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Abstract

We show that essentially every correlated equilibrium of any finite game with complete information with four players can be implemented as a perfect Bayesian equilibrium of an extended game, in which before choosing actions in the underlying game, players exchange cheap talk messages. In particular, we improve on the result of Bárány (1992) and Gerardi (2004). And our result generalizes to sequential equilibria and to games with incomplete information, i.e. to the set of (regular) communication equilibria.

KEYWORDS: unmediated communication; sequential rationality; correlated equilibria; communication equilibria; communication protocols.

JEL CLASSIFICATION: C72; D82.

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1 The setup and the main result

Let $\Gamma =< I = \{1,2,3,4\}, (A_i)_{i\in I}, (g_i)_{i\in I} >$ be a finite 4-player game of complete information, where $I = \{1,2,3,4\}$ is the set of players, A_i is the finite set of actions available to player $i \in I$, $A = \prod_{i\in I} A_i$ is the set of action profiles, and $g_i : A \to \mathbb{R}$ is the payoff function of player $i \in I$. We let $A_{-i} = \prod_{j\neq i} A_i$ denote the set of profiles of actions of players different from i. The set of probability distributions over a finite set X is denoted by $\Delta(X)$.

We consider the cheap talk extension of Γ, where before choosing actions in Γ (the action phase), players communicate with each others for finitely many stages either through pairwise private communication channels or by making public announcements (the communication phase). During the communication phase, players exchange "cheap" messages in that they do not affect directly their payoffs. More precisely, at each stage of the communication phase, each player simultaneously[1] sends a private message from a finite set $\mathcal{M}$ to all the other players, and/or make a public announcement from the same set $\mathcal{M}$. This specification of the extended game is without loss of generality, since players can send an "empty" message by mixing with positive probability among all the possible messages. The description of the set $\mathcal{M}$ will be part of the construction. A history of length $n > 0$ for player i is given by $h_i^n = (m_{-i,i}^t, m_{i,-i}^t, p^t)_{0\leq t\leq n}$ where $m_{-i,i}^t = (m_{j,i}^t)_{j\in I\backslash\{i\}}$ denotes the private messages received by player i from players $-i$, $m_{i,-i}^t = (m_{i,j}^t)_{j\in I\backslash\{i\}}$ denotes the private messages sent by player i to players $-i$, $p^t = (p_j^t)_{j\in I}$ denotes the profile of public announcements made at stage t by all players, and $(m_{-i,i}^0, m_{i,-i}^0, p^0) \equiv \emptyset$. The set of histories of length n for player i is denoted by H_i^n, and let $H^n = \prod_{i\in N} H_i^n$.

A communication protocol or a communication strategy profile $c = (c_i)_{i\in I}$ of length n, where $c_i = (c_i^1, \ldots, c_i^t, \ldots, c_i^n)$, specifies for each player i which private message to send to each player $m_{i,-i}^t \in \mathcal{M}^{I-1}$ and which public announcement to make $p_i^t \in \mathcal{M}$ at stage t for $0 < t \leq n$ given a history $h_i^{t-1} \in H_i^{t-1}$: that is, for each player i, each $j \in I\backslash\{i\}$ and each $0 \leq t \leq n$, $c_i^t = (m_{i,j}^t, p_i^t)_{j\in I\backslash\{i\}}$, $m_{i,j}^t : H_i^{t-1} \to \Delta(\mathcal{M})$, and $p_i^t : H_i^{t-1} \to \Delta(\mathcal{M})$.

In stage $n+1$ players choose actions according to the decision rule $d_i : H_i^n \to \Delta(A_i)$ in Γ as a function of the realized and observed communication history $h_i^n \in H_i^n$. Let $d = (d_i)_{i\in I}$. Each player i then receives his payoff according to g_i. Clearly, there is an induced distribution on $H^n \times A$ which we denote by $P^{c,d}$.

Solution concept. Our solution concept is weak perfect equilibrium as defined in MWG, henceforth PBE. The belief maintaining our equilibria are super plausible as we will discuss it later.

Let $PBE(\Gamma)$ be the set of outcomes in Γ induced by perfect Bayesian equilibria of finite cheap talk extensions of Γ: a probability distribution $\mu \in \Delta(A)$ is in $PBE(\Gamma)$ if and only if there exists a cheap-talk extension of Γ and a perfect Bayesian equilibrium of that extension that induces μ.

[1]This assumption is not necessary, only the penultimate stage of communication and in case of certain deviations the last stage of communication should be done simultaneously.

A probability distribution $\mu \in \Delta(A)$ is a correlated equilibrium of Γ if and only if:

$$\sum_{a \in A} \mu(a) \left(g_i(a) - g_i(a_{-i}, \delta_i(a_i))\right) \geq 0 \, \forall i \in I, \, \forall \delta_i \, : \, A_i \to A_i.$$

We say that a correlated equilibrium μ is *rational* if for every action profile in A, the probability $\mu(a)$ is a rational number. Let $C(\Gamma)$ be the set of rational correlated equilibria of Γ.

The main result. Our main theorem is the following.

Theorem 1. *Let Γ be a finite normal-form game with four players, and let $\mu \in C(\Gamma)$ then $\mu \in PBE(\Gamma)$.*

Remark 1. Our result generalizes to sequential equilibrium and to the case of incomplete information games, i.e. to the set of (regular) communication equilibria.

2 Proof of the theorem

The proof is constructive. First we introduce an auxiliary protocol à la Bárány (1992). Then we construct several other protocols. When a public message of a player or his private message to another at a certain stage is not specified then it is assumed that this player babbles, i.e. uses a completely mixed behavioral strategy over $\mathcal{M}$. $\mathcal{M}$ is chosen to be finite but large enough so that players can send all the messages specified by the equilibrium at once at any stage of the communication. Note however that most of the communication can be done sequentially (politely). We will point out those stages where simultaneity is important, basically the last two stages of the communication. The length of the communication phase n is chosen to be large enough and is determined by the length of the longest protocol plus one. It is because all the protocols are assumed to be run simultaneously, what is important however is that their last stages are performed simultaneously (within and across the protocols) at stage $n-1$ (as there will be an extra round of communication). In equilibrium and also after certain out of equilibrium histories in the nth stage all the players babble and only after certain out of equilibrium histories it is specified that what messages the players should send in stage n.

We summarize in several lemmas the important properties of the different protocols which are then used to prove that our construction is indeed an equilibrium, given out of equilibrium behavior, and that it induces the desired correlated equilibrium outcome. Importantly, we discuss players beliefs and equilibrium strategies out of equilibrium as well.

Let $\mu \in \Delta A$ be an arbitrary correlated equilibrium distribution of Γ with rational entries. Let E be a finite set which is partitioned into $(E_a)_{a \in A}$ in such a way that $|E_a|/|E| = \mu(a)$ for all $a \in A$. For all $i \in I$ let $pr_i : E \to A_i$ such that $pr_i(e) = a_i$ if and only if $e \in E_a$. Latin letters are element of E (e.g. $e \in E$) and Greek letters are bijections (or permutations) from E to itself so their inverse exists. We write $\alpha\beta$ for the composition of two such functions (or the product of two permutations) and by abusing notation we write $\alpha e \in E$ denoting the image of e under α (i.e. instead of $\alpha(e)$). All random choices are specified to be uniform over the specified finite sets.

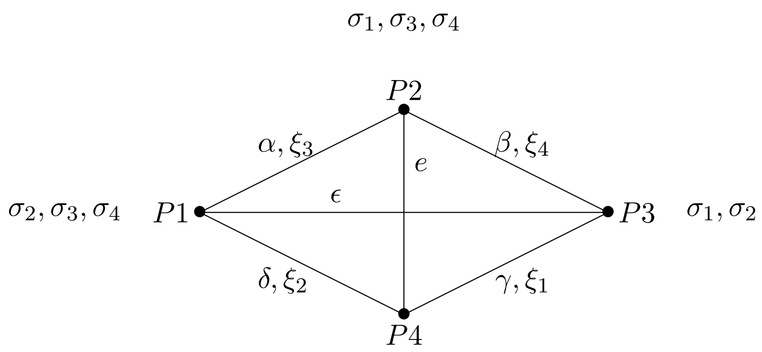
2.1 Auxiliary protocol à la Bárány (1992): $\mathcal{B}^+$

stage 0 **free choices of** $\alpha, \beta, \gamma, \delta, \epsilon, (\xi_i)_{i \in I}$ **and** $e \in E$**:**
- 1 chooses α, ξ_3 and sends it to 2
- 2 chooses β, ξ_4 and sends it to 3
- 3 chooses γ, ξ_1 and sends it to 4
- 1 chooses δ, ξ_2 and sends it to 4
- 1 chooses ϵ and sends it 3
- 2 chooses e and sends it to 4

choices of $(\sigma_i)_{i \in I}$**:**
- 2 chooses σ_1 and sends it to 3
- 1 chooses σ_2 and sends it to 3
- 1 chooses σ_3 and sends it to 2
- 1 chooses σ_4 and sends it to 2

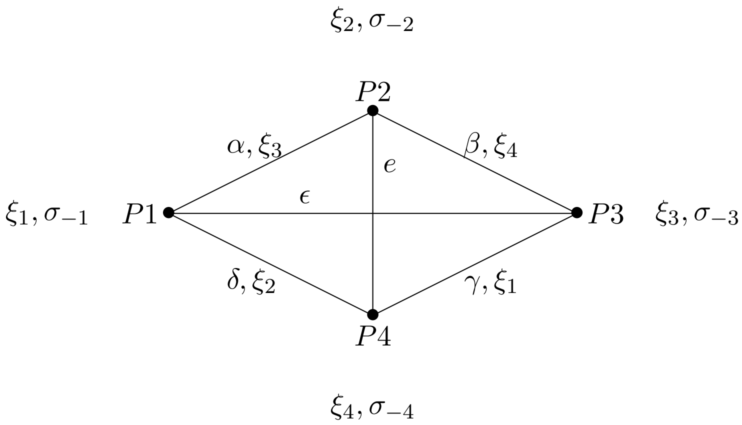
So we have the following picture representing the knowledge of the players:

Figure 1: Random permutations known by the players at the end of stage 0.



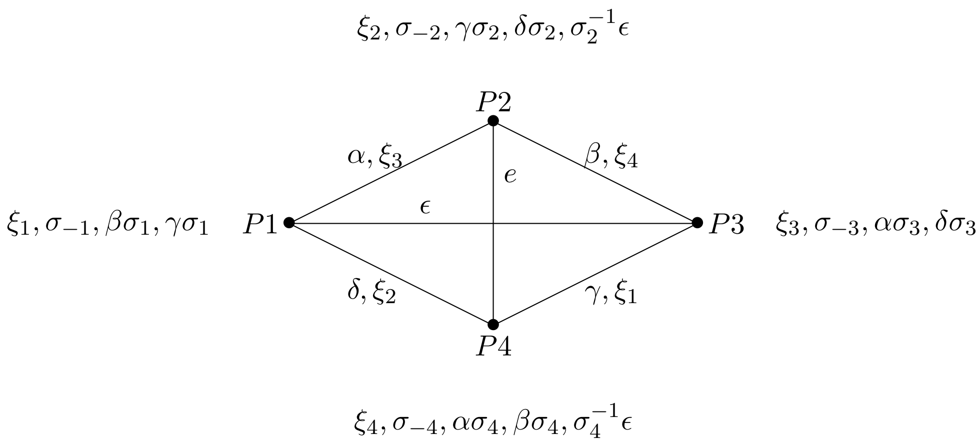
In the following stages all the messages are sent by two players. First, in stage 1 σ_i-s and ξ_i-s are distributed in such a way that i learns ξ_i and only $-i$ learn σ_i and we denote the knowledge of i about the σ_j-s with σ_{-i}. So we get to the following table:

Figure 2: Random permutations known by the players at the end of **stage 1**.



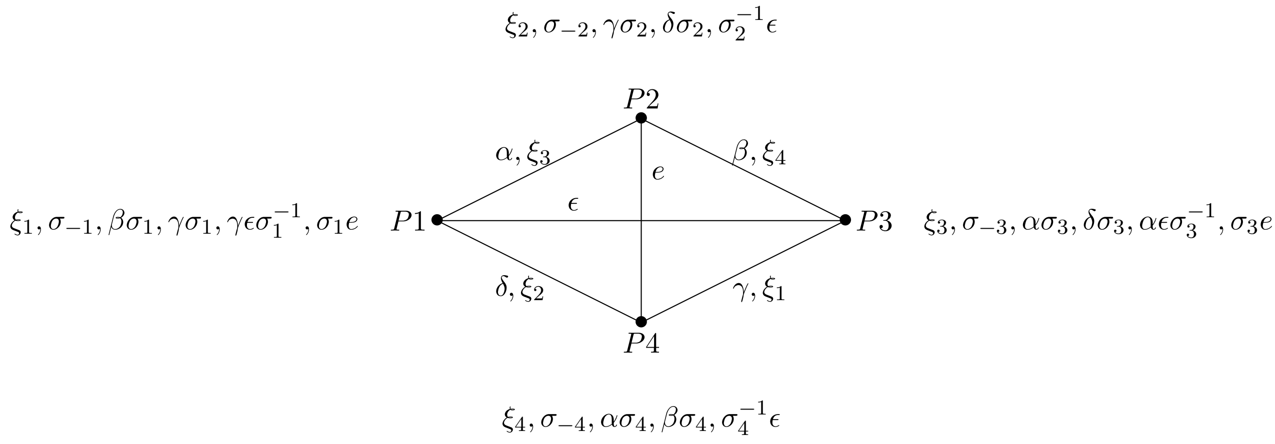
After stage 2 we get to the following table:

Figure 3: Random permutations known by the players at the end of **stage 2**.



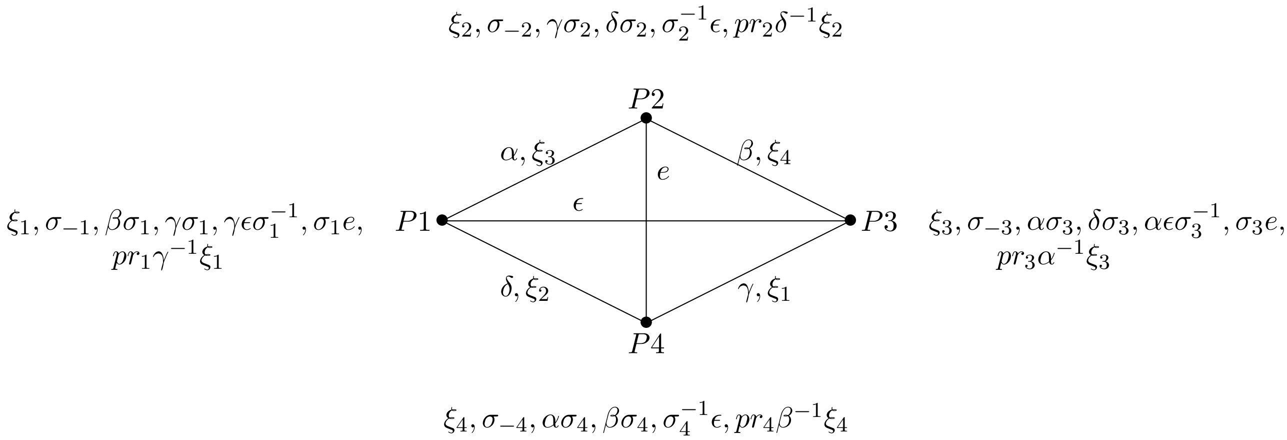
Notice that players 2 and 4 can already calculate $\beta\epsilon e$ and $\delta\epsilon e$ respectively and they can also calculate $\gamma\epsilon$ and $\alpha\epsilon$ respectively. In the third stage players 1 and 3 learn what they need to calculate $\gamma\epsilon e$ and $\alpha\epsilon e$ respectively:

Figure 4: Random permutations known by the players at the end of **stage 3**.



Finally, in the last stage (stage 4) player i learns the appropriate transformation of ξ_i to be able to calculate $pr_i\epsilon(e)$. Notice that this is need only because these messages will be publicly announced later.

Figure 5: Random permutations known by the players at the end of **stage 4**.



We denote by $\mathcal{B}$ the protocol $\mathcal{B}^+$ without the last stage (4) messages which we call the *codes for decision rules*. Suppose that every player i chooses his own computed action $pr_i\epsilon e$ at the action stage of the extended game, then the protocol, together with these decision rules d induce a distribution $P \in \Delta(H \times A)$.

Lemma 1. *From Theorem 1 in Bárány (1992) we have the following properties:*

1. *$P(a) = \mu(a)$ for all $a \in A$,*
2. *For every i, a_{-i} and h_i which has positive probability under P: $P(a_{-i}|h_i) = \mu(a_{-i}|d_i(h_i))$, and for any subhistory h_i^t: $P(a|h_i^t) = \mu(a)$.*

3. *Unilateral deviations in randomization (which can only happen in stage 0) do not affect properties (1) and (2),*

4. *From stage 1 on any message which is sent by some player i to player k is also sent by some player $j \neq i$ to player k and hence unilateral deviations from stage 1 on are detected instantaneously with probability 1.*

5. *Messages in the last stage (4), i.e. the codes for decision rules can be announced publicly and simultaneously without affecting any of the above properties.*

Proof. Properties (1) and (4) follow by construction. Property (2) and (3) follow from the fact that even in case of a unilateral deviation in randomization $\epsilon(e)$ is distributed uniformly and it is independent of the rest of the random variables. This is because $\epsilon(e)$ is chosen jointly by players 1 and 2 (see the detailed proof in Bárány (1992) and in Gerardi (2004)). Property (5) follows from the slight modification of Bárány (1992) by the usage of the "covering" permutations $(\xi_i)_{i \in I}$. □

In what follows we derive 11 protocols from $\mathcal{B}$ all of which are run independently in an arbitrary order. When all these protocols terminated (in stage $n-2$) we add a last stage (stage $n-1$) to each of them in which players communicate simultaneously, possibly publicly according to the last stage of $\mathcal{B}^+$ together possibly with some other messages with which players can check whether there was any deviation or not. All of these protocols will be run independently so any stage 0 randomization is independent across the protocols. Finally, we add one more stage the nth stage of communication in case of certain deviations happened before.

2.2 The master protocol: $\mathcal{P}0$

Consider the protocol $\mathcal{B}$ and modify it as follows. Leave stage 0 unchanged and if a message m in a later stage is sent by players i and j to k in $\mathcal{B}$, say $i < j$, then let only i send the message m to k and let j choose a permutation λ_m randomly, we call them *key generators*, and send it to i and to k. We say that in this case m *was sent with the key* $\lambda_m m$ between i, j and k.

Let $\mathcal{P}0^+$ denote the protocol in which after $\mathcal{P}0$ is over, in stage $n-1$ the players announce publicly and simultaneously (important!) the followings:

1. their messages as in the last stage (4) of $\mathcal{B}^+$, i.e. the codes for decision rules,

2. for all the m-s which were sent with a key in $\mathcal{P}0$ between some i, j and k, players i, j, k compute and announce the key $\lambda_m m$.

In case of no deviation in $\mathcal{P}0^+$ the code for every player decision is announced by two other player and they must coincide. Also all the keys of all the messages must coincide across the players between whom a message was sent with a key. In case of deviation in $\mathcal{P}0$ one of those triplets must be incompatible with each other with probability 1, i.e. there must be a message which was sent with a key between i, j and k and the publicly announced keys for this message should differ across these players. Deviations during only the public announcement are also detected by these incompatibilities or by the differing codes for a player's decision rule. Hence we have the following lemma.

Consider the corresponding decision rules and the induced distribution.

Lemma 2. *All the properties of $\mathcal{B}^+$ but property (4), as stated in lemma 1, are inherited to $\mathcal{P}0^+$. Instead of property (4) we have that any private history in $\mathcal{P}0$ has positive probability and any unilateral deviation is detect with probability 1 in $\mathcal{P}0^+$ at the end of the protocol.*

Remark 2. In case no deviaton was detected in $\mathcal{P}0^+$, which will be the case in equilibrium, players babble in stage n and then players choose their action according to the result of $\mathcal{P}0^+$.

2.3 The protocol when i and j do not talk to each other: $\mathcal{P}ij$

Consider now the master protocol $\mathcal{P}0$ and modify it as follows. Whenever a message m is sent from i to j in $\mathcal{P}0$ let i choose a random permutation η_m, which we will call a *splitting code* and send η_m to k and send $\eta_m m$ to l and then the protocol requires k and l to forward these messages to j. Let us stress that m in the previous sentence can be any message which is sent in $\mathcal{P}0$ including the random key generators and the stage 0 messages, in short any message which is prescribed by $\mathcal{P}0$. Let us apply similar changes when a message is sent from j to i in $\mathcal{P}0$. We say that such m messages are *sent in a split* $(\eta_m, \eta_m m)$ between i and j through k and l.

Let $\mathcal{P}ij^+$ denote the protocol in which after $\mathcal{P}ij$ is over, in stage $n-1$ the players announce publicly and simultaneously (important!) their messages just as in $\mathcal{P}0^+$ namely the codes for decision rules and the keys $\lambda_m m$ which are corresponding to messages m and key generators λ_m including those which were sent in a split between i and j through k and l. Notice that there were no keys generated for the splits $(\eta_m, \eta_m m)$ but there is λ_m key generator for m and they both might be sent in a split (however not at the same time).

Consider the corresponding decision rules and the induced distribution.

Lemma 3. *All the properties of $\mathcal{P}0, \mathcal{P}0^+$, as stated in lemma 2, are inherited to $\mathcal{P}ij, \mathcal{P}ij^+$ respectively. An additional property we have is that i and j never send direct messages to each other in $\mathcal{P}ij$.*

Remark 3. In case a deviation was detected in $\mathcal{P}0^+$, but there is an i, j such that no deviation was detected in $\mathcal{P}ij^+$ then (after babbling in stage n) players will choose their actions according to the result of that protocol. If there a more such protocols then players play according to the first (according to some commonly known order) such protocol.

2.4 The protocol when i does not talk: $(\mathcal{P}i)$

Consider again the protocol $\mathcal{B}^+$ and modify it as follows. In case player i sends some random message to any player j in stage 0 in $\mathcal{B}^+$, let now instead in $\mathcal{P}i^+$ player j select randomly and send the selected object to i. Also, in case player i sends some message in the rest of stages in $\mathcal{B}$, let player i remain silent in $\mathcal{P}i^+$. Finally, when players $j, k \in -i$ are specified to send the same message to some player l (which can be also i) then it is only the player with the smaller index who sends the message and the other remains silent.

Let $\mathcal{P}i$ denote the protocol without the last stage of $\mathcal{P}i^+$, i.e. without sending the codes for the decision rules and let us postpone the last stage of $\mathcal{P}i^+$ to stage $n-1$. We note that the last stage of $\mathcal{P}i^+$ is prescribed to be performed through private channels (not important just simplifies later!) as in $\mathcal{B}^+$. Consider the corresponding decision rules and the induced distribution.

Lemma 4. *All the properties of $\mathcal{B}^+$ but property (4), as stated in lemma 1, are inherited to $\mathcal{P}i^+$. Instead of property (4) we have that any private history in $\mathcal{P}i^+$ has positive probability and no deviation from the rules are ever detected.*

Remark 4. In case deviations were detect in $\mathcal{P}0^+$ and in all the six $\mathcal{P}ij^+$ protocols, after an additional stage (stage n) communication, the players will be able to identify the deviator, say player i, and choose actions according to the result of the protocol in which the deviator was not sending any message, i.e. according to $\mathcal{P}i^+$.

2.5 Putting the 11 protocol together: The main protocol and the decision rules

Notice that protocols $\mathcal{P}0$ and $\mathcal{P}i$ for all i has the same length as $\mathcal{B}$, i.e. 4 stages. $\mathcal{P}ij$ protocols are however longer due to the messages which are sent in a split. We needed to synchronize the only last stage (the $^+$ stage) across all the protocols, and within the protocols, which we have already prescribed to be performed at stage $n-1$. Notice that in case players would know that according to which protocol they will choose their final action, then they knew already their own actions in stage $n-1$. The main protocol then continues with stage n communication and with the choices of actions in stage $n+1$ as already hinted by remarks 2,3 and 4 above on which we elaborate now.

Case 0: on the path

On the equilibrium path players should babble in stage n and then choose actions according to the results of $\mathcal{P}0^+$. It follows that if players were on the path at stage $n-1$ no stage n deviation can suprise them anymore because every message has positive probability.

Case 1: off the path

Notice that, by the point (4) of lemmas 2,3 and 4 above, players may find themselves in an out of equilibrium information set for the first time only in stage $n-1$, where their belief will be consistent with the assumption that there was only a single deviator. This means that they believe that at least one of the $\mathcal{P}i^+$ protocols is unaffected by the deviator and that in protocols where no deviation was detected are also not manipulated (remember only unilateral deviations are detected with probability one).

Now we describe players stage n equilibrium communication strategies and how they choose actions after any possible realized history. We also specify the players' out of equilibrium beliefs. So suppose that players are off the equilibrium path in stage $n-1$.

Case 1a

In case no deviation was detected in $\mathcal{P}0^+$ or there is an i, j such that no deviation was detected in $\mathcal{P}ij^+$ players babble in stage n and actions are chosen according to results of the first (non-faulty) protocol (according to some commonly known order) where no deviation was detected. Players believe that there was a single deviator along the first $n-1$ stages. Hence they believe that the selected protocol is indeed not manipulated and form their beliefs about other players' action according to point 2 of lemma 1.

Case 1b

In case deviations were detect in $\mathcal{P}0^+$ and in all the six $\mathcal{P}ij^+$ protocol, then equilibrium (continuation) strategies require each player to report truthfully all the messages they have sent and received along the first $n-2$ stages but only in this 7 faulty protocols but not those of $\mathcal{P}i^+$. Additionally, deviator(s) of the first $n-1$ stages are required to send a special message which we call *confession* and those who did not deviate in the first $n-1$ stages should send the message *no confession*.

To define, how players choose actions in Case 1b we only have to consider histories which are consistent with the assumption that a single player has deviated in the first $n-1$ stages and that in the nth stage nobody has deviated or a single player has deviated (which can be of course the same as before or even a different player). After other histories an arbitrary Nash equilibrium of the underlying game is played.

1. In case of a single confession by player i players choose actions according to the result of $\mathcal{P}i^+$ and they believe that indeed it was player i who deviated. Hence they believe that the selected protocol is indeed not manipulated and form their beliefs about other players' action according to point 2 of lemma 1.

2. In case of a double confession by players i, j it must be that both players have deviated and they also know who has deviated in the first $n-1$ stages and who in stage n, however this, without any further calculations, is unknown to the non-deviant players k, l who are surprised again. These players can trust in each others stage n report and given that they together have controlled all the messages in protocol $\mathcal{P}ij^+$ they can determine the order of deviators and choose actions according to the result of $\mathcal{P}i^+$ or $\mathcal{P}j^+$ depending on who turned out to be the deviator along the first $n-1$ stages. Of course, so do players i and j as well.

3. Finally, in case of no confession, note that it must be (by unilateral deviation) that the first deviator has deviated again and the others reported in stage n honestly. Then players consider stage n reported messages of all the protocols (but the $\mathcal{P}i^+$ protocols about which there were no reports in stage n) and stage $n-1$ public announcements. They identify a deviator according to the following rules below, and play according to the result of $\mathcal{P}i^+$ if i was the identified deviator believing again that this protocol was not manipulated by another player (by unilateral deviations).

The deviator is identified as follows. We say that two players are in conflict if there is a message about which they disagree: the receiver of a message reported a received message m and the sender of that message reported a sent message $m' \neq m$ in stage n.

(a) If no players are in conflict then the deviator is identified by assuming that all the players reported honestly in stage n. This assumption comes from assuming a unilateral deviation in stage n.

(b) Notice that the deviator must be in conflict with at least two non-deviants and non-deviants are never in conflict with each other. It is not possible that the deviator, say i, is only conflict with an another player j. It is because in protocol $\mathcal{P}ij^+$ they do not communicate with each other and yet we know that deviation was detected in this protocol (remember we are still in Case 1b). So there must be a (unique) player who is in conflict with at least two other players in which case players identify this player as the deviator.

We have completed the full description of the main protocol. In what follows we prove that this is indeed a PBE and implements the desired correlated equilibrium with which we conclude the proof our theorem.

2.6 Verifying equilibrium conditions

In our putative equilibrium described in the previous paragraph (Case 0) players choose actions following the results of $\mathcal{P}0^+$ which according to point 1 of lemma 2 implements μ and according to point 2 of lemma 2 it is sequentially rational for the players to play according to their calculated actions, since μ is a correlated equilibrium. A deviator expects the same payoff if he does not manipulate $\mathcal{P}0^+$ and all the six $\mathcal{P}ij^+$, because otherwise the protocol prescribes (Case 1a) to choose actions according to one of the non-manipulated protocols (remember that by points 4 of lemma 2 and 3 deviations are detected with probability 1) which again promises the same payoff as on the path (by points 1 of lemmas 2 and 3) and again by points 2 of lemmas 2 and 3 it is sequentially rational for all the players (including the deviator) to play according to their calculated actions. Hence, the only chance of deviator i to obtain a better payoff is to manipulate $\mathcal{P}0^+$, all the six $\mathcal{P}ij^+$, and some or all of the $\mathcal{P}j^+$ protocols (where $j \in -i$) and drive the communication to Case 1b. In this case the deviator should not confess, because that triggers Case 1b1 and play will be according to $\mathcal{P}i^+$ which the deviator unable to manipulate and again according to point 1 of lemma 4 μ is implemented and by point 2 of lemma 4 it is sequentially rational for all the players to play according to their calculated actions. So deviator i must trigger Case 1b3 (he cannot get to 1b2) and hope that he can avoid getting identified. But players $-i$ honestly report in stage n so i can only induce Case 1b3a or 1b3b and clearly he will be identified as a deviator.

Finally, we have to check sequential rationality of all the players when they find themselves out of the equilibrium path in stage $n-1$ and they are in Case 1b (otherwise we have checked already sequential rationality). We have also seen that the deviator cannot do better than by confessing because in any case he will be identified as the deviator. However, some player $j \neq i$ may be so unlucky, that his calculated action in protocol $\mathcal{P}i^+$ (which he knows already in stage $n-1$) promises him a very bad payoff and he would prefer to play according to some other protocol, say according to $\mathcal{P}j^+$ or a Nash equilibrium, by convincing the others that he was deviating in the previous stages. Player j's only chance is to deviate in stage n, confess and reach Case1b2 because i cannot deviate there (it is j who is deviating, so Nash plays are not available) and i also confesses (and reports honestly). Remember, if j is not confessing but he only dishonestly reports in stage n, players end up in Case 1b1 and still $\mathcal{P}i^+$ will be used to determine final actions. However, as we pointed out in Case 1b2, k and l will identify i as the early deviator and play will sequentially rationally follow $\mathcal{P}i^+$.[2] Q.E.D.

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[2]Aside: i does not even need to report honestly in stage n.