A User-Centric Model of Connectivity in Street Networks

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Abstract

Modelling street network connectivity is a fundamental research problem in transportation science. Here, we argue that the connectivity of a street network cannot be defined independently from the users of that street network. That is, different users will experience different levels of connectivity depending on their corresponding travel behaviour. In this work, we propose a model of connectivity that is defined with respect to a user's travel behaviour within a given street network. We demonstrate that many real-world problems can be posed as instances of an optimisation problem defined with respect to this model. This includes optimising a user's home location with respect to their connectivity and optimising the location of a facility with respect to the connectivity of its users. We prove the above optimisation problem is NP-hard and present an integer programming solution that scales to large problem instances.

Keywords: street network, connectivity, optimisation

1. Introduction

The purpose of a street network is to facilitate the transportation of people and goods between spatial locations. Consequently, the quality of a given street network can be defined as its usefulness with respect to performing this activity. Given the potential time and monetary savings, planning practitioners commonly attempt to model and optimise the quality of street networks

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[1, 2]. Formally defining a measure of street network quality is challenging; however, most agree that an important feature of a high-quality street network is good connectivity [3]. Street network connectivity is a concept that is difficult to define formally. Koohsari [4] defines street connectivity as the directness and availability of alternative routes between home and local destinations. Tal et al. [5] state that street network connectivity is a function of proximity to destinations and the directness of routes to those destinations.

Many models of street network connectivity have previously been proposed [6, 7]. Usually, these models are based on computing summary statistics relating to geometrical and/or topological features. For example, a frequently used model is the average number of street intersections per unit of area, where this statistic is considered to be positively correlated with connectivity [8]. However, the connectivity of a street network cannot be defined independently from the users of that street network and their corresponding travel behaviour. That is, the connectivity experienced by a user is a function of many parameters specific to that user such as where they live, where they travel, what times of the day they travel and what modes of transportation they use. For example, two users living at the same address may experience contrasting levels of connectivity if their travel behaviour is different (in that they frequently visit different destinations). This insight suggests that existing models of street network connectivity, which are defined independently of users' travel behaviour, fail to capture an important aspect of connectivity.

To overcome this limitation, we propose a model of street network connectivity that is defined with respect to a user's travel behaviour within a given street network. Specifically, the travel behaviour of a user is modelled as a set of trips between origin-destination pairs, plus the relative frequencies of these trips. This model of travel behaviour is, in turn, used to model the street network connectivity with respect to the user in question. For example, consider the case where a given user's travel behaviour contains a single trip between their home and place of work. In this case, the connectivity in question could be improved by selecting a home location better connected to the place of work. In this work, the connectivity of a given user travel behaviour is modelled as the mean street network distance of the corresponding trips. This approach makes the simplifying assumption that connectivity is solely measured in terms of street network distance. However, the model can be generalised to consider other dimensions of connectivity such as access to public transportation.

In this work, we will demonstrate that many real-world problems can be posed as instances of an optimisation problem defined with respect to the proposed model. Such problems include optimising a user's home location with respect to their connectivity and optimising the location of a facility with respect to the connectivity of its users. We subsequently prove the above optimisation problem to be NP-hard and present an integer programming approach for solving it. We also demonstrate that existing models of street network connectivity based on the concept of route directness can be interpreted in terms of our proposed model.

The structure of this paper is as follows. In Section 2 we review related works on modelling street network connectivity and optimising a model of travel behaviour. In Section 3 we then present the proposed model of street network connectivity. In Section 4 we describe how this model can be used to optimise a user's travel behaviour with respect to connectivity. In doing so we prove this problem to be NP-hard and suggest an appropriate integer programming solution method. In Section 5 we present an evaluation of this method on a large set of problem instances. In Section 6 we then describe several applications of the proposed model to real-world problems. Finally, Section 7 concludes the paper and describes directions for future research.

2. Related Works

As described in the introduction, existing models of street network connectivity are defined with respect to a given street. This contrasts with our proposed model, which is defined with respect to both a given street network plus a given user. This section presents a review of related works on modelling street network connectivity and optimising a model of travel behaviour.

Street networks are commonly modelled as graphs. Therefore models of graph connectivity can be directly applied [9]. For example, vertex- and edge-connectivity are models of graph connectivity defined as the number of vertices and edges (respectively) that must be removed to disconnect the graph. However, these models do not consider the effects of spatial properties. As a result, other models have been proposed. Dill [3], for example, suggests several models of connectivity including:

- *Maximum city block length*, where a city block is a group of buildings surrounded by streets;
- *Maximum city block area*;
- *City block density*, defined as the number of city blocks per unit of area;

- *Intersection density*, defined as the number of intersections per unit of area;
- *Connected node ratio*, defined as the ratio of the number of intersections to the number of intersections plus cul-de-sacs;
- *Link node ratio*, defined as the ratio of the number of street segments to the number of intersections plus cul-de-sacs;
- *Grid pattern*, which tells us whether or not the street network has a grid pattern;
- *Route directness*, defined as the average network distance between locations divided by the corresponding straight-line distance; and
- *Effective walking area*, defined as the average number of parcels within a given distance.

In other work, Stangl [10] proposed a model of connectivity based on street intersection density. This model is similar to the aforementioned intersection density but also uses additional heuristics to count intersections (specifically, intersections that connect or are within isolated areas are ignored). In a later work, Stangl [6] identified some drawbacks to this model and proposed a new model similar to the route directness measure. Stangl [11] then further developed this model by combining several additional heuristics to improve robustness.

Tal et al. [5] proposed a model of connectivity based on the concept of a "pedshed", which is defined as the area within a given distance of a given point. Peponis et al. [12], meanwhile, proposed the "metric reach" and "directional distance" models of connectivity. Metric reach is defined as the length of the network within a specified network distance of a given point. Directional distance is defined as the average number of directional changes made when navigating between locations. In other work, Ellis et al. [13] presents an evaluation of several connectivity models. This includes a model called "directional reach", which is similar to the metric reach measure proposed by Peponis et al. [12], albeit with an additional constraint relating to the number of possible changes in direction.

In more recent work, Knight et al. [7] have considered three models of connectivity, called "connectivity index", "intersection density" and "street density". The connectivity index is defined as the ratio of the number of streets to the number of intersections. Intersection density and street density are equivalent to the models proposed by Dill [3]. It is determined that all three models are strongly correlated with many features unrelated to connectivity. They therefore conclude that these models are not robust. Stangl [14] has also considered several connectivity models related to the city block models of Dill [3], highlighting some drawbacks of these models and, in turn, proposing a new "block" model. This is defined as the average maximum Euclidean distance between locations on the boundary of an area surrounded by streets.

Several authors have also proposed models of connectivity based on the concept of "directness", which is defined as the ratio of the straight-line distance to the street network distance between pairs of locations [15, 1]. A limitation of these approaches is that the pairs of locations in question are usually defined in an ad hoc manner and do not accurately reflect the travel behaviour of users [3]. For example, Hess [15] has modelled the connectivity of a given region as the mean directness between random points outside the region and the centre of the region. Similarly, Randall et al. [1] has modelled the connectivity of a given region as the mean directness between each residential building in the region and a specific location, such as a school.

Other works have also defined models of connectivity with respect to specific services such as schools [16] and fire stations [17]. A closely related concept to this is "centrality", where the centrality of a vertex measures its importance with respect to other vertices in the graph. Some standard measures of centrality include degree centrality, betweenness centrality, and closeness centrality. Curado et al. presented an analysis of different centrality measures applied to street networks [18].

There exist several works that have considered the problem of optimising a model of travel behaviour. The shortest path problem is an optimisation problem where the objective is to minimise the distance travelled by an agent subject to the travel behaviour constraint that the agent begins and ends their journey at specified locations. The Travelling Salesman Problem (TSP) is a classic optimisation problem where the objective is to minimise the distance travelled by an agent subject to the travel behaviour constraint that the agent visits a set of locations in any order before returning to the first location [19]. The Vehicle Routing Problem (VRP) is a generalisation of the TSP [20]. The VRP is an optimisation problem where the objective is to minimise the distance travelled by a set of agents subject to the travel behaviour constraint that the agents visit a set of locations in any order. There exist several variants of the VRP. The VRP with Time Windows adds that additional travel behaviour constraint that some locations can only be visited within corresponding time windows. The VRP is commonly used to model the travel behaviour of delivery vehicles. One instance of this is the Capacitated VRP which adds that additional travel behaviour constraint that each agent has a maximum carrying capacity. A related optimisation is the Pick-up and Delivery Problem (PDP) where the objective is to minimise the distance travelled by an agent subject to the travel behaviour constraint that the agent visits a given pickup location before visiting a corresponding delivery location [21]. A problem related to both the TSP and VRP is the Orienteering Problem [22]. In this optimisation problem, the objective is to maximise the number of locations visited subject to the travel behaviour constraint that the agent can travel at most a specified distance. The facility location problem is an optimisation problem where the objective is to decide the location of a given facility to minimise the distance travelled by agents that have a travel behaviour that involves visiting the facility [23].

3. Model of Connectivity

In this section, we describe our proposed model of street network connectivity. This model is defined with respect to a model of user travel behaviour. In Section 3.1 we describe this model of user travel behaviour and in Section 3.2 we define an equivalence relation for travel behaviours. In this equivalence relation, two travel behaviours are considered equivalent if they provide the same utility to the user in question. Finally, in Section 3.3 we describe the proposed model of connectivity. Several transportation-related problems can be reduced to optimising instances of this model. This optimisation assumes that the street network is fixed but that the user in question can optimise their travel behaviour with respect to this street network to improve their connectivity while maintaining the same utility.

3.1. Travel Behaviour Model

The proposed model of travel behaviour is defined with respect to a street network, modelled as an arc-weighted directed multi-graph G^s. We define a given street network $G^s = (V^s, A^s)$, where the set of vertices V^s model street intersections and dead ends, and the set of arcs A^s model individual street segments. The weight assigned to each arc is the length of the corresponding road segment [24]. Path lengths between origin and destination vertices are defined with respect to these weights. For this work we assume that there exists a path between all pairs of vertices in G^s; that is, the graph is strongly connected. For $v, v' \in V^s$, we denote the length of the shortest path in G^s

Figure 1: The street network of central Cardiff, Wales. Vertices and arcs are represented by black dots and grey lines respectively.

from v to v' as $w_{v,v'}$. For example, Figure 1 displays the Cardiff city street network modelled as a graph G^s.

Here, we model the relative frequency of trips made by a user between two vertices as an element of the set:

$$T = \{(p, v, v') : p \in [0, 1], v \in V^s, v' \in V^s\} \tag{1}$$

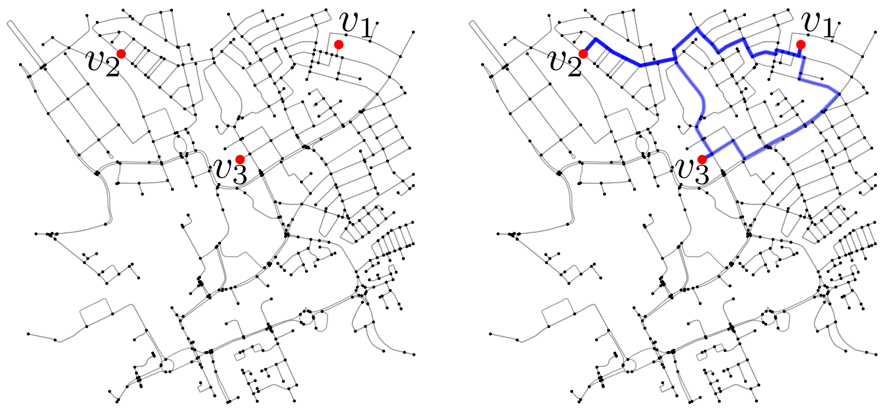
Specifically, $(p, v, v') \in T$ models the fact that the relative frequency of trips made by the user in question from origin vertex v to destination vertex v' is p. For example, $(0.5, v, v') \in T$ models the fact that half of the trips made by the user in question are from vertex v to vertex v'.

Let 2^T denote the power set of T (the set of all subsets of T). The *travel behaviour* of an individual user is modelled as an element of the set:

$$B = \{b : b \in 2^T, \sum_{(p,v,v') \in b} p = 1.0\} \tag{2}$$

Specifically, $\{(p_1, v_1, v_1'), \ldots, (p_n, v_n, v_n')\} \in B$ models the fact that the travel behaviour of the user in question is modelled by n distinct trips in which the sum of the relative frequencies is 1.

To illustrate this model of travel behaviour, consider the street network displayed in Figure 2(a) and a user who travels between their home, place



(a) (b)

Figure 2: In (a), a user travels between their home, their place of work, and a supermarket. These are shown by the red vertices v_1, v_2 and v_3 respectively. The blue lines in (b) show the shortest paths between the vertex pairs (v_1, v_2), (v_2, v_1), (v_2, v_3) and (v_3, v_1).

of work and a supermarket. These three locations are represented by the red vertices v_1, v_2 and v_3 respectively in Figure 2(a). Now let the travel behaviour of this user be modelled by $\{(0.4, v_1, v_2), (0.2, v_2, v_1), (0.2, v_2, v_3), (0.2, v_3, v_1)\}$. This models the fact that the user makes many of their trips from their home to their place of work and back while also making some trips from their place of work to the supermarket and from the supermarket to their home.

3.2. Equivalence Classes of Travel Behaviour

In this section, we define a set of equivalence classes on the set of travel behaviours B whereby travel behaviours having equal utility belong to the same equivalence class. Two travel behaviours have equal utility if and only if they contain corresponding trips between locations of equal type. In this context, the set of different location types will be defined by the user in question. For example, a user may define a given type to equal supermarkets that satisfy their grocery shopping requirements. Toward this goal of defining equivalence classes on the set of travel behaviours, we first define an equivalence relation on the set of vertices V^s and an equivalence relation on the set of trips T.

Let S be a set of vertex types. The elements in this set will be specific to the user in question. For example, an element in this set may correspond to a residential property that meets the user's accommodation needs or a supermarket that meets the user's shopping needs. Let type : $V^s \to S$ be a mapping from vertices to their corresponding type and let $\sim_V$ be an equivalence relation on the set of vertices V^s whereby $v \sim_V v'$ if and only if $\text{type}(v) = \text{type}(v')$. That is, two vertices are equivalent if and only if they have equal types. Given a vertex $v \in V^s$, the corresponding equivalence class $[v]_V$ is the set containing all vertices equivalent to v defined as:

$$[v]_V = \{v' \in V^s : v \sim_V v'\}. \tag{3}$$

Now let $\sim_T$ be an equivalence relation on the set of trips T, whereby $(p_1, v_1, v'_1) \sim_T (p_2, v_2, v'_2)$ if and only if $p_1 = p_2$, $\text{type}(v_1) = \text{type}(v_2)$ and $\text{type}(v'_1) = \text{type}(v'_2)$. That is, two trips are equivalent if and only if they have equal relative frequencies, origin vertices with equal types and destination vertices with equal types. Given a trip $t \in T$, the corresponding equivalence class $[t]_T$ is now the set containing all trips equivalent to t defined as:

$$[t]_T = \{t' \in T : t \sim_T t'\}. \tag{4}$$

Given $\sim_V$ and $\sim_T$ we can now define a set of equivalence classes on the set of travel behaviours as follows. Let $\sim_B$ be an equivalence relation on the set of travel behaviours B whereby $b \sim_B b'$ if and only if there exists a bijection τ from the set b to the set b' such that, for all $t \in b$, it holds that $t \sim_T \tau(t)$. That is, two travel behaviours are equivalent if and only if they contain equivalent sets of trips. Given a travel behaviour $b \in B$, the corresponding equivalence class $[b]_B$ is the set containing all travel behaviours equivalent to b defined as:

$$[b]_B = \{b' \in B : b \sim_B b'\}. \tag{5}$$

To illustrate travel behaviour equivalence classes, consider again the travel behaviour $\{(0.4, v_1, v_2), (0.2, v_2, v_1), (0.2, v_2, v_3), (0.2, v_3, v_1)\}$ from Figure 2. Now consider the travel behaviour $\{(0.4, v_4, v_2), (0.2, v_2, v_4), (0.2, v_2, v_3), (0.2, v_3, v_4)\}$ where the vertex v_1 has been replaced by the vertex v_4, which is of the same type. In this context, v_1 and v_4 have the same type if they both are residential properties that meet the user's accommodation needs. Given this, both travel behaviours belong to the same equivalence class.

Given a travel behaviour $b \in B$, the number of elements in the corresponding equivalence class $[b]_B$ is defined as:

$$\prod_{v \in \{v : (p,v,v') \in b\} \cup \{v' : (p,v,v') \in b\}} |[v]_V| \tag{6}$$

Informally, the number of elements in an equivalence class $[b]_B$ equals the product of the number of vertices with type equal to type(v) for each vertex v in b. To illustrate this computation consider again the travel behaviour $b = \{(0.4, v_1, v_2), (0.2, v_2, v_1), (0.2, v_2, v_3), (0.2, v_3, v_1)\}$ where $[v_1]_V = \{v_1, v_4\}$, $[v_2]_V = \{v_2\}$ and $[v_3]_V = \{v_3\}$. In this case $\{v : (p, v, v') \in b\} \cup \{v' : (p, v, v') \in b\} = \{v_1, v_2, v_3\}$ and the size of the equivalence class is $2 \times 1 \times 1 = 2$.

In the related works section, we reviewed previous works on optimising a model of travel behaviour. The model of travel behaviour proposed in this work is distinct from these previous models in the following two ways. Firstly, the proposed model represents the relative frequency of different trips. On the other hand, previous models generally assume that all trips have the same relative frequency. For example, the model of travel behaviour in the Travelling Salesman Problem (TSP) assumes that the agent in question travels to each location exactly once. Secondly, the proposed model of travel behaviour allows for a variable number of different location types to be defined. On the other hand, previous models generally assume a fixed number of different location types are defined. For example, the model of travel behaviour in the facility location problem assumes that a single location type of suitable facility locations is defined. The above two distinctive features of the proposed model mean that it can be used to represent more complex travel behaviours.

3.3. Connectivity Model

In this section, we now define our model of street network connectivity. Specifically, we define the connectivity of a given travel behaviour as the expected or average trip distance using a real-valued map $\mathcal{C}$ defined as follows:

$$\begin{aligned} \mathcal{C} : B &\to \mathbb{R} \\ b &\mapsto \sum_{(p,v,v') \in b} p \cdot w_{v,v'} \end{aligned} \tag{7}$$

Elements of B that are mapped by $\mathcal{C}$ to smaller values are considered to have greater connectivity. That is, users with travel behaviours having smaller mean or expected travel distance will experience greater connectivity.

To illustrate the model $\mathcal{C}$ of connectivity consider again the example travel behaviour $\{(0.4, v_1, v_2), (0.2, v_2, v_1), (0.2, v_2, v_3), (0.2, v_3, v_1)\}$ represented in Figure 2(a). The shortest path between each pair of vertices in this example is represented in Figure 2(b) using blue lines. The connectivity of this travel behaviour is defined by the model to equal $\mathcal{C}(\{(0.4, v_1, v_2), (0.2, v_2, v_1), (0.2, v_2, v_3), (0.2, v_3, v_1)\}) = 1,381$. That is, the expected trip distance in question is 1,381 meters. Now consider an alternative travel behaviour that is equal to the above travel behaviour except that the vertex v_1 has been replaced by the vertex v_4 shown in Figure 3(a). The shortest path between each pair of vertices in this new travel behaviour is represented in Figure 3(b) using a blue line. From inspection, it is clear that some of the shortest paths in this travel behaviour are longer than those in the previous travel behaviour. This is because the pairs of vertices in question are now further apart, and the paths in question are less direct. The connectivity of this new travel behaviour is defined to equal $\mathcal{C}(\{(0.4, v_4, v_2), (0.2, v_2, v_4), (0.2, v_2, v_3), (0.2, v_3, v_4)\}) = 2,267$. This value is greater than the previous travel behaviour and indicates a larger mean travel distance.

Note that this example of connectivity makes the simplifying assumption that connectivity is solely measured in terms of street network distance. However, the model can be generalised to consider other dimensions of connectivity such as access to public transportation. For example, consider the case where we replace shortest path distances with the smallest travel times in the proposed model of travel behaviour. In this case, the connectivity of a given travel behaviour would be improved if the trips in question were aligned with fast public transportation links.

4. Optimisation of Connectivity

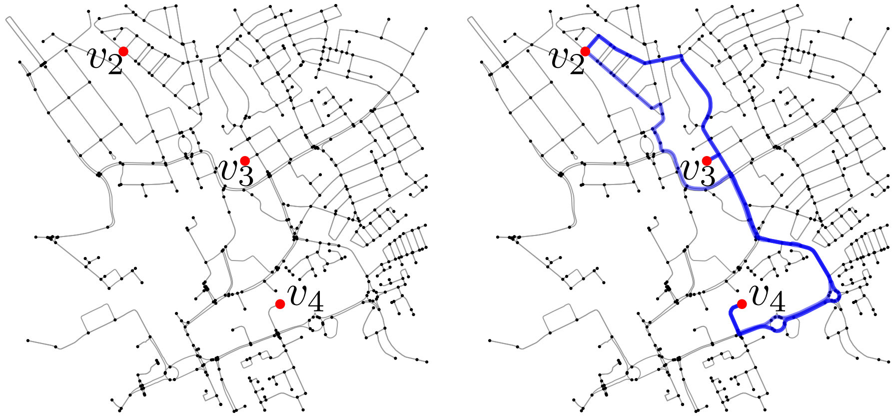
In this section, we describe how our proposed model can be used to optimise travel behaviour with respect to connectivity. In the next subsection, we formally define this optimisation problem, which we call CONNECT, and prove that is, in fact, NP-hard. In Section 4.2 we then propose two integer programming formulations of the problem.

4.1. Problem Definition and Complexity

A user travel behaviour $b \in B$ is optimised with respect to connectivity by solving the following optimisation problem entitled CONNECT:

$$\arg\min_{b' \in [b]_B} \mathcal{C}(b'). \tag{8}$$

Recall from earlier that $G^s = (V^s, A^s)$ is our street network graph. Without loss of generality, also assume that the vertices in b are labelled $v_1, \ldots, v_k$.



(a) (b)

Figure 3: In (a), a user travels between their home, their place of work, and a supermarket, shown by the red vertices v_1, v_2 and v_3 respectively. The blue lines in (b) show the shortest paths between the vertex pairs (v_4, v_2), (v_2, v_4), (v_2, v_3) and (v_3, v_4).

Now define a new arc-weighted directed k-partite graph $G^p = (V^p, A^p)$ with respect to G^s and b. The set of vertices V^p in this new graph is partitioned into k non-empty independent sets $V_1, \ldots, V_k$, where the elements of V_i correspond to all vertices in G^s with the same type as v_i. The arc set in this graph is now $A^p = \{(v, v') : v, v' \in V^p \wedge v \in V_i \wedge v' \in V_j \wedge (p, v_i, v_j) \in b\}$. Finally, the weights $w_{v,v'}$ of each arc $(v, v') \in A^p$ are set to the length of the shortest path from v to v' in G^s, multiplied by the corresponding probability in b. An example of this process is shown in Figure 4.

Given this new graph G^p, a feasible candidate solution for CONNECT is gained by selecting exactly one vertex from each independent set V_i (for $i \in \{1, \ldots, k\}$). Such a solution corresponds to a single member of the equivalence class $[b]_B$. More formally, a candidate solution is represented as a sequence of vertices $S = (u_1, u_2, \ldots, u_k)$ in which $u_i \in V_i$, $\forall i \in \{1, \ldots, k\}$. The aim is to identify a solution that minimises the objective function

$$\sum_{\forall (p, v_i, v_j) \in b} w_{u_i, u_j}. \tag{9}$$

Note that the total number of candidate solutions for this problem is $\prod_{i=1}^{k} |V_i|$. This matches the size of the equivalence class $[b]_B$, defined in Equation (6).

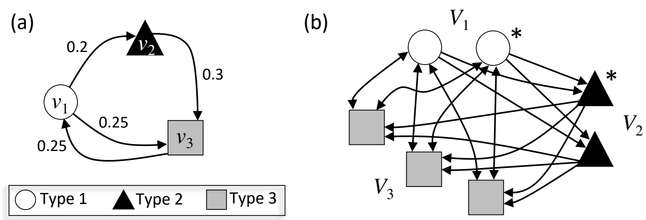


Figure 4: Part (a) shows a travel behaviour $b = \{(0.2, v_1, v_2), (0.25, v_1, v_3), (0.3, v_2, v_3), (0.25, v_3, v_1)\}$, giving $k = 3$. Part (b) shows the corresponding k-partite graph G^p. Here, each independent set V_i comprises all vertices in the street network G^s with the same type as v_i. As an example, the weight of the arc between the two vertices marked by asterisks is calculated as 0.2 multiplied by the length of the shortest path between the corresponding vertices in G^s.

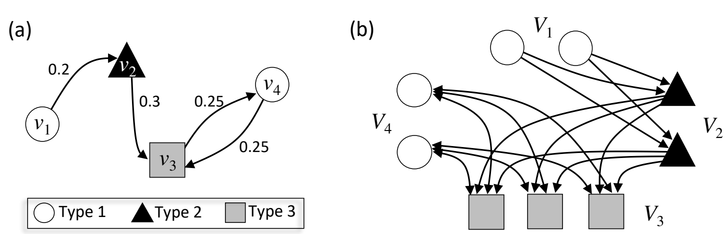


Figure 5: Part (a) shows a travel behaviour b involving $k = 4$ vertices, where v_1 and v_4 have the same type. Part (b) shows the resultant graph G^p.

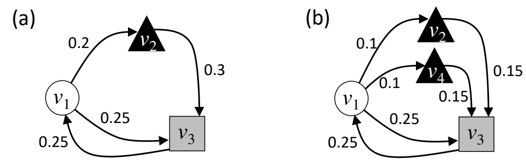


Figure 6: Part (a) shows an example travel behaviour with three vertices. In Part (b), the additional vertex v_4 has been added to allow two occurrences of vertices of type 2 in a solution. In this case, the original probabilities have been distributed evenly between v_2 and v_4, though other options are permissible.

In this model, it is permissible for a travel behaviour to involve several vertices of the same type. In Figure 5, for example, vertices v_1 and v_4 (and therefore independent sets V_1 and V_4) are both of type 1. Given a pair of independent sets V_i and V_j of the same type, it is possible that an optimal solution $S = (u_1, \ldots, u_k)$ will feature vertices u_i and u_j that refer to the same vertex (location) in the street network G^s. If desired this can be avoided by adding further arcs to G^p: specifically, an arc (v, v') of weight $w_{v,v'} = \infty$ should be added in all cases where $v \in V_i$, $v' \in V_j$, and v and v' refer to the same vertex in G^s.

Similarly, it is also possible to stipulate that a solution S should contain more occurrences of a particular type than are present in the travel behaviour b. To do this, we can simply make copies of the appropriate vertices in b and adjust the associated probabilities as desired. An example is shown in Figure 6.

We now prove that the CONNECT problem is NP-hard. To do this we consider the problem of identifying a clique of size k in a simple k-partite graph G_k. We call this latter problem k-CLIQUE-G_k, which is already known to be NP-hard due to the existence of a polynomial-time reduction method from 3-CNF-SAT [25]. It is therefore sufficient to give a polynomial-time reduction from k-CLIQUE-G_k to the CONNECT problem.

Theorem 1. *CONNECT is NP-hard.*

Proof. To show that k-CLIQUE-$G_k \propto$ CONNECT, consider an arbitrary k-partite graph $G_k = (V_k, E_k)$. By definition, the vertices of G_k can be partitioned into k independent sets $I_1, I_2, \ldots, I_k$; consequently, identifying a clique of size k in G_k involves selecting exactly one vertex from each independent set I_i for $i \in \{1, \ldots, k\}$.

Now create a new edge-weighted graph G. This is a copy of G_k in which all edges are given a zero weight. Next, convert G into a complete, edge-weighted k-partite graph by adding edges of weight one between any pair of vertices $\{v, v'\}$ that are (a) nonadjacent in G_k and (b) belong to different independent sets I_i and I_j in G_k.

It is now clear that G_k contains a clique of size k if and only if G features a zero-cost solution for the CONNECT problem. This implies that the task of identifying a k-clique in G_k is a special case of CONNECT. Hence, CONNECT is also NP-hard. □

4.2. An Integer Programming Solution Method

The fact that CONNECT is NP-hard tells us that we cannot hope to find an exact algorithm for solving it that operates in polynomial time (unless P = NP). In this section, we propose two integer programming (IP) formulations for the CONNECT problem, which are then tackled using branch-and-bound. The advantage of using IP is that, during a run, refinements are made to the upper and lower bounds of the optimal cost value. If these bounds become equal, branch-and-bound can then halt with a certificate of optimality. On the other hand, this approach features scaling-up issues, as we will demonstrate below.

As noted, an instance of CONNECT is defined by the arc-weighted, directed graph $G^p = (V^p, A^p)$, where V^p is partitioned into independent sets $V_1, \ldots, V_k$. To define the IP, let $\mathbf{D}_{|V^p|\times|V^p|}$ be a distance matrix for G^p. That is,

$$D_{uv} = \begin{cases} w_{u,v} & \text{if } (u,v) \in A^p \text{ and} \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Now, using the binary decision variable X_u, where

$$X_u = \begin{cases} 1 & \text{if } u \in S \text{ and} \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

the objective is to minimise the cost function

$$\sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{\forall u \in V_i} \sum_{\forall v \in V_j} X_u X_v D_{uv} \tag{12}$$

subject to

$$\sum_{\forall u \in V_i} X_u = 1 \qquad \forall i \in \{1, \ldots, k\}. \tag{13}$$

Here, Equation (13) ensures that exactly one vertex is selected from each independent set V_i (for $i = 1, \ldots, k$) in G^p, as required.

Note that the above model involves just $|V^p|$ variables and k constraints. On the other hand, the objective function given by Equation (12) contains $X_u X_v$, making this a *quadratic* integer program. Although modern commercial IP solvers can handle such formulations, the use of quadratic objective functions can sometimes hinder performance. An option for linearising this model is to introduce an additional parameter $\mathbf{A}^b_{k \times k}$ corresponding to the adjacency matrix for b

$$A^b_{ij} = \begin{cases} 1 & \text{if } (p, v_i, v_j) \in b \wedge p > 0 \text{ and} \\ 0 & \text{otherwise,} \end{cases} \tag{14}$$

together with an auxiliary binary decision variable Y_{ijuv}, where

$$Y_{ijuv} = \begin{cases} 1 & \text{if there exists an arc from vertex } u \in V_i \text{ to vertex } v \in V_j \text{, and} \\ 0 & \text{otherwise,} \end{cases} \tag{15}$$

and the following constraints:

$$0 \leq A^b_{ij} + X_u + X_v - 3Y_{ijuv} \leq 2 \qquad \forall i, j \in \{1, \ldots, k\}, \forall u, v \in V^p. \tag{16}$$

These additional constraints ensure that the correct binary values are assigned to the variables Y_{ijuv} based on the values of the X variables. They also allow us to restate the objective function in the required linear form:

$$\sum_{i=1}^{k} \sum_{j=1}^{k} \sum_{\forall u \in V_i} \sum_{\forall v \in V_j} Y_{ijuv} A^b_{ij} D_{uv}. \tag{17}$$

Note, however, that this formulation is much larger than the previous, involving a total of $|V^p|^2 k^2 + |V^p|$ variables and $2|V^p|^2 k^2 + k$ constraints.

5. Optimisation Results

In this section, we present an evaluation of the two proposed IP formulations. Two types of problem instances are considered: those based on Euclidean distances and those based on lengths of shortest paths in a real-world street network.

In our trials, individual problem instances were generated by first forming a travel behaviour involving k vertices. Specifically, a travel behaviour b of the prescribed size was formed by taking a random sample of arcs (without replacement) from the set $\{(v_i, v_j) : i, j \in \{1, \ldots, k\} \wedge i \neq j\}$, ensuring that each vertex v_i (for $i \in \{1, \ldots, k\}$) was included in b at least once. Each probability in b was then set to $1/|b|$.

To generate a Euclidean problem instance, the graph $G^p = (V^p, A^p)$ was formed by first placing a prescribed number of vertices randomly into the unit square. Each vertex was then also allocated to an independent set V_i (for $i \in \{1, \ldots, k\}$), ensuring that the number of vertices in each independent set differed by at most one. The arcs of G^p were then set to $A^p = \{(v, v') : v, v' \in V^p \wedge v \in V_i \wedge v' \in V_j \wedge (p, v_i, v_j) \in b\}$. Finally, the arc weights $w_{v,v'}$ were set to the Euclidean distance between v and v', multiplied by the associated probability in b, as described earlier. For the geographic instances, the same process was followed except that each vertex in V^p was placed at a randomly selected location on a real-world street network. Arc-weights $w_{v,v'}$ were then set to the length of the shortest paths between these points, calculated using Dijkstra's algorithm, multiplied by the associated probability.

In our trials, problem instances were generated using $k \in \{5, 10, 50\}$, $|b| \in \{1k, 2k, 3k, 4k\}$, and $|V^p| \in \{50, 100, \ldots, 3000\}$. For each combination of these values, eleven Euclidean and geographic instances were generated, giving 15,840 problem instances in total. For the geographic instances, a 35,000-vertex street network of Cardiff, Wales was used. This was generated using the osmnx Python library [26], and all arc distances are stated in meters. Both IP formulations were implemented using Gurobi 11.0.1 and given time limits of 60 seconds per instance.[1] A full listing of our results and source code can be found online [27].

The figures in Table 1 show the scaling-up properties of the two IP formulations by giving, for each set of generated problem instances, the smallest problem size in which optimality was not achieved. These demonstrate that

[1]Executed on Windows 64-bit 3.0 GHz machines with 16 GB of RAM.

k	5				10				50			
$\|b\|$	$1k$	$2k$	$3k$	$4k$	$1k$	$2k$	$3k$	$4k$	$1k$	$2k$	$3k$	$4k$
Euclidean Instances												
Quadratic	2150	1850	1800	1800	1950	1200	900	750	900	600	450	350
Linear	300	300	300	300	150	150	150	150	50	50	50	50
Geographic Instances												
Quadratic	2000	1700	1750	1700	2350	1350	1050	950	1300	600	450	350
Linear	300	300	300	300	150	150	150	150	50	50	50	50

Table 1: Smallest values of $|V^p|$ for which the IP methods failed to find optimal solutions for any of the generated problem instances within the 60-second time limit.

the quadratic model can cope with much larger problem instances than the linear model. This difference seems due to the larger number of variables and constraints involved in the latter, which places much higher demands on computer memory; indeed, in our trials, the linear model often halted with an "out of memory" error for larger values of $|V^p|$ and k. There also appears to be little difference in these patterns between the Euclidean and geographic problem instances.

To further examine these scaling-up properties, Figure 7 shows the time required by the quadratic IP formulation to find optimal solutions across the various problem instances. We see that problem instances of up to $|V^p| = 1800$ vertices are solved within the time limit, though these times depend on the parameters used to generate the problems. Specifically, instances generated with lower values of $|V^p|$ and k, lead to fewer variables and constraints in the IP, giving shorter solution times. Similarly, the use of smaller travel behaviours (values for $|b|$) leads to fewer non-zero values in the distance matrix $\mathbf{D}$, giving a problem that is easier to solve. Again, any differences in behaviour between the Euclidean and geographic problem instances appear to be minor.

Figure 8 shows further characteristics of the quadratic IP by comparing the best upper and lower bounds generated within the time limit. Here, the upper bound gives the cost of the best observed feasible (integral) solution, whereas the lower bounds give the minimum possible cost, derived using relaxed versions of the problem. When these bounds are equal, as is the case for the smaller problem sizes of Figure 8, this indicates that provably optimal solutions have been found for the associated instances within the time limit in all cases. For larger problems, however, we see that the gaps between these

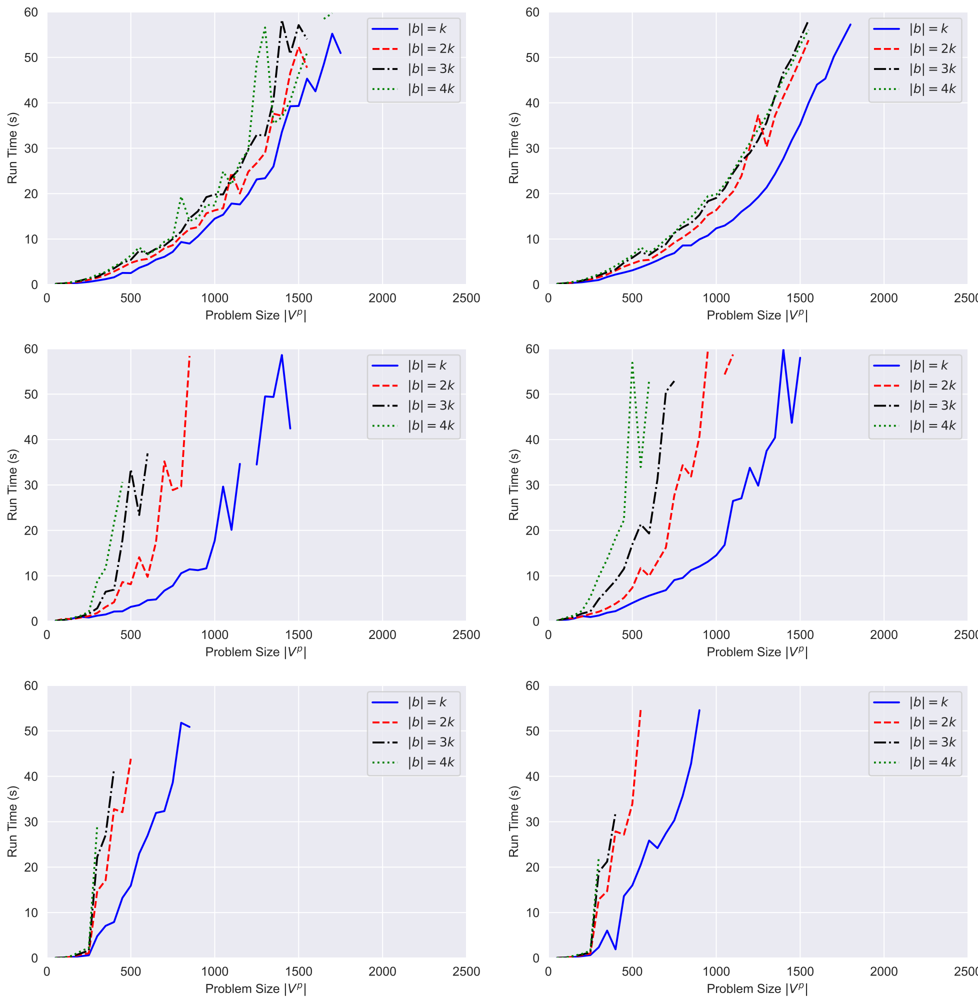


Figure 7: CPU time required by the quadratic IP to find optimal solutions for Euclidean (left) and geographic (right) instances for $k = 5$ (top), $k = 10$ (middle) and $k = 50$ (bottom), using various values of $|b|$ and $|V^p|$. Each point in the charts is the median across the 11 associated problem instances.

bounds widen, indicating an increased problem difficulty. Indeed, for larger problem instances and values of $|b|$, positive lower bounds have not been generated in many cases.

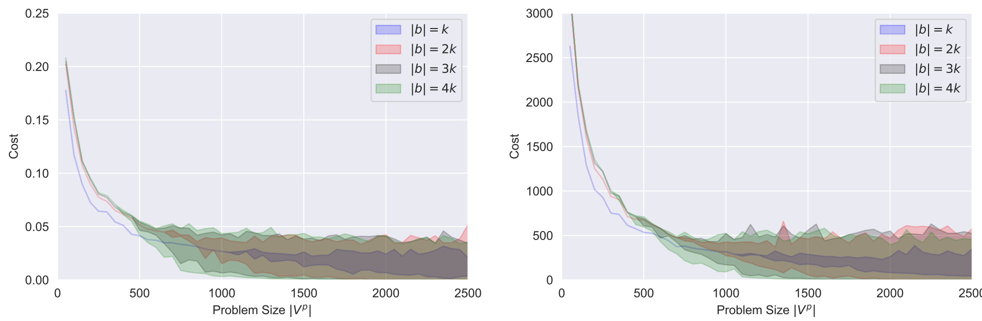


Figure 8: Cost bounds generated by the quadratic IP with Euclidean (left) and geographic (right) instances for $k = 10$, using various values of $|b|$ and $|V^p|$. Each point in the charts is the mean across the 11 associated problem instances.

6. Applications of the Model

In this section, we demonstrate how the proposed model of connectivity can be used to model and subsequently solve several real-world problems. Specifically, we consider the problems of optimising a user's home location, optimising the location of a facility, and modelling the connectivity of a city street network. The diverse nature of these applications illustrates the wide applicability of our proposed model. This contrasts with most existing models of connectivity, which are only applicable to the single problem of modelling the connectivity of a city street network. Each of the above three problems plus corresponding solutions are described in the following subsections.

6.1. Selecting a Home

In this section, we demonstrate how the proposed model of connectivity can be used to provide a solution to the problem of a user selecting a residential property that maximizes their connectivity. This is a problem commonly faced by people when renting or buying a new home. For a given street network $G^s = (V^s, A^s)$, let $U \subset V^s$ be the set of candidate residential properties that meet the user's accommodation needs. Here, we are making the simplifying assumption that residential properties are located at graph vertices. The user wishes to select that element of U that maximizes their connectivity. Let $b \in B$ be the user travel behaviour defined with respect to $v \in U$, selected uniformly at random from U. The connectivity of b models the connectivity with respect to selecting v to be the residential property. Given this, the equivalence class $[b]_B$ contains the same number of elements as U. Each element of $[b]_B$ models the travel behaviour with respect to selecting a corresponding element of U to be the residential property. Let $b' \in B$ be the travel behaviour that solves the optimisation problem defined in Equation (8) with respect to the equivalence class $[b]_B$. The element of U contained in b' is the residential property that maximizes the user's connectivity.

To illustrate the above, consider the street network displayed in Figure 9(a) where vertices $v_1, \ldots, v_6$ are indicated. Let $U = \{v_1, v_4\}$, with vertex v_2 as the user's work location and vertices v_3, v_5 and v_6 as the supermarkets that meet the user's shopping needs. Using $b = \{(0.4, v_4, v_2), (0.3, v_2, v_4), (0.2, v_4, v_3), (0.1, v_3, v_4)\}$, let $[b]_B$ be the equivalence class of the user's travel behaviour. This equivalence class states that that the user plans for 40% of their trips to be from their residential property to work, 40% from work to their residential property, 10% from their residential property to a supermarket, and 10% from the supermarket back to their residential property.

Now, let $b' \in B$ be the travel behaviour that solves the optimisation problem defined in Equation (8) with respect to the equivalence class $[b]_B$ defined above. Figure 9(b) displays the shortest paths corresponding to each trip in this travel behaviour. The element of U contained in b' is the vertex v_1 and this is the residential property that maximizes the user's connectivity. From the above figure, we see that this residential property is near to both the user's work location v_2 and a supermarket v_5.

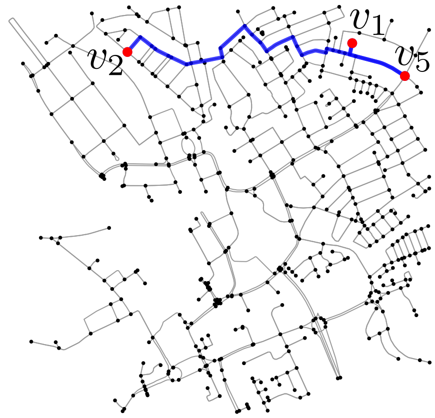
6.2. Facility Location

We now demonstrate how our proposed model can be used to provide a solution to the facility location problem. This problem involves determining an optimal location for a given facility with respect to user accessibility [28]. For a given street network $G^s = (V^s, A^s)$, let $F \subset V^s$ be a set of candidate locations where the facility might be located. Also, let $U \subset V^s$ be the set of facility users' home locations, and let $\lambda : U \to [0, 1]$ be a map of the corresponding relative frequencies (such that $\sum_{u \in U} \lambda(u) = 1.0$). Given this, we define the following travel behaviour b, where v is selected uniformly at random from F:

$$b = \{(\lambda(u), u, v) : u \in U\} \tag{18}$$

The connectivity of this travel behaviour models the accessibility with respect to placing the facility at location v. Recall from the proposed model of

(a)



(b)

Figure 9: In (a), the vertices v_1 and v_4 are residential properties that meet the user's accommodation needs, v_2 is the user's work location, and v_3, v_5 and v_6 are supermarkets that satisfy the user's shopping needs. The blue lines in (b) show the shortest paths between each pair of vertices in the optimal travel behaviour.

connectivity that "type" is a mapping (type : $V^s \to S$) from vertices to their corresponding type. Also let this mapping be defined such that each element of U is mapped to a unique type and each element of F is mapped to the same type. Given this, the equivalence class $[b]_B$ contains the same number of elements as U. Each element of $[b]_B$ now models the travel behaviour with respect to placing the facility at a corresponding element of U. Let $b' \in B$ be the travel behaviour that solves the optimisation problem defined in Equation (8) with respect to the equivalence class $[b]_B$. The element of F contained in b' is therefore the facility location that maximizes accessibility.

To illustrate this, consider the city street network displayed in Figure 10(a), where the sets of red and blue vertices correspond to the sets F and U respectively. The vertex contained in b' is the lowest blue vertex in Figure 10(a) and this is the facility location that maximizes accessibility. We can see from the figure that this location is the most spatially central of the three vertices in F. The shortest path between each pair of vertices in b' is represented using blue lines in Figure 10(b).

6.3. Modelling City Connectivity

As a final example, we now show how our model can be used to measure the connectivity of a given city street network. Here, we define the connec-

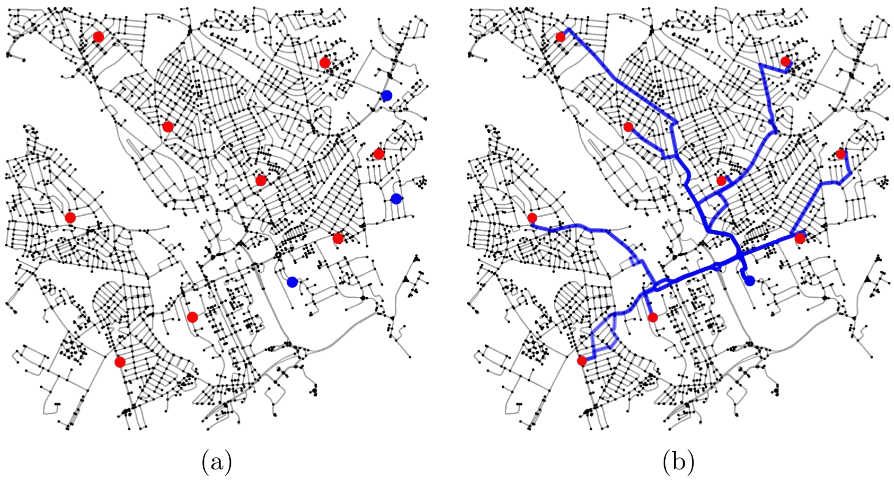


Figure 10: In (a), the sets of blue and red vertices correspond to candidate facility locations and user home locations respectively. Part (b) shows the shortest paths between each pair of vertices in the optimal travel behaviour.

tivity of a city street network to be the connectivity of the corresponding mean or average user travel behaviour. Population travel behaviour data is usually very sensitive because, from such data, one can infer much private information regarding the individuals in question [29]. Although some organizations such as Google and the UK Office for National Statistics (ONS) do have population travel behaviour data, this data is not publicly available. For example, the UK ONS states that such data is "protected by the highest level of access limitation and are available only to approved researchers via the Virtual Microdata Laboratory (VML)".[2]

To address this challenge, for a given city street network we propose to approximate the corresponding mean user travel behaviour as follows. First, we select 2000 pairs of spatial coordinates uniformly at random within the street network bounding box and map these to the corresponding pair of nearest vertices. Next, for each of these pairs of vertices (v, v') we add the trip $(1/2000, v, v')$ to the travel behaviour in question. Given this user travel behaviour, we evaluate the connectivity of the street network in question by

[2]`https://www.ons.gov.uk/census/2011census/2011censusdata/originanddestinationdata/secureoriginanddestinationtables`

computing the connectivity of this travel behaviour.

Broadly speaking, the proposed model of city connectivity will determine that a city with more direct routes between pairs of vertices has greater connectivity. In fact, in this particular case, it is closely related to the models of city connectivity based on the concept of directness described in Section 2. Recall that, directness is defined as the ratio of the straight line distance to the street network distance between pairs of vertices. Street networks with greater directness between pairs of vertices will therefore be determined to have greater connectivity by the proposed model. This relationship provides a useful way to interpret existing models of city connectivity based on directness in terms of the proposed model of travel behaviour. That is, these existing models implicitly assume a travel behaviour consisting of trips between random locations in the network. At present, a standard framework for comparing different models of city connectivity does not exist, therefore it is difficult to quantitatively compare the proposed model to other existing models.

To illustrate our proposed model of city street network connectivity we considered the set of street networks corresponding to all 66 UK cities. For each city, we selected a location in the city centre and extracted the street network within a ten-kilometre bounding box centred at this location. Table 2 displays the names of the UK cities in question plus the number of vertices and arcs in the corresponding graph representations. Figure 11 displays the street networks for the cities of Edinburgh and Hull.

Given the stochastic nature of the proposed model of connectivity, we computed the connectivity of each city ten times. Table 3 displays the mean and standard deviation statistics of these values for each city. Note that the relative magnitude of the standard deviation values is not significant, suggesting that the proposed model is a robust estimate. From these statistics, we see that the city of Edinburgh has the greatest connectivity. That is, the street network distance between random pairs of vertices is the smallest in this street network. On the other hand, the city of Hull is determined to have the lowest connectivity.

7. Conclusions

This work has proposed a model of street network connectivity that is defined with respect to user travel behaviour. To the authors' knowledge, this is the first such model to consider travel behaviour in this way; consequently,

City	$\vert V^s\vert$	$\vert A^s\vert$	City	$\vert V^s\vert$	$\vert A^s\vert$
Aberdeen	9,403	21,808	Liverpool	32,130	73,907
Armagh	2,226	5434	London	47,049	111,353
Bangor	3,327	7,426	Manchester	45,384	104,503
Bath	8,987	19,821	Newcastle	30,269	67,663
Belfast	18,743	42,478	Newport	10,997	23,747
Birmingham	38,129	84,886	Newry	3,757	8,503
Bradford	31,348	71,174	Norwich	13,074	28,113
Brighton	6,663	16,048	Nottingham	23,707	52,196
Bristol	23,446	52,146	Oxford	7,667	16,716
Cambridge	8,131	17,454	Perth	3,113	6,897
Canterbury	6,263	13,719	Peterborough	10,389	21,608
Cardiff	15,198	33,605	Plymouth	10,236	22,695
Carlisle	4,555	10,224	Portsmouth	13,679	30,304
Chelmsford	6480	13982	Preston	13998	30701
Chester	9,319	20,353	Ripon	2,148	4,796
Chichester	5,403	11,932	St Albans	13,126	28,887
Coventry	13,667	29,955	St Asaph	6,154	13,667
Derby	12,834	27,689	St Davids	338	787
Derry	5,549	12,147	Salford	42,071	96,544
Dundee	7,385	17,018	Salisbury	3,138	6,887
Durham	11,851	26,133	Sheffield	22,260	50,846
Edinburgh	15,166	34,973	Southampton	18,854	40,807
Ely	3,658	7,720	Stirling	5,585	12,221
Exeter	8,430	18,361	Stoke-on-Trent	16,803	37,608
Glasgow	30,435	70,423	Sunderland	18,744	42,155
Gloucester	10,577	22,629	Swansea	10,268	22,599
Hereford	3,736	8,361	Truro	3,201	7,271
Inverness	4,842	10,329	Wakefield	21,430	46,702
Hull	14,363	30,774	Wells	3,663	8,349
Lancaster	5,167	11,871	Westminster	47,193	111,630
Leeds	29,734	67,011	Winchester	5,910	12,749
Leicester	19,045	41,882	Wolverhampton	25,580	57,243
Lichfield	11,348	24,306	Worcester	8,040	17,587
Lincoln	8,062	17,544	York	7,774	16,817
Lisburn	11,220	25,271			

Table 2: The names of the 66 UK city considered plus the number of vertices ($|V^s|$) and arcs ($|A^s|$) in the corresponding graph representations.

there are several opportunities for future research and development. We now consider three such options.

One of the major challenges of applying the proposed model in practice is that it requires an accurate model of user travel behaviour. User travel behaviour data is highly sensitive because it is possible to infer many pri-

(a) (b)

Figure 11: The street networks for the UK cities of Edinburgh and Hull are displayed in (a) and (b) respectively, where street segments are represented by black lines.

vate details about the user in question [29]. In cases where such data is not available, the proposed model is not applicable or can only be applied using artificial travel behaviour that represents an approximation to the real behaviour. In this work, we used the latter approach when modelling the connectivity of a given city street network. Developing better approximations of user travel behaviour represents a useful direction for future research that, if successful, would result in a more accurate model of connectivity.

Our proposed model of connectivity assumes that the connectivity between two locations is exclusively a function of the network distance between the locations in question. However, the connectivity between two locations is also a function of other factors including street type and traffic volume that, in turn, influence travel time. For example, two locations connected by streets with low traffic might be considered as being more connected than those with high congestion. Furthermore, the proposed model assumes zero travel time or cost between street network intersections, which is not the case at busy intersections [30]. Developing a better model of connectivity between locations also represents a useful direction for future research that, if successful, would result in a more accurate model of connectivity.

Although we have shown our formulation of CONNECT to be NP-hard, we have seen that we can quickly solve fairly large instances of the problem using a compact quadratic IP formulation with a contemporary off-the-shelf solver. There also exist certain special cases of this problem that can be solved

City	Connectivity	City	Connectivity
Edinburgh	10,910 ± 124	Newcastle	13,560 ± 112
Sunderland	11,275 ± 135	St Albans	13,566 ± 147
St Davids	11,279 ± 171	Lichfield	13,588 ± 225
Aberdeen	11,489 ± 154	Salisbury	13,619 ± 248
Brighton	12,076 ± 167	Chichester	13,713 ± 193
Swansea	12,637 ± 126	Bath	13,718 ± 299
Lancaster	12,647 ± 185	Durham	13,726 ± 254
Birmingham	12,662 ± 157	Winchester	13,727 ± 241
London	12,689 ± 189	Stirling	13,777 ± 239
Cardiff	12,727 ± 123	Derby	13,790 ± 175
Armagh	12,812 ± 180	Oxford	13,805 ± 154
Wolverhampton	12,823 ± 147	Exeter	13,825 ± 238
Plymouth	12,881 ± 192	Hereford	13,853 ± 221
Lisburn	12,908 ± 205	York	13,881 ± 151
Westminster	12,914 ± 213	Nottingham	13,903 ± 213
Manchester	13,076 ± 148	Chelmsford	13,933 ± 153
Bradford	13,138 ± 144	Ely	13,955 ± 90
Sheffield	13,166 ± 227	Worcester	13,980 ± 256
Leeds	13,193 ± 182	Cambridge	14,075 ± 144
Coventry	13,228 ± 199	Ripon	14,076 ± 142
Leicester	13,264 ± 113	Liverpool	14,140 ± 188
Portsmouth	13,284 ± 196	Newport	14,210 ± 116
Carlisle	13,288 ± 201	Chester	14,215 ± 114
Bristol	13,318 ± 123	Preston	14,281 ± 196
St Asaph	13,338 ± 121	Derry	14,304 ± 223
Wells	13,368 ± 149	Peterborough	14,356 ± 165
Glasgow	13,375 ± 157	Gloucester	14,546 ± 247
Newry	13,414 ± 211	Perth	14,685 ± 197
Salford	13,416 ± 155	Dundee	14,710 ± 186
Stoke-on-Trent	13,443 ± 204	Bangor	14,987 ± 201
Wakefield	13,466 ± 218	Inverness	15,293 ± 212
Norwich	13,478 ± 166	Truro	15,538 ± 204
Lincoln	13,506 ± 207	Southampton	16,259 ± 252
Canterbury	13,512 ± 259	Hull	18,750 ± 221
Belfast	13,540 ± 156		

Table 3: The mean (± standard deviation) connectivity values for each UK city as determined by the proposed model of city connectivity.

in polynomial time. For example, if a travel behaviour b corresponds to a directed path, then, as illustrated in Figure 12, a graph can be constructed whose optimal solution corresponds to the shortest x-y-path. Further research may also reveal other special cases that are polynomial-time solvable.

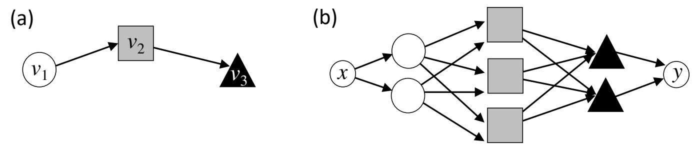


Figure 12: Part (a) shows a travel behaviour corresponding to a path. Part (b) shows the corresponding arc-weighted, directed k-partite graph $G^p = (V^p, A^p)$ where $V^p = (V_1, V_2, V_3)$, plus two additional vertices x and y. The shortest x-y-path corresponds to the optimal solution.

References

[1] T. A. Randall, B. W. Baetz, Evaluating pedestrian connectivity for suburban sustainability, Journal of Urban Planning and Development 127 (1) (2001) 1–15.

[2] S. Labi, A. Faiz, T. U. Saeed, B. N. T. Alabi, W. Woldemariam, Connectivity, accessibility, and mobility relationships in the context of low-volume road networks, Transportation research record 2673 (12) (2019) 717–727.

[3] J. Dill, Measuring network connectivity for bicycling and walking, in: 83rd annual meeting of the Transportation Research Board, Washington, DC, 2004, pp. 11–15.

[4] M. J. Koohsari, T. Sugiyama, K. E. Lamb, K. Villanueva, N. Owen, Street connectivity and walking for transport: role of neighborhood destinations, Preventive medicine 66 (2014) 118–122.

[5] G. Tal, S. Handy, Measuring nonmotorized accessibility and connectivity in a robust pedestrian network, Transportation research record 2299 (1) (2012) 48–56.

[6] P. Stangl, The pedestrian route directness test: A new level-of-service model, Urban Design International 17 (3) (2012) 228–238.

[7] P. L. Knight, W. E. Marshall, The metrics of street network connectivity: their inconsistencies, Journal of Urbanism: International Research on Placemaking and Urban Sustainability 8 (3) (2015) 241–259.

[8] A. Shashank, N. Schuurman, Unpacking walkability indices and their inherent assumptions, Health & place 55 (2019) 145–154.

[9] K. S. Sahitya, C. Prasad, Modelling structural interdependent parameters of an urban road network using gis, Spatial Information Research (2019) 1–8.

[10] P. Stangl, J. M. Guinn, Neighborhood design, connectivity assessment and obstruction, Urban Design International 16 (4) (2011) 285–296.

[11] P. Stangl, Overcoming flaws in permeability measures: modified route directness, Journal of Urbanism: International Research on Placemaking and Urban Sustainability 12 (1) (2019) 1–14.

[12] J. Peponis, S. Bafna, Z. Zhang, The connectivity of streets: reach and directional distance, Environment and Planning B: Planning and Design 35 (5) (2008) 881–901.

[13] G. Ellis, R. Hunter, M. A. Tully, M. Donnelly, L. Kelleher, F. Kee, Connectivity and physical activity: using footpath networks to measure the walkability of built environments, Environment and Planning B: Planning and Design 43 (1) (2016) 130–151.

[14] P. Stangl, Block size-based measures of street connectivity: A critical assessment and new approach, Urban Design International 20 (1) (2015) 44–55.

[15] P. M. Hess, Measures of connectivity [streets: old paradigm, new investment], Places 11 (2) (1997).

[16] S. Handy, R. G. Paterson, K. Butler, Planning for street connectivity, Getting from here to there. American Planning Association, Planning Advisory Service (Planning Advisory Service report, no. 515), Chicago (2003).

[17] K. Kiran, J. Corcoran, P. Chhetri, Measuring the spatial accessibility to fire stations using enhanced floating catchment method, Socio-Economic Planning Sciences 69 (2020) 100673.

[18] M. Curado, L. Tortosa, J. F. Vicent, G. Yeghikyan, Analysis and comparison of centrality measures applied to urban networks with data, Journal of Computational Science (2020) 101127.

[19] D. L. Applegate, The traveling salesman problem: a computational study, Vol. 17, Princeton university press, 2006.

[20] K. Braekers, K. Ramaekers, I. Van Nieuwenhuyse, The vehicle routing problem: State of the art classification and review, Computers & industrial engineering 99 (2016) 300–313.

[21] M. W. Savelsbergh, M. Sol, The general pickup and delivery problem, Transportation science 29 (1) (1995) 17–29.

[22] P. Vansteenwegen, W. Souffriau, D. Van Oudheusden, The orienteering problem: A survey, European Journal of Operational Research 209 (1) (2011) 1–10.

[23] R. Z. Farahani, M. SteadieSeifi, N. Asgari, Multiple criteria facility location problems: A survey, Applied mathematical modelling 34 (7) (2010) 1689–1709.

[24] P. Corcoran, P. Mooney, Characterising the metric and topological evolution of openstreetmap network representations, The European Physical Journal Special Topics 215 (1) (2013) 109–122.

[25] T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein, Introduction to algorithms, MIT press, 2009.

[26] G. Boeing, Osmnx: New methods for acquiring, constructing, analyzing, and visualizing complex street networks, Computers, Environment and Urban Systems 65 (2017) 126–139.

[27] R. Lewis, P. Corcoran, Code and data sets, `https://zenodo.org/records/11504139`, accessed: 2024-06-06 (2024).

[28] A. Gagarin, P. Corcoran, Multiple domination models for placement of electric vehicle charging stations in road networks, Computers & Operations Research 96 (2018) 69–79.

[29] P. Corcoran, P. Mooney, A. Gagarin, A distributed location obfuscation method for online route planning, Computers and Security 95 (2020).

[30] R. Lewis, Algorithms for finding shortest paths in networks with vertex transfer penalties, Algorithms 13 (11) (2020) 269.