# Effects of porosity on the fatigue performance of polymethyl methacrylate bone cement: An analytical investigation

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# Effects of porosity on the fatigue performance of polymethyl methacrylate bone cement: an analytical investigation

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**Abstract:** Porosity has been shown to affect the fatigue life of bone cements, but, although vacuum mixing is widely used to reduce porosity in the clinical setting, results have been mixed and the effects of porosity are not well understood. The aim of this study was to investigate the effects of porosity using stress analysis and fracture mechanics techniques.

The stress concentrations arising at voids in test specimens were found using analytical solutions and boundary element methods. The fatigue life of specimens containing voids of various sizes was predicted using fracture mechanics techniques.

For spherical voids that do not occupy a significant proportion of the cross-section, the resulting stress concentration is independent of void size and too small to account for the observed crack initiation. Cracks must therefore initiate at additional stress raisers such as radiopacifier particles or additional voids. For large voids, the stress increases as the remaining cross-section of the specimen decreases, and this may account for much of the observed reduction in fatigue strength in hand-mixed cement.

Although crack initiation may be largely independent of void size, there is an effect on crack growth rate. Cracks are predicted to grow faster around larger voids, since they remain in the stress concentration around the void for longer. This effect may account for the relationship between porosity and fatigue life that has been observed in samples without large voids.

Since porosity appears to affect crack growth more than initiation, it may be less damaging in high-cycle clinical fatigue, which may be predominantly initiation controlled, than in short laboratory tests.

**Keywords:** bone cement, fatigue, polymethyl methacrylate (PMMA)

## 1 INTRODUCTION

Polymethyl methacrylate (PMMA) bone cement is currently the most successful method for fixing hip prostheses [**1**] and is widely used in many other applications including other joint replacements and vertebroplasty. There is evidence that fatigue failure of the cement occurs [**2**], and cement failure is implicated in other forms of aseptic loosening. For example, localized failure of the cement is part of the process of failure of the bone cement interface. Cracks in the cement typically run in a radial direction relative to the femoral stem [**2**], possibly as a result of shrinkage stresses [**3**], and cracks that penetrate the cement mantle can provide a path for fluid pressure and/or wear particles from the joint capsule to reach the bone cement interface, causing osteolysis. Minimizing the risk of fatigue failure of the cement is therefore the focus of considerable research.

Fatigue cracks in bone cement have typically been found to initiate at internal defects, such as pores or agglomerations of barium sulphate opacifier [**4**]. In smooth test specimens there are few stress raisers on the outer surface, and so internal defects are the most likely site for crack initiation. A great deal of attention has been focused on the role of porosity. Several studies [**5**, **6**] have shown that the fatigue life of test specimens is increased when vacuum mixing systems are used to minimize air entrapment, which can produce large pores. The use of vacuum mixing is now widespread, and has other advantages in minimizing monomer inhalation during mixing. However, the Swedish hip register has shown a higher incidence of early aseptic loosening when vacuum-mixed cement is used [**7**], and this implies that the situation is somewhat more complicated than has been generally assumed. Ling and Lee [**8**] reviewed various clinical studies of the effects of vacuum mixing and found no evidence of any improvement in clinical performance; in fact, in some studies there was a higher revision rate when vacuum mixing was used.

There have been many published studies of the effects of porosity on cement fatigue performance [**5**, **6**], but in general these have involved only simple S–N tests at relatively high loads. Not surprisingly, considering the simplistic nature of the tests and the wide variety of specimen designs, cement types and preparation techniques, and loading conditions, the results have been somewhat contradictory. Some authors have found increased fatigue life when vacuum mixing is used [**5**, **6**], while others have found little or no effect with some mixing systems [**9**]. There has perhaps been a tendency to over-generalize from the results of these studies.

In an attempt to reduce the number of complicating factors in these tests, Cristofolini *et al.* [**10**] tested a series of specimens of varying porosity, which was characterized before testing using radiography. They found that pores larger than 1 mm in diameter were often the site of failure, and that more consistent results were obtained when these specimens were discarded. Based on this finding, they proposed that specimens with pores $>1$ mm in diameter should be discarded whenever bone cement is fatigue tested, and argued that these large pores are in themselves detrimental to the cement. This paper stimulated some discussion, with a reply from Prendergast *et al.* [**11**] arguing that, since such pores are common *in vivo*, it would be inappropriate to remove them when testing cement samples. However, this argument ignores the very different size, shape, and stress distribution of the *in vivo* cement mantle.

Other studies by Lewis *et al.* [**12**] and by Dunne *et al.* [**6**] have also shown detrimental effects of large flaws in cement samples. Since the pores in question occupied a substantial proportion of the cross-section of the samples, this is perhaps not surprising. However, it remains to be shown that large pores in themselves weaken the cement, aside from the reduction in specimen cross-section, as has sometimes been argued on the basis of fracture mechanics considerations.

The aim of this study was therefore to use stress analysis and fracture mechanics techniques to investigate the effects of pores on crack initiation and growth, and hence to gain an insight into the effects of porosity on fatigue life. Analytical techniques have the potential to provide a more general understanding of the effects of factors such as specimen size, which can be addressed only in a very limited way in an experimental study.

## 2 METHODS

For the purpose of this study, the experiments of Baleani *et al.* [**13**] were used as a basis for analysis. This group have also published fatigue crack growth data for the same cement [**14**], which was used for crack growth calculations.

Solutions for the stresses around a spherical internal pore are summarized by Peterson [**15**]. Timoshenko and Goodier [**16**] give an analytical solution for the stress concentration factor around such a void under uniform uniaxial tension. In this case, the maximum principal stress occurs around the equator of the void and is greater than the remote uniform stress by a stress concentration factor that depends only on the Poisson's ratio

$$K_t = \frac{27 - 15\nu}{14 - 10\nu} \tag{1}$$

This stress concentration is rather localized, and little interaction would be expected between the voids in bone cement. Sternberg and Sadowsky [**17**] considered the case of two voids, one diameter apart, in an infinite continuum, and found only a 5 per cent increase in stress compared with a single void. Although this solution was for biaxial loading, it gives an indication of the lack of interaction between adjacent voids.

For small voids, therefore, the maximum stress was calculated by multiplying the nominal stress by the stress concentration factor defined above. Larger voids can significantly reduce the net cross-sectional area, and so the nominal stress was recalculated on the basis of the reduced cross-sectional area. For very large voids that occupy a substantial proportion of the cross-section, the assumption of an infinite body is no longer appropriate, and so a numerical method was used instead.

Quarter-symmetry models of the gauge length of the tensile specimens used by Baleani *et al.* [**13**] were used ($5\ \text{mm} \times 3.5\ \text{mm}$ cross-section), containing a central void. These were modelled using Franc3D boundary element software (Cornell Fracture Group, Cornell University, http://www.cfg.cornell.edu), with its associated solid modelling program (OSM) and boundary element solver (BES). Approximately 700 linear boundary elements were used, as shown in

Fig. 1. Care was taken to ensure a ring of well-formed elements in the area of interest; this is important if accurate results are to be obtained. Although a finite element model would have been equally effective, the use of boundary elements allowed rapid generation and solution of three-dimensional models with a high degree of mesh refinement in the areas of interest, and is also advantageous for modelling crack growth.

To calculate the fatigue crack growth rate, a simple Paris law relationship was used

$$\frac{da}{dN} = C\,\Delta K^{m} \tag{2}$$

where $da/dN$ is the crack growth rate in metres per cycle and $\Delta K$ is the stress intensity range. The empirical constants $C$ and $m$ were found from the data presented by Piazza *et al.* [**14**] to be $2 \times 10^{-6}$ m/cycle and 5.48 respectively; $\Delta K$ is in MPa $m^{1/2}$.

To calculate the stress intensity of a crack growing around a void, three separate procedures were used. For a small semicircular crack growing from a void, where the radius of the crack, $a$, is less than a tenth of the radius of the void, $r$, the stress intensity is given by the standard solution for a semicircular edge crack [**18**] multiplied by the stress concentration factor defined above

$$K = 0.713(K_t \sigma)\sqrt{\pi a} \tag{3}$$

where $\sigma$ is the remote stress. For a crack that has grown much larger than the void, the stress intensity is that for a simple circular internal crack [**18**]

$$K = (2/\pi)\sigma\sqrt{\pi(r+a)} \tag{4}$$

This solution is accurate when $a > 2r$; for intermediate values a linear interpolation was used, based on a previously presented boundary element model [**19**]. Figure 2 shows the results of this interpolation together with the numerical results.

To find the number of cycles for the crack to grow to a given length, these formulae for $K$ were substituted into the Paris law equation [equation (2) above], replacing the stress $\sigma$ with the stress range $\Delta\sigma$. For the short and long crack cases [equations (3) and (4)], the various constants were substituted into the equations, which were then solved by separating the variables and integrating in the usual way. For the short crack case this gave

$$N = -1518.4\Delta\sigma^{-5.48}(a_f^{-1.74} - a_i^{-1.74}) \tag{5}$$

where $N$ is the number of cycles required for the crack to grow from the initial crack length $a_i$ to the final crack length $a_f$. Similarly, for the long crack case

$$N = -148\,240\Delta\sigma^{-5.48}(a_f^{-1.74} - a_i^{-1.74}) \tag{6}$$

For intermediate crack lengths, the integration was carried out numerically by breaking the crack growth into several portions, with the stress intensity

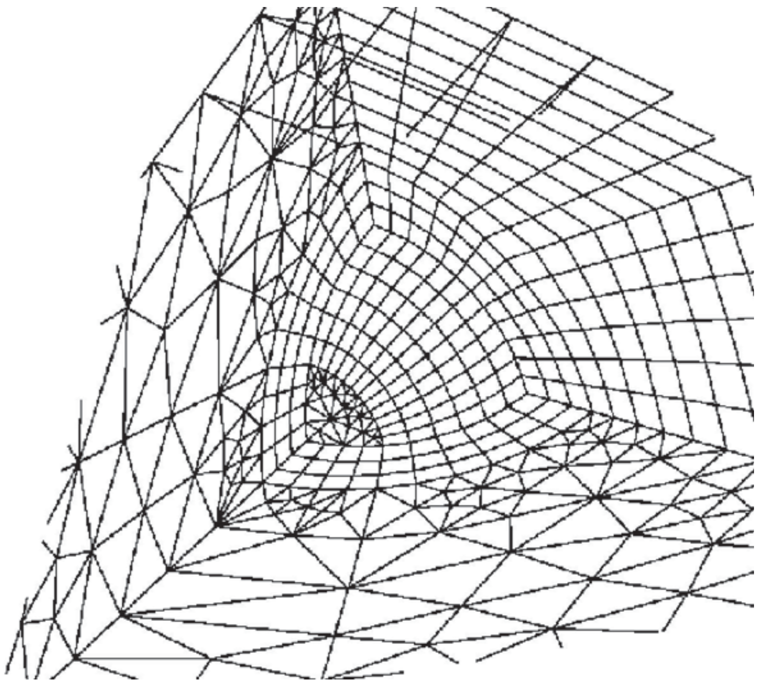

**Fig. 1** Typical boundary element mesh representing a void in the centre of a tensile specimen. Only one-eighth of the central section of the gauge length needs to be represented owing to symmetry; the picture shows the mesh around the void

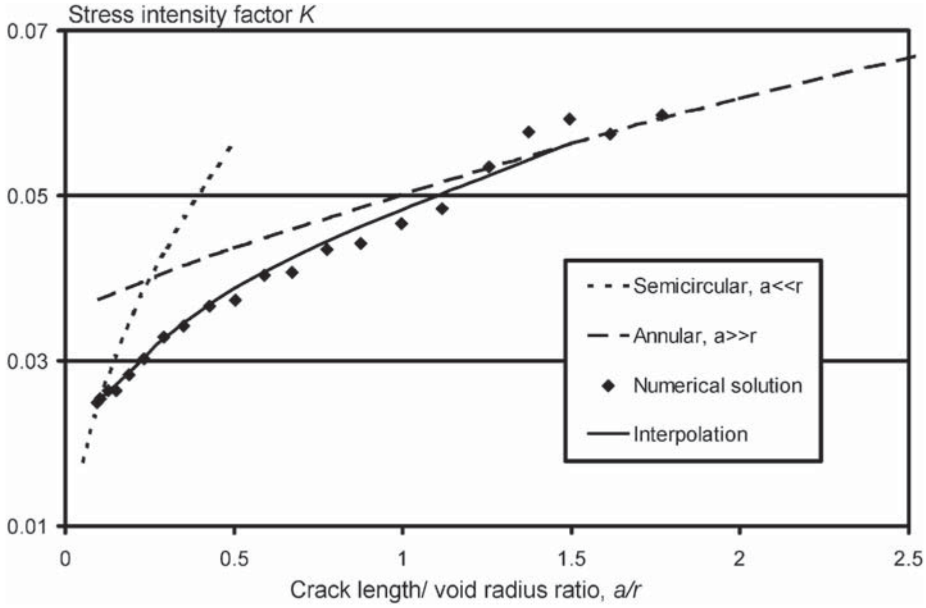


**Fig. 2** Stress intensity factors for a crack growing from a spherical void in a continuum, showing short and long crack approximations, interpolation for intermediate values, and numerical results. The stress intensity values shown are for a 1 mm diameter void and an applied stress of 2 MPa

assumed to be constant for each. The number of steps used was increased until convergence was achieved, which required 50 steps. The full procedure to calculate the number of cycles required for a crack to grow from a given initial length to a given final length around a void of given radius was therefore as follows.

1. Calculate the maximum crack length for which the short crack solution is valid ($a = r/10$), and find the number of cycles to reach this length using equation (5).
2. Calculate the minimum crack length for which the long crack solution is valid ($a = 2r$), and find the number of cycles required to reach this length numerically.
3. Calculate the remaining cycles required to reach the final length using equation (6).

## 3 RESULTS

For a Poisson's ratio of 0.33, equation (1) gives a stress concentration factor of 2.06. Note that this is independent of the size of the void.

Figure 3 shows a typical stress distribution around a void in a boundary element model. The stress concentration is very localized around the equator of the void. Figure 4 shows the stress concentration factor as a function of the size of the void in a 5 mm × 3.5 mm tensile specimen. The stress does not increase significantly until the void is about 1 mm in diameter; the boundary element and analytical solutions are in good agreement, except for the largest void sizes when the void approaches the edges of the specimen. For intermediate void sizes, a close approximation to the actual stress was found using the stress concentration factor and correcting for the actual cross-sectional area.

The stress around voids of various sizes in specimens of 10 × 4 mm cross-section was calculated in the same way, using the actual cross-sectional area, and this is shown in Fig. 5 with some results obtained by Cristofolini *et al.* [**10**] for comparison. There is an excellent correlation between the predicted stress and the proportion of voids that caused failure in the tests of Cristofolini *et al.*

The number of cycles for a crack to grow from an initial size of 20 μm to a final diameter of 2 mm is shown as a function of void size in Fig. 6. It is evident that the fatigue life is reduced as the void size increases, even though the stress concentration factor around the void and the number of cycles for crack initiation are unchanged.

## 4 DISCUSSION

### 4.1 Initiation of cracks at pores

It has been widely assumed that pores act as sites for crack initiation in bone cement owing to their stress-raising effect, and also that larger pores will

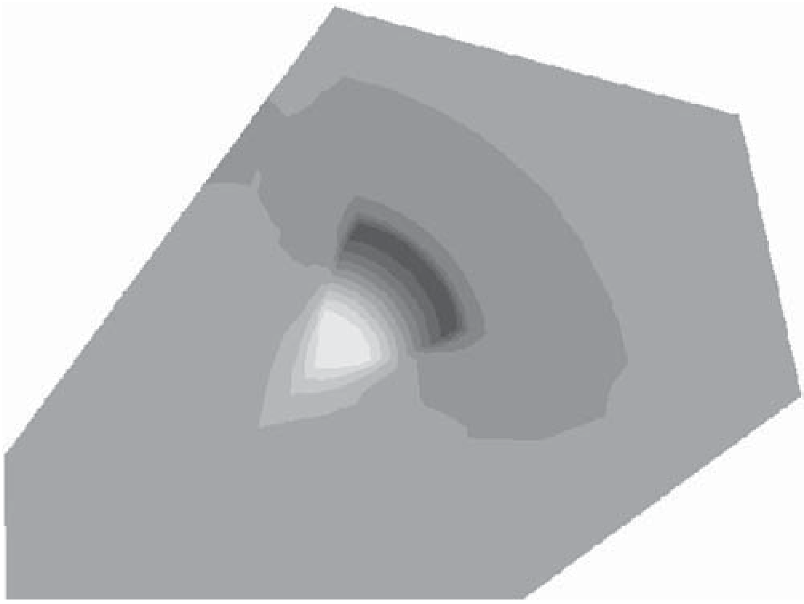

**Fig. 3** Contour plot showing a typical stress distribution around a void. The stress concentration is highly localized around the equator of the void, and the adjoining outside surface has little effect

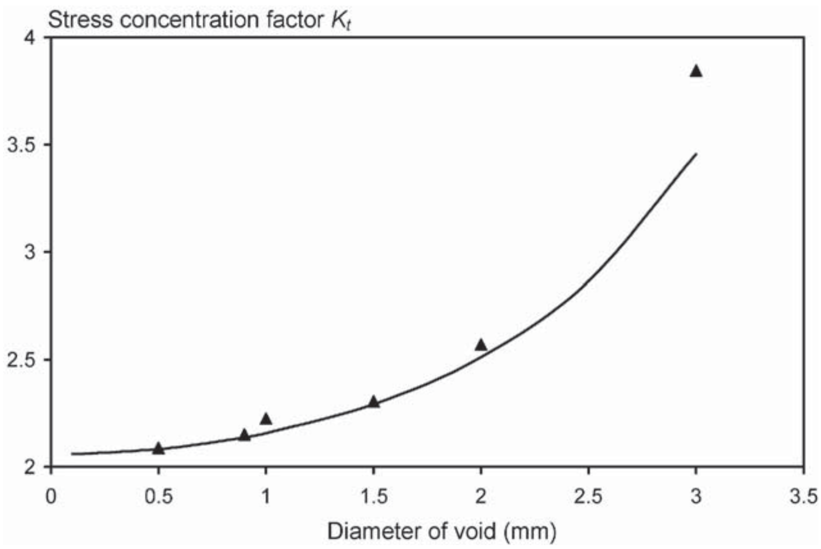


**Fig. 4** Stress concentration factor as a function of void size for central voids in a specimen of $5 \times 3.5$ mm cross-section. The solid line shows the approximate analytical solution, using the actual cross-sectional area, and the points show boundary element results

have a more damaging effect than smaller ones, based on the fracture mechanics relationship between the size of sharp defects and the failure stress. However, voids are not sharp cracks, and it can be seen in equation (1) that the stress concentration around a smooth void depends only on the Poisson's ratio and not on the void diameter. Using an appropriate value of Poisson's ratio for bone cement, the resulting stress concentration factor is only 2.06, which is insufficient to cause crack initiation. For example, Baleani *et al.* [**13**] applied stresses in the range 12–18 MPa, which would result in stresses at the voids in some cases below 25 MPa. Since the yield stress of the cement is typically in excess of 80 MPa, this is insufficient to cause crack initiation.

There must therefore be further stress concentrations at the edges of the voids or elsewhere, in order to initiate fatigue cracks. These might include the rough surface of the inside of the voids, caused

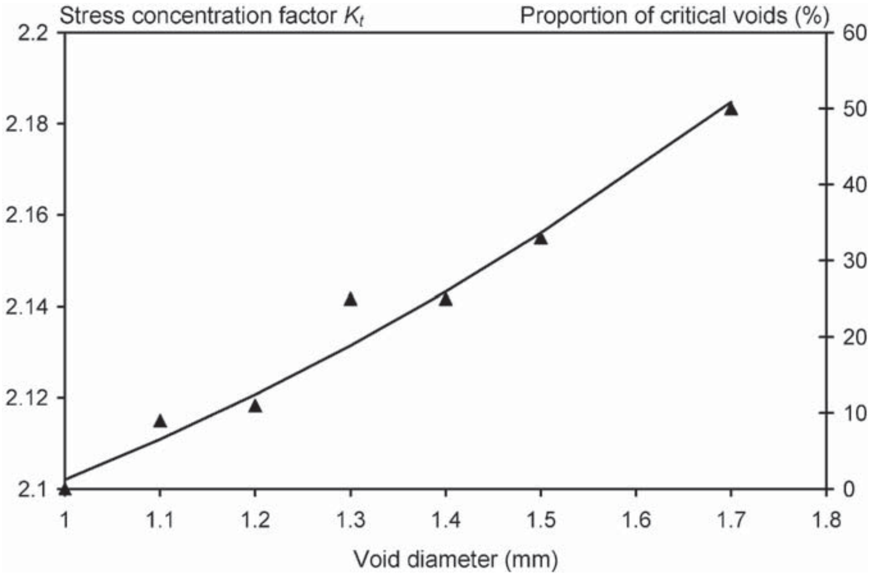


**Fig. 5** Approximate stresses around voids in specimens of 10 × 4 mm cross-section. Data from Cristofolini *et al.* [**10**] showing the proportion of critical voids of various sizes is included for comparison. There is an excellent correlation between the predicted stress and the proportion of the voids that caused failure

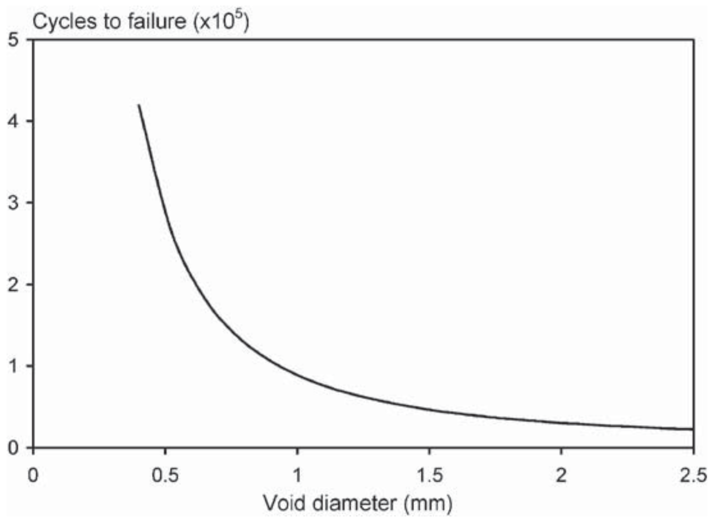


**Fig. 6** Number of cycles required for a 0.02 mm crack originating at the equator of a spherical void to grow to 2 mm as a function of the radius of the void. Crack growth is faster around the larger voids because the crack takes longer to grow out of the stress concentration created by the void

by the prepolymerized PMMA beads in the cement, and adjacent barium sulphate particles. In either case, the stress concentration produced will be largely independent of the size of the void.

A further point to consider is that fatigue cracks in PMMA initiate by crazing. Since this involves localized cavitation of the material, a substantial tensile hydrostatic stress is needed [**20**]. Precise values for fatigue crack initiation in PMMA are not readily available in the literature, but Gearing and Anand [**20**] give a criterion for craze initiation after prolonged static loading

$$\sigma_1 \geqslant 45.60 + \frac{785.56}{\sigma_H}$$
$$\sigma_H = \tfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \tag{7}$$

For a small spherical void with an applied stress of 12 MPa, this gives a critical maximum principal stress of 122.3 MPa, much higher than the actual value of 24.96 MPa. Even under fatigue conditions, this is insufficient to initiate a crack. Secondary stress concentrations that have sharp corners, such as radiopacifier particles, may cause higher triaxial stresses as well as a higher maximum principal stress, and hence are more likely sites for crack initiation immediately adjacent to a void.

If cracks initiate at secondary stress concentrations, and the stress concentration due to the voids is independent of void size, the size of the voids may have little or no effect on the *initiation* of cracks. However, there is a possible effect on the *growth* of cracks around voids, which is discussed below.

## 4.2 Effect of large voids

In the preceding section it was argued that there is no intrinsic link between the size of a void and the likelihood of crack initiation. However, for large voids occupying a significant fraction of the cross-sectional area of the specimen, a further stress concentration will arise, over and above that which would be caused by a similar void in an infinite continuum. This effect was analysed by two methods, firstly by using the actual cross-section around the void to calculate the stress and secondly by using boundary element models. The results of this analysis are shown in Fig. 4, and it is clear that in these specimens the stress around voids smaller than about 1 mm diameter was roughly constant and independent of the void size. For larger voids, there was an increase in stress. The boundary element results confirm that the analytical approximation using the actual cross-sectional area gives accurate stresses for all but the largest voids.

This approximation was used to estimate the stresses around the voids in the rather larger specimens used by Cristofolini *et al.* [**10**], as shown in Fig. 5. On the same graph, the results of Cristofolini *et al.* for the proportion of critical (crack-initiating) voids are shown for comparison. There is an excellent correlation between the predicted additional stress concentration and the number of critical voids. This additional stress concentration would seem to account for these results, and also to allow a useful generalization of the recommendations of Cristofolini *et al.* The presence of the large voids effectively reduces the specimen cross-sectional area; for specimens of different shapes and sizes, samples containing voids larger than about 2 per cent of the cross-section should be rejected. Also, this explanation does not tend to support the contention of Prendergast *et al.* [**11**] that large voids are an integral part of the cement and should be included in the test specimens, since the effect of these voids seems to be largely a function of the specimen design and reveals little about the properties of the cement itself.

The results of James *et al.* [**5**] also corroborate the argument that porosity is most significant when it occupies a substantial fraction of the cross-sectional area. They found that, for uncentrifuged, hand-mixed cement, there was a very strong negative correlation between porosity measured in the fracture surface and fatigue life. These specimens had a porosity in the range of 2–33 per cent. However, for centrifuged specimens with a porosity of 2.5–6.5 per cent, there was only a weaker correlation ($R = -0.7$) and a much smaller gradient. There was still a decrease in fatigue life with increased porosity, however, and this may be explained by the effects of voids on fatigue crack growth.

## 4.3 Effect of void size on crack growth rate

It was argued above that the size of voids in the cement may have little effect on the initiation of fatigue cracks, unless the voids occupy a significant proportion of the cross-section of the specimen. For polystyrene, a similar brittle, amorphous polymer, it was found by Botsis and Huang [**21**] that both the size of the initial crack and the number of cycles for crack initiation had an exponential relationship with the applied stress. It would be expected, therefore, that, since the stress concentration is similar regardless of void size, the number of cycles to crack initiation and the size of the initial crack would also be constant (of the order of 20 μm). However, a further effect arises as the crack grows around the void. For short cracks, the stress intensity rises more quickly than for longer cracks, as seen in Fig. 2. Comparing equations (5) and (6), the only difference is in the initial constant which is two orders of magnitude greater for long cracks. Short cracks will thus grow much more quickly than long ones over a given range of crack length; 'short' and 'long' in this context are relative to the size of the void, so that cracks around larger voids will spend more time in the stress concentration around the void and hence grow more quickly. The effect of this is shown in Fig. 6, which shows the number of cycles required for a 20 μm crack to grow to 2 mm in length as a function of the void size. It is clear that large voids will lead to more rapid failure than smaller ones.

These results provide a further explanation of the results of Cristofolini *et al.* [**10**] and others [**2**, **4–6**, **9**, **10**]. For voids smaller than about 1 mm diameter, the life rises rapidly, while for larger voids the life is relatively short. This may explain why Cristofolini *et al.* found no failures starting from voids smaller than 1 mm in diameter, and also provides a further explanation for the sharp increase in the number of critical voids between 1.1 and 1.7 mm in diameter.

This effect is also interesting, because it provides some explanation of the large numbers of microcracks that are sometimes seen in bone cement, especially around voids. Small cracks grow quickly in the stress concentration around a void, but then slow down as they outgrow the stress concentration, resulting in large numbers of 'dormant', very slow-growing microcracks.

A further interesting implication is that porosity may be less damaging in high-cycle fatigue (and hence *in vivo*) than in laboratory tests. Published laboratory studies [**5**] have typically used relatively high loads leading to failure in $10^3$–$10^5$ cycles. In this situation, crack initiation is rapid and the fatigue life is mainly determined by the number of cycles required for crack growth. In high-cycle fatigue, however, where failure occurs in $10^6$–$10^8$ cycles, the life is usually determined primarily by the number of cycles required for crack initiation. In this situation the effect of void size may be less pronounced.

### 4.4 Improving the fatigue life of bone cements

Recently, efforts to improve the fatigue life of bone cements have focused on reducing porosity, since this has been shown significantly to improve fatigue performance in the laboratory. However, once the misleading effect of large voids is removed, the effect of void size is much less significant. Additional stress raisers, such as barium sulphate particles, are needed to initiate cracks from voids, and some studies [**4**] have shown cracks predominantly initiating from these features even when there are large voids nearby. There is clearly scope for improvements in fatigue performance through the use of alternative radiopacifiers [**22**] or increased fatigue crack propagation resistance [**23**].

### 4.5 Relationship between laboratory tests and *in vivo* fatigue life

The present results suggest that in some cases the observed effects of porosity on fatigue life are primarily due to large voids occupying a significant proportion of the cross-section of the specimen. It is not clear how these results relate to clinical practice. The shape of the cement mantle is quite different, and it is likely that different sizes and shapes of voids will be formed, depending on the cementing technique. Also, it is possible that the cement mantle *in vivo* may be more tolerant of damage, since it has more possibilities for load redistribution when one part of the cement is damaged. As an extreme example, in the case of a tapered, polished femoral stem, even extensive cement cracking can be accommodated by distal migration and wedging of the stem.

A further complicating factor is the effect of voids on crack growth, which may be more important in low-cycle, high-load laboratory tests and less relevant in high-cycle clinical failure. There are also issues regarding variable-amplitude loading that have not been investigated to any significant extent; for example, clinical cracks may be initiated by accidental overload many years after implantation. Stress relaxation and creep are also important [**24**]; periods of prolonged loading will cause greater deformation, while periods with little load may allow some recovery and relaxation of stresses.

Although it is difficult to predict the behaviour of bone cement *in vivo*, the analysis presented here may provide an explanation of the discrepancy between laboratory fatigue tests, which have shown substantial effects of porosity on fatigue life, and actual clinical results, where no such correlation has yet been reported [**8**]. It appears that the effect of porosity in the laboratory may be largely governed by the specimen design and test method, and so it is perhaps not surprising that there is little or no evidence that the same effects occur in clinical practice.

## 5 CONCLUSIONS

1. The stress concentration produced by a smooth, circular void is independent of the size of the void unless it significantly reduces the overall cross-section. Typically, the stress concentration factor is around 2, which is insufficient to initiate cracks in PMMA under common fatigue loads. Crack initiation therefore requires some further stress raiser, such as a radiopacifier particle, a rough surface inside the void, or an adjacent smaller void.
2. For large voids, the stress increases owing to the reduction in cross-section. Test samples with voids that occupy a significant proportion of the cross-section should therefore be rejected – the strength of these will depend on the size of the specimen and therefore reveals little about the properties of the cement.
3. Although void size may not have much effect on crack initiation, crack growth around a large void will be significantly quicker since the crack remains in the stress concentration caused by the void for a larger proportion of its life.
4. While large voids would be expected to have a

detrimental effect on fatigue life, small voids may be of limited significance compared with other stress concentrations such as agglomerations of radiopacifier particles, which are also implicated in crack initiation from voids. Future work on improving the fatigue resistance of cements may usefully concentrate on reducing these other stress concentrations and improving the resistance of the cement matrix to crack propagation, rather than focusing exclusively on porosity.

5. The precise relationship between the results of laboratory tests and the actual clinical situation is unknown. Much of the observed effect of porosity on specimen fatigue life may be attributed to the presence of large voids occupying a significant proportion of the cross-sectional area, and it is not clear how this would relate to the clinical situation. Also, it is not clear that the effect of voids on crack growth would be as important in high-cycle fatigue *in vivo*, since this may be predominantly controlled by crack initiation.

## ACKNOWLEDGEMENTS

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## APPENDIX

### Notation

| | |
|---|---|
| $a_i$, $a_f$ | initial and final crack lengths (m) |
| $C$ | Paris law constant |
| $da/dN$ | crack growth rate – increase in crack length per cycle (m/cycle) |
| $K$ | stress intensity factor (Pa m$^{1/2}$) |
| $K_t$ | stress concentration factor |
| $m$ | Paris law constant |
| $N$ | number of loading cycles |
| $r$ | radius of a void (m) |
| $\Delta K$ | cyclic stress intensity factor range (Pa m$^{1/2}$) |
| $\Delta\sigma$ | cyclic stress range (Pa) |
| $v$ | Poisson's ratio |
| $\sigma$ | applied stress (Pa) |
| $\sigma_H$ | hydrostatic stress (Pa) |
| $\sigma_1$, $\sigma_2$, $\sigma_3$ | principal stresses (Pa) |