

This is an Open Access document downloaded from ORCA, Cardiff University's institutional repository: <https://orca.cardiff.ac.uk/id/eprint/38149/>

This is the author's version of a work that was submitted to / accepted for publication.

Citation for final published version:

Disney, Stephen Michael and Towill, Denis Royston 2003. The effect of vendor managed inventory (VMI) dynamics on the Bullwhip Effect in supply chains. *International Journal of Production Economics* 85 (2) , pp. 199-215. 10.1016/S0925-5273(03)00110-5

Publishers page: [http://dx.doi.org/10.1016/S0925-5273\(03\)00110-5](http://dx.doi.org/10.1016/S0925-5273(03)00110-5)

Please note:

Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher's version if you wish to cite this paper.

This version is being made available in accordance with publisher policies. See <http://orca.cf.ac.uk/policies.html> for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.



The Effect of Vendor Managed Inventory (VMI) Dynamics on the Bullwhip Effect in Supply Chains

S.M. Disney and D.R. Towill

Logistics Systems Dynamics Group, Cardiff Business School, Cardiff University,
Aberconway Building, Colum Drive, Cardiff. CF10 3EU, UK. E-mail:
DisneySM@Cardiff.ac.uk, Tel: +33 (0)29 2087 6083, Fax: +44(0)29 20874301

Abstract

The paper compares the expected performance of a Vendor Managed Inventory (VMI) supply chain with a traditional "serially-linked" supply chain. The emphasis of this investigation is the impact these two alternative structures have on the "Bullwhip Effect" generated in the supply chain. We pay particular attention to the manufacturer's production ordering activities via a simulation model based on difference equations. VMI is thereby shown to be significantly better at responding to volatile changes in demand such as those due to discounted ordering or price variations. Inventory recovery as measured by the Integral of Time * Absolute Error (ITAE) performance metric is also substantially improved via VMI. Noise bandwidth, that is a measure of capacity requirements, is then used to estimate the order rate variance in response to random customer demand. Finally, the paper simulates the VMI and traditional supply chain response to a representative retail sales pattern. The results are in accordance with "rich picture" performance predictions made from deterministic inputs.

Key Words: Bullwhip, Forrester Effect, VMI, Supply Chains, Inventory Control.

Word Count: Total 7744, Main body 5162

1. Introduction

This paper is concerned with the comparison of a Vendor Managed Inventory (VMI) supply chain to a traditional "serially-linked" supply chain. The particular emphasis of this paper is the impact the two supply chain structures have on the "Bullwhip Effect", (Lee, Padmanabhan and Whang, [1,2]) generated within the supply chain. The performance is investigated using difference equations forming a simulation model. Focusing on a one supplier, one customer relationship special attention is given to the manufacturer's production scheduling activities. The latter is known to

be one well-established source of bullwhip (which we term the Forrester effect after the seminal work of Jay Forrester, [3]). A number of standard ways of reducing bullwhip have been examined by Wikner et al [4], van Ackere et al [5], and summarised by Towill [6]. Furthermore these methods actually work in the real-world, as demonstrated by Towill and McCullen [7]. They found that, for a global mechanical precision product supply chain, bullwhip was typically reduced via an appropriate BPR Programme by 50% and simultaneously stock turn improvements of 2:1 were observed.

Vendor Managed Inventory (VMI) is of particular interest in the bullwhip context. Potentially VMI offers two possible sources of bullwhip reduction. Firstly there is the elimination of one layer of decision-making. Secondly we have the elimination of some information flow time delays. Since removing both factors reduces distortion they can be utilised to damp down bullwhip. Hence herein we provide an overview both VMI and the traditional supply chain in which the latter is used as our performance benchmark. We also describe how bullwhip, and particularly the Forrester effect, can arise in the real world. The difference equations used to model the VMI and the traditional supply chains are described in detail. Optimum parameter settings from previous analytic and field research are also reviewed as possible starting points for the simulation studies.

The "rich picture" resulting from using step response tests are conclusive in indicating bullwhip reduction via VMI. As we have shown previously (Mason-Jones et al, [8]) this "rich picture" gives considerable insight into system response under a wide range of conditions. This includes the well-known supply chain phenomenon of rogue ordering. For example a large positive spike of advance orders may appear, only to be followed by an equally large drop some time in the future, i.e. a net change of zero. Simulating the unit step input is also very useful as it is a very simple non-stationary input, from which many qualitative and quantitative performance aspects may be inferred. Many of these insights are difficult to achieve in an analytical approach only where stationary characteristics are typically studied. Such a waveform emulates price discounting, as illustrated by Fisher [9]. Inventory recovery is assessed via the use of the Integral of Time x Absolute Error (ITAE) performance metric. VMI is shown to be substantially better in reducing ITAE following a step

demand by the customer. A step response is simply the integral of the impulse (or the spike induced by rogue ordering due to promotions). Thus, in a linear system the impulse response is directly related to the step. However, the step input has the advantage that it is accumulative and slight differences that off-set responses may be readily identified. It is thus our demand signal of choice. Order rate variance is conveniently estimated via the calculation of noise bandwidth. It is shown that the performance benefits predicted from the "rich picture" and order rate variance analysis are confirmed via simulation of the supply chain responses to a typical retail sales pattern.

2. Overview of a traditional supply chain

A supply chain is a system consisting of material suppliers, production facilities, distribution services, and customers who are all linked together via the downstream feed-forward flow of materials (deliveries) and the upstream feedback flow of information (orders), as shown in Figure 1 (Stevens, [10]). In a traditional supply chain each "player" is responsible for his own inventory control and production or distribution ordering activities. One fundamental characteristic and problem that all players in a traditional supply chain (such as retailers, distributors, manufacturers, raw material suppliers) must solve is "just how much to order the production system to make (or the suppliers to supply) to enable a supply chain echelon to satisfy its customers' demands". This is the classic production/ inventory control problem.

According to Axsäter [11], "the purpose of a production/ inventory control system (the method used to control inventory levels and production rates) is to transform incomplete information about the market place into co-ordinated plans for production and replenishment of raw materials". Practitioners tackle the production/inventory control problem by inspecting data relating to demands, inventory levels and orders in the pipeline and either, in a structured, mathematical way (for example, by using a decision support system with a well (or poorly!) designed replenishment rule), or in a less formal way (by using their own experience and judgement), place orders up the supply chain. In the real world, the ordering process is frequently biased according to who is perceived as the most important customer, or simply in favour of those found to be most troublesome.



Figure 1. Schematic of a Traditional Supply Chain

The structure of the traditional supply chain has developed partly as a result of the need for a company to be in control of its own assets and partly because, until recently, it has been uneconomic to pass vast amounts of information around. The traditional supply chain is characterised by each player in the supply chain basing his production orders or delivery orders **solely** on his sales to his customer, on his inventory levels and, sometimes, on WIP targets. Each echelon in the supply chain only has information about what their immediate customers want and not on what the end customer wants. This does not allow suppliers to gain any insight into what their customers are ordering to cover their own inventory based Customer Service Level (CSL) and cost requirements and what the customers are ordering to satisfy immediate customer demand (Kaipia et al [12]). This lack of visibility of real demand can and does cause a number of problems in a supply chain if it is not properly designed and even then fluctuations cannot be completely eliminated.

3. Overview of a VMI supply chain

In reacting to this scenario, many companies have been compelled to improve their supply chain operations by sharing demand and inventory information with their suppliers and customers. Different industries and market sectors have coined different terms for VMI, but most are based essentially on the same idea. VMI is a supply chain strategy where the vendor or supplier is given the responsibility of managing the customer's stock. For clarity the terms "distributor" for the customer in the VMI relationship and "manufacturer" for the supplier or vendor in the VMI relationship will be used.

VMI has become more popular in the grocery sector in the last 15 years due to the success of retailers such as Wal-Mart, Andel [13] and Stalk, Evans and Shulman [14]. Additionally, it is only relatively recently that the necessary information and communication technology has become economically available to enable the strategy,

although Holmström [15] has shown that it can be enabled via fax or emails and spreadsheets. Disney, Holmström, Kaipia and Towill [16] have implemented VMI in a supply chain using data available from a popular ERP system and a spreadsheet based decision support system. Moreover, VMI is not a new strategy; it was eloquently discussed by Magee ([17], pp298) in a presentation of a conceptual framework for designing a production control system. Quoting directly from the text (as it very concisely portrays what VMI actually is):

“Frequently there is argument as to who should control inventories. For example, should it be the sales organisation or (some) other unit that draws on the stocks and wants to be sure they are there, or the operation that supplies the stock point and wants to feed it economically? There is probably no resolution to this question as stated; the difficulty is that both have a legitimate interest. It is possible to restate the question slightly and reach a solution. The user has to be sure the material that he requires will be there. He has corresponding responsibility to state what his maximum and minimum requirements will be. Once these limits are accepted as reasonable, the supplier has the responsibility of meeting demand within these limits, making whatever use he can of the flexibility the inventory provides. Thus both have a share in the responsibility for and control over a stock unit. One specifies what the maximum and minimum demands on the stock unit will be; the other has the responsibility of keeping the stock unit replenished but not overloaded as long as demand stays within the specified limits”, Magee [17].

VMI comes in many different forms. Familiar names are Quick Response (QR), (Lee, So and Tang [18]), Synchronized Consumer Response (SCR), Continuous Replenishment (CR), Efficient Consumer Response (ECR), (Cachon and Fisher, [19]), Rapid Replenishment (RR), Collaborative Planning, Forecasting and Replenishment (CPFR), (Holmström et al [20]) and Centralised Inventory Management (CIM), (Lee, Padmanabhan and Whang, [1]), depending on sector application, ownership issues and scope of implementation. However, in essence, they are all specific as applications of VMI as summarised conceptually in Figure 2.

behaviour. It is the impact of VMI control on the Forrester induced Bullwhip Effect that is the subject of this paper. The effect that the other sources of bullwhip, as described by Lee et al [1,2] have on the dynamics of the orders in VMI supply chains has been discussed elsewhere in Disney and Towill [23].

Traditional supply chains are extremely prone to bullwhip. Stalk and Haut [24], provide a detailed description of the Bullwhip Effect found in a clothing supply chain, and this has been summarised by Towill and McCullen [7] as shown in Figure 3. This particular supply chain suffered from the Forrester Effect, with the demand variation typically increasing by an order of 2 to 1 at each level of the supply chain. There is significant evidence that the Bullwhip Effect is prevalent in many real world supply chains. It is not just a phenomenon of interest to academics, but also a source of money haemorrhaging out of supply chains via stock holding charges, production ramp up/ramp down costs etc (McCullen and Towill, [25]).

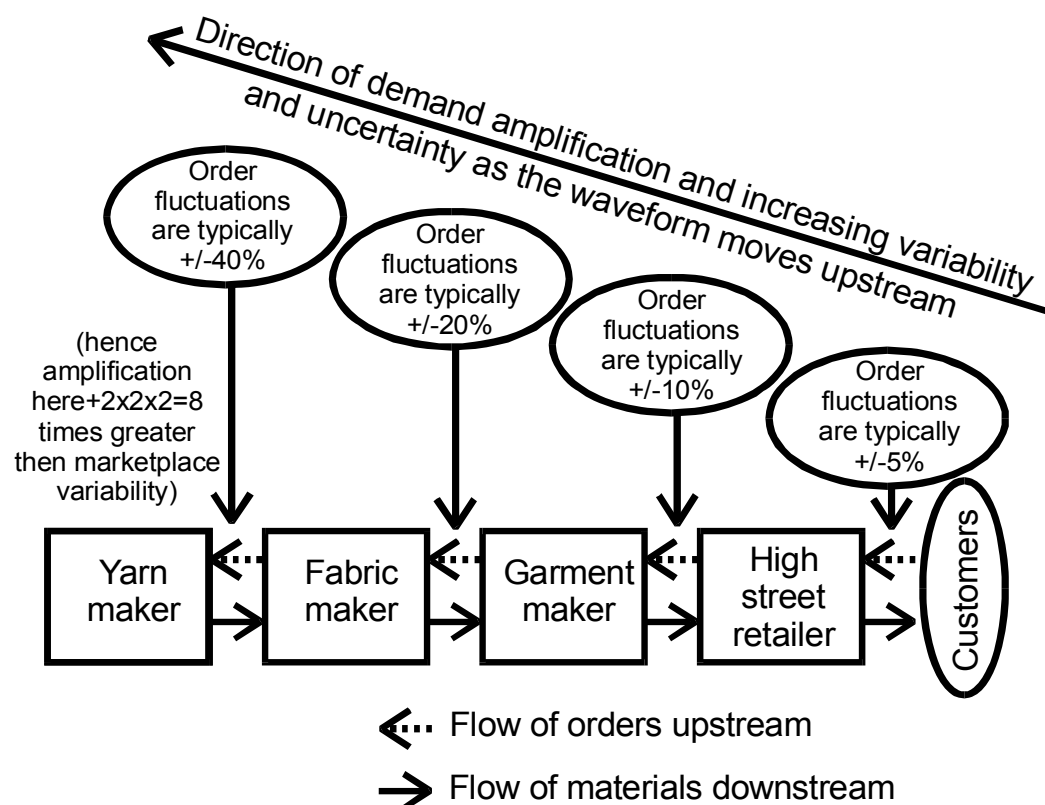


Figure 3. The Bullwhip Effect in a Traditional Retail Supply Chain
(Taken from Towill and McCullen 1999)

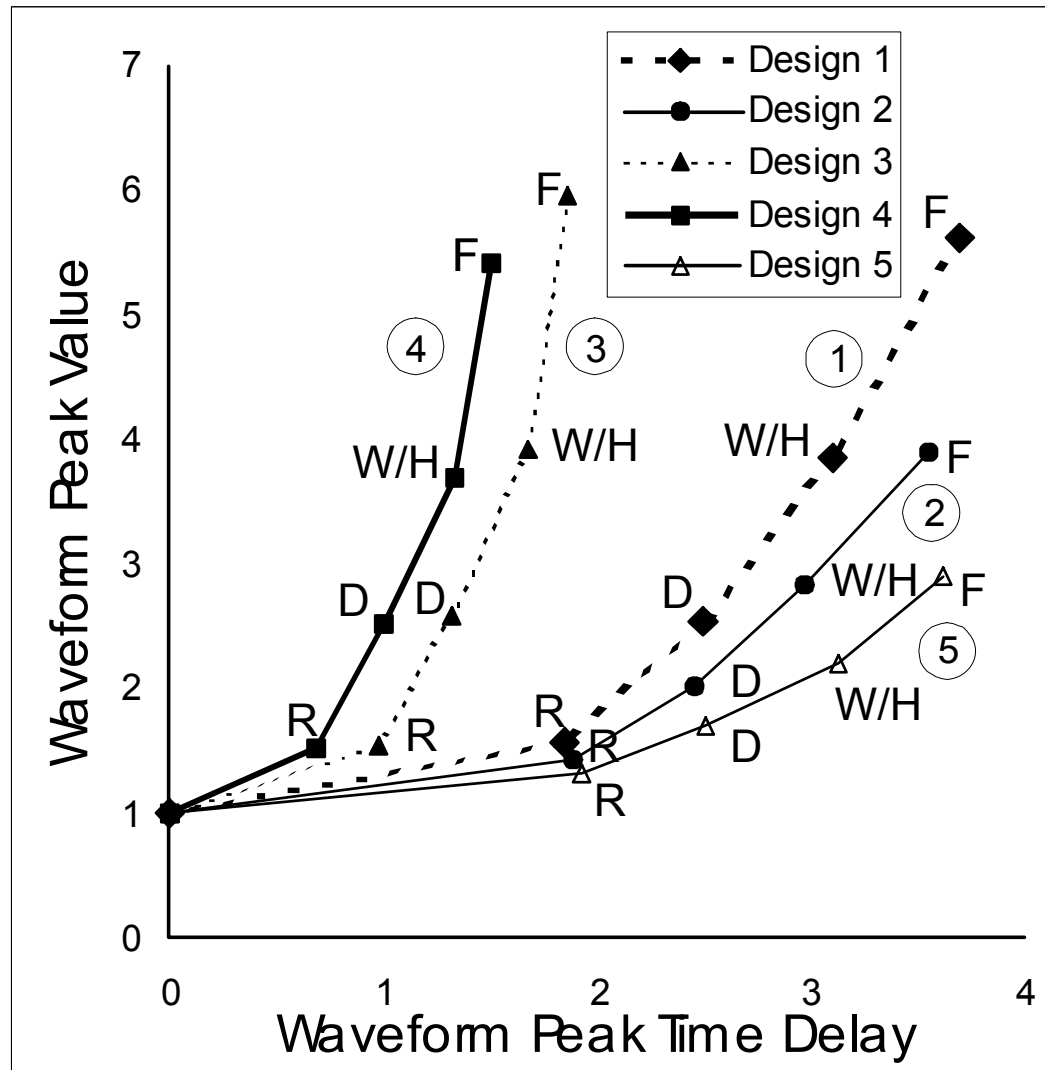
It is, however, much harder to quantify the impact of Bullwhip on the profitability of a company, although Metters [26] has studied the problem with a linear programming approach. He reports that elimination of the Bullwhip Effect can result in a 5% increase in profitability using managerially relevant parameter settings and that this saving can be even higher in a capacity limited supply chain. Stalk and Haut [24] also report that the production on-costs (the costs associated with ramping up and down the production level) is proportional to the cube of the deviation about the mean of the production order rate. So variations within the factory are reckoned to be extremely expensive.

The Forrester Effect is noticeable in traditional supply chains as a particular player over-orders in response to genuine changes in demand to account for inventory deviations that result from the production / distribution lead-time. This over-ordering is then amplified up the supply chain, creating wide fluctuations in the demand signal as it passes through the supply chain. This effect can be very concisely portrayed via "Propagation curves" and has been used in previous contributions (Mason-Jones, et al, [8] and Van Aken [27]) to clearly show how order variance amplifies up the supply chain. The propagation curves in Figure 4, taken from Mason-Jones et al [8], show how demand is amplified (measured as peak value) as it is passed up the supply chain, against the response time of supply chain players take to reach their peak value (peak time delay). The figure shows five traditional supply chain designs, with a range of parameter settings that reflect good solutions for a traditional supply chain;

- without pipeline control (Design 1) based on "hardware analogues",
- with pipeline control (Design 2) based on "hardware analogues",
- Sterman's design (Design 4) with pipeline controls based on results from his analysis of the Beer Game (Sterman [22]),
- and two alternative solutions with pipeline controls (Designs 3 and 5).

Clearly much damping of the bullwhip effect can be obtained via careful selection of an appropriate Decision Support System. However, other advances to move towards the Seamless Supply Chain concept (Towill, [6]) may be even more profitable

(Wikner et al, [4]; van Ackere et al, [5]). We shall see later in the paper where VMI sits within the performance improvement portfolio.



Key: R= Retailer, D=Distributor, W/H=Warehouse, F=Factory

Figure 4. Propagation Curves, Illustrating the Forrester Effect in Different Supply Chain Designs
(Mason-Jones, Naim and Towill, 1997)

5. Description of the VMI Supply Chain Simulation Model

The difference equations required to model our version of the VMI scenario are shown in Appendix 1. These difference equations can quickly be turned into a mathematical model of the VMI supply chain by using z-transforms. The formulation and exploitation of such a mathematical model is not presented in this contribution due to space restrictions but can be found in Disney [28] and Disney and Towill [29,

30]. Reference [29] is a comprehensive analysis of the stability of a VMI supply chain and a single echelon of a traditional supply chain. In [30] the VMI model is studied to determine "good" parameter values for a range of circumstances. Thus we are able to compare one VMI system to another. What this paper uniquely contributes is to take a selection of these VMI systems and compares them with a traditional two-stage supply chain as a benchmark. Hence we investigate the benefits or otherwise of moving from the traditional supply chain with all its real world faults) to a VMI scenario. We know the VMI system is representative of a real world application, Disney, Holmström, Kaipia and Towill [16], however this comparison is restricted to simulation of these models. Herein, the difference equation representation will be exploited. The difference equations may be quickly realised through "spreadsheet" applications such as Microsoft Excel. Difference equations can also be implemented in standard computer languages with relative ease, as shown in Table 1. The equations in Appendix 1 describe the VMI supply chain when individual stock holding points and transportation despatches are modelled explicitly, whereas for simplicity the pseudo code in Table 1 models inventory and transportation as based on virtual consumption. Of course, the two systems are exactly the same when focusing on the production order rate, (ORATE). A fixed production lead-time of 4 time-units will be used throughout this paper.

The specific "flavour" of VMI that the difference equations represent in Appendix 1 is termed VMI-APIOBPCS, or *Vendor Managed Inventory, Automatic Pipeline, Inventory and Order Based Production Control System*. The VMI term in VMI-APIOBPCS reflects the most significant fact about a VMI supply chain, i.e. that the distributor (the customer in the VMI relationship) passes inventory information and Point of Sales (POS) data to their suppliers rather than placing replenishment orders, (Kaipia et al [12], Cottrill [31]). The actual inventory at the customer is then compared to a re-order point that has been agreed on by both parties. This re-order point is set to ensure adequate availability without building up excessive stocks. It triggers a replenishment order that is delivered to the customer if the actual inventory is below the re-order point every planning period. Each party also agrees the order-up-to point, O. The dispatches between the two echelons are equal to the order-up to level, O, minus the re-order point, R, and within this framework the dispatches can be of a constant or varying size.

The re-order point is set dynamically to reflect changes in demand. This is done by exponentially smoothing (over T_q time units) the sales signal and multiplying it by a constant (G) that ensures appropriate customer service levels at the distributor, taking into account the transportation lead-time between the two parties in the supply chain. Exponential smoothing was chosen as the forecasting mechanism because it is; simple to implement in computer systems (requiring less data storage), readily understood and the most favoured technique by both industrialists and academics. It should be noted that the net change in the re-order point from one time period to another is added to the sales signal and the vendor treats this as a demand. So, when demand is increasing and the distributors re-order point grows, the supplier or vendor treats the stock (re-order point) requirements at the distributor as demand and incorporates that into his forecasts and stock levels, as he clearly should do. Obviously, the negative argument also applies, i.e. when the re-order point is reducing in size over time, demand signals to the manufacture and the system inventory levels reflect this.

| Pseudo Code | Description |
|--|---|
| Set all variables to zero; | Set all variables to zero |
| GET T_a , T_i , T_q , T_w and G ; | Get input from user |
| WHILE [$t < 160$, | Do while time increment is less then 160..... |
| CONS=IF [$t > 2$, 1., 0.]; | Demand is 0 until time equals 2, when it then equals 1 |
| $R = R + ((1/(1+T_q)) * ((G * CONS) - R))$; | Calculate the re-order point R |
| $DR = R - PR$; | Calculate the net change in the re-order point R |
| $PR = R$; | Store current R as previous R |
| $VCON = DR + CONS$; | Virtual consumption = actual consumption plus net change in R |
| $AVCON = AVCON + ((1/(1+T_a)) * (VCON - AVCON))$; | Set manufacturers forecast, $AVCON$ |
| $COMRATE = CR_4$; $CR_4 = CR_3$; $CR_3 = CR_2$; $CR_2 = CR_1$; $CR_1 = ORATE$; | Set manufacturers completion rate as a delayed function of the order rate (production delay = 4 time units, the extra unit delay is to ensure the proper order of events is obeyed) |
| $AINV = AINV - VCON + COMRATE$; | Calculate inventory levels |
| $EINV = ((0 - AINV) / T_i)$; | Set inventory contribution to $ORATE$ |
| $DWIP = AVCON * 4$; | Set target Work In Progress (WIP) |
| $WIP = WIP + ORATE - COMRATE$; | Calculate actual WIP |
| $EWIP = ((DWIP - WIP) / T_w)$; | Set WIP contribution to $ORATE$ |
| $ORATE = AVCON + EINV + EWIP$; | Calculate the production order rate, $ORATE$ |
| $t++$]; | Increment time and return to start of loop |

Table 1. Pseudo Code Representation of VMI-APIOBPCS in Response to a Step Input

The term APIOBPCS in VMI-APIOBPCS (John, Naim and Towill, [32]) refers to the structure of the ordering decision used by the supplier or vendor in the VMI relationship to schedule production (or distribution if that is the suppliers business). APIOBPCS can be expressed in words as "Let the production targets be equal to the sum of a forecast (average consumption smoothed over T_a time units) of perceived demand (that is actually in VMI the sum of the stock adjustments at the distributor and the actual sales), plus a fraction ($1/T_i$) of the inventory discrepancy between actual and target levels of finished goods, plus a fraction ($1/T_w$) of the discrepancy between target WIP and actual WIP". This is a well-known production-scheduling rule that can be "tuned" to reflect a wide range of scheduling strategies. The APIOBPCS system has been studied before by a number of authors, John, Naim and Towill [32], Mason-Jones et al [8], Towill, Cheema and Evans [33] etc. It also has exactly the same structure as Sterman's Anchoring and Adjustment heuristic used by him to model his Beer Game data (Naim and Towill, [34]).

6. Description of the Traditional Supply Chain Simulation Model

The APIOBPCS model, John, Naim and Towill [32], was chosen to represent a traditional supply chain. This was due to a number of reasons. Firstly it was felt important that it is desirable that like (APIOBPCS) is compared to like as much as possible (VMI-APIOBPCS) in order to gain as much understanding as possible on the fundamental structure of VMI. Secondly, APIOBPCS was chosen for VMI and the traditional supply chain, as it is recognised as good practice, Edghill, Olsmats and Towill [35] incorporates all commonly available forms of information, represents human behaviour (Sterman, [22] and Naim and Towill [34]) and is a well-understood member of the IOBPCS (Towill, [36]) family. The APIOBPCS model can be expressed in words as outlined in the previous section. It incorporates three variables;

- T_a , a parameter that describes how quickly demand is tracked in the forecasting mechanism,
- T_i , a parameter that describes of much of the discrepancy between actual inventory and target inventory levels should be added to the production/distribution order rate and

- Tw, a parameter that describes how much of the discrepancy between actual WIP and target WIP levels should be added to the production/ distribution order rate.

Individual echelons, or APIOBPCS models, can be linked together to form a supply chain, by coupling the ORATE signal of the consuming echelon to the CONS signal of the supplying echelon, as recognised by Burns and Sivazlian [37] and further exploited by Towill and del Vecchio [38]. The difference equations required for modelling a two-level APIOBPCS supply chain (for example in a spreadsheet) are shown in Appendix 2. Table 2 shows how the system can be implemented in a computer language such as C++ or Visual Basic. Like the VMI model the production and distribution delays are assumed to be of four time units.

| Pseudo Code | Description |
|--|---|
| Set all variables to zero; | Set all variables to zero |
| GET TaS, TiS, TwS, Ta, Ti, Tw; | Get input from user |
| WHILE[t<160, | Do while time increment is less then 160..... |
| CONS=IF[t>2,1,0.]; | Demand is 0 until time equals 2, when it then equals 1 |
| AVCONS=AVCONS+((1/(1+TaS))*(CONSS-AVCONS)); | Set distributors forecast, AVCON |
| COMRATES=CR4S;CR4S=CR3S; CR3S=CR2S;CR2S=CR1S;CR1S=ORATES; | Set distributors completion rate as a delayed function of the distributors order rate (production delay = 4 time units, the extra unit delay is to ensure the proper order of events is obeyed) |
| AINVS=AINVS-CONSS+COMRATES; | Calculate distributors inventory levels |
| EINVS=((0-AINV)/TiS); | Set distributors inventory contribution to ORATE |
| DWIPS=AVCONS*4; | Set distributors target Work In Progress (WIP) |
| WIPS=WIPS+ORATES-COMRATES; | Calculate distributors actual WIP |
| EWIPS=((DWIPS-WIPS)/TwS); | Set distributors WIP contribution to ORATE |
| ORATES=AVCONS+EINVS+EWIPS; | Calculate the distributors order rate, ORATE |
| AVCON=AVCON+((1/(1+Ta))*(ORATES-AVCON)); | Set manufacturers forecast, AVCON |
| COMRATE=CR4; CR4=CR3; CR3=CR2; CR2=CR1; CR1=ORATE; | Set manufacturers completion rate as a delayed function of the order rate (production delay = 4 time units, the extra unit delay is to ensure the proper order of events is obeyed) |
| AINV=AINV-ORATES+COMRATE; | Calculate manufacturers inventory levels |
| EINV=((0-AINV)/Ti); | Set manufacturers inventory contribution to ORATE |
| DWIP=AVCON*4; | Set manufacturers target Work In Progress (WIP) |
| WIP=WIP+ORATE-COMRATE; | Calculate manufacturers actual WIP |
| EWIP=((DWIP-WIP)/Tw); | Set manufacturers WIP contribution to ORATE |
| ORATE=AVCON+EINV+EWIP; | Calculate the manufacturers order rate, ORATE |
| t++]; | Increment time and return to start of loop |

Table 2. Pseudo Code Representation of a Two Level APIOBPCS Supply Chain in Response to a Step Input

7. Step Response Comparison of the Forrester Effect

To investigate the Bullwhip Effect, the Factory Order Rate response of the two supply chain structures to a step input will be used. Understanding the dynamic response to a step input will yield insight into how the system will be affected by promotions. As there are an infinite number of designs for VMI and traditional supply chains that might be compared, previous best practise designs will be used to compare the two supply chains via the step response. The following designs were chosen to represent good designs of a traditional supply chain with a production lead-time of 4 time periods;

- John et al [32] recommended settings ($T_a=8$, $T_i=4$, $T_w=8$). This was derived using classical control theory and simulation. It can be regarded as a very conservative design relating back to "best practice" in hardware control systems.
- Disney et al [39] recommended settings ($T_a=8$, $T_i=4$, $T_w=15$). This was based on a Genetic Algorithms search, using Laplace transforms, simulation with the aim of minimising the Forrester Effect, inventory holding, selectivity, whilst maximising robustness to errors in estimation of WIP levels and production lead-times.
- Naim and Towill [34] values of ($T_a=8$, $T_i=4$, $T_w=4$). These were derived from improving Sterman's [22] Beer Game derived optimum settings.
- Disney [28] recommended settings ($T_a=4$, $T_i=7$, $T_w=28$). This was based on the full solution based search using z-transforms and simulation aimed at minimising the Forrester Effect, inventory holding, whilst maximising Customer Service Levels.

| Operational Setting | | Parameters of "Optimum" VMI System | | | |
|---------------------|----------|------------------------------------|-------|-------|-------|
| G^{\sim} | $W^{\#}$ | T_a | T_i | T_q | T_w |
| 1 | 0.01 | 1 | 3 | 1 | 3 |
| 1 | 1 | 6 | 7 | 6 | 42 |
| 1 | 100 | 18 | 23 | 6 | 63 |
| 4 | 0.01 | 1 | 14 | 1 | 14 |
| 4 | 1 | 4 | 14 | 4 | 63 |
| 4 | 100 | 22 | 27 | 6 | 63 |

$\sim G$ = Gain on exponential forecasts used to calculate the re-order point level

$\# W$ = Weighting function to trade-off production capacity requirements against stock requirements

Table 3. Sample Optimum Parameter Values for VMI System

As outlined earlier, the VMI strategy has 5 key parameters (T_q , G , T_a , T_i , T_w) that determine the dynamic response of the system. The terms T_a , T_i , T_q and T_w depend on the parameter G that is independently set to reflect the desired CSL given the transportation lead-time between the manufacturer and the distributor, via the re-order point equation. A full-scale optimisation procedure (Disney [28]) has been applied to these parameters for a range of ratios of production adaptation costs (due to the Forrester Effect) to the associated inventory holding costs and for different values of the re-order point G . The resulting optimal parameter settings for T_a , T_i , T_q and T_w for the case when $G=1$ and 4 are shown in Table 3. In this Section it is sufficient to illustrate the VMI system step response for the case where production adaptation and inventory holding costs were given equal importance for the two designs chosen to represent good solutions for a VMI supply chain. Hence the "good" settings for the VMI supply chain used were;

- The optimum parameter setting when the distributor has a re-order point level set at 1 planning periods average demand, (i.e. $G=1$, $T_a=6$, $T_i=7$, $T_q=6$, $T_w=42$)
- The optimum parameter setting when the distributor has a re-order point level set at 4 planning periods average demand, (i.e. $G=4$, $T_a=4$, $T_i=14$, $T_q=4$, $T_w=63$)

It can be seen from inspection of Figure 5 that the VMI design outperforms the traditional supply chain, with less peak overshoot, faster settling time and a generally quicker response. We will now investigate the comparison further.

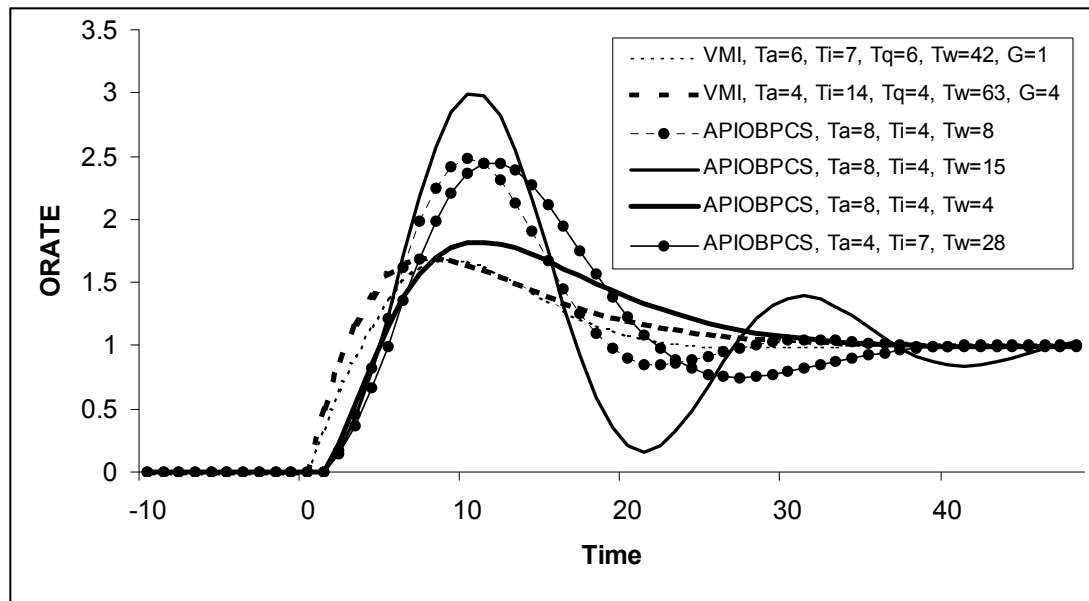


Figure 5. Step Response For VMI And Traditional Supply Chains

8. General Dynamic Comparison of VMI and Traditional Supply Chains

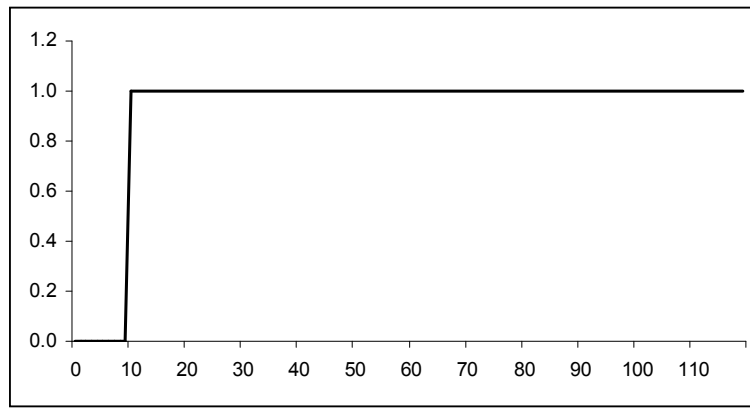
In Table 4 we have compared VMI and traditional supply chains across a range of performance metrics. The peak ORATE overshoot is the simple measure of bullwhip and has already been met in Fig. 5. Note that for completeness Table 4 includes three optimal solutions for each of the two values of G (1.0 and 4.0). These are for ratios of production adaptation/inventory holding costs $W = 0.01$; $W = 1.0$; and $W = 100$. The reason for this is that $W = 0.01$ approximates an agile system; $W = 100$ approximates a lean (level scheduling) system; whilst $W = 1.0$ is a compromise solution. As noted by Christopher and Towill [40] there are occasions where “agile” is the best business solution, and where “lean” is the best business solution, and where some “mix” is required.

Table 4. Bullwhip and Stock Performance Trade-Offs can be found at the end of the document

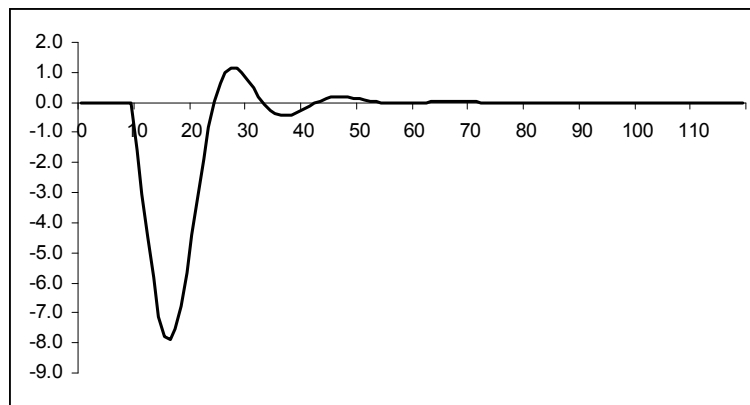
For the optimal VMI supply chains, the bullwhip is reasonably unaffected by varying W for a given value of G . This is because the optimisation programme (Disney, [27]) drives the VMI parameters to yield the best possible response, (as we have seen in Table 3, the parameter settings to achieve this goal are substantially different). If the

peak ORATE overshoot is 2.5, then X is a bullwhip effect of 150% and so on. So comparing the optimal VMI system with the nearest equivalent traditional supply chain i.e. $G = 1$, $W = 1.0$, and with VMI optimal parameter setting, we see VMI reduces the bullwhip effect from 144% to 69%. Some authors (for example Chen, Ryan and Simchi-Levi, [41]) use the ratio of order and sales variance as a bullwhip measure others (for example Fransoo and Wouters [42]) have been using ratios involving the standard deviation. Whilst both are conceptually similar, the variance ratio is preferred as this can be calculated directly from a system transfer function, Disney and Towill [43] or efficiently enumerated via the difference equations. Hence in Table 4 we have included an estimate of variance obtained via evaluation of system noise bandwidth (Towill, [36]). This bullwhip measure has been reduced from 0.93 (Traditional supply chain) to 0.46 (VMI system), almost a factor of 2 to 1. So the effect on both bullwhip measures using VMI is a great improvement.

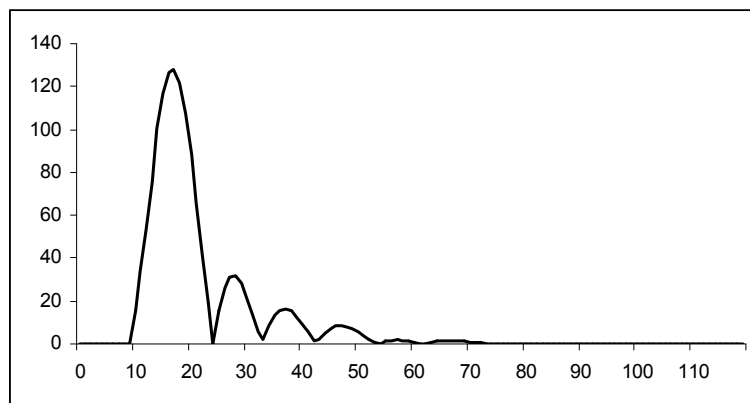
We use the composite measure of Integral of Time * Absolute Error (ITAE) to compare dynamic inventory performance. ITAE is much used as a means of ranking comparative performance of competitive hardware control systems (Towill, [44]). The reasoning is that large instantaneous errors are unavoidable and should not be penalised. On the other hand persistent errors occurring after a long time are to be avoided so should be heavily penalised, hence the time weighting to achieve this effect. Although ITAE may be regarded as an intuitive performance measure its minimisation does normally result in a good system design [44]. Figure 6 shows how ITAE is computed and thus how a transient waveform may be converted into a single number. The first observation to make on ITAE in Table 4 is that the values for the VMI systems are very much dependent on the re-order point G . For the like-for-like comparison between VMI ($G = 1$, $W = 1$) and the traditional supply chain the ITAE is always substantially lower for the VMI system. Hence the inventory recovery dynamics are much improved by adopting VMI.



(a) Step in Demand



(b) Corresponding Inventory Behaviour



(c) Product of Time Absolute Error in inventory levels

Figure 6. Computation of ITAE as a Measure of Inventory Response

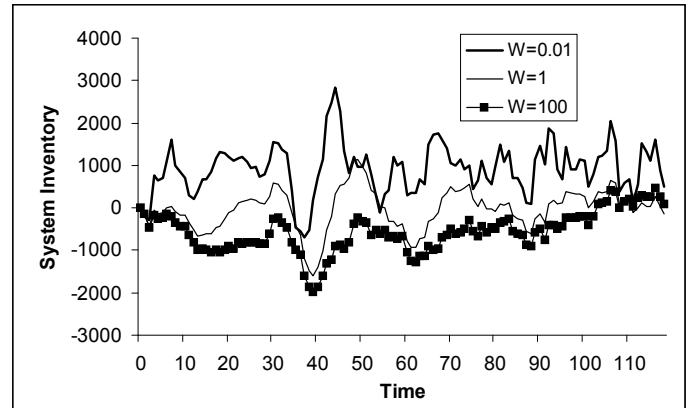
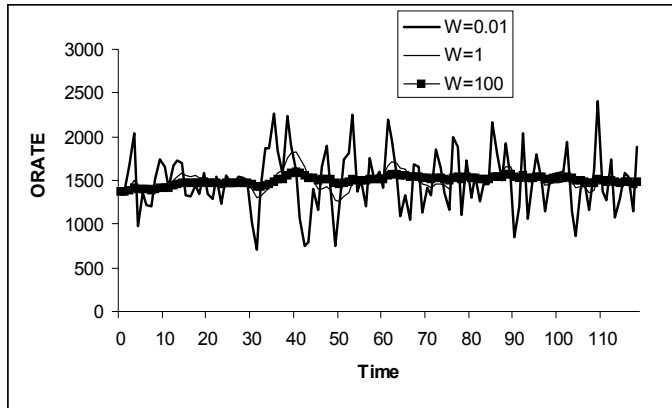
We are determining the availability resulting from setting the target inventory to 10, in response to an i.i.d. normally distributed demand pattern. In the VMI scenario this target inventory refers to the system inventory (that is the distributors stock plus the goods in transit and the manufacturers stock level minus the reorder point) and for traditional supply chain this refers to the manufacturers target stock position. The

demand signal was chosen to be a standard normal distribution with a mean of 0 and a standard deviation / variance of 1. This scenario can be efficiently enumerated via difference equations in a spreadsheet environment. The availability measure specifically refers to the probability that inventory levels are above zero (that is the chance stock is available to be shipped to the distributor).

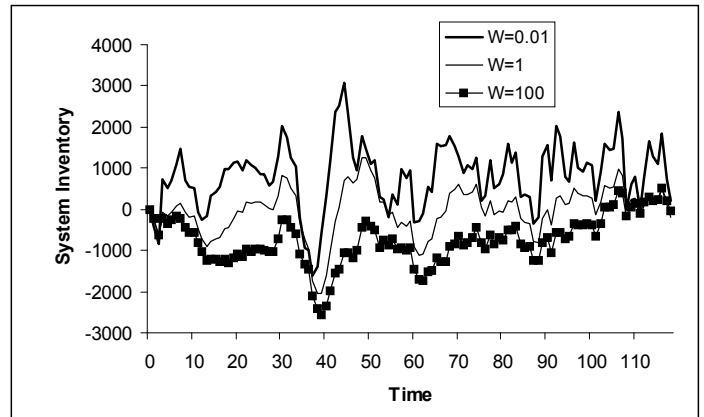
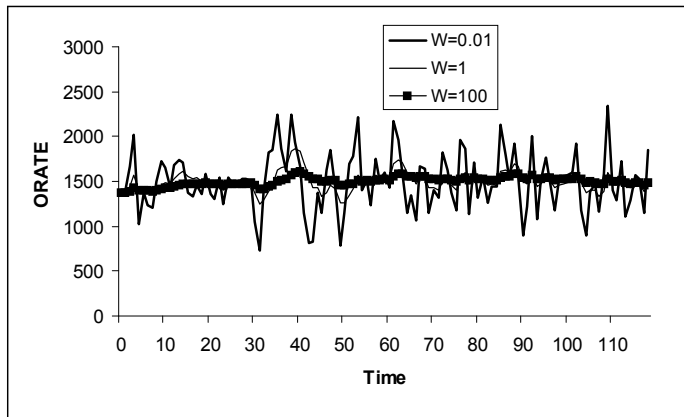
9. Simulated Responses to Typical Retail Sales Time Series

To give some idea of the differences in behaviour in response to real-world disturbances, we have simulated responses to a typical retail sales pattern. The corresponding ORATE variances have already been listed in Table 4 (σ^2 calculated from the time series), but we also include to enable a visual comparison. Note that if the retail sales pattern had been random white noise (or an independently and identically distributed normal distribution), then this σ^2 would be exactly equal to that evaluated via noise bandwidth. So the ORATE variances would lead us to expect significantly different responses for the various systems. This is indeed the case, as is quite obvious from Fig. 7 which shows ORATE and the associated stock movements about the target inventory level.

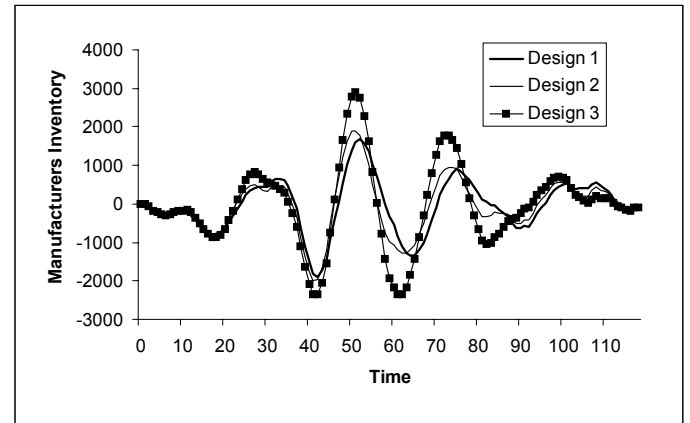
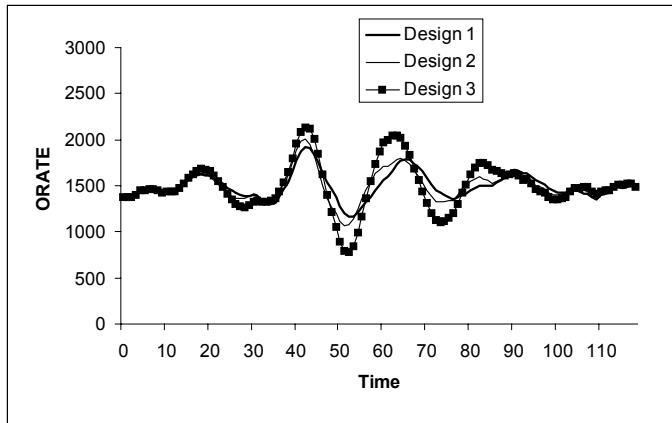
The first noteworthy point is that varying the weighting function W does produce an "agile" response, a "lean" response, and a "compromise" response. Also note that this family of responses is generated for both values of the re-order point G . However, *all* of the traditional supply chains simulated have excessive swings in ORATE. Furthermore, these designs exhibit resonances that are clearly a property of the system design, and not of the excitation caused by the sales time series (Towill, [36]). In fact the business manager can be wrongfooted and assume that there is seasonality present in the demand. This it is the well-known "second" Forrester Effect of "rogue seasonality" (Berry and Towill, [45]). We have studied the responses further and summarized the results in Table 5 below.



Optimal VMI designs when $G=1$



Optimal VMI designs when $G=4$



Traditional supply chain designs

Figure 7. Simulated Responses of Sample Optimal VMI and Traditional Supply Chains to a Typical Retail Sales Pattern

| System | | | Inventory | | | Orders | | |
|-------------------|----------|--------|--------------------------|---------------------|----------------|--------------------------|---------------------|----------------|
| Supply chain type | Settings | | No. of direction changes | Range of the swings | Variance Ratio | No. of direction changes | Range of the swings | Variance Ratio |
| VMI | G=1 | W=0.01 | 62 | 3553 | 16.89 | 76 | 1692 | 5.634 |
| | | W=1 | 49 | 2730 | 11.53 | 70 | 560 | 0.502 |
| | | W=100 | 63 | 2463 | 10.99 | 62 | 219 | 0.108 |
| | G=4 | W=0.01 | 66 | 4708 | 29.87 | 76 | 1604 | 5.07 |
| | | W=1 | 59 | 3300 | 17.84 | 72 | 618 | 0.62 |
| | | W=100 | 63 | 3074 | 16.44 | 68 | 231 | 0.11 |
| Traditional | Design | 1 | 21 | 3567 | 23.39 | 24 | 737 | 1.01 |
| | | 2 | 23 | 3898 | 24.77 | 28 | 937 | 1.31 |
| | | 3 | 17 | 5243 | 53.07 | 22 | 1351 | 3.18 |

Table 5. Summary of the simulated responses

As expected, the rogue seasonality is visible in the manufacturers inventory records. The swings in inventory are clearly due to system-induced excitation; there are less changes in direction in the inventory levels and large swings have occurred as is evident in the range and variance of those swings. Indeed, if we look at the autocorrelation function of inventory and orders, the VMI supply chains typically have a negative correlation at one periods delay, and then slight positive and negative correlation thereafter, whereas the traditional supply chain has significant autocorrelation across many periods. In the "agile" system both inventory and ORATE behaviour are distinctly "sharp-edged". However, it must be pointed out that the $G = 1$ VMI supply chain is relatively smooth in behaviour. It also has substantially smaller swings in both ORATE and inventory compared to any of the traditional supply chains. The superiority of the VMI design is therefore both widespread and considerable.

10. Conclusions

VMI is a well-established supply strategy that has found favour in a number of market sectors. In many cases this has occurred in response to a feeling by the retailer that it would be a good thing to delegate further responsibility to the vendor. As a concept it

can be dated back to the classical contribution of Magee [17]. He first raised the issue of which player should control the supply pipeline. It is intuitively obvious that better sight and hence understanding of both information flow and material flow should lead to better business performance. In particular it is possible to "condense" the pipeline so that its behaviour approaches that of a single echelon. This is achieved by using a constant (that is one that does not change over time) re-order point at the distributor. As we have demonstrated via a VMI simulation model this is indeed the case.

We have compared the bullwhip performance of a number of VMI supply chains with two-level supply chains. In all cases there is substantial reduction in bullwhip (typically halving the effect). This is true irrespective of the bullwhip measure used. In the paper we have concentrated on the two measures of peak order rate to a step input (a "rich picture" approach) and order rate variance. The latter is widely used in industry. From our perspective it also has the advantage of being predictable from system noise bandwidth. Under certain circumstances this is amenable to an analytic solution, which we have developed but not exploited here. In other cases it is possible to take substantial computational short cuts to calculate variance and availability. ITAE has been used as a composite measure of inventory dynamics. Here VMI also offers a substantial improvement in performance.

Finally, as Berry and Towill [45] have shown, managers need to be aware that in practice there are potentially two Forrester effects. The first (demand amplification) is universally known. But the second (rogue seasonality) can equally likely be induced by the system dynamics. Hence a periodicity may appear in the ordering waveforms that is not present in the marketplace demand. Our simulations have shown that particularly in the stock records rogue seasonality is induced by the traditional supply chain in response to typical retail demand waveforms. In contrast this is observed to be less of a problem with the VMI system.

References

- [1] Lee, H.L., Padmanabhan, P. and Whang, S., "Information distortion in a supply chain: the Bullwhip Effect", *Management Science*, Vol. 43, 1997, pp543-558.
- [2] Lee, H.L., Padmanabhan, V., Whang, S., "The Bullwhip Effect in supply chains", *Sloan Management Review*, Spring, 1997, pp. 93-102.
- [3] Forrester, J., "Industrial dynamics", Cambridge MA, MIT Press, 1961.
- [4] Wikner, J., Naim, M.M. and Towill, D.R., "The system simplification approach in understanding the dynamic behaviour of a manufacturing supply chain", *Journal of System Engineering*, Vol. 2, 1992, pp164-178.
- [5] Van Ackere, A., Larsen, E.R. and Morecroft, J.D.W., "Systems thinking and business process redesign: An application to the Beer Game", *European Management Journal*, Vol. 11, No. 4, 1993, pp412-423.
- [6] Towill, D.R., "Forridge - Principles of good material flow", *Production Planning and Control*, Vol. 8, No. 7, 1997, pp622-632.
- [7] Towill D.R. and McCullen, P., "The impact of an agile manufacturing programme on supply chain dynamics", *International Journal Logistics Management*, December, Vol. 10, No. 1, 1999, pp83-96.
- [8] Mason-Jones R., Naim, M.M. and Towill, D.R., "The impact of pipeline control on supply chain dynamics", *International Journal of Logistics Management*, Vol. 8, No. 2, 1997, pp47-62.
- [9] Fisher, M., Hammond, J., Obermeyer, W. and Raman, A., "Configuring a supply chain to reduce the cost of demand uncertainty", *Production and Operations Management*, Vol. 6, No. 3, 1997, pp211-225.
- [10] Stevens, G., "Integrating the supply chain", *International Journal of Physical Distribution and Logistics Management*, Vol. 19, No. 8, 1989, pp3-8.
- [11] Axsäter, S., "Control theory concepts in production and inventory control", *International Journal of Systems Science*, Vol. 16, No. 2., 1985, pp161-169.
- [12] Kaipia, R., Holmström, J. and Tanskanen, K., "VMI: what are you losing if you let your customer place orders", Working Paper, Dept of Industrial Management, Helsinki University of Technology, Finland, 2000.
- [13] Andel, T., "Manage inventory, own information", *Transport and Distribution*, Vol. 37, Issue 5, 1996, pp54-58.
- [14] Stalk, G., Evans, P. and Shulman, L.E., "Competing on capabilities: The new rules of corporate strategy", *Harvard Business Review*, March-April, 1992, pp57-69.
- [15] Holmström, J., "Business process innovation in the supply chain: a case study of implementing Vendor Managed Inventory", *European Journal of Purchasing and Supply Management*, Vol. 4, 1998, pp127-131.
- [16] Disney, S.M., Holmström, J., Kaipia, R., Towill, D.R., "Implementation of a VMI production and distribution control system", *International Symposium of Logistics*, Salzburg, Austria, July 2001.
- [17] Magee, J.F., "Production planning and inventory control", McGraw-Hill Book Company, New York, 1958, pp80-83.
- [18] Lee, H.L., So, K.C. and Tang, C.S., "The value of information sharing in a two-level supply chain", *Management Science*, Vol. 46, No. 5, 2000, pp626-643.
- [19] Cachon, G. and Fisher, M., "Campbell soup's continuous replenishment program: evaluation and enhanced inventory decision rules", *Productions and Operations Management*, Vol. 6, No.3, 1997, pp266-276.
- [20] Holmström, J., Framling, K., Kaipia, R. and Saranen, J., "Collaborative Planning Forecasting and Replenishment: New solutions needed for mass

- collaboration", Working Paper, Dept. of Industrial Management, Helsinki University of Technology, Finland, 2000.
- [21] Schmenner, R.W., "Looking ahead by looking back: Swift, even flow in the history of manufacturing", *Production and Operations Management*, Vol. 10, No. 1, Spring 2001, pp87-96.
 - [22] Sterman, J., "Modelling managerial behaviour: Misperceptions of feedback in a dynamic decision making experiment", *Management Science*, Vol. 35, No. 3, 1989, pp321-339.
 - [23] Disney, S.M. and Towill, D.R., "The effect of VMI on the Bullwhip Effect", *International Conference on Production Research*, July 30- August 2, Prague, Czech Republic, 2001.
 - [24] Stalk, G. and Hout, T.M., "Competing against time: How time-based competition is reshaping global markets", The Free Press, New York, 1990.
 - [25] McCullen, P. and Towill, D.R., "Practical ways of reducing bullwhip: The case of the Glosuch supply chain", *Control*, Dec/Jan 2001, pp24-30.
 - [26] Metters, R., "Quantifying the Bullwhip Effect in supply chains", *Journal of Operations Management*, Vol. 15, 1997, pp89-100.
 - [27] Van Aken, J.E., "On the control of complex industrial organizations", London, Martinus Nijhoff Social Sciences Division, 1978.
 - [28] Disney, S.M., "The production and inventory control problem in Vendor Managed Inventory supply chains", PhD Thesis, Cardiff Business School, Cardiff University, UK, 2001.
 - [29] Disney, S.M. and Towill, D.R., "A discrete linear control theory model to determine the dynamic stability of Vendor Managed Inventory", *International Journal of Production Research*, Vol. 40, No. 1, pp179-204
 - [30] Disney, S.M. and Towill, D.R., "Optimisation of the Dynamic Response of Vendor Managed Inventory", *Computers and Industrial Engineering*, Vol. 43, No. 1-2, pp27-58.
 - [31] Cottrill, K., "Reforging the supply chain", *Journal of Business Strategy*, November/ December 1997, pp35-39.
 - [32] John, S., Naim, M.M., Towill, D.R., "Dynamic analysis of a WIP compensated decision support system", *International Journal of Manufacturing Systems Design*, Vol. 1, No. 4, 1994, pp283-297.
 - [33] Towill, D.R., Evans, G.N. and Cheema, P., "Analysis and Design of an Adaptive Minimum Reasonable Inventory Control System", *Production Planning and Control*, Vol. 8, No. 6, 1997, pp545-557.
 - [34] Naim, M.M. and Towill, D.R., 1995, "What's in the Pipeline?", 2nd *International Symposium on Logistics* 11-12, July, pp135-142.
 - [35] Edghill, J., Olsmats, C. and Towill, D., "Industrial case-study on the dynamics and sensitivity of a close-coupled production-distribution system", *International Journal of Production Research*, Vol. 26, No. 10, pp1681-1693.
 - [36] Towill, D.R., "Optimisation of an inventory and order based production control system", *Proceedings of the Second International Symposium on Inventories*, Budapest, Hungary, 1982.
 - [37] Burns, J.F. and Sivazlian, B.D., "Dynamic analysis of multi-echelon supply systems", *Computer and Industrial Engineering*, Vol. 2, 1978, pp181-193.
 - [38] Towill, D.R. and del Vecchio, A., "Application of filter theory to the study of supply chain dynamics", *Production Planning and Control*, Vol. 5, No. 1, 1994, pp82-96.

- [39] Disney, S.M., Naim, M.M. and Towill, D.R., "Dynamic simulation modelling for lean logistics", *International Journal of Physical Distribution and Logistics Management*, Vol. 27, Issue 3, 1997, pp174-96.
- [40] Christopher, M. and Towill, D.R., "Marrying the lean and agile paradigms", *Proceedings of the 7th EUROMA Conference*, Ghent, Belgium, June 4-7, 2000, in "Operations Management - "Crossing borders and boundaries: The changing role of operations"", edited by Van Dierdonck, R. and Vereecke, A., 2000 pp-needed.
- [41] Chen, F., Ryan, J.K. and Simchi-Levi, D., "The impact of exponential smoothing forecasts on the Bullwhip Effect", *Naval Research Logistics*, Vol. 47, 2000, pp269-286.
- [42] Fransoo, J.C. and Wouters, M.J.F., "Measuring the Bullwhip Effect in the supply chain", *Supply Chain Management: An International Journal*, Vol. 5, No. 2, 2000, pp78-89.
- [43] Disney, S.M. and Towill, D.R., "Solving the bullwhip problem in supply chains", submitted to *Omega: The International Journal of Management Science*, 2001.
- [44] Towill, D.R., "Transfer function techniques for control engineers", Iliffe Books, London, 1970.
- [45] Berry, D. and Towill, D.R. "Reduce Costs ~ Use a More Intelligent Production and Inventory Planning Policy", *BPICS Control*, Nov 1995, pp 26-30.

Appendix 1. The difference equations required the VMI-APIOBPCS model when inventory are treated separately and transportation despatches are modelled explicitly

| Description | Difference Equations | Eq. No |
|--|---|--------|
| Forecasted Re-order point at the distributor | $R_t = R_{t-1} + \frac{1}{1+Tq} ((G * CONS_t) - R_{t-1})$ | (1.1) |
| Order-up-to point at the distributor | $O_t = R_t + TQ_t,$ | (1.2) |
| Distributor's inventory level | $DINV_t = DINV_{t-1} - CONS_t + DES_{t-T},$ | (1.3) |
| Goods In Transit between factory and distributor | $GIT_t = \sum_{i=t-T+1}^{i=t} DES_i$, where T is the transportation lead-time, | (1.4) |
| Despatches | $DES_t = \begin{cases} TQ_{t-1} & \text{if } DINV_{t-1} + GIT_{t-1} < R_{t-1} \\ 0 & \text{if } DINV_{t-1} + GIT_{t-1} \geq R_{t-1} \end{cases},$ | (1.5) |
| Transport quantity | $TQ_t = CONS_t$ or ETQ_t , nominally set to equal 4 in Figure 8 | (1.6) |
| System inventory levels | $SINV_t = FINV_t + GIT_t + DINV_t - R_t,$ | (1.7) |
| Factory inventory levels | $FINV_t = FINV_{t-1} + COMRATE_t - DES_t,$ | (1.8) |
| Virtual consumption | $VCON_t = CONS_t + dSS_t,$ | (1.9) |
| Net changes in the distributor's re-order point | $dSS_t = R_t - R_{t-1},$ | (1.10) |
| Forecasted consumption for the factory | $AVCON_t = AVCON_{t-1} + \frac{1}{1+Ta} (VCON_t - AVCON_{t-1}),$ | (1.11) |
| Desired WIP | $DWIP_t = AVCON_t * \bar{T_p},$ | (1.12) |
| Actual WIP | $WIP_t = WIP_{t-1} + ORATE_t - COMRATE_t,$ | (1.13) |
| Error in WIP | $EWIP_t = DWIP_t - WIP_t$ | (1.14) |
| Order rate | $ORATE_t = AVCON_{t-1} + \frac{EINV_{t-1}}{Ti} + \frac{EWIP_{t-1}}{Tw},$ | (1.15) |
| Completion rate | $COMRATE_t = ORATE_{t-(Tp)},$ | (1.16) |
| Error in system inventory levels | $EINV_t = TINV_t - SINV_t.$ | (1.17) |
| Typical Test Input | $CONS_t = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases},$ for a step input | (1.18) |
| Typical Target inventory | $TINV_t = 0$ | (1.19) |

Appendix 2. Difference equations required for the two level APIOBPCS model

These difference equations (where the subscript 1 denoted the distributor variables and subscript 2 denotes the manufacturer variables) are for modelling a two level APIOBPCS model are;

| Description | Difference Equations | Eq. No |
|--|--|--------|
| Distributor's actual WIP | $WIP_t = WIP_{t-1} + ORATE_t - COMRATE_t,$ | (2.1) |
| Distributor's completion rate | $COMRATE_t = ORATE_{t-(Tp_1)},$ | (2.2) |
| Distributor's desired WIP | $DWIP_t = AVCON_t * \overline{Tp_1},$ | (2.3) |
| Distributor's error in system inventory levels | $EINV_t = TINV_t - SIN V_t.$ | (2.4) |
| Distributor's error in WIP | $EWIP_t = DWIP_t - WIP_t$ | (2.5) |
| Distributor's forecasted consumption for the factory | $AVCON_t = AVCON_{t-1} + \frac{1}{1 + Ta_1}(CONS_t - AVCON_{t-1})$ | (2.6) |
| Distributor's inventory levels | $AINV_t = AINV_{t-1} + COMRATE_t - CONS_t,$ | (2.7) |
| Distributor's order rate | $ORATE_t = AVCON_{t-1} + \frac{EINV_{t-1}}{Ti_1} + \frac{EWIP_{t-1}}{Tw_1},$ | (2.8) |
| Distributor's typical target inventory | $TINV_t = 0$ | (2.9) |
| Manufacturer's Actual WIP | $MWIP_t = MWIP_{t-1} + MORATE_t - MCOMRATE_t,$ | (2.10) |
| Manufacturer's Completion rate | $MCOMRATE_t = MORATE_{t-(Tp_2)},$ | (2.11) |
| Manufacturer's Desired WIP | $MDWIP_t = MAVCON_t * \overline{Tp_2},$ | (2.12) |
| Manufacturer's error in inventory levels | $MEINV_t = MTINV_t - MAINV_t.$ | (2.13) |
| Manufacturer's Error in WIP | $MEWIP_t = MDWIP_t - MWIP_t$ | (2.14) |
| Manufacturer's forecasted consumption for the manufacturer | $MAVCON_t = MAVCON_{t-1} + \frac{1}{1 + Ta_2}(ORATE_t - MAVCON_{t-1}),$ | (2.15) |
| Manufacturer's Inventory levels | $MAINV_t = MAINV_{t-1} + MCOMRATE_t - ORATE_t,$ | (2.16) |
| Manufacturer's Order rate | $MORATE_t = MAVCON_{t-1} + \frac{MEINV_{t-1}}{Ti_2} + \frac{MEWIP_{t-1}}{Tw_2},$ | (2.17) |
| Manufacturer's typical target inventory | $MTINV_t = 0$ | (2.18) |
| Typical test input | $CONS_t = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}, \text{ for a step input}$ | (2.19) |

| System Performance | | Optimal VMI supply chain | | | | | | | Traditional supply chain | | | | | | | | | |
|--------------------|--|--------------------------|--------|-------|--------|--------|-------|--------|--------------------------|--------------------------------------|----|-------|-------------------------------|----|-------|------------------------------------|----|----|
| | | G=0 | G=1 | | | | G=4 | | | Design 1 “G=0, W=1 equivalent” | | | Design 2 John et al (1994) | | | Design 3 Disney et al (1997) | | |
| | | | | | | | | | | Ta | Ti | Tw | Ta | Ti | Tw | Ta | Ti | Tw |
| | | W=1 | W=0.01 | W=1 | W=100 | W=0.01 | W=1 | W=100 | 4 | 7 | 28 | 8 | 4 | 8 | 8 | 4 | 15 | |
| Bullwhip Measures | Peak ORATE overshoot | 1.6 | 2.5 | 1.69 | 1.21 | 2.45 | 1.70 | 1.22 | 2.44 | | | 2.48 | | | 2.99 | | | |
| | Noise Bandwidth/ π | 0.45 | 5.52 | 0.46 | 0.08 | 4.96 | 0.59 | 0.09 | 0.93 | | | 1.1 | | | 2.32 | | | |
| | σ^2 (calculated from time series) | 0.5 | 5.63 | 0.5 | 0.11 | 5.07 | 0.62 | 0.12 | 1.01 | | | 1.31 | | | 3.18 | | | |
| Stock performance | Maximum deficit | -5.99 | -5.96 | -6.80 | -11.03 | -8.87 | -8.75 | -13.37 | -8.88 | | | -8.7 | | | -9.56 | | | |
| | ITAE | 1302 | 134 | 411 | 19589 | 1158.2 | 1458 | 29066 | 2884 | | | 2066 | | | 3230 | | | |
| | %Availability | 99.98 | 99.68 | 99.91 | 99.87 | 97.8 | 99.32 | 99.28 | 98.64 | | | 98.74 | | | 94.89 | | | |

Table 4. Bullwhip and Stock Performance Trade-offs For VMI and Traditional Supply Chain