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Planning maintenance when resources are limited: a study of periodic opportunistic replacement

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We study an aged-based replacement policy with two control limits. The first triggers opportunistic replacement and the second triggers a guaranteed replacement. The policy is novel because: the instances for component replacement are restricted to instances of time, which we call slots, that arise periodically; and a slot provides an opportunity for replacement with a particular probability. The policy models contexts in which maintenance is periodic, and resources are limited or execution of maintenance is not guaranteed. The policy is important for practice because it is simple and reflects the common reality of time-based maintenance planning. Long-run cost per unit time and average availability are calculated in a renewal-reward framework. Numerical study indicates that, if opportunities are rare, guaranteed replacement is beneficial and opportunities should be taken early in the life of a system. Using the policy, a maintainer can evaluate the cost–benefit of investing more resources to reduce the time between slots. Specific analysis and policy comparisons can be carried out using a web-application developed by the authors.

Keywords: maintenance; reliability; replacement; availability; opportunistic maintenance.

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1. Introduction

1.1. *Motivation*

There are very many mathematical maintenance policies in the published literature. These policies are becoming increasingly complicated and there is little evidence that they are used in practice (Fraser *et al.*, 2015). Thus, as theory develops, the gap between modelling and its practical application widens (Holmström *et al.*, 2009). We aim to close this theory-practice gap by developing a model of a policy that is motivated by the practical necessities of simplicity and applicability. To this end, we propose and study a policy with a periodic structure that possesses the positive attributes of block-replacement, age-based replacement, and opportunistic replacement. Also, we develop a web-application that makes it possible for non-expert users to calculate decision variables for a range of different parameters. This is a step towards bridging the gap, making policies more accessible and applicable.

In the policy we study, a system is visited when it is aged is $(i = 1,2, \ldots)$ but not in between. The instances is $(i = 1, 2, \ldots)$ are called *slots* (Budai *et al.*, 2006; Cavalcante *et al.*, 2021; Melo *et al.*, 2023), and at a slot an opportunity to maintain the system arises with probability q. In this way, q models limited availability of resources or unforeseen conditions that prevent the execution of maintenance. Thus, one can imagine a vessel periodically visiting a wind farm with limited time and spares, and in changeable weather, and selecting to maintain only some turbines (and selected components therein) (Ma *et al.*, 2022; Irawan *et al.*, 2023; O'Neil *et al.*, 2023). In this way, opportunities are a subset of the slots, and we might call a slot that is an opportunity a "visit". Alternatively, in a different similar practical context, a maintenance team may be visiting installations in remote locations, such as groundwater well-heads or power or telecoms systems (Manco *et al.*, 2022; Alotaibi *et al.*, 2023; Sanoubar *et al.*, 2023), with a finite stock of spares and limited time and personnel for the execution of component replacement. Or, an OEM or third-party maintainer may be contracted to visit sites periodically (Wang, 2010).

The system ages and its lifetime is a random variable, X. Further, at a slot, the state (either operating or failed) of the system is known precisely. Then, a sensible policy replaces the system at the first opportunity on or after min (X, Ws), for some W, the decision variable. At the W-th slot, the window for preventive maintenance opens. Prior to this slot, replacement, if carried out, is corrective only. Nonetheless, preventive replacement at age Ws is not guaranteed because not all slots are opportunities. Practically, the policy provides decision support for a maintainer, with many assets, limited resources, and limited access to the assets, who wish to plan interventions that are neither too early nor too late in the life of the system.

1.2. *Importance*

The policy is important because it is operationally simple. Unlike the classic age-based replacement policy, replacements of assets cannot become unsynchronised (Barlow & Proschan, 1964). Also, the policy has more in common with modified block replacement (Berg & Epstein, 1976) than block replacement, the latter being made cost-inefficient by premature replacement. This simplicity means the policy is scalable. The policy is also flexible because replacement is opportunistic. Thus, a maintainer can prioritise production or missions (e.g. Broek *et al.*, 2021; Wei *et al.*, 2023) by postponing interventions (e.g. De Jonge *et al.*, 2015; Berrade *et al.*, 2017; Alotaibi *et al.*, 2020; Wang *et al.*, 2020; Bautista *et al.*, 2022), achieve economies of scale when there exist fixed set-up costs (Li *et al.*, 2020; Do *et al.*, 2022), and reduce inventory costs (e.g. Basten & Ryan, 2019).

Slots may correspond to seasonal stoppages or reduced demand. For example, assets used in agricultural production are scarcely studied but interesting because usage is intermittent and intensive

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(Hu *et al.*, 2020). The timing of overhauls for large energy-production assets is an old problem (e.g. Hara *et al.*, 1966) much studied (Kralj and Petrovic, 1987), but tactical maintenance planning for small-scale power-grids is emerging (Korkovelos *et al.*, 2020). Slots may be determined by the number of assets, geographical location, and transportation resources (e.g. Gundegjerde *et al.*, 2015). The frequency of slots itself might be optimised, with s chosen to minimise a cost-rate. The decision-maker may control the frequency of slots ($1/s$), e.g. a maintainer could lease additional vessels for windfarm maintenance. In this paper we assume s is fixed. Finally, the notion of fixed periodicity of maintenance interventions is relevant to many maintenance planning scenarios, particular where interventions can be delayed (e.g. Ahmadi *et al.*, 2023).

In our modelling framework, the lifetime of the system need not be specified in continuous time. We could use a discrete distribution that is “IFR”, e.g. a negative binomial distribution. This can simplify computation. But we will cost downtime continuously, so that if the system fails at time t and is replaced at slot j then the downtime is $(js - t)$and the unavailability (downtime-rate) for the period $[0, js]$ (renewal cycle) is $\{(js - t)\ /js\}$. Costing downtime in this way requires X to be specified continuously.

In summary then, the policy proposed in this study is important because it is simple to apply in practice, both technically and managerially. We think the simplification that the fixed frequency of slots provides is very useful for practice and that the model is a step towards bridging the gap between theory and practice in maintenance planning. The policy is to our knowledge also new, demonstrating that non-simplicity and generality are not necessary requirements for novelty.

1.3. *Literature review*

Recent works on opportunistic maintenance modelling consider many industrial contexts: offshore wind turbines (Li *et al.*, 2020; Wang *et al.*, 2021; Xia *et al.*, 2021a); railway infrastructure (Bakhtiary *et al.*, 2021); power transmission (Dong *et al.*, 2022); machining (Gan *et al.*, 2021); nuclear power plant (Zhang *et al.*, 2023a); battery systems (Chen *et al.*, 2022), manufacturing (Salmasnia & Shabani, 2023; Shi *et al.*, 2023; Zhang *et al.*, 2023b), and so on. Furthermore, different configurations and concepts are adopted, including multi-component systems (Vu *et al.*, 2020; Najafi *et al.*, 2021; Shen *et al.*, 2021; Jamali & Pham, 2022; Dinh *et al.*, 2023; Dinh *et al.*, 2024), series–parallel systems (Zhou & Shi, 2019; Xia *et al.*, 2021b), economic dependence (Jiang *et al.*, 2022), imperfect maintenance (Zhou *et al.*, 2015; Liu *et al.*, 2021), reinforcement learning (Valet *et al.*, 2022), spare-parts provisioning (Zhang *et al.*, 2021; Zhu & Zhou, 2023; Zhu *et al.*, 2024). The fundamentals of the policies modelled in these works can be traced back to Sethi (1977). For recent reviews of opportunistic maintenance, see De Jonge & Scarf (2020). Moreover, a practical framework for implementing opportunistic maintenance is presented by Ab-Samat & Kamaruddin (2019).

Our approach is different from these works in the following two aspects. Firstly, opportunities arise only at predetermined time slots, so that maintenance actions can occur only at discrete times. Secondly, we model two phases, both subject to opportunistic replacement, but in the first phase action (replacement) depends on the state while in the second it does not. The former aspect, the notion of discrete, periodic slots, was proposed and studied by Melo *et al.* (2023) and the latter aspect (corrective and preventive phases) was modelled recently by Misaii *et al.* (2022). However, as we indicate above, we draw together both these strands into a flexible maintenance policy that is relevant and practical.

Discrete time policies themselves have received little recent attention, but classic works do consider them (e.g. Nakagawa & Osaki, 1977). In this regard, discretizing provides predictability and makes managing activities easier, and reduces the computational burden of optimization (Briš *et al.*, 2017). Further, periodic maintenance remains the dominant maintenance strategy in many industries e.g. marine

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(Lazakis *et al.*, 2018), nuclear (Khurmi *et al.*, 2021), aerospace (Meissner *et al.*, 2021), manufacturing (Mehmeti *et al.*, 2018).

Regarding the practical application of maintenance policy modelling generally, there is limited evidence of impact (Alsyouf, 2009). While there is evidence that industry seeks to implement novel maintenance strategies (Pinjala *et al.*, 2006), the effectiveness of innovation in maintenance upon productivity is difficult to measure and claims that the implementation of novel maintenance policies can reduce operational costs, improve workplace safety, and increase of production efficiency are not supported in the published literature (Da Costa & Cavalcante, 2022). The gap between theory and application is wide, and there is little evidence that it is closing, despite the existence of works that seek to close the gap (e.g. Scarf *et al.*, 2024c). Therefore, and notwithstanding the large growth in sophisticated maintenance policy solutions (Zonta *et al.*, 2020), which can be costly to implement, there remains a need for models that can deliver maintenance solutions that are practical and relevant. Thus, this paper attempts to bridge this gap with a simple, intuitive model of periodic visits to a system or fleet of systems by a maintainer with limited resources.

1.4. *Structure of the paper*

The structure of the paper is as follows. Next, we describe the system, the failure model, and the policy. Then in Sub-section II.A, we develop expressions for decision criteria in a renewal-reward framework. Sub-section II.B discusses policy variations and special cases. Section III describes a numerical study of the behaviour of the policy. We point there to the web-application that allows users to explore practical contexts. The paper ends with a summary of the policy and discussion of limitations of the study and future work.

2. The replacement policy

We consider a two-state (operating or failed), critical (failure immediately apparent), one-component system. On failure, the component is replaced (instantaneously) and the system is as-new (renewal). The time in the good state (age at failure) is a random variable X with density $f(x)$, distribution function $F(x)$, and reliability function $R(x)$.

The policy has two phases, a corrective replacement phase followed by an opportunistic replacement phase. Slots (potential opportunities for replacement) arise periodically every s time units. The policy, called the $\{W, M\}-$policy, has two integer-valued decision variables (control limits), W and M, $(0 < W \leq M)$. The first phase $[0, Ws)$ is the interval from when the system is aged 0 until immediately before the W-th slot. The second phase is the interval $[Ws, Ms]$. In the first phase, the system (component) is replaced (renewal) at a slot if and only if the system is failed and an opportunity arises at the slot.

In the second phase, the system is replaced (renewed) at a slot regardless of the state of the system (good or failed) if and only if an opportunity arises, with the exception of the final slot at Ms, at which it is replaced with probability 1 if the system attains that age. Note, the replacement at Ms may be corrective (on failure) or preventive. Thus, the policy is opportunistic over the interval $[0, Ms)$, non-opportunistic at Ms, and in the first phase a replacement (if it occurs) is corrective, and in the second phase replacement may be corrective or preventive. As replacements can occur only at opportunities at the periodic slots, we say the policy is “quasi = periodic”. The decision process for the policy is illustrated in Fig. 1. Note, although failures are immediately revealed, replacement is not immediate. Instead, the system waits in the failed state until a subsequent opportunity arises and an opportunity can only arise at a slot. Downtime (the period of time the system is failed) is costed (see below). Furthermore, the costs incurred at an

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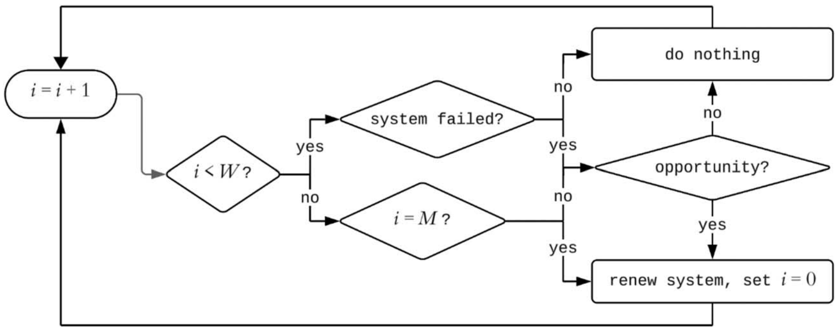

FIG. 1. The replacement decision process at a slot for the quasi-periodic opportunistic policy.

opportunistic replacement and at the age limit (final slot at Ms) vary because the system may have failed prior to the replacement. The logic in Fig. 1 does not show costs.

Now we describe the process of opportunities in detail. An opportunity arises at the j-th slot ($j = 1, ..., M - 1$) with probability q, regardless of the phase and the state of the system, and independently of the history of opportunities. Thus, opportunities arise as a pure Bernoulli process. Thus, if the system fails at age t such that $(i - 1)\,s < t < is$, that is, it fails between the $(i - 1)$-th and the i-th slots, then the system is renewed at the i-th slot (at age is) with probability q, or renewed at the next slot at age $(i + 1)\,s$ with probability $(1 - q)\,q$, or at age $(i + 2)\,s$ with probability $(1 - q)^2 q$, and so on, until age Ms when it is renewed regardless of whether an opportunity arises at the final slot at age Ms or not. Note, "at age js" is synonymous with "at slot j" and "at the j-th slot".

Thus, the policy models the reality in which the maintainer can only visit the system periodically, and even so, and despite the state of the system, maintenance may not be possible because resources are limited. This limitation on resources may arise because the maintainer manages many assets with finite capacity for spares and personnel. Furthermore, the maintainer is inclined to preventively replace older assets (in the second phase) and places an age-limit (Ms) on replacement. The existence of such an age-limit implies that the maintainer must prioritise maintenance for assets aged Ms.

The cost parameters are assumed fixed. The rate of cost of downtime is c_{D} (cost per unit of time); the cost of a corrective replacement at an opportunity is c_{F}. The cost of a preventive replacement at an opportunity is c_{P}. The cost of a replacement at age M incurs an additional (unit) cost c_{M}, to guarantee replacement. Thus, cost of a corrective replacement at age Ms is $c_{\mathrm{F}} + c_{\mathrm{M}}$ and the cost a preventive replacement at age Ms is $c_{\mathrm{P}} + c_{\mathrm{M}}$. Modelling costs in this way (with three different factors) provides flexibility. The policy can model realities in which: high availability is important or prevention of failure events themselves is important or both; scheduled interventions are prioritised above unscheduled interventions. In regard of the latter, this is a different way of looking at opportunistic interventions. Typically, these are merely assumed to cost-saving. However, in our model, it is uncertainty that characterises opportunistic interventions.

2.1. *Derivation of the cost-rate*

We seek the value of (W,M) that minimises the long-run cost per unit time (cost-rate). Exactly four renewal scenarios are possible. In each renewal scenario $k = 1, ..., 4$, we find: (i) the probability that the renewal scenario occurs, denoted by P_k; (ii) the contribution to the expected age of the system at renewal (contribution to the length of a renewal cycle), denoted by L_k; and (iii) the contribution to the

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expected cost of renewal (system replacement) and downtime over the renewal cycle (contribution to the cost of a renewal cycle), denoted by C_k. We then use these to calculate the cost-rate and operational reliability. Each renewal scenario has distinct events and unit costs associated with these events. Note, the contribution to the expected downtime over a renewal cycle, denoted by D_k, can be calculated by setting $c_{\mathrm{F}} = 0$ and $c_{\mathrm{D}} = 1$ in the expressions for C_k.

In the first renewal scenario, the system fails in an interval between slots and is correctively replaced at a subsequent slot, but not necessarily the immediately following slot (because an opportunity for replacement is required and an opportunity arises with probability $q < 1$). Thus, the age at failure, x, is such that $(i-1)\,s < x < is$, $(i = 1, ..., M-1)$, and the renewal cycle length is an integer multiple of s, js, $(j = 1, ..., M-1)$, the downtime is $js - x$, and the cost is $c_{\mathrm{F}} + c_{\mathrm{D}}\,(js - x)$. This scenario occurs with probability,

$$P_1 = \sum_{i=1}^{W-1}\sum_{j=i}^{M-1}(1-q)^{j-i}q\int_{(i-1)s}^{is} f(x)dx + \sum_{i=W}^{M-1}\sum_{j=i}^{M-1}(1-q)^{j-W}q\int_{(i-1)s}^{is} f(x)dx.$$

Note, there are two terms here because the two phases are different in respect of corrective opportunistic replacement. In the first phase, corresponding to the first term, failure arises in the i-th interval in the first phase, and then the replacement occurs at the j-th slot such that the first$j-i$ slots following failure are not opportunities (with probability $(1-q)^{j-i}$) and the j-th slot is necessarily an opportunity (with probability q). In the second phase, corresponding to the second term, failure arises in the final interval in the first phase or in any interval except the last in the second phase. For this to occur the slots in the second phase preceding failure could not have provided opportunities, otherwise the system would have been replaced preventively (because this is demanded by the policy—see Fig. 1). Thus, all slots in this second phase that precede the slot at which replacement occurs must be non-opportunities. This accounts for the coefficient $(1-q)^{j-W}$ in the second term. Note, calculated this way, the W-th slot is necessarily the first slot at which a preventive opportunistic replacement can occur, so the second phase starts precisely at Ws. In this scenario, the renewal cycle length contribution is

$$L_1 = \sum_{i=1}^{W-1}\sum_{j=i}^{M-1}(1-q)^{j-i}qjs\int_{(i-1)s}^{is} f(x)dx + \sum_{i=W}^{M-1}\sum_{j=i}^{M-1}(1-q)^{j-W}qjs\int_{(i-1)s}^{is} f(x)dx,$$

and the renewal cycle cost contribution is

$$C_1 = \sum_{i=1}^{W-1}\sum_{j=i}^{M-1}(1-q)^{j-i}q\int_{(i-1)s}^{is}\left\{c_{\mathrm{F}} + c_{\mathrm{D}}\,(js-x)\right\}\ f(x)dx + \sum_{i=W}^{M-1}\sum_{j=i}^{M-1}(1-q)^{j-W}q\int_{(i-1)s}^{is}\left\{c_{\mathrm{F}} + c_{\mathrm{D}}\,(js-x)\right\}\ f(x)dx.$$

In the second renewal scenario, preventive opportunistic replacement occurs at slot j in the second phase,$(j = W, ..., M-1)$. Necessarily, the system must survive to this j-th slot and the preceding slots in the second phase must not have been opportunities. Note, preventive opportunistic replacement cannot occur in the first phase, and in this scenario there is no downtime. Thus, the probability of this scenario is

$$P_2 = \sum_{j=W}^{M-1}(1-q)^{j-W}qR(js).$$

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Further, the renewal cycle length contribution is

$$L_2 = \sum_{j=W}^{M-1} (1-q)^{j-W} qjsR(js),$$

and the renewal cycle cost contribution is

$$C_2 = \sum_{j=W}^{M-1} (1-q)^{j-W} qc_{\mathrm{P}} R(js),$$

recalling that $R(js)$ is the probability that the system survives to the j-th slot at age js.

The third scenario is another failure scenario, but here replacement occurs at Ms. That is, if the failure arises in the first phase, then there are no subsequent opportunities, and if the failure arises in the second phase the slots in the second phase that both precede it and follow it must not be opportunities. Thus, the phases must be handled slightly differently, and we obtain two terms in the probability of this renewal scenario. Thus,

$$P_3 = \sum_{i=1}^{W-1} (1-q)^{M-i} \int_{(i-1)s}^{is} f(x)dx + \sum_{i=W}^{M} (1-q)^{M-W} \int_{(i-1)s}^{is} f(x)dx.$$

In this scenario, the renewal cycle length contribution is

$$\begin{aligned} L_3 = {} & \textstyle\sum_{i=1}^{W-1} (1-q)^{M-i} Ms \int_{(i-1)s}^{is} f(x)dx \\ & + \textstyle\sum_{i=W}^{M} (1-q)^{M-W} Ms \int_{(i-1)s}^{is} f(x)dx, \end{aligned}$$

And the renewal cycle cost contribution is

$$\begin{aligned} C_3 = {} & \textstyle\sum_{i=1}^{W-1} (1-q)^{M-i} \int_{(i-1)s}^{is} \left\{c_{\mathrm{F}} + c_{\mathrm{M}} + c_{\mathrm{D}} \left(Ms - x\right)\right\} f(x)dx \\ & + \textstyle\sum_{i=W}^{M} (1-q)^{M-W} \int_{(i-1)s}^{is} \left\{c_{\mathrm{F}} + c_{\mathrm{M}} + c_{\mathrm{D}} \left(Ms - x\right)\right\} f(x)dx. \end{aligned}$$

In the fourth and final scenario, the guaranteed preventive replacement occurs at Ms. Thus, the system must survive to Ms (with probability $R(Ms)$) and the slots in the second phase must not be opportunities. This scenario occurs with probability

$$P_4 = (1-q)^{M-W} R(Ms),$$

and the renewal cycle length contribution is

$$L_4 = (1-q)^{M-W} MsR(Ms),$$

and the renewal cycle cost contribution is

$$C_4 = (1-q)^{M-W} \left(c_{\mathrm{P}} + c_{\mathrm{M}}\right) R(Ms).$$

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Then, we can check that no other scenarios can arise ($\sum_{k=1}^{4} P_k = 1$). Next, we can calculate the value of our decision criterion, the cost-rate:

$$C_\infty(W, M) = \sum\nolimits_{k=1}^{4} C_k \Big/ \sum\nolimits_{k=1}^{4} L_k. \tag{1}$$

Also, the average unavailability is given by

$$U_\infty(W, M) = \left(D_1 + D_3\right) \Big/ \sum\nolimits_{k=1}^{4} L_k,$$

and the operational reliability is

$$\mu_\infty(W, M) = \sum\nolimits_{k=1}^{4} L_k \Big/ \left(P_1 + P_3\right),$$

by defining the operational reliability as the mean time between failures, and on the basis that the mean time between failures is the ratio of the expected cycle length to the probability that a renewal cycle ends in failure (Scarf *et al.*, 2005). The operational reliability is useful when setting a reliability constraint, so that the cost-rate can be minimised subject to meeting a specified level of "safety".

2.2. *Related policies*

If $W = M = \infty$, then the policy is failure-based replacement, albeit with replacements occurring only at the periodic slots. For this policy there is only one renewal scenario, scenario 1 above, whence the renewal cycle length is js, $(j = 1, 2, ...)$, the downtime is $js - x$, the cost is $c_\mathrm{F} + c_\mathrm{D}(js - x)$, and the cost-rate is given by

$$C_\infty^\mathrm{F} = \frac{\sum_{i=1}^{\infty} \sum_{j=i}^{\infty} (1-q)^{j-i} q \int_{(i-1)s}^{is} \left\{c_\mathrm{F} + c_\mathrm{D}(js - x)\right\} f(x)dx}{\sum_{i=1}^{\infty} \sum_{j=i}^{\infty} (1-q)^{j-i} q js \int_{(i-1)s}^{is} f(x)dx}.$$

Here, for evaluation, the limits on the summations must be finite, L say, but sufficiently large that $F(Ls) \approx F((L-1)s)$. The unavailability is given by

$$U_\infty^\mathrm{F} = \frac{\sum_{i=1}^{\infty} \sum_{j=i}^{\infty} (1-q)^{j-i} q \int_{(i-1)s}^{is} (js - x) f(x)dx}{\sum_{i=1}^{\infty} \sum_{j=i}^{\infty} (1-q)^{j-i} q js \int_{(i-1)s}^{is} f(x)dx},$$

and the operational reliability (mean time between operational failures) is given by

$$\mu_\infty^\mathrm{F} = \frac{1}{\sum_{i=1}^{\infty} \sum_{j=i}^{\infty} (1-q)^{j-i} q js \int_{(i-1)s}^{is} f(x)dx}.$$

Other policies are also special cases. When $M = \infty$, there are the two renewal scenarios 1 and 2 only. We call this the W-policy. The cost-rate for this policy is most simply evaluated by letting $M \to \infty$ in

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Equation 1. This policy is interesting despite its cost-inefficiency relative to the general policy because it is simpler.

When $W = M$, there is one phase of corrective replacement and then preventive replacement at Ms. We call this the quasi-periodic age-replacement policy. Practically, the policy operates in the following way: when the maintainer visits the site of the systems, scheduled preventive replacements are executed and unscheduled (corrective) replacements are executed if time and resource remains. The cost-rate, the unavailability, and the mean time between operational failures, can be calculated given the expected renewal cycle length, which is

$$L^{\mathrm{A}} = \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(1-q)^{j-i}qjs\int_{(i-1)s}^{is} f(x)dx$$
$$+ Ms\left\{\int_{(M-1)s}^{Ms} f(x)dx + R(Ms)\right\},$$

and the expected cost of a renewal cycle, which is

$$C^{\mathrm{A}} = \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(1-q)^{j-i}q\int_{(i-1)s}^{is}\left\{c_{\mathrm{F}} + (is-x)\,c_{\mathrm{D}}\right\}f(x)dx$$
$$+ \int_{(M-1)s}^{Ms}\left\{c_{\mathrm{F}} + c_{\mathrm{M}} + (Ms-x)\,c_{\mathrm{D}}\right\}f(x)dx + \left(c_{\mathrm{P}} + c_{\mathrm{M}}\right)R(Ms)\Big\}.$$

Further, when $q = 1$, the policy with $W = M$ is a modified (discretised) age-based replacement policy. That is, replacement would occur correctively at the first slot following failure or at the M-th slot, whichever is sooner. Replacement at the M-th slot could be corrective (if the system fails after the $(M-1)$ −th slot) or preventively (otherwise). Setting $c_{\mathrm{F}} >> c_{\mathrm{P}}$ and $c_{\mathrm{M}} = 0$ allows the evaluation of the cost-disbenefit (relative to the classic age-based replacement policy) of restricting replacements to the periodic slots is $(i = 1, 2, ...)$. Such an evaluation can be made for differing values of c_{D}. Thus, Scarf *et al*. (2024a) compares the cost-rate of discretised age-based replacement, which is

$$C_{\infty}^{\mathrm{P}} = \frac{\sum_{i=1}^{M}\int_{(i-1)s}^{is}\left\{c_{\mathrm{F}} + (is-x)\,c_{\mathrm{D}}\right\}f(x)dx + c_{\mathrm{P}}R(Ms)}{\sum_{i=1}^{M} is\int_{(i-1)s}^{is}f(x)dx + MsR(Ms)},$$

with the cost-rate of the classic policy, which is

$$C_{\infty}^{\mathrm{Classic}} = \frac{c_{\mathrm{F}}F(T) + c_{\mathrm{P}}R(T)}{\int_0^T R(x)dx}.$$

The $\{W, M\}$ −policy, and hence its special cases, do not need to distinguish a critical system from a protection system. That is, the model can be used for both. Thus, at a slot, test the system. If failed, replace at next opportunity. Once system reaches age Ws, replace at next opportunity regardless of its state. The difference is merely how the rate of cost of downtime, c_{D}, is evaluated.

The policy might also be generalised so that opportunities arise with different probabilities in the event of failure replacement and preventive replacement. Thus, $q_{\mathrm{F}} > q_{\mathrm{P}}$ in an obvious notation. In this way, a maintainer may put more resources towards corrective replacement than preventive replacement.

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TABLE 1. *Parameter values in the base case*

β	η	c_P	c_D	c_M	c_F	q	s
3	10	1	0.5	1	1	0.2	1

Then, setting $q_F = 1$ would allow a fair comparison of the policy with the classic opportunistic policy set in continuous time proposed by Dekker & Plasmeijer (2001), wherein opportunities arise according to a Poisson process with rate λ and the critical age limits for opportunistic and preventive replacements are t and T, respectively. Nonetheless, we briefly compare this continuous policy with our proposed policy (with fixed q) in the next section. This comparison goes some way towards assessing the marginal cost of discretisation in opportunistic age-based replacement, that is, of maintenance only at slots. This marginal cost of discretisation has been evaluated for age-based replacement by Scarf *et al.* (2024a).

Finally, we make a point about extension of the model to a multi-component system: discretisation of instances for maintenance simplifies the grouping problem (Do *et al.*, 2015; Vu *et al.*, 2015).

3. Numerical study

This section describes the numerical study of the behaviour of the policy. Parameter values for the base case are shown in Table 1. A range of parameter values are studied; these are the cases in Table 2; the base case is case 3. These values are used for two main reasons: firstly to demonstrate interesting policy behaviour, and secondly because these values are somewhat motivated by the context of a non-repairable component in a wind turbine in a windfarm (Kang & Soares, 2020). The unit of time is arbitrary, and in the base case we set s, the interval between slots, to unity, noting that it is the ratio of η to s that determines how fast time "passes". The unit of cost is also arbitrarily chosen so that c_P, the cost of a preventive replacement at an opportunity, is unity. A Weibull distribution is used to describe the lifetime of a component: $R(x) = \exp\left\{-(x/\eta)^{\beta}\right\}$.

In the base case, we set $c_P = c_F$. In this way, downtime (unavailability) cost $\left(c_D\right)$ accounts for the financial impact of failure. Nonetheless, there are other risks associated with a failure beyond downtime. These may include reputational damage, safety concerns, and unforeseen consequences of failure. It may be important to consider these additional risks, whence $c_P < c_F$ (cases 9–10, 19–25). We set q ($q = 0.2$) as quite small, with the interpretation that resources are scare and consequently opportunities are rare. This is the likely reality when owners seek to reduce maintenance costs. Characteristic lifetime (of a component) is moderately long (relative to s) ($\eta = 10$) and moderately variable ($\beta = 3$). The cases in Table 2 then explore alternatives, broadly halving or doubling values in the base case.

Optimal policy is found using a crude search over $(M, W) : W < M \in \{1, ..., 50\}$. The search is coded in Python code including the libraries SciPy and NumPy. The optimisation step takes 10 minutes on a standard PC. For $M > W \geq 50$ increments in the cost-rate are $< 10^{-4}$. Note, search over this large range is necessary despite the very unlikely survival of the system to old age (e.g.$R(33) < 10^{-15}$). This is because renewal can be delayed for quite some time after failure when q is small. For example, when $q = 0.2$, the probability that a failure persists for at least 20 slots is > 0.01 ($0.8^{20} = 0.012$).

A web-application (see the data statement at the end of the paper) makes it possible for non-expert users to calculate decision variables for parameter values motivated by their own contexts. This has two purposes: to bridge the gap between model development of policy and the use of policy in practice; and to

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TABLE 2. *Study of the cost-minimum policy*

	parameters						$\{W, M\}$ −policy					$\{\infty, \infty\}$ −policy		
case	β	c_D	c_M	c_F	q	s	W^*	M^*	C^*_∞	U^*_∞	μ^*_∞	C^F_∞	U^F_∞	μ^F_∞
1	1	0.5	1	1	0.2	1	≥49	≥50	0.225	0.310	14.5	0.224	0.311	14.5
2	2	0.5	1	1	0.2	1	8	20	0.237	0.275	15.2	0.243	0.337	13.4
3*	3	0.5	1	1	0.2	1	6	14	0.223	0.193	17.3	0.242	0.335	13.4
4	3	0.0	1	1	0.2	1	≥49	≥50	0.074	0.335	13.4	0.074	0.335	13.4
5	3	0.25	1	1	0.2	1	10	≥50	0.157	0.305	14.7	0.158	0.335	13.4
6	3	1	1	1	0.2	1	5	9	0.292	0.099	21.2	0.410	0.335	13.4
7	3	0.5	0.5	1	0.2	1	7	11	0.208	0.154	16.3	0.242	0.335	13.4
8	3	0.5	2	1	0.2	1	6	≥50	0.225	0.245	18.3	0.242	0.335	13.4
9	3	0.5	1	2	0.2	1	5	≥50	0.277	0.227	19.7	0.316	0.335	13.4
10	3	0.5	1	4	0.2	1	3	≥50	0.371	0.195	23.0	0.465	0.335	13.4
11	3	0.5	1	1	0.1	1	5	11	0.259	0.184	16.8	0.312	0.515	18.4
12	3	0.5	1	1	0.4	1	9	≥50	0.176	0.139	14.3	0.183	0.183	10.9
13	3	0.5	0.5	1	0.4	1	9	≥50	0.176	0.139	14.3	0.183	0.183	10.9
14	3	0.5	2	1	0.4	1	9	≥50	0.176	0.139	14.3	0.183	0.183	10.9
15	3	0.5	1	2	0.4	1	6	≥50	0.232	0.094	21.0	0.274	0.183	10.9
16	3	0.5	1	4	0.4	1	4	≥50	0.313	0.064	30.7	0.457	0.183	10.9
17	3	0.5	1	1	0.2	0.5	16	≥50	0.182	0.146	15.4	0.190	0.201	11.2
18	3	0.5	1	1	0.2	2	3	6	0.260	0.214	16.2	0.307	0.502	17.9
19	2	0.25	1	2	0.2	1	9	≥50	0.229	0.298	15.1	0.234	0.337	13.4
20	2	0.25	1	4	0.2	1	4	≥50	0.318	0.210	21.4	0.382	0.335	13.4
21	3	0.25	1	2	0.4	1	7	≥50	0.207	0.110	18.0	0.229	0.183	10.9
22	3	0.5	1	1	1.0	1	15	16	0.132	0.051	9.70	0.133	0.053	9.4
23	3	0.5	1	2	1.0	1	8	≥9	0.205	0.026	18.2	0.239	0.053	9.4
24	2	0.12	1	1.5	0.2	1	21	≥50	0.153	0.336	13.4	0.153	0.337	13.4
25	2	0.12	1	2	0.2	1	12	≥50	0.189	0.318	14.1	0.190	0.337	13.4
26	2	0.12	1	2	0.4	1	11	≥50	0.203	0.160	12.9	0.206	0.184	10.9

Cost-rate, average unavailability, and mean time between operational failure for various cases for the $\{W, M\}$ −policy and failure-based replacement ($W = M = \infty$). In all cases: $\eta = 10$, $c_P = 1$ (unit of cost). *base case.

address the limitation of a numerical study that can only investigate some parameter values and describe some behaviours in a specific rather than a general way.

3.1. *Interpretation of the results*

We interpret the results in Table 2 case by case. Generally, here, we can get a sense of the existence of a cost-minimising $\{W, M\}$ −policy or otherwise (finite W^* or M^*), the advantages (or otherwise) of the optimum policy relative to failure-based replacement (in terms of the differences in the cost-rates, average unavailabilities and MTBOFs).

Throughout, the average unavailabilities are quite high (of the order of 20%). This is because s is quite large relative to η. It reduces in case 17, ($s = 0.5$), and cases 12–16,21,26 ($q = 0.4$), being cases with greater resource for maintenance; notice here the $\{W, M\}$ −policy is not preferred to the W−policy ($M^* \geq 50$). This is because more frequent slots and/or a larger opportunity probability imply more

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opportunities so that scheduling a final replacement at Ms is unnecessary. This effect persists for different values of c_{M}(cases 13,14), noting that $c_{\mathrm{M}}$is a redundant parameter when M is large.

A similar effect (related to the existence of M^*) is seen when c_{F} is larger (cases 9,10,19-21,24–26). As c_{F} increases W^* decreases, negating the need for the scheduled replacement. We might conclude broadly that the $W-$policy is preferable to the $\{W, M\}-$policy for practice, because it is simpler and almost always as good as the $\{W, M\}-$policy.

When resources are not scarce ($q = 1$), cases 22,23, the policy is indifferent to M. In this case, provided W^* exists, replacement is guaranteed at W^*s. Now the average unavailability is small because the system never waits long in the failed state. Furthermore, if an opportunity is guaranteed at every slot ($q = 1$), then it does not make sense that the cost of replacement at early slots is less than replacement at the M-th slot. This point is relevant to the justification of the costs, so that an interesting cost formulation may set $c_{\mathrm{M}} = 0$ and $c_{\mathrm{F}}, c_{\mathrm{P}} \sim g(q)$, that is, replacement costs depend on the opportunity probability, so that a maintainer who has more resources (larger q) spends more on maintenance.

Notice also in case 22, the cost-rate is low while the MTBOF is also low. Again, the system never waits too long in the failed state and the maintainer is not concerned about failure beyond the resulting downtime (because $c_{\mathrm{P}} = c_{\mathrm{F}}$). Therefore, the MTBOF is not a very good metric for measuring policy performance, and it is better to focus on the cost-rate and the average unavailability. Other contexts, which the reader can explore in the web-application may be different.

Some other effects are also apparent. If the cost of failure is large, the second phase starts sooner (small W^*) (cases 9,10). The same effect is seen as the rate of cost of downtime varies (cases 4–6). Further, when c_{D} is small the $W-$policy is preferred to the $\{W, M\}-$policy. Indeed, when $c_{\mathrm{D}} = 0$ (case 4), it appears that neither W^* nor M^* to exist, so that the preferred policy is failure-based replacement and the only renewal scenario is corrective replacement at an opportunity. This is what we would expect when there are no consequences of failure. Of course, in practice the case $c_{\mathrm{D}} = 0$ and $c_{\mathrm{P}} = c_{\mathrm{F}}$ is absurd because there is no requirement for the system, but the policy insists on replacement of the system when it is failed. So, we can regard this case as providing confirmatory evidence of expected behaviour. The same point applies for case 1, when lifetime is exponential and there is no benefit to preventive replacement.

Robustness of the cost-minimum policy is studied in Fig. 2. Broadly, this shows that the cost-rate is less sensitive to W than to M when q is small, and the effects are opposite when q is larger. This is as we would expect because when opportunities are rare, knowing exactly when the window of opportunity should open is less important than when the final scheduled replacement should take place, and vice versa. We can also see that the $\{W, M\}-$policy is very much preferred to failure-based replacement when the rate of cost of downtime is high and the difference between the policies is greatest when opportunities are rare (small q).

Policy metrics are studied in detail in Fig. 3. Here, for the cost-minimum $\{W, M\}-$policy, we show the length of the first phase (W^*s), the relative lengths of the first (corrective) and second (preventive) phases (W^*/M^*), and the cost-rate and the average unavailability, for various values of the time between visits (s), the rate of cost of downtime $\left(c_{\mathrm{D}}\right)$, and the opportunity probability (q). When the opportunities are more frequent, the window of opportunity opens later, and the cost-rate and unavailability are both lower, as expected. When c_{D} is lower, the effects are likewise, except for the unavailability which is higher, obviously, because unavailability is less important. These effects are the same for different values of s. The overall picture in respect of (W^*/M^*) is more complicated. However, broadly, when q is small, the window of opportunity opens relatively sooner (small W^*) than when q is larger. The interaction between c_{D} and q is perhaps because the value of M^* is highly variable.

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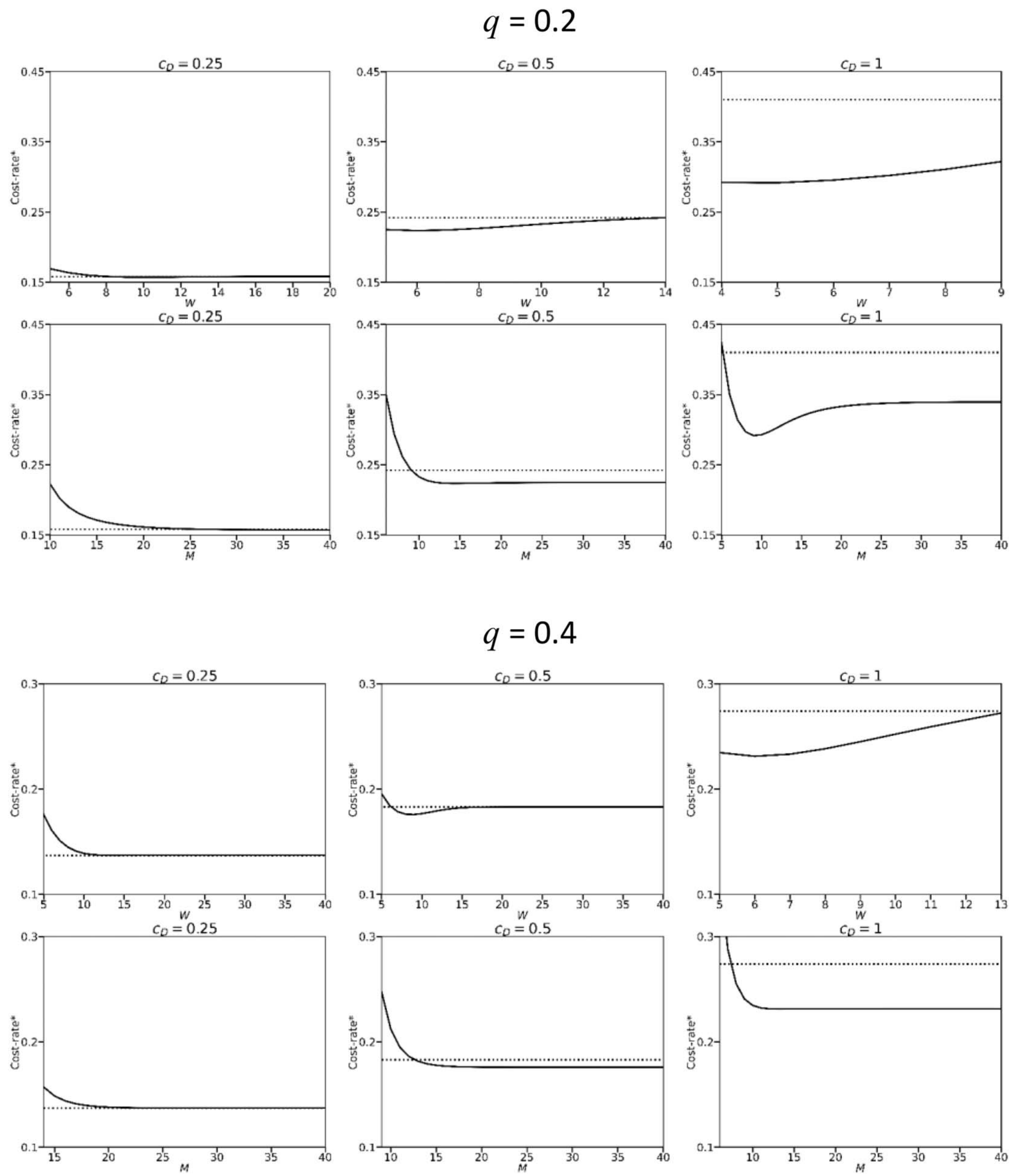

FIG. 2. For the $\{W, M\}$ −policy, cost-rate versus W (with $M = M^{*}$) (top row) and cost-rate versus M (with $W = W^{*}$) (bottom row) for various values of q (opportunity probability) and c_{D} (downtime cost-rate).

Figure 4 confirms the insensitivity of the policy to values of c_{M} when q is large, as discussed above. When q is small, varying c_{M} has the expected effect. When c_{M} is largest, the window of opportunity for preventive replacement opens earliest.

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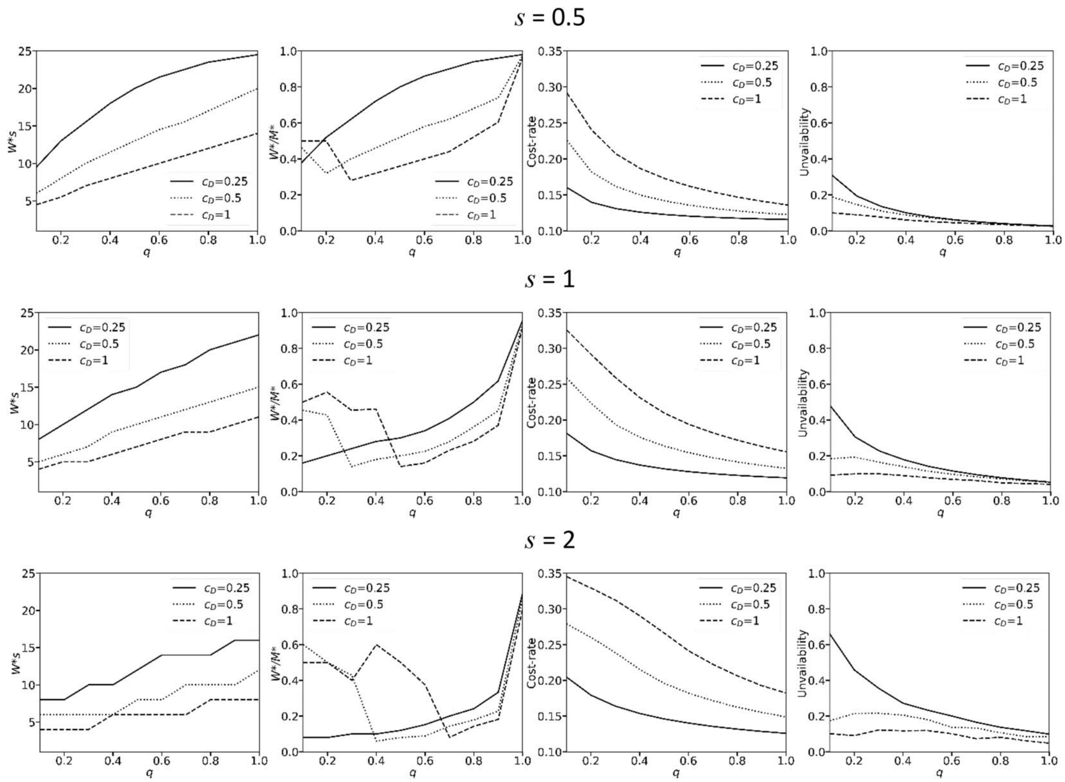

FIG. 3. For the $\{W, M\}$ −policy, behaviour of optimal policy as a function of q (opportunity probability) for various values of c_{D} and s. Other parameter values as in the base case.

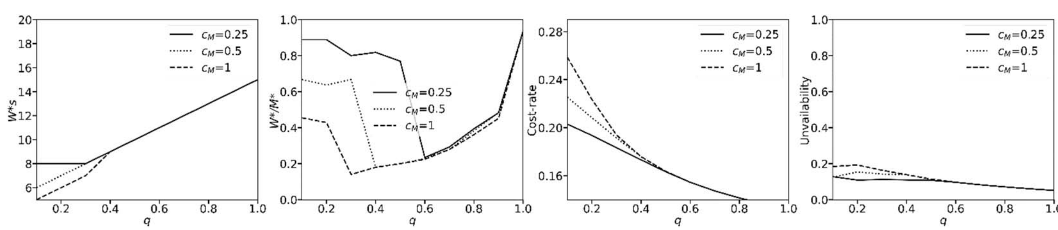

FIG. 4. For the $\{W, M\}$ −policy, behaviour of optimal policy as a function of q (opportunity probability) for $s = 1$ and various values of c_{M}. Other parameter values as in the base case.

3.2. *Comparison of policies*

In Fig. 5 we compare the performance of the $\{W, M\}$ −policy with policies that are special cases. The extent to which the $\{W, M\}$ −policy is preferred depends on c_{D}. The threshold at which preventive maintenance is preferred to corrective maintenance is quite high, c_{D} $\sim$0.25, whence the cost of a replacement is equivalent to 4 time-periods (e.g. years) of downtime.

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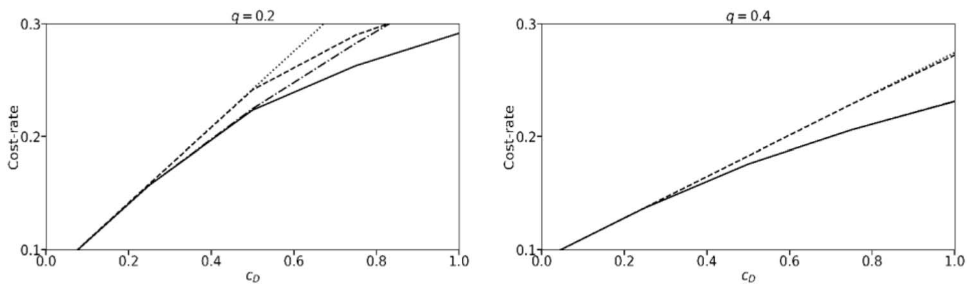

FIG 5. Cost-rate versus c_{D} (rate of cost of downtime) for various values of q (opportunity probability) for the $\{W, M\}$ −policy (——) (quasi-periodic opportunistic replacement) and policies that are special cases: $M = \infty$ (-·-) ($W-$policy); $W = M$(---) (quasi-periodic age-based replacement); $W = M = \infty$ (. . . .) (failure-based replacement). Other parameter values as base case.

For moderately large q, the $W-$policy performs as well as the $\{W, M\}$ −policy, so that preventive replacement is carried out only if resources permit, noting that finite M^* implies otherwise (guaranteed replacement at M^*s). This makes practical sense. For example, for a fleet of turbines in an offshore windfarm, with periodic visits to the farm and limited resources, lost production may not be large enough to justify the additional resource that guaranteed availability would require. Thus, the operator may be prepared to tolerate some turbines that are unavailable for relatively long periods. As the rate of cost of downtime increases, or opportunities are rarer, or both, the less the operator will be prepared to wait for an opportunity for replacement. In a finite horizon context, where a fleet has a limited operational life, then the $W-$policy would become even more attractive because in the $\{W, M\}$ −policy M^* would likely be very sensitive to the horizon length.

3.3. *Comparison with a continuous policy*

Dekker & Plasmeijer (2001) study the classic (continuous) opportunistic age-replacement policy in which an item (a critical one-component system) is replaced on failure, at an opportunity if that arises when the item is aged at least t, and preventively at age T, whichever occurs soonest. Opportunities arise according to a Poisson process with rate λ (the expected number of opportunities per unit of time). Figure 6 shows some results when this policy is compared with our proposed quasi-periodic policy. In the comparison, the slot interval s and the downtime cost-rate c_{D} vary. In so doing, for a fair comparison, it is necessary to set $\lambda = q/s$ since q/s is the expected number of opportunities per unit of time for the quasi-periodic policy.

Firstly, we observe that as s decreases the cost-rates converge. This is because when s is small the quasi-periodic policy is near-continuous, and we would expect the cost-rate for the continuous policy and the quasi-periodic policy to be similar. Also, there is negligible downtime in the quasi-periodic policy, so that variation in the downtime cost-rate has little effect.

Secondly, when the downtime cost-rate is reasonably large, the cost-inefficiency of the quasi-periodic policy is apparent. This is of the order of 10% for moderate s. This is the marginal cost of discretisation. Note, the quasi-periodic policy does not replace on failure immediately, so comparison with the continuous policy must be treated with caution. This is apparent particularly when $c_{\mathrm{D}} = 0$, wherein less frequent maintenance (larger s) has lower cost-rate because downtime, and hence a period in the failed state prior to renewal is not penalised.

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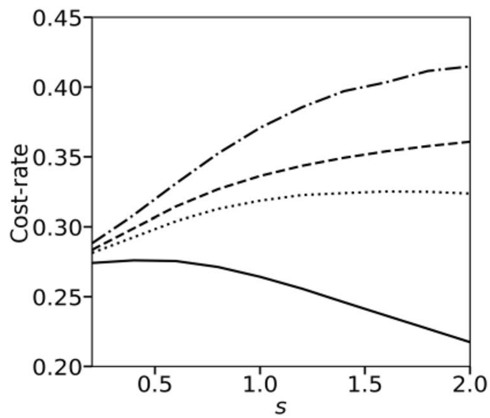

FIG. 6. Cost-rates for the continuous policy (---) and quasi-periodic policy (– – – – $c_D = 0$; ■■■■■■■■ $c_D = 0.25$; ■ — ■ — $c_D = 0.5$) as a function of s, with $q = 0.2$ fixed and λ varying. Other parameters: $\eta = 10$, $\beta = 3$, $c_P = 1$ (unit of cost), $c_M = 1$ and $c_F = 4$.

4. Discussion

This paper develops a model for a novel replacement policy motivated by practical necessities of simplicity and applicability. The policy has periodic structure. Times (slots) at which replacement can be executed, if resources permit, occur periodically with frequency $1/s$. A proportion q of slots provides an opportunity for corrective or preventive replacement. The model mimics the reality in which resources are limited or execution of maintenance is not guaranteed, due to, for example, bad weather, lack of spares or personnel, etc. An offshore windfarm or remote telecommunications installations or power micro-grids, and the non-repairable components therein, provide motivating contexts.

The system is modelled as a one-component, critical system. The policy has two phases: the first in the early life of the system is corrective; the second in the later life is preventive. Decision variables W and M define the durations of these phases. Failures are corrected (by component replacement) at the first opportunity following failure. Any downtime incurred is measured. In the second phase, preventive replacement occurs at an available opportunity regardless of the system state. This is the $\{W, M\}$ −policy. The cost-rate and average availability are calculated. Contributions to the cost arise from downtime and replacements.

We study the behaviour of optimal policy numerically for a range of values of model parameters. In particular, interest lies in broad decision support regarding the efficacy of preventive replacement and how the availability of opportunities (and hence resources for maintenance) impact upon policy performance. We compare the $\{W, M\}$ −policy to policies that are special cases, indicating circumstances in which preventive replacement out-performs corrective replacement. Modified (discretised) age-based replacement is a special case of the $\{W, M\}$ −policy.

Generally, we find that if slots are infrequent and/or opportunities are rare, the operator must tolerate quite high average unavailability. Conversely, more maintenance resources are required to achieve lower unavailability, and the model can quantify this resource-availability trade-off. Furthermore, the rarer are opportunities the more beneficial it is to impose some certainty on replacement through a guaranteed replacement and the sooner, in the life of a system, the maintainer would favour taking opportunities for preventive replacement. Conversely, when opportunities are not rare, guaranteed replacement is not

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cost-beneficial. Specific analysis and policy comparisons can be carried out using a web-application developed by the authors.

For practice, the $W-$policy may be preferred to the $\{W, M\}-$policy because it performs nearly as well for a range of parameter values and it is simpler to operationalise. Indeed, all the policies we describe are operationally simple, because the schedule of slots is fixed. Such a fixed schedule is justified when regulation prescribes it or when limited resources imply it. The simplicity of the policy means the models can facilitate the transfer of knowledge to managerial practice in the manner discussed by Tayur (2024). Furthermore, the model can inform broad decisions about whether preventive replacement is effective and what is an appropriate level of resourcing for replacement, for the former by comparing the $\{W, M\}-$policy and $W-$policy with failure-based replacement, and for the latter by quantifying how the cost-rate and average availability vary with q, the resource-dependent opportunity probability.

There is some scope to measure the marginal cost of discretisation, by comparing the cost-rate of the $\{W, M\}-$policy with that of opportunistic age-replacement, the $\{t, T\}-$policy, which is continuous. However, the marginal cost of discretisation varies with the downtime cost-rate, noting that there is no downtime in the continuous policy. So, the comparison must be treated with some caution. Nonetheless, we demonstrate that the $\{W, M\}-$policy and the $\{t, T\}-$policy have similar cost-rates when the slot interval s is small.

It is clear that q is an important aspect of the proposed quasi-periodic policy. In reality, this parameter may be difficult to estimate. Further, a maintainer may prioritise corrective replacement over preventive replacement, putting more resources towards the former. This generalisation can be accommodated by specifying q_{F} and q_{P}. However, W influences the ratio of corrective to preventive replacement which in turn would, in practice, influence q_{F} and q_{P}. This problem could be simulated by specifying a fleet of systems, a fixed resource, and rules that prioritise corrective replacement over preventive replacement. This would be a different but worthwhile study, as would a study of the policy with $q_{\mathrm{F}} = 1$. Also, in a non-stationary framework (finite horizon), q may decrease as demand for maintenance grows, and a dynamic version the policy could be interesting. Nonetheless, we can justify the current study because it provides both broad decision support and guidance about how fuller, more specific studies might be developed.

Finally, we note that fixing the period of slots has further advantages for modelling a multi-component extension. Indeed, it would be interesting to study a case in which different critical components have different replacement costs. Then, with limited resources, one would obviously prioritise less costly replacements. The interaction with spare-parts provisioning (Scarf *et al.*, 2024c) and flexible repair policies (Cha *et al.*, 2023) would also be interesting. The model of the policy can also be used in the context of a protection system; Scarf *et al.* (2024b) consider this type of system but without opportunities.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Data statement

The data presented in this paper were calculated using code written by the authors. A demonstrator of this code is here: https://github.com/randomprototype/QPOP_SOFTWARE With this code, the reader can verify the results and explore new cases.

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