A Modified Fixed-Point Iteration Algorithm for Magnetic Field Computation With Hysteresis Models

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The fixed-point iteration method is widely used in electromagnetic field analysis involving hysteresis property due to its strong robustness, but it has the problem of low computational efficiency. In this paper, a modified fixed-point iteration algorithm is proposed where the convergence factor is adaptively adjusted for different time steps according to the variation of residuals, instead of being globally static in traditional ways. Numerical analysis for a C-shaped iron core characterized by inverse vector Preisach model is performed using the traditional method and the proposed method respectively. The efficiency, stability and applicability of both methods are assessed in which the proposed approach shows superior performance.

***Index Terms*—Fixed-point iteration, nonlinear hysteresis, Preisach model, finite element analysis (FEA).**

I. Introduction

History-dependent hysteresis nonlinearity is an intrinsic magnetic property of magnetic materials, which is closely related to some important magnetic parameters such as remanence and coercivity. In electromagnetic field analysis, accounting for hysteresis property is essential to enhance the accuracy of predicting magnetic flux and core loss [1-2].

However, when dealing with multi-valued hysteresis problem, the existence of local extrema in the solution space can cause instability in calculating the Jacobian matrix when the Newton-Raphson iteration method is employed, leading to divergence [3]. In such cases, the fixed-point iteration method is commonly utilized due to its robustness [4]. The nonlinear magnetic field problem with fixed-point iteration is then formulated as:

$$H(B) = \nu_{\text{FP}} B + M(B) \tag{1}$$

where $\boldsymbol{H}(\boldsymbol{B})$ represents the hysteretic nonlinear relationship between magnetic field intensity $\boldsymbol{H}$ and magnetic flux density $\boldsymbol{B}$. $\boldsymbol{M}(\boldsymbol{B})$ is a magnetization-like residual term. ν_{FP} is the so called fixed-point coefficient.

The classical fixed-point iteration method can always ensure convergence when the ν_{FP} is set as half the reluctivity of the vacuum for the entire ferromagnetic region and all time steps. This approach, known as global method, nevertheless exhibits poor convergence rate, particularly at low inductions. To address this issue, Dlala and Arkkio introduced local fixed-point iteration method (LFPM), where the ν_{FP} for each finite element is updated once at each time step and then held constant during the following iterations [5-6]. For 2D models, the ν_{FP} at the current time step k is usually computed from the converged results of last time step as follows:

$$\nu_{\text{FP}}^{(k)} = C\frac{\dfrac{dH_x}{dB_x}^{(k-1)} + \dfrac{dH_y}{dB_y}^{(k-1)}}{2} = C\frac{\nu_{xx} + \nu_{yy}}{2} \tag{2}$$

where ν_{xx} and ν_{yy} are the differential reluctivity determined by material models and the converged solution provided by the previous time step. C is convergence factor whose value must be greater than 1. This approach has shown dramatic improvement in reducing iteration number and improving the convergence rate. However, convergence is no longer guaranteed if C is not appropriately chosen.

Several variants of the local fixed-point method have been proposed to enhance its stability. For instance, Mathekga introduced a smoothing strategy for the local fixed-point coefficient to effectively mitigate numerical instability arising from perturbations like "peaks" or "troughs" in differential reluctance calculation [7]. A diagonal tensor-form local fixed-point coefficient is proposed by Li to better accommodate to the anisotropic materials models [8]. In addition, Zhou suggested an adaptive strategy alternating between two correction strategies based on B or H to accelerate the convergence of global method [9]. Other researchers proposed hybrid methods combining the fixed-point method and Newton-Raphson method to address hysteresis problems. These approaches leverages the strengths of both methods through modifying the formula of constitutive law, combining their solution correction method or alternating them in different convergence stages [10-12]. Hybrid algorithms share the superiority of strong adaptability, while their convergence rate proves to be a compromise.

The work presented in this paper concerns the application of fixed-point iteration method in time-stepping finite element analysis involving hysteresis nonlinearity. The constant convergence factor C in LFPM is adaptively adjusted according to the residual variations during iterations. Thus ν_{FP} for each time step is more accurately estimated since it is no longer excessively amplified by globally chosen C according to (2). Based on this acceleration algorithm and an in-house developed finite element package, the numerical analysis for a C-shaped iron core is carried out where the iron core material is characterized by inverse vector Preisach model. The results show that the proposed method can effectively improve computational efficiency and stability.

II. Modified Fixed-Point Iteration Algorithm

Current studies indicate that the key of local fixed-point method is the choice of C (i.e., ν_{FP}). Employing a large C can enhance convergence for all time steps at the cost of increasing iteration number and computation time. Conversely, a small coefficient may cause difficulty in the convergence of certain time steps, especially at around the magnetization reversal points. For now, the value of C is usually determined by trial and error method based on empirical judgment.

To address this issue, the authors propose an adaptive algorithm for the LFPM by dynamically adjusting C and ν_{FP} in response to convergence difficulties encountered during specific time steps. The formula is expressed as:

$$\nu_{FP}^{(k)} = C^{(k)} \frac{\frac{dH_x}{dB_x}^{(k-1)} + \frac{dH_y}{dB_y}^{(k-1)}}{2} = C^{(k)} \frac{\nu_{xx} + \nu_{yy}}{2} \quad (3)$$

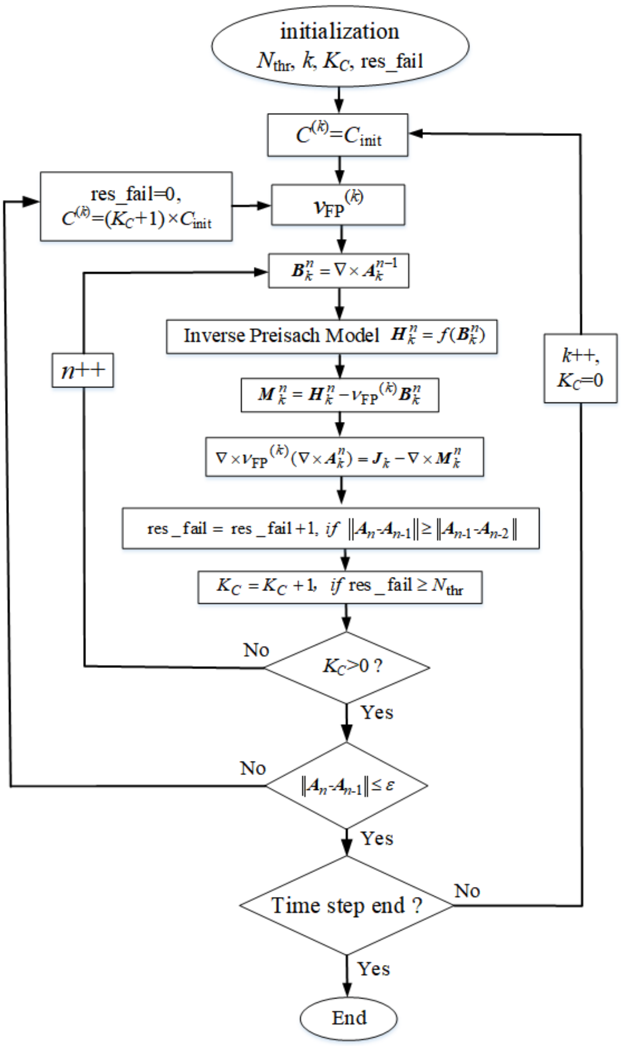


Fig. 1 Procedure of electromagnetic FEA using the modified fixed-point iteration algorithm.

In the proposed method, a small coefficient C_{init} (2 in this paper) is employed in the initial iterations of each time step so as to achieve fastest convergence. If the solution is not converged after several iterations and the cumulative count of residual failure reaches the preset updating threshold (N_{thr}), then it can be inferred that the current fixed-point coefficient is insufficient for the convergence of this time step. In such case, C is increased accordingly and this time step is resolved until convergence is achieved. With the adaptive refinement of C, the need for search optimal convergence factor as in traditional LFPM is eliminated, thus the flexibility and applicability of this method is improved. The computational procedure of the proposed algorithm is depicted in Fig. 1. Note that n represents iteration number while k represents time step number. The residual failure is indicated by res_fail and its cumulative count is recorded by K_C.

III. Validation

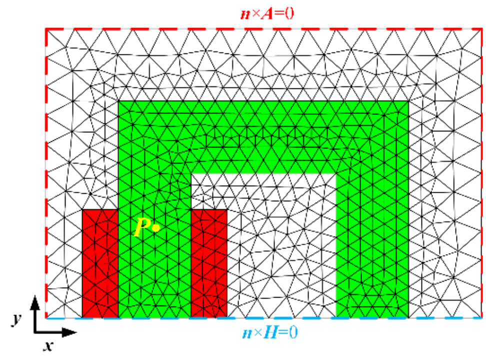
The performance of the proposed algorithm is evaluated by simulating a C-shaped iron core model, based on an in-house electromagnetic analysis package developed by the authors' team. A mesh of the problem is shown in Fig. 2 (a). The magnetic properties of the iron core is characterized by inverse vector Preisach model. The relative permeability for coil and air region is set to 1. The corresponding mesh and material data are listed in Table I.

The analysis is performed using the magnetic vector potential formulation. The coil region is fed by sinusoidal current source. As a boundary condition, zero vector potential is applied to the top, left and right boundaries. Simulation is run with 125 time steps for 1.25 electrical period and a cap of 500 iterations for each step is preset. Both traditional LFPM with different convergence factors and proposed algorithm with different updating thresholds are implemented and examined.

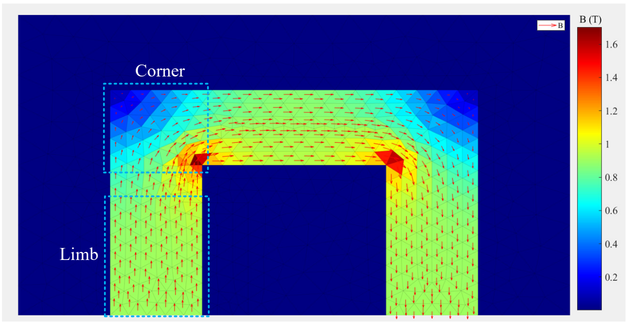
TABLE I
Main Parameters of the Iron Core Model

Material	Elements number	Material model
Iron core	490	inverse vector Preisach
Coil	96	μ_r=1
Air	417	μ_r=1

Fig. 2 (b) shows the distribution of $\boldsymbol{B}$ in the core region at time step k = 60. It can be seen that magnetic flux is concentrated at the inner corner, and runs through core limbs mainly along the y-axis direction with a relatively uniform amplitude. Fig. 3 depicts the hysteresis loops and magnetic flux density at point P in the y-axis direction at two different excitation levels respectively. As illustrated in the figure, the magnetization starts from the demagnetization state and the hysteresis phenomena is well characterized by the inverse vector Preisach model. The $\boldsymbol{B}$ waveform remains sinusoidal at low induction (0.65T) and shows distortion at high magnetic flux density level (1.40T) due to nonlinearity as the material gradually approaching saturation.



(a)



(b)

Fig. 2 The analyzed FEM model. (a) Mesh of the C-shaped iron core model (b) flux density vector plot at time step 60

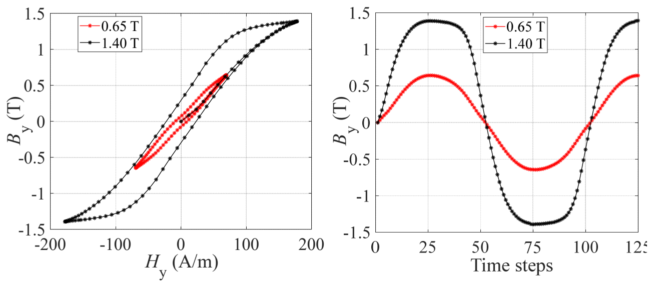
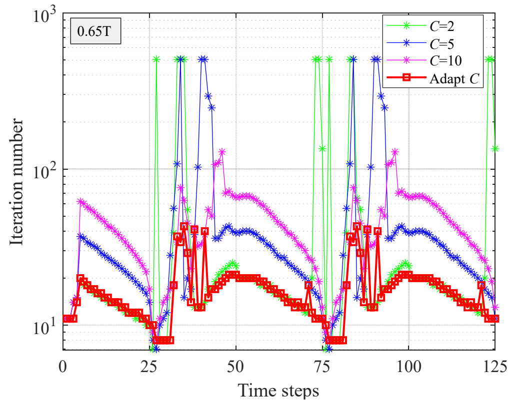


Fig. 3 Numerical solution at point *P* (a) Hysteresis loops in the *y* direction (b) ***B*** component along *y* axis

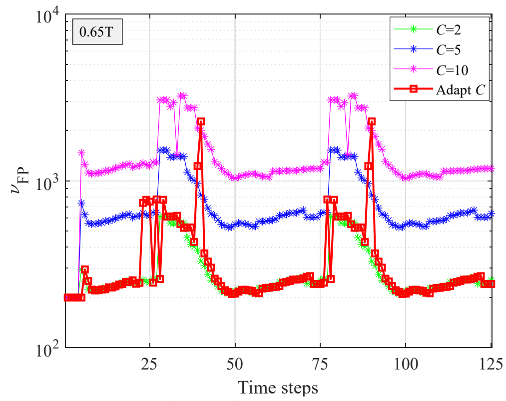
Fig. 4 (a) shows the number of iterations of all time steps by the LFPM with different convergence factor values and the proposed algorithm (with $N_{\text{thr}} = 5$, abbreviated as Adapt *C*) under low induction. It can be seen that for the LFPM, when *C* takes a value of 2 or 5, the number of iterations of some time steps reach the preset maximum which implies non-convergence, especially for those instants around the reversals of magnetization history such as k = 25 and 75. When *C* takes 10, the convergence of all time steps is guaranteed, but the iterations numbers required for most time steps are significantly increased. The above analysis reveals the contradiction between convergence and computational costs of traditional LFPM since the adoption of constant *C* fails to accommodate every time step optimally while considering global stability.

In contrast, the proposed algorithm can guarantee the convergence for all time steps while maintaining as few iterations as possible. This is due to the fact that when convergence difficulty is detected in some time steps, the convergence factor and the fixed-point coefficient is automatically adjusted, and this time step is resolved. Through this method, the number of iterations and computational time is significantly reduced. Fig. 4 (b) illustrates the fixed-point coefficients at point *P* calculated by the two methods, and the notable difference near the magnetization reversals indicates the effectiveness of the modified method.

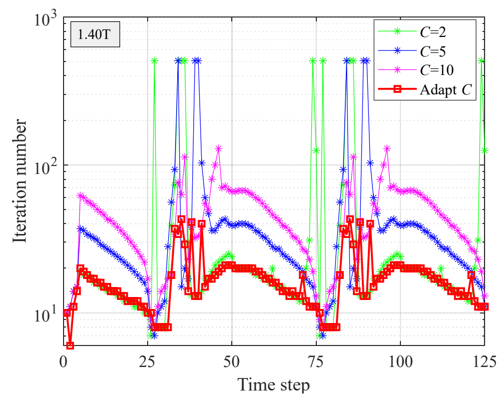
Similarly, Fig. 4 (c-d) shows the number of iterations of all time steps and the calculated ν_{FP} respectively at point *P* by the LFPM with different convergence factor values and the proposed algorithm (with $N_{\text{thr}} = 5$) under high induction. It can be seen that the typical convergence behavior is similar to that of low induction. In general, the proposed method can effectively reduce the number of iterations at both high and low magnetic flux densities.



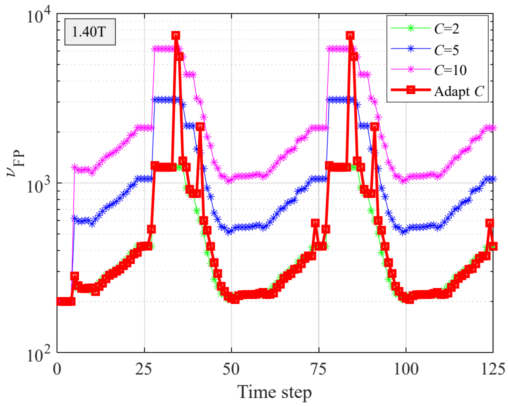
(a) Number of iteration at 0.65T



(b) ν_{FP} of point *P* at 0.65T



(c) Number of iteration at 1.40T



(d) ν_{FP} of point P at 1.40T

Fig. 4 Comparison of convergence behavior of two methods.

Table II shows the non-convergent time steps and total computational time using the proposed method compared with those using the LFPM under different excitation levels. It can be seen that the improper value of C is closely related to the convergence failure behaviors, consequently deteriorating the computational efficiency. When C is set to 2, the numbers of non-converged time step at 0.65T, 1.4T and 1.7T reach 11, 10 and 6 respectively. The number of iterations of these time steps reaches the preset 500 times, far exceeding the numbers of other normal converged time steps (with iteration numbers less than 100), and therefore the total computational time is increased. Similar results are observed when C takes 5. When C is 10, global convergence is reached with relatively small calculation effort. For the proposed method, possible non-convergence in LFPM is eliminated for all time steps and different induction levels regardless of the value of N_{thr}, and the overall computational time is significantly reduced compared with the LFPM. In other words, the proposed method circumvents the extra parameter optimization process required by the traditional method, thus reducing the complexity of application and enhancing the suitability of this method.

TABLE II
COMPARISON OF THE CONVERGENCE BEHAVIOR

Methods		0.65T		1.40T		1.70T	
		Non-converged Time steps	Total time (s)	Non-converged Time steps	Total time (s)	Non-converged Time steps	Total time (s)
LFPM	C=2	11	1742.2	10	1522.0	6	1132.1
	C=5	6	1520.8	6	1259.2	3	852.3
	C=10	0	1421.5	0	1253.3	0	728.5
Adapt C	N_{thr}=2	0	328.1	0	306.4	0	276.2
	N_{thr}=5	0	437.9	0	427.9	0	388.6
	N_{thr}=10	0	790.4	0	737.1	0	417.5

IV. Conclusions

This paper proposes a modified fixed-point iteration algorithm for FEA with hysteretic media featured by adaptively correcting the convergence factor for different time steps according to the residual variations. The performance of this algorithm is evaluated by a numerical example of a C-shaped iron core model. It is shown that the algorithm can effectively reduce the number of iterations required near reversals in the magnetization history, and thus improve computational efficiency. Moreover, the empirically determined C in traditional methods is instead directly controlled by the residual evolution, thereby the adaptability and applicability are enhanced simultaneously.

Acknowledgment

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