Exchange Rate Risk and Deviations from Purchasing Power Parity[♦]

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Abstract

This paper proposes a new solution to the purchasing power parity (PPP) puzzles, arguing that investors' higher-order risk attitudes, combined with higher-order uncertainty about nominal exchange rates, as reflected by skewness and kurtosis, drive a risk premium that leads to deviations from PPP. Analyzing US dollar exchange rates against the currencies of three major net exporting countries to the US—Canada, Japan, and the European Union—we find that the skewness of the expected nominal exchange rate is the most significant and statistically robust moment-based factor influencing these deviations. Our estimates further suggest that only low to moderate exchange rate risks generate risk premia that contribute to these PPP deviations.

Keywords: Purchasing Power Parity, risk-aversion, exchange rate, uncertainty; downside risk

JEL classification: D81, G15, F31, F41

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1. Introduction

The existing literature on purchasing power parity (PPP) puzzles primarily attributes deviations from PPP to market imperfections. In this paper, we combine two significant areas of research—purchasing power parity (PPP) puzzles and the impact of investors' higher-order risk attitudes on asset returns—and demonstrate, both theoretically and empirically, that moment-based factors such as the skewness and kurtosis of nominal exchange rates, which capture higher-order uncertainty, drive deviations from PPP. Our empirical analysis enables us to propose that exchange rate risk, as measured by the skewness of expected nominal exchange rates, is a dominant factor in solving the first two PPP puzzles. To the best of our knowledge no other study has so-far explained deviations from PPP on the basis of higher moments of the exchange rate distribution.

Arghyrou et al. (2011) -AGP hereafter- establish that there is a link between the variance of future floating exchange rates and deviations from the law of one price.[1] The finance literature has long emphasized investors' higher-order risk attitude and corresponding moment-based measures of risk such as asset-related skewness and kurtosis as important factors which determine asset returns. The literature suggests that assets with return distributions that are characterized by positive skewness and low kurtosis are attractive to risk-averse investors because they periodically pay large returns, and the downside risk is low.[2] Several studies show that risk measures such as systematic and idiosyncratic skewness and kurtosis induce risk-premia which drive asset prices and returns. Studies by Jondeaua and Rockinger (2003) and

[1] AGP present a theoretical framework where nominal exchange rate uncertainty, captured by the variance of future nominal exchange rate returns, leads risk-averse consumers without access to hedging instruments to pay domestic importers of a foreign good a positive premium over the price quoted by the foreign producer of the good, when both prices are expressed in domestic currency. AGP provide empirical evidence from a near-perfect market (on-line air tickets) supporting their theoretical model.

[2] The empirical literature which highlights higher-moment market risk in asset pricing is mainly concerned with the co-skewness and co-kurtosis with market portfolios. This literature includes among others the work of Kraus and Litzenberger (1976), Friend and Westerfield (1980), Barone-Adesi (1985), Sears and Wei (1988), Fang and Lai (1997), Harvey and Siddique (2000), Dittmar (2002), Langlois (2020) and Schneider et al. (2020). Studies that relate the idiosyncratic skewness of securities with portfolio decisions include, among others, Brunnermeier et al. (2007), Mitton and Vorkink (2007), Barberis and Huang (2008), Boyer et al. (2010) and Conrad et al. (2013).

Brunnermeier et al. (2010), document the association of higher moments of the nominal exchange rate with stock indices and interest rate differentials, respectively. Engle (2011) finds that the skewness of asset returns becomes increasingly negative as the horizon extends, raising the likelihood of large-scale defaults and episodes of systemic risk.

Motivated by those findings and the large volume of foreign exchange (forex) trading, this paper extends the AGP framework and shows that in the absence of complete hedging against exchange rate risk, the skewness and kurtosis of future nominal exchange rates -exchange rate skewness and kurtosis hereafter- are factors which drive deviations from the direct generalization of the law of one price, the PPP.[3] In testing our theory, we exclude the post-Brexit referendum period which also includes the COVID-19 pandemic period that followed, as these were unique events that caused significant and atypical fluctuations in exchange rates. These fluctuations were mainly driven by short-term investor behavior and market sentiment rather than by long-term economic fundamentals.

Our underlying argument is that the asymmetry of the nominal exchange rate distribution, combined with consumers' and investors' aversion to exchange rate risk, induces premia that create wedges between domestic and foreign prices, leading to deviations from PPP. Exchange rate risk at date t arises from the fact that risk-averse buyers purchase goods and services in foreign prices with payments completed or cleared in date $t + n$ with $n > 0$. Extending the AGP framework, we show that risk-averse buyers pay risk premia to domestic importers and/or domestic producers of homogeneous goods which depend on the skewness and kurtosis of the floating nominal exchange rate. Due to the prohibitive cost of fully hedging against any exchange rate risk, market participants hedge only against extreme risks that may potentially harm them the most. We argue that exchange rate risk-premia incorporated in relative domestic

[3] Approximately $5 trillion ($220 billion) of foreign exchange transactions take place per day (hour). Most of the trading involves the US dollar, the Euro and the Yen.

prices of net importer countries drive permanent deviations from the law of one price (LOOP), thus explaining the first PPP puzzle. Persistent shocks to the higher moments of the expected exchange rate distribution induce persistent deviations from the LOOP, explaining the second PPP puzzle. Finally, under a perfectly credible fixed exchange rate regime, the premium reduces to zero. This is consistent with the findings of previous studies supporting PPP under fixed exchange rate regimes.[4]

To test the empirical validity of our theory, we employ a model with time-varying coefficients using US dollar real exchange rates against the Canadian dollar, the Euro, and the Japanese yen—currencies of countries with which the US is a major trade partner and net importer. We thus treat the US as the domestic market. Our analysis covers the period from 1973 to 2016, using monthly data. Our benchmark series of real exchange rates utilize CPI series, while PPI-based real exchange rates are also employed for robustness. Our findings indicate that exchange rate skewness is a significant and statistically robust moment-based risk determinant driving deviations from PPP. Our estimates further suggest that only low and moderate risks influence PPP, as measured by the asymmetry of the exchange rate distribution. Specifically, an increase in skewness, which corresponds to an increase in extreme exchange rate risk and a decrease in low and moderate risk, consistently raises the real exchange rate against all three currencies. Although it may seem that extreme exchange rate risk is not priced by risk-averse agents, we argue that this is because extreme risk is hedged in advance, while agents only price future unhedged risk. If extreme appreciations of the exchange rate are hedged, domestic agents only price low and moderate risks, which decrease as skewness increases, inducing positive deviations from PPP. Hence, in response to moderate and low risks, domestic agents tend to reduce the risk premium incorporated in domestic prices over foreign ones. According to our estimates, a unit increase in skewness leads to an average

[4] See, for example, Gaillot (1970) and Taylor and McMahon (1988).

deviation from PPP of about one percent, mainly for transaction clearing horizons of one or two months ahead. Such deviation is substantial given that the standard deviation of skewness is significantly higher than unity in most cases. As previously suggested, the high cost of insuring against exchange rate risk may deter complete hedging, leaving only lower-level risks unhedged. This aligns with findings in the literature indicating that the use of derivatives for hedging is only moderate. We therefore propose exchange rate skewness as a solution to the first two PPP puzzles. According to our estimates, the variance of future nominal exchange rate returns is consistently found to be statistically insignificant in explaining deviations from PPP. While most coefficients on expected kurtosis are statistically insignificant, in the few cases where they are significant, their signs support the argument that only changes in lower-level risks drive deviations from PPP. Our main results also hold when real exchange rates are calculated using the producer price index (PPI). To address concerns about potential bias in the confidence intervals, we estimate rolling samples of fixed length, which demonstrate the stability and robustness of these intervals. Finally, we show that the results remain unaffected after controlling for other macroeconomic variables, such as GDP growth, interest rates, money supply, and capital inflows.

The remainder of this paper is structured as follows: Section 2 reviews the literature on the PPP puzzles. Section 3 presents our argument that higher moments of nominal exchange rates drive deviations from PPP. Section 4 describes our data and provides estimates of the higher moments in the distribution of future nominal exchange rates. Section 5 examines the time-varying effects of these higher moments on real exchange rates. Section 6 assesses the robustness of the confidence intervals, while Section 7 discusses the hedging of extreme exchange rate risks and provides a broader analysis of the implications of our main findings. Finally, Section 8 offers concluding remarks.

2. Literature Review

The main theoretical explanations for the empirical failure of PPP, which is defined in the literature as the first PPP puzzle according to Sarno (2005), include shifts in the demand for domestic goods relative to foreign ones, such as changes in consumer preferences or fiscal shocks as discussed by Alesina and Perotti (1995). Other explanations involve shifts in relative productivity, as originally suggested by Balassa (1964) and Samuelson (1964). Studies by Engel and Rogers (1996) and Betts and Devereux (2000) argue that deviations from PPP arise due to market imperfections, such as trade barriers and price rigidities, which limit the pass-through from exchange rate changes to prices. Additionally, studies by Parsley and Wei (2007) emphasize the role of non-tradable goods in driving deviations from PPP in CPI-based measures.

As highlighted by Rogoff (1996), while several studies argue that real exchange rates tend to converge to PPP in the long run, estimates suggest that the speed of convergence is extremely slow. The excessive persistence of deviations from PPP is defined in the literature as the second PPP puzzle. Dumas (1992) and Sercu et al. (1995) show that deviations from the LOOP are non-mean reverting as long as transaction costs are small enough relative to arbitrage-trading costs but become quickly mean-reverting when arbitrage profits exceed the costs of arbitrage trading imposing the LOOP. Empirical studies, reviewed by Sarno (2005), provide substantial evidence for non-linear PPP adjustment, justifying the aforementioned failure of standard linear time-series techniques to validate long-run PPP.[5] A common characteristic of the

[5] A related, but different explanation for the two PPP puzzles has been offered by Imbs et al (2005). They show that heterogeneous adjustment dynamics to the LOOP across the individual components of a basket of goods, caused by varying impediments to arbitrage and nominal rigidities across different goods, result in a positive bias in standard panel and time-series estimates of persistence of shocks to the aggregate real exchange rate. They also show that persistent adjustment heterogeneity at the disaggregated level is fully compatible with the nonlinear dynamics observed in the movements of aggregate real exchange rates.

explanations for the two PPP puzzles discussed above is that they refer to movements of the first moment of nominal or real exchange rate.

Although theoretical models provide insights into exchange rate dynamics, predicting exchange rate fluctuations remains challenging. This difficulty is exemplified by the Meese and Rogoff (1983a,b) puzzle, which shows that a random walk model often performs better than economic models in forecasting exchange rates. In her comprehensive review, Rossi (2013) examines the various factors influencing the predictive power of exchange rate models. She highlights that the effectiveness of these models can vary significantly depending on factors such as the choice of predictors, the time horizon, and the evaluation methods used. Rossi's analysis suggests that while certain predictors, such as Taylor-rule fundamentals and net foreign assets, may offer predictive value under specific circumstances, the overall challenge of achieving consistent accuracy across different contexts remains unresolved, with many models failing to consistently outperform the random walk benchmark. In the current study, we assume that the nominal exchange rate follows a random walk, as is commonly found in the literature.

Iseringhausen (2020) emphasizes the role of time-varying skewness in crash risk for major exchange rates, highlighting that skewness can help improve forecasts of risk measures. Recent research by Lilley et al. (2022) finds that specific proxies for global risk—such as credit spreads, S&P 500 returns, and implied volatility in option markets—exhibit comovement with nominal exchange rates, primarily since 2007. This finding underscores the growing importance of global risk factors in explaining exchange rate dynamics. In this study, we extend the framework of Arghyrou et al. (2011) to generalize the concept of risk beyond these specific proxies in relation to real exchange rates. We argue that higher-order uncertainty about future nominal exchange rates, captured by higher moments of the exchange rate distribution—such as skewness and kurtosis—plays a crucial role in driving deviations from PPP. Specifically, changes in the distribution of nominal exchange rates correspond to varying levels of risk as perceived by risk-averse agents. Our assumption is that complete hedging is costly and unachievable; thus, investors hedge only against extreme exchange rate risks that could cause the most significant damage, leaving low and moderate risks unhedged. The substantial use of various forex hedging instruments, constitutes evidence that exchange rate risk is an important factor for traders. Bodnar et al. (1998) report that 83% of the largest firms and 45% of medium size firms hedge exchange rate risk using derivatives. The empirical analysis of Allayannis and Weston (2001) shows that firms hedge about 22% of their exchange rate exposure and that the use of foreign currency derivatives increases total firm value by 4.87% on average.[6] Survey data from the Foreign Exchange Committee (FXC) covering the months of October and April for the years 2004 to 2021 suggests that 55-60% of forex transactions involve outright forwards, forex swaps and currency options and only 40-45% of the total volume are spot transactions.[7] In theory, forex traders can fully insure against any exchange rate risk using derivatives such as options, forward and futures contracts. In practice however, complete hedging of forex risk can be unachievable due to high insurance fees. Stulz (1996) and Cooper and Melo (1999) note that the cost of hedging may rise with the probability of default, limiting the ability to hedge. Similarly, Brown (2001) presents evidence that hedging costs increase as forex market volatility rises, which in turn limits the degree of hedging.[8] Hence, it is reasonable to assert, not only that exchange rate risk is a factor of PPP but also that forex traders, hedge only against the most painful outcomes that is, extreme risk. Our empirical analysis is a direct test of the former and an indirect test of the latter.

[6] Guay and Kothari (2002) argue that the percentage of the hedged exchange rate exposure reported by Allayannis and Weston (2001) is overstated because foreign sales used to scale the amount of foreign currency derivatives understates exchange rate exposure. However, they do not report any numbers for the fraction of sales in their long sample.
[7] The survey covers all transactions from trades in United States, Canada, and Mexico.
[8] Similar findings are reported by Adam et al. (2017), while Nguyen et al. (2007) also highlight size along with leverage as determinants of the use of derivatives.

3. Higher moments of the nominal exchange rate and PPP

Consider a net importer (home) country and a foreign country which consume homogeneous good x in a perfectly competitive market; x may represent a basket of goods rather than a single good. Without loss of generality, we normalise x to unity and assume zero transportation and transaction costs since the idea is to demonstrate that there are deviations from PPP even in the absence of such costs. There is a risk-averse representative agent that purchases $x = 1$, either from the domestic market or directly from the foreign market. In the first case, the agent's ex-post net real wealth per unit of the domestic currency is given by $w_t - 1$, whereas in the second case it is given by $w_t - S_{t+d}P_t^f P_t^{-1}$, where P_t and P_t^f denote the domestic and foreign price indices, respectively, and S_{t+d} denotes the nominal exchange rate at time $t + d$ when the transaction is cleared.[9] In the first case, there is no uncertainty regarding wealth after the purchase whereas in second case, the agent's ex-post net wealth is subject to some degree of uncertainty since S_{t+d} is not known at the time of the purchase.[10] The representative agent has expectations about S_{t+d}, given information up to period t. Consistent with findings from previous literature, the growth rate of the nominal exchange rate between periods t and d is assumed to be random with a time-varying variance:

$$\frac{S_{t+d}-S_t}{S_t} \approx \ln(S_{t+d}) - \ln(S_t) = \varepsilon_{t+d},$$

where $E_t\varepsilon_{t+d} = 0$ and $E_t\varepsilon_{t+d}^2 = \sigma_{t+d}^2$.[11] Although the exact value of S_{t+d} is not known in period t, the agent has full information regarding the distribution of S_{t+d}. The underlying

[9] In other words, $w_{t+d} = w_t - 1$ in the first case and $w_{t+d} = w_t - S_{t+d}P_t^f P_t^{-1}$ in the second case.

[10] A few examples related to individual consumers would include online purchases where the price is expressed only in foreign currency and the transaction clears at a later stage and appears on bank statements several days after; Payments in foreign currency by travellers, using credit cards issued in the traveller's country of origin; Payments made by checks, payable at foreign banks. Such checks usually take several weeks to clear (6 weeks or more).

[11] In most of the cases in our sample (discussed in the following section) the random walk hypothesis, for various d's, cannot be rejected by the data. Even for the rest of the cases where the random walk is rejected, the processes are nearly random walks.

assumption is that the representative agent dislikes any level of transaction risk of high future appreciations of the foreign currency and thereby uses hedging instruments to reduce it. Hedging is incorporated in a simplified way, allowing the representative agent to hold insurance instruments of total cost $H(S_t)$ that ensure compensation if certain high level appreciations of the foreign currency occur in period $t+d$, where the first and second derivatives of H are given by $H'(S_t) > 0$ and $H''(S_t) < 0$, respectively.[12] The representative agent buys insurance against high level exchange rate risk at the beginning of the period, no matter whether the purchase of *x* will be from the domestic or the foreign market. To keep the notation simple, wealth, *w*, is always net of the real insurance fee H/P.

The utility function of the representative agent residing in the home country is defined over wealth w and it is given by $u(w)$ which is continuously differentiable, increasing in w, and strictly concave. Following AGP, in equilibrium, prices must be such that the domestic agent is indifferent between buying *x* from the domestic market, via either a domestic importer or a domestic producer, and buying it directly from a foreign supplier that is,[13]

$$u(w_t - 1) = E_t u(w_t - S_{t+d} P_t^f P_t^{-1}), \tag{1}$$

where the left-hand-side of (1) corresponds to the utility of the domestic agent from buying a unit of *x* domestically and the right-hand-side corresponds to the expected utility of the domestic agent if he buys a unit of *x* directly from a foreign supplier and the transaction clears in period *t* + *d*. Following AGP, we approximate (1) around $\bar{w}_t = w_t - S_t P_t^f P_t^{-1}$:

$$-u'^{(\bar{w}_t)\pi_t} + \frac{u''(\bar{w}_t)}{2}\pi_t^2 - \frac{u'''(\bar{w}_t)}{3!}\pi_t^3 + \frac{u''''(\bar{w}_t)}{4!}\pi_t^4 + o(\pi_t) \approx \frac{u''(\bar{w}_t)}{2}\left(\frac{S_t P_t^f}{P_t}\right)^2 \sigma_{t+d}^2 -$$

$$\frac{u'''(\bar{w}_t)}{3!}\left(\frac{S_t P_t^f}{P_t}\sigma_{t+d}\right)^3 \frac{E_t \varepsilon_{t+d}^3}{\sigma_{t+d}^3} + \frac{u''''(\bar{w}_t)}{4!}\left(\frac{S_t P_t^f}{P_t}\sigma_{t+d}\right)^4 \frac{E_t \varepsilon_{t+d}^4}{\sigma_{t+d}^4} + o\left(\frac{E_t \varepsilon_{t+d}^n}{\sigma_{t+d}^n}\right), \tag{2}$$

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[12] It is reasonable to assume that domestic buyers (consumers or firms) focus only on hedging extreme exchange rate risk as low and moderate risk might be too costly to insure against.

[13] Although we do not model the decision of domestic importers/producers, we implicitly assume that if they attempt to reduce the price in order to dominate the market, then the foreign producers will respond by decreasing the foreign price as well, and vice-versa.

where $\pi_t = (P_t - S_t P_t^f)/P_t$ is the exchange rate induced risk-premium, per unit of the domestic price that the risk-averse domestic agent is willing to pay to the domestic importer or producer due to the uncertainty about the floating nominal exchange rate. Assuming that $\pi_t^2 \approx 0$, $o(\pi_t) \approx 0$ and $o(\frac{E_t \varepsilon_{t+d}^n}{\sigma_{t+d}^n}) \approx 0$, equation (2) reduces to

$$\pi_t \approx \alpha_{1t}\sigma_{t+d}^2 + \alpha_{2t}Skewness_{t+d} + \alpha_{3t}Kurtosis_{t+d}, \tag{3}$$

where $skewness_{t+d} = E_t\varepsilon_{t+d}^3/\sigma_{t+d}^3$, $kurtosis_{t+d} = E_t\varepsilon_{t+d}^4/\sigma_{t+d}^4$,

$$\alpha_{1t} = \frac{-\mathcal{U}_2(\bar{w}_t)}{2}\left(\frac{S_t P_t^f}{P_t}\right)^2, \alpha_{2t} = \frac{\mathcal{U}_3(\bar{w}_t)}{3!}\left(\frac{S_t P_t^f}{P_t}\sigma_{t+d}\right)^3 \text{ and } \alpha_{3t} = \frac{-\mathcal{U}_4(\bar{w}_t)}{4!}\left(\frac{S_t P_t^f}{P_t}\sigma_{t+d}\right)^4,$$

with $\mathcal{U}_2(\bar{w}_t) = u''(\bar{w}_t)/u'(\bar{w}_t)$, $\mathcal{U}_3(\bar{w}_t) = u'''(\bar{w}_t)/u'(\bar{w}_t)$ and $\mathcal{U}_4(\bar{w}_t) = u''''(\bar{w}_t)/u'(\bar{w}_t)$.[14] AGP assume that agents are concerned only about the overall exchange rate risk, as captured by σ_{t+d}^2. In this paper, we argue that agents may also behave differently against high level exchange rate risk as captured by skewness and kurtosis of the nominal exchange rate. Equation (3) can also be rearranged in terms of the logarithm of the real exchange rate, $Q_t = S_t P_t^f/P_t$, as follows:

$$q_t \equiv lnQ_t \approx -\alpha_{1t}\sigma_{t+d}^2 - \alpha_{2t}Skewness_{t+d} - \alpha_{3t}Kurtosis. \tag{4}$$

Approximations (3) and (4) suggest that the risk-premium as well as the real exchange rate are driven by the agent's second-order risk attitude, u'', as well as her higher-order risk attitude, u''' and u''''. The underlying assumption is that the agent does not insure against exchange rate risk by omitting to insure against high level risk or extreme risk. Since the cost of insurance is high, part of the risk remains unhedged. To simplify the exposition of our analysis we separate exchange rate risk into two categories, extreme risk and moderate risk which incorporates both moderate risk and low-level risk. It is reasonable to assume that, given the high cost of insurance, the agent would only insure against the most severe outcomes,

[14] AGP did not consider terms that involve third and fourth powers, and thus did not examine the role of expected skewness and kurtosis.

specifically, extreme exchange rate risk. These assumptions are consistent with findings in the literature, which suggest that firms hedge only a small percentage of their exchange rate exposure (Allayannis and Weston, 2001), and that hedging is limited due to its high cost (e.g., Stulz, 1996; Cooper and Melo, 1999; Brown, 2001). Under these assumptions, the higher-order properties of the agent's utility function depend on whether the agent is hedged against extreme exchange rate risk ($H(S_t) > 0$) or not ($H(S_t) = 0$), and can be summarized as follows:

$$u(w_t) = \begin{cases} \bar{u}(w_t) \; if \; H(S_t) = 0 \\ \underline{u}(w_t) \; if \; H(S_t) > 0 \end{cases} \text{ such that } u''' \begin{cases} \geq 0 \; if \; H = 0 \\ < 0 \; if \; H > 0 \end{cases} \text{ and } u'''' \begin{cases} \leq 0 \; if \; H = 0 \\ > 0 \; if \; H > 0. \end{cases}$$

As noted by Courbage and Rey (2010), within an expected utility framework, a common assumption is "complete properness", i.e. the feature where successive derivatives of the utility function alternate in signs.[15] The utility function of a prudent agent is characterized by $u'''(w_t) > 0$ if $H(S_t) = 0$ and $u'''(w_t) < 0$ if $H(S_t) > 0$, while the utility function of a temperate agent is characterized by $u''''(w_t) < 0$ if $H(S_t) = 0$ and $u''''(w_t) > 0$ if $H(S_t) > 0$. As skewness increases, extreme exchange rate appreciations become more likely while moderate and small appreciations become less likely. In other words, as skewness increases, extreme risk increases while moderate risk reduces because mass from the 'middle' of the distribution flows into the left and extreme right. A prudent forex trader would be willing to pay a risk premium, incorporated in domestic prices, against the highest unhedged risk since the degree of 'pain' (utility loss) involved by adding risk increases as wealth w_t decreases (Eeckhoudt and Schlesinger, 2013). The premium is paid to domestic importers of foreign-produced *x* or to domestic producers of *x* that deliver a certain level of net wealth for the agent at the end of a *d*-period horizon.[16] It follows that if extreme exchange rate risk is unhedged

[15] The term "complete properness" is used by Pratt and Zeckhauser (1987), while Caballé and Pomansky (1996) refer to it as "mixed risk aversion". This is the case with the commonly used constant relative risk aversion (CRRA) utility function, which implies that both α_{2t} and α_{3t} are strictly positive.

[16] As skewness increases, large positive shocks in the value of the future exchange rate, S_{t+d}, become more likely. If the agent is not hedged against those shocks, they are translated into large negative shocks on the agent's remaining (post-purchase) wealth if the agent buys *x* directly from the foreign supplier. If, on the other hand, the agent is hedged against extreme positive shocks of the foreign exchange, the latter become irrelevant and so as

($H(S_t) = 0$), a prudent agent ($u'''(w_t) > 0$) will react with $\alpha_{2t} > 0$, whereas if extreme exchange rate risk is hedged $H(S_t) > 0$, a prudent agent ($u'''(w_t) < 0$) will react with $\alpha_{2t} < 0$.[17] On the one hand, under unhedged extreme appreciations of the exchange rate, we would expect a decrease in the real exchange rate following an increase in skewness as the agent would be willing to pay a higher risk premium due to the higher likelihood of extreme appreciations of the foreign currency. On the other hand, if extreme appreciations of the exchange rate are a priori hedged, we would expect an increase in the real exchange rate following an increase in skewness as the agent would be willing to pay a lower risk premium since moderate risk is reduced. Hence, in the former case, the prudent agent would be pricing unhedged extreme risk, while in the latter case, the agent would be pricing moderate risk.

Kurtosis measures the thickness of the two tails of the exchange rate distribution. As in the case of skewness, the effect of kurtosis on the risk premium depends on whether extreme risk is a priori hedged or not. In the absence of hedging ($H(S_t) = 0$), if the representative agent exhibits downside risk aversion (temperance), she will have a utility function characterized by $u''''(\overline{w}_t) < 0$ and a positive risk premium response ($\alpha_{3t} > 0$) to an increase in kurtosis. The latter is induced by the fact that the agent dislikes the fatter right tale that translates to increased probability of extreme appreciations of the foreign currency.[18] If, on the other hand, extreme

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skewness increases, only the increased probability of negative moderate shocks in the value of the future exchange rate matter because the risk-averse agent cares only about the highest unhedged risk.

[17] Our reasoning regarding the case $u''' < 0$ differs from that of Eeckhoudt et al. (2005) who argue that it is possible that a risk-averse agent exhibits an imprudent behavior, by insuring risk at an unfair premium in the face of a non-insurable future risk, e.g. $u(w) = \upsilon_1 w - \upsilon_2 w^3$, with $\upsilon_1 > 0$ and $\upsilon_2 > 0$, implies that $u'' < 0$ and $u''' < 0$. We argue that in economies with highly developed options markets which allow agents to hedge extreme exchange rate risk, the case where $u''' < 0$ characterizes a prudent agent who pays risk premia against the highest a priori unhedged risk, which is moderate or low risk.

[18] The impact of kurtosis is distinctively different than that of skewness. If the distribution of S_{t+d} is symmetric with excess kurtosis, the probability of extreme outcomes is higher than under the normal distribution: All else equal, increasing kurtosis implies an increase in the probability both of an extreme positive and an extreme negative deviation of the future exchange rate from its expected value, respectively implying an increased probability both of a large negative and a large positive wealth shock for the agent. In the absence of hedging, if the risk-averse agent is of temperate nature, downside wealth risk will carry higher weight in her decisions than upside wealth risk. An extended illustration and discussion on prudence and temperance can be found in Eeckhoudt and Schlesinger (2013).

risk from excessively fat right tails (extreme appreciations of the foreign currency) is eliminated via hedging ($H(S_t) > 0$), the utility function of the temperate agent will be characterized by $u''''(\bar{w}_t) > 0$ and a negative risk premium response ($\alpha_{3t} < 0$) to an increase in kurtosis. That is, as kurtosis increases, low and moderate risk is reduced while depreciations of the foreign currency become more likely, inducing a negative effect on the risk premium.

4. Data and the distribution of future nominal exchange rate returns

4.1. Data

The empirical analysis that follows employs a dataset of nominal exchange rates which includes end of month closing bilateral nominal exchange rates for Canadian dollars, Euro and Japanese Yen per US dollars, from January 1973 to October 2016, except for the Euro which starts from January 1982.[19]

To mitigate the potential impact of short-term market disruptions, our research sample concludes in October 2016. This decision prevents the inclusion of tumultuous periods following extreme events, as found by various studies (e.g., Iglesias, 2022), such as the Brexit referendum and the onset of the COVID-19 pandemic. These events significantly affected exchange rates that would have impacted the right tail of our sample and required special treatment, unnecessarily complicating matters. The Brexit vote on June 23, 2016, resulted in substantial fluctuations in the British pound against other leading currencies due to speculation. Although our sample does not directly encompass exchange rates involving the British pound, the global repercussions of this event are evident in the performance of other currencies such as the Japanese Yen and Euro (Dao et al., 2019; Janjusevic and Chegeni, 2020). The COVID-19 pandemic, which began in 2020, also caused substantial disruptions to exchange rates

[19] Prior to January 1999, the Euro/USA exchange rate refers to a synthetic euro exchange rate which is calculated by weighting the bilateral exchange rates of the (then) eleven euro-area countries using weights based on country trade shares. When we run our main regression in the following section, we start the sample from January 1997.

through altered capital flows, unprecedented policy interventions, and changes in exchange rate determinants (Konstantakis et al., 2021; Beckmann and Czudaj, 2022). Jamal and Bhat (2022) argue that the pandemic altered market expectations for future exchange rates in COVID-19 hotspots by demonstrating a unidirectional causal relationship between COVID deaths and exchange rates.

To further evaluate the appropriateness of ending our sample in October 2016, we extended the sample period for all three real exchange rates to July 2024, dividing the data into two subsamples: our benchmark sample, which ends in October 2016, and the latest subsample, from November 2016 to July 2024, referred to as Sample 2. Implementing the Chow test at the levels of the real exchange rates, the hypothesis of no structural break in October 2016 is rejected at the 1% significance level for all three currency pairs. Additionally, we find that the variances of all three real exchange rate pairs differ significantly between the two subsamples. Specifically, the 10-month rolling variances of the JPN/USA, EU/USA, and CA/USA real exchange rate pairs are, respectively, 37%, 104%, and 18% higher in the benchmark sample compared to Sample 2. Given these findings, and to avoid potential distortions in our analysis, we exclude the periods surrounding the Brexit referendum (June 2016) and the onset of the COVID-19 pandemic (2020). These events would have had a substantial impact on the right tail of our sample due to elevated volatility (Bollerslev et al., 2015), structural breaks (Perron, 2006), and unprecedented policy responses (Altavilla et al., 2023) that can create non-linear effects on exchange rates and complicate the isolation of higher moments related to skewness and kurtosis of nominal exchange rates. These would require additional treatment, unnecessarily complicating our econometric approach by focusing on the right tail of the sample. Furthermore, our evidence using pre-Brexit referendum data suggests that only low and moderate risks drive deviations from PPP. For these reasons, in this study we avoid complexities in our econometric method and data handling by excluding the extreme event periods from 2016 onward, relying on relatively more stable market conditions.

We define the net importer country as the country which has trade deficits in goods. The data on trade balances are obtained from the United States Census Bureau. We do not consider trade in services as it includes an element of financial investment income. Since the US has trade deficit in goods against all three countries in most months of the sample, we consider the US as the home country and the net importer. The US has a bilateral trade surplus with Canada only in 5 months of our sample (May 2015 and March 2016 to June 2016) and with the EU only in 1 month of sample (February 1997) and none with Japan. Consumer price indexes for Canada, the Euro area, Japan and the US are included to calculate the CPI-based real exchange rate, Q_t and Producer price indexes are included to calculate the PPI-based exchange rate as a robustness test. The real exchange rate is defined as in section 3 that is, $Q_t = S_t P_t^f / P_t$. Due to data availability, the CPI based real exchange rate for the euro area starts from 1983M10 while PPI based real exchange rate for euro area starts from 1996M01. The data source is the IMF's International Financial Statistics databank.

4.2. Estimating higher moments of nominal exchange rate distributions

For notational convenience, let $\theta_t(d) \equiv E_t \varepsilon_{t+d}^2 = \sigma_{t+d}^2$, with the corresponding skewness and kurtosis, defined in section 2, as $Sk_t(d)$ and $Ku_t(d)$, respectively. We estimate $\theta_t(d)$, $Sk_t(d)$ and $Ku_t(d)$, following the methodology of León et al. (2005), by considering the following system of equations:[20]

$$\theta_t(d) = \mu_{\theta 0} + \mu_{\theta 1} \varepsilon_{t+d}^2 + \mu_{\theta 2} \theta_{t-1}(d) \tag{5}$$

$$Sk_t(d) = \mu_{s0} + \mu_{s1} \eta_{t+d}^3 + \mu_{s2} Sk_{t-1}(d) \tag{6}$$

[20] León et al. (2005), extends Harvey and Siddique (1999), by assuming time-varying kurtosis. León et al. (2005) also assume that $d = 1$.

$$Ku_t(d) = \mu_{k0} + \mu_{k1}\eta_{t+d}^4 + \mu_{k2}Ku_{t-1}(d) \tag{7}$$

for $d \geq 1$, where

$$\varepsilon_{t+d}(d)|I_t \sim (0, \theta_t(d)), \eta_{t+d} = \varepsilon_{t+d}/\sqrt{E_t\varepsilon_{t+d}^2} \equiv \varepsilon_{t+d}\theta_t^{-\frac{1}{2}}(d),$$

with $\eta_{t+d} \sim (0,1)$ and I_t is a set that contains all information up to date *t*. Following León et al. (2005), we abstract from unnecessary constants so that the logarithm of the likelihood function of the conditional distribution of $\varepsilon_{t+d} = \theta_t^{\frac{1}{2}}(d)\eta_{t+d}$ is given by

$$L_t = -\frac{1}{2}ln\theta_t(d) - \frac{1}{2}\eta_{t+d}^2 + ln\,\psi^2(\eta_{t+d}) - ln\Gamma_t(d),$$

where,

$$\psi(\eta_{t+d}) = \left[1 + \frac{Sk_t(d)}{3!}(\eta_{t+d}^3 - 3\eta_{t+d}) + \frac{Ku_t(d)-3}{4!}(\eta_{t+d}^4 - 6\eta_{t+d}^2 + 3)\right],$$

and

$$\Gamma_t(d) = 1 + \frac{Sk_t^2(d)}{3!} + \frac{(Ku_t(d)-3)^2}{4!}.$$ [21]

The estimates of (5)-(7) are provided in the appendix.[22] Figures 1 to 3, display the estimated time series for the higher moments of the future exchange rate distribution for different transaction clearing periods, as defined by d. Our theory is expected to hold for relatively short transaction clearing periods as it is rather unlikely that sellers will agree to be paid several periods ahead with an unspecified exchange rate. Therefore, in the analysis that follows, we limit the transaction clearing period to a maximum of 3 months, presenting estimates for $d = 1, 2$ and 3.

The volatilities of the variance, skewness and kurtosis increase with *d* in most of the cases. That is, as the transaction clearing period is lengthened, the uncertainty about the future nominal exchange rate increases, the distribution of the nominal exchange becomes more

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[21] Refer to León et al. (2005) for the details regarding the derivation of the p.d.f.

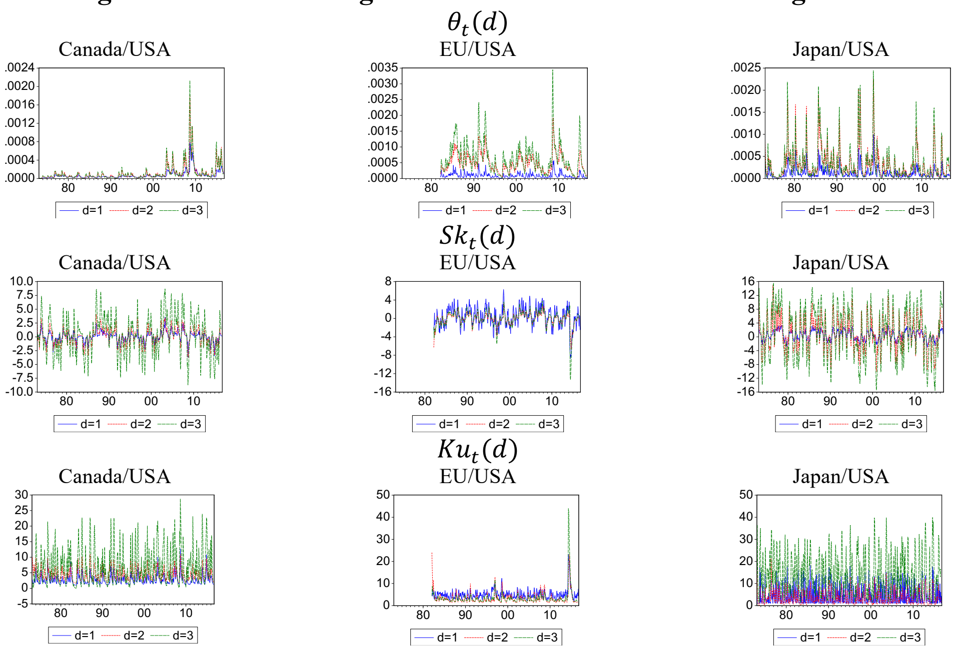
[22] The standard errors of the estimated coefficients correspond to the Bollerslev-Wooldridge robust standard errors and are calculated through $Var(\hat{\beta}) = J^{-1}(\beta_0)I(\beta_0)J^{-1}(\beta_0)$, where $J(\beta_0)$ is the negative of the expected Hessian matrix and $I(\beta_0)$ is the information matrix which can be calculated by the outer product of the score vector.

asymmetric, and the tails of the distribution become heavier. The only exceptions are the skewness and the kurtosis of the EU/USA bilateral exchange rate which are, on average, more volatile when the transaction clearing period is short, i.e. $d = 1$. This implies that extreme values of the EU/USA exchange rate are, on average, less likely if the transaction clearing period is long, i.e. $d = 3$, apart from the momentous event of the oil price collapse which occurred in the mid 2014.[23] All moments capture the event while $d = 3$ exhibits a much larger swing than $d = 2$ and $d = 1$, not only for the variance but also for skewness and kurtosis, with skewness being negative in almost all periods from mid 2014 until the end of 2015. Negative skewness suggests that the markets expect that appreciations of the Euro, the Canadian dollar and the Yen against US dollar are more likely, while the probability of extreme appreciations of the US dollar are non-zero. The fact that $d = 3$ exhibits larger swings than $d = 1$ and $d = 2$ for most moments, suggests several things. In the event of a significant oil price decline, not only exchange rate uncertainty increases as the transaction clearing horizon extends but also markets expect that most of the probability mass will be even more concentrated to the right tail of the exchange rate distribution. Extreme depreciations and appreciations of the foreign currencies (Euro, Canadian dollar and Yen) are even more likely, with a particular emphasis on extreme depreciations. The large negative skewness and fatter tails of the distribution, observed as a consequence of the oil price collapse, can be attributed to the fact that the US economy is relatively more exposed to oil price fluctuations than the EU. The variances of all exchange rates exhibit a sharp upswing in September of 2008, and in the case of EU/USA $d = 3$ a month before, following the Lehman Brothers episode. The skewness reaches very negative values in September 2008 for CA/USA and EU/USA and significantly positive values for JP/USA. This implies that following the episode, markets expected a high probability of US dollar depreciations relative to both the Canadian dollar and the Euro without ruling out the

[23] The oil price collapse of mid 2014 was one of the three biggest declines in oil prices since World War II.

possibility of extreme depreciations of both the Euro and Canadian dollar. On the contrary, following the Lehman episode, the markets expected a high probability of US dollar appreciations relative to the Yen, without ruling out the possibility extreme appreciations of the latter. As in the cases of the other two moments, kurtosis for all exchange rates and transaction clearing horizons, exhibit increases in September, following the Lehman episode, with more evident the increases in the exchange rates of CA/USA and JP/USA. The EU/USA exchange rate increased in September, but the increase was not as sharp as the upswings of the other two exchange rates because the former began increasing from July. Across the sample, exchange rates exhibit positive skewness about 59% of the time and excess kurtosis about 58% of the time. [24]

Figure 1: Estimated Higher Moments of Bilateral Exchange Rate



[24] In particular, the skewness of the CA/USA exchange rate is positive 57%, 55% and 57% of the time at $d = 1$, $d = 2$ and $d = 3$, respectively. The skewness of the EU/USA exchange rate is positive 59%, 52% and 56% of the time at $d = 1$, $d = 2$ and $d = 3$, respectively, while the skewness of the JP/USA exchange rate is positive 62%, 61% and 54% of the time at $d = 1$, $d = 2$ and $d = 3$, respectively. Our estimates also indicate that the exchange rate series exhibit excess kurtosis 0.27%, 0.69% and 0.73% of the time for CA/USA, 0.91%, 0.37% and 0.39% of the time for EU/USA, and 0.45%, 0.46% and 0.92% for JP/USA, respectively at $d = 1$, $d = 2$ and $d = 3$.

5. Time-varying effects of higher moments

Stationarity tests indicate that the bilateral real exchange rates for all three countries exhibit unit root behavior, indicating non-stationarity in their levels. To obtain stationary series and ensure consistency with stationary regressors, we use the first difference of the real exchange rates as the dependent variable. This transformation achieves stationarity and allows us to specify a regression model that includes four stationary variables, as outlined below:[25]

$$\Delta lnQ_t = \alpha_{0t} + \alpha_{1t}\sigma_{t+d}^2 + \alpha_{2t}Sk_{t+d} + \alpha_{3t}Ku_{t+d} + u_t, \tag{8}$$

where ΔlnQ_t is the first difference of the logarithm of the real exchange rate and u_t is a random error term.[26] Since all variables in (8) are stationary, the time-varying coefficients are estimated using the semi-parametric approach of Robinson (1989). Parameter α_{it} is defined as $\alpha_{it} = \alpha_i(z_t)$ for $i = 0, 1, 2, 3$, where z_t is a smoothing variable such that $z_t = \tau = t/T$ and $\alpha_i(z_t) = f(\tau)$ for $t = 1,2, \dots T$. The time-varying coefficients are obtained by combining ordinary least squares and a local polynomial kernel estimator, known as the Nadaraya-Watson approach, as described in Casas and Fernandez-Casal (2019).[27] Following Cifarelli and Paladino (2015), we argue that any potential measurement errors in our regressors are more than compensated by

[25] Unit roots cannot be rejected for either country at the 5% significance level. The unit root results are available upon request.

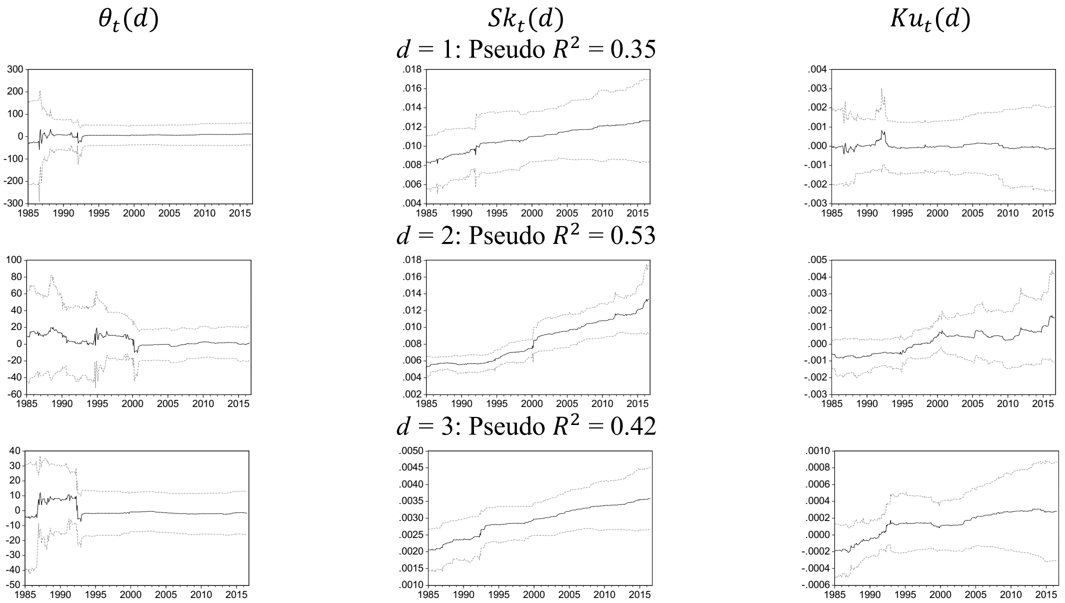
[26] The percentage of months where the US deficit becomes surplus is tiny. Even if intercept dummies are included, effectively taking those observations off the sample, the results remain the same.

[27] Assuming that $\alpha_i(z_t)$ is twice differentiable, it can be linearly approximated around z, using the Taylor Rule as $\alpha_i(z_t) \approx \alpha_i(z) + \alpha_i'(z)[z_t - z]$, where $\alpha_i'(z)$ is the first derivative with respect to z. Then, estimates for $\alpha_i(z)$ and $\alpha_i'(z)$ are obtained by solving the minimization problem $\arg\min \sum_{t=1}^{T}[\Delta lnQ_t - x_t^{\mathsf{T}}\theta_0 - (z_t - z)x_t^{\mathsf{T}}\theta_1]^2\, K_b(z_t - z)$, where $x_t^{\mathsf{T}} = [1, \sigma_{t+d}^2, Skewness_{t+d}, Kurtosis_{t+d}\,]$ and $K_b(z_t - z)$ is the Epanechnikov kernel with bandwidth b. The latter is selected by leave-one-out cross-validation apart from a few cases where such procedure implies relatively large bandwidths, inconsistent with most of the other cases and oversmoothed (constant) estimates. These are the cases of CPI-based EU/US $d = 3$, the three PPI-based EU/US cases and the two PPI-based cases of JP/US $d = 1$ and $d = 2$. For the first case, we choose the bandwidth to be equal to unity so that it is roughly equal to the other two CPI-based $d = 3$ cases and the other two CPI-based EU/US cases. For the PPI-based cases, we choose bandwidths which are equal to the corresponding values of the CPI-based cases. In this way, we avoid oversmoothed estimates. Further details on the estimation procedure can be found in Casas and Fernandez-Casal (2019).

the greater accuracy of the estimates of the conditional higher moments provided by the GARCH-type model.[28]

Figures 2 to 4 display the estimated time-varying coefficients for different transaction clearing horizons, along with 95% confidence intervals. The intercept coefficient is estimated to be about zero, thereby it is not reported to save on space. The time-varying confidence intervals for the variance cover zero in all exchange rates and transaction clearing horizons. The only case where the confidence interval is marginally statistically insignificant is for the JP/USA exchange rate at $d = 1$. In this case, the lower limit of the confidence interval is close to zero, while the upper limit remains in the positive region. The estimates on the variance effect essentially suggest that first-order uncertainty is not statistically significant in driving the real exchange rate and deviations from PPP, that is deviations of Q_t from unity.

Figure 2: Estimated time-varying coefficients: Canada – CPI



The coefficients on skewness on the other hand, are positive and statistically significant, as they never cover zero, for all three exchange rates and transaction clearing

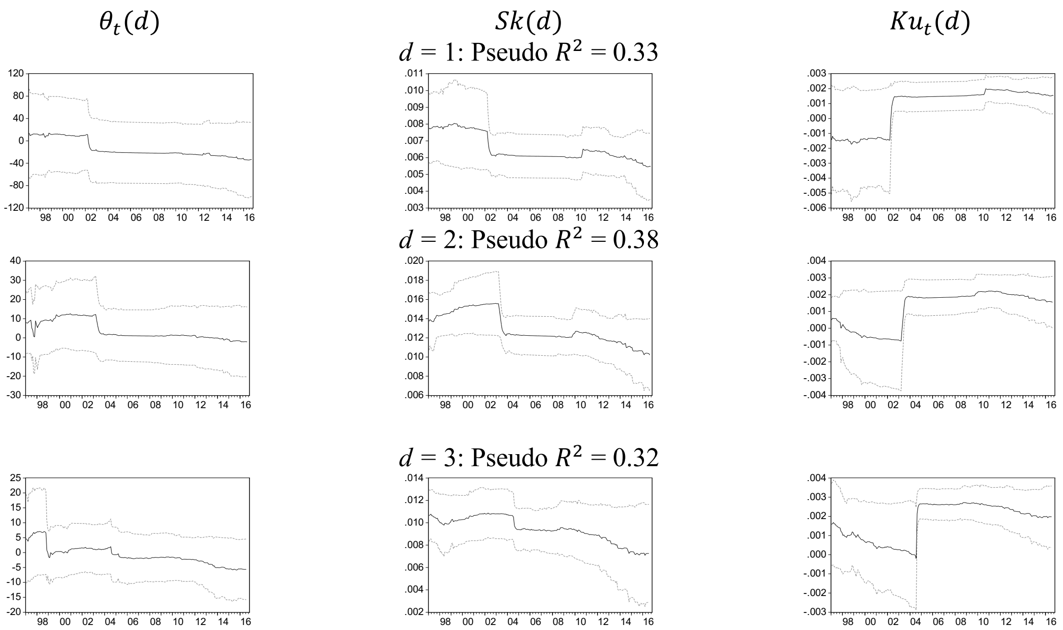
[28] The use of GARCH measures of variance, derived from preliminary estimates, as regressors is not uncommon in the literature, e.g. French et al. (1987), Ramachandran (2004) and Ramachandran and Srinivasan (2007).

horizons. This result indicates that increases in skewness tend to increase the gap between the foreign price measured in US dollars and the domestic price. In other words, as skewness increases, homogeneous goods become more expensive abroad relative to the US in US dollars. According to our theory, this phenomenon is attributed to the reduction in unhedged moderate and low exchange rate risk, leading to a lower risk premium embedded in domestic prices. That is, when transactions are settled in the future, moderate appreciations of the US dollar become more likely, making it more probable that US agents purchasing goods directly from abroad, paying relatively less for those goods. Consequently, prudent US agents tend to pay a lower risk premium to domestic importers and producers in response to the diminished low and moderate risks, based on the assumption that extreme appreciations of foreign currencies are hedged a priori. This framework suggests that the perceived exchange rate movements by US agents translate into cost savings and a competitive advantage in the domestic market, as the need to allocate additional resources for moderate risk coverage is reduced. Moreover, the estimates suggest that the positive effect of skewness on the real exchange rate is rising over time in the case of CA/USA and decreasing over time in the cases of EU/USA and JP/USA. The increasing economic integration between Canada and the USA, highlighted by agreements such as NAFTA, has led to synchronized economic activities and a reduction in moderate exchange rate risks for US agents over time. In contrast, moderate and low exchange rate risks for US agents have tended to increase over time with the Euro and Yen. This differential impact likely reflects the varying degrees of economic relationships between the Eurozone and Japan with the US, as well as differences in policy stability compared to Canada, which influence how exchange rate risks are perceived and managed by US agents.

The coefficient on skewness for the EU/USA exchange rate shows a sharp decrease in early 2002, which corresponds to the significant drop in inflation in the EU starting in

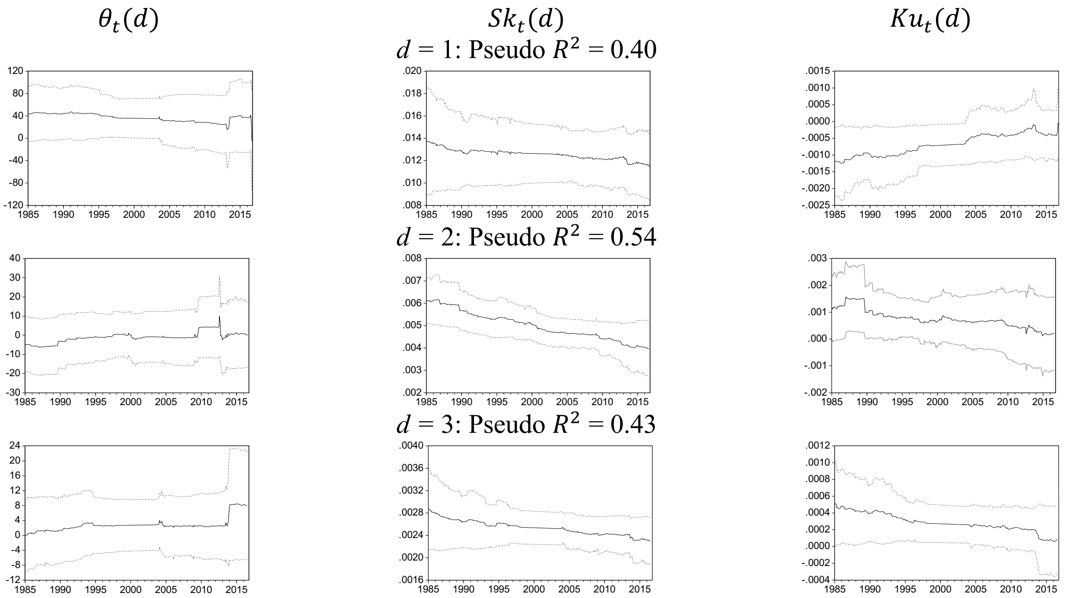
April, relative to the increase in US inflation during the same month. The fact that EU products became relatively cheaper decreased the impact of the reduction in moderate exchange rate risk. As table 1 suggests, a unit increase in skewness implies an average increase of the logarithm of the real exchange rate, or deviation from PPP, of about one percent, mainly for transaction clearing horizons of one or two months ahead. This is a non-negligible increase as the standard deviation of the measure of skewness in most cases is higher than unity. When the mean coefficient is significantly lower than unity, the standard deviation of skewness is significantly higher than unity (e.g. CA/USA and JP/USA for $d = 3$ and JP/USA for $d = 2$). The means of the 95% confidence intervals indicate that the coefficients are estimated with precision, suggesting high reliability and accuracy. This implies that the model's main findings regarding the effect of skewness are robust and not likely due to random chance, thereby increasing confidence in the validity of the conclusions. The robustness of the results is analyzed further in section 6.

Figure 3: Estimated time-varying coefficients: EU – CPI



Most of the confidence intervals of the coefficients on kurtosis cover zero, apart from a few cases where the coefficients are positive and marginally statistically significant, with lower limits that are positive but very close to zero. These cases include the following: EU/USA after May 2002 for all transaction clearing horizons; JP/USA for $d = 2$ in 80% of the months between July 1, 1986, and May 1, 1997; and JP/USA for $d = 3$ in almost all the months between January 1, 1985, and March 1, 2007.[29] These results are consistent with the argument that extreme exchange rate risk is hedged. Specifically, under the assumption that the risk from extreme appreciations of the foreign currency is already priced in (making the right tail of the exchange rate distribution irrelevant), the focus shifts to the left tail of the distribution. As kurtosis increases and the left tail becomes longer and fatter, the likelihood of significant foreign currency depreciations rises. This increased likelihood leads to a lower risk premium that US agents are willing to pay, resulting in lower domestic prices relative to foreign ones and positive deviations from PPP.

Figure 4: Estimated time-varying coefficients: Japan – CPI



[29] In the case of EU/USA after May 2002, the mean coefficients on skewness (in percentages) and corresponding mean of 95% confidence intervals (also in percentages) are 0.16 (0.06, 0.26) for $d = 1$, 0.17 (0.05, 0,3) for $d = 2$ and 0.17 (0.08, 0.33) for $d = 3$. The standard deviations of kurtosis are 2.37, 2.85 and 3.79 for the cases of $d = 1$, $d = 2$ and $d = 3$, respectively. While the estimated skewness coefficients do not cover zero and are positive, the confidence intervals for kurtosis are wider compared to those for skewness coefficients.

Table 1: Means of coefficients on skewness and standard deviations of skewness - CPI[*]

	$d = 1$	$d = 2$	$d = 3$
		Canada/USA	
skewness	1.1	0.8	0.3
	(0.8, 1.4)	(0.68, 1)	(0.2, 0.4)
s.d. of skewness	0.83	1.34	3.35
		European Union/USA	
skewness	0.65	1.3	0.9
	(0.5, 0.8)	(1.04, 1.54)	(0.68, 1.2)
s.d. of skewness	2.16	1.28	1.84
		Japan/USA	
skewness	1.3	0.5	0.3
	(1, 1.5)	(0.4, 0.6)	(0.21, 0.29)
s.d. of skewness	1.28	4.16	7.08

*The table reports the mean of the time-varying coefficients (α_{2t}) in percentages and standard deviations of estimated skewness across the sample using CPI based real exchange rates. In parenthesis, are the means of the upper and the lower bounds of the 95% confidence intervals, displayed in figures 2-4.

Finally, the measure of the goodness of fit of the regressions, suggests that in all cases the transaction clearing horizon of two months fits the data the best. The main results from CPI-based real exchange rates are also confirmed when the real exchange rate is computed using the Producer Price Index (PPI). Although there are a few cases where the estimated coefficients on variance become positive without covering zero, the confidence intervals are extremely wide. Additionally, there are instances where the coefficients on kurtosis become negative for certain periods without covering zero. However, these results provide very weak evidence that kurtosis contributes positively to the risk premium, as the coefficients are not only very small but also cover zero in most periods. Table 2 displays the means of the coefficients on skewness, the means of the 95% confidence intervals, and the standard deviations of the skewness estimates using PPI-based real exchange rates. Figures displaying estimated coefficients from PPI based real exchange rates are reported in the appendix.

Table 2: Means of coefficients on skewness and standard deviations of skewness - PPI*

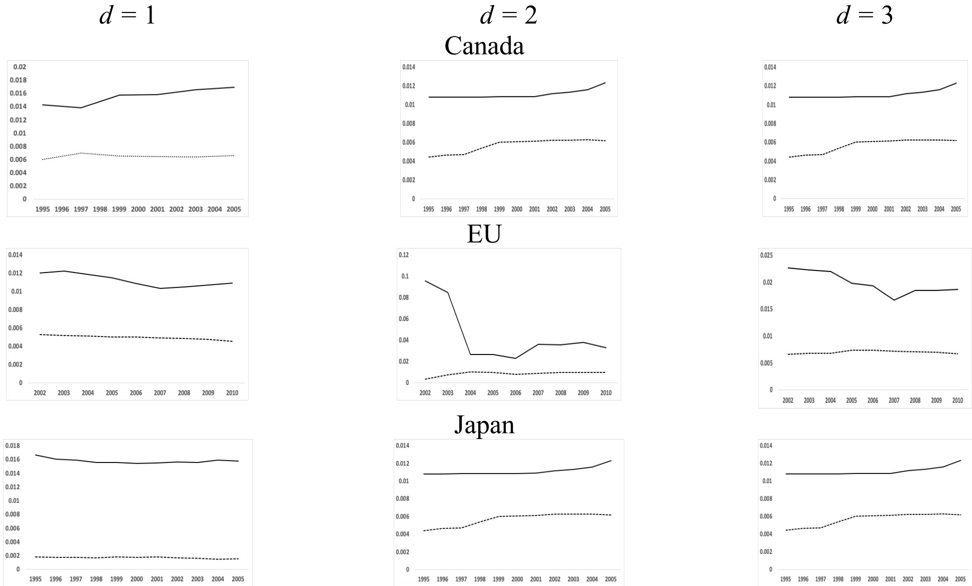
	$d = 1$	$d = 2$	$d = 3$
		Canada/USA	
skewness	0.8	0.6	0.2
	(0.62, 0.93)	(0.5, 0.8)	(0.15, 0.3)
s.d. of skewness	0.83	1.34	3.35
		European Union/USA	
skewness	0.6	1.06	0.8
	(0.45, 0.71)	(0.8, 1.32)	(0.6, 1)
s.d. of skewness	2.16	1.28	1.84
		Japan/USA	
skewness	1.2	0.5	0.24
	(0.94, 1.45)	(0.4, 0.6)	(0.19, 0.31)
s.d. of skewness	1.28	4.16	7.08

*The table reports the mean of the time-varying coefficients (α_{2t}) in percentages and standard deviations of estimated skewness across the sample using PPI based real exchange rates. In parenthesis, are the means of the upper and the lower bounds of the 95% confidence intervals, displayed in figures A1-A3 in the appendix.

6. Robustness of results

As noted by Cifarelli and Paladino (2015), any potential errors-in-variables distortion is more than compensated by the greater accuracy of estimates provided by GARCH-type procedures, while the use of such estimates as regressors is not uncommon in the literature. Our main results implied by the time-varying estimates are reinforced by the fact that they hold for both CPI-based and PPI-based real exchange rates. Evaluating potential generated regressor bias in models with time-varying coefficients is currently an unknown territory. Such bias may affect the confidence intervals of the estimated coefficients in equation (8), as the issue lies in the fact that the uncertainty associated with estimating the regressors in (8) is not accounted for when estimating the coefficients in (8). To address this issue, we assess the stability of the confidence intervals for the estimated coefficients by estimating a rolling sample of fixed length and reporting the 97.5th percentile of the upper bound and the 2.5th percentile of the lower bound over a specified period. By utilizing these percentiles, we effectively exclude the most extreme 5% of values at both the upper and lower ends of the distribution. We focus specifically on the confidence intervals of CPI-based skewness coefficients which are the highlight of our main results.

Figure 5: Percentiles of Confidence Intervals of Skewness Coefficients from Rolling Samples*



*The focus period is set to January 1995 through December 2005 for CA/USA and JP/USA, and from January 2002 to December 2010 for EU/USA. The rolling samples for CA/USA and JP/USA are fixed to 252 observations, starting from January 1, 1985, and rolling over for one year until and including January 1, 1994. For EU/USA, the rolling sample is fixed to 168 observations, starting from January 1, 1997, and rolling over for one year until and including January 1, 2002. The percentiles reported in the figures are the 97.5th for the upper bound of the confidence interval and the 2.5th for the lower bound. The real exchange rates are computed using CPI.

Since the CPI-based results are indicative and to conserve space, we do not report the results from PPI-based exchange rates; however, these are available upon request. Figure 5 demonstrates that the confidence intervals for all bilateral real exchange rates at all transaction clearing horizons do not cover zero and always lie within the positive region. This result provides strong support for the stability of the confidence intervals and the validity of the finding that US agents respond to moderate and low exchange rate risks by increasing (decreasing) the risk premium incorporated in domestic prices when such risks increase (decrease).

To assess the robustness of our benchmark model findings, we extend the regression by including additional U.S. macroeconomic variables as controls: real GDP, the 1-year Treasury yield, M2 money supply, U.S. foreign net long-term capital flows, referred to as net capital flows and U.S. Monetary Policy Uncertainty (MPU). Data for these variables were obtained from *Datastream*. For MPU, we used the index developed by Baker et al. (2016), which tracks the frequency of articles on monetary policy uncertainty published in 10 major U.S. newspapers. Real GDP, M2, and MPU are expressed in logarithms, and all variables, except for net capital flows and MPU, are first-differenced to ensure stationarity. To conserve space, and since the additional variables do not significantly affect our main conclusions, we present only Table A2 in the appendix. This table provides the mean estimates for skewness and the control variables, along with the corresponding confidence intervals. After including these control variables, apart from changes in the magnitude of the estimates in some cases, skewness consistently remains positive, with no evidence of substantial endogeneity effects undermining our main conclusions. The coefficients for variance and kurtosis remain statistically insignificant in most cases, consistent with the benchmark results, as their confidence intervals typically cover zero. Both real GDP and M2 are statistically insignificant, with average confidence intervals covering zero in all cases except real GDP for CAD/USD $d = 2$. The Treasury yield appears to be statistically significant in the cases of EUR/USD for $d = 2$ and $d = 3$. In these instances, an increase in the yield leads to a decrease in the real exchange rate. This result is intuitive, as higher bond yields make domestic assets more attractive, leading to increased demand for these assets, a depreciation of the foreign currency, and a subsequent decrease in the real exchange rate. Moreover, the U.S. monetary policy uncertainty variable appears to have mixed result - it is consistently insignificant in all cases for CAD/USD and EUR/USD however statistically significant in the cases of JPY/USD for all values of d. In these instances, an increase in the U.S monetary policy uncertainty leads to an increase in the real exchange rate. This result is intuitive, as higher U.S monetary policy uncertainty make domestic assets less attractive, leading to decreased demand for these assets, an appreciation of the Japanese currency, and a subsequent increase in the real exchange rate. The mixed results depend on how closely a foreign country's monetary policy aligns with that of the U.S. Compared to Canada and the European Union, Japanese monetary policy differs significantly from most developed countries. While many advanced economies adopt broadly similar monetary policy frameworks, Japan stands out due to its persistent economic challenges and reliance on unconventional measures. This distinction makes Japanese assets more attractive during periods of high monetary policy uncertainty in the U.S. Despite the statistical significance of the Treasury yield and the U.S. monetary policy uncertainty in these specific cases, the main conclusion regarding the significance of skewness in driving deviations from PPP remains robust. Figures showing estimates for the time-varying parameters of all variables are available upon request.

7. Hedging hypothesis and discussion

Direct hedging in Forex trading, which involves holding both long and short positions on the same currency pair simultaneously, is considered illegal in the U.S. However, holding forex options which enable recouping the difference between the market price and the strike price when extreme appreciations of the forex occur or using other derivatives when extreme appreciations of foreign currencies are expected is a perfectly legitimate strategy.[30] We argue that US agents, especially large corporations that purchase foreign goods and services denominated in foreign currencies, hold derivative portfolios to hedge the exchange rate risk but only on a limited basis, focusing on extreme appreciations of foreign currencies.

[30] The evidence suggests that option derivatives is a significant component of derivatives portfolios. For instance, when the foreign currency significantly appreciates, foreign call options enable domestic (US) agents to recoup the price difference and thus fully hedge extreme exchange rate risk, while moderate and low risks remain unhedged. Specifically, domestic agents can purchase foreign exchange at the exercise price which is lower than the market price and then sell it at the market price. Futures and forward contracts also enable domestic agents to eliminate extreme appreciations of the foreign exchange when the latter are expected. In principle, low and moderate exchange rate risks can also be hedged, however this would imply higher overall insurance fees than any premium paid to domestic importers or producers. For instance, domestic agents can use put options and sell foreign exchange at the exercise price which is higher than the market price and then purchase at the market price.

Operational hedging is another option that agents can adopt when extreme appreciations of foreign currencies are expected.[31] Although direct testing of this hypothesis is challenging, existing empirical evidence from corporate derivatives supports it. Guay and Kothari (2002) find that the use of corporate derivatives, not inclusive to exchange rate risk, appears to be moderate by taking into account firm size, operating and investing cash flows.[32] Although increased use of derivatives among more geographically diverse firms is observed, the magnitudes of the derivatives positions are found to be quite small. Furthermore, Brown (2001), provides evidence that a large US based corporation would be more concerned with downside exchange rate risk than upside risk. As Brown's case study suggests, the cost of initiating and maintaining a derivatives program is not trivial. Therefore, the assumption of hedging extreme downside risk only, is justified by the fact that large-scale hedging would be too expensive and detrimental for either businesses and/or consumers.

Our findings underscore the significant role of exchange rate uncertainty, as evidenced by higher-order moments, particularly skewness, in driving deviations from PPP. Specifically, they highlight the impact of low and moderate exchange rate risks in disrupting the balance between domestic and foreign prices. This has important implications for both policymakers and market participants. For policymakers, understanding that the skewness of exchange rates can induce systematic deviations from PPP due to moderate risks suggests a need for more nuanced approaches in monitoring and stabilizing currency markets, particularly in countries heavily engaged in international trade. Regulatory bodies might consider enhancing transparency and further reducing transaction costs in forex markets to mitigate the impacts of skewness-driven exchange rate risks. For market participants, such as multinational

[31] For instance, an agent may postpone or cancel a purchase when an extreme appreciation of foreign exchange is expected to occur (e.g. see Boyabatlı and Toktay, 2004).
[32] Specifically, Guay and Kothari (2002), present detailed evidence on the cash flow and market value sensitivities of financial derivatives portfolios to extreme simultaneous changes in interest rates, currency exchange rates, and commodity prices using an extended random sample of large non-financial corporations.

corporations and investors, our findings imply that hedging only against extreme risks does not diminish the significance of exchange rate risk, given the complexities of moderate unhedged risks. This underscores the importance of developing more sophisticated risk management tools that account for the asymmetry in exchange rate distributions. By focusing on these higher moments such as exchange rate skewness, market participants can better anticipate and manage potential deviations from PPP, thereby optimizing their international investment and pricing strategies. Ultimately, our study provides a critical lesson: in a world of incomplete markets and asymmetric risk distributions, traditional models of exchange rate behavior must be augmented with measures of risk based on higher moments to more accurately forecast and manage deviations from economic fundamentals such as PPP.

8. Concluding remarks

We extend the theoretical framework of Arghyrou et al. (2011), which posits that transactions clearing on future dates lead to deviations from the law of one price due to uncertainty about future nominal exchange rates. We expand their framework to a macroeconomic context incorporating PPP and demonstrate that deviations from PPP are driven not only by first-order uncertainty, as described by Arghyrou et al., but also by higher-order uncertainty, as measured by the skewness of the exchange rate distribution. We further argue that, due to high insurance costs, only extreme exchange rate risks can realistically be fully hedged through options markets. Our framework suggests that, even when extreme exchange rate risks are hedged, the presence of low and moderate risks can lead to significant risk premia being incorporated into domestic prices, thereby causing deviations from PPP. To test our theory, we use a model with time-varying coefficients, analyzing bilateral exchange rates between the US and three of its main net exporting partners: Canada, the EU, and Japan. Our estimates indicate that higher-order uncertainty, as measured by skewness, is a significant factor driving deviations from PPP. Furthermore, our findings support the hypothesis that US agents hedge a priori against extreme exchange rate risk and are willing to pay risk premia only for unhedged low and moderate risks. Thus, the skewness of future nominal exchange rates emerges as a crucial factor in explaining deviations from PPP and offers a potential solution to the PPP puzzles.

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Appendix

Table A1: GARCHSK Estimation Parameters for Bilateral Exchange Rates: 1973:1-2016:10*

	Canada/USA			European Union/USA			Japan/USA		
	d=1	d=2	d=3	d=1	d=2	d=3	d=1	d=2	d=3
$\theta_t(d)$*100									
$\mu_{\theta 0}$	0.000	0.001	0.008	0.017	0.009	0.209	0.009	0.057	0.051
	(0.000)	(0.000)	(0.003)	(0.001)	(0.002)	(0.007)	(0.000)	(0.003)	(0.001)
$\mu_{\theta 1}$	10.306	7.652	18.629	39.083	4.845	70.928	0.859	20.587	17.086
	(0.100)	(0.100)	(3.272)	(0.327)	(0.522)	(1.507)	(0.023)	(1.736)	(1.686)
$\mu_{\theta 2}$	60.737	64.704	30.073	21.657	48.313	-26.708	69.145	53.103	47.038
	(32.985)	(32.985)	(20.287)	(18.374)	(77.284)	(0.337)	(0.467)	(1.165)	(9.005)
$Sk_t(d)$									
μ_{s0}	0.0763	-0.401	-0.714	0.001	0.829	0.033	0.026	0.093	-0.140
	(0.026)	(0.026)	(1.469)	(0.002)	(0.986)	(0.013)	(0.010)	(0.145)	(0.133)
μ_{s1}	0.137	0.182	0.065	-0.002	0.203	-0.468	0.018	-0.175	0.005
	(0.002)	(0.002)	(0.021)	(0.001)	(0.05)	(0.371)	(0.007)	(0.345)	(0.002)
μ_{s2}	-0.065	-0.029	-0.191	0.016	-0.053	0.476	0.831	0.330	-0.978
	(0.001)	(0.001)	(0.251)	(0.014)	(0.036)	(0.036)	(0.532)	(0.085)	(0.006)
$Ku_t(d)$									
μ_{k0}	1.287	1.014	1.346	1.381	1.874	3.950	1.307	2.332	3.140
	(0.488)	(0.488)	(4.875)	(11.142)	(0.885)	(1.193)	(0.346)	(2.709)	(0.314)
μ_{k1}	0.075	0.122	0.003	0.029	0.094	-2.770	0.027	-0.730	-0.526
	(0.000)	(0.000)	(0.000)	(0.005)	(0.007)	(2.132)	(0.001)	(0.851)	(0.411)
μ_{k2}	0.057	-0.003	0.498	0.485	-0.024	0.023	0.478	0.262	0.186
	(0.001)	(0.001)	(1.879)	(4.192)	(0.027)	(0.024)	(0.267)	(0.207)	(0.154)

*For the European Union, the sample begins from 1983M10 for the CPI, while for the PPI it begins from 1996M01.

Table A2: Means of selected coefficients from extended regression - CPI*

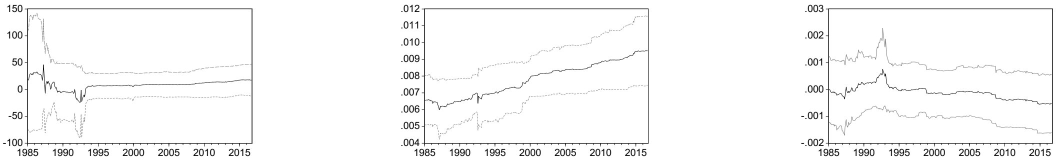
	$d = 1$	$d = 2$	$d = 3$
		Canada/USA	
skewness	0.582	0.343	0.302
	(0.407 0.756)	(0.233 0.453)	(0.161 0.443)
real GDP	-28.420	-29.49	-28.20
	(-57.990 1.152)	(-55.58 -3.399)	(-60.88 4.478)
1-year Treasury yield	-0.100	-0.099	-0.176
	(-0.684 0.483)	(-0.689 0.490)	(-0.787 0.435)
M2	-25.510	-15.20	-25.23
	(-74.51 23.49)	(-64.74 34.35)	(-81.99 31.53)
net capital flow	0.000	0.000	0.000
	(-0.000 0.000)	(-0.000 0.000)	(-0.000 0.000)
monetary policy	0.156	-0.232	-0.230
uncertainty	(-0.498 0.186)	(-0.562 0.098)	(-0.539 0.078)
		European Union /USA	
skewness	0.917	0.776	0.258
	(0.825 1.008)	(0.643, 0.908)	(0.204 0.312)
real GDP	-22.88	-17.16	-19.58
	(-61.84, 16.08)	(-61.62, 27.31)	(-76.27 37.11)
1-year Treasury yield	-0.881	-1.193	-1.156
	(-1.830, 0.069)	(-2.028, -0.358)	(-2.231 -0.081)
M2	-15.45	8.578	8.382
	(-98.46, 67.56)	(-70.74, 87.89)	(-105.10 201.80)
net capital flow	0.000	-0.000	-0.000
	(-0.000 0.000)	(-0.000 0.000)	(-0.000, 0.000)
monetary policy	0.007	-0.100	-0.003
uncertainty	(-0.415 0.429)	(-0.734 0.535)	(-0.616 0.611)
		Japan/USA	
skewness	3.309	2.220	3.590
	(2.317 4.301)	(1.684 2.757)	(2.235 4.944)
real GDP	14.88	24.81	36.75
	(-52.22 81.99)	(-44.04 93.65)	(-21.83 95.32)
1-year Treasury yield	-0.825	-0.760	-0.793
	(-2.261 0.611)	(-1.620, 0.099)	(-1.729 0.142)
M2	-21.970	-21.06	-16.60
	(-119.10 75.18)	(-126.20 84.12)	(-123.10 89.93)
net capital flow	0.000	0.000	0.000
	(-0.000 0.000)	(-0.000 0.000)	(-0.000, 0.000)
monetary policy	0.804	1.072	1.203
uncertainty	(0.018 1.591)	(0.508 1.636)	(0.548 1.858)

*The table reports the means of selected time-varying coefficients (in percentages) along with the means of the corresponding 95% confidence intervals (in parenthesis) from regression of ΔlnQ_t on a constant, σ^2_{t+d}, Sk_{t+d} and Ku_{t+d}, extended with additional controls. These controls include the first differences of the logarithms of U.S. real GDP and U.S. M2, the first difference of the 1-year U.S. Treasury yield, the level of U.S. foreign net long-term capital flows, and the logarithms of U.S. monetary policy uncertainty. The dependent variable is the first difference of the logarithm of the CPI-based real exchange rate.

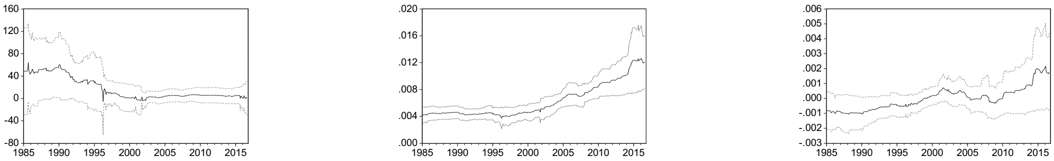
Figure A1: Estimated time-varying coefficients: Canada – PPI

$Variance_{t+d/t}$ $Skewness_{t+d/t}$ $Kurtosis_{t+d/t}$

d = 1: Pseudo R^2 = 0.32



d = 2: Pseudo R^2 = 0.49



d = 3: Pseudo R^2 = 0.40

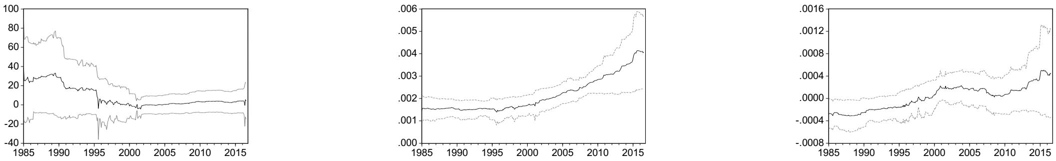
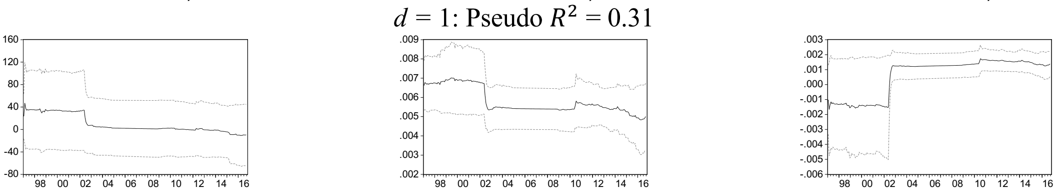


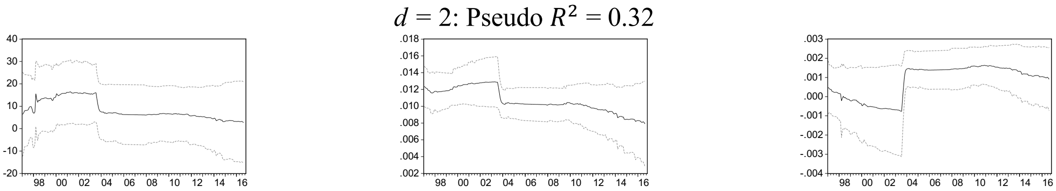
Figure A2: Estimated time-varying coefficients: EU – PPI

$Variance_{t+d/t}$ $Skewness_{t+d/t}$ $Kurtosis_{t+d/t}$

d = 1: Pseudo R^2 = 0.31



d = 2: Pseudo R^2 = 0.32



d = 3: Pseudo R^2 = 0.26

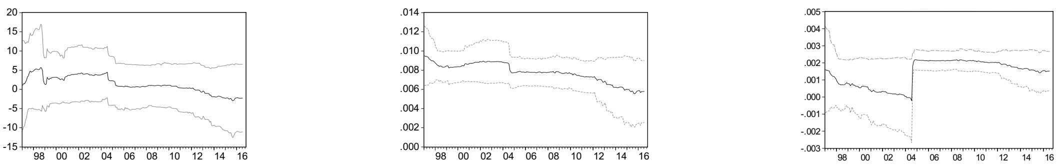
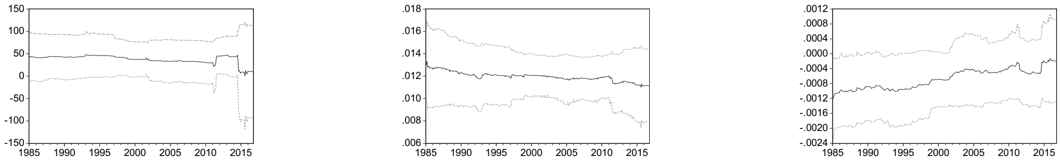


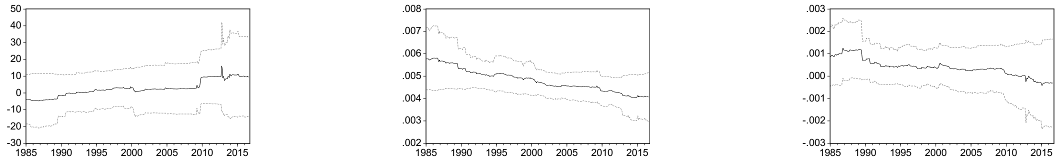
Figure A3: Estimated time-varying coefficients: Japan – PPI

$Variance_{t+d/t}$ $Skewness_{t+d/t}$ $Kurtosis_{t+d/t}$

d = 1: Pseudo R^2 = 0.38



d = 2: Pseudo R^2 = 0.51



d = 3: Pseudo R^2 = 0.42