

Identifying the host galaxy of gravitational wave signals

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One of the goals of the current LIGO-GEO-Virgo science run is to identify transient gravitational wave (GW) signals in near real time to allow follow-up electromagnetic (EM) observations. An EM counterpart could increase the confidence of the GW detection and provide insight into the nature of the source. Current GW-EM campaigns target potential host galaxies based on overlap with the GW sky error box. We propose a new statistic to identify the most likely host galaxy, ranking galaxies based on their position, distance, and luminosity. We test our statistic with Monte Carlo simulations of GWs produced by coalescing binaries of neutron stars and black holes, one of the most promising sources for ground-based GW detectors. Considering signals accessible to current detectors, we find that when imaging a single galaxy, our statistic correctly identifies the true host $\sim 20\%$ to $\sim 50\%$ of the time, depending on the masses of the binary components. With five narrow-field images the probability of imaging the true host increases from $\sim 50\%$ to $\sim 80\%$. When collectively imaging groups of galaxies using large field-of-view telescopes, the probability improves from $\sim 30\%$ to $\sim 60\%$ for a single image and from $\sim 70\%$ to $\sim 90\%$ for five images. For the advanced generation of detectors (circa 2015 +), and considering binaries within 100 Mpc (the reach of the galaxy catalogue used), the probability is $\sim 40\%$ for one narrow-field image, $\sim 75\%$ for five narrow-field images, $\sim 65\%$ for one wide-field image, and $\sim 95\%$ for five wide-field images, irrespective of binary type.

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I. INTRODUCTION

The next decade should see the first direct detection of gravitational waves with the global network of gravitational wave (GW) detectors. The two LIGO detectors [1,2] are located in Louisiana and Washington state, USA, the joint French-Italian Virgo detector [3] in Pisa, Italy, and the German-British detector GEO-600 [4] in Hannover, Germany. The 2005–2007 science run of LIGO-GEO-Virgo saw the two LIGO detectors take data at design sensitivity. Since 2009 LIGO and Virgo have been taking data with improved sensitivities, and GEO also recommenced data taking in 2010. It is expected that in the advanced detector era (circa 2015 +) LIGO and Virgo will operate at sensitivities more than 10 times greater than initial LIGO, thereby increasing the volume of monitored universe by more than a factor of 1000 [5,6]. In addition new detectors, such as LCGT in Japan [7] and AIGO in Australia [8], are being planned.

Electromagnetic identification of a GW might not only confirm a GW detection, but also improve parameter extraction, and, by independently identifying the source's position and time, lower the signal-to-noise ratio (SNR) required for a confident detection. Joint GW-EM observations could also address specific questions such as the nature of short hard γ -ray bursts and allowing a more precise measurement of H_0 ; see Bloom *et al.* [9] for an overview.

GW detectors are nonimaging detectors with a large field of view; their antenna response is greater than half maximum over 65% of the sky. Source localization for short-lived signals therefore requires multiple detectors, in

order to use the measured time delay between detectors as well as the amplitude of the measured signal in each detector to triangulate a sky location. Several methods of localization have been investigated [10–20]. Fairhurst [18] gives the following approximation for the timing accuracy of a GW signal:

$$\sigma_t \sim \frac{1}{2\pi\sigma_f\rho}, \quad (1)$$

where σ_f is the effective bandwidth of the signal and ρ is the SNR. For nominal values $\sigma_f = 100$ Hz and $\rho = 8$, timing accuracies are on the order of 0.1 ms. This can be compared to the light travel time between detectors, 10–30 ms for the LIGO-Virgo network. For example, for a binary coalescence signal at the threshold of detectability, Fairhurst [18] estimates a best-case localization of 20 deg² (90% containment), and a typical localization of twice this.

The locating and observing optical counterparts to unmodeled pulses (LOOC UP) project [21] consists of reconstructing the sky position of candidate GW signals and making prompt follow-up observations using wide field-of-view cameras. However, given the large sky error box associated with GW signals, identifying the source of a GW signal is not trivial; over 100 galaxies can be found within an error box out to 100 Mpc. We therefore present a ranking statistic for identifying the most likely host galaxy based on galaxy distance and luminosity and the sky position error box. Using Monte Carlo simulations of GW signals, we demonstrate that this ranking statistic can correctly identify the host galaxy for a significant

fraction of GW signals detectable by the initial and advanced LIGO-Virgo networks.

II. SOURCES OF GWS AND OPTICAL TRANSIENTS

Because of the strongly relativistic nature of GW sources, many systems that would produce detectable GW emission should also be observable electromagnetically. Cutler and Thorne [22] provide a brief review of sources of GWs observable by ground-based detectors. Mergers of binary neutron stars (NS-NS) or binaries consisting of a neutron star and a stellar mass black hole (NS-BH) are the best understood in terms of GW range and expected rate [23], and are the most likely sources for producing both detectable GW signals and optical transients. They are also the favored progenitor model for short γ -ray bursts [24].¹ These events will emit a significant proportion of their binding energy in GWs at frequencies to which the current and next generation of GW detectors are sensitive. The distance to which a GW signal can be detected depends on the masses of the binary components. Assuming fiducial masses of $1.4M_{\odot}$ for neutron stars and $10M_{\odot}$ for black holes, the current LIGO observatories can detect NS-NS binary systems with $\rho \geq 8$ out to a maximum distance of approximately 30 Mpc, and NS-BH systems out to 65 Mpc. With this sensitivity, Abadie *et al.* [23] estimate the most likely rate of detectable signals at $\sim 0.02 \text{ yr}^{-1}$ for NS-NS and $\sim 0.004 \text{ yr}^{-1}$ for NS-BH systems. Estimates of the optical emission due to the radioactive decay of heavy elements synthesized in the merger ejecta predict a peak magnitude of approximately 20 or brighter at 65 Mpc [25–27]. For advanced LIGO (circa 2015 +) the GW range increases to approximately 450 Mpc for NS-NS systems and 930 Mpc for NS-BH systems, with most likely rate estimates of $\sim 40 \text{ yr}^{-1}$ and $\sim 10 \text{ yr}^{-1}$, respectively. This could require optical imaging to magnitude 25–26 for the most distant sources. Restricting to sources within 100 Mpc (the reach of our galaxy catalogue, discussed below), the expected detection rates are $\sim 5 \text{ yr}^{-1}$ for NS-NS and $\sim 0.2 \text{ yr}^{-1}$ for NS-BH.²

Another possible EM-GW source is core-collapse supernovae; however, even for the advanced detectors, GW emission from these systems is likely to be detectable only for supernovae occurring within our galaxy [29], for which the rate is $\sim 0.02 \text{ yr}^{-1}$. We therefore focus our attention on NS-NS and NS-BH systems.

¹The γ emission might itself be used to identify the host galaxy for those cases where the emission is beamed towards us.

²The fraction of detections that occur within 100 Mpc may be estimated as $(100 \text{ Mpc}/R_{\text{SM}})^3$, where R_{SM} is the “sensemon range” [23,28]. R_{SM} is the radius of a sphere whose volume is the effective volume in which a source can be detected, taking into account all possible sky locations and binary orientations. R_{SM} is a factor of 2.26 smaller than the maximum detection distance.

III. GW CATALOGUE

We simulate GW signals coming from known external galaxies, using the Gravitational Wave Galaxy Catalogue [30] of White *et al.* [31]. This catalogue contains approximately 53 000 galaxies out to a distance of 100 Mpc. There are 22 000 galaxies within 65 Mpc, the maximum distance to which a $1.4\text{--}10.0M_{\odot}$ NS-BH system can be detected with $\rho \geq 8$ by initial LIGO, and 7300 galaxies within 30 Mpc, the maximum distance for a NS-NS binary. White *et al.* estimate the catalogue to have a completeness of 60% to 100 Mpc, 75% to 50 Mpc, and a completeness consistent with 100% out to 40 Mpc. Approximately 50% of the galaxies have a defined type in the de Vaucouleurs classification [32]; these account for 80% of the total luminosity in the catalogue.

IV. RANKING STATISTIC

A LIGO-Virgo GW error box can contain over 100 galaxies out to 100 Mpc. Imaging all to search for an EM counterpart will likely be impractical. This motivates considering ways to rank the galaxies by their likelihood of hosting the source of the observed GW event.

We expect a nearby galaxy to be more likely *a priori* to be the host of a detectable GW signal source than a more distant galaxy. Furthermore, larger galaxies contain more potential sources. We therefore propose to rank each galaxy as the possible host for a GW signal by the following statistic:

$$R = e^{-(\chi^2/2)} \frac{L}{d^\alpha}. \quad (2)$$

Here L is the luminosity of the putative host galaxy, d is the distance to the galaxy, α is a constant, and χ^2 is the chi-squared match between the measured and predicted time of arrival of the signal in each detector [12], given by

$$\chi^2 = \sum_i \frac{(t_i - p_i)^2}{\sigma_i^2}. \quad (3)$$

Here σ_i is the timing uncertainty in each detector, t_i is the measured arrival time, p_i is the predicted arrival time based on the sky direction of the putative host galaxy, and the sum is taken over all detectors. We include $\exp(-\chi^2/2)$ in our ranking statistic as this is the likelihood associated with a Gaussian timing error in each detector. It determines which galaxies have sky positions consistent with the observed time delays between detectors; i.e., it represents the GW triangulation error box.³

³For the LIGO-Virgo network that we will simulate, the χ^2 sky map is mirror symmetric through the plane of the detectors, thus usually yielding two error boxes. In principle, the measured signal SNRs can be used to break this degeneracy and determine which box contains the correct sky location. For our tests, we use both boxes. Therefore, a more sophisticated GW analysis than that assumed here may reduce the number of galaxies that need to be imaged by up to a factor of 2.

We scale R with luminosity because we assume the luminosity of each galaxy to be approximately proportional to the number of sources within it. The $d^{-\alpha}$ factor favors intrinsically weak signals from nearby galaxies as being more likely than strong signals from distant galaxies. More generally, if we assume the rate of GW events of intrinsic amplitude h_0 within each galaxy to be of the form

$$\frac{dN}{dh_0} \sim h_0^{-\alpha}, \quad (4)$$

then, since the received amplitude h is $h \propto h_0 d^{-1}$, the correct distance weighting is $d^{-\alpha}$. In our simulations we test $\alpha = 1, 2, 3$. We find $\alpha = 2$ gives marginally better performance for the initial LIGO detectors, and $\alpha = 1$ the best for advanced LIGO. However, the variation in the probability of identifying the host galaxy is only a few percent; we conclude that our ranking is not sensitive to the precise distance weighting used.

For comparison, we also test ranking based purely on the error box, with no luminosity or distance weighting:

$$R = e^{-(\chi^2/2)}. \quad (5)$$

This statistic is poor at identifying the host galaxy; the probability of correct identification is a factor of 2–4 lower (depending on binary mass) than when including the L/d weighting.

V. SIMULATIONS

To evaluate how well our ranking statistic identifies the true host galaxy of a GW signal, we simulate how GWs will appear in a realistic search. We consider inspiralling NS-NS and NS-BH binaries. The strength of their GWs has a well-defined dependence on the system’s mass, distance, and inclination of the binary orbital axis to the line of sight. We study three different mass pairs: 1.4–1.4 M_\odot NS-NS, 1.4–5.0 M_\odot NS-BH, and 1.4–10.0 M_\odot NS-BH systems. The orientations are random and isotropic. The true host galaxy is selected randomly with weight proportional to the galaxy luminosity and with an additional weighting based on galaxy type as discussed below.

We simulate the LIGO-Hanford, LIGO-Livingston, and Virgo network, assuming all three detectors to have sensitivity given by the initial LIGO design [1], or the advanced LIGO design [5]. For each GW, we compute the received SNR in each detector based on the binary mass and distance, and the detector sensitivity to that sky direction and binary orientation. We also compute the timing uncertainty using Eq. (1). The measured amplitudes and times are “jittered” by additive Gaussian errors to simulate the detector noise background. To be considered detected, a GW needs to have an SNR of $\rho \geq 8$ in at least two detectors, and a quadrature-sum SNR over all three detectors ≥ 12 . For each Monte Carlo run we generate enough binaries to give approximately 800 detected signals.

While our ranking statistic [Eq. (2)] treats all galaxy types equally, the rate of binary coalescences is likely to be different in different galaxy types. O’Shaughnessy *et al.* [33] estimate the rate of NS-NS and NS-BH mergers in elliptical and spiral galaxies for a large range of plausible binary evolution scenarios. They produce a total of 488 samples of merger rates, and find the relative rate in spirals and ellipticals to vary widely in their models. We account for this uncertainty in our simulations by performing 50 separate Monte Carlo runs for each waveform type; in each run, the relative rate of mergers in spirals and ellipticals is determined by a random draw from the models by O’Shaughnessy *et al.* We treat lenticular galaxies as equivalent to ellipticals and irregular galaxies as spirals for these simulations. For those galaxies without a specified type, one is assigned randomly in proportion to the number of galaxies of each type in the catalogue. In all, 70% of the galaxies are treated as spiral, and 30% as elliptical galaxies.

Finally, to simulate the effect of measurement errors in the galaxy catalogue we also jitter the luminosity and distance of each galaxy by a random amount consistent with the stated uncertainties. This is done by creating a second copy of the galaxy catalogue and using this jittered catalogue for signal generation (keeping the original catalogue for ranking).

After the GW signals are generated, we compute the χ^2 match [Eq. (3)] between the predicted and the measured GW arrival time at each detector. We then rank all the galaxies as potential hosts for each GW using Eq. (2). The distribution of ranks assigned to the true host galaxy for each GW then tells us the probability of observing the true host as a function of the number of galaxies imaged. This probability is shown in Fig. 1. We find that for a narrow field-of-view telescope [$O(10)$ arcmin, sufficient to image one galaxy at 10 Mpc] the probability of the true host being the top-ranked galaxy is $50 \pm 3\%$ for a 1.4–1.4 M_\odot NS-NS system, $32 \pm 2\%$ for a 1.4–5.0 M_\odot NS-BH, and $21 \pm 3\%$ for a 1.4–10.0 M_\odot NS-BH system. When imaging the five highest-ranked galaxies, the chances of including the true host increase to $78 \pm 3\%$, $63 \pm 3\%$, and $48 \pm 3\%$, respectively. For the advanced LIGO detectors, and considering only binaries within 100 Mpc, the probabilities are approximately independent of binary type: $39 \pm 3\%/43 \pm 4\%/40 \pm 3\%$ for one image and $72 \pm 3\%/75 \pm 3\%/73 \pm 3\%$ for five images. In each case the uncertainties are dominated by the range of possible relative rates for mergers in spiral versus elliptical galaxies.

We note that the success rate for initial LIGO is highest for NS-NS systems, and decreases with increasing binary mass. This is due to two factors. The effective bandwidth σ_f is larger for low-mass systems, giving smaller timing uncertainties [see Eq. (1)]. Furthermore, less massive binaries are detectable to smaller distances, hence there are fewer potential hosts for these systems, so the probability of imaging the true host increases. Indeed, in the NS-NS

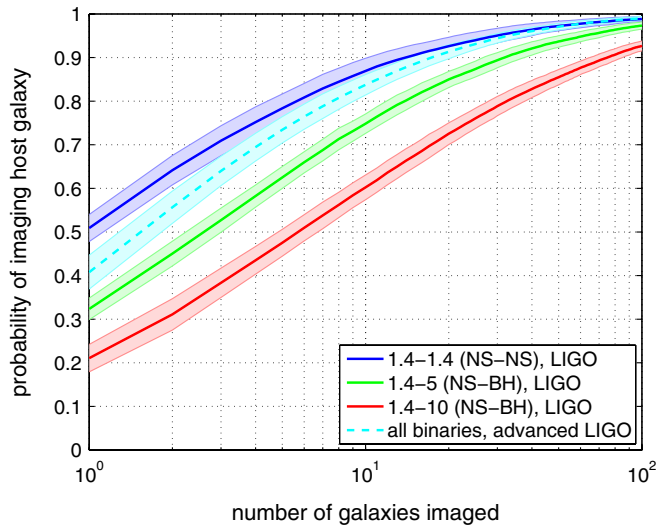


FIG. 1 (color online). Narrow field-of-view case. The probability of imaging the true host galaxy for each type of binary system versus the number of images taken. The shaded regions denote the 1-sigma uncertainty in the probability estimate.

simulations for current detectors, we find that 10% of all detected signals are due to only ten galaxies: PGC047885, NGC0224 (Andromeda galaxy), NGC4594 (Sombrero galaxy), ESO468-020, NGC0253, NGC5457 (Pinwheel galaxy), NGC6964, PGC2802329, PGC009892, and NGC4472. It may therefore be worthwhile to take reference images of these “most promising” galaxies *before* the GW search is performed, to allow immediate identification of an EM transient when one of these galaxies is selected for follow-up of a GW signal.

For the advanced LIGO detectors, we find that the probability of imaging the true host galaxy is approximately the same for all binary types. This is due to the restriction to signals originating within a fixed distance of 100 Mpc. Higher-mass systems give larger SNR ρ at a fixed distance; this offsets the effect of their lower effective bandwidth σ_f in the timing uncertainty [Eq. (1)].

The LOOC UP program [21] is currently using wide field-of-view telescopes to image potential host galaxies, including TAROT [34], QUEST [35], and SkyMapper [36], as well as narrow-field telescopes such as Zadko [37]. Depending on the length of exposure (between 60 and 180 s) and the filter used, these telescopes have limiting magnitudes ranging from 17 to 22, sufficient to detect the EM emission from binary mergers predicted by Metzger *et al.* [27] to 15–150 Mpc. The wide-field telescopes can image several square degrees at once, allowing multiple galaxies to be observed simultaneously and therefore increasing the probability of observing the true host in a given number of exposures. We simulate imaging with a 3–4 deg² field-of-view telescope by grouping galaxies which lie within 1 deg of one another when computing the probability of imaging the host. That is, we consider

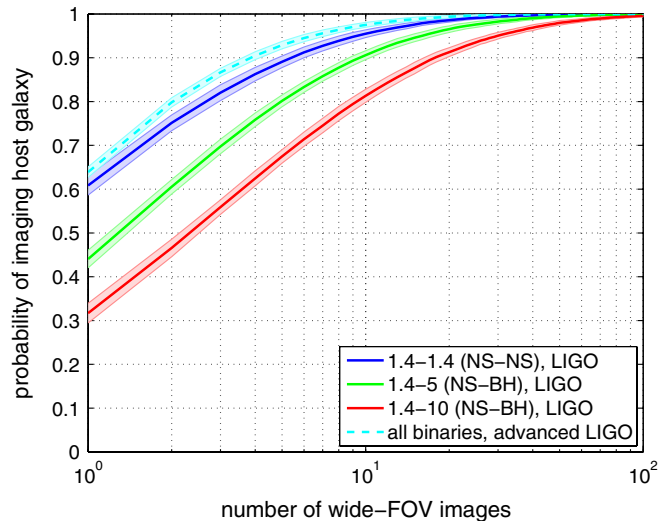


FIG. 2 (color online). Wide field-of-view case. The probability of imaging the true host galaxy for each type of binary system versus the number of images taken. The shaded regions denote the 1-sigma uncertainty in the probability estimate.

the true host as having been imaged if it lies within 1 deg of any of the N top-ranked galaxies, where N is the number of wide-field images taken. The results are shown in Fig. 2. We find that for initial LIGO, for 1.4–1.4 M_{\odot} /1.4–5.0 M_{\odot} /1.4–10.0 M_{\odot} systems the chances of observing the true host are $61 \pm 2\%/44 \pm 2\%/32 \pm 2\%$ for one image and $89 \pm 1\%/80 \pm 1\%/67 \pm 2\%$ for five images. These are a factor of about 1.2 better than the narrow field-of-view results. For the advanced LIGO detectors the probabilities are $64 \pm 1\%/68 \pm 1\%/64 \pm 1\%$ for one image and $93 \pm 1\%/94 \pm 1\%/92 \pm 1\%$ for five wide-field images, a factor of 1.3–1.5 better than in the narrow field-of-view case.

VI. SUMMARY AND CONCLUSION

We propose a ranking statistic for identifying the host galaxy of gravitational wave signals. The ranking is based on the galaxy distance, luminosity, and overlap with the GW sky position error box. We have tested the statistic by simulating GW signals from coalescing binaries of neutron stars and black holes and using the Gravitational Wave Galaxy Catalogue of White *et al.* [31]. For the current LIGO-Virgo network we find the probability of the true host being within the field of view of the top-ranked galaxy ranges from $\sim 20\%$ to $\sim 60\%$, depending on the masses of the binary components and whether the observations are made with narrow field-of-view or wide field-of-view telescopes. The probability of the true host being in the top 5 ranges from $\sim 50\%$ to $\sim 90\%$. For the advanced LIGO-Virgo network, and restricting to binaries within 100 Mpc (the range of our catalogue), the probability of the true host being the top-ranked ranges from $\sim 40\%$ to $\sim 70\%$, and in the top 5 from $\sim 70\%$ to $>90\%$. In general our ranking

statistic favors larger, closer galaxies as the most probable hosts, and so performs best for GWs from nearby galaxies.

Our simulations account for uncertainties in the relative rate of mergers in galaxies of different types, as well as uncertainties in the measured properties of the galaxies (distance, luminosity, and type). We find these effects change the probability of imaging the true host by only a few percent. We have also verified that the ranking is not sensitive to the precise distance weighting used. We believe the main source of systematic error that we have not accounted for is the incompleteness of the galaxy catalogue. That is, some fraction of detectable GWs will originate in galaxies that are not included in the catalogue, and so the true host cannot be given a ranking. Our estimated probabilities for successful imaging should be multiplied by the catalogue completeness, estimated as 75% to 50 Mpc and higher at smaller distances. For comparison, our simulations reveal that 90% of the GWs detectable by current instruments originate from galaxies within 21/34/44 Mpc for 1.4–1.4 M_{\odot} NS-NS/1.4–5.0 M_{\odot} NS-BH/1.4–10.0 M_{\odot} NS-BH binaries.

The approximation [Eq. (1)] used for timing errors has been shown to underestimate the error for low-SNR signals by up to 20% [18]. Increasing the timing error for all detectors and all signals by 20% was found to change the probabilities by only a few percent. A more important factor is that our simulations treat the advanced Virgo detector as identical to advanced LIGO; using the design proposed in [6] reduces the probabilities by 5%–10% due to the lower distance sensitivity of the advanced Virgo design.

Finally, let us comment briefly on the applicability of our ranking statistic to the advanced LIGO [5] and Virgo [6] detectors. Advanced LIGO will have maximum ranges of ~ 450 Mpc for NS-NS systems and ~ 930 Mpc for NS-BH systems. This improved sensitivity presents two challenges for host identification: there are many more galaxies in a typical sky position error box; and we lack comprehensive galaxy catalogues to these distances. While our technique appears to be promising for the most close-by binaries (within 100 Mpc, expected at a rate of a few per year), more extensive catalogues will be required to apply it to the majority of detected signals. More generally, further investigation is needed of strategies for host galaxy identification in the advanced detector era.

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