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Asymmetric Information and Credit Rationing in a Model of Search

Cemil Selcuk

Department of Economics, Cardiff Business School, Cardiff University, Cardiff CF10 3AT, UK; selcukc@cardiff.ac.uk

Abstract: This paper presents a competitive search model focusing on the impact of asymmetric information on credit markets. We show that limited entry by lenders results in endogenous credit rationing, which, in turn, plays a key role in managing adverse selection and prevents the credit market from collapsing.

Keywords: asymmetric information; credit rationing; directed search

JEL Classification: D82; D43; G20

1. Introduction

In credit markets, informational asymmetry arises when financiers cannot assess the success potential of entrepreneurs, leading to adverse selection. When entrepreneurs differ in terms of their probability of success (Stiglitz & Weiss, 1981), this information gap creates a scenario similar to what Akerlof (1970) termed the "lemons problem", which can ultimately cause the credit market to collapse.

With this issue in mind, this paper focuses on how market participants interact with each other and how this may impact the issue of adverse selection. Traditional models frequently assume a Walrasian market, where matches between creditors and entrepreneurs are instantaneous and frictionless. By contrast, our paper adopts a directed/competitive search model in the tradition of Guerrieri et al. (2010), capturing the decentralized and frictional characteristics, which are typical in real-world credit markets.

We consider an economy with homogeneous, risk-neutral lenders and heterogeneous entrepreneurs seeking financing for investment projects. Entrepreneurs differ in their likelihood of success: high types have a greater chance of success, while low types are less likely to succeed. Lenders simultaneously and independently post contracts specifying collateral and interest rate terms. After observing all available contracts, borrowers direct their search to the most favorable option. Each lender can finance only one project. Given the decentralized nature of the matching process and the capacity of each lender, some lenders may receive no applications, and some borrowers may not secure financing.

Under complete information, where lenders can accurately distinguish between entrepreneur types, contracts are tailored to each type's risk profile. High-type entrepreneurs receive favorable contracts with relatively low collateral and interest requirements. Low types, on the other hand, face contracts that account for their greater risk. Crucially, the separating equilibrium comes with full credit allocation without rationing. This is because, in equilibrium, sufficiently many lenders enter the market for each type. The favorable terms incentivize low types to misrepresent themselves; however, with complete information, this is not an issue as lenders can tell who is who.

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Under incomplete information, lenders cannot identify the different types, so they design separating contracts with interest rates and collateral requirements that encourage self-selection. The key problem is to disincentivize low types from misrepresenting themselves. This issue is resolved in two ways: First, high-type contracts are fully collateralized, i.e., lenders demand the maximum possible collateral as part of such contracts. While this demand does not deter high types, it discourages low types due to their greater risk of failure. Second, and more importantly, only a limited number of lenders enter the high-type market and offer credit, restricting the availability of such contracts. This limitation in supply results in credit rationing and discourages low types from making an application.[1] Ultimately, credit rationing, coupled with fully collateralized loan requirements, prevents the credit market from collapsing because otherwise, it is impossible to distinguish between the different types and prevent adverse selection.

Endogenous credit rationing is not always enough to keep the market operational, though: an intermediate scenario arises where low types have success probabilities too small to justify tailored contracts for them, but still large enough that they pursue high-type contracts.[2] In this region, lenders cannot prevent them from applying for high-type contracts. Consequently, the market collapses, as lenders cannot offer any contract without being exposed to adverse selection.

Related Literature. Our paper is broadly related to the vast literature on asymmetric information in credit markets, which has evolved significantly since Akerlof (1970)'s seminal work on the "lemons problem", e.g., see Stiglitz and Weiss (1981), Bester (1985), or Besanko and Thakor (1987), among many others. We differ from these studies by considering a fundamentally different market setup that incorporates search and matching, as opposed to a frictionless Walrasian market.

Our model is based on a competitive search framework in the tradition of Burdett et al. (2001), Wright et al. (2021), or more recently Selcuk (2024). The paper by Guerrieri et al. (2010) incorporates asymmetric information into a competitive search setup and explores how adverse selection in such an environment can be managed. Our paper extends this line of research by specifically analyzing credit markets, where the interaction between search frictions and asymmetric information creates equilibrium outcomes with credit rationing.

Additionally, our work is related to the literature on search models in credit markets, such as Vesala (2007), who examine market liquidity and competition under asymmetric information; Dong et al. (2016), who analyze credit rationing under search frictions (but with no asymmetric information); and Davoodalhosseini (2019), who further examines the efficiency of the equilibrium in Guerrieri et al. (2010). We contribute to this line of work by focusing on endogenous credit rationing and collateral requirements, and how they can be used to manage informational asymmetry in a decentralized market setting.

2. Model Setup

The economy consists of a continuum of heterogeneous borrowers (entrepreneurs) and homogeneous lenders. Each lender has \$1 available to lend and must choose between entering the credit market or staying out. Entering the market involves lending to an entrepreneur who undertakes a risky project. Alternatively, by staying out, a lender can invest in a risk-free asset with a guaranteed return of $1 + t$.

Each entrepreneur requires \$1 to finance their project. If successful, a project yields a payoff of $1 + g$. The probability of success depends on the entrepreneur's type: high-type entrepreneurs have a success probability of p_h, while low-type entrepreneurs have a success probability of p_l, with $p_h > p_l$.

Entrepreneurs possess illiquid assets, which cannot be used directly for financing but can be pledged as collateral to secure a loan from lenders. Lenders offer contracts defined by an interest rate $r \in (0, g)$ and a collateral requirement c. The collateral requirement can range from 0 to \$1, corresponding to no collateral or full collateralization, or any value in between.

The search process for matching lenders with borrowers occurs in the form of directed search (Guerrieri et al., 2010): First, lenders simultaneously announce contract terms (r, c). After observing the available contracts, borrowers select one to apply. Once matching takes place and the contracts are awarded, projects commence, and eventually, payoffs are realized. Successful projects return $1 + g$, while failed projects lead to forfeiture of the pledged collateral.

Our focus will generally be on separating equilibria, where some creditors target high types while others cater to low types. Let θ_i represent the lender-to-entrepreneur ratio (also called "market tightness") in the market for type $i = h, l$ entrepreneurs.[3] These parameters are determined endogenously through free entry by lenders. For instance, if no lender is willing to lend to low-type borrowers, then $\theta_l = 0$, which effectively shuts down the market for low types. This decision would naturally impact the market for high types as well (more on this later). The probability that a borrower finds a lender in market i is given by

$$\mu(\theta_i) = \min\{\theta_i, 1\}.$$

Similarly, the probability that a lender finds an entrepreneur is $\mu(\theta_i)/\theta_i$. If $\theta_i > 1$, all entrepreneurs are matched with lenders, though some lenders will remain unmatched. Conversely, if $\theta_i < 1$, all lenders are assured a match with an entrepreneur, while some entrepreneurs will not secure financing.[4] This shortage of credit for entrepreneurs when $\theta_i < 1$ reflects credit rationing. Thanks to the functional form of $\mu(\theta_i)$, every participant on the "short side" of the market is guaranteed a match. Importantly, the short side is not exogenous, but instead, it is determined endogenously via θ_h and θ_l.

3. Complete Information

With complete information, lenders can distinguish between borrower types. The utility function of a type i borrower is given by

$$U_i(r_i, c_i, \theta_i) = \mu(\theta_i)[p_i(1 + g - r_i) + (1 - p_i)(1 - c_i)] + 1 - \mu(\theta_i). \quad (1)$$

With probability $\mu(\theta_i)$, the borrower can access credit. If the project is successful (with probability p_i), the borrower retains a payoff of $1 + g$ after repaying the interest r_i. Conversely, if the project fails (with probability $1 - p_i$), the borrower forfeits their collateral c_i and retains $1 - c_i$. The term $1 - \mu$ accounts for the scenario where the borrower cannot find a lender, in which case they keep their entire collateral value of 1.

The profit for a lender targeting type i borrowers is equal to

$$\pi_i(r_i, c_i, \theta_i) = \frac{\mu(\theta_i)}{\theta_i}[p_i(1 + r_i) + (1 - p_i)c_i] + 1 - \frac{\mu(\theta_i)}{\theta_i}. \quad (2)$$

The term μ/θ_i represents the probability that a lender meets a borrower. If a borrower is found and the project succeeds (with probability p_i), the lender receives back their initial loan amount 1 plus interest r_i. If the project fails (with probability $1 - p_i$), the lender retrieves the collateral c_i. Finally, the term $1 - \mu/\theta_i$ captures the situation where the lender cannot find a borrower, in which case they retain their capital amount of 1.[5]

We focus on a separating equilibrium in which some creditors target high types, while others cater to low types. A representative lender operating in market i solves

$$\overline{U}_i = \max_{\theta_i, r_i, c_i} U_i(r_i, c_i, \theta_i) \quad \text{subject to:}$$
$$\text{(i)} \quad \pi_i(r_i, c_i, \theta_i) = 1 + t, \tag{P–CI}$$
$$\text{(ii)} \quad U_i(r_i, c_i, \theta_i) \geq 1.$$

Note that there are two problems in (P–CI), one for high types and one for low types. The solution to these problems defines the separating equilibrium, which is characterized by the market tightness for each borrower type, θ_i, and contract terms r_i and c_i, such that creditors compete to offer each type the highest possible payoff, denoted by $\overline{U}_i$. This competition is subject to two key constraints: (i) a free entry condition, which ensures that each lender earns a profit equal to the return from the risk-free asset[6], and (ii) a participation constraint, which guarantees that borrowers will only take out a loan if their expected utility from receiving credit is at least as large as their utility from holding onto their illiquid collateral. Note that we do not need an incentive compatibility constraint here because, with complete information, creditors can identify borrower types.

Proposition 1. *Credit is available to types $i = h, l$ as long as their probability of success is sufficiently high, i.e., $p_i \geq \frac{1+t}{1+g}$. Assuming this condition, the optimal level of entry in each market is $\theta_i^* = 1$, ensuring that credit rationing does not occur for either type. The optimal contracts (r_i^*, c_i^*) satisfy*

$$r_i^* = \frac{t}{p_i} + \frac{1-p_i}{p_i}(1 - c_i^*). \tag{3}$$

Sellers targeting high types offer credit with lower interest rates and/or reduced collateral requirements, while those targeting lower types offer credit with higher interest rates and/or stricter collateral conditions, as shown in Figure 1. Lower types have the incentive to misrepresent themselves to take out the more favorable contracts, however, with complete information, this is not an issue for lenders.

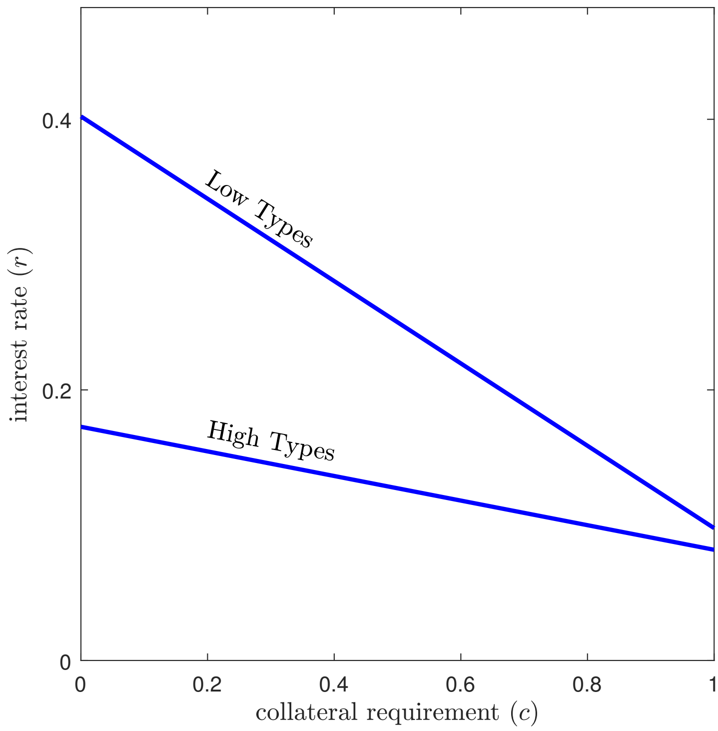

Figure 1. Credit contracts with complete information.

An important feature of the complete information setting is the fact that $\theta_h^* = \theta_l^* = 1$, which allows access to credit for both types without rationing. Thanks to their ability to distinguish between different entrepreneur types, lenders have no incentive to limit credit access; however, as we will see, this changes in a setting with incomplete information.

The participation condition $p_i \geq (1+t)/(1+g)$ determines whether a borrower will enter the market, where g represents the growth potential of a successful project and t reflects the return from the risk-free asset. If the growth rate g significantly exceeds t, even borrowers with a lower probability of success are inclined to participate due to the high potential payoff. Conversely, if t approaches g, only those with a high probability of success will find it worthwhile to participate.

Finally, partial market participation is also possible: if $p_l < \frac{1+t}{1+g} < p_h$ then only high types are offered credit contracts, and low types are excluded from the market. This exclusion poses no issue (i.e., low types do not attempt to pursue high-type contracts) as creditors can reliably identify different types.

4. Incomplete Information

We now examine the setting with incomplete information, where each borrower's type is private information, making it impossible for creditors to verify their likelihood of success. As in the previous section, we focus on a separating equilibrium. A representative lender catering to low types solves

$$\begin{aligned} \overline{U}_l = \max_{\theta_l, r_l, c_l} & \; U_l(r_l, c_l, \theta_l) \quad \text{subject to:} \\ \text{(i)} & \quad \pi_l(r_l, c_l, \theta_l) = 1 + t, \\ \text{(ii)} & \quad U_l(r_l, c_l, \theta_l) \geq 1, \\ \text{(iii)} & \quad U_h(r_l, c_l, \theta_l) \leq \overline{U}_h. \end{aligned} \tag{P-Low}$$

This setup resembles the complete information problem in (P–CI) but includes an incentive constraint. If a high-type borrower poses as a low type, they receive payoff $U_h(r_l, c_l, \theta_l)$; if they choose their own contract, then they obtain $\overline{U}_h$. The incentive constraint in (iii) is required to discourage them from opting for the low-type contracts.

Similarly, the problem for a lender catering to high types is given by

$$\begin{aligned} \overline{U}_h = \max_{\theta_h, r_h, c_h} & \; U_h(r_h, c_h, \theta_h) \quad \text{subject to:} \\ \text{(i)} & \quad \pi_h(r_h, c_h, \theta_h) = 1 + t, \\ \text{(ii)} & \quad U_h(r_h, c_h, \theta_h) \geq 1, \\ \text{(iii)} & \quad U_l(r_h, c_h, \theta_h) \leq \overline{U}_l. \end{aligned} \tag{P-High}$$

Here, the incentive constraint (iii) is key once again: if a low type attempts to pass as a high type they obtain $U_l(r_h, c_h, \theta_h)$. By choosing their designated contract, however, they receive $\overline{U}_l$. To ensure incentive compatibility we need $U_l(r_h, c_h, \theta_h) \leq \overline{U}_l$.

Now consider the larger problem (P) of solving (P-Low) and (P-High) simultaneously. A separating equilibrium is a tuple $(\theta_h, \theta_l, r_h, r_l, c_h, c_l)$ that solves the general problem (P). In what follows, we characterize this equilibrium in detail. Start with (P-Low). We conjecture, to be verified later in the Appendix A, that high types would not want to join a contract for low types, thus the incentive constraint (iii) is slack. In the absence of (iii), (P-Low) is identical to (P–CI), which we solved in the previous section. The solution, therefore, entails

$$\theta_l^* = 1, \quad r_l^* = \frac{t}{p_l} + \frac{1-p_l}{p_l}(1-c_l), \quad \overline{U}_l = p_l(1+g) - t. \tag{4}$$

We now turn to the second problem involving high types.

Lemma 1. *The incentive constraint in (P-High) must hold with equality.*

The proof is by contradiction: consider an outcome where the incentive constraint (iii) is slack. Absent (iii), the problem in (P-High) is the same as the problem in (P–CI), therefore the resulting contracts are also identical. Recall that in (P–CI) the contract for high types is more favorable, as such low types have an incentive to misrepresent themselves. In Appendix A, we show that without the incentive constraint, lower types would indeed choose to pass as high types, causing the equilibrium to collapse; a contradiction. Therefore (iii) must hold with equality. We can now proceed to characterize the equilibrium.

Proposition 2. *Under incomplete information, there exists a separating equilibrium characterized by the following:*

- *Low types experience no credit rationing, i.e., $\theta_l^* = 1$. Their contracts satisfy (4), but c_l^* is capped at $\bar{c}_l$.*
- *High types face credit rationing, with $\theta_h^* < 1$, where*

$$\theta_h^* = \frac{p_l(1+g) - t - 1}{p_l(1+g) - \frac{p_l}{p_h}t - 1}.$$

Their credit offer entails $c_h^* = 1$ and $r_h^* = t/p_h$.

The equilibrium is illustrated in Figure 2, showing the credit offers available to each type. Contracts for high types require full collateralization ($c_h^* = 1$), which is less discouraging for them given their higher probability of success, while lower types are more cautious about such collateral requirements.[7] However, full collateralization alone does not fully prevent lower types from applying. The equilibrium also includes credit rationing for high-type contracts ($\theta_h^* < 1$). Together, these features satisfy incentive compatibility and ensure the market remains functional for all participants.

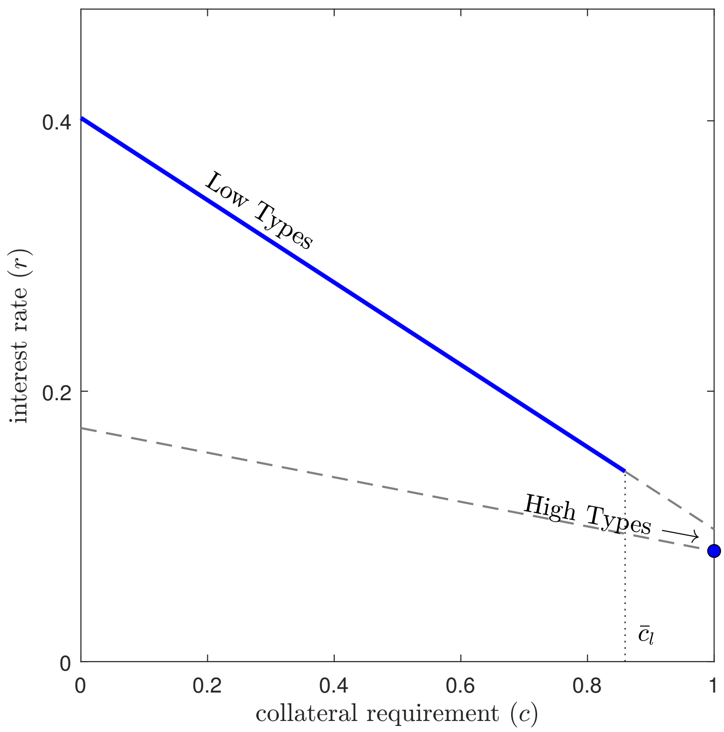

Figure 2. Separating equilibrium with credit rationing.

Moreover, it is easy to verify that θ_h^* decreases as p_l falls. This implies that as the difference between entrepreneur types widens—reflected by a lower p_l—credit availability for high types becomes more limited due to intensified credit rationing. Indeed, as the gap between types grows, the terms offered to low types worsen, giving them greater incentive to mimic high types. In response, creditors further limit the availability of high-type contracts. In this context, credit rationing serves as an effective risk management tool for lenders.

The contracts for low types must satisfy condition (4) as well as $c_l^* < \bar{c}_l$ to prevent high types from pursuing them. While the high-type contract features more attractive terms, its limited availability has the potential to make the more accessible low-type contracts seem appealing. By capping collateral for low types, thereby keeping interest rates high, creditors remove any incentive for high types to opt for the readily available low-type contracts.

As in the complete information scenario, the existence of equilibrium depends on the condition that $p_l \geq (1+t)/(1+g)$. However, a key difference arises here. In the complete information context, if low types do not meet this threshold, the market for them would collapse, but the market for high types would continue to operate since creditors can distinguish between the two. In the current setup, however, the two markets are interconnected. Therefore, if p_l falls below the threshold, it could result not only in the shutdown of the low-type credit market but potentially in the collapse of the entire market. In what follows, we explore this relationship.

4.1. Shutdown

Proposition 3. *Suppose low types' probability of success p_l is below $(1+t)/(1+g)$. Then the credit market shuts down either entirely or partially.*

- *If p_l is even below $1/(1+g-t/p_h)$ then low types avoid the credit market altogether while the market for high types remains operational. High types face no credit rationing, i.e., $\theta_h^* = 1$. Their contracts satisfy (3), but c_h^* is bounded below by $\underline{c}_h$.*
- *If p_l is above $1/(1+g-t/p_h)$ then low types cannot be prevented from applying to contracts designed for high types. Consequently, the separating equilibrium ceases to exist, leading to a complete shutdown of the entire credit market.*

If p_l is even below the threshold $1/(1+g-t/p_h)$, the market remains partially open, with only high types receiving credit offers. In this case, low types stay out of the market, while high types are offered contracts with collateral requirements bounded below by a minimum level, $\underline{c}_h$. This lower bound on collateral serves to discourage low types from making an application. In this region, lower type entrepreneurs recognize that their probability of success is so low that pursuing a contract intended for high types is simply not worthwhile. The considerable collateral requirement, combined with the minimal chance of success, exposes them to significant risk. If they fail, they would forfeit their collateral, ultimately leaving them in a worse position. Thus, when p_l is sufficiently small, the market functions partially, providing credit exclusively to high types. Figure 3 illustrates this scenario.

If, however, p_l lies above $1/(1+g-t/p_h)$ but still below $(1+t)/(1+g)$, then the market experiences a complete shutdown. The shutdown region lies in an intermediate zone where low types' probability of success is too small to warrant a contract tailored specifically for them (if they identify as low types), yet not so small that they refrain from attempting to acquire contracts designed for others. In this region, the entire market collapses, as lenders cannot offer any contract without being exposed to adverse selection.

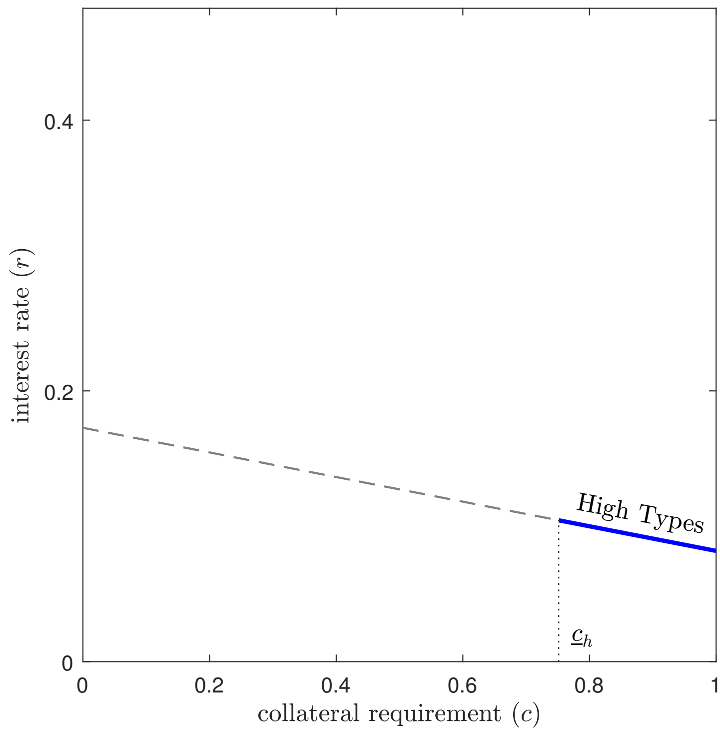

Figure 3. Partial shutdown—availability for high types only.

4.2. Pooling Contracts

Up to this point, our focus has been on a separating equilibrium, where each borrower type was offered a distinct credit contract. We now turn our attention to the possibility of a "pooling equilibrium", in which all types receive the same contract. We will first identify the conditions under which such an outcome is feasible. Subsequently, we will demonstrate that separating contracts dominate pooling contracts, meaning that in the parameter space where a separating equilibrium is viable, the pooling equilibrium will cease to exist (subject to a condition on the percentage of high types among the borrowers).

Suppose that in the existing pool of customers, fraction α are high types, while the remaining $1-\alpha$ are low types. Creditors recognize that they cannot observe the types of borrowers. Instead of working with p_h and p_l, they consider the "average" borrower to have a probability of success given by

$$\bar{p} = \alpha p_h + (1-\alpha)p_l.$$

Based on this weighted probability, creditors then offer a generic, average contract to all entrepreneurs. As we will demonstrate, there are parameter regions where this outcome can indeed materialize. However, when it does occur, it benefits low types at the expense of high types. In other words, such an outcome effectively cross-subsidizes lower types while disadvantaging higher types. This dynamic is crucial because it will be key to demonstrating that the pooling equilibrium collapses once separating contracts become available.

The profit of a creditor is given by

$$\tilde{\pi} = \frac{\mu(\theta)}{\theta}[\bar{p}(1+r) + (1-\bar{p})c] + 1 - \frac{\mu(\theta)}{\theta}. \quad (5)$$

This is similar to Equation (2) in the previous section, but with the key difference that the creditor does not observe individual p_is and thus uses $\bar{p}$ as a proxy for the probability. Also, θ, r, and c are not type-indexed. The payoff for a type i buyer is given by

$$\tilde{U}_i = p_i(1+g-r) + (1-p_i)(1-c). \quad (6)$$

Since lenders cater to a pool, the parameters r and c are uniform for all borrowers. However, the probability of success, p_i, remains type-specific, as each borrower type knows their own success probability. For a pooling equilibrium to emerge, we need $\tilde{U}_i \geq 1$, since each entrepreneur's outside option is to simply retain their illiquid collateral and walk away with a utility of 1.

Lemma 2. *A pooling equilibrium exists if $\bar{p} \geq (1+t)/(1+g)$.*

Our goal is to compare the separating and pooling equilibria. A separating equilibrium with active markets for both types exists only if $p_l > (1+t)/(1+g)$. Since $\bar{p}$ is greater than p_l, the condition for a pooling equilibrium is satisfied wherever a separating equilibrium exists. Note that because $\bar{p} > p_l$, the parameter region supporting a separating equilibrium is actually broader than the one supporting a pooling equilibrium.

Proposition 4. *Low types are strictly better off in a pooling equilibrium than in a separating equilibrium. In contrast, high types are worse off in the pooling equilibrium if the fraction of high types is less than a threshold $\bar{\alpha}$.*

The pooling equilibrium offers contracts based on the pooling probability $\bar{p}$, which is higher than p_l but lower than p_h. Consequently, low types receive more favorable terms than they would in a separating equilibrium and are clearly better off. As for high types, the opposite is true: they receive worse terms in a pooling equilibrium than they would in a separating equilibrium. However, there is a caveat: in the separating equilibrium, high types face limited availability via credit rationing. Although the terms of a separating equilibrium are more favorable, the limited availability makes the relative performance of the pooling equilibrium dependent on the fraction α.

Corollary 1 (of Proposition 4)**.** *If $\alpha \geq \bar{\alpha}$, then the pooling equilibrium is feasible. Conversely, if $\alpha < \bar{\alpha}$, the pooling equilibrium will cease to exist as high types opt for separating contracts instead.*

The existence of a pooling equilibrium depends on the presence of a sufficiently large percentage of high types. When α is high, the average probability of success $\bar{p}$ is close to p_h. This makes the pooling contract favorable for high types and reduces the incentive for them to seek out separate contracts. In this case, the pooling equilibrium is feasible. However, if α falls below the threshold $\bar{\alpha}$, then $\bar{p}$ is close to p_l, which makes the terms of the pooling equilibrium much less appealing for high types. Consequently, they gravitate toward separating contracts (despite their limited availability). This self-selection by high types disrupts the pooling equilibrium and leads to its breakdown.

4.3. Broader Implications for Policymakers

Our results establish that in equilibrium, credit availability can be intentionally limited and accompanied by very high collateral requirements. Even though such an outcome may be restrictive, it plays an important role in ensuring that the market remains functional under asymmetric information. Regulations or policies that try to make credit more widely available or lower collateral requirements could, therefore, unintentionally disrupt this equilibrium dynamics and lead to adverse selection and potential market collapse.

The key takeaway for financial market regulators is that non-intervention may often be the optimal approach in such market settings. By recognizing that credit rationing and demanding collateral requirements are equilibrium features rather than market failures, regulators can avoid unnecessary and harmful interventions. These insights highlight the importance of understanding market mechanisms before attempting to address inefficiencies in credit markets.

5. Conclusions

In this paper, we investigate a credit market with asymmetric information, where lender–borrower interactions are characterized by a decentralized search and matching process. Under complete information, markets for both types are operational and no credit rationing takes place in that there is enough entry into each market.

With incomplete information, however, lenders limit their participation in the high-type market, thus rationing credit for these borrowers. Credit rationing and the requirement for full collateral on high-type contracts deter low types from misrepresenting themselves. This helps sustain a separating equilibrium, ensuring that, even with information asymmetry and lenders' inability to differentiate types, both markets remain active. This setup not only addresses the challenges of asymmetric information but also shows how credit rationing acts as a key risk management tool in practice.

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Conflicts of Interest: The author declares no conflict of interest.

Appendix A. Omitted Proofs

Proof of Proposition 1. Recall that π_i is given by (2). Solving $\pi_i = 1 + t$ yields

$$r_i = \frac{t\theta_i}{\mu(\theta_i)p_i} + \frac{1 - p_i}{p_i}(1 - c_i). \quad \text{(A1)}$$

Substituting this expression into (1) and noting that $\mu(\theta_i) = \min\{\theta_i, 1\}$, there are two cases:

- Case 1. If $\theta_i \geq 1$, then $\mu(\theta_i) = 1$ and the maximization problem becomes

  $$\overline{U}_i = \max_{\theta_i, c_i} p_i(1 + g) - t\theta_i.$$

  The objective function decreases in θ_i. It follows that $\theta_i^* = 1$.
- Case 2. If $\theta_i \leq 1$, then $\mu(\theta_i) = \theta_i$, and the maximization problem becomes

  $$\overline{U}_i = \max_{\theta_i, c_i} \theta_i[p_i(1 + g) - 1 - t] + 1.$$

  If $p_i \geq \frac{1+t}{1+g}$, the expression inside the square brackets is positive, thus $\theta_i^* = 1$.

Substituting $\theta_i^* = 1$ into (A1) yields the relationship (3) in the body of the proposition. Substituting (3) into $\overline{U}_i$ yields the expected utility of type i borrowers in equilibrium as

$$\overline{U}_i = p_i(1 + g) - t.$$

Borrowers participate if $\overline{U}_i \geq 1$, which is equivalent to $p_i \geq (1 + t)/(1 + g)$. □

Proof of Lemma 1. By contradiction, suppose the incentive constraint (iii) in (P-High) is slack. In the absence of (iii), the problem in (P-High) is identical to the problem in (P–CI). The solution, therefore, entails

$$\theta_h = 1, \quad r_h = \frac{t}{p_h} + \frac{1 - p_h}{p_h}(1 - c_h), \quad \overline{U}_h = p_h(1 + g) - t. \quad \text{(A2)}$$

If a low type pretends to be a high type and attempts to join the above contract, they earn

$$\begin{aligned} U_l(r_h, c_h, \theta_h) &= \mu(\theta_h)[p_l(1+g-r_h) + (1-p_l)(1-c_h)] + 1 - \mu(\theta_h) \\ &= p_l(1+g) - \frac{p_l}{p_h}(1+t) + 1 - \frac{p_h - p_l}{p_h} \cdot c_h \end{aligned}$$

The first line: if a low type decides to join a contract for a high type, they face market tightness θ_h, interest rate r_h, and collateral requirement c_h; however, in calculating their payoff U_l they still rely on their own probability of success, p_l. The second line obtains after substituting for the expressions in (A2). If the buyer, instead, joins in the contract designed for low types, then per (4) they earn

$$\overline{U}_l = p_l(1+g) - t. \tag{A3}$$

It is straightforward to show that $U_l(r_h, c_h, \theta_h) > \overline{U}_l$ if

$$\frac{p_h - p_l}{p_h} \cdot (1 - c_h + t) > 0.$$

This inequality always holds because $p_h > p_l$ and $c_h \leq 1$. This means that low types would want to pose as high types and purchase those contracts instead of their own; this is a contradiction. So we cannot have an equilibrium where the (iii) is slack. □

Proof of Proposition 2. Solving constraint (i) yields

$$r_h = \frac{t\theta_h}{\mu(\theta_h)p_h} + (1 - c_h)\frac{1 - p_h}{p_h}. \tag{A4}$$

With this relationship the objective function becomes

$$\overline{U}_h = \quad \max_{\theta_h, c_h} \mu(\theta_h)[(1+g)p_h - 1] + 1 - t\theta_h.$$

Since $\mu(\theta_h) = \min\{1, \theta_h\}$ raising θ_h above 1 reduces $\overline{U}_h$. The implication is that the optimal θ_h must be less than or equal to 1, and therefore, $\mu(\theta_h) = \theta_h$. Problem (P-High) can, therefore, be rewritten as

$$\overline{U}_h = \max_{\theta_h, c_h} \theta_h[(1+g)p_h - 1 - t] + 1 \quad \text{subject to:}$$

$$\theta_h\left[(1+g)p_l - \frac{p_l}{p_h}(1+t) - \frac{(p_h - p_l)c_h}{p_h}\right] + 1 = \overline{U}_l.$$

Solving the constraint for θ_h we have

$$\theta_h = \frac{\overline{U}_l - 1}{p_l(1+g) - \frac{p_l}{p_h}(1+t) - \frac{p_h - p_l}{p_h}c_h}. \tag{A5}$$

Substitute this into the objective function

$$\overline{U}_h = \max_{c_h} \frac{[p_h(1+g) - 1 - t](\overline{U}_l - 1)}{p_l(1+g) - \frac{p_l}{p_h}(1+t) - \frac{p_h - p_l}{p_h}c_h} + 1.$$

Since $p_h > p_l$ the objective function rises in c_h; thus, the optimal value satisfies $c_h^* = 1$. The optimal values of θ_h and r_h follow from (A4) and (A5). Noting that $\overline{U}_l$ is given by (4), we have

$$\theta_h^* = \frac{p_l(1+g) - t - 1}{p_l(1+g) - \frac{p_l}{p_h}t - 1}, \quad r_h^* = \frac{t}{p_h}, \quad \overline{U}_h = \theta_h[p_h(1+g) - 1 - t] + 1. \tag{A6}$$

Borrowers participate if $\overline{U}_h \geq 1$ and $\overline{U}_l \geq 1$. Noting (4) and (A6), these are satisfied if both p_h and p_l exceed the threshold $\frac{1+t}{1+g}$. Since $p_h > p_l$, the relevant restriction pertains to the low types. Finally, we need to verify our earlier conjecture that the incentive constraint in (P-Low) is slack, i.e., high types would not want to pretend to be low types. This requires

$$\overline{U}_h > [p_h(1+g-r_l) + (1-p_h)(1-c_l)].$$

Substitute for r_l via (4) and the inequality holds if $\Delta > 0$, where

$$\Delta(c_l) := \theta_h[p_h(1+g) - 1 - t] - (1+g)p_h + \frac{p_h}{p_l}(t+1) - c_l \cdot \frac{p_h - p_l}{p_l}.$$

Note that Δ falls in c_l. It is straightforward to check that $\Delta(1) < 0$. Furthermore,

$$\Delta(0) = \frac{p_h - p_l}{p_l} \cdot [(1+g)p_l - 1 - t].$$

Note that $\Delta(0) > 0$ as we assume p_l to exceed the threshold $(1+t)/(1+g)$. By the Intermediate Value Theorem, there exists some critical $\bar{c}_l$ satisfying $\Delta(\bar{c}_l) = 0$, solving yields $\bar{c}_l = \theta_h^*$, which is given by (A6). It follows that for all $c_l < \theta_h^*$, we have $\Delta(c_l) > 0$, which means that the incentive constraint is slack. This completes the proof. □

Proof of Proposition 3. Start with (P-Low). Substituting constraint (i) into the objective function, (P-Low) can be rewritten as

$$\overline{U}_l = \max_{\theta_l, c_l} \theta_l[p_l(1+g) - 1 - t] + 1.$$

Since $p_l < (1+t)/(1+g)$, it is clear that the objective function falls in θ_l, which means that the optimal value is $\theta_l = 0$, i.e., no seller offers a contract to low types. Thus $\overline{U}_l = 1$. With this, (P-High) becomes

$$\overline{U}_h = \max_{\theta_h, c_h} \theta_h[(1+g)p_h - 1 - t] + 1 \quad \text{subject to:}$$
$$\theta_h\left[(1+g)p_l - \frac{p_l}{p_h}(1+t) - \frac{(p_h - p_l)c_h}{p_h}\right] \leq 0.$$

The first line is obtained after substituting constraint (i) in (P-High) into the objective function. Note that constraint (i) implies that the (r_h, c_h) pairs ought to satisfy (3). The second line is obtained after substituting $\overline{U}_l = 1$ into the incentive constraint (iii) in (P-High). Call the expression inside the square brackets in the incentive constraint $\Delta(c_h)$ and note that Δ falls in c_h. The upper bound for c_h is 1. Thus, if

$$\Delta(1) = p_l(1+g) - \frac{p_l}{p_h}t - 1 > 0 \Leftrightarrow p_l > \frac{1}{1+g-t/p_h},$$

then the solution entails $\theta_h = 0$ as low types cannot be prevented from joining the contract. The implication is that the market for high types, too, shuts down as creditors cannot feasibly satisfy incentive compatibility. If, however,

$$\Delta(1) < 0 \Leftrightarrow p_l < \frac{1}{1+g-t/p_h},$$

then there exists a unique $\underline{c}_h \in (0,1)$ satisfying $\Delta(\underline{c}_h) = 0$. Basic algebra reveals

$$\underline{c}_h = \frac{p_l[p_h(1+g) - (1+t)]}{p_h - p_l}.$$

Since Δ falls in c_h, we have $\Delta(c) \leq 0$ for all $c \geq \underline{c}_h$. The optimal solution entails picking any c above $\underline{c}_h$ and setting $\theta_h = 1$ as the objective function is increasing in θ_h and the upper bound is 1. With $c > \underline{c}_h$, the incentive constraint for low types is satisfied, i.e., they are better off staying out of the credit market. This completes the proof. □

Proof of Lemma 2. First, note that the market tightness parameter θ is relevant only for screening purposes. In a pooling outcome, all buyers are treated uniformly, rendering screening unnecessary. Thus, $\theta = 1$ and therefore $\mu = \min\{\theta, 1\} = 1$.

Creditors' profit $\tilde{\pi}$ is given by (5). The free entry condition remains as before, leading to

$$\tilde{\pi} = 1 + t \Leftrightarrow r = \frac{t}{\bar{p}} + \frac{(1-\bar{p})}{\bar{p}}(1-c).$$

Substituting r into $\tilde{U}_i$, given by (6), we have

$$\tilde{U}_i = p_i(1+g) - \frac{p_i}{\bar{p}}t - \frac{(1-\bar{p})p_i}{\bar{p}}(1-c) + (1-p_i)(1-c). \tag{A7}$$

For the pooling equilibrium to exist, we need $\tilde{U}_i \geq 1$, which is equivalent to

$$\bar{p} > \frac{p_i(1+t-c)}{p_i(1+g) - c}.$$

Assuming $p_i > \frac{1+t}{1+g}$, the right-hand side falls with respect to c, so the strictest case is $c = 0$, yielding $\bar{p} > (1+t)/(1+g)$. So if $\bar{p}$ exceeds this threshold then the pooling outcome is feasible. □

Proof of Proposition 4. The first claim obtains if $\tilde{U}_l$ exceeds $\overline{U}_l$, where the first expression is given by (A7) and the second by (A3). After substituting, we have

$$\tilde{U}_l > \overline{U}_l \Leftrightarrow \frac{\bar{p} - p_l}{\bar{p}}(t + 1 - c) > 0.$$

Noting that $\bar{p} > p_l$ and that $1 > c$, the inequality holds, establishing the first claim. As for the second claim, define $\Delta(\alpha) := \tilde{U}_h - \overline{U}_h$, which are given by (A6) and (A7), and note that Δ rises in $\bar{p}$, thus in α. In one extreme $\alpha = 1$ we have $\bar{p} = p_h$, and therefore,

$$\Delta(1) = (p_h(1+g) - t - 1)(1 - \theta_h).$$

The first term is positive since $p_h > \frac{1+t}{1+g}$ and the second term is positive since $\theta \leq 1$. Thus, $\Delta(1) > 0$. Similarly, when $\alpha = 0$ we have $\bar{p} = p_l$. Going through similar steps, it is straightforward to establish that $\Delta(0) < 0$. By the Intermediate Value Theorem, there exists some $\bar{\alpha} \in (0,1)$ such that $\Delta(\bar{\alpha}) = 0 \Leftrightarrow \tilde{U}_h = \overline{U}_h$. It follows that $\tilde{U}_h < \overline{U}_h$ when $\alpha < \bar{\alpha}$. This completes the proof. □

Notes

1 In our setup, credit rationing refers to the idea that credit availability is restricted, meaning not all applicants receive credit. A related concept involves offering a smaller amount than requested, which we do not consider here.

2 When entrepreneurs recognize that their likelihood of failure is considerably high, then the risk of forfeiting collateral deters them from pursuing credit by mimicking another type.

[3] Our reference to "market i" intends to represent two distinct contract types, one tailored to high types and the other to low types. This does not imply the concept of submarkets as in Moen (1997) restricting participants from making applications beyond a submarket. For instance, in our setup, low-type borrowers are not restricted from making an application to high-type contracts.

[4] While the functional form for the matching function simplifies the analysis, generalizing the matching function could provide additional insights into the role of search frictions. For instance, stronger search frictions might intensify credit rationing by limiting borrower–lender matches, whereas weaker frictions could ease such constraints, potentially affecting the existence of a separating equilibrium. For the sake of tractability, we refrain from such considerations in this paper.

[5] We assume that if a lender allocates capital to the credit market but is unable to find a suitable borrower, the capital cannot be redirected in time to take advantage of the risk-free investment option yielding $1 + t$. This assumption is without loss of generality; indeed, the results would still hold if we assumed that the lender could reallocate their capital. However, such a scenario would introduce additional terms involving t, complicating the analysis unnecessarily.

[6] Condition (i) further ensures that creditors earn the same expected profit no matter which type they target, so there is no arbitrage opportunity.

[7] Note that contracts for low types may involve zero collateral (accompanied with high interest rates). While zero-collateral contracts may seem unconventional for lower types, this approach is justified here as there is no moral hazard: borrowers use funds solely for their projects.

References

Akerlof, G. A. (1970). The market for lemons: Quality uncertainty and the market mechanism. *Quarterly Journal of Economics*, *84*(3), 488–500. [CrossRef]

Besanko, D., & Thakor, A. V. (1987). Competitive equilibrium in the credit market under asymmetric information. *Journal of Economic Theory*, *42*(1), 167–182. [CrossRef]

Bester, H. (1985). Screening vs. rationing in credit markets with imperfect information. *American Economic Review*, *75*(4), 850–855.

Burdett, K., Shi, S., & Wright, R. (2001). Pricing and matching with frictions. *The Journal of Political Economy*, *109*(5), 1060–1085. [CrossRef]

Davoodalhosseini, S. M. (2019). Constrained efficiency with adverse selection and directed search. *Journal of Economic Theory*, *183*, 568–593. [CrossRef]

Dong, F., Wang, P., & Wen, Y. (2016). Credit search and credit cycles. *Economic Theory*, *61*(2), 215–239. [CrossRef]

Guerrieri, V., Shimer, R., & Wright, R. (2010). Adverse selection in competitive search equilibrium. *Econometrica*, *78*(6), 1823–1862.

Moen, E. R. (1997). Competitive search equilibrium. *The Journal of Political Economy*, *105*(2), 385–411. [CrossRef]

Selcuk, C. (2024). All-pay vs. standard auctions when competing for budget-constrained buyers. *Economics Letters*, *242*, 111886. [CrossRef]

Stiglitz, J. E., & Weiss, A. (1981). Credit rationing in markets with imperfect information. *American Economic Review*, *71*(3), 393–410.

Vesala, T. (2007). Financial matching, asymmetric information and entrepreneurial risk taking. *The Scandinavian Journal of Economics*, *109*(3), 469–485. [CrossRef]

Wright, R., Kircher, P., Julien, B., & Guerrieri, V. (2021). Directed search and competitive search equilibrium: A guided tour. *Journal of Economic Literature*, *59*(1), 90–148. [CrossRef]