooting*

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Abstract: This paper develops an open economy New Keynesian model in which shocks to monetary policy generate delayed nominal exchange rate overshooting. We show analytically that delayed overshooting is a consequence of heterogeneity in nominal price rigidities. Immediately after a contractionary monetary shock, the reaction of firms with relatively flexible prices generates a strong response of inflation, alongside a currency appreciation. Overtime, as firms with relatively less flexible prices adjust, the appreciation continues, but is subsequently followed by a depreciation. In a calibrated version of the model, with heterogeneity in price rigidity matched with micro-evidence, the peak response of the nominal exchange rate to a monetary policy shock occurs at around 4 quarters.

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1. Introduction

Dornbusch (1976) suggests that, a monetary contraction, which raises the domestic nominal interest rate, leads to a greater appreciation of the domestic currency in the short-run than in the long-run, a phenomenon known as exchange rate overshooting. A general conclusion from this analysis is that when prices are sticky, monetary shocks can generate large, immediate movements in the nominal exchange rate. Empirical studies, however, find evidence of delayed exchange rate overshooting, whereby a monetary contraction leads to a gradual appreciation of the domestic currency, followed by a gradual depreciation (Eichenbaum and Evans, 1995).[1] Typically, the estimated delay in exchange rate overshooting is around 4 quarters.

While Dornbusch's original analysis, and subsequent empirical studies, are based primarily on the dynamics of the nominal exchange rate, the existing New Keynesian literature, perhaps following the lead of Chari *et al.* (2002), has focused on the dynamics of the real exchange rate. In this paper, we take on the challenge of examining whether an open economy New Keynesian model - the modern version of the Dornbusch (1976) model - can generate the type of nominal exchange rate dynamics reported in empirical studies of monetary policy in the open economy.

We start our analysis by considering a standard open economy New Keynesian model. By standard, we mean one with Calvo local-currency sticky-prices, complete international financial markets, and an interest rate setting rule, as, for example, in Engel (2019). We first show that the standard open economy New Keynesian model fails to generate delayed overshooting. A one-off contractionary home monetary policy shock - a rise in the exoge-

[1] Also see Kim and Roubini (2000), Faust and Rogers (2003), Scholl and Uhlig (2008), and Bjornland (2009). Schmitt-Grohe and Uribe (2018) have recently drawn a distinction between the role of temporary and permanent monetary shocks.

nous component of the interest rate rule - leads to an immediate appreciation of the home currency. If the exogenous component of monetary policy is not too persistent, there is immediate overshooting, whereas, if it is relatively persistent, the long-run response of the exchange rate is greater than the short-run response, and there is no overshooting.

Understanding the immediate overshooting result is an important first step in our analysis. We note that the response of the nominal exchange rate to the shock is determined by the sum of the response of relative inflation across countries and the exogenous component of monetary policy.[2] The persistence of the latter is exogenous. The response of relative inflation to the monetary shock is explained by the change in reset prices. If many firms have the option to reset their price (prices are relatively flexible), inflation is relatively responsive to the shock. If the exogenous component of monetary policy is sufficiently persistent, the shock dominates the adjustment process, and there is no overshooting. On the contrary, if few firms have the option to reset their price (prices are relatively sticky), inflation is slow to adjust, and the endogenous policy response to falling prices dominates the adjustment process. In this case, the initial appreciation is followed by a continuous depreciation, and there is immediate overshooting of the nominal exchange rate.

We take the standard open economy New Keynesian model and replace Calvo pricing with Multiple Calvo (MC) pricing. With MC pricing, there are many sectors, each with a different Calvo-style price contract, and Calvo pricing is a special case of MC pricing when all sectors face the same degree of price rigidity. The open economy New Keynesian model with MC pricing provides an explanation for delayed exchange rate overshooting.[3] When

[2] This is a direct result of the uncovered interest rate parity condition combined with home and foreign interest rate rules in which only inflation enters. We refer to this policy regime as inflation targeting.

[3] Whilst it is entirely possible that other frictions - i.e., those which affect asset markets - can lead to delayed nominal overshooting, we focus on frictions in the goods market, in keeping with the standard New Keynesian approach.

we calibrate the model under positive trend inflation, and match heterogeneity in price rigidity with the micro-evidence compiled by Bils and Klenow (2004), the peak response of the nominal exchange rate occurs at around 4 quarters after the shock to monetary policy. This is, for example, close to the empirical results reported in Scholl and Uhlig (2008) - both for a VAR specified with sign restrictions and one with a recursive identification scheme, for the US-German, US-UK, and US-Japan bilateral exchange rate.[4]

To understand why heterogeneity in price rigidities affects the response of the nominal exchange rate to a monetary policy shock it suffices to consider a two-sector example in which firms in one sector have flexible prices and firms in a second sector have Calvo sticky-prices.[5] Again, we need only consider the response of relative inflation across countries. With two sectors, when a shock hits the economy, the initial response of inflation is mainly determined by firms in the flexible-price sector. This is because firms in the flexible-price sector are more responsive to shocks and the effect of changes in prices in the sticky-price sector on inflation are relatively small. Moreover, with heterogeneity in price rigidities, sectoral relative prices also play a role in determining inflation, and changes in sectoral relative prices affect inflation with a delay.

Because the initial process of price adjustment is dominated by firms in the flexible-price sector, and because this sector is relatively responsive to shocks, with MC pricing, the initial response of inflation is larger than what might be suggested by the average level of price-stickiness in the economy. In this sense, there is an 'overshooting' of inflation, alongside the initial currency appreciation. What matters for the on-going response of the exchange rate is that, as firms with sticky-prices begin to adjust to the shock, sectoral relative prices change,

[4]Kim *et al.* (2017) also use a VAR with sign restrictions. They find that only during the Volcker period, from 1979 to 1987, when inflation was relatively high, did the exchange rate exhibit a hump-shaped pattern of adjustment to monetary policy shocks.

[5]This is also the case for which we solve the model with heterogeneity in price rigidities in closed-form.

and the appreciation continues. When the change in sectoral relative prices dissipates the currency begins to depreciate toward its long-run level. In other words, the fact that, in the short-run, the response of inflation is mainly determined by flexible-prices, whilst in the long-run, it is dominated by the sluggish, sticky-price sectors, means that inflation is more sensitive to shocks in the short-run than in the long-run, and this generates a delayed response of the nominal exchange rate to a monetary policy shock.

Our results also allow us to understand the role of trend inflation for nominal exchange rate dynamics when there are monetary shocks. We trace the implications of trend inflation for delayed nominal overshooting back to the volatility of inflation. Trend inflation has a stronger effect on firms with relatively stickier prices because such firms make larger price changes, when given the opportunity to do so, in order to protect their prices against future inflation; in effect, they become more forward looking as trend inflation rises. Such forward-lookingness acts to magnify the mechanism that generates delayed nominal overshooting and produces delayed overshooting in the real exchange rate. Overall, trend inflation leads to an earlier, more pronounced delay in the peak response of the nominal exchange rate.

It is important to note that the mechanism we outline above, not only gives rise to delayed overshooting in our model, but improves the empirical performance of New Keynesian models, in general. MC pricing accounts for the heterogeneity in price rigidity evident in micro-data and is specifically designed to address criticisms directed at New Keynesian models.[6] Estimating MC pricing within the Smets and Wouters (2007) framework, Kara (2015) shows that while both versions of the model match the aggregate data equally well, prices in the MC model are more volatile than those under Calvo pricing, and MC pricing matches the data on reset price inflation better than the Calvo model. Price volatility in the MC

[6]Chari *et al.* (2009) note that standard New Keynesian models, such as Smets and Wouters (2007), require large price shocks to match inflation data. Bils, Klenow and Malin (2009) find that such large price shocks lead to implausible reset price dynamics.

model arises due to the presence of sectors with relatively flexible prices and the required size of exogenous price shocks, used to match price data, are smaller in the MC model than in the corresponding Calvo model. The fact that the MC model produces price dynamics that are closer to the data, and those price dynamics, in turn, generate an empirically relevant nominal exchange rate response to monetary policy shocks, reinforce our insight that prices matter for nominal exchange rate determination.

In the model we develop, uncovered interest rate parity (UIP) holds, and the exchange rate fluctuations we identify are based on what Engel (2015) refers to as the traditional asset markets approach. We show that delayed overshooting can occur when UIP holds. There is a large theoretical literature that studies how the nominal exchange rate responds to monetary shocks when UIP fails to hold. For example, in Gourinchas and Tornell (2004), systematic distortions in investors' beliefs lead to delayed overshooting, and in Bacchetta and van Wincoop (2010), there are infrequent portfolio decisions because investors face a fixed interval of changing portfolios. In Valchev (2020), excess returns arise as compensation for differences in the convenience yields of bonds - the non-pecuniary benefit of holding safe and liquid assets that can serve as substitute for money - denominated in different currencies.[7] In general, under inflation targeting, the change in the nominal exchange rate can be decomposed into relative inflation across countries, excess returns, and the exogenous process for monetary policy. If we were to eliminate the heterogeneity in our model, and instead impose a standard Calvo pricing structure, delayed overshooting would require a violation of UIP, with a specific pattern of excess returns.[8]

[7]There is also an important literature that demonstrates how models with complete markets can generate endogenous risk premia using the long-run risks model of Bansal and Yaron (2004). For example, see Bansal and Shaliastovich (2013).

[8]There is a distinction between the failure of UIP and delayed overshooting. The former is an unconditional statement about nominal interest rates and changes in the exchange rate, whereas the latter is a statement about their joint conditional response to a common unanticipated monetary innovation. We

Our analysis builds on a large open economy New Keynesian literature. Closest to our paper is Carvalho and Nechio (2011), who focus on the role of aggregation for the purchasing power parity puzzle (the persistence and volatility of the real exchange rate), and Carvalho *et al.* (2019) and Engel (2019), who focus on real exchange rate persistence, and the source of persistence in the nominal interest rate, when monetary policy is specified as an interest rate setting rule. Relative to these papers, we focus primarily on the path of adjustment of the nominal exchange rate, and more specifically, we consider exchange rate overshooting. We show that persistence in the exogenous component of interest rate policy can reduce the likelihood of delayed nominal overshooting but that delayed nominal overshooting is insensitive to interest rate inertia. We also find that trend inflation raises the potential for delayed overshooting.

Finally, we relate our quantitative analysis of an open economy to research that focuses on the role of trend inflation in a closed economy. Ascari and Ropele (2009) and Ascari and Sbordone (2014) have emphasized the importance of trend inflation for determinacy.[9] More recently, however, using a medium-scale New Keynesian model, Ascari *et al.* (2019) have shown that there are potentially large interactions between trend inflation and shocks to the marginal efficiency of investment. Although we focus exclusively on a shock to monetary policy, we also find that trend inflation has important cyclical implications. We emphasize that trend inflation matters when there is heterogeneity in price rigidities: multiple sectors, each of which differ in their duration of price spells.

The remainder of the paper is organized as follows. In section 2, we develop an open economy New Keynesian model with heterogeneity in price rigidities. In section 3, we characterize nominal exchange rate overshooting (in a one sector economy) and delayed overshooting (in

discuss the role of excess returns in section 4.

[9] The empirical relevance of the New Keynesian Phillips curve under trend inflation is discussed in Cogley and Sbordone (2008).

a two sector economy). In section 4, we calibrate our model to be consistent with micro-evidence on price rigidities and study the implications of monetary policy shocks. Section 5 concludes.

2. Model Economy

In this section, we develop an open economy New Keynesian economy with heterogeneity in nominal price rigidities. There are two identical countries - home and foreign - each populated by a continuum of households and firms with mass normalized to one. In each country, a representative household supplies labor to firms and consumes a basket of home and foreign goods. Households have access to a complete set of internationally traded state-contingent securities. Firms are divided into $i = 1, ..., N$ sectors and all firms serve the domestic and export market. The only frictions in our economy are monopolistic competition in the goods market and Calvo local-currency price stickiness.

Since some features of our model are standard, in what follows, we focus on the role of sectoral relative prices and trend inflation for firm pricing equations. Consumption, output, and the nominal price of the home/foreign output are denoted by the subscript $s = h, f$. Asterisks denote foreign country variables. A complete description of the economy is provided in Appendix A.

2.1. Households

The representative household consumes differentiated goods from each sector, with cumulative budget shares, $\widehat{\alpha}_i = \sum_{k=1}^{i} \alpha_k$, where $\widehat{\alpha}_0 = 0$ and $\widehat{\alpha}_N = 1$. The unit interval for sector i is $[\widehat{\alpha}_{i-1}, \widehat{\alpha}_i]$ and there is a constant elasticity of substitution aggregator over goods,

$$c_{s,t} = \left\{ \sum_{i=1}^{N} \int_{\widehat{\alpha}_{i-1}}^{\widehat{\alpha}_i} \left[c_{i,s,t}(z) \right]^{(\varepsilon-1)/\varepsilon} dz \right\}^{\varepsilon/(\varepsilon-1)} \tag{1}$$

where $s = h, f$. A firm is indexed by z, the parameter $\varepsilon > 1$ measures the elasticity of substitution, and $\sum_{i=1}^{N} \alpha_i = 1$. Given the form of preferences in equation (1), we express the demand curve for $i = 1, ..., N$ as, $c_{i,s,t}(z) = [p_{i,s,t}(z) / p_{s,t}]^{-\varepsilon} c_{i,s,t}$, with price index, $p_{s,t}^{1-\varepsilon} = \sum_{i=1}^{N} \int_{\widehat{\alpha}_{i-1}}^{\widehat{\alpha}_i} [p_{i,s,t}(z)]^{1-\varepsilon} dz$.

The household consumes home ($c_{h,t}$) and foreign ($c_{f,t}$) goods according to the following aggregator,

$$c_t = \left[\omega^{1/\nu} c_{h,t}^{(\nu-1)/\nu} + (1-\omega)^{1/\nu} c_{f,t}^{(\nu-1)/\nu}\right]^{\nu/(\nu-1)} \tag{2}$$

where $\omega \in (0, 1)$ is a measure of openness to trade and $\nu > 0$ measures the elasticity of substitution. Given the form of preferences in equation (2), the demand curves are, $c_{s,t} = \omega (p_t/p_{s,t})^{-\upsilon} c_t$ where $p_t = \left[\omega p_{h,t}^{1-\upsilon} + (1-\omega) p_{f,t}^{1-\upsilon}\right]^{1/(1-\upsilon)}$ is the consumer price index. The first-order conditions of the household optimization problem can be expressed as,

$$M_{t,t+1} = \frac{\beta c_{t+1}^{-\sigma} / c_t^{-\sigma}}{p_{t+1}/p_t} \quad \text{and} \quad E_t M_{t,t+1} = \frac{1}{R_t} \tag{3}$$

and,

$$\frac{c_{f,t}}{c_{h,t}} = \frac{1-\omega}{\omega} \left(\frac{p_{h,t}}{p_{f,t}}\right)^{\nu} \quad \text{and} \quad w_t = \delta L_t^{\eta} c_t^{\sigma} \tag{4}$$

where $R_t = 1 + i_t > 1$ is the short-term gross nominal interest rate (with $M_{t,t} \equiv 1$), $w_t \equiv W_t/p_t$ is the wage rate in units of consumption, and the parameter σ (η) measures the inverse of the intertemporal elasticity of substitution in consumption (Frisch elasticity of labor supply, L_t, with respect to wages).[10]

2.2. Firms

In sector i, each period, firm z has a probability $1 - \gamma_i$ of being able to reset its price. Each firm has a linear technology in labor and sells their product in the domestic and export

[10]The parameter $\delta > 0$ is a weight in the utility function that allows us to calibrate steady-state labor supply. It plays no other role in our analysis.

market. The profit from domestic sales for firm z is, $\vartheta_{i,h,t}(z) = [p_{i,h,t}(z) - W_t]\, c_{i,h,t}(z)$, and the profit from export sales is, $\vartheta^{\star}_{i,h,t}(z) = \left[e_t p^{\star}_{i,h,t}(z) - W_t\right] c^{\star}_{i,h,t}(z)$, where e_t is the nominal exchange rate, defined as the price of one unit of foreign currency in units of home currency.

Firm z chooses the optimal reset price in the domestic and export market, $X_{i,h,t}(z)$ and $X^{\star}_{i,h,t}(z)$, respectively, to maximize expected discounted profits,

$$E_t \sum_{j=0}^{\infty} \gamma_i^j Q_{t,t+j} \left[\frac{\vartheta_{i,t+j}(z)}{p_{t+j}}\right] \quad \text{and} \quad E_t \sum_{j=0}^{\infty} \gamma_i^j Q_{t,t+j} \left[\frac{\vartheta^{\star}_{i,t+j}(z)}{p^{\star}_{t+j}}\right] \tag{5}$$

where $Q_{t,t+j} \equiv \beta^j \left(c_{t+j}/c_t\right)^{-\sigma}$ is a (real) stochastic discount factor and $\beta \in (0,1)$ is the subjective discount factor of the representative household.

The home currency domestic and export market real reset prices (units of consumption) for firm z are given by the following expressions:

$$x_{i,h,t}(z) = \frac{\varepsilon}{\varepsilon - 1}\frac{\psi_{i,h,t}}{\phi_{i,h,t}} \qquad \text{where} \qquad \begin{array}{l} \psi_{i,h,t} = w_t c_t^{1-\sigma} + \gamma_i \beta E_t \left(\frac{\pi_{h,t+1}^{\varepsilon-\upsilon}}{\pi_{t+1}^{-\upsilon}}\psi_{i,h,t+1}\right) \\ \phi_{i,h,t} = c_t^{1-\sigma} + \gamma_i \beta E_t \left(\frac{\pi_{h,t+1}^{\varepsilon-\upsilon}}{\pi_{t+1}^{1-\upsilon}}\phi_{i,h,t+1}\right) \end{array} \tag{6}$$

and,

$$x^{\star}_{i,h,t}(z) = \frac{\varepsilon}{\varepsilon - 1}\frac{\psi^{\star}_{i,h,t}}{\phi^{\star}_{i,h,t}} \qquad \text{where} \qquad \begin{array}{l} \psi^{\star}_{i,h,t} = w_t c_t^{-\sigma} c_t^{\star} + \gamma_i \beta E_t \left[\frac{\left(\pi^{\star}_{h,t+1}\right)^{\varepsilon-\nu}}{\left(\pi^{\star}_{t+1}\right)^{-\upsilon}}\psi^{\star}_{i,h,t+1}\right] \\ \phi^{\star}_{i,h,t} = \left(c_t^{\star}\right)^{1-\sigma} + \gamma_i \beta E_t \left[\frac{\left(\pi^{\star}_{h,t+1}\right)^{\varepsilon-\nu}}{\left(\pi^{\star}_{t+1}\right)^{1-\nu}}\phi^{\star}_{i,h,t+1}\right] \end{array} \tag{7}$$

for $i = 1, ..., N$. In equations (6) and (7), $\frac{\varepsilon}{\varepsilon-1} > 1$ is the flexible-price markup, and $\pi_{t+1} \equiv p_{t+1}/p_t$ is the rate of home consumer price inflation. Analogous definitions are applied for home producer price inflation, $\pi_{h,t}$, and foreign consumer and producer price inflation, $\pi^{\star}_t$ and $\pi^{\star}_{f,t}$, respectively. We express the optimal reset prices in equations (6) and (7) as the ratio of marginal cost (ψ) to marginal revenue (ϕ). Inflation terms appear in both the cost and revenue functions because forward-looking firms realize that the optimal price set in period t may remain fixed for a number of periods and that inflation erodes their markup over time. Hence, firms use future expected inflation rates to discount future marginal costs, and the higher the future expected rate of inflation, the higher the relative weight on expected future costs (Ascari and Sbordone, 2014).[11]

Associated with the firm optimal pricing equations, there are two sector-specific dynamic equations. The first dynamic equation explains the sector-level producer price versus the consumer price index and the second equation explains dispersion in firm-level producer prices. For example, for domestic sales in the home economy - i.e., those conditions relevant for equations in (6), the evolution of the average sector price is,

$$\rho_{i,h,t}^{1-\varepsilon} = \gamma_i \left(\frac{\rho_{i,h,t-1}}{\pi_t} \right)^{1-\varepsilon} + (1-\gamma_i)\, x_{i,h,t}^{1-\varepsilon} \tag{8}$$

where $\rho_{i,h,t} \equiv p_{i,h,t}/p_t$. This condition links the average price with the reset price. The dynamic equation for price dispersion is,

$$\Delta_{i,h,t} = (1-\gamma_i) \left[x_{i,h,t} \left(\frac{p_t}{p_{h,t}} \right) \right]^{-\varepsilon} + \gamma_i \pi_{h,t}^{\varepsilon} \Delta_{i,h,t-1} \tag{9}$$

where $\Delta_{i,h,t} \equiv \int_0^1 \left(p_{i,h,t}(z) / p_{h,t} \right)^{-\varepsilon} dz$. Whilst equations (8) and (9) are relatively standard, what matters for our analysis is that the γ_i parameters are used to match facts consistent with micro-data.

2.3. Households and Firms in the Foreign Economy

The foreign economy is identical to the home economy and foreign household total consumption, $c_t^\star$, is defined over foreign-produced ($c_{f,t}^\star$) and home-imported ($c_{h,t}^\star$) goods. Foreign households also have access to home-currency state-contingent securities and a non-traded foreign-currency riskless asset, which allows us to write the stochastic discount factor as,

[11] In the open economy, inflation rates for both the price of home goods (in local currency terms) and the overall consumption basket affect the reset price, and the influence of these terms is determined by both the elasticity of substitution between home and foreign differentiated goods ($\upsilon > 0$) and the elasticity of substitution between differentiated goods ($\varepsilon > 1$).

$Q^\star_{t,t+1} \equiv \beta \left(c^\star_{t+1}/c^\star_t\right)^{-\sigma} \left(e_t p^\star_t / e_{t+1} p^\star_{t+1}\right)$. With no impediments to trade in financial markets, $Q^\star_{t,t+1} = Q_{t,t+1}$.

The nominal exchange rate is,

$$e_t = q_t \left(\frac{p_t}{p^\star_t}\right) \tag{10}$$

and the international risk sharing condition is,

$$q_t = \left(\frac{c_t}{c^\star_t}\right)^\sigma \tag{11}$$

where $q_0 \left(c_0/c^\star_0\right)^{-\sigma} = 1$.

Foreign production is consumed domestically and is exported, with reset prices, $X^\star_{i,f,t}(z)$ and $X_{i,f,t}(z)$, set in local currency, with each sector, i, having a different Calvo hazard rate.

2.4. Specification of Monetary Policy

Monetary policy in each economy is conducted using an interest rate setting rule. In the home economy,

$$\widehat{i}_t = \rho_i \widehat{i}_{t-1} + (1-\rho_i)\left(\phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t\right) + \widehat{\upsilon}_t \tag{12}$$

where a circumflex denotes the log deviation of a variable from its steady state value and y_t is GDP. The parameter ρ_i is an interest rate smoothing parameter and $\widehat{\upsilon}_t$ is the exogenous component of monetary policy. Throughout our analysis we assume the exogenous component of monetary policy follows an AR(1) process, such that $\widehat{\upsilon}_t = \rho \widehat{\upsilon}_{t-1} + \widehat{\varepsilon}_t$, where $\rho \in [0, 1)$. The foreign economy uses the same interest rate rule, targeting the lagged nominal interest rate, consumer price inflation and GDP.

3. Heterogeneity in Price Rigidities and Delayed Nominal Exchange Rate Overshooting

In this section, we study the role of heterogeneity in price rigidities for delayed nominal exchange rate overshooting analytically. We first explain why, in a one sector model, it is not possible to generate delayed exchange rate overshooting. In response to a contractionary home monetary policy shock, after an initial appreciation, the home currency either continues to appreciate, or begins to depreciate. If ρ measures the persistence of the exogenous component of monetary policy and γ the mass of firms that cannot reset their price each period, overshooting (a continuous appreciation) requires $\rho < \gamma$ ($\rho > \gamma$).

In a two sector model, we characterize the conditions under which delayed overshooting occurs. With two sectors, inflation is not simply a weighted sum of contemporaneous reset prices in both sectors, but rather, it also depends, with a one period lag, on the sectoral relative price. Delayed overshooting occurs when, after an initial appreciation, the home currency continues to appreciate, but at some point depreciates towards its long-run level. The condition which characterizes a sustained, but temporary, appreciation of the home currency is, $\frac{\rho^{t+1}-z_1^{t+1}}{\rho^t-z_1^t} > \gamma$, where z_1 is increasing in the mass of flexible price firms, and importantly, is equal to zero without such firms.

Throughout the analysis below we assume that monetary policy targets inflation. Since uncovered interest rate parity (UIP) holds in our model, we express the expected change in the nominal exchange rate as a function of relative CPI inflation across countries and the exogenous component of monetary policy,

$$E_t \Delta \widehat{e}_{t+1} = \phi_\pi \widehat{\pi}_t^R + \widehat{\upsilon}_t \tag{13}$$

where $\widehat{\pi}_t^R \equiv \widehat{\pi}_t - \widehat{\pi}_t^\star$ is relative inflation. The shock we study is a one-off change in $\widehat{\varepsilon}_t$ at period $t = 0$ which, all else equal, acts to raise the home nominal interest rate. Nominal exchange rate dynamics are determined by the persistence of the exogenous component of monetary policy and by how price rigidities influence the adjustment of inflation across countries. The latter depends on the distribution of price-setting.

3.1. Nominal Exchange Rate Dynamics in a One Sector Economy

In this section, we collapse our model to one in which there is a single sector where firms have a Calvo probability γ. We make two further simplifications. We assume an equal weight is placed on home and foreign consumption in total consumption (no home-bias) and that utility from total consumption (labor) is logarithmic (linear). Second, we assume the steady-state level of inflation is zero.[12]

With Calvo-pricing, for the home economy, we have the following conditions for reset prices,

$$\widehat{x}_{s,t} = (1-\gamma\beta)\,\widehat{c}_t + \gamma\beta E_t\left(\widehat{\pi}_{t+1} + \widehat{x}_{s,t+1}\right) \qquad \text{for} \qquad s = h, f \tag{14}$$

where $\widehat{x}_{h,t}$ ($\widehat{x}_{f,t}$) is the domestic (import) reset price in home currency and $\widehat{c}_t = \widehat{w}_t$ is marginal cost. Current reset prices depend on their expected future values, $E_t\widehat{x}_{s,t+1}$, current home consumption, $\widehat{c}_t$, and expected future home inflation, $E_t\widehat{\pi}_{t+1}$. Because domestic and import reset prices depend on common terms it is immediate that $\widehat{x}_{f,t} = \widehat{x}_{h,t}$.

Reset prices are linked to international relative prices in the following way,

$$\widehat{\rho}_{s,t} = \gamma\left(\widehat{\rho}_{s,t-1} - \widehat{\pi}_t\right) + (1-\gamma)\,\widehat{x}_{s,t} \tag{15}$$

where $\widehat{\rho}_{h,t} \equiv \widehat{p}_{h,t} - \widehat{p}_t$ ($\widehat{\rho}_{f,t}$) is the relative price of the domestic (imported) good in home currency. Again, inflation enters these equations in a common way, and since $\widehat{x}_{f,t} = \widehat{x}_{h,t}$, it must be that $\widehat{\rho}_{f,t} = \widehat{\rho}_{h,t}$. This result means that the consumer price index, which can be written as, $0 = \frac{1}{2}\left(\widehat{\rho}_{h,t} + \widehat{\rho}_{f,t}\right)$, implies $\widehat{\rho}_{f,t} = \widehat{\rho}_{h,t} = 0$. In other words, in response to monetary policy shocks, there can be no dynamics in international relative prices.[13]

[12] These restrictions are dropped in the quantitative section. Moreover, if we allow home-bias in consumption, we can generate similar results with Calvo producer-currency sticky-prices as opposed to Calvo local-currency sticky-prices.

[13] This point is discussed in detail in Benigno (2002), where it is shown (Proposition 1, page 493) that price movements offset each other as long as the duration of price contracts are the same within and across countries.

We now return to equation (15), which implies,

$$\widehat{\pi}_t = \left(\frac{1-\gamma}{\gamma}\right)\widehat{x}_{s,t} \tag{16}$$

and provides a link between home inflation and reset prices for the domestic and imported good. Eliminating the reset price from equation (14) generates the standard open economy New Keynesian Phillips curve, which, in relative terms is,

$$\widehat{\pi}_t^R = \zeta\widehat{c}_t^R + \beta E_t\widehat{\pi}_{t+1}^R \tag{17}$$

where $\widehat{c}_t^R \equiv \widehat{c}_t - \widehat{c}_t^\star$ is relative aggregate consumption and $\zeta \equiv (1-\gamma)(1-\gamma\beta)/\gamma$ is the slope of the New Keynesian Phillips curve. Since, by UIP, the expected change in the nominal exchange rate is a function of relative inflation across countries and the exogenous component of monetary policy - equation (13) - we can solve for the nominal exchange rate, which we summarize in the following Proposition.

Proposition 1 Assume $\beta \to 0$. In a one sector economy, the nominal exchange rate is determined by,

$$\Delta\widehat{e}_t = \frac{(\gamma-\rho)\,\rho^t}{\Lambda}\widehat{\varepsilon}_0 \tag{18}$$

for $t \geq 1$ where $\Lambda \equiv \gamma\rho\left[(1-\rho) + \frac{1-\gamma}{\gamma}(\phi_\pi - \rho)\right] > 0$ and $\widehat{\varepsilon}_0 > 0$ is the initial shock to monetary policy.

Proof See Appendix. ■

Proposition 1 establishes that delayed overshooting of the nominal exchange rate to a monetary policy shock is ruled-out in a one-sector economy. The home currency will either appreciate continuously ($\rho > \gamma$), or immediately overshoot it's long-run level ($\rho < \gamma$), which implies the initial appreciation is followed by a sustained depreciation. We can see this point by noting that the initial response of the exchange rate (in period $t = 0$) is,

$$\widehat{e}_0 = -\frac{\rho}{\Lambda}\widehat{\varepsilon}_0$$

In subsequent periods ($t > 0$), the change in the nominal exchange rate, $\Delta\widehat{e}_t$, is either always positive or negative. The response of the exchange rate depends on the parameter γ, which is a measure of price-stickiness, and on the parameter ρ, which measures the persistence of the exogenous component of monetary policy. If $\rho > \gamma$, then $\Delta\widehat{e}_t < 0$, so, when the shock is sufficiently persistent ($\rho > \gamma$), the exchange rate appreciates monotonically; whereas, when $\rho < \gamma$, there is immediate overshooting; i.e., $\widehat{e}_0 < 0$ and $\Delta\widehat{e}_t > 0$ for all $t > 0$.[14]

To explain the difference between these two possibilities we re-express the New Keynesian Phillips curve as, $\widehat{\pi}_t^R = \zeta \sum_{j=0}^{\infty} \beta^j E_t \widehat{c}_{t+j}^R$, where ζ is falling in γ.[15] In the standard New Keynesian model, the Phillips curve is purely forward-looking, and the maximum effect of the shock on inflation occurs upon impact. Monetary policy shocks affect inflation through changes in consumption and the response of inflation to a change in consumption is determined by the extent of price-stickiness.

We first consider the case of $\rho > \gamma$ where the exogenous process for monetary policy is relatively persistent. A contractionary monetary policy shock leads to a fall in consumption and inflation alongside an appreciation of the home currency. Since prices are sticky, inflation adjusts sluggishly to the shock, and the initial appreciation is sustained until the exchange rate reaches its new steady-state value. This pattern of adjustment means there is no overshooting. In the case of $\rho < \gamma$, as before, the monetary shock results in a drop in inflation and an appreciation of the home currency. However, since the effect of the shock is relatively temporary, and inflation adjusts quickly, there is no force in the model that would sustain the initial appreciation. As a consequence, the home currency begins to depreciate to its new steady-state level, a pattern of adjustment which gives rise to immediate

[14] In either case, we can verify $\widehat{e}_1 < 0$. The long-run response of the nominal exchange rate to the shock can be written as, $\widehat{e} = \left(\frac{1-\gamma}{1-\rho}\right)\widehat{e}_0$.

[15] Recall, $\zeta \equiv (1-\gamma)(1-\gamma\beta)/\gamma$. In Propositions 1 (and 2, below), we impose $\beta \to 0$ to sharpen our exposition. This does not affect any of the points we make in this section.

overshooting. Overall, these findings suggest that, whilst the standard model can produce immediate exchange rate overshooting, delayed overshooting is ruled out.

3.2. Inflation and Exchange Rate Dynamics with MC Pricing

We now focus on the simplest case of our model with two sectors. Firms in sector 1 have flexible prices ($\gamma_1 = 0$) and comprise a fraction $\alpha_1 = \alpha$ of all firms. Firms in sector 2 have local-currency Calvo sticky-prices ($\gamma_2 = \gamma$).[16]

The optimal reset price for domestic sales in the flexible-price sector (sector 1) is $\widehat{x}_{1,h,t} = \widehat{c}_t$. With flexible prices, the relative price and the reset price are equated contemporaneously, such that, $\widehat{\rho}_{1,h,t} = \widehat{x}_{1,h,t}$. By analogy, in the foreign economy, it must be that $\widehat{\rho}^{\star}_{1,f,t} = \widehat{x}^{\star}_{1,f,t} = \widehat{c}^{\star}_t$, and because the law of one price holds in sector 1, the reset price of imported goods in the home economy is determined by $\widehat{x}_{1,f,t} = \widehat{x}^{\star}_{1,f,t} + \widehat{q}_t$. Finally, international risk-sharing equates the real exchange rate with relative consumption and $\widehat{x}_{1,f,t} = \widehat{c}_t$. As such, the sector 1 reset prices of domestic and imported goods are the same when expressed in a common currency, $\widehat{x}_{1,h,t} = \widehat{x}_{1,f,t}$.

The Calvo sticky-price sector (sector 2) is identical to the one sector model described above and so the results that apply there still hold. In particular, the reset prices of domestic and imported goods are equal, i.e., $\widehat{x}_{2,h,t} = \widehat{x}_{2,f,t}$. Again, this is because, in addition to their own future value, reset prices only depend on aggregate home consumption and home inflation. The difference between sector 1 and sector 2 is that, in sector 2, because there is Calvo-pricing, it is not possible to equate the relative price and the reset price contemporaneously.

Sectoral relative prices in the home economy are linked in the following way,

$$\widehat{\rho}_{s,t} = \alpha\widehat{\rho}_{1,s,t} + (1-\alpha)\,\widehat{\rho}_{2,s,t} \tag{19}$$

[16] This version of the model collapses to a standard one-sector open economy when $\alpha = 0$.

where $\widehat{\rho}_{s,t}$ are international relative prices, defined above. Compared to the one sector economy, since we have only added a single flexible-price sector, it is still the case that there is synchronization of international relative prices, conditional on monetary policy shocks, and so $\widehat{\rho}_{s,t} = 0$.

Using the above information, along with the condition that determines Calvo price-dynamics, we arrive at the following expression for inflation in a two sector economy,

$$\widehat{\pi}_t = \frac{\alpha}{(1-\alpha)\gamma}\widehat{x}_{1,s,t} + \frac{1-\gamma}{\gamma}\widehat{x}_{2,s,t} - \frac{\alpha}{1-\alpha}\widehat{\rho}_{1,s,t-1} \tag{20}$$

which is a generalization of equation (16). With two sectors, when a shock hits the economy, the response of inflation is mainly determined by the flexible-price sector. There are three reasons for this. First, firms in the flexible-price sector are more responsive to shocks. Second, the effect of reset prices on inflation in the sticky-price sector is small. This is because only a fraction $\frac{1-\gamma}{\gamma}$ of firms adjust their price in each period. Finally, and as a consequence of heterogeneity in price rigidities, sectoral relative prices become relevant since profit-maximizing firms do not want their price to deviate too much from the prices of other firms in the economy. Changes in sectoral relative prices affect inflation with a delay.

Because the initial part of price adjustment is dominated by the flexible-price sector and because this sector is relatively responsive to shocks, with MC pricing, the initial response of inflation is larger than what might be suggested by the average level of price-stickiness in the economy. In this sense, there is an 'overshooting' of inflation, which is later corrected, as firms want keep their prices close to other firms' prices, a point captured by the fact that sectoral relative prices enter equation (20) with a negative sign. It is this correction, over time, that sustains the initial appreciation of the home currency. However, as the influence of sectoral relative prices dissipates, the home currency begins to depreciate toward its long-run level.

Before providing a formal characterization of our main result it is useful to derive an expression for inflation dynamics. Eliminating the optimal reset prices and the sectoral relative price in equation (20) we explain home inflation in terms of aggregate consumption. Taking the same steps for the foreign economy, relative inflation is determined by the following expression,

$$\widehat{\pi}_t^R = \frac{\alpha(1+\beta)+\zeta}{1-\alpha}\widehat{c}_t^R + \beta E_t \widehat{\pi}_{t+1}^R - \frac{\alpha}{1-\alpha}\left(\widehat{c}_{t-1}^R + \beta E_t \widehat{c}_{t+1}^R\right) \tag{21}$$

which is a two sector open economy New Keynesian Phillips curve. The important point of note from equation (21) is that current relative inflation depends on the one-period lag of relative consumption. We know that consumption enters this expression with a lag term because of sectoral relative prices. Finally, the presence of flexible-price firms also affects the slope of the New Keynesian Phillips curve, and with a greater mass of flexible-price firms, current inflation is more sensitive to relative consumption contemporaneously.

3.3. Delayed Exchange Rate Overshooting in a Two Sector Economy

We now provide a formal characterization of our main result

Proposition 2 Assume $\beta \to 0$. In a two sector economy, the nominal exchange rate is determined by,

$$\Delta\widehat{e}_t = \frac{(\gamma-\rho)\,\rho^t - (\gamma - z_1)\, z_1^t}{(z_1-\rho)\,(\rho - z_2)}\widehat{\varepsilon}_0 \tag{22}$$

for $t \geq 1$ where z_1 and z_2 are (stable and unstable) roots of $\gamma_f z^2 - z + \gamma_b = 0$, $\gamma_f > 0$ and $\gamma_b > 0$ are composite parameters defined in the Appendix, and $\widehat{\varepsilon}_0 > 0$ is the initial shock to monetary policy.

Proof See Appendix. ■

It is worth remembering that $\alpha = 0$ is the case without heterogeneity in price rigidities. Imposing this condition we find $z_1 = 0$ and $z_2 = \frac{1+\phi_\pi\zeta}{1+\zeta} > 1$ and equation (22) collapses to equation (18) in Proposition 1. In general, $z_1 < 1$ is increasing in α, because the more flexible-price firms there are, the more inertia in inflation is created through sectoral relative prices. As we discuss above, this is reflected in both home inflation - equation (20) - and the two sector open economy New Keynesian Phillips curve - equation (21). A final observation from Proposition 2 is that the initial change in the exchange rate that results from the shock is, $\widehat{e}_0 = \frac{1}{\rho - z_2}\widehat{\varepsilon}_0 < 0$, which only depends on z_2 and the persistence of the exogenous component of monetary policy, ρ.[17]

Given the initial response of the exchange rate to the monetary shock the simplest way to express delayed overshooting is as follows:

$\Delta\widehat{e}_1 < 0, ..., \Delta\widehat{e}_{T-1} < 0, \Delta\widehat{e}_T > 0, \Delta\widehat{e}_{T+1} > 0, ...$

In words, following the initial appreciation of the home currency, there needs to a period of continued appreciation, followed by a period of depreciation. Thus, what matters is that the initial appreciation continues for at least one period. Given the required pattern of adjustment for the nominal exchange rate, Proposition 2 delivers a clear condition that needs to be satisfied for delayed nominal exchange rate overshooting to occur. Delayed overshooting requires $(\gamma - \rho)\,\rho^T > (\gamma - z_1)\,z_1^T$ to hold for $T \geq 1$. Only in this case, is there a gradual appreciation, followed by a gradual depreciation.

To understand the condition for delayed overshooting we isolate the mass of flexible-price firms using the following expression,

$$\frac{\rho^{T+1} - z_1^{T+1}}{\rho^T - z_1^T} > \gamma \tag{23}$$

where $z_1 < \rho$. Equation (23) says that, at some point after the shock to monetary policy, it needs to be the case that the combination of ρ and α (via z_1) is greater than the price-stickiness in sector 2. The simplest way to understand this condition is to suppose that

[17] The long-run response of the nominal exchange rate, given by, $\widehat{e} = \left(\frac{1-\gamma}{1-\rho}\right)\frac{1}{1-z_1}\widehat{e}_0$, is also a straightforward generalization of the one sector model.

$\rho = 0$. In this case, there must be immediate overshooting as $z_1 < \gamma$. This condition is conceptually similar to the condition in a one sector economy that requires $\rho < \gamma$ to hold for immediate overshooting. More generally, this means delayed overshooting will not occur with either a low ρ or a low α. That is, there needs to be some persistence in the exogenous process for monetary policy and some firms with flexible prices.

To generate further insights, consider the case in which $T = 1$, where equation (23) reduces to $\rho + z_1 > \gamma$. This condition corresponds to the one additional period of home currency appreciation delayed overshooting requires. Now recall, in the one sector economy (i.e., when $z_1 = 0$), a continuous appreciation occurs when $\rho > \gamma$. Given that z_1 is increasing in α, a higher share of flexible-price firms in the economy helps generate delayed overshooting in the nominal exchange rate. This insight holds true more generally, for $T > 1$, and it is possible to verify that, given ρ, a higher share of flexible prices increases the right-hand side of equation (23), helping to satisfy this condition.

We also illustrate our point by solving for the peak response of the exchange rate. Suppose the condition in equation (23) holds with equality at period $t = T$. This condition characterizes $\Delta\widehat{e}_T = 0$ and allows us to solve for the timing of the peak response of the nominal exchange rate in the following way,

$$T = \frac{\ln(\gamma - \rho) - \ln(\gamma - z_1)}{\ln(z_1) - \ln(\rho)} \tag{24}$$

which is increasing in z_1, and therefore α, the fraction of flexible-price firms. In this case, it is useful to consider a simple numerical example. Suppose $\phi_\pi = 1.5$ and $\gamma = 0.75$, which are standard values, and that 25% of firms in the economy have flexible prices ($\alpha = 0.25$). This parameterization results in $z_1 = 0.24$. Using these values we solve for the required level of persistence of the exogenous component of monetary policy such that there is delayed overshooting. For example, solving for $T = 1$ ($T = 2$), we find $\rho > 0.51$ ($\rho > 0.69$).[18]

[18] We can verify this corresponds to the example of $T = 1$ where $\rho + z_1 > \gamma$ needs to be satisfied. Since

Our example shows that, if the exogenous process for monetary policy is not sufficiently persistent, there will be immediate overshooting.

4. Exchange Rate Dynamics in a Calibrated Model

In this section, we use a calibrated version of our model to understand the potential quantitative implications of heterogeneity in price rigidities for nominal exchange rate dynamics.

4.1. Calibration

The average post-1979 annual rate of US inflation was 3.46 percent and the average nominal interest rate during this period was 4.90 percent. This implies an average annual real return of 1.44 percent. We set $\beta = 0.9856^{1/4}$ to match this statistic.[19] Our calibration of the model then proceeds in two steps. First, we assign parameters to hazard rates, sector shares, and the elasticity of substitution between differentiated goods (that is, γ_i and α_i and ε) using micro evidence on prices provided by Bils and Klenow (2004), hereafter BK. Second, we assign standard values to the parameters of the model that are not directly related to heterogeneity in price rigidities.

The share of each sector in our economy is calibrated based on BK. This set of parameters replace the parameter α in our analysis above. BK report the frequency of price changes for around 300 product categories, which covers 70 percent of the US consumer price index. Following Kara (2015), we aggregate up from their 300 sectors so that we have 9 sectors with distinct reset probabilities. The aggregation is performed by forming probability focal points in increments of 0.1 percentage points $(0.2, ..., 1)$. The BK reset probabilities are

$\gamma = 0.75$ implies $z_1 = 0.24$, based on other parameter choices, it is immediate that we require $\rho > 0.51$ to generated delayed overshooting of one period.

[19]Data for inflation are taken from the World Bank (indicator code: FP.CPI.TOTL.ZG) and for US interest rates we use the Federal Reserve Bank of St. Louis (FRED); mnemonic FEDFUNDS. The figure of 1.44 percent for the real interest rate is similar to the empirical estimates reported in Del Negro *et al.* (2019).

rounded to 0.1 percentage point. Next, we allocate the BK reset probabilities to these focal points. The sectors are scaled by the share in expenditure that is allocated to each focal point. The economy-wide mean frequency of price adjustment is around 0.4 with the share of flexible contracts at 34 percent.

Given the distribution of prices we target a long-run average markup. At the sectoral level, the markup is increasing in both trend inflation and the elasticity of substitution, such that the more sticky the sector, the stronger the effect of trend inflation on the markup. The long-run sector i markup is, $\mu_i = [\varepsilon / (\varepsilon - 1)] [(1 - \beta\gamma_i\pi^{\varepsilon-1}) / (1 - \beta\gamma_i\pi^{\varepsilon})]$, where $\varepsilon / (\varepsilon - 1) > 1$ is the standard markup arising from monopolistic competition. For the 34 percent of firms with flexible prices in our model we set $\varepsilon = 10$ such that the flexible-price sector markup is 11 percent. Given sectoral shares, and conditional on the sector-specific value of γ, we adjust ε such that the average markup across sectors is also 11 percent.

Following the parameterization in Carvalho and Nechio (2011), we set $\nu = 1.5$, $\omega = 0.9$, and $\sigma^{-1} = 1/3$. These parameters determine the Armington elasticity, the extent of home-bias in consumption, and the intertemporal elasticity of substitution in consumption, respectively. We set $\eta^{-1} = 0.2$, which is the Frisch elasticity of labor supply with respect to wages. Parameters associated with the policy rule are also standard, and are set at $\rho_i = 0.7$, $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. Finally, we are forced to take a stance on the persistence of the exogenous component of the interest rate in the home economy. As a benchmark, we assume the persistence is 0.68.

4.2. Impulse Response Functions

In this section, we focus on the impulse response functions for the nominal exchange rate $(\widehat{e}_t)$ and the price level $(\widehat{p}_t)$.[20] Figure 1 reports the impulse responses for the Calvo and MC pricing models to a one-off reduction in the exogenous component of the nominal interest

[20]We only need consider these key variables because, when $\widehat{p}^{\star}_t \simeq 0$, we can construct the real exchange

rate of 1 percent under our benchmark calibration. The solid (red) line reports the case with trend inflation and the dashed (blue) line reports the case with zero trend inflation.

===== Figure 1 =====[21]

We start by considering the IRFs for the Calvo pricing model. The Calvo model cannot generate delayed overshooting in the nominal exchange rate. On impact, both the price level and the nominal exchange rate increase and gradually move toward to their new long-run value (which in the latter case is a continuous depreciation of the home currency). Figure 1 further indicates that, with a single sector, trend inflation has little role to play in the dynamic adjustment of the nominal exchange rate.

Figure 1 also shows that heterogeneity in price rigidities leads to delayed nominal exchange rate overshooting. The explanation for delayed overshooting was discussed above in our analytic model. After the shock, firms with flexible prices are first to react. Because flexible-price firms dominate the initial part of the price-setting process, there is overshooting in inflation, alongside the initial currency depreciation. Overtime, as firms with sticky-prices adjust to the shock, the currency continues to depreciation, up until a point where the lag effect of sectoral relative prices dissipates sufficiently. Then the currency appreciates toward it's long-run level. Positive trend inflation interacts with this mechanism. In particular, without trend inflation, the peak response of the nominal exchange rate occurs at around 8 quarters, whereas, with positive trend inflation, there is a more pronounced peak, which occurs at around 4 quarters. To understand why, recall the optimal reset prices, given by

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rate, $\widehat{q}_t = \widehat{e}_t - \widehat{p}_t^R$, the relative nominal interest rate, $\widehat{i}_t^R = \Delta\widehat{e}_{t+1}$, relative inflation, $\widehat{\pi}_t^R = \widehat{p}_t^R - \widehat{p}_{t-1}^R$, and the relative real interest rate, $\widehat{r}_t^R = \widehat{i}_t^R - \widehat{\pi}_{t+1}^R$.

[21] Notes: Figure 1 presents impulse response functions for the nominal exchange rate and price level for the Calvo and MC model. IRFs are plotted with and without trend inflation. The horizontal axis is quarters and the vertical axis is percent deviation from the steady-state.

equations (6) and (7), which show that firms use future expected inflation rates to discount future marginal costs. The higher the future expected rate of inflation, the higher the relative weight on these costs. As inflation rises, firms effectively become more forward-looking, giving more weight to future than to present economic conditions. The result is an earlier, more pronounced delay in the peak response of the exchange rate.

In general, the change in the nominal exchange rate can be decomposed into relative interest rates and excess returns (violations of UIP). When interest rates are used to target inflation, the nominal exchange rate is determined by relative inflation, excess returns, and the exogenous process for monetary policy. Since our analysis assumes UIP holds, excess returns are zero, and the change in the exchange rate maps directly into the relative interest rate. Thus, a loosening of home monetary policy, which causes a home currency depreciation, implies that the relative interest rate first rises above, and then falls below, its long-run value. This process occurs alongside rising home prices. The reason the interest rate responds in this particular way is that heterogeneity in price rigidities, and the presence of firms with flexible prices, generates sensitivity in the initial response of prices to monetary shocks. This suggests that a violation of UIP is not a necessary condition for delayed overshooting.

The empirical literature that documents delayed overshooting, and which focuses on the impulse response of the exchange rate to a monetary shock, has identified a potential role for excess returns. For example, in Eichenbaum and Evans (1995), a positive innovation to the Fed Funds rate, causes a rise in the US interest rate, an appreciation of the US Dollar, and an increase in the returns to investing in short-term US versus foreign securities (Figure III, page 995). This conditional violation of UIP tends to be short-lived (at around 6 months) and occurs prior to the peak response of the nominal exchange rate (at over 2 years, in this particular specification).[22] On the contrary, our mechanism stresses the adjustment of prices over a number of periods, and is consistent with the empirical response of inflation. If we

[22] Kim and Roubini (2000), Faust and Rogers (2003), Scholl and Uhlig (2008), and Bjornland (2009) discuss

were to specify a standard New Keynesian model with Calvo pricing, delayed overshooting would require a violation of UIP, with a specific path of adjustment for excess returns.

We now consider the role played by the specification of interest rate policy. Carvalho *et al.* (2019) have shown that a model with MC-type pricing can generate a hump shaped response of the real exchange rate to a monetary policy shock. However, they further show that allowing for inertia in the policy rule (ρ_i in our notation) eliminates delayed overshooting in the real exchange rate, even when there is high persistence in the exogenous component of monetary policy (specifically, $\rho_i = 0.927$).[23] Analytically, we have discussed the role of persistence in the exogenous component of monetary policy for generating delayed nominal exchange rate overshooting. Given these insights, we focus our attention on the role of persistence in the exogenous component of monetary policy and policy inertia.

Figure 2 plots impulse responses for the nominal exchange rate assuming different degrees of persistence in the exogenous component of monetary policy and policy inertia.

===== **Figure 2** =====[24]

Two results stand out from Figure 2. First, interest rate inertia does not affect the extent of delayed nominal exchange rate overshooting. In fact, qualitatively, the path of the nominal exchange rate is almost identical with no policy inertia ($\rho_i = 0$) and our benchmark case

the conditional path of excess returns. Unconditionally, the violation in UIP is such that higher interest rates are associated with higher (lower) excess returns in the short-run (long-run). See Engel (2016) and Valchev (2020).

[23] Coibion and Gorodnichenko (2012) present empirical evidence of the source on interest rate persistence.

[24] Notes: Figure 2 presents IRFs for the nominal exchange rate for the case considered by Carvalho *et al.* (2019), our baseline case ($\rho = 0.68$), and zero persistence ($\rho = 0$), as a point of reference. IRFs are plotted with and without trend inflation. The horizontal axis is quarters and the vertical axis is percent deviation from the steady-state.

($\rho_i = 0.7$). What does change, however, is the size of the overall response, which, at longer horizons, is around three times larger once policy inertia is introduced. Second, simply adding persistence to the exogenous process for the interest rate, whilst allowing for a delay in the peak response of the real exchange rate (not reported), eliminates the same phenomenon for the nominal rate, since it only acts to raise the long-run response to the shock. Thus, delayed nominal overshooting occurs with and without interest rate inertia, when the persistence of the exogenous process for monetary policy is not too strong.[25]

Finally, in tests we do not report here (available upon request), we checked the robustness of our conclusions to an alternative price dataset provided by Nakamura and Steinsson (2008). These authors argue that, when analyzing the macroeconomic effects of price-stickiness, it is important to distinguish between sale and non-sale price changes. With sales, price changes, on average, are more frequent than without, and this difference is reflected in the distribution of the frequency of price adjustment. In particular, with sales, the share of flexible prices is around 39% (34% in the BK distribution), whereas when sale prices are excluded, the share of flexible prices reduces to 22%. In all cases, the model with MC pricing generates delayed overshooting in the nominal exchange rate. While the model calibrated with the price data including sales produces more or less the same pattern of delayed overshooting as the BK distribution, the model with the price data excluding sales produces an earlier peak response of the nominal exchange rate. These findings are consistent with the notion that flexible prices play a significant role in influencing nominal exchange rate dynamics.

4.3. International Correlations

So far, we have discussed the dynamic response of the nominal exchange rate to a one-

[25]Ultimately, as Engel (2019) discusses, the dynamics of the exchange rate are sensitive to the source of interest rate persistence, whether that be inertia in the interest rate or persistence of the exogenous process for monetary policy.

off shock to home monetary policy. In this section, we briefly consider the international correlation between key variables implied by our model whilst maintaining this single-shock approach. To do so, we take the benchmark calibration and pick a cross-country correlation for home and foreign monetary policy shocks that matches the cross-correlation of GDP in the data. Our motivation for undertaking this exercise is to better understand the role of pricing assumptions in a model in which monetary shocks account for the observed cross-correlation in GDP.[26]

===== Table 1 =====[27]

Column (1) of Table 1 presents the statistics implied by the data, while Columns (2) and (3) report those from the Calvo and MC-pricing models, respectively. We first consider cross-correlations. Overall, we find that the MC model has relatively little impact on the cross-correlation of consumption (and other variables) relative to the Calvo model. This is not surprising, since adding MC-pricing to the baseline New Keynesian model does not affect the endogenous international spillover of monetary shocks, which are based on a standard, and well-accepted, transmission mechanism.

For both the Calvo model and the model with MC pricing, the correlation of the real exchange rate with the real interest rate differential is the correct sign - implying a country with a

[26]The same approach is taken in Chari *et al.* (2002), page 556. In general, we do not target empirical moments since matching unconditional moments from the data would require a richer model and additional shocks.

[27]Notes: The empirical moments we report are from three sources. The serial correlation in the real exchange rate, the correlation between the real and nominal exchange, and the cross-correlation of consumption and GDP are reported in Chari *et al.* (2002); Table 6. The cross-correlation in inflation is reported in Wang and Wen (2007); Table 1. The remaining statistics are from Valchev (2020); Table 2. All model-based statistics we report have been HP filtered with a smoothing parameter of $1,600$.

high interest rate tends to have a stronger currency. Whilst the Data column of Table 1 reports unconditional correlations there is evidence that this feature of the data may hold conditionally. For example, Eichenbaum and Evans (1995) report a value of $\mathsf{corr}\left(e_t, \widehat{i}_t\right)$ between -0.27 and -0.11 (Table IIIa, page 997). Although this negative correlation is a feature of the data, models that generate violations of UIP based on an endogenous risk premium, and which may offer a potential alternative explanation for delayed overshooting, produce a positive correlation (Engel, 2016). In contrast, Valchev (2020), who focuses on convenience yields, rather than the risk premium, generates a conditional correlation of -0.93, when monetary policy shocks are uncorrelated across countries. Finally, we report serial correlations, of which the real exchange rate has received the most attention. It has been well-documented - for example, in Chari *et al.* (2002) and Carvalho and Nechio (2011) - that the monetary model cannot explain the PPP puzzle. The same logic applies to our model with MC pricing. However, the focus of our analysis is the conditional response to monetary shocks, and as Itskhoki and Mukhin (2020) have recently shown, the solution to this puzzle (and others) may require combining financial shocks with productivity and monetary shocks.

5. Conclusions

This paper studies the response of the nominal exchange rate to monetary shocks using an open economy New Keynesian model. We show that heterogeneity in price rigidities play an important role in determining the response of the exchange rate to monetary policy shocks. In particular, such heterogeneity can lead to delayed nominal overshooting, a phenomenon documented in the empirical literature, such as Eichenbaum and Evans (1995). When heterogeneity in price rigidity is matched with micro-evidence, and when long-run inflation is calibrated to historical values, the peak response of the nominal rate is around 4 quarters. Whilst the importance of heterogeneity in price rigidities and trend inflation have been noted separately for the response of real variables to monetary shocks, we show that the interaction between these two factors has implications for nominal exchange rate dynamics.

Appendix A

In this Appendix we present details omitted in the main text for the model setting of section 2. The home household's intertemporal utility function is $E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \delta \frac{L_t^{1+\eta}}{1+\eta} \right)$. Utility is maximized choosing consumption bundles of the home and foreign good, $c_{h,t}$ and $c_{f,t}$, internationally traded home currency state-contingent securities, B_{t+1}, and labor supply, subject to the flow budget constraint,

$$p_{h,t} c_{h,t} + p_{f,t} c_{f,t} + E_t \left[M_{t,t+1} B_{t+1} \right] = W_t L_t + B_t + \int \vartheta_t (z) \, dz + T_t \tag{25}$$

where $M_{t,t+1}$ is a nominal stochastic discount factor that prices, in period t, any financial asset portfolio with state-contingent payoff B_{t+1} at the beginning of period $t+1$, $\int \vartheta_t (z) \, dz$ is aggregate profit, and T_t is a lump-sum transfer.

The problem for the firm is to choose the reset price, $X_{i,h,t}(z)$, to solve the following unconstrained problem,

$$\max E_t \sum_{j=0}^{\infty} Q_{t,t+j} \gamma^j \left\{ \frac{X_{i,h,t}(z)}{p_{t+j}} \left[\frac{X_{i,h,t}(z)}{p_{h,t+j}} \right]^{-\varepsilon} - \frac{W_{t+j}}{p_{t+j}} \left[\frac{X_{i,h,t}(z)}{p_{h,t+j}} \right]^{-\varepsilon} \right\} c_{h,t+j} \tag{26}$$

where p_t is the consumer price index defined in the main text. Since all firms that have the opportunity to reset their price are identical, we drop the z index, and define $x_{i,h,t} \equiv X_{i,h,t}/p_t$ as the real reset price (units of consumption). We express the solution to the firms problem as,

$$x_{i,h,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\psi_{i,h,t}}{\phi_{i,h,t}} \qquad \text{where} \qquad \begin{array}{c} \psi_{i,h,t} \equiv E_t \sum_{j=0}^{\infty} (\gamma_i \beta)^j \left(\frac{\pi_{h,t,t+j}^{\varepsilon-\nu}}{\pi_{t,t+j}^{-\upsilon}} \right) \delta L_{t+j}^{\eta} c_{t+j} \\ \phi_{i,h,t} \equiv E_t \sum_{j=0}^{\infty} (\gamma_i \beta)^j \left(\frac{\pi_{h,t,t+j}^{\varepsilon-\nu}}{\pi_{t,t+j}^{1-\nu}} \right) c_{t+j}^{1-\sigma} \end{array} \tag{27}$$

In the main text we express $\psi_{i,h,t}$ and $\phi_{i,h,t}$ recursively following Ascari and Sbordone (2014). The unconstrained optimization problem for exporting is to choose $X_{i,h,t}^{\star}(z)$ to maximize,

$$E_t \sum_{j=0}^{\infty} \gamma_i^j Q_{t,t+j} \left\{ \frac{q_{t+j} X_{i,h,t}^{\star}(z)}{p_{t+j}^{\star}} \left[\frac{X_{i,h,t}^{\star}(z)}{p_{h,t+j}^{\star}} \right]^{-\varepsilon} - w_{t+j} \left[\frac{X_{i,h,t}^{\star}(z)}{p_{h,t+j}^{\star}} \right]^{-\varepsilon} \right\} c_{h,t+j}^{\star} \tag{28}$$

With the real reset price defined in units of consumption in the destination market; i.e., $x^{\star}_{h,t} \equiv X^{\star}_{h,t}/p^{\star}_{t}$, we find,

$$x^{\star}_{i,h,t} = \frac{\varepsilon}{\varepsilon - 1}\frac{\psi^{\star}_{h,t}}{\phi^{\star}_{h,t}} \qquad \text{where} \qquad \begin{array}{l} \psi^{\star}_{i,h,t} \equiv E_t \sum_{j=0}^{\infty} (\gamma_i \beta)^j \left[\frac{\left(\pi^{\star}_{h,t,t+j}\right)^{\varepsilon-\nu}}{\left(\pi^{\star}_{t,t+j}\right)^{-\nu}}\right] \delta L^{\eta}_{t+j} c^{\star}_{t+j} \\ \phi^{\star}_{i,h,t} \equiv E_t \sum_{j=0}^{\infty} (\gamma_i \beta)^j \left[\frac{\left(\pi^{\star}_{h,t,t+j}\right)^{\varepsilon-\nu}}{\left(\pi^{\star}_{t,t+j}\right)^{1-\nu}}\right] \left(c^{\star}_{t+j}\right)^{1-\sigma} \end{array} \tag{29}$$

using $p^{\star}_t = p^{\star}_{t+j}/\pi^{\star}_{t,t+j}$ and $q_{t+j} = e_{t+j}p^{\star}_{t+j}/p_{t+j} = \left(c^{\star}_{t+j}/c_{t+j}\right)^{-\sigma}$. Foreign conditions are derived analogously. For reference, we note, $\vartheta^{\star}_{i,f,t}(z) = p^{\star}_{i,f,t}(z)\, y^{\star}_{i,f,t}(z) - W^{\star}_t l^{\star}_{i,f,t}(z)$ and $\vartheta_{i,f,t}(z) = \frac{p_{i,f,t}(z)}{e_t} y_{i,f,t}(z) - W^{\star}_t l_{i,f,t}(z)$ are profits of foreign firms (for domestic and export sales, respectively).

Total resources in the domestic economy are,

$$\begin{aligned} L_t &= \sum_{i=1}^{N} \int_{\widehat{\alpha}_{i-1}}^{\widehat{\alpha}_i} c_{i,h,t}(z) + \sum_{i=1}^{N} \int_{\widehat{\alpha}_{i-1}}^{\widehat{\alpha}_i} c^{\star}_{i,h,t}(z) \\ &= \left[\omega \Delta_{h,t} + (1-\omega)\Delta^{\star}_{h,t} \left(\frac{e_t p^{\star}_{h,t}}{p_{h,t}}\right)^{-\nu} q_t^{\nu - 1/\sigma}\right] \left(\frac{p_{h,t}}{p_t}\right)^{-\nu} c_t \end{aligned} \tag{30}$$

where we define $\Delta_{h,t} \equiv \sum_{i=1}^{N} \alpha_i \Delta_{i,h,t}$ and $\Delta^{\star}_{h,t} \equiv \sum_{i=1}^{N} \alpha_i \Delta^{\star}_{i,h,t}$. In the foreign economy,

$$L^{\star}_t = \left[\omega \Delta^{\star}_{f,t} + (1-\omega)\Delta_{f,t} \left(\frac{p_{f,t}}{e_t p^{\star}_{f,t}}\right)^{-\nu} q_t^{-\nu + 1/\sigma}\right] \left(\frac{p^{\star}_{f,t}}{p^{\star}_t}\right)^{-\nu} c^{\star}_t$$

where $\Delta^{\star}_{f,t} \equiv \sum_{i=1}^{N} \alpha^{\star}_i \Delta^{\star}_{i,f,t}$ and $\Delta_{f,t} \equiv \sum_{i=1}^{N} \alpha^{\star}_i \Delta_{i,f,t}$. The foreign economy is also characterized by demand for goods and labor supply conditions analogous to the home economy.

Home-currency state-contingent securities are traded internationally and there are no impediments to trade in financial markets, $M_{t,t+1} = M^{\star}_{t,t+1} \equiv \beta \left(c^{\star}_{t+1}/c^{\star}_t\right)^{-\sigma} \left(e_t p^{\star}_t / e_{t+1} p^{\star}_{t+1}\right)$. Defining the real exchange rate as, $q_t = e_t \left(p^{\star}_t/p_t\right)$, the international risk sharing condition is, $q_t = \left(c^{\star}_t/c_t\right)^{-\sigma}$, where $q_0 \left(c_0/c^{\star}_0\right)^{-\sigma} = 1$.

Appendix B

Our analytical results in section 3 are derived for a two sector economy. In this appendix, we present the details and the proofs of Proposition 1 and 2.

B.1. Inflation Dynamics in the Two Sector Economy

Assume $\sigma = 1$, $\eta = 0$, $\omega = 1/2$, $\delta = 1$ Household labor-leisure conditions are, $W_t/p_t = c_t$ and $W_t^\star/p_t^\star = c_t^\star$ with $q_t = c_t/c_t^\star$. Sectoral demand functions are

$$c_{s,t} = \frac{1}{2}\left(p_{s,t}/p_t\right)^{-\upsilon} c_t \quad \text{and} \quad c_{s,t}^\star = \frac{1}{2}\left(p_{s,t}^\star/p_t^\star\right)^{-\upsilon} c_t^\star \tag{31}$$

where $p_t = \left[(1/2)\, p_{h,t}^{1-\upsilon} + (1/2)\, p_{f,t}^{1-\upsilon}\right]^{1/(1-\upsilon)}$ and $p_t^\star = \left[(1/2)\, p_{h,t}^{\star 1-\upsilon} + (1/2)\, p_{f,t}^{\star 1-\upsilon}\right]^{1/(1-\upsilon)}$ are the home and foreign CPI. Firms are divided into sectors: a fraction α of firms are free to change their price each period and a fraction $1-\alpha$ of firms have a Calvo technology. In the home economy,

$$p_{s,t}^{1-\varepsilon} = \underbrace{\int_0^\alpha \left[p_{1,s,t}(z)\right]^{1-\varepsilon} dz}_{\text{flexible prices}} + \underbrace{\int_\alpha^1 \left[p_{2,s,t}(z)\right]^{1-\varepsilon} dz}_{\text{Calvo sticky-price}} \tag{32}$$

Similar equations hold for $p_{s,t}^\star$.

Now consider equation (6) in the text. In linearized form, for sector i, we have, $\widehat{x}_{i,h,t} = \widehat{\psi}_{i,h,t} - \widehat{\phi}_{i,h,t}$, where,

$$\widehat{\psi}_{i,h,t} = \left(1 - \gamma_i \beta \pi^\varepsilon\right) \widehat{c}_t + \theta \beta \pi^\varepsilon E_t \left[\left(\varepsilon - \upsilon\right) \widehat{\pi}_{h,t+1} + \upsilon \widehat{\pi}_{t+1} + \widehat{\psi}_{i,h,t+1}\right] \tag{33}$$

and,

$$\widehat{\phi}_{i,h,t} = \beta \gamma_i \pi^{\varepsilon-1} E_t \left[\widehat{\phi}_{i,h,t+1} + \left(\varepsilon - \upsilon\right) \widehat{\pi}_{h,t+1} - \left(1 - \upsilon\right) \widehat{\pi}_{t+1}\right] \tag{34}$$

The sector average price is connected to the reset price by,

$$\widehat{x}_{i,h,t} = \frac{1}{\left(1-\gamma_i\right)\left(x_{2,h}/\rho_{i,h}\right)^{1-\varepsilon}} \left(\widehat{\rho}_{i,h,t} - \gamma_i \pi^{\varepsilon-1} \widehat{\rho}_{i,h,t-1}\right) + \frac{\gamma_i \pi^{\varepsilon-1}}{\left(1-\gamma_i\right)\left(x_{i,h}/\rho_{i,h}\right)^{1-\varepsilon}} \widehat{\pi}_t \tag{35}$$

where $\left(x_{i,h}/\rho_{i,h}\right)^{1-\varepsilon} = \left(1 - \gamma_i \pi^{\varepsilon-1}\right) / \left(1 - \gamma_i\right)$.

Impose $\pi = 1$ as in the main text. The four $i = 1$ reset price equations in sector 2 ($\gamma_2 = \gamma$) are such that,

$$\widehat{\rho}_{2,i,t} = \gamma \left(\widehat{\rho}_{2,i,t-1} - \widehat{\pi}_t\right) + (1-\gamma)\,\widehat{x}_{2,h,t} \quad \text{and} \quad \widehat{\rho}^{\star}_{2,i,t} = \gamma \left(\widehat{\rho}^{\star}_{2,i,t-1} - \widehat{\pi}^{\star}_t\right) + (1-\gamma)\,\widehat{x}^{\star}_{2,i,t} \tag{36}$$

where $\widehat{x}_{2,i,t} = (1-\gamma\beta)\,\widehat{c}_t + \gamma\beta\left(\widehat{\pi}_{t+1} + \widehat{x}_{2,i,t}\right)$ and an analogous expression for $\widehat{x}^{\star}_{2,i,t}$. The four $i = 2$ reset price equations in sector 1 ($\gamma_1 = 0$) are such that,

$$\widehat{\rho}_{1,h,t} = \widehat{x}_{1,h,t} \quad \text{and} \quad \widehat{\rho}_{1,f,t} = \widehat{x}_{1,f,t} \quad \text{and} \quad \widehat{\rho}^{\star}_{1,f,t} = \widehat{x}^{\star}_{1,f,t} \quad \text{and} \quad \widehat{\rho}^{\star}_{1,h,t} = \widehat{x}^{\star}_{1,h,t} \tag{37}$$

where $\widehat{x}_{1,h,t} = \widehat{c}_t$ and $\widehat{x}^{\star}_{1,f,t} = \widehat{c}^{\star}_t$. Since the law of one price holds in sector 1, we also have $\widehat{x}^{\star}_{1,h,t} = (\widehat{x}_{1,h,t} - \widehat{q}_t)$ and $\widehat{x}_{1,f,t} = \widehat{x}^{\star}_{1,f,t} + \widehat{q}_t$, where $\widehat{q}_t$ is the real exchange rate. In the home economy, sectoral relative prices are connected by,

$$\widehat{p}_{s,t} - \widehat{p}_t = \alpha\widehat{\rho}_{1,s,t} + (1-\alpha)\,\widehat{\rho}_{2,s,t} \tag{38}$$

where the CPI is $\widehat{p}_t = \frac{1}{2}\left(\widehat{p}_{h,t} + \widehat{p}_{f,t}\right)$. Using this in the reset price equations we conclude $\left(\widehat{p}_{f,t-1} - \widehat{p}_{h,t-1}\right) = \frac{1}{\gamma}\left(\widehat{p}_{f,t} - \widehat{p}_{h,t}\right) = 0$, and therefore $\widehat{\rho}_{s,t} = 0$, following Benigno (2002), Proposition 1.

Finally, corresponding to our local-currency pricing assumption, there are 4 conditions which determine CPI inflation as a function of the reset price, which collapse to 2 conditions, given $\widehat{\rho}_{s,t} = 0$. These are:

$$\widehat{\pi}_t = \frac{1-\gamma}{\gamma}\widehat{x}_{2,s,t} + \frac{\alpha}{1-\alpha}\left(\frac{1}{\gamma}\widehat{x}_{1,s,t} - \widehat{x}_{1,s,t-1}\right) \tag{39}$$

with analogous conditions for the foreign economy. Defining $\widehat{\pi}^R_t \equiv \widehat{\pi}_t - \widehat{\pi}^{\star}_t$, these conditions, along with reset price equations, generate equation (21) in the main text.

B.2. Proof of Proposition 1

Equation (21) in the main text collapses to $\widehat{\pi}^R_t = \zeta\widehat{c}^R_t + \beta E_t\widehat{\pi}_{t+1}$ when $\alpha = 0$. With inflation targeting, the UIP condition is, $E_t\Delta\widehat{e}_{t+1} = \phi_\pi\widehat{\pi}^R_t + \widehat{\upsilon}_t$. The real exchange rate ($\widehat{q}_t \equiv \widehat{e}_t - \widehat{p}^R_t$)

and inflation ($\widehat{\pi}_t^R \equiv \widehat{p}_t^R - \widehat{p}_{t-1}^R$) then imply,

$$\widehat{q}_t = \left(\frac{1+\zeta}{1+\phi_\pi\zeta}\right) E_t\widehat{q}_{t+1} - \left(\frac{1}{1+\phi_\pi\zeta}\right)\upsilon_t \tag{40}$$

once we impose $\beta \to 0$. We solve this equation forward. Using $E_t\widehat{\upsilon}_{t+j} = \rho^j\widehat{\upsilon}_t$ generates $\widehat{q}_t = -\frac{\widehat{\upsilon}_t}{1-\rho+\zeta(\phi_\pi-\rho)}$. The nominal exchange rate is then,

$$\Delta\widehat{e}_t = \Delta\widehat{q}_t + \widehat{\pi}_t^R = \Delta\widehat{q}_t + \zeta\widehat{q}_t = -\left[\frac{\Delta\widehat{\upsilon}_t + \zeta\widehat{\upsilon}_t}{1-\rho+\zeta\left(\phi_\pi-\rho\right)}\right] \tag{41}$$

which is consistent with the presentation in chapter 7 of Gali (2015). Applying the definition of $\Lambda > 0$ in the main text and noting $\widehat{\upsilon}_t = \rho^t\widehat{\upsilon}_0$ for $t \geq 1$ generates equation (18) in Proposition 1 when $\widehat{\upsilon}_0 = \widehat{\varepsilon}_0$. We solve for the level of the nominal exchange rate as, $\widehat{e}_t = \widehat{e}_{-1} + \sum_{j=0}^{t}\Delta\widehat{e}_j$, and the long-run response of the exchange rate is, $\widehat{e} = \Delta\widehat{e}_0 + \sum_{j=1}^{\infty}\Delta\widehat{e}_j = -\frac{\Lambda\zeta}{1-\rho}\widehat{\varepsilon}_0$.

B.3. Proof of Proposition 2

Equation (21) in the main text collapses to $\widehat{\pi}_t^R = -\frac{\alpha}{1-\alpha}\widehat{c}_{t-1}^R + \frac{\alpha+\zeta}{1-\alpha}\widehat{c}_t^R$ when $\beta \to 0$ is imposed. Using the UIP condition and the definition of the real exchange rate,

$$\begin{aligned} \widehat{q}_t &= \gamma_b\widehat{q}_{t-1} + \gamma_f E_t\widehat{q}_{t+1} + \kappa\widehat{\upsilon}_t \\ \gamma_b &\equiv \frac{\alpha\phi_\pi}{1+\phi_\pi\left(\zeta+\alpha\right)} \;\; ; \;\; \gamma_f \equiv \frac{1+\zeta}{1+\phi_\pi\left(\zeta+\alpha\right)} \;\; ; \;\; \kappa \equiv -\frac{1-\alpha}{1+\phi_\pi\left(\zeta+\alpha\right)} \end{aligned} \tag{42}$$

This is a second-order difference equation with forward and backward-looking components, the solution to which is,

$$\widehat{q}_t = z_1\widehat{q}_{t-1} + \frac{\kappa}{z_2\gamma_f}\sum_{j=0}^{\infty}\left(\frac{1}{z_2}\right)^j E_t\widehat{\upsilon}_{t+j} = z_1\widehat{q}_{t-1} - \left(\frac{1-\alpha}{1+\zeta}\frac{1}{z_2-\rho}\right)\widehat{\upsilon}_t \tag{43}$$

and where $z_{1,2} = \left[1 \mp \sqrt{\left(1-4\gamma_f\gamma_b\right)}\right]\frac{1}{2\gamma_f}$. Re-express this solution as,

$$\widehat{q}_t = z_1^{t+1}\widehat{q}_{-1} + \left(1-\alpha\right)\gamma\frac{\rho^{t+1}-z_1^{t+1}}{\left(\rho-z_1\right)\left(\rho-z_2\right)}\widehat{\upsilon}_0 \tag{44}$$

where $\widehat{\upsilon}_s = \rho^{s-t}\widehat{\upsilon}_t$ and $\sum_{s=0}^{t}\left(\frac{\rho}{z_1}\right)^s = \left[\left(\frac{\rho}{z_1}\right)^{t+1} - 1\right]\left(\frac{\rho}{z_1} - 1\right)^{-1}$ and $\widehat{q}_{-1} = 0$ is the initial real exchange rate.

Recall that the definition of the real exchange rate implies $\Delta\widehat{e}_t = \Delta\widehat{q}_t + \widehat{\pi}_t^R$ and that in a two sector model relative inflation is itself a function of the current and lagged real exchange rate, $\widehat{\pi}_t^R = \frac{\alpha+\zeta}{1-\alpha}\widehat{q}_t - \frac{\alpha}{1-\alpha}\widehat{q}_{t-1}$. Using these conditions,

$$\Delta\widehat{e}_t = \left(\frac{1+\zeta}{1-\alpha}\right)\widehat{q}_t - \left(\frac{1}{1-\alpha}\right)\widehat{q}_{t-1} = \left[\frac{\left(\rho^{t+1} - z_1^{t+1}\right) - \gamma\left(\rho^t - z_1^t\right)}{\left(\rho - z_1\right)\left(\rho - z_2\right)}\right]\widehat{\varepsilon}_0 \tag{45}$$

which is the same as equation (22) in Proposition 2. The period $t = 0$ reaction of the nominal exchange rate to the shock is $\Delta\widehat{e}_0 = \widehat{e}_0 = \frac{1}{\rho - z_2}\widehat{\varepsilon}_0 < 0$. The long-run response the shock is $\widehat{e} = \Delta\widehat{e}_0 + \sum_{j=1}^{\infty}\Delta\widehat{e}_j = \frac{1}{\rho-z_2}\frac{1-\gamma}{(1-\rho)(1-z_1)}\widehat{\varepsilon}_0$.

We also characterize the roots $z_{1,2}$. First, as $\alpha \to 0$ we find $\gamma_f \equiv \frac{1+\zeta}{1+\phi_\pi\zeta}$ and $\gamma_b \equiv 0$. In this case, $z_1 = 0$ and $z_2 = \frac{1}{2\gamma_f}$, as we report in the main text. In general, we have,

$$z_{1,2} = \frac{\chi \mp \sqrt{\left(\chi^2 - 4\left(1+\zeta\right)\alpha\phi_\pi\right)}}{2\left(1+\zeta\right)} \quad ; \quad \chi \equiv 1 + \phi_\pi\left(\zeta + \alpha\right) \tag{46}$$

For stability, we require $z_1 < 1$, and this is equivalent to,

$$\chi^2 - 4\left(1+\zeta\right)\alpha\phi_\pi > \left[\chi - 2\left(1+\zeta\right)\right]^2 \Leftrightarrow \phi_\pi > 1$$

which is a standard condition for determinacy. It is also the case that $\frac{\partial z_1}{\partial\alpha} > 0$.

To generate equation (23) in the main text we express the nominal exchange rate in terms of the initial change, $\Delta\widehat{e}_t = \left[\frac{\left(\rho^{t+1} - z_1^{t+1}\right) - \gamma\left(\rho^t - z_1^t\right)}{\rho - z_1}\right]\Delta\widehat{e}_0$. Since, $\rho > z_1$, then delayed overshooting requires $\left(\rho^{t+1} - z_1^{t+1}\right) > \gamma\left(\rho^t - z_1^t\right)$ for at least one period. That is, we require $\frac{\rho^{T+1} - z_1^{T+1}}{\rho^T - z_1^T} > \gamma$ for $T > 1$ periods. Define $t = T$ such that $\Delta\widehat{e}_T = 0$. This generates $\left(\frac{z_1}{\rho}\right)^T = \frac{\rho - \gamma}{z_1 - \gamma}$. Taking logs of this expression, we solve for T as in the main text; equation (24). Finally, in the main text, we present a numerical example assuming values $\{\phi_\pi, \gamma, \alpha\} = \{1.5, 0.75, 0.25\}$.

Using equation (46) these values imply $z_{1,2} = \{0.241, 1.165\}$. We then ask what the lower bound on ρ is such that $T \geq 1$ (the upper-bound is $\rho < \gamma$). Solving for $T = 1$ in condition (24) implies $\rho = 0.51$.

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Figure 1: Dynamic Path of the Nominal Exchange Rate and Domestic Price Level in the Calvo and Multiple Calvo Models

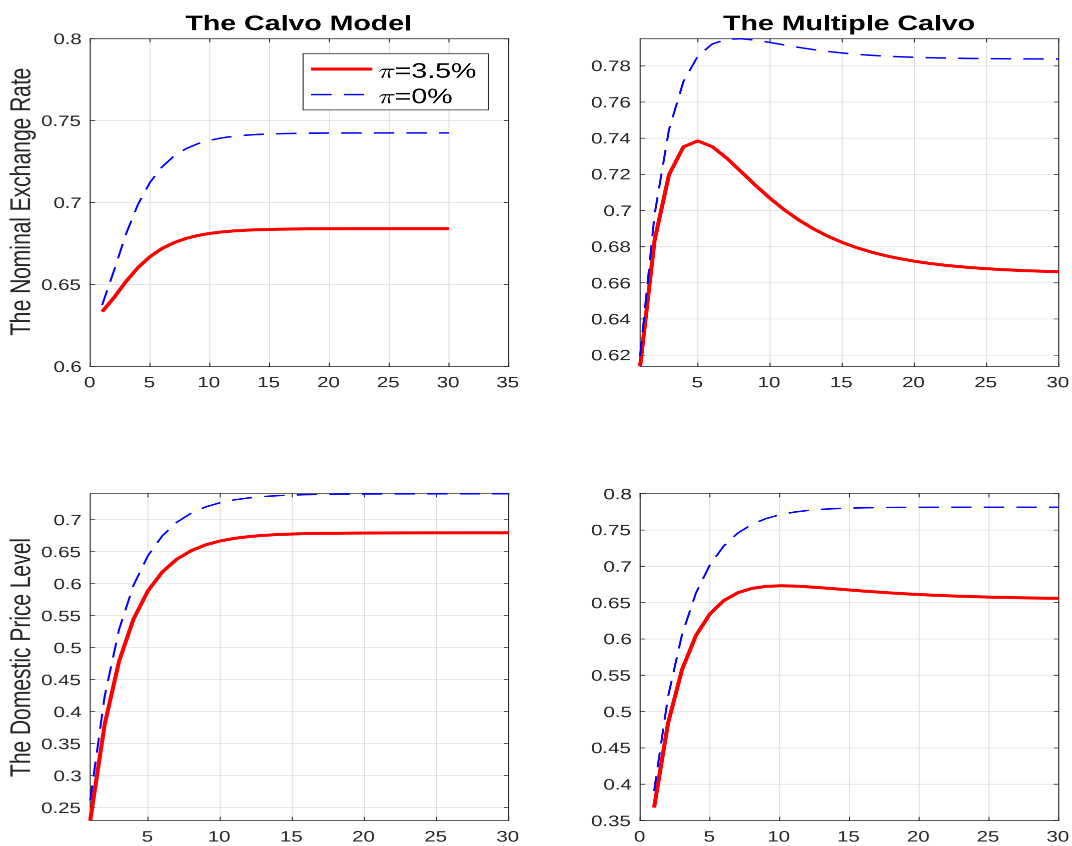

Figure 2: Dynamic Path of the Nominal Exchange Rate in the Multiple Calvo model with Alternative Parameter Values

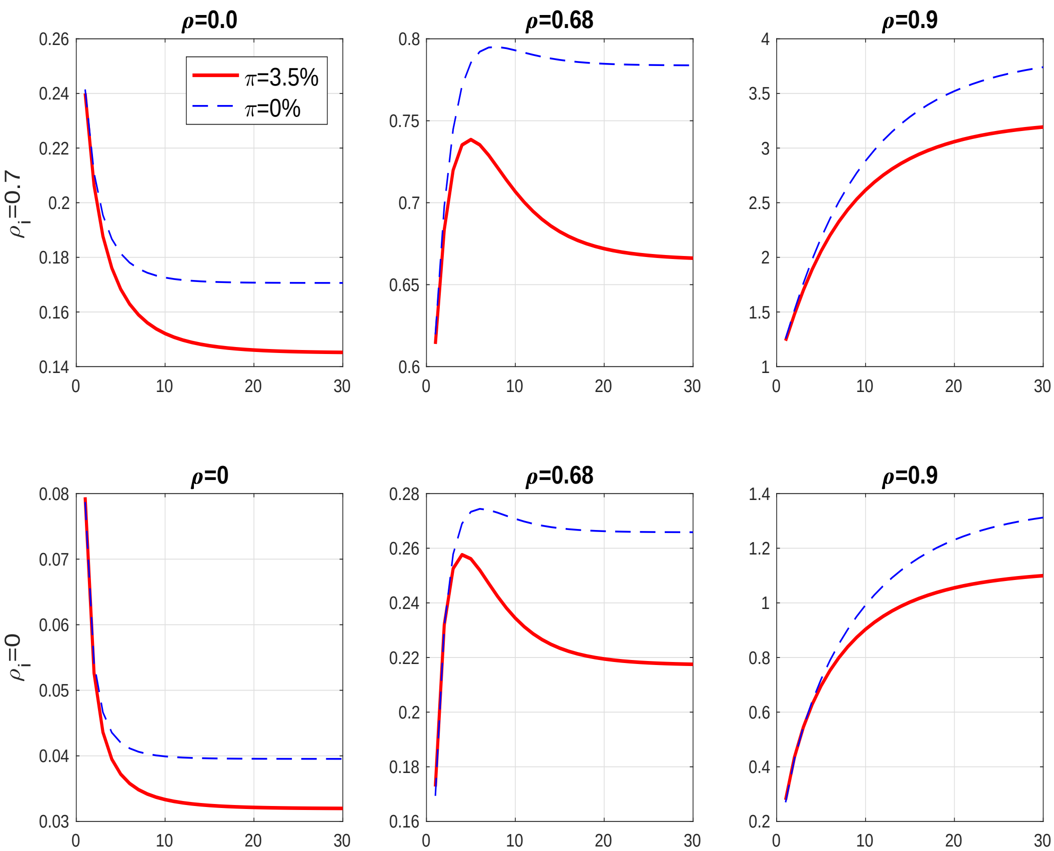

Table 1: International Correlations from the Data and for the Model

	Data	Calvo	Multiple Calvo
	(1)	(2)	(3)
Moments	Correlations		
	Cross-Country		
$\mathsf{corr}(\widehat{y}_t, \widehat{y}_t^\star)$	0.60	0.60	0.60
$\mathsf{corr}(\widehat{c}_t, \widehat{c}_t^\star)$	0.38	0.41	0.38
$\mathsf{corr}(\widehat{\pi}_t, \widehat{\pi}_t^\star)$	0.62	0.43	0.41
$\mathsf{corr}\left(\widehat{i}_t, \widehat{i}_t^\star\right)$	0.68	0.47	0.40
$\mathsf{corr}(\widehat{r}_t, \widehat{r}_t^\star)$	0.49	0.43	0.41
	with the Real Ex. Rate		
$\mathsf{corr}(\Delta\widehat{e}_t, \Delta\widehat{q}_t)$	0.99	0.92	0.93
$\mathsf{corr}(q_t, \widehat{r}_t - \widehat{r}_t^\star)$	-0.17	-0.99	-0.99
	Serial Correlations		
$\rho\left(\Delta\widehat{e}_t\right)$	0	-0.06	0.09
$\rho\left(\widehat{i}_t - \widehat{i}_t^\star\right)$	0.74	0.76	0.47
$\rho\left(\widehat{q}_t\right)$	0.83	0.49	0.66
$\rho\left(\widehat{r}_t - \widehat{r}_t^\star\right)$	0.14	0.47	0.67