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FIGHTING COLLUSION: AN IMPLEMENTATION THEORY APPROACH*

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A competition authority (CA) has an objective, which specifies what output profile firms need to produce as a function of production costs. These costs change over time and are only known by the firms. The objective is repeatedly implementable if the firms cannot collude and deceive the CA in equilibrium. We identify necessary and sufficient conditions for repeated implementation when firms can only announce prices and quantities. We use these conditions to study when the competitive output is implementable. We extend the analysis to the case when the firms can also supply hard evidence.

1. INTRODUCTION

We study the prevention of collusion by firms as a repeated implementation problem. Specifically, we consider an infinite-horizon oligopoly model with discounting. Each period firms produce a homogenous good and face an exogenously given demand, while their production costs change randomly from one period to another. A competition authority (CA) has a certain objective about the firms' output. For example, it wants them to produce the competitive equilibrium output in every period. This objective depends on the production costs, which are unknown to the CA, but commonly known by the firms. The firms will try to benefit from the information asymmetry and instead produce closer to the collusive output. The question that we address is whether the CA can design a game (aka regime) such that the output in every period of every equilibrium coincides with the one given by the CA's objective. That is, we study a full implementation problem.

Repeated implementation has been studied in a general setup by Lee and Sabourian (2011), Mezzetti and Renou (2017), and Āzacis and Vida (2019). If we wanted to apply the results of those papers to the oligopoly problem, the firms would be required to report information about their own and every competitor's costs to the CA. However, it is not how firms normally compete.

The novelty of our article is that we restrict what the firms can report: Each firm only announces the quantity it wants to produce and the price at which it wants to sell. Furthermore, we require that there exists an equilibrium such that on the equilibrium path, in every period, the firms produce exactly those quantities that they announce. We refer to this requirement as

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forthrightness. Without forthrightness, announced prices and quantities could stand for anything, including the costs of other firms, in which case we would be back in the framework of the aforementioned papers.

We can interpret the forthrightness requirement as a restriction on what the CA can do. Specifically, the CA does not intervene in the market as long as the announced prices and quantities are consistent with its objective. We define firms' messages as consistent if the announced quantities coincide with the CA's objective for some realization of costs, and additionally all firms announce the price that equates the announced quantities with the demand. However, once the messages are not consistent, the CA steps in and tells the firms what to produce. In practice, CAs also do not intervene unless there is an evidence of possible anti-competitive behavior.

When firms announce consistent messages, it can still be that the announced quantities do not coincide with the CA's objective given the actual realized costs, in which case the CA fails to implement its objective. Therefore, we study conditions on the CA's objective that ensure that the firms do not collude on consistent but untruthful messages in equilibrium. Throughout we consider pure-strategy subgame perfect equilibria (SPE), although we argue later that all our results remain true if instead we looked for pure-strategy Nash equilibria (NE). Following the implementation literature, we now refer to the CA's objective as a social choice function (SCF). In the main body, we focus on implementation of SCFs in the simpler symmetric setup, while we relegate the general case with asymmetric firms to the Appendix. In the latter case, we show that a condition, which we call pq-dynamic monotonicity, is both necessary and sufficient for repeated implementation of SCF (Proposition A0). Intuitively, the condition guarantees that whenever the firms collude, there always exists a firm that has incentives to act as a whistle-blower by triggering inconsistent messages, which then induce the CA to intervene.

Mezzetti and Renou (2017) show that a condition called dynamic monotonicity is necessary for repeated implementation when the messages can be arbitrary. pq-dynamic monotonicity is a strengthening of dynamic monotonicity due to the restriction of messages to price–quantity pairs and due to the forthrightness requirement. Whether or not messages are consistent only depends on the outputs specified by the SCF and not directly on the realized costs. Therefore, if the SCF specifies the same outputs for two different cost realizations, the CA cannot infer the cost realization from the consistent messages. This makes it harder to provide the right incentives for the whistle-blower because the CA also needs to ensure that no firm acts as a whistle-blower when the other firms are in fact reporting truthfully.

The rest of the analysis is restricted to the symmetric setup. We find that in this setup, pq-dynamic monotonicity is equivalent to two simpler conditions—pq-monotonicity and pq-stationary monotonicity (Proposition 1). The first condition ensures that there cannot be an equilibrium in which the firms report consistent but untruthful messages only for one period but otherwise they behave truthfully. The second condition ensures that there cannot be an equilibrium in which the firms report (contingent on their costs) the same consistent but untruthful messages in every period. Because it is enough to consider these two types of collusive behavior, checking for pq-dynamic monotonicity in the symmetric setup is much easier.

Equipped with these results, we next investigate repeated implementation of the SCF that requires the firms to produce the competitive output for any cost realization. We show that the competitive SCF satisfies pq-monotonicity under some fairly mild assumptions (Proposition 2). However, competitive SCF can fail pq-stationary monotonicity because of a high discount factor (Example 1) or because of the forthrightness requirement (Example 2). However, when the cost functions exhibit strictly increasing differences, we show that the competitive SCF is implementable for all values of the discount factor less than one, if the number of firms exceeds some threshold. For a given number of firms, we show that the competitive SCF is implementable if the discount factor is low enough, and we identify a condition when it is also implementable for discount factors close to one (Proposition 3). Interestingly, an SCF can be implementable for low and high values of the discount factor, but not be implementable for intermediate values (Example 3).

We also briefly consider another SCF that for every cost realization, maximizes the aggregate output subject to firms breaking even. Although the competitive SCF is in general implementable for discount factors close to zero, we find that this other SCF might not be implementable for any value of the discount factor (Proposition 4) or only implementable for the intermediate values (Example 4).

Finally, motivated by the literature on leniency programs (discussed below), we extend the model by incorporating hard evidence. The CA can request the firms to back any reports with supporting evidence. Clearly, more SCFs can be implemented with evidence because it can help when either pq-monotonicity or pq-stationary monotonicity fails. We show that so-called evidence monotonicity is necessary and sufficient for repeated implementation in the symmetric setup (Propositions 5 and 6). If the firms can hide their profits but cannot exaggerate them, then the competitive SCF satisfies evidence monotonicity and, hence, is repeatedly implementable (Example 5).

1.1. *Related Literature.* Nowadays CAs are increasingly relying on leniency programs to combat collusion by firms. Under a leniency program, a firm that helps to uncover and prosecute a cartel, receives a reduction in fines. Starting with Motta and Polo (2003), Spagnolo (2004), Aubert et al. (2006), and Harrington (2008), there is a large and growing literature that analyzes the impact of leniency programs on collusion.[1] A typical model in this literature assumes that collusion creates hard evidence that the CA can discover with some probability, in which case colluding firms face large fines. By offering reduced fines, the CA can incentivize firms to come forward before it has even launched investigation or before the investigation has produced enough evidence.

Our modeling approach differs in several ways from that literature. First of all, the above leniency models can be viewed as moral hazard models (collusion as unobservable action), whereas ours is an adverse selection model (collusion to misrepresent the unobservable costs). Second, the leniency models assume verifiable information, whereas in our model, the CA must rely on nonverifiable information (or partially verifiable information in the case of hard evidence) that the firms supply. Third, in the leniency models, the (sometimes implicit) stage game is given (e.g., a Cournot model), whereas in our model, the CA designs it. Similar to the leniency models, the whistle-blower needs to be rewarded, but instead of using transfers, it is done through altering its future profits.

Because in our model, the CA can influence the price the firms can charge or the quantities they can produce, these firms can also be viewed as regulated companies such as utilities. Over past decades, regulators across the world have been adopting performance benchmarking of the regulated firms in order to improve their efficiency and service quality.[2] That is, firms are rewarded for being efficient, while their efficiency is measured against that of other firms. However, such benchmarking can also create incentives for the firms to coordinate information that they provide to the regulators. Although information revelation by regulated firms is a widely studied subject (see, e.g., Laffont and Tirole, 1993), previous work on collusion by firms that are subject to benchmarking, has only been done by Tangerås (2002) and Dijkstra et al. (2017). In particular, Tangerås (2002) finds the implementable SCF that maximizes the welfare in a static duopoly model with side-payments.

Our analysis builds on several strands of implementation literature. Repeated implementation has been studied by Lee and Sabourian (2011), Lee and Sabourian (2015), Mezzetti and Renou (2017), and Āzacis and Vida (2019). As noted, Mezzetti and Renou (2017) introduce

[1] See also the survey by Rey (2003).

[2] See, for example, the international comparison of benchmarking methods in the electricity sector by Jamasb and Pollitt (2000) or the assessment of benchmarking in the telecommunications, water, and energy sectors in the United Kingdom by Dassler et al. (2006), among many others.

the dynamic monotonicity condition that is closely related to the monotonicity conditions of this article. Further, Lee and Sabourian (2011) show that if an SCF is not efficient, then it is not repeatedly implementable for high enough discount factors. In Remark 2, we comment on the differences in assumptions that allow us to implement inefficient SCFs even when the discount factor is high. Chassang and Ortner (2019) and Ortner et al. (2022) also employ the insights behind the results in Lee and Sabourian (2011) to study cartel detection.

Implementation when the announcements are restricted to prices and quantities has previously been studied in static economies. The seminal work in this area is by Dutta et al. (1994), Sjöström (1994), and Saijo et al. (1996). pq-monotonicity resembles the necessary conditions that were identified in these papers. Bull and Watson (2004), Ben-Porath and Lipman (2012), Kartik and Tercieux (2012), and Banerjee et al. (2023) study one-shot implementation with evidence. In particular, Kartik and Tercieux (2012) introduce the notion of evidence monotonicity. Compared to this literature, we are the first to study implementation with price–quantity announcements and with evidence in the repeated setup.

The rest of the article is organized as follows. We describe the basic elements of the model in Section 2 and define the repeated implementation problem in Section 3. We study necessary and sufficient conditions for repeated implementation in the symmetric setup in Section 4 and in the general case in Appendix A. We investigate repeated implementation of the competitive SCF in Section 5 and that of "break-even SCF" in Section 6. In Section 7, we consider the extension to hard evidence. Although we study a specific implementation problem, we argue in Section 8 that our results can also be applied to other problems like multigood exchange economies. We also discuss alternative solution concepts in this section. We conclude in Section 9. Finally, most of the proofs and extensions are relegated to either the Appendix or the Online Appendix.

2. THE SETUP

There are $n \geq 2$ firms in a market and they interact for an infinite number of periods.[3] Let $I = \{1, \dots, n\}$ denote the set of firms. The periods are indexed as $0, 1, 2, \dots$, with period 0 being the initial period. The firms produce a homogenous product. Each period the firms face the same inverse demand function $p = p(Q)$, where $Q = \sum_{i=1}^{n} q_i$ is the aggregate output and $q_i \in \mathbb{R}_+$ is the output of firm $i \in I$. It is assumed that $p(Q)$ is continuous in Q and strictly decreasing for all Q such that $p(Q) > 0$.

Production costs depend on the state of the world. Let Θ be the set of possible states of the world, which is assumed to be finite. At the start of each period, a new state $\theta \in \Theta$ is drawn independently and identically according to a distribution function l. Each state $\theta \in \Theta$ is realized with a strictly positive probability, $l(\theta) > 0$. The cost function of firm i is $c_i(q_i, \theta)$, which is assumed to be continuous and strictly increasing in q_i with $c_i(0, \theta) = 0$. Let $c_i'(q_i, \theta)$ denote the marginal cost function of firm i when $c_i(q_i, \theta)$ is differentiable in q_i.

The profit of firm i in any period with state θ is given by $\pi_i(q, \theta) = p(Q)q_i - c_i(q_i, \theta)$, where $q = (q_1, \dots, q_n)$ denotes an output profile. Also, q_{-i} is obtained by omitting the output of firm i from q. Throughout we maintain the following assumption:

ASSUMPTION A1. *For each $i \in I$, there exists $\theta \in \Theta$ and q such that $\pi_i(q, \theta) > 0$.*

This assumption ensures that every firm wants to produce a positive quantity at least in some state.

Firms discount their future profits using a common discount factor δ. Let v_i denote a continuation value of firm i that it expects from the next period on, and let $v = (v_1, \dots, v_n)$. If in some period with state θ, the firms produce an output profile q and expect v in continuation, then the present discounted profit of firm i is given by $(1 - \delta)\pi_i(q, \theta) + \delta v_i$. Firms want

[3] We briefly study a finite horizon setup in Section E in the Online Appendix.

to maximize their present discounted profits. Also, for v_i to be feasible, it must be that $v_i \leq \overline{v}_i$ where $\overline{v}_i := \max_{\phi:\Theta\to\mathbb{R}^n_+} \sum_{\theta\in\Theta} l(\theta)\pi_i(\phi(\theta),\theta)$ denotes the highest continuation value that firm i can attain. Clearly, to attain $\overline{v}_i$, firm i must be made a monopolist. Assumption A1 guarantees that $\overline{v}_i > 0$ for all i. Let $V_i = \{v_i | v_i \leq \overline{v}_i\}$ for all i and $V = \times_{i\in I} V_i$.

The objective of CA is captured by an SCF, which specifies the desired output profile for every state of the world, $f : \Theta \to \mathbb{R}^n_+$. For example, in Section 5, we will focus on SCF that selects a competitive output in every state of the world. It is assumed that f does not change over time and that the CA never observes the state of the world of any period. Throughout we also maintain the following assumption:

ASSUMPTION A2. *f is such that $p\left(\sum_{i\in I} f_i(\theta)\right) > 0$ for all θ.*

This assumption together with the assumption that $p(Q)$ is strictly decreasing in Q as long as $p(Q) > 0$ ensures that there is a one-to-one relationship between the price and the aggregate output in the range of f.

3. REPEATED IMPLEMENTATION

In order to implement an SCF, the CA uses a regime, which tells what mechanism to select in each period as a function of past play. We now define mechanisms and regimes formally. A mechanism is a pair (M, g) where M is a message space, which is the same for all firms, and $g : M^n \to \mathbb{R}^n_+$ is an outcome function that specifies an output profile for each profile of messages. Since later we will fix the message space M, we denote the mechanism (M, g) with its output function g. Let m_i be a generic message of firm i and let $m = (m_1, \dots, m_n)$ be a generic message profile. m_{-i} is obtained by omitting the message of firm i from m. It is assumed that in each period, the firms send their messages simultaneously.

From now on, a superscript to any variable will indicate the period. Thus, θ^t and m^t indicate period t state of the world and period t profile of messages, respectively. In period $t > 0$, a history of states is $\zeta^t = (\theta^0, \dots, \theta^t)$, a history of messages is $\mu^t = (m^0, \dots, m^{t-1})$, and a history is $h^t = (\mu^t, \zeta^{t-1})$. Let $\zeta^0 = \theta^0$ and $\zeta^{-1} = \mu^0 = h^0 = \emptyset$. For any t, let H^t denote the set of all possible period t histories.

The mechanism that the CA employs can differ from one period to another. A regime, r, describes which mechanism is selected after every possible message history: $g^t = r(\mu^t)$. Occasionally, it will be more convenient to condition r on h^t, with the understanding that $r(\mu^t, \zeta^{t-1}) = r(\mu^t, \hat{\zeta}^{t-1})$ for any $\zeta^{t-1}, \hat{\zeta}^{t-1}$. We assume that the CA commits to a regime at the start of period 0 and that the firms know which regime the CA employs.

The firms learn the period t state of the world before sending period t messages. At the end of period t, they also learn the period t messages of other firms. A (pure) strategy s_i of firm i specifies a message $m^t_i = s_i(h^t, \theta^t)$ for every t, h^t, and θ^t. We will condition strategies interchangeably on (h^t, θ^t) and (μ^t, ζ^t). Let $s = (s_1, \dots, s_n)$ denote a strategy profile. Given t, h^t, and s, let $\rho(h^t | h^t, s) = 1$ and for any $\tau > t$, let

$$\rho(h^\tau | h^t, s) = \begin{cases} \rho(h^{\tau-1} | h^t, s) l(\theta) & \text{if } \mu^\tau = \left(\mu^{\tau-1}, s\left(h^{\tau-1}, \theta\right)\right) \text{and } \zeta^{\tau-1} = \left(\zeta^{\tau-2}, \theta\right), \\ 0 & \text{otherwise.} \end{cases}$$

For any t and h^t, the continuation value of firm i before it knows period t state and given that the firms follow strategies s is

$$v_i(s|h^t) = (1-\delta) \sum_{\tau=t}^{\infty} \sum_{h^\tau \in H^\tau} \delta^{\tau-t} \rho(h^\tau | h^t, s) l(\theta^\tau) \pi_i(g^\tau(s(h^\tau, \theta^\tau)), \theta^\tau),$$

where $g^\tau = r(h^\tau)$.[4]

We use pure-strategy SPE as a solution concept, although as we argue in Subsection 8.1.1, all results remain true if instead we use pure-strategy NE. A strategy profile s is an SPE of r if for all i, t, h^t, and s_i', it is true that $v_i(s|h^t) \geq v_i((s_i', s_{-i})|h^t)$. Note that this formulation also implies that firm i will not want to deviate from s_i once it learns θ^t.

DEFINITION 1. A regime r *repeatedly implements* f in SPE if the set of SPE is nonempty and for every SPE s, we have that $r(h^t)(s(h^t, \theta^t)) = f(\theta^t)$ for all t, θ^t, and h^t such that $\rho(h^t|h^0, s) > 0$.

f is *repeatedly implementable* in SPE if there exists a regime that repeatedly implements it in SPE.

3.1. *Price–Quantity Mechanisms.* Message spaces of the mechanisms that are employed in the implementation literature are often large and unnatural. For example, the canonical mechanism for one-shot implementation in NE would require each firm to report a state (effectively, a profile of cost functions), an output profile, and an integer.[5] In order to address this unattractive feature of the mechanisms, similar to Dutta et al. (1994) and Saijo et al. (1996), we require that the CA only uses price–quantity mechanisms, where the messages of the firms are announcements about their prices and quantities. Thus, the message space of every firm is $M = \mathbb{R}_{++} \times \mathbb{R}_+$. A typical message $m_i = (p_i, q_i)$ will have the following interpretation: p_i is firm i's announcement of the market price and q_i is its output. By announcing a price and a quantity, each firm is also telling how much the other firms should jointly produce. Note that we do not allow the firms to announce zero price, which we can do because of Assumption A2.

Without further restrictions on the mechanisms, however, what firms announce can differ from what they actually produce. This, in turn, means that their announcements can potentially encode additional information, for example, about the output of each of their rivals. In order to prevent such information smuggling, we require the so-called forthrightness from the regime.[6] It says that there exists an SPE such that on the equilibrium path, in every period, the firms announce and produce exactly those quantities that the CA wants the firms to produce in that period according to f, and they all announce the price that equates demand with the announced quantities.

Formally, for every $\theta \in \Theta$, let $m(\theta)$ denote the message profile such that $m_i(\theta) = (p, q_i)$ for all $i \in I$ where $(q_1, \ldots, q_n) = f(\theta)$ and $p = p\left(\sum_{i \in I} q_i\right)$. We refer to $m(\theta)$ as the messages that are consistent with state θ.

DEFINITION 2. A regime r satisfies *forthrightness* w.r.t. f if there exists an SPE s s.t. $s(h^t, \theta^t) = m(\theta^t)$ and $r(h^t)(m(\theta^t)) = f(\theta^t)$ for all t, θ^t, and h^t s.t. $\rho(h^t|h^0, s) > 0$.

Two observations are in order about the above definition. First, it requires that there exists an equilibrium such that in every period, the firms announce messages that are consistent with the true state of that period. However, the definition does not rule out the existence of other equilibria in consistent but untruthful messages or even in messages that are not consistent with any state. In the next section, we introduce conditions on the SCF that will guarantee that these other equilibria do not exist. Second, the definition implies that in any period, the firms produce what they announce if their current and all past messages are consistent with some (not necessarily true) states of the world. Only if their messages are ever inconsistent with any state, from then on their outputs can differ from what they announce. In order to

[4] vs also depend on r, but we do not make this dependence explicit.

[5] For the description of the canonical mechanism, see the proof of Theorem 3 in Maskin (1999). Mezzetti and Renou (2017) and Āzacis and Vida (2019) also use similar mechanisms for repeated implementation.

[6] For one-shot implementation in exchange economies, Saijo et al. (1996) also impose forthrightness on price–quantity mechanisms, whereas Dutta et al. (1994) refer to this property as truthfulness.

motivate this feature of forthrightness, we can think that the CA will intervene in the market once it observes firms' behavior that is clearly incompatible with its objective.

4. NECESSARY AND SUFFICIENT CONDITIONS

Suppose that in every period, firms announce messages that are consistent with some state. It can be, however, that in some period with state θ, firms announce $m(\theta')$ for some θ' such that $m(\theta') \neq m(\theta)$. In order to capture such a situation, we introduce a concept of deception. It is convenient to distinguish between static and dynamic deceptions. A static deception tells which message profile from $\{m(\theta)|\theta \in \Theta\}$ to announce for every $\theta' \in \Theta$, whereas a dynamic deception specifies which static deception to use in any given period as a function of history of states.[7] Repeated implementation of SCF requires that the firms cannot sustain any dynamic deception in equilibrium. We next provide conditions on SCFs that guarantee that.

Mezzetti and Renou (2017, Theorem 1) have proven that for an arbitrary message space M, an SCF must satisfy the so-called dynamic monotonicity in order to be repeatedly implementable. Dynamic monotonicity says that for every dynamic deception that results in an undesirable output profile after some history, there exists a firm that has incentives to deviate from this deception after some (possibly different) history. In Appendix A, we define a version of dynamic monotonicity when $M = \mathbb{R}_{++} \times \mathbb{R}_{+}$, that we call pq-dynamic monotonicity, and show in Proposition A0 that it is necessary and sufficient for repeated implementation when the regime satisfies forthrightness. Checking whether an SCF satisfies pq-dynamic monotonicity, though, can be a daunting task because there is an infinity of possible dynamic deceptions.[8]

Similar to most of the literature that studies collusion in the repeated setup, we restrict attention to a symmetric environment in the main body. It turns out that in this case, pq-dynamic monotonicity is equivalent to two simpler monotonicity conditions, which are easier to check. It is because in the symmetric setup, it is enough to consider only two types of dynamic deceptions. The first type of dynamic deceptions are the ones where the firms only deceive for one period but otherwise they behave truthfully. The second type are stationary deceptions according to which the same static deception is used in every period. For each of the two types of deceptions, we define a corresponding notion of monotonicity (which are special cases of pq-dynamic monotonicity) and show that both notions of monotonicity are not only necessary but together they are also sufficient for an SCF to be repeatedly implementable in the symmetric setup.

We first introduce additional notation. Let $\alpha : \Theta \to \Theta$ denote a static deception where $\alpha(\theta) = \theta'$ says that the firms act as if the state was θ', although the true state is θ. Let $v_i(f \circ \alpha) := \sum_{\theta\in\Theta} l(\theta)\pi_i(f(\alpha(\theta)), \theta)$ denote the expected profit of firm i when the firms use the deception α. Let $v_i(f) := v_i(f \circ \alpha)$ when α is the identity map. As usual, $v(f)$ and $v(f \circ \alpha)$ denote vectors of continuation values.

Let $L_i(q, v, \theta) := \{(q', v') \in \mathbb{R}^n_+ \times V | (1-\delta)\pi_i(q, \theta) + \delta v_i \geq (1-\delta)\pi_i(q', \theta) + \delta v'_i\}$ denote the lower contour set of firm i in state θ at quantity–value pair (q, v). $L_i(q, v, \theta)$ consists of all those quantity-value pairs that give to firm i weakly lower (present discounted) profit than (q, v) in state θ. Given any output profile q, let $f^{-1}(q) := \{\theta \in \Theta | q = f(\theta)\}$ be the set of states that are consistent with q given f, and let $\Lambda^f_i(q) := \cap_{\theta\in f^{-1}(q)} L_i(q, v(f), \theta)$.

The first condition on f that we introduce, is pq-monotonicity. The intuition behind it is as follows. Suppose f is repeatedly implementable in SPE by a regime that satisfies forthrightness. Consider the strategy profile s given in Definition 2. Take any period, say, period 0 and any state of that period, say, θ'. According to s, the firms announce $m(\theta')$. Because s is an SPE, any deviation from $m_i(\theta')$ by firm i must result in an output–value profile $(q, v) \in$

[7] A formal definition of dynamic deception is given in Appendix A.

[8] Though, in Āzacis and Vida (2019, Theorem 1), we provide an alternative characterization of dynamic monotonicity, which allows to test dynamic monotonicity of SCF using numerical methods.

$L_i(f(\theta'), v(f), \theta')$. However, given that other firms announce $m_{-i}(\theta')$, the CA can only infer that one of the states in $f^{-1}(f(\theta'))$ has occurred. In order to ensure that firm i does not have incentives to deviate in any of these states, (q, v) must belong to $\Lambda_i^f(f(\theta')) := \bigcap_{\theta'' \in f^{-1}(f(\theta'))} L_i(f(\theta''), v(f), \theta'')$.

Now, consider a static deception α such that $\Lambda_i^f(f(\alpha(\theta))) \subseteq L_i(f(\alpha(\theta)), v(f), \theta)$ holds for all i and θ. Then, there exists an SPE, in which on the equilibrium path, the firms announce $m(\alpha(\theta))$ in period 0, but announce $m(\theta)$ in any other period whenever the state of that period is θ. Intuitively, the assumed set inclusions ensure that no firm i will deviate from $m(\alpha(\theta))$ in period 0 when the state is θ because it prefers the equilibrium output–value pair $(f(\alpha(\theta)), v(f))$ to anything it can get in $\Lambda_i^f(f(\alpha(\theta)))$. Neither will it deviate in subsequent periods when the firms are truthful because it prefers $(f(\theta), v(f))$ to anything in $\Lambda_i^f(f(\theta))$. Therefore, if f is repeatedly implementable, it must be that $f(\alpha(\theta)) = f(\theta)$ for all θ. Hence, we have the following condition.

DEFINITION 3. f satisfies *pq-monotonicity* if for any $\alpha : \Theta \to \Theta$, (a) implies (b):

a. $\Lambda_i^f(f(\alpha(\theta))) \subseteq L_i(f(\alpha(\theta)), v(f), \theta)$ for all $i \in I$ and $\theta \in \Theta$,
b. $f(\alpha(\theta)) = f(\theta)$ for all $\theta \in \Theta$.

pq-monotonicity resembles Condition W^* in Saijo et al. (1996), which is necessary for one-shot implementation with quantity mechanisms in exchange economies. Condition W^* is a strengthening of Maskin monotonicity due to forthrightness, and it is similar here. If the forthrightness was not required from the regime, then the firms could be made to send different equilibrium messages in different states (effectively, to announce the states). Then, if firm i deviates, the CA can infer a unique state, say, $\alpha(\theta)$ from the messages of other firms. Therefore, $\Lambda_i^f(f(\alpha(\theta)))$ can be replaced with $L_i(f(\alpha(\theta)), v(f), \alpha(\theta))$ in the above definition. Because $\Lambda_i^f(f(\theta)) \subseteq L_i(f(\theta), v(f), \theta)$ holds, f can satisfy *pq*-monotonicity if forthrightness is not imposed, but fail it when forthrightness is imposed. The same is also true about the next definition of monotonicity.

The second condition on f is *pq*-stationary monotonicity. If the firms adopt the same static deception α in every period, they expect $v(f \circ \alpha)$ instead of $v(f)$ in the continuation. Now, if $\Lambda_i^f(f(\alpha(\theta))) \subseteq L_i(f(\alpha(\theta)), v(f \circ \alpha), \theta)$ holds for all i and θ, then there exists an SPE, in which on the equilibrium path, the firms announce $m(\alpha(\theta))$ in every period when the state of that period is θ. The assumed set inclusions again ensure that no firm i will deviate from $m(\alpha(\theta))$ in any period with state θ because it prefers $(f(\alpha(\theta)), v(f \circ \alpha))$ to anything it can get in $\Lambda_i^f(f(\alpha(\theta)))$. Therefore, if f is repeatedly implementable, it must again be that $f(\alpha(\theta)) = f(\theta)$ for all θ. Hence, we obtain the following condition.

DEFINITION 4. f satisfies *pq-stationary monotonicity* if for any $\alpha : \Theta \to \Theta$, (a) implies (b):

a. $\Lambda_i^f(f(\alpha(\theta))) \subseteq L_i(f(\alpha(\theta)), v(f \circ \alpha), \theta)$ for all $i \in I$ and $\theta \in \Theta$,
b. $f(\alpha(\theta)) = f(\theta)$ for all $\theta \in \Theta$.

Both notions of monotonicity coincide when $\delta = 0$, but otherwise they are different. In the next section, we will show that the competitive SCF can satisfy *pq*-monotonicity, but fail *pq*-stationary monotonicity. One can also construct an opposite example when an SCF satisfies *pq*-stationary monotonicity, but fails *pq*-monotonicity.

Both monotonicity conditions are not only necessary but also sufficient for repeated implementation when all firms are identical and the CA wants them to produce identical outputs. Hence, we now introduce the following assumption.

ASSUMPTION A3. *For every θ, $c_i(\cdot, \theta) = c_j(\cdot, \theta)$ and $f_i(\theta) = f_j(\theta)$ for all $i, j \in I$.*

Even though the firms are identical, their interests are not fully aligned because given Assumption A1, each firm prefers to be the only firm in the market.

PROPOSITION 1. *Suppose Assumption A3 holds. f is repeatedly implementable in SPE with a regime that satisfies forthrightness w.r.t. f if and only if f satisfies pq-monotonicity and pq-stationary monotonicity.*[9]

In order to prove the sufficiency part of Proposition 1, we construct a regime and show that it satisfies forthrightness w.r.t. f and that it implements f in SPE if Assumption A3 holds and if f satisfies both pq-monotonicity and pq-stationary monotonicity. Here, we informally describe the regime. We first introduce additional terminology. Recall that m is consistent with some state or consistent, for short, if there exists θ such that $m = m(\theta)$. Suppose m is not consistent. Then for each firm i, we ask if there exists m_i' such that (m_i', m_{-i}) is consistent. If such m_i' exists, we say that firm i is a whistle-blower. Note that given an inconsistent m, there can be simultaneously several whistle-blowers or none at all.[10]

The regime employs two mechanisms, $\hat{g}$ and $\check{g}$. $\hat{g}$ is similar to the mechanisms that appear in Saijo et al. (1996). This mechanism is selected in period 0 and in any future period as long as the messages in all previous periods have been consistent. Thus, suppose $\hat{g}$ is selected in period t. If period t messages m^t are consistent, then the firms produce what they announce. If m^t is not consistent, then the period t outputs and the continuation values depend on the number of whistle-blowers. If firm i is a unique whistle-blower, then it chooses an output–value pair in $\Lambda_i^f(f(\theta))$ where θ is a state such that $m_{-i}^t = m_{-i}(\theta)$.[11] If there are multiple whistle-blowers, then they all are severely punished. If there is no whistle-blower (but m^t is not consistent), then one of the firms becomes a monopolist forever. Also, after inconsistent m^t, $\check{g}$ is selected forever. $\check{g}$ is a dictatorial mechanism and, similar to Lee and Sabourian (2011), its purpose is to ensure that any of the whistle-blowers or the monopolist receives the right continuation value.

The regime has the property that on the equilibrium path, the firms always announce consistent messages and, hence, produce the quantities that they announce. There is no equilibrium in which the firms ever announce inconsistent messages because each firm has incentives to become a monopolist or, in case of multiple whistle-blowers, each whistle-blower has incentives to avoid the punishment. pq-monotonicity and pq-stationary monotonicity, in turn, ensure that the firms never deceive when announcing consistent messages in equilibrium. Specifically, we invoke pq-stationary monotonicity to show that the continuation values of the firms must be strictly less than $v(f)$ if they ever deceive after period 0. Because any firm i can guarantee itself a continuation value of $v_i(f)$, no deception after period 0 is sustainable, whereas pq-monotonicity rules out any deceptions in period 0. Hence, it follows that the regime satisfies forthrightness and f is repeatedly implemented. The regime has multiple SPE but they only differ in what happens off the equilibrium path.

We apply the result of Proposition 1 to study repeated implementation of the competitive SCF in the next section.

5. REPEATED IMPLEMENTATION OF THE COMPETITIVE SCF

In order to define the competitive SCF and to study its implementability, we make the following simplifying assumption:

[9] We relegate all lengthy proofs to the Appendix, which also contains Proposition A0.

[10] For example, when $n = 2$, the former happens when $m = (m_1(\theta), m_2(\theta'))$ and $m(\theta) \neq m(\theta')$, whereas the latter happens when m is such that $m_i \neq m_i(\theta)$ for both i and any θ.

[11] From definitions of $m(\theta)$ and $\Lambda_i^f(q)$, it follows that $\Lambda_i^f(f(\theta))$ is the same for all θ such that $m_{-i}^t = m_{-i}(\theta)$.

ASSUMPTION A4. *$c_i(q_i, \theta)$ is continuously differentiable and convex in q_i for all i and θ. $p(Q)$ is differentiable for all Q s.t. $p(Q) > 0$. For each θ, the solution q^* to the system of equations:*

$$p(Q) \leq c_i'(q_i, \theta), q_i \geq 0, (p(Q) - c_i'(q_i, \theta))q_i = 0 \text{ for } i = 1, \dots, n, \tag{1}$$

is unique with $q_i^* > 0$ for all i.

Given Assumption A4, the competitive SCF, which we denote by f^c, is such that for every θ, $f^c(\theta)$ is a solution to (1).

PROPOSITION 2. *If Assumption A4 holds, then f^c satisfies pq-monotonicity.*

In the next two examples, we show why f^c might fail to be pq-stationary monotonic. Both examples satisfy Assumptions A3 and A4. The message of the first example is that it is harder to satisfy pq-stationary monotonicity when δ is large because firms put more weight on the continuation value in their discounted profits.

EXAMPLE 1. Let $n = 2$, $\Theta = \{\theta_1, \theta_2\}$, $l(\theta_1) = 0.5$, $p = \max\{0, 2700 - 60Q\}$, $c_i(q_i, \theta_1) = 90q_i^2$, and $c_i(q_i, \theta_2) = 60q_i^2 + q_i^3$ for $i = 1, 2$. The competitive SCF is $f_i^c(\theta_1) = 9$ and $f_i^c(\theta_2) = 10$ for $i = 1, 2$. The firms are better off if they adopt the following deception in every period: $\alpha(\theta_1) = \alpha(\theta_2) = \theta_1$ because $\pi_i(f^c(\theta_1), \theta_2) = 8,991 > \pi_i(f^c(\theta_2), \theta_2) = 8,000$.

We show that f^c does not satisfy pq-stationary monotonicity for large enough δ. In order to incentivize firm i to deviate when the firms follow deception α, but not to deviate when they behave truthfully, we need to find (q, v_i) s.t.

$$(1 - \delta)\pi_i(f^c(\theta_1), \theta_1) + \delta v_i(f^c) \geq (1 - \delta)\pi_i(q, \theta_1) + \delta v_i, \tag{2}$$
$$(1 - \delta)\pi_i(f^c(\theta_1), \theta_2) + \delta v_i(f^c \circ \alpha) < (1 - \delta)\pi_i(q, \theta_2) + \delta v_i. \tag{3}$$

After combining both inequalities and simplifying, one arrives at the following necessary condition:

$$\begin{aligned}(1 - \delta)(c_i(q_i, \theta_1) - c_i(q_i, \theta_2)) > (1 - \delta)&(c_i(f_i^c(\theta_1), \theta_1) - c_i(f_i^c(\theta_1), \theta_2)) \\ &+ \delta l(\theta_2)(\pi_i(f^c(\theta_1), \theta_2) - \pi_i(f^c(\theta_2), \theta_2)).\end{aligned} \tag{4}$$

$c_i(q_i, \theta_1) - c_i(q_i, \theta_2)$ is maximized for $q_i = 20$. Thus, setting $q_i = 20$ and evaluating costs and profits at the specified values, we find that the above inequality will not be satisfied for $\delta \geq \frac{4598}{5589} \approx 0.8227$.

Let us write the r.h.s. of (2) and (3) as $w_i - (1 - \delta)c_i(q_i, \theta)$ for $\theta \in \Theta$ where $w_i := (1 - \delta)p(Q)q_i + \delta v_i$. Also, let us denote the l.h.s. of (2) and (3) as $v_i(\theta_1, \theta_1)$ and $v_i(\theta_2, \theta_1)$, respectively. Then, we can conveniently illustrate (2) and (3) when they hold with equalities, in the (q_i, w_i) space, which is done in Figure 1 for two values of δ. When $\delta = 0$, the iso-profit lines intersect and we can find (q_i, w_i) that gives the right incentives to firm i. (We can pick any point above the dotted red line, but below the solid blue line.) But when $\delta = 0.9$, the lines do not cross and we cannot satisfy (3) without violating (2).

REMARK 1. Collusion models usually assume that in the absence of collusion, firms play NE strategies of a static oligopoly model. In general, the NE outcome will differ from the competitive outcome. Therefore, one might ask if the result would be qualitatively different if instead the CA's objective was, for example, for the firms to produce the NE output of the static Cournot model. In the above example, it is given by $f_i^n(\theta_1) = 7.5$ and $f_i^n(\theta_2) \approx 8.3095$ for $i = 1, 2$. However, a similar result still applies: f^n is not repeatedly implementable for high enough values of δ. That is, if we use f^n instead of f^c to evaluate (4), we find that pq-stationary monotonicity of f^n is not satisfied for $\delta \geq 0.9530$.

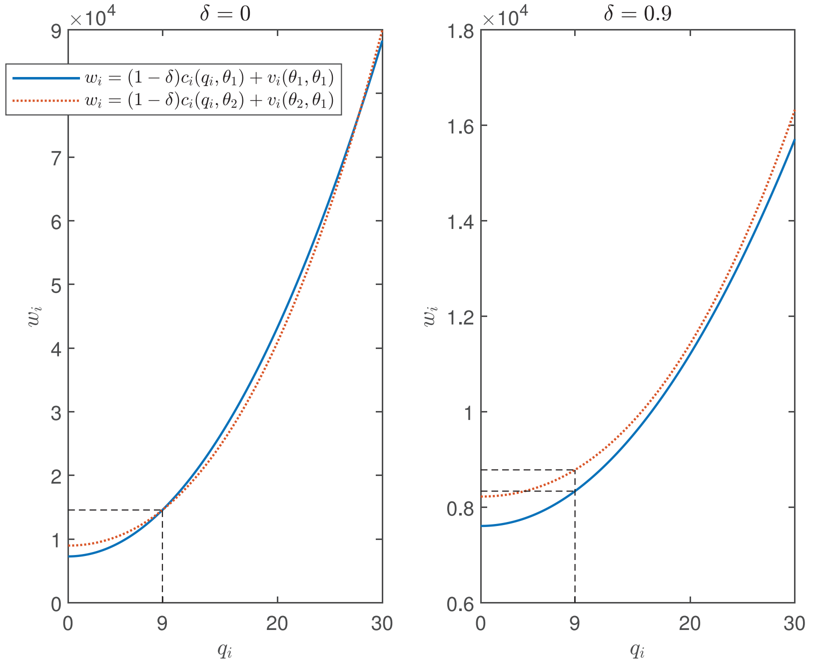

FIGURE 1

ILLUSTRATION OF EXAMPLE 1

The next example shows that pq-stationary monotonicity of f^c might fail because of the requirement that a regime satisfies forthrightness.

EXAMPLE 2. Let $n = 5$, $\Theta = \{\theta_1, \theta_2, \theta_3\}$, $l(\theta_1) = l(\theta_2) = \frac{1}{3}$, $p = \max\{0, 22 - Q\}$, and for all $i \in I$, the cost functions are as follows:

$$c_i(q_i, \theta_1) = \begin{cases} 2q_i & \text{if} \quad q_i \leq 6, \\ \frac{2q_i^3 + 4\cdot 6^3}{3\cdot 6^2} & \text{if} \quad q_i > 6, \end{cases}$$
$$c_i(q_i, \theta_2) = \begin{cases} 4q_i & \text{if} \quad q_i \leq 5, \\ \frac{2q_i^2 + 2\cdot 5^2}{5} & \text{if} \quad q_i > 5, \end{cases}$$
$$c_i(q_i, \theta_3) = \begin{cases} 4q_i & \text{if} \quad q_i \leq 11, \\ \frac{q_i^4 + 3\cdot 11^4}{11^3} & \text{if} \quad q_i > 11. \end{cases}$$

All these cost functions are continuously differentiable (to be consistent with Assumption A4). The competitive SCF is $f_i^c(\theta_1) = 4$ and $f_i^c(\theta_2) = f_i^c(\theta_3) = 3.6$ for all $i \in I$. The firms are better off if they adopt the following deception: $\alpha(\theta_1) = \alpha(\theta_2) = \alpha(\theta_3) = \theta_2$ because $\pi_i(f^c(\theta_2), \theta_1) = 7.2 > \pi_i(f^c(\theta_1), \theta_1) = 0$. Also, $v_i(f^c) = 0$ and $v_i(f^c \circ \alpha) = 2.4$.

In order to eliminate deception α, we need to find (q, v) s.t. for some i, $(q, v) \in \Lambda_i^{f^c}(f^c(\alpha(\theta_1)))$ but $(q, v) \notin L_i(f^c(\alpha(\theta_1)), v(f^c \circ \alpha), \theta_1)$ or

$$0 \geq (1-\delta)\pi_i(q, \theta_2) + \delta v_i,$$
$$0 \geq (1-\delta)\pi_i(q, \theta_3) + \delta v_i,$$
$$7.2(1-\delta) + 2.4\delta < (1-\delta)\pi_i(q, \theta_1) + \delta v_i.$$

Figure 2 plots the above expressions for $\delta = 0.85$ when they hold with equalities, in the (q_i, w_i) space where $w_i = (1-\delta)p(Q)q_i + \delta v_i$. The figure shows that it is impossible to find such a

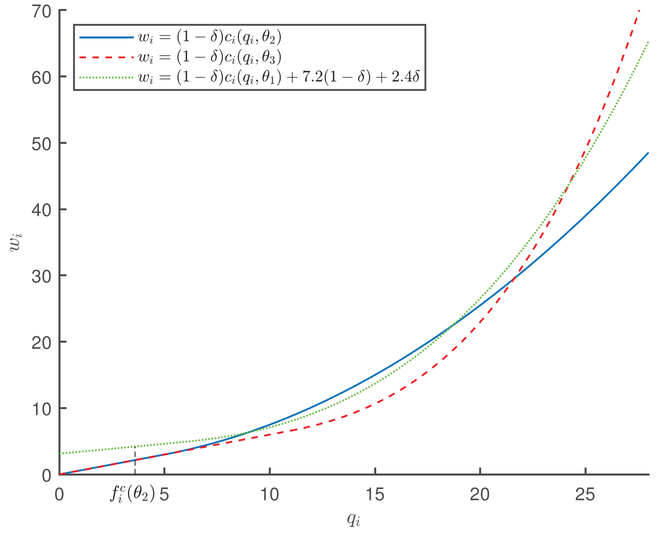

FIGURE 2

FAILURE OF pq-STATIONARY MONOTONICITY IN EXAMPLE 2 WHEN $\delta = 0.85$

pair (q, v) that would satisfy all three inequalities: (q, v) must lie below both the solid blue line and the dashed red line, but above the dotted green line. We conclude that f^c is not pq-stationary monotonic.

Note, however, if we did not impose forthrightness and allowed the firms to send different messages (on the equilibrium path) in states θ_2 and θ_3, then we could eliminate both deception $\alpha(\theta_1) = \theta_2$ and $\alpha(\theta) = \theta$ for $\theta = \theta_2, \theta_3$ (because the solid blue line intersects the dotted green line) and deception $\alpha'(\theta_1) = \theta_3$ and $\alpha'(\theta) = \theta$ for $\theta = \theta_2, \theta_3$ (because the dashed red line intersects the dotted green line).[12,13] Thus, here we have an example of f^c that is repeatedly implementable when forthrightness is not imposed, but not implementable when forthrightness is imposed.

We now introduce an assumption that rules out situations like in Examples 1 and 2 when the isoprofit lines never intersect.

ASSUMPTION A5. *$\Theta \subset \mathbb{R}_{++}$, $c_i(q_i, \theta) = \theta c(q_i)$ for all $\theta \in \Theta$ and $i \in I$, $c(0) = 0$, $p(0) > \max_{\theta \in \Theta} \theta c'(0)$, $c'(q_i) > 0$, and $c''(q_i) \geq 0$ for all $q_i \geq 0$. $p(Q)$ is twice differentiable and $p''Q + 2p' \leq 0$ for all Q s.t. $p(Q) > 0$.*

Assumption A5 implies that the cost function, which is the same for all firms, exhibits strictly increasing differences. Increasing differences guarantee that the isoprofit lines corresponding to any two states always intersect, which is needed in order for the firms to have the right incentives to deviate from deceptions. However, because with higher δ, the firms put less weight on their current costs, the output level of firm i at which its isoprofit lines intersect, is increasing in δ and becomes unboundedly large as δ approach 1. Therefore, the assumption that the firms can produce arbitrary large quantities will matter for the next proposition.

[12] That is, f^c satisfies dynamic monotonicity as defined in Mezzetti and Renou (2017).

[13] v_i must also be feasible, that is, $v_i \leq \overline{v}_i$ must hold where $\overline{v}_i = 83.8775$. According to Figure 2, it is enough to consider $w_i < 70$ or $v_i < 70/\delta \approx 82.3529$. Thus, the feasibility of v_i is satisfied. This is the reason why we set $n = 5$.

Note also that Assumption A5 is a special case of Assumption A4, except that it only guarantees that there is a unique *symmetric* solution to (1).[14] Therefore, whenever Assumption A5 is invoked, we require that f^c is such that for every θ, $f^c(\theta)$ is a symmetric solution to (1), that is, $p(nf_i^c(\theta)) = \theta c'(f_i^c(\theta))$ for all i and θ. Also, let f^m denote an SCF that assigns the symmetric cartel output to the firms in every state. That is, given Assumption A5, $f_i^m(\theta)$ is a solution to $p(nf_i^m(\theta)) + p'(nf_i^m(\theta))nf_i^m(\theta) = \theta c'(f_i^m(\theta))$ for all i and θ. ($c'' \geq 0$ and $p''Q + 2p' \leq 0$ ensure that the second order condition is satisfied.)

PROPOSITION 3. *Suppose Assumption A5 holds.*

1. *There exists* $\underline{\delta} > 0$ *s.t.* f^c *satisfies pq-stationary monotonicity for all* $\delta \in [0, \underline{\delta})$.
2. *Suppose* $\overline{v}_1 > v_1(f^c) + \frac{\max_\theta \theta(v_1(f^m) - v_1(f^c))}{\min_{\theta,\theta'|\theta>\theta'} \theta - \theta'}$ *holds. Then, there exists* $\overline{\delta} < 1$ *s.t.* f^c *satisfies pq-stationary monotonicity for all* $\delta \in (\overline{\delta}, 1)$.
3. *There exists* $\overline{n}$ *s.t.* f^c *satisfies pq-stationary monotonicity for all* $\delta \in [0, 1)$ *if* $n > \overline{n}$.

Proposition 2 implies that f^c satisfies pq-stationary monotonicity for $\delta = 0$. Part 1 of Proposition 3 says that f^c continues to satisfy pq-stationary monotonicity for δs sufficiently close to 0. Part 2 of Proposition 3 requires more explanation. In order to produce closer to the cartel output, the firms will exaggerate their costs. In order to incentivize, say, firm 1 to deviate from this deception, the CA asks it to produce more in the current period, which hurts its profit, but the firm is rewarded with a higher continuation value v_1 in the following periods. If the true cost is lower than what the other firms claim it to be, then indeed firm 1 will want to deviate. However, the higher δ is, the more firm 1 must produce in the current period and the higher v_1 must be in the continuation, which might violate the feasibility constraint, $v_1 \leq \overline{v}_1$. Part 2 of Proposition 3 provides a condition that guarantees that the feasibility constraint is satisfied for sufficiently large δs. The condition basically requires that $v_1(f^m) < \frac{\theta - \theta'}{\theta}\overline{v}_1 + \frac{\theta'}{\theta} v_1(f^c)$ for all $\theta, \theta' \in \Theta$ such that $\theta > \theta'$. Finally, part 3 of Proposition 3 holds because $v_1(f^c)$ and $v_1(f^m)$ are decreasing in n, which helps to satisfy the feasibility constraint (for all values of δ) and, hence, also pq-stationary monotonicity of f^c.

REMARK 2. Lee and Sabourian (2011) have shown in their Theorem 1 that an SCF f is not repeatedly implementable for large enough discount factors if f is not efficient in the range, meaning, there exists a deception α such that $v(f \circ \alpha) > v(f)$.[15] In general, f^c is not efficient from the firms' perspective because $v(f^m) > v(f^c)$. The result in part 2 of Proposition 3 that f^c can even be implemented when δ is arbitrary close to 1, however, does not contradict Theorem 1 of Lee and Sabourian (2011) because they assume bounded payoffs: There exists some finite K s.t. $\max_{i \in I, \theta \in \Theta, q \in \mathbb{R}_+^n} |\pi_i(q, \theta)| < K$. Our proof relies on the assumption that the firms can produce arbitrary large quantities, which means that the current period profit is unbounded from below. Note, though, that the large output is not used to punish firms, but rather to provide the incentives to deviate from deceptions. If instead the firms faced capacity constraints, then Theorem 1 of Lee and Sabourian (2011) would apply and the implementation of f^c would fail for large δ's.

It also follows from the proof of Proposition 3 that under Assumption A5, f^c can only fail to be pq-stationary monotonic because of the constraint on the feasible continuation values. However, if we allow for monetary transfers from the CA to firms, this constraint will not apply. Hence, we have the following corollary.

[14] That is, Assumption A5 does not rule out the existence of additional *asymmetric* solutions to (1). For example, there is a continuum of solutions to (1) in Example 3. Proposition 2, however, remains true if we replace Assumption A4 with A5; see Footnote 27.

[15] We study the implementation of SCFs, which are efficient in the range, in Section F of the Online Appendix.

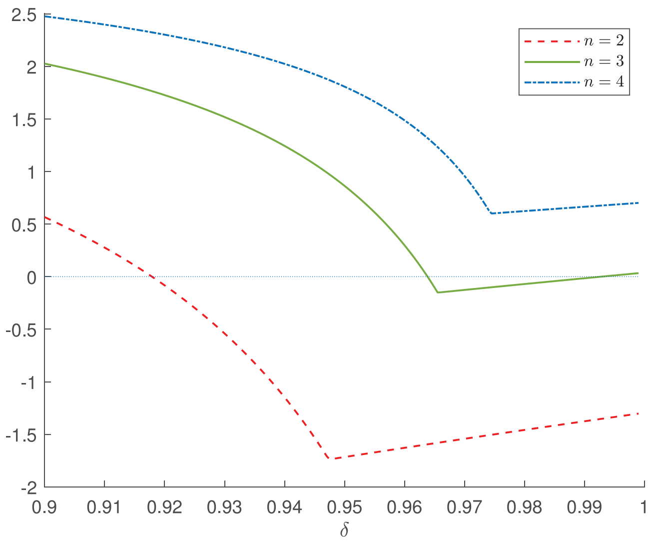

FIGURE 3

FEASIBILITY CONSTRAINT IN EXAMPLE 3

COROLLARY 1. *Suppose Assumption A5 holds. If monetary transfers from the CA to firms are feasible, then* f^c *satisfies pq-stationary monotonicity for all* $\delta \in [0,1)$.

In the final example of the section, we show that there can be $0 < \underline{\delta} < \overline{\delta} < 1$ such that f^c is not *pq*-stationary monotonic when $\delta \in [\underline{\delta}, \overline{\delta}]$, but it is *pq*-stationary monotonic when $\delta \in [0, \underline{\delta}) \cup (\overline{\delta}, 1)$. Thus, in general, it is not true that if an SCF is *pq*-stationary monotonic for some δ, then it is also *pq*-stationary monotonic for any smaller δ. The example also shows that even if δ is close to 1, the number of firms that is needed for f^c to satisfy *pq*-stationary monotonicity does not need to be large.

EXAMPLE 3. Let $\Theta = \{\theta_1, \theta_2\}$ with $a > \theta_1 > \theta_2 > 0$, $p = \max\{0, a - Q\}$, $c_i(q_i, \theta) = \theta q_i$ for all $i \in I$ and $\theta \in \Theta$. Thus, Assumption A5 is satisfied. The competitive SCF is $f_i^c(\theta) = \frac{a-\theta}{n}$ for all $i \in I$ and $\theta \in \Theta$. The firms are better off if they adopt the following deception: $\alpha(\theta_1) = \alpha(\theta_2) = \theta_1$ because $\pi_i(f^c(\theta_1), \theta_2) = (\theta_1 - \theta_2)\frac{a-\theta_1}{n} > \pi_i(f^c(\theta_2), \theta_2) = 0$. Note that $v_i(f^c) = 0$, $v_i(f^c \circ \alpha) = l(\theta_2)(\theta_1 - \theta_2)\frac{a-\theta_1}{n}$, and $\overline{v}_i = l(\theta_1)\left(\frac{a-\theta_1}{2}\right)^2 + l(\theta_2)\left(\frac{a-\theta_2}{2}\right)^2$.

From the proof of Proposition 3, f^c is *pq*-stationary monotonic if $v_1^* < \overline{v}_1$ where v_1^* is given by (C5). In the example, $v_1^* < \overline{v}_1$ takes the following form:

$$l(\theta_1)\left(\frac{a-\theta_1}{2}\right)^2 + l(\theta_2)\left(\frac{a-\theta_2}{2}\right)^2 - l(\theta_2)(a-\theta_1)\frac{(\theta_1 - \max\{0, a - q_1^*\})q_1^*}{nq_1^* - a + \theta} > 0,$$

where $q_1^* = \frac{a-\theta_1}{n}\frac{1-\delta l(\theta_1)}{1-\delta}$ is given by (C4) when it holds with equality.

We plot the l.h.s. of the above inequality against δ in Figure 3 for $a = 10$, $\theta_1 = 8$, $\theta_2 = 5.8$, $l(\theta_1) = 0.5$, and several values of n. Thus, when $n = 2$, f^c is *pq*-stationary monotonic for all $\delta < \underline{\delta} \approx 0.9179$ and not monotonic otherwise. When $n = 3$, f^c is *pq*-stationary monotonic for all $\delta \in [0,1)\backslash[\underline{\delta}, \overline{\delta}]$ and not monotonic otherwise, where $\underline{\delta} \approx 0.9638$ and $\overline{\delta} \approx 0.9929$. When $n \geq 4$, f^c is *pq*-stationary monotonic for all $\delta < 1$.[16]

[16] The kinks in the figure correspond to $q_1^* = a$ and, hence, $p = 0$.

5.1. *Buyer as Another Participant.* In a well-known survey of price fixing conspiracies in the United States, Hay and Kelley (1974, table 1) report that in 14% of cases (7 out of 49 cases), the conspiracy was uncovered after a complaint by a customer. Carree et al. (2010) have surveyed all formal decisions by the European Commission on antitrust cases during 1957–2004 and have found that almost 100 of these cases started after a complaint as opposed to 29 cases that started with a leniency application (see their table 1). Thus, complaints play an important role in detecting anticompetitive behavior. Although a detailed breakdown of complaints is not available, some of these complaints are filed by the customers who are the victims of the anticompetitive practices. Therefore, in Section F of the Online Appendix, we consider an extension to the model where a representative buyer also participates in the regime by sending messages to the CA.

We prove there that if an SCF is efficient, then it only needs to satisfy pq-monotonicity in order to be repeatedly implementable by a forthright regime. For this result, we even do not need to invoke Assumption A3 about the symmetric setup. Because $f^c(\theta)$ maximizes the sum of firms' profits and the representative buyer's utility in every state θ, f^c is clearly an efficient SCF.[17] Furthermore, the result of Proposition 2 that f^c satisfies pq-monotonicity given Assumption A4, continues to hold when we add the buyer as another participant in the regime. (The regime can always ignore the buyer's messages.) Therefore, we have the following result:

COROLLARY 2. *Suppose Assumption A4 holds and the buyer is also one of the participants in the regime. Then, f^c is repeatedly implementable in SPE with a regime that satisfies forthrightness w.r.t. f^c.*

The result effectively says that it is easier to implement the competitive output if the CA does not exclusively rely on whistle-blowing firms, but also solicits messages from buyers.

6. REPEATED IMPLEMENTATION OF THE BREAK-EVEN OUTPUT

We now briefly consider another appealing SCF. Suppose that in every state, the CA wants to maximize the aggregate output subject to firms breaking even. Formally, let the SCF, which we denote as f^b, be defined as follows: For every θ, $f^b(\theta)$ is such that $\pi_i(f^b(\theta), \theta) = 0$ for all i and there does not exist another output profile q such that $\pi_i(q, \theta) = 0$ for all i and $\sum_i q_i > \sum_i f_i^b(\theta)$.

We know from Proposition 2 that f^c is implementable when $\delta = 0$ and so, by continuity, it is also implementable for sufficiently small values of δ. The same is not necessarily true in the case of f^b. The following proposition tells that if the domain of possible cost functions is sufficiently rich, then f^b does not satisfy pq-stationary monotonicity for any value of δ and, hence, it cannot be repeatedly implemented.

PROPOSITION 4. *Suppose there exist θ' and θ'' s.t. for all $i \in I$, $c_i'(q_i, \theta'') - c_i'(q_i, \theta')$ is strictly increasing in q_i and $c_i'(f_i^b(\theta'), \theta'') = c_i'(f_i^b(\theta'), \theta')$. Then f^b does not satisfy pq-stationary monotonicity for any value of δ.*

Note that the assumptions in the proposition are compatible with Assumption A4.

PROOF OF PROPOSITION 4. First of all, given the definition of f^b, the assumptions about $c_i'(q_i, \theta'') - c_i'(q_i, \theta')$, and that $c(0, \theta') = c(0, \theta'') = 0$, we have that $\pi_i(f^b(\theta'), \theta'') > 0$ for all i, from which it follows that $f^b(\theta') \neq f^b(\theta'')$.

Suppose firms use the following deception in every period: $\alpha(\theta'') = \theta'$ and $\alpha(\theta) = \theta$ for all $\theta \neq \theta''$. Note that $v_i(f^b) = 0$ and $v_i(f^b \circ \alpha) > 0$ for all i. In order to eliminate this deception,

[17] But note that f^c is not efficient from the perspective of firms. Therefore, to implement f^c in the benchmark model, we additionally need that it satisfies pq-stationary monotonicity.

we need i, θ, and (q, v_i) s.t.

$$0 \geq (1-\delta)\pi_i(q, \alpha(\theta)) + \delta v_i,$$
$$(1-\delta)\pi_i(f^b(\alpha(\theta)), \theta) + \delta v_i(f^b \circ \alpha) < (1-\delta)\pi_i(q, \theta) + \delta v_i.$$

Clearly, the inequalities cannot be satisfied if $\theta \neq \theta''$. Thus, set $\theta = \theta''$. Furthermore, the above inequalities together imply the following inequality:

$$(1-\delta)(c_i(f_i^b(\theta'), \theta'') - c_i(f_i^b(\theta'), \theta')) > (1-\delta)(c_i(q_i, \theta'') - c_i(q_i, \theta')) + \delta v_i(f^b \circ \alpha).$$

However, $v_i(f^b \circ \alpha) > 0$ and, due to the assumptions, $c_i(q_i, \theta'') - c_i(q_i, \theta')$ is minimized for $q_i = f_i^b(\theta')$ for all i. Therefore, the inequality cannot be satisfied for any i and f^b fails pq-stationary monotonicity. And this is true for all values of δ. □

We saw in Example 3 that the range of δ's for which an SCF satisfies pq-stationary monotonicity, can be disconnected. In the following example, we show that the range of δs for which an SCF fails pq-stationary monotonicity, can also be disconnected. That is, we show that f^b is not implementable for both small and large values of δ, but is implementable for intermediate ones.

EXAMPLE 4. Let $n = 2$, $\Theta = \{\theta_1, \theta_2\}$, $l(\theta_1) = \frac{1}{2}$, $p = \max\{0, 12 - Q\}$, and for $i = 1, 2$, the cost functions are $c_i(q_i, \theta_1) = 4q_i$ and

$$c_i(q_i, \theta_2) = \begin{cases} \frac{q_i^2}{2} & \text{if} \quad q_i \leq 6, \\ 27 - (q_i - 9)^2 & \text{if} \quad 9 \geq q_i > 6, \\ 27 + (q_i - 9)^2 & \text{if} \quad q_i > 9. \end{cases}$$

($c_i(q_i, \theta_2)$ is a strictly increasing and continuously differentiable function.) The SCF is $f_i^b(\theta_1) = 4$ and $f_i^b(\theta_2) = 4.8$ for $i = 1, 2$. The firms are better off if they adopt the following deception in every period: $\alpha(\theta_1) = \alpha(\theta_2) = \theta_1$ because $\pi_i(f^b(\theta_1), \theta_2) = 8$ and $v_i(f^b \circ \alpha) = 4$. Note that $c_i'(f_i^b(\theta_1), \theta_1) = c_i'(f_i^b(\theta_1), \theta_2)$.

In order to incentivize firm i to deviate when the firms follow deception α, but not to deviate when they behave truthfully, we need to find (q, v_i) s.t.

$$0 \geq (1-\delta)\pi_i(q, \theta_1) + \delta v_i,$$
$$8(1-\delta) + 4\delta < (1-\delta)\pi_i(q, \theta_2) + \delta v_i.$$

Figure 4 plots the above expressions when they hold with equalities, in the (q_i, w_i) space for two values of δ where $w_i = (1-\delta)p(Q)q_i + \delta v_i$. When $\delta = 0.7$, the iso-profit lines do not intersect and we cannot find (q, v_i) that would satisfy the above inequalities. When $\delta = 0$, the iso-profit lines intersect, but any (q, v_i) that satisfies the above inequalities, violates the feasibility constraint that the firm cannot earn more than its monopoly profit, $w_i \leq (1-\delta)p(q_i)q_i + \delta \overline{v}_i$ where $\overline{v}_i = 20$. That is, we cannot find (q_i, w_i) that lies below the solid blue and dash-dotted green lines, but above the red dashed line.

One can verify numerically that there exists a (small) range of δs for which the feasibility constraint and the above inequalities can be simultaneously satisfied and, consequently, f^b implemented. Specifically, f^b is repeatedly implementable when $\delta \in \left(\frac{29}{65}, \frac{1}{2}\right)$, whereas for all other values of δ, it is not implementable. Thus, interestingly, the range of δs for which an SCF fails pq-stationary monotonicity, can be disconnected.

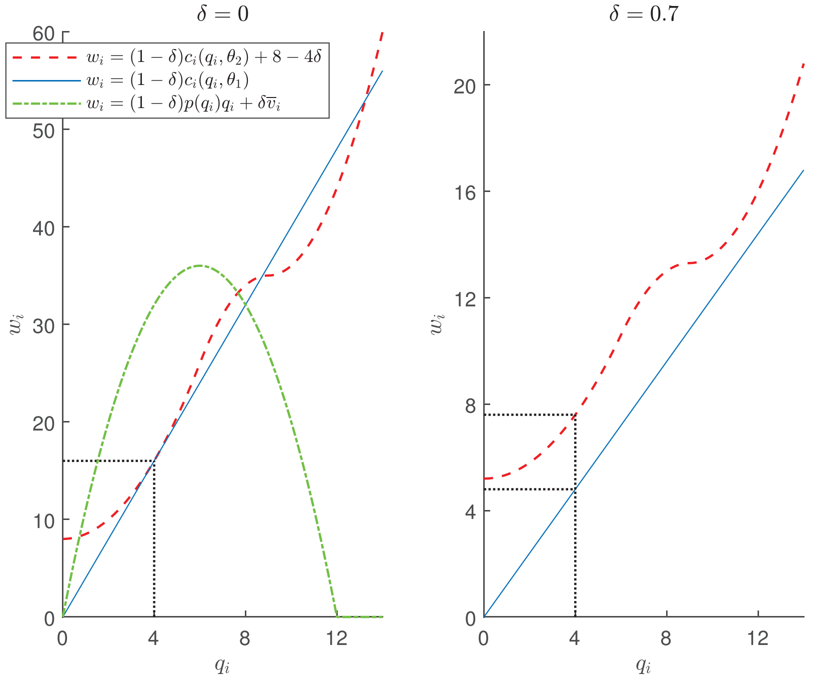

FIGURE 4

ILLUSTRATION OF EXAMPLE 4

7. HARD EVIDENCE

The literature that studies the impact of leniency programs on collusion, explicitly or implicitly assumes the existence of hard evidence, which proves collusion by firms (see, e.g., Motta and Polo, 2003; Spagnolo, 2004; Aubert et al., 2006). Therefore, we now extend the model of Section 3 by assuming that the firms possess hard evidence, which varies with the state of the world. Intuitively, the existence of hard evidence can help with implementation exactly when either pq-monotonicity or pq-stationary monotonicity of f fails. We build on Kartik and Tercieux (2012) who studied one-shot implementation with evidence, and we show that a condition, called evidence monotonicity, is necessary and sufficient for repeated implementation of SCFs (under certain assumptions). It says that for any deception that violates either pq-monotonicity or pq-stationary monotonicity of f, it must be that either the deception cannot be supported with evidence or a firm can supply evidence that would not be available if the firms were not deceiving.

Thus, we now assume that in each period t, firm $i \in I$ possesses a set of evidence $E_i(\theta^t) \neq \emptyset$, which only depends on the state of the world of that period, $\theta^t \in \Theta$. A generic piece of evidence of firm i is denoted as e_i. In state θ, firm i can only present evidence that it has, that is, $e_i \in E_i(\theta)$. We refer to $\{E_i(\theta)\}_{i \in I, \theta \in \Theta}$ as an evidence structure. For simplicity, the evidence structure does not change over time. Let $E(\theta) = E_1(\theta) \times \ldots \times E_n(\theta)$ for every θ and $E = \cup_{\theta \in \Theta} E(\theta)$, with a generic element $e = (e_1, \ldots, e_n)$. Besides the firms' messages, the outcome function of a mechanism now also depends on the evidence supplied by the firms, $g : M^n \times E \to \mathbb{R}^n_+$.[18] We also slightly modify μ^t by including the evidence that is provided by the firms, in the history of messages. That is, let now $\mu^t = (\mu^{t-1}, (m^{t-1}, e^{t-1}))$ for all $t > 0$. No

[18] This formulation assumes that firm i can only submit a single piece of evidence. In order to allow it to submit more than one piece of evidence, we can assume that if $e_i, e_i' \in E_i(\theta)$ for some θ, then there is also $e_i'' \in E_i(\theta)$ such that $e_i'' = \{e_i\} \cup \{e_i'\}$.

other changes in notation are required. In particular, the definitions of regime and strategies remain the same (given the modified history of messages).

We refer to $e : \Theta \to E$ s.t. $e(\theta) \in E(\theta)$ for all θ as an evidence function.

DEFINITION 5. A regime r satisfies *forthrightness* w.r.t. f if there exists an SPE s s.t. $s(h^t, \theta^t) = (m(\theta^t), e(\theta^t))$ and $r(h^t)(m(\theta^t), e(\theta^t)) = f(\theta^t)$ for all t, θ^t, and h^t s.t. $\rho(h^t|h^0, s) > 0$ and e is some evidence function.

Let $\mathcal{A}^f$ be the set of all deceptions, which violate either pq-monotonicity or pq-stationary monotonicity of f, that is, for any $\alpha \in \mathcal{A}^f$, part (a) in either Definition 3 or 4 holds, but part (b) does not.

DEFINITION 6. f satisfies *evidence monotonicity* if there is an evidence function e s.t. for every $\alpha \in \mathcal{A}^f$, there exist $i \in I$ and $\theta \in \Theta$ s.t. either $e_i(\alpha(\theta)) \notin E_i(\theta)$ or $E_i(\theta) \nsubseteq E_i(\alpha(\theta))$.

In order to keep matters simple, we make the following assumption:

ASSUMPTION A6. *f is s.t. $f(\theta) \neq f(\theta')$ for all $\theta, \theta' \in \Theta$.*

Given this assumption, we can replace $\Lambda_i^f(f(\alpha(\theta)))$ with $L_i(f(\alpha(\theta)), v(f), \alpha(\theta))$ in Definitions 3 and 4.

PROPOSITION 5. *Suppose f satisfies Assumption A6. If f is repeatedly implementable in SPE with a regime that satisfies forthrightness w.r.t. f, then f satisfies evidence monotonicity.*[19]

PROPOSITION 6. *Suppose Assumptions A3 and A6 hold. If f satisfies evidence monotonicity, then f is repeatedly implementable in SPE with a regime that satisfies forthrightness w.r.t. f.*

Even though we assume the symmetry of firms and the SCF, we do not require that the evidence structure is also symmetric in Proposition 6.

We finish this section by providing an example of natural evidence structure, which guarantees that f^c is repeatedly implementable.

EXAMPLE 5. Suppose that f^c is such that Assumption A5 holds. (Hence, Assumption A6 also automatically holds.) Assume that the evidence structure is such that for any $\theta, \theta' \in \Theta$, if $\pi_i(f^c(\theta'), \theta) > \pi_i(f^c(\theta'), \theta')$ for all $i \in I$, then $E_j(\theta) \nsubseteq E_j(\theta')$ for some $j \in I$. The motivation for this assumption is that firm j can prove in state θ that it can earn higher profits with the output profile $f^c(\theta')$ than in state θ'. (It seems reasonable to assume that it is easier to underreport profits than to overreport them.) We argue that in this case, f^c satisfies evidence monotonicity. We already know that f^c satisfies pq-monotonicity. Therefore, $\mathcal{A}^{f^c}$ only contains those deceptions that violate pq-stationary monotonicity of f^c.

Take any deception $\alpha \in \mathcal{A}^{f^c}$ such that $v_i(f^c \circ \alpha) > v_i(f^c)$ for all i. (Note that we are in the symmetric setup.) There must exist a state θ such that $\alpha(\theta) > \theta$, that is, the firms exaggerate their costs in order to produce less. Therefore, $\pi_i(f^c(\alpha(\theta)), \theta) > \pi_i(f^c(\alpha(\theta)), \alpha(\theta))$ holds for all $i \in I$. Given the assumed evidence structure, $E_j(\theta) \nsubseteq E_j(\alpha(\theta))$ for some $j \in I$. From the definition of evidence monotonicity, any such α cannot violate evidence monotonicity of f^c.

Take now any deception $\alpha \in \mathcal{A}^{f^c}$ such that $v_i(f^c) \geq v_i(f^c \circ \alpha)$ for all i. In this case, $L_i(f^c(\alpha(\theta)), v(f^c \circ \alpha), \theta) \subseteq L_i(f^c(\alpha(\theta)), v(f^c), \theta)$ holds for all i and θ. Therefore, if pq-stationary monotonicity of f^c fails for this α, then pq-monotonicity of f^c must also fail for the same α, which is a contradiction.

[19] The proofs of this section are similar to those of Propositions A0 and 1 and, therefore, are relegated to the Online Appendix.

We conclude that f^c satisfies evidence monotonicity given the assumed evidence structure and, according to Proposition 6, f^c is repeatedly implementable in SPE with a regime that satisfies forthrightness w.r.t. f^c. Finally, we can choose any evidence function e because the above argument does not rely on some evidence not being feasible (i.e., $e_i(\alpha(\theta)) \notin E_i(\theta)$ for some i, α, and θ).

8. DISCUSSION

8.1. *Alternative Solution Concepts.*

8.1.1. *Nash equilibrium.* Because we consider a dynamic setup, it is natural to use SPE as a solution concept. However, the results we obtain are in fact stronger because the propositions remain true if instead we use NE as a solution concept. For instance, in the proof of the necessity of pq-dynamic monotonicity in Proposition A0, we argue that if a strategy profile $\tilde{s}$ fails to implement an SCF, there must exist a profitable deviation *on the path* of play. Therefore, $\tilde{s}$ not only fails to be an SPE, but also an NE. From this it follows that if an SCF is repeatedly implementable in NE, then it must also satisfy pq-dynamic monotonicity. The same is true about the other definitions of monotonicity in the article. Likewise, in the proof of sufficiency in Proposition 1, it is still true that only the mechanism $\hat{g}$ will be selected on the path of any NE play (i.e., the result of Lemma B2 extends to NE). pq-monotonicity and pq-stationary monotonicity then ensure that only the desired output profiles are produced on the equilibrium path in any NE. Again, the same is true for the other proofs of sufficiency in the article. Mezzetti and Renou (2017, p. 266) made a similar observation about the equivalence of NE and SPE for their results.

8.1.2. *Mixed strategies.* The regime that is used to prove the sufficiency part in Proposition 1 employs mechanism $\hat{g}$, which contains the so-called modulo game. Its purpose is to eliminate pure-strategy equilibria in inconsistent messages. However, because of this modulo game, there can be mixed-strategy equilibria. Although we only consider implementation in pure strategies, we now briefly discuss how one can tackle mixed-strategy equilibria. We base our discussion on Lee and Sabourian (2015) who show how one can rule out undesirable mixed-strategy equilibria in the repeated setup if firms have a preference for less complex strategies.

If firms randomize when facing mechanism $\hat{g}$, there is a positive probability that they will announce inconsistent messages, in which case the regime singles out one of the firms, say, firm i and determines its target continuation value, v_i. The regime in Proposition 1 is designed so that in the continuation, firm i expects a profit of v_i in each of future periods. Alternatively, the continuation regime could be modified so that firm i still expects v_i in the continuation, but some of the periods it does not produce anything and gets a profit of 0.[20] In these latter periods, the other firms can be inflicted very large losses by requiring them to produce very large outputs. Therefore, if the firms were to earn profits higher than $v(f)$ in a mixed-strategy SPE, they must announce consistent messages with a sufficiently high probability and so face $\hat{g}$ again in the next period. Furthermore, those losses can be chosen so that in any mixed-strategy SPE, the firms randomize with different probabilities in different periods whenever they face $\hat{g}$. If the firms have a preference for simpler strategies, any firm will opt for a strategy that gives the same profit, but assigns the same probabilities in different periods. Potentially low profits and complexity of mixed-strategy equilibria help to rule them out; for details, see Lee and Sabourian (2015).[21]

[20] It is possible that $v_i = \overline{v}_i$, which is given to firm i in order to incentivize it to deviate from some undesirable message profile. However, because the incentives to deviate must be strict and because its discounted profit is continuous in v_i, it is always possible to give $v_i < \overline{v}_i$ while still preserving the right incentives.

[21] In a recent paper, Chen et al. (2022) study one-shot implementation in mixed-strategy NE when transfers and lotteries are available. Because the continuation values can play the role of transfers, the insights of their paper could

8.2. *Other Applications.* This study considers repeated implementation in a specific environment that has been motivated by the industrial organization literature on collusion. However, certain results also extend to other environments. For example, there exists extensive literature on one-shot implementation that selects the Walrasian equilibrium allocation in multi-good economies in every state; for references, see Corchón (1996, subsection 5.6) or Jackson (2001, subsection 3.7). One can adapt our model to study implementation of the Walrasian SCF in the repeated setup. Suppose there are L perishable goods that cannot be saved from one period to another. I now is a set of consumers. Each period consumer $i \in I$ receives the same endowment $\omega_i \in \mathbb{R}^L_+$. $q_i \in \mathbb{R}^L_+$ now is an L-dimensional consumption bundle of consumer i and his utility from consuming the bundle is $\pi_i(q_i, \theta)$ in state θ. With these modifications, we can study whether an SCF $f : \Theta \to \{q \in \mathbb{R}^{nL}_+ | \sum_{i\in I} q_i = \sum_{i\in I} \omega_i\}$ and, in particular, the Walrasian SCF is repeatedly implementable.

Based on this modified environment, we make a couple of observations. First, it is easy to see that the definitions of forthrightness and monotonicity are context-free. The same is true about the proof of the necessity of pq-dynamic monotonicity in Proposition A0.[22] Hence, it follows that in the repeated exchange economy, an SCF must still satisfy pq-dynamic monotonicity if it is implementable with a forthright regime. Only the definitions of consistent messages, $m(\theta)$ for $\theta \in \Theta$, and the intersection of lower contour sets would depend on the application. For example, assuming interior allocations in exchange economies, consistent messages $m(\theta)$ for each θ are such that $m_i(\theta) = (p_i, q_i)$ with $p_i = \nabla\pi_i(q_i, \theta)$ for all $i \in I$ and $(q_1, \dots, q_n) = f(\theta)$, where $\nabla\pi_i(q_i, \theta)$ denotes the (normalized) gradient vector. If f is the Walrasian SCF, then with consistent messages, all consumers announce the same Walrasian equilibrium price vector. The intersection of lower contour sets now is $\Lambda^f_i(q, \theta) := \cap_{\theta' \in f^{-1}(q,\theta)} L_i(q, v(f), \theta')$ where $f^{-1}(q, \theta) := \{\theta' \in \Theta | q = f(\theta')$ and $\nabla\pi_i(q_i, \theta') = \nabla\pi_i(q_i, \theta)\}$.

Second, the regime that is used to prove the sufficiency of pq-dynamic monotonicity is specific to the application that we consider. Among other things, the proof assumes the existence of a sufficiently bad outcome $\overline{q}_i$, which firm i receives if it is one of several whistle-blowers in some period. Therefore, the situation when there are multiple whistle-blowers, will never arise on the equilibrium path. Saijo et al. (1996) require that in exchange economies, the mechanism always selects a balanced allocation: $g(m) \in \{q \in \mathbb{R}^{nL}_+ | \sum_{i\in I} q_i = \sum_{i\in I} \omega_i\}$ for all m. In this case, when everyone is a whistle-blower, the outcome $(\overline{q}_1, \dots, \overline{q}_n)$ cannot be too bad and, in fact, it might be an equilibrium outcome in some state. Because of that, Saijo et al. (1996) show that for an SCF to be implementable, besides a version of Maskin monotonicity, it must additionally satisfy Condition PQ. We expect similar additional necessary conditions in the repeated setup once we impose further restrictions on the regime besides forthrightness.

9. CONCLUDING REMARKS

To our knowledge, we are the first to study repeated collusion by firms as an implementation problem. The main message is that if either of the necessary monotonicity conditions fails, the CA cannot achieve its objective. There are no restrictions on the regime r in Proposition 1 (or in Proposition A0 when the symmetry is not assumed) once a deviation from the equilibrium path has occurred. However, in a more realistic setup, the CA would face constraints on rewards and punishments that it can apply. Clearly, even less objectives can be implemented in this case. Said differently, our results characterize the largest set of objectives that the CA can possibly implement. Other takeaways are that the CA should also encourage buyers to act as whistle-blowers and that the CA should exploit the variation in the available evidence (e.g., in profits) across the states.

also be used in the repeated setup. However, in their mechanism, agents report states (even multiple times when transfers are small), which we do not allow.

[22] We focus here on pq-dynamic monotonicity because restricting attention to a symmetric setup in exchange economies is not interesting.

This research can be extended in several interesting directions. For example, one could identify further desirable restrictions, besides forthrightness, that r needs to satisfy, and study necessary and sufficient conditions for repeated implementation in their presence. Also, one could allow the evidence structure in Section 7 to change over time depending on the past messages or outputs. Further, in the model, the CA controls the output, while the firms can only influence it indirectly via their messages. In a more complex model with several profit-relevant variables, the firms could directly decide on some of these variables, whereas the other variables would be controlled by the CA. Finally, in the exchange economy that we briefly discuss in the previous section, it is not immediate how the necessary conditions change if the consumers can save which affects their feasible consumption in the future.

APPENDIX A: pq-DYNAMIC MONOTONICITY

Here, we define pq-dynamic monotonicity and prove that it is necessary and sufficient for the repeated implementation of SCF using forthright regimes.

A dynamic deception allows firms to deceive differently in different periods. Thus, a dynamic deception β specifies a state $\tilde{\theta}^t \in \Theta$ for every t and $\zeta^t = (\zeta^{t-1}, \theta^t)$: $\beta(\zeta^{t-1}, \theta^t) = \tilde{\theta}^t$.[23] Given t and ζ^t, let $l(\zeta^t|\zeta^t) = 1$ and for any $\tau > t$, let

$$l(\zeta^{\tau+1}|\zeta^t) = \begin{cases} l(\zeta^\tau|\zeta^t)l(\theta) & \text{if } \zeta^{\tau+1} = (\zeta^\tau, \theta), \\ 0 & \text{otherwise.} \end{cases}$$

For any t and ζ^t, the discounted value of future profits of firm i if the firms follow the dynamic deception β is

$$v_i(f \circ \beta(\zeta^t)) = (1-\delta) \sum_{\tau > t} \sum_{\zeta^{\tau-1}} \sum_{\theta^\tau} \delta^{\tau-t-1} l(\zeta^{\tau-1}|\zeta^t) l(\theta^\tau) \pi_i(f(\beta(\zeta^{\tau-1}, \theta^\tau)), \theta^\tau).$$

DEFINITION A1. f satisfies *pq-dynamic monotonicity* if for any β, (a) implies (b):

a. $\Lambda_i^f(f(\beta(\zeta^{t-1}, \theta^t))) \subseteq L_i(f(\beta(\zeta^{t-1}, \theta^t)), v(f \circ \beta(\zeta^{t-1}, \theta^t)), \theta^t)$ for all t, $i \in I$, and $(\zeta^{t-1}, \theta^t) \in \Theta^{t+1}$,
b. $f(\beta(\zeta^{t-1}, \theta^t)) = f(\theta^t)$ for all t and $(\zeta^{t-1}, \theta^t) \in \Theta^{t+1}$.

Clearly, pq-monotonicity and pq-stationary monotonicity are special cases of pq-dynamic monotonicity. The former restricts attention to dynamic deceptions β s.t. $\beta(\zeta^t, \theta) = \theta$ for all $\theta \in \Theta$, $\zeta^t \in \Theta^{t+1}$, and $t \geq 0$, whereas the latter restricts attention to β s.t. $\beta(\zeta^t, \theta) = \beta(\zeta^{-1}, \theta)$ for all $\theta \in \Theta$, $\zeta^t \in \Theta^{t+1}$, and $t \geq 0$. Also, if we replace $\Lambda_i^f(f(\beta(\zeta^{t-1}, \theta^t)))$ with $L_i(f(\beta(\zeta^{t-1}, \theta^t)), v(f), \beta(\zeta^{t-1}, \theta^t))$, we obtain the definition of dynamic monotonicity (Mezzetti and Renou, 2017). Because $\Lambda_i^f(f(\beta(\zeta^{t-1}, \theta^t))) \subseteq L_i(f(\beta(\zeta^{t-1}, \theta^t)), v(f), \beta(\zeta^{t-1}, \theta^t))$ for all t, i, and (ζ^{t-1}, θ^t), pq-dynamic monotonicity implies dynamic monotonicity.

PROPOSITION A0. *f is repeatedly implementable in SPE with a regime that satisfies forthrightness w.r.t. f if and only if f satisfies pq-dynamic monotonicity.*

PROOF OF PROPOSITION A0.

Necessity. Suppose f is repeatedly implementable in SPE with a regime that satisfies forthrightness w.r.t. f. Thus, there exists an SPE s s.t. $s(h^t, \theta^t) = m(\theta^t)$ and $r(h^t)(m(\theta^t)) =$

[23] Firms could also condition their deception on the history of messages that they send and on the history of mechanisms that are selected by the regime. However, because the firms select their messages deterministically and the regime also selects the mechanisms deterministically, it is sufficient to condition the deception only on the history of states.

$f(\theta^t)$ for all t, θ^t, and h^t s.t. $\rho(h^t|h^0, s) > 0$. Fix this s. Given s, we define another SPE $\hat{s}$, which we will then use to prove that f must satisfy pq-dynamic monotonicity. We first explain $\hat{s}$ in words. $\hat{s}$ only differs from s when a unilateral deviation from the equilibrium play occurs. Specifically, suppose that $\hat{\theta}, \tilde{\theta} \in f^{-1}(f(\theta))$ for some θ. Suppose that firm i deviates from s in period t. Then, the continuation values that the firms expect starting period $t+1$, can differ depending if period t state is $\hat{\theta}$ or $\tilde{\theta}$ because the firms can condition their future play on period t state. $\hat{s}$ will have a property that the continuation values are the same whether period t state is $\hat{\theta}$ or $\tilde{\theta}$. The reason why such an equilibrium exists is that because of forthrightness, the CA cannot infer from the messages whether the state is $\hat{\theta}$ or $\tilde{\theta}$ and, hence, cannot offer different mechanisms in the continuation.

We now proceed with defining $\hat{s}$ formally. First, let $\hat{s}(h^t, \theta^t) = s(h^t, \theta^t) = m(\theta^t)$ for all t, θ^t, and h^t s.t. $\rho(h^t|h^0, s) > 0$. Next, take any t, θ^t, $h^t = (\mu^t, \zeta^{t-1})$ s.t. $\rho(h^t|h^0, s) > 0$. Suppose that period t state is some $\hat{\theta}^t \in f^{-1}(f(\theta^t))$. Note that $s(h^t, \hat{\theta}^t) = m(\theta^t)$. Take any firm i. We now define the output–value pair that firm i induces by announcing some $m_i \neq m_i(\theta^t)$:

$$(q, v(\hat{\theta}^t)) := (r(\mu^t)(m_i, m_{-i}(\theta^t)), v(s|((\mu^t, (m_i, m_{-i}(\theta^t))), (\zeta^{t-1}, \hat{\theta}^t)))).$$

Although, given $(m_i, m_{-i}(\theta^t))$, q does not depend on $\hat{\theta}^t$, $v(\hat{\theta}^t)$ can depend on it since firms can condition their future messages on the state in period t. Firm i will not have incentives to deviate from $m_i(\theta^t)$ in state $\hat{\theta}^t$ if $(q, v(\hat{\theta}^t)) \in L_i(f(\theta^t), v(f), \hat{\theta}^t)$. Although it is not guaranteed that $(q, v(\tilde{\theta}^t)) \in \Lambda_i^f(f(\theta^t))$ for all $\tilde{\theta}^t \in f^{-1}(f(\theta^t))$, this must be true for the state in which $v_i(\tilde{\theta}^t)$ is minimal among $\tilde{\theta}^t \in f^{-1}(f(\theta^t))$. Suppose that this state is $\hat{\theta}^t$, that is, $(q, v(\hat{\theta}^t)) \in \Lambda_i^f(f(\theta^t))$ holds.

$\hat{s}$ will be defined in such a way that for every $\tilde{\theta}^t \in f^{-1}(f(\theta^t))$, the continuation value after the history $((\mu^t, (m_i, m_{-i}(\theta^t))), (\zeta^{t-1}, \tilde{\theta}^t))$ is $v(\hat{\theta}^t)$ instead of $v(\tilde{\theta}^t)$. Thus, let $\zeta^t = (\zeta^{t-1}, \tilde{\theta}^t)$ for any $\tilde{\theta}^t \in f^{-1}(f(\theta^t))$ and let $\hat{\zeta}^t = (\zeta^{t-1}, \hat{\theta}^t)$. For all $\tau > t$ and all θ^τ, let $\zeta^\tau = (\zeta^{\tau-1}, \theta^\tau)$ and $\hat{\zeta}^\tau = (\hat{\zeta}^{\tau-1}, \theta^\tau)$. Let $\mu^{t+1} = (\mu^t, (m_i, m_{-i}(\theta^t))$. For all $\tau > t+1$ and all m^τ, let $\mu^{\tau+1} = (\mu^\tau, m^\tau)$. Let $\hat{s}$ be such that $\hat{s}(\mu^\tau, \zeta^\tau) = s(\mu^\tau, \hat{\zeta}^\tau)$ for all $\tau > t$. Intuitively, $\hat{s}$ does not depend on which exact state in $f^{-1}(f(\theta^t))$ is realized in period t (given h^t and $(m_i, m_{-i}(\theta^t))$). By construction, it follows that $v(\hat{s}|(\mu^{t+1}, \zeta^t)) = v(s|(\mu^{t+1}, \hat{\zeta}^t))$ and

$$(r(\mu^t)(m_i, m_{-i}(\theta^t)), v(\hat{s}|(\mu^{t+1}, \zeta^t))) \in \Lambda_i^f(f(\theta^t)). \tag{A1}$$

Since s are SPE strategies of the subgame starting after history $(\mu^{t+1}, \hat{\zeta}^t)$ and because profits from period $t+1$ onwards do not depend on period t history of states, it must be that $\hat{s}$ forms SPE strategies of the subgame starting after history (μ^{t+1}, ζ^t).

Now, repeat the above process for all t, θ^t, h^t s.t. $\rho(h^t|h^0, s) > 0$, and all i and $m_i \neq m_i(\theta^t)$. The only remaining histories are the ones that occur after there has been a multilateral deviation from the equilibrium path of s. For any such history h^t in any period t and any θ, let $\hat{s}(h^t, \theta) = s(h^t, \theta)$.

The constructed $\hat{s}$ is also SPE of the game induced by r. By construction, (A1) holds for all t, θ^t, $h^t = (\mu^t, \zeta^{t-1})$ s.t. $\rho(h^t|h^0, s) > 0$, and all i and $m_i \neq m_i(\theta^t)$, which means that no unilateral deviation from the equilibrium path is profitable. Also, from the definition of $\hat{s}$, it follows that $\hat{s}$ forms NE strategies in the subgames that are reached after a (unilateral or multilateral) deviation from the equilibrium path.

We now use $\hat{s}$ to prove that f must satisfy pq-dynamic monotonicity. Suppose, on the contrary, that f does not satisfy pq-dynamic monotonicity. That is, there exists β such that in Definition A1, part (a) holds, but part (b) does not. Fix this β. Given $\hat{s}$ and β, we now define another strategy profile $\tilde{s}$. We first explain $\tilde{s}$ in words. Under $\tilde{s}$, the firms follow strategies $\hat{s}$, but they deceive according to β as long as no deviation has occurred. If a deviation ever occurs, the firms start conditioning $\tilde{s}$ on the true states, but they still behave as if the states that were drawn up to and including the period of first deviation, were the ones given by the deception.

Formally, $\tilde{s}$ is defined recursively with the help of a couple of auxiliary variables. Let $\tilde{\zeta}^0 = \beta(\zeta^0)$ and $d(\mu^0, \zeta^0) = 1$. The period 0 strategies are $\tilde{s}(\mu^0, \zeta^0) = \hat{s}(\mu^0, \tilde{\zeta}^0)$. Also, let $d(\mu^1, \zeta^1) = d(\mu^0, \zeta^0) \cdot \mathbf{1}_{\{(m^0,\zeta^0)=(\hat{s}(\mu^0,\tilde{\zeta}^0),\zeta^0)\}}$, where $\mathbf{1}_{\{X\}}$ is an indicator function taking value 1 if X is true and 0 otherwise. In period 1, if $d(\mu^1, \zeta^1) = 1$, then $\tilde{\zeta}^1 = (\tilde{\zeta}^0, \beta(\zeta^1))$; otherwise, $\tilde{\zeta}^1 = (\tilde{\zeta}^0, \theta^1)$. Let $\tilde{s}(\mu^1, \zeta^1) = \hat{s}(\mu^1, \tilde{\zeta}^1)$ and $d(\mu^2, \zeta^2) = d(\mu^1, \zeta^1) \cdot \mathbf{1}_{\{((\mu^1,m^1),\zeta^1)=((\mu^1,\hat{s}(\mu^1,\tilde{\zeta}^1)),\zeta^1)\}}$. Suppose we have defined the variables up to period $t-1$. In period t, if $d(\mu^t, \zeta^t) = 1$, then $\tilde{\zeta}^t = (\tilde{\zeta}^{t-1}, \beta(\zeta^t))$; otherwise, $\tilde{\zeta}^t = (\tilde{\zeta}^{t-1}, \theta^t)$. Let $\tilde{s}(\mu^t, \zeta^t) = \hat{s}(\mu^t, \tilde{\zeta}^t)$ and $d(\mu^{t+1}, \zeta^{t+1}) = d(\mu^t, \zeta^t) \cdot \mathbf{1}_{\{(\mu^t,m^t),\zeta^t)=((\mu^t,\hat{s}(\mu^t,\tilde{\zeta}^t)),\zeta^t)\}}$.

By assumption, there exist τ and $\zeta^\tau = (\zeta^{\tau-1}, \theta^\tau)$ such that $f(\beta(\zeta^\tau)) \neq f(\theta^\tau)$. Therefore, the constructed strategy profile $\tilde{s}$ selects an undesirable outcome on its path. Since it is assumed that r implements f, $\tilde{s}$ cannot be an SPE. Because the future profits do not depend on the history of states, by construction, $\tilde{s}$ implies NE play in the subgames that follow after a deviation from the path of $\tilde{s}$ has occurred. That is, if $\tilde{s}$ are not SPE strategies, it must be because there is a profitable deviation on the path. Thus, there exist t, θ^t, $h^t = (\mu^t, \zeta^{t-1})$ s.t. $\rho(h^t|h^0, s) > 0$, i, and m_i such that

$$
\begin{aligned}
(q, v) &= (r(\mu^t)(m_i, m_{-i}(\beta(\zeta^t))), v(\tilde{s}|((\mu^t, (m_i, m_{-i}(\beta(\zeta^t)))), \zeta^t))) \\
&\notin L_i(f(\beta(\zeta^t)), v(f \circ \beta(\zeta^t)), \theta^t).
\end{aligned}
$$

By part (a) in Definition A1, $(q, v) \notin \Lambda_i^f(f(\beta(\zeta^t)))$. Also, from the definition of $\tilde{s}$, there exists $\tilde{\zeta}^{t-1}$ such that

$$
(q, v) = (r(\mu^t)(m_i, m_{-i}(\beta(\zeta^t))), v(\hat{s}|((\mu^t, (m_i, m_{-i}(\beta(\zeta^t)))), (\tilde{\zeta}^{t-1}, \beta(\zeta^t))))).
$$

However, this contradicts (A1), which must hold for all t, $\tilde{\theta}^t$, $\tilde{h}^t$ s.t. $\rho(\tilde{h}^t|h^0, s) > 0$, and all i and $m_i \neq m_i(\tilde{\theta}^t)$, including $\tilde{\theta}^t = \beta(\zeta^t)$ and $\tilde{h}^t = (\mu^t, \tilde{\zeta}^{t-1})$. It follows that for any β, if part (a) in Definition A1 holds, then part (b) must also hold when f is repeatedly implementable. That is, f must satisfy pq-dynamic monotonicity.

Sufficiency. The proof is similar to the one of Proposition 1 below and, therefore, it is relegated to the Online Appendix. □

APPENDIX B: PROOFS OF SECTION 4

Before proving Proposition 1, we introduce some additional notation. Let $\partial\Lambda_i^f(q)$ denote the boundary of the set $\Lambda_i^f(q)$. In particular, if $(q', v) \in \partial\Lambda_i^f(q)$, then there does not exist v' s.t. $(q', v') \in \Lambda_i^f(q)$ and $v_i < v_i' \leq \overline{v}_i$. Note that $\Lambda_i^f(f(\theta)) \subseteq L_i(f(\theta), v(f), \theta')$ if and only if $\partial\Lambda_i^f(f(\theta)) \subset L_i(f(\theta), v(f), \theta')$. For each i, let $\overline{q}_i$ be such that $(1-\delta)\pi_i((\overline{q}_i, 0_{-i}), \theta) < (1-\delta)\pi_i(f(\theta'), \theta) + \delta \min\{0, v_i(f)\}$ for all (θ, θ'). $\overline{q}_i$ represents a bad outcome for firm i. If $\pi_i(f(\theta'), \theta) > 0$ for all (θ, θ'), then we can choose $\overline{q}_i = 0$. Otherwise, $\overline{q}_i$ must be large enough for the firm to earn a negative profit. Also, if $v_i(f) \geq 0$, $\overline{q}_i$ can be chosen independently of the value of δ. Only when $v_i(f) < 0$, $\overline{q}_i$ must increase in δ. Let $Q_{-i} := \sum_{j \neq i} q_j$, $\hat{\pi}_i(q_i, Q_{-i}, \theta) := \pi_i(q, \theta)$, $q_i(Q_{-i}, \theta) := \arg\max_{q_i} \hat{\pi}_i(q_i, Q_{-i}, \theta)$ and $\overline{v}_i(Q_{-i}) := \sum_\theta l(\theta)\hat{\pi}_i(q_i(Q_{-i}, \theta), Q_{-i}, \theta)$. Note $\overline{v}_i(0) = \overline{v}_i$. Let $\underline{v}_i(q_i) := \sum_\theta l(\theta)\pi_i((q_i, 0_{-i}), \theta)$. Finally, let $|X|$ denote the cardinality of set X.

PROOF OF PROPOSITION 1.

Necessity. Because both pq-monotonicity and pq-stationary monotonicity are implied by pq-dynamic monotonicity, their necessity follows directly from Proposition A0.

Sufficiency. We start by defining a regime. After any history, the regime will call one of two mechanisms and this will be done with the help of an auxiliary mapping d.

Regime r. Let $d(\mu^0) = (0, 0)$. For any $t \geq 0$ and μ^t, if $d(\mu^t) = (0, 0)$, then $r(\mu^t) = \hat{g}$; otherwise, $r(\mu^t) = \breve{g}$. How d is determined for $t > 0$ and μ^t is given in the description of mechanisms below.

Mechanism $\hat{g}$. Suppose the mechanism is called after the message history μ^t. Given $m = ((p_1, q_1), \dots, (p_n, q_n))$, let $D(m) := \{i \in I | \exists m_i' \text{ s.t. } (m_i', m_{-i}) = m(\theta) \text{ for some } \theta\}$ denote the set of potential deviators when the firms announce m. Because of Assumption A3, $m_i(\theta) = m_j(\theta)$ for all $i, j \in I$ and $|D(m)| \in \{0, 1, n\}$. $|D(m)| = n$ and $m \neq m(\theta)$ for any θ can only occur when $n = 2$. Otherwise, when $n > 2$, $|D(m)| = n$ only if $m = m(\theta)$ for some θ.

The outcome function of the mechanism is as follows:

- i If $m = m(\theta)$ for some $\theta \in \Theta$, then $\hat{g}(m) = f(\theta)$. Set $d^* = (0, 0)$.
- ii If $m \neq m(\theta)$ for any $\theta \in \Theta$ and $D(m) = \{i\}$, then $\hat{g}(m) = q'$ s.t. $q_i' = q_i$ and $q_j' = \max\{0, (p^{-1}(p_i) - q_i)/(n-1)\}$ for all $j \neq i$. Let v be s.t. $(q', v) \in \partial \Lambda_i^f(\hat{q}_i, q_{-i})$ where $\hat{q}_i = q_j$ for some $j \neq i$.[24] Set $d^* = (i, v_i)$.
- iii If $m \neq m(\theta)$ for any $\theta \in \Theta$ and $|D(m)| = n = 2$, then $\hat{g}(m) = (\overline{q}_1, \overline{q}_2)$. Set $d^* = (1, \underline{v}_1(\overline{q}_1))$.
- iv If $m \neq m(\theta)$ for any $\theta \in \Theta$ and $D(m) = \emptyset$, then $\hat{g}(m) = (q_i, 0_{-i})$ where $i = \left(\sum_{j \in I} \lceil p_j \rceil\right)$ (mod n) + 1 is the winner of the "modulo game".[25] Set $d^* = (i, \overline{v}_i)$.

Let $\mu^{t+1} = (\mu^t, m)$ and $d(\mu^{t+1}) = d^*$.

Mechanism $\breve{g}$. Suppose the mechanism is called after the message history μ^t and $d(\mu^t) = (i, v_i)$. Given $m = ((p_1, q_1), \dots, (p_n, q_n))$, if $\max\{0, \underline{v}_i(\overline{q}_i)\} < v_i \leq \overline{v}_i$, then $\breve{g}(m) = (q_i, q_{-i}')$ s.t. $\overline{v}_i(\sum_{j \neq i} q_j') = v_i$. (It is irrelevant how $\sum_{j \neq i} q_j'$ is divided among $j \neq i$.) If $v_i \leq \max\{0, \underline{v}_i(\overline{q}_i)\}$, then $\breve{g}(m) = (q_i', 0_{-i})$ s.t. $\underline{v}_i(q_i') = v_i$.

Let $\mu^{t+1} = (\mu^t, m)$ and $d(\mu^{t+1}) = d(\mu^t)$.

LEMMA B1. *There exists an SPE s s.t. $s(h^t, \theta^t) = m(\theta^t)$ and $r(h^t)(m(\theta^t)) = f(\theta^t)$ for all t, θ^t, and h^t s.t. $\rho(h^t | h^0, s) > 0$.*

PROOF OF LEMMA B1. For all t, θ^t, and $h^t = (\mu^t, \zeta^{t-1})$, let s be defined as follows:

- If $d(\mu^t) = (0, 0)$, then $s_i(h^t, \theta^t) = m_i(\theta^t)$ for all i.
- If $d(\mu^t) = (i, v_i)$, then $s_i(h^t, \theta^t) = (\cdot, q_i(Q_{-i}, \theta^t))$ where Q_{-i} is s.t. $\overline{v}_i(Q_{-i}) = \max\{0, v_i\}$. (The first coordinate in i's message can be any p_i.) $s_j(h^t, \theta^t)$ for $j \neq i$ can be anything.

If the firms follow the specified strategies, then the mechanism $\hat{g}$ is selected for every t, and the outcome is $f(\theta^t)$ for every θ^t, that is, the desired output is implemented in every period. Next, we verify that no firm has incentives to deviate from s. First we consider deviations in the subgames off the path and next we consider deviations in the subgames on the path.

Consider any t, θ^t, and $h^t = (\mu^t, \zeta^{t-1})$ s.t. $d(\mu^t) = (i, v_i)$ for some $i \in I$ and $v_i \leq \overline{v}_i$. Thus, the firms face the mechanism $\breve{g}$. Since the firms are also facing exactly the same problem in all future periods irrespective of what their period t messages are, the best that firm i can do is to announce $m_i = (\cdot, q_i(Q_{-i}, \theta^t))$ where Q_{-i} is s.t. $\overline{v}_i(Q_{-i}) = \max\{0, v_i\}$. Any messages by other firms are optimal because their messages do not affect the outcome. (If $v_i \leq \max\{0, \underline{v}_i(\overline{q}_i)\}$, then the message of firm i also does not affect the outcome.) Also note that given s, after history h^t, but before period t state is realized, firm i expects exactly the continuation value of v_i.

Consider any t, θ^t, and $h^t = (\mu^t, \zeta^{t-1})$ s.t. $d(\mu^t) = (0, 0)$, in which case the firms face the mechanism $\hat{g}$. Given that other firms follow s, if firm i deviates, the messages will fall under part (ii) or possibly part (iii) of the mechanism. By construction, any deviation will result in

[24] Given q', v_i is the same for all v s.t. $(q', v) \in \partial \Lambda_i^f(\hat{q}_i, q_{-i})$. Thus, by announcing (p_i, q_i), firm i not only specifies q, but also v_i. The exact v_{-i} is irrelevant because the present discounted profit of firm i does not depend on it.

[25] $\lceil p_j \rceil$ denotes the smallest integer greater than or equal to p_j.

an output–value pair $(q', v) \in \Lambda_i^f(f(\theta^t))$ and, therefore, is not profitable. We conclude that s is indeed an SPE. It also follows that r satisfies forthrightness w.r.t. f. □

In the continuation, for any t and $h^t = (\mu^t, \zeta^{t-1})$ s.t. $d(\mu^t) = (i, v_i)$ for some $i \in I$ and $v_i \leq \overline{v}_i$, it should be understood that firm i behaves as specified in the second bullet point in the proof of Lemma B1. This guarantees that it receives the continuation value of v_i, which is the best it can get.

LEMMA B2. *There does not exist an SPE s s.t. $r(h^t) = \check{g}$ for some t and h^t s.t. $\rho(h^t|h^0, s) > 0$.*

PROOF OF LEMMA B2. If $\check{g}$ is played on the equilibrium path, there must be some $\tau < t$, θ^τ, and $h^\tau = (\mu^\tau, \zeta^{\tau-1})$ s.t. $\rho(h^\tau|h^0, s) > 0$, $d(\mu^\tau) = (0, 0)$, and $s(h^\tau, \theta^\tau) \neq m(\theta)$ for any θ. That is, period τ messages fall under parts (ii)-(iv) of mechanism $\hat{g}$. If $s(h^\tau, \theta^\tau)$ falls under part (ii), any firm $j \notin D(s(h^\tau, \theta^\tau))$ expects strictly less than its (discounted) monopoly profit $(1-\delta)\pi_j((q_j(0, \theta^\tau), 0_{-j}), \theta^\tau) + \delta\overline{v}_j(0)$, which because of Assumption A1, is strictly positive. To see that its profit is indeed strictly less than its monopoly profit, note that if $d(\mu^\tau, s(h^\tau, \theta^\tau)) = (i, v_i)$ with $v_i \neq 0$, then firm i must be producing a positive output in every period $\tau' > \tau$ and firm $j \neq i$ cannot be a monopolist, whereas if $v_i = 0$, then from the description of $\check{g}$, it follows that $q_j^{\tau'} = 0$. Firm j can obtain its monopoly profit by deviating to a message that triggers part (iv) of $\hat{g}$ and wins the modulo game. Similarly, if $s(h^\tau, \theta^\tau)$ falls under part (iv), each firm has incentives to win the modulo game.

If $n = 2$ and $s(h^\tau, \theta^\tau)$ falls under part (iii), there exists θ s.t. $s_2(h^\tau, \theta^\tau) = m_2(\theta) = (p_2, q_2)$. Firm 1 can trigger part (ii) and obtain profits arbitrarily close to $(1-\delta)\pi_1(f(\theta), \theta^\tau) + \delta v_1(f)$ by announcing $m_1 = (p_2 + \epsilon, q_2)$ where ϵ is a small positive number. (If it announced $m_1 = (p_2, q_2)$, then because of Assumption A3, part (i) of $\hat{g}$ would apply.) By definition of $\overline{q}_1$, these profits are strictly higher than $(1-\delta)\pi_1((\overline{q}_1, \overline{q}_2), \theta^\tau) + \delta\underline{v}_1(\overline{q}_1)$ that firm 1 obtains if part (iii) of $\hat{g}$ applies. We conclude that s s.t. $r(h^t) = \check{g}$ for some t and h^t s.t. $\rho(h^t|h^0, s) > 0$ cannot be an SPE. □

LEMMA B3. *In any SPE s, $s(h^t, \theta^t) = m(\theta^t)$ and $r(h^t)(m(\theta^t)) = f(\theta^t)$ for all t, θ^t, and h^t s.t. $\rho(h^t|h^0, s) > 0$.*

PROOF OF LEMMA B3. From Lemma B2, we know that in any SPE s, the mechanism $\hat{g}$ is always selected on the equilibrium path. Therefore, for every t, θ^t, and h^t s.t. $\rho(h^t|h^0, s) > 0$, it must be that $s(h^t, \theta^t) = m(\tilde{\theta}^t)$ for some $\tilde{\theta}^t$. Fix some SPE s. Given s, dynamic deception β is defined as follows. For every t and $\zeta^t = (\zeta^{t-1}, \theta^t)$, let $\beta(\zeta^t) = \tilde{\theta}^t$ where $\tilde{\theta}^t$ is s.t. $s(\mu^t, \zeta^t) = m(\tilde{\theta}^t)$ and μ^t is the history of messages that is induced by s and ζ^{t-1}. In fact, as long as no deviation from s has occurred, we can think that firms' strategies are given by β. Also, given any t, θ^t, and ζ^{t-1}, firm i expects a profit of

$$(1-\delta)\pi_i(f(\beta(\zeta^{t-1}, \theta^t)), \theta^t) + \delta v_i(f \circ \beta(\zeta^{t-1}, \theta^t))$$

if the firms follow s or, equivalently, β.

We can think that for a given t and ζ^t, $\beta(\zeta^t, \cdot)$ specifies a static deception that the firms use in period $t+1$ and which we denote as α^{ζ^t}. (Thus, the static deception that the firms use in period 0 is denoted $\alpha^{\zeta^{-1}}(\cdot) := \beta(\zeta^{-1}, \cdot)$.) Since Θ is assumed to be finite, the number of possible static deceptions is also finite. Among all static deceptions α^{ζ^t} for all $t \geq -1$ and ζ^t, find the one for which $v(f \circ \alpha^{\zeta^t})$ is maximal. (Recall that the firms are symmetric.) Suppose this deception is played in period $\tau + 1$ when period τ history of states is ζ^τ and let us denote this deception simply as $\alpha(\cdot)$. Further, suppose that $\alpha(\theta) \notin f^{-1}(f(\theta))$ for some θ. Since f satisfies pq-stationary monotonicity, then for any i (since firms are symmetric) and some θ', there exists $(q, v) \in \partial\Lambda_i^f(f(\alpha(\theta')))$ s.t.

$$\begin{aligned}(1-\delta)\pi_i(q,\theta') + \delta v_i &> (1-\delta)\pi_i(f(\alpha(\theta')),\theta') + \delta v_i(f\circ\alpha)\\ &\geq (1-\delta)\pi_i(f(\beta(\zeta^\tau,\theta'),\theta') + \delta v_i(f\circ\beta(\zeta^\tau,\theta')).\end{aligned}$$

Because firm i only cares about the total output of its competitors, we can assume that q is s.t. $q_j = q_k$ for all $j, k \neq i$. Further, firm i can secure (q, v_i) after the history (ζ^τ, θ') by announcing $(p(\sum_{j\in I} q_j), q_i)$, which triggers part (ii) of $\hat{g}$. Since (q, v_i) gives higher profit to firm i, it wants to deviate from the deception β, which contradicts that s is an SPE.

Thus, among the static deceptions that are selected by β, any deception α that results in the highest continuation value, is s.t. $\alpha(\theta) \in f^{-1}(f(\theta))$ for all θ and, hence, $v(f\circ\alpha) = v(f)$ holds. That is, $v(f)$ is the maximal continuation value that the firms can obtain by deceiving. (Therefore, if α is s.t. $v(f\circ\alpha) = v(f)$, then $\alpha(\theta) \in f^{-1}(f(\theta))$ for all θ.) It also follows that for all $t \geq -1$ and ζ^t, α^{ζ^t} is s.t. $v(f\circ\alpha^{\zeta^t}) \leq v(f)$.

Now, if for some $t \geq 0$ and $\zeta^t = (\zeta^{t-1}, \theta^t)$, α^{ζ^t} is s.t. $v(f\circ\alpha^{\zeta^t}) < v(f)$, then $v(f\circ\beta(\zeta^t)) < v(f)$ also holds. We claim that any firm i again has a profitable deviation: it can trigger part (ii) of $\hat{g}$ in period t and obtain profits arbitrarily close to $(1-\delta)\pi_i(f(\beta(\zeta^t)),\theta^t) + \delta v_i(f)$ by announcing $m_i^t = (p_j + \epsilon, q_j)$ for a small ϵ where $(p_j, q_j) := m_j(\beta(\zeta^t))$ for any $j \neq i$. Thus, it is true that for all $t \geq 0$ and ζ^t, $\alpha^{\zeta^t}(\theta) \in f^{-1}(f(\theta))$ for all θ.

The only remaining case is that $\alpha^{\zeta^{-1}}$ is s.t. $v(f\circ\alpha^{\zeta^{-1}}) < v(f)$. (Here, we cannot invoke the argument of the previous paragraph because a firm would need to deviate before the start of period 0, which we do not allow.) Because $v(f\circ\alpha^{\zeta^{-1}}) < v(f)$, it must be that $\alpha^{\zeta^{-1}}(\theta) \notin f^{-1}(f(\theta))$ for some θ. Since f satisfies pq-monotonicity, then for any i, there exists $(q, v) \in \partial\Lambda_i^f(f(\alpha^{\zeta^{-1}}(\theta)))$ s.t.[26]

$$\begin{aligned}(1-\delta)\pi_i(q,\theta) + \delta v_i &> (1-\delta)\pi_i(f(\alpha^{\zeta^{-1}}(\theta)),\theta) + \delta v_i(f)\\ &= (1-\delta)\pi_i(f(\beta(\zeta^{-1},\theta),\theta) + \delta v_i(f\circ\beta(\zeta^{-1},\theta)).\end{aligned}$$

Again, we can assume that q is s.t. $q_j = q_k$ for all $j, k \neq i$ because the profit of firm i only depends on $\sum_{j\in I\setminus\{i\}} q_j$. Firm i can secure (q, v_i) by triggering part (ii) of $\hat{g}$ in state θ of period 0 and, therefore, it will want to deviate from the deception β. Thus, $\alpha^{\zeta^{-1}}(\theta) \in f^{-1}(f(\theta))$ must hold for all θ.

We have shown that for s to be an SPE, it must be the case that for all $t \geq 0$ and $\zeta^t = (\zeta^{t-1}, \theta^t)$, $\alpha^{\zeta^{t-1}}(\theta^t) = \beta(\zeta^{t-1}, \theta^t) \in f^{-1}(f(\theta^t))$ holds. Since $m(\theta) = m(\theta')$ for all $\theta, \theta' \in f^{-1}(f(\theta^t))$, it follows that $s(h^t, \theta^t) = m(\beta(\zeta^{t-1}, \theta^t)) = m(\theta^t)$ for all t, θ^t, and h^t s.t. $\rho(h^t|h^0, s) > 0$. Also, the description of $\hat{g}$ implies that $r(h^t)(s(h^t, \theta^t)) = f(\theta^t)$ for all t, θ^t, and h^t s.t. $\rho(h^t|h^0, s) > 0$. Finally, since s was an arbitrary SPE, the same applies to all SPE and, hence, r implements f in SPE. □

□

APPENDIX C: PROOFS OF SECTION 5

PROOF OF PROPOSITION 2. In order to prove that f^c satisfies pq-monotonicity, we will show that for any θ, θ', $f^c(\theta) \neq f^c(\theta')$ implies that there exists firm i, for which $\Lambda_i^{f^c}(f^c(\theta)) \nsubseteq L_i(f^c(\theta), v(f), \theta')$ holds. That is, there exists (q, v) s.t. $(q, v) \in \Lambda_i^{f^c}(f^c(\theta))$, but $(q, v) \notin L_i(f^c(\theta), v(f), \theta')$. As it turns out, we can choose $v = v(f)$, while q is s.t. $\sum_j q_j = \sum_j f_j^c(\theta)$. To see it, consider Figure C1, which shows iso-profit lines of firm i as a function of its own

[26] It is enough to consider $\alpha^{\zeta^{-1}}$ s.t. $\alpha^{\zeta^{-1}}(\theta') = \theta'$ for all $\theta' \neq \theta$ because the firms do not deceive after period 0 anymore.

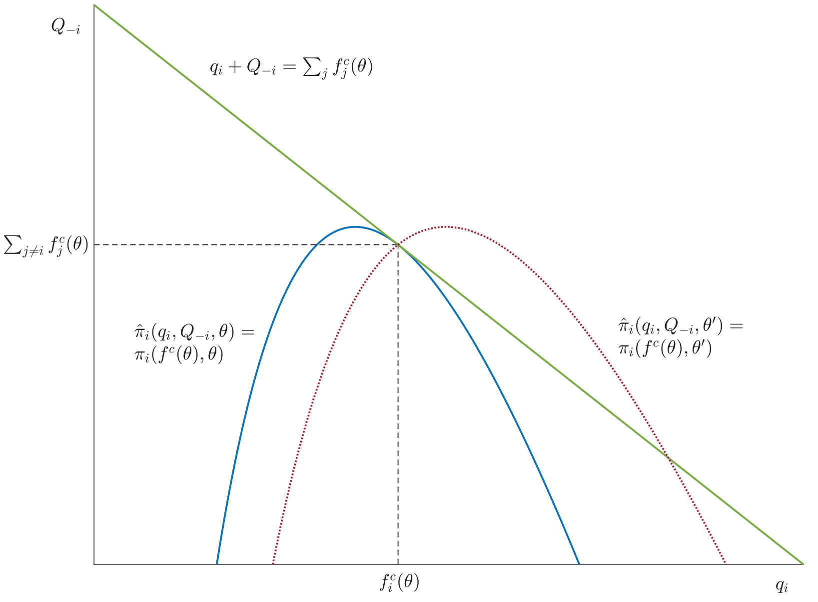

FIGURE C1

pq-MONOTONICITY OF f^c

output, q_i, and the aggregate output of all the other firms, $Q_{-i} := \sum_{j\neq i} q_j$. One can think that in the competitive equilibrium of state θ, firm i is choosing q_i and Q_{-i} to maximize its profit $\hat{\pi}_i(q_i, Q_{-i}, \theta) := \pi_i(q, \theta)$ subject to the constraint $q_i + Q_{-i} = \sum_j f_j^c(\theta)$, meaning that the aggregate output and, hence, price remain constant. The optimum is at $(f_i^c(\theta), \sum_{j\neq i} f_j^c(\theta))$. The slope of iso-profit line is

$$\frac{dQ_{-i}}{dq_i} = -\frac{p'(Q)q_i + p(Q) - c_i'(q_i, \theta)}{p'(Q)q_i},$$

which is equal to -1, the slope of the constraint, in the competitive equilibrium where $p(\sum_j f_j^c(\theta)) = c_i'(f_i^c(\theta), \theta)$ holds because $f_i^c(\theta) > 0$. The same is true in all other states of the world, for which $f^c(\theta)$ remains the competitive equilibrium outcome. It follows that the points on the constraint $q_i + Q_{-i} = \sum_j f_j^c(\theta)$, together with $v = v(f)$, belong to $\Lambda_i^f(f(\theta))$.

Now, suppose that $f^c(\theta)$ is not a competitive equilibrium outcome in state θ'. Then, because the solution to (1) is unique according to Assumption A4, there exists a firm, say, firm i, for which $p(\sum_j f_j^c(\theta)) \neq c_i'(f_i^c(\theta), \theta')$.[27] It means its iso-profit line through point $(f_i^c(\theta), \sum_{j\neq i} f_j^c(\theta))$ is crossing the constraint in state θ'. Therefore, given the assumption that $f_j(\theta) > 0$ for all j, we can find a point on the constraint that firm i strictly prefers to $(f_i^c(\theta), \sum_{j\neq i} f_j^c(\theta))$. It follows that there exists q s.t. $\sum_j q_j = \sum_j f_j^c(\theta)$ and $(q, v(f)) \in \Lambda_i^{f^c}(f^c(\theta))$ but $(q, v(f)) \notin L_i(f^c(\theta), v(f), \theta')$. We can conclude that f^c satisfies pq-monotonicity. □

PROOF OF PROPOSITION 3. Since we are in a symmetric environment, we will consider the incentives of firm 1 to deviate from a deception. We start with a couple of observations. First, we know from Proposition 2 that f^c satisfies pq-stationary monotonicity for $\delta =$

[27] If we also impose Assumption A3 and, hence, only consider symmetric competitive equilibrium outcomes, then $f^c(\theta) \neq f^c(\theta')$ implies $p(\sum_j f_j^c(\theta)) \neq c_i'(f_i^c(\theta), \theta')$ without assuming uniqueness. That is, we do not need to rule out the existence of additional asymmetric equilibrium outcomes.

0. Therefore, in the following, we assume that $\delta \in (0,1)$. Second, because of Assumption A5, $0 < f_1^c(\theta) < f_1^c(\theta')$ if $\theta > \theta'$. One implication is that $f^{c-1}(f^c(\theta)) = \{\theta\}$ and $\Lambda_1^{f^c}(f^c(\theta)) = L_1(f^c(\theta), v(f^c), \theta)$ for all $\theta \in \Theta$.

Consider any stationary deception α s.t. $\alpha(\theta') \neq \theta'$ for some $\theta' \in \Theta$. This deception would result in an undesirable output profile in state θ'. In order to incentivize firm 1 to deviate when the firms deceive according to α, but not to deviate when they behave truthfully, we need to find (q, v_1) s.t.

$$\text{(C1)} \qquad (1-\delta)\pi_1(f^c(\theta), \theta) + \delta v_1(f^c) \geq (1-\delta)\pi_1(q, \theta) + \delta v_1,$$

$$\text{(C2)} \qquad (1-\delta)\pi_1(f^c(\theta), \theta') + \delta v_1(f^c \circ \alpha) < (1-\delta)\pi_1(q, \theta') + \delta v_1,$$

for some θ, θ' s.t. $\alpha(\theta') = \theta \neq \theta'$. Suppose that $v_1(f^c \circ \alpha) \leq v_1(f^c)$ holds. Then, we can choose $v_1 = v_1(f^c)$ and, according to Proposition 2, we can find q that satisfies (C1) and (C2). Therefore, in the continuation, we assume that $v_1(f^c \circ \alpha) > v_1(f^c)$ holds. It means that there must exist θ, θ' s.t. $\alpha(\theta') = \theta > \theta'$. That is, the firms will increase their profits if they produce less than the competitive output, which requires that they exaggerate their costs. Next, we ask when firm 1 will have incentives to deviate from α in this particular state θ'.

We can equivalently rewrite (C1) and (C2) as:

$$\begin{aligned} &(1-\delta)[\theta(c(q_1) - c(f_1^c(\theta)) + p(nf_1^c(\theta))f_1^c(\theta) - p(Q)q_1] + \delta v_1(f^c) \geq \delta v_1 \\ \text{(C3)} \qquad &>(1-\delta)[\theta'(c(q_1) - c(f_1^c(\theta)) + p(nf_1^c(\theta))f_1^c(\theta) - p(Q)q_1] + \delta v_1(f^c \circ \alpha). \end{aligned}$$

(C3) tells what values v_1 can take, given q. It can, however, be that any v_1 that satisfies the inequalities in (C3), is not feasible because it exceeds the ex ante monopoly profit of firm 1, $\overline{v}_1$. It is easier to satisfy the feasibility constraint if we set $q_{-1} = 0_{-1}$, which we can always do. That is, if (q, v_1) satisfies (C3), then so does $((q_1, 0_{-1}), v_1')$ where v_1' is given by $\delta(v_1' - v_1) = (1-\delta)(p(Q) - p(q_1))q_1 \leq 0$. Thus, in the continuation, we assume that $q_{-1} = 0_{-1}$ and $p(Q) = p(q_1)$ in (C3).

If we take the right-most and left-most expressions in (C3) and simplify, we find that q_1 must satisfy the following inequality:

$$\text{(C4)} \qquad (1-\delta)(\theta - \theta')c(q_1) > (1-\delta)(\theta - \theta')c(f_1^c(\theta)) + \delta(v_1(f^c \circ \alpha) - v_1(f^c)).$$

From (C4), it follows that $q_1 > f_1^c(\theta)$ because $\theta > \theta'$ and $v_1(f^c \circ \alpha) > v_1(f^c)$.

To sum up, if we can select q_1 that satisfies (C4) and v_1 that satisfies (C3) (with $q_{-1} = 0_{-1}$), then we can incentivize firm 1 to deviate from deception α in state θ'. Now, for any $\delta \in (0,1)$, we can always find q_1 that satisfies (C4). Therefore, we only need to determine when we can find $v_1 \leq \overline{v}_1$ that satisfies (C3).

Let q_1^* and v_1^* be the values of q_1 and v_1, respectively, such that both inequalities in (C3) are in fact equalities. (Hence, (C4) also holds with equality.) Using (C4) when $q_1 = q_1^*$ and $v_1 = v_1^*$, we can eliminate δ in the first line of (C3) to obtain that

$$\text{(C5)} \qquad v_1^* = v_1(f^c) + \frac{v_1(f^c \circ \alpha) - v_1(f^c)}{\theta - \theta'}\left\{\theta - \frac{p(q_1^*)q_1^* - p(nf_1^c(\theta))f_1^c(\theta)}{c(q_1^*) - c(f_1^c(\theta))}\right\}.$$

If $v_1^* < \overline{v}_1$, then we can always find $q_1 > q_1^*$ and $v_1 \leq \overline{v}_1$ that satisfy (C3) and (C4). Therefore, next we study when $v_1^* < \overline{v}_1$ holds.

Part 1: The term in the curly brackets of (C5) can be written as

$$\frac{-(p(q_1^*) - p(nf_1^c(\theta)))q_1^* + \theta[c(q_1^*) - c(f_1^c(\theta)) - c'(f_1^c(\theta))(q_1^* - f_1^c(\theta))]}{c(q_1^*) - c(f_1^c(\theta))}$$

where we have used that $p(nf_1^c(\theta)) = c'(f_1^c(\theta))$. When q_1^* is sufficiently close to $f_1^c(\theta)$, the above expression is negative because the expression in the square brackets is close to zero and $p(q_1^*) > p(nf_1^c(\theta))$. Hence, $v_1^* < \overline{v}_1$ holds. From (C4), there is an increasing one-to-one relationship between δ and q_1^*. Therefore, we can conclude that there exists $\underline{\delta} > 0$ such that $v_1^* < \overline{v}_1$ is satisfied for $\delta \in (0, \underline{\delta})$. Since α and θ' were arbitrary, this completes the proof of the first part of the proposition.

Part 2: For any $\epsilon > 0$, we can pick $\tilde{q}_1$ such that the term in the curly brackets of (C5) is less than $\theta + \epsilon$ for all $q_1^* > \tilde{q}_1$. If $\overline{v}_1 > v_1(f^c) + \frac{\max_\theta \theta(v_1(f^m) - v_1(f^c))}{\min_{\theta,\theta'|\theta>\theta'} \theta - \theta'}$ holds, then there exists $\tilde{q}_1$ or, equivalently, $\overline{\delta}$ such that $v_1^* < \overline{v}_1$ is satisfied for all $q_1^* > \tilde{q}_1$ or, equivalently, for all $\delta \in (\overline{\delta}, 1)$. This proves the second part of the proposition.

Part 3: Finally, the term in the curly brackets of (C5) can be upper-bounded, for example, with $\theta + \frac{p(nf_1^c(\theta))f_1^c(\theta)}{c(nf_1^c(\theta)) - c(f_1^c(\theta))}$ for all $q_1^* > f_1^c(\theta)$ or, equivalently, all $\delta \in (0, 1)$. One can verify that as long as $f_1^c(\theta) > 0$, $\frac{df_1^c(\theta)}{dn} < 0$, whereas $\frac{dnf_1^c(\theta)}{dn} \geq 0$. Thus, the term in the curly brackets is decreasing in n. Likewise, $\frac{dv_1(f^m)}{dn} < 0$ and $\frac{dv_1(f^c)}{dn} \leq 0$; the latter with strict inequality if $v_1(f^c) > 0$. Since $v_1(f^m) \geq v_1(f^c \circ \alpha)$, $v_1(f^c \circ \alpha)$ must also tend to 0 as n increases. We conclude that there exists $\overline{n}$ such that $v_1^* < \overline{v}_1$ is satisfied for all $n > \overline{n}$. This proves the final part of the proposition. □

SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

REFERENCES

AUBERT, C., P. REY, and W. E. KOVACIC, "The Impact of Leniency and Whistle-Blowing Programs on Cartels, " *International Journal of Industrial Organization* 24(6) (2006), 1241–66.

ĀZACIS, H., and P. VIDA, "Repeated Implementation: A Practical Characterization, " *Journal of Economic Theory* 180 (2019), 336–67.

BANERJEE, S., Y.-C. CHEN, and Y. SUN, "Direct Implementation with Evidence," *Theoretical Economics*, forthcoming (2023).

BEN-PORATH, E., and B. L. LIPMAN, "Implementation with Partial Provability," *Journal of Economic Theory* 147(5) (2012), 1689–724.

BULL, J., and J. WATSON, "Evidence Disclosure and Verifiability," *Journal of Economic Theory* 118(1) (2004), 1–31.

CARREE, M., A. GÜNSTER, and M. P. SCHINKEL, "European Antitrust Policy 1957-2004: An Analysis of Commission Decisions," *Review of Industrial Organization* 36(2) (2010), 97–131.

CHASSANG, S., and J. ORTNER, "Collusion in Auctions with Constrained Bids: Theory and Evidence from Public Procurement," *Journal of Political Economy* 127(5) (2019), 2269–300.

CHEN, Y.-C., T. KUNIMOTO, Y. SUN, and S. XIONG, "Maskin Meets Abreu and Matsushima," *Theoretical Economics* 17(4) (2022), 1683–717.

CORCHÓN, L. C., *The Theory of Implementation of Socially Optimal Decisions in Economics* (London: Palgrave Macmillan, 1996).

DASSLER, T., D. PARKER, and D. S. SAAL, "Methods and Trends of Performance Benchmarking in UK Utility Regulation," *Utilities Policy* 14(3) (2006), 166–74.

DIJKSTRA, P. T., M. A. HAAN, and M. MULDER, "Industry Structure and Collusion with Uniform Yardstick Competition: Theory and Experiments," *International Journal of Industrial Organization*, 50 (2017), 1–33.

DUTTA, B., A. SEN, and R. VOHRA, "Nash Implementation Through Elementary Mechanisms in Economic Environments," *Economic Design* 1 (1994), 173–203.

HARRINGTON, J. E., "Optimal Corporate Leniency Programs," *The Journal of Industrial Economics* 56(2) (2008), 215–46.

HAY, G. A., and D. KELLEY, "An Empirical Survey of Price Fixing Conspiracies," *The Journal of Law and Economics* 17(1) (1974), 13–38.

JACKSON, M. O., "A Crash Course in Implementation Theory," *Social Choice and Welfare* 18(4) (2001), 655–708.

JAMASB, T., and M. POLLITT, "Benchmarking and Regulation: International Electricity Experience," *Utilities Policy* 9(3) (2000), 107–30.

KARTIK, N., and O. TERCIEUX, "Implementation with Evidence," *Theoretical Economics* 7(2) (2012), 323–55.

LAFFONT, J.-J., and J. TIROLE, *A Theory of Incentives in Procurement and Regulation* (Cambridge, MA: MIT Press, 1993).

LEE, J., and H. SABOURIAN, "Efficient Repeated Implementation," *Econometrica* 79(6) (2011), 1967–94.

———, and ———, "Complexity and Repeated Implementation," *Journal of Economic Theory* 158 (2015), 259–92.

MASKIN, E., "Nash Equilibrium and Welfare Optimality," *The Review of Economic Studies* 66(1) (1999), 23–38.

MEZZETTI, C., and L. RENOU, "Repeated Nash Implementation," *Theoretical Economics* 12(1) (2017), 249–85.

MOTTA, M., and M. POLO, "Leniency Programs and Cartel Prosecution," *International Journal of Industrial Organization* 21(3) (2003), 347–79.

ORTNER, J. M., S. CHASSANG, K. KAWAI, and J. NAKABAYASHI "Screening Adaptive Cartels," Working Paper 30219, National Bureau of Economic Research, 2022.

REY, P., "Towards a Theory of Competition Policy," in M. Dewatripont, L. P. Hansen, and S. J. Turnovsky, eds., *Advances in Economics and Econometrics: Theory and Applications, Eight World Congress*, Volume 2, Chap. 3 (Cambridge: Cambridge University Press, 2003), 82–132.

SAIJO, T., Y. TATAMITANI, and T. YAMATO, "Toward Natural Implementation," *International Economic Review* 37(4) (1996), 949–80.

SJÖSTRÖM, T., "Implementation by Demand Mechanisms," *Economic Design* 1 (1994), 343–54.

SPAGNOLO, G., "Divide Et Impera: Optimal Leniency Programs," CEPR Discussion Paper 4840, Center for Economic Policy Research, 2004.

TANGERÅS, T. P., "Collusion-Proof Yardstick Competition," *Journal of Public Economics* 83(2) (2002), 231–54.