The Cross-Sectional Determinants Of US Stock Returns

by

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Abstract

In this thesis, we investigate the relationship between the US stock returns and downside risk in a cross-sectional context. When the classic market model with a moving window approach is adopted, downside risk estimated coefficients exhibit a positive impact on stock returns. However, when two other non-linear time-varying models: the cubic piecewise polynomial function (CPPF) and the Fourier Flexible Form (FFF) models are adopted, downside risk estimated coefficients show a negative impact on stock returns. Cross-sectionally, the risk estimated coefficients of the two non-linear models produce a much better fit than the classic market model. The predictive power for future stock returns of downside risk estimated coefficients are found to be weak. Two more risk factors: commodity market risk and Aruoba-Diebold-Scotti (ADS) business condition index risk (both downside and upside versions thereof), are shown to have a significant effect on stock returns.

Key words: US stocks, downside risk, cross-section, time-varying, cubic piecewise polynomial function, Fourier Flexible Form, commodity market, Aruoba-Diebold-Scotti business condition index

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Introduction

This thesis aims to examine the cross-sectional relationship between US stock returns and risk. The whole study is built on the classic Capital Asset Pricing Model (CAPM), but differentiates in a number of ways. Firstly, from Chapter 2, the constant beta assumption of the single market factor model is relaxed. From Chapter 2 to Chapter 4, the conventional beta is calculated at each point in time through this study. Specifically, in Chapter 3 and Chapter 4, risk premia are smoothly adjusted and beta is allowed to be time-varying in order to show the variation between risk and return. Secondly, conventional beta is not the only risk factor considered. Rather, this factor is intentionally divided into downside risk and upside risk factors with corresponding downside beta and upside beta. Thirdly, the return of the stock market portfolio is not the only risk factor employed in the model. Specifically, in Chapter 4, the commodity market price index and the Aruoba-Diebold-Scotti (ADS) business condition index are employed as risk factors. Fourthly, rather than treating each observation equally in the sample, in Chapter 3 and Chapter 4, two non-linear models are introduced: the cubic piecewise polynomial function (CPPF) model and the Fourier Flexible Form (FFF) model. With both models, risk premia are smoothly adjusted by the nature of the models. By doing so, the best fit of time-varying beta estimates can be decided according to the corresponding Akaike Information Criteria (AIC). Finally, all the risk factor coefficients and stock returns are examined using the Fama-Macbeth (1973) regression methodology to discover the cross-sectional relationship among them.

The thesis was initially motivated by Ang et al's (2006) study, which examines the relationship between stock returns and downside beta. However, in their study, only the conventional single market factor model is adopted, and the data used is not rigorous.[1] Apart from that, there are few studies in the literature that specifically focus on the cross-sectional risk and return relationship with beta broken down into downside and upside components. Introducing non-linear models into this study is another innovation. There is a long history of using non-linear models in asset pricing, but few studies employ them within a downside risk context.

The contribution of this thesis to the asset pricing literature is that, firstly, it explores the alternative risk measurements of classic beta. Specifically, decomposing classic beta into downside and upside, and cross-sectionally examining the risk-return relationship can highlight the sensitivity of return to risk on both downside and upside. Secondly, allowing beta, downside beta and upside beta to be time-varying, and employing two non-linear models to give the data more flexibility can improve the goodness of fit, and enhance the effectiveness of the asset pricing model. Thirdly, rather than taking market risk as the only risk factor, two more risk factors; commodity market risk and real business condition risk are examined, and the impacts of both factors (downside or upside components thereof) on stock returns are documented.

[1] In Ang et al's (2006) study, dataset and data frequency are not consistent through the study, only the dataset in favor of the expected result are chosen.

The whole thesis focuses on the cross-sectional determinants of US stock returns. Each chapter of this thesis is closely linked and arranged in the following order: Chapter 1 contains a literature review of cross-sectional asset pricing. Chapters 2 to 4 contain empirical work. Specifically, Chapter 2 focuses on the downside risk and classic market factor model with time-varying beta obtained by using a moving window approach. Chapter 3 introduces the two non-linear models to obtain flexible time-varying downside risk. Chapter 4 is based on Chapter 3 but employs commodity market risk and real business condition risk as additional risk factors. This thesis ends with a conclusion, appendix and references.

Chapter 1

Literature review on cross-sectional asset pricing

1.1 Introduction

In this chapter, a review of cross-sectional asset pricing literature is conducted.[2] This starts with factor selection, and moves to cross-sectional methodologies, then the empirical evidence on two well-known asset pricing models, the CAPM and the Arbitrage Pricing Theory (APT), are reviewed. Prior to concluding, the literature on return anomalies is considered.

1.2 Factor selection

Choosing which specific factor to employ and how many factors to use are the essential steps to building an asset pricing model. Among the vast array of asset pricing literature, there are mainly three approaches to factor selection. According to Goyal's study (2012), the first approach is by following certain theories and finance or economic intuition. The most well-known asset pricing model determined by this approach is the CAPM proposed by Sharpe (1964), Linnter (1965) and Mossin (1966). They predict that the market portfolio return is the only factor relating to the expected returns of stocks. The theory was then developed significantly by Merton (1973) when he proposed the intertemporal capital asset pricing model (ICAPM), which states that any risk factor relating to future investment can be employed as a state variable in the model. The advancement of theory is that asset pricing models are not limited to one risk factor and multivariate asset pricing models are introduced in finance literature.

[2] The literature review covers the three main asset classes: equities, bonds and money market instruments, while the empirical work of this thesis focus on the equities.

Following Merton's study, there are a large number of studies that focus on employing state variables in an asset pricing model. For instance, Breeden (1979) proposed the consumption based capital asset pricing model (CCAPM), which takes the covariance between stock returns and the marginal utility of consumption into consideration. In addition to the CCAPM, Lettau and Ludvigson (2001) introduce the consumption to wealth ratio as a state variable. Moreover, Chen et al. (1986) posit macroeconomic variables such as inflation, production growth and oil price as state variables in their asset pricing model.

The second approach to factor selection is based on statistical analysis. This approach is motivated by Ross's (1976) APT.[3] The main concept of the APT is that the expected return of any asset can be modeled as a linear function of various risk factors, and each factor loading represents the sensitivity of the factor upon the expected return of the asset. The APT is relatively a broad concept, and difficult to apply without certain restrictions. Following the concept of the APT, Anderson (1984) proposed factor analysis, which employed the factor depending on the covariance of asset returns and factors. Factor analysis has been further developed and demonstrated by Lehmann and Modest (1988, 2005). An alternative statistical method is principal component analysis proposed and developed by Connor and Korajczyk (1986, 1993). The principal component analysis aims to extract the principal component from each variable, specifically to a large sample of cross-sectional returns with a short sample

[3] Statistically, the APT is under the 2nd approach, however, a few study also apply the APT on factors selected by economic intuition, such as Chen (1980) and Chen et al. (1986).

size.

The third approach to factor selection is based on firm characteristics. The motivation for using firm characteristics is due to the return anomalies, and the most representative one is the Fama-French three-factor model (1993), which is motived by the size and value anomalies. Moreover, Carhart (1997) argued that a momentum factor should be posited in the Fama-French three-factor model aiming to capture the momentum anomaly.[4]

There is also a literature related to factor selection, which goes beyond the above mentioned approaches.[5] For instance, Amihud and Mendelson (1986) point out that liquidity is a significant factor in asset pricing, and they find that the bid-ask spread is significantly priced. Chordia et al. (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001), Acharya and Pedersen (2005) and Korajczyk and Sadka (2008) also find supportive evidence that liquidity can be a factor in asset pricing. Moreover, Easley and O'Hara (1987) suggest that information risk can be a risk factor in asset pricing. Easley et al. (1996, 2002), Easley and O'Hara (2004) and Duarte and Young (2009) find that the probability of informed trading is priced in a cross-sectional asset pricing setting. Furthermore, Malkiel and Xu (1997, 2002), Bali and Cakici (2008), Fu (2009) and Fink et al. (2011) show that idiosyncratic volatility can explain

[4] In Carhart (1997), the momentum factor is described as the equally weighted average of firms with the highest 30% lagged twelve months returns minus the ones with the lowest 30% over the same period.

[5] The literature relating to other factors that are priced is vast. Consequently, only a brief selection of examples from this literature are considered.

cross-sectional stock returns.

1.3 Methodology

To demonstrate the methodology of cross-sectional asset pricing, some notation has to be presented for the convenience of later illustration. From the asset perspective, denote the return of asset i at time t by R_{it}, then denote the number of assets by N and sample size by T. It is quite usual to encounter unbalanced panel data, which is for a large number of assets during sample size T, the return is not available for each asset at each point in time. In this case, sorting assets into portfolios based on certain characteristics is widely used by researchers, for instance, Jegadeesh (1990) sorts stocks into ten portfolios based on the return forecast of individual stocks. And more recently, Fama and French (2011) assign stocks into four portfolios according to the location of the stock exchange. Nonetheless, the notations defined at the beginning of this section are still adopted to demonstrate the methodology of cross-sectional asset pricing.

For a multi-factor asset pricing model in general, we further denote that the return of risk factor k at time t by F_{kt}. Then define μ to be a $N \times 1$ vector of expected returns to the assets, and λ to be a $K \times 1$ vector of risk premia. The aim of an asset pricing model is to discover and explain the relationship between the risk factors and returns. Specifically, for cross-sectional asset pricing models, the aim is to reveal the variation of expected returns among all assets. According to Goyal's survey (2012), the general proposal of asset pricing model is

$$\mu = \tau\lambda_0 + B\lambda\ , \tag{1.1}$$

where τ is the $N\times 1$ vector of ones, λ_0 is the constant, and B is the $N\times K$ factor loading matrix. Equation (1.1) is not only a general proposal of asset pricing, but also the original version of a number of multi-factor asset pricing models, for instance Ross (1976) APT model, and Merton's (1973) ICAPM.

Moreover, given the risk-free asset return R_f, and a $K\times 1$ vector of risk factor return μ_F, equation (1.1) can be further modified to

$$\mu = \tau R_f + B(\mu_F - \tau R_f), \tag{1.2}$$

where the well-known CAPM can be treated as a special case of equation (1.2). It can be written as follows:

$$\mu_i = R_f + \beta_i(\mu_M - R_f), \tag{1.3}$$

where the return of the market portfolio μ_M is introduced, and the risk-free rate R_f can be moved to the left hand side of the equation to be expressed in a capital budgeting format. Notably β_i is the market beta of asset *i* defined by

$$\beta_i = \text{cov}(R_{it}, R_{Mt}) / \text{var}(R_{Mt}). \tag{1.4}$$

Specifically for cross-sectional asset pricing, the above assumptions and notations still hold. Although according to Campbell et al.'s study (1997), while it is possible to use time-series regression to obtain coefficient estimates of risk factors and corresponding pricing errors, using the cross-sectional regression approach can reduce the complexity and simplify the whole process.

The main idea is a two-stage regression. In stage one, the estimates of all risk factors are obtained by a using time-series regression as follows:

$$R_t = \alpha + BF_t + \varepsilon_t \,. \tag{1.5}$$

In the second stage, a cross-sectional regression of average asset returns at each point in time upon corresponding beta estimates

$$\overline{R_T} = B\lambda + \phi, \tag{1.6}$$

where $\overline{R_T}$ is defined as the average asset returns over the sample size T and ϕ is the pricing error. It can be seen from equation (1.6) that the beta estimates from stage one become the independent variables in stage two.

Notably, for the convenience of demonstration, there is no intercept in stage two. However, there could be an intercept in equation (1.6) if we assume that the zero-beta rate is different from the risk-free rate, and the intercept would be the difference between these two rates, while in equation (1.6), the null hypothesis is that the zero-beta rate is the same as the risk-free rate. Moreover, the cross-sectional pricing error, given by ϕ, is the average of the residual of the time series pricing error, but not the time series residual which is commonly assumed to have a mean of zero (given by ε). Furthermore, the estimate and variance of the factor loading λ can be obtained by ordinary least squares (OLS) regression as follows:

$$\lambda = (B^{'}B)^{-1}B^{'}\overline{R_T}, \tag{1.7}$$

$$\mathrm{var}(\lambda) = \frac{1}{T}(B^{'}B)^{-1}(B^{'}\textstyle\sum_{\varepsilon} B)(B^{'}B)^{-1}, \tag{1.8}$$

and the estimate and variance of the pricing error ϕ are given by

$$\phi = \overline{R_T} - B\lambda, \tag{1.9}$$

$$\mathrm{var}(\phi) = \frac{1}{T}(I_N - B(B'B)^{-1}B)\Sigma_\varepsilon(I_N - B(B'B)^{-1}B)', \qquad (1.10)$$

where Σ_ε is given by

$$E(\phi'\phi) = \frac{1}{T}\Sigma_\varepsilon. \qquad (1.11)$$

1.4 Fama-Macbeth Regression

The Fama-Macbeth (1973) regression is the most widely used cross-sectional asset pricing approach. This approach is motived by the two-stage regression demonstrated in the previous section with further modification. Following the two-stage regression, in stage one, all beta estimates are obtained by running the time-series regression

$$R_t = \alpha + BF_t + \varepsilon_t. \qquad (1.12)$$

In stage two, the beta estimates obtained in stage one are treated as independent variables, then a cross-sectional regression of asset returns in each period upon beta estimates is run as follows:

$$R_t = \lambda_{0t} + B\lambda_t + \phi_t, \qquad (1.13)$$

where an intercept λ_{0t} is included to represent the difference between the zero-beta rate and risk-free rate, and λ_t is the factor loading to be estimated at time t. In the second stage, OLS, generalized least squares (GLS) and weighted least squares (WLS) approaches can be applied. According to Litzenberger and Ramaswamy (1979) and Shanken (1985), estimates resulting from all three approaches converge to the same limit. The reason for that is, compared to the two-stage approach, the Fama-Macbeth regression runs the cross-sectional regression at each point in time, while the former one just runs the regression once using average asset returns in stage two.

Unlike the two-stage approach, the estimates of the factor loading and pricing error associated with the Fama-Macbeth regression are given by

$$\lambda = \frac{1}{T}\sum_{t=1}^{T}\lambda_t, \tag{1.14}$$

and

$$\phi = \frac{1}{T}\sum_{t=1}^{T}\phi_t, \tag{1.15}$$

where both estimates are the average of cross-sectional estimates in each period. Moreover, the variance of both estimates are given by

$$\text{var}(\lambda) = \frac{1}{T^2}\sum_{t=1}^{T}(\lambda_t - \lambda)(\lambda_t - \lambda)', \tag{1.16}$$

and

$$\text{var}(\phi) = \frac{1}{T^2}\sum_{t=1}^{T}(\phi_t - \phi)(\phi_t - \phi)'. \tag{1.17}$$

Notably, by using the Fama-Macbeth regression, the variances of both estimates are not computed at each point in time, but computed as the variance of the average estimates obtained from stage two.

To sum up, there are a number of advantages of using the Fama-Macbeth regression as the approach in cross-sectional asset pricing. First of all, the unbalanced data problem can be easily overcome, and it is not necessary to sort assets into portfolios. Secondly, it allows time-varying estimates regardless of the sample size and number of assets, for instance Fama and French (1992) used rolling betas in their studies and applied the Fama-Macbeth regression. Thirdly, the possible autocorrelation among returns (and consequently in factor loading estimates) can be avoided by a number of econometric techniques, for instance adjusted by Newey-West (1987) robust standard errors which is used extensively in later chapters.

1.5 Empirical evidence on asset pricing models

1.5.1 Empirical evidence on the CAPM

Since the 1950s, the empirical tests on asset pricing models have followed theoretical developments. It is sensible to begin with reviewing the tests on the CAPM, literally the first well-known asset pricing model.

As a single factor asset pricing model, the CAPM was not widely accepted when it was first proposed. Right after the CAPM was published in the finance literature, Lintner (1965) and Douglas (1969) doubted that the market factor would be the only factor related to asset returns. However, with a comprehensive econometric demonstration, Miller and Scholes (1972) argued that the conclusion Lintner and Douglas made was not solid enough since their derivation of beta was not rigorous. Then it became the fashion in the literature to group assets into portfolios based on certain standards. For instance, Fama and Macbeth (1973) sorted stocks into portfolios based on their beta value. In their study they also found supportive evidence that asset returns are directly related to their beta values but not residuals, which is quite consistent with the initial idea of the CAPM. However, a more recent study by Ang and Kristensen (2011) shows that using portfolios instead of individual betas would lead to the loss of information of each individual asset and result in inefficient estimates. The above studies also pointed out that asset returns and beta are not tightly related, while the Jensen’s (1968) alpha is significantly positive for low beta assets and negative for high beta assets.

The critique to the CAPM took other forms, Roll (1977) argues that it is impossible to construct a real market portfolio because it is not possible to measure returns to all assets in the market. However, counter arguments have been presented, for instance, Stambaugh (1982) chooses various assets as a proxy in the CAPM and finds that the market proxy could be different if the target assets vary. More solid evidence was found by Shanken (1987) and Kandel and Stambaugh (1987). In their studies they show that if the market proxy and true market returns are correlated at 70%, then either of them in the CAPM will be equally significant. Although the controversy surrounding the CAPM seems to have reduced recently, as a milestone of asset pricing models, the challenge to CAPM will never end.

1.5.2 Empirical evidence on the APT

Compared to the single factor CAPM, the APT introduced by Ross (1976) is another well-known asset pricing model in the finance literature. Consistent with other asset pricing models, the APT agrees with the intuition behind CAPM, however, the main assumptions of the CAPM about utility theory and mean-variance derivation are not adopted by the APT. There is no requirement in the APT that states that the market portfolio has to be mean variance efficient (Roll and Ross, 1980). The mean-variance or investors' utility assumptions are replaced by the process of generating assets returns.

Although there are not a great number of empirical studies on the APT compared to the CAPM, there still exists some empirical research related to the multiple factor return generating models which contribute to the formulation of the APT. Farrar (1962) and King (1966) conducted their research focusing on multiple factors analytic methods, and employed the factors based on industry influence. Although no test was conducted for such effects, their research still can be considered as a valuable rudiment of multiple generating factors.

Brennan (1971) however, was truly related to the APT. Brennan (1971) adopted the approach that decomposes the idiosyncratic disturbance for a market portfolio regression. He discovered that there were two factors contributing to idiosyncratic disturbance, rather than one representing the true return generating process and that it was not possible to conduct a cross-sectional test on the CAPM due to it being a single factor model. Brennan's (1971) study can be considered as the prime motivation for the APT.

The empirical study by Gehr (1975) is the first empirical study directly related to the APT. He used a similar approach as the APT but with a smaller data set (24 industry indices and 41 individual securities) to conduct his study. The more famous APT introduced later by Ross (1976) carried on and extended Gehr's (1975) study by giving more accurate definitions, with more comprehensive analysis. Nevertheless, Gehr's (1975) study can still be recognized as the beginning of the APT.

After the APT was formulated by Ross (1976), more empirical studies were conducted by researchers in order to discover a deeper understanding of the multiple factor return generating process. Rosenberg and Marathe (1977) analyzed the so called extra market component of returns, and employed descriptor variables to track the change in the CAPM's parameters. They found that the descriptor was equal to the factor loading in the multiple factors model, so it proved the single factor model is not powerful enough to explain the return generating process. However, Rosenberg and Marathe's (1977) study did not focus on the separate influences of those factors; they only decompose the beta in the CAPM into several constitutive parts without a clear definition and test.

Moreover, some more recent studies are more or less related to the multiple generating factors. Unfortunately, none of those studies has shown a clearer return generating process than the APT. Langetieg (1978) and Lee and Vinso (1980) found evidence that more than one market factor generates returns. Kryzanowski (1979) even did a formal test that showed there existed additional factors affecting the generation returns. However, he also found that the additional factor cannot be treated as equal to the market factor. More recently, Clare and Thomas (1994) found that, under the APT, return on the market is only significantly priced when stocks are sorted into portfolio according to their market value than the value of the beta. Priestley (1996) argued that the conventional factor analysis in the APT may lead to false inference regarding to the statistical significance, and an alternative approach to generate the unexpected component is proposed. Nonetheless, factor analysis is still the main stream of factor selection of the APT.

The first step to testing the APT empirically is to conduct a factor analysis to determine the number of factors in the APT and estimate the factor loadings. Factor analysis, as a statistical tool, is separate from the performance of the APT, and is used to obtain the estimates of coefficients in the APT. Based on previous studies by Roll and Ross (1980), Reinganum (1981), Weinstein and Brown (1983), the optimal number of factors in the APT is no more than five. Therefore, in Chen's study (1983), five factors were selected to conduct the empirical test on the APT.

As all factors in the APT are unobservable, OLS regression is difficult to apply in factor loading estimation; hence, factor analysis is adopted. Chen (1980) found that it is easy to estimate the factor loadings of a subset of assets by using the variance-covariance matrix and extend those estimates to the whole sample. The procedure of Chen's method is described as follows:

Let R_i and R_p be the *ith* and *pth* assets in the portfolio and $i = 1, 2, 3 \ldots .k, k+1$, and $p \in i$, hence,

$$Cov(R_p, R_i) = b_{p1}b_{i1}Var(\delta_1) + \ldots . + b_{pk}b_{ik}Var(\delta_k)$$

$$= b_{p1} b_{i1} \sigma_1{}^2 + ... + b_{pk} b_{ik} \sigma_k{}^2 , \tag{1.18}$$

and in matrix form, the above equation can be written as follows

$$\begin{bmatrix} b_{11}\sigma_1^2 & b_{12}\sigma_2^2 & \cdots & b_{1k}\sigma_k^2 \\ b_{21}\sigma_1^2 & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ b_{k1}\sigma_1^2 & b_{k2}\sigma_2^2 & \cdots & b_{kk}\sigma_k^2 \end{bmatrix} \begin{bmatrix} b_{p1} \\ b_{p2} \\ \vdots \\ b_{pk} \end{bmatrix} = \begin{bmatrix} Cov(R_p, R_1) \\ \vdots \\ \vdots \\ Cov(R_p, R_k) \end{bmatrix}, \tag{1.19}$$

since the coefficient matrix is non-singular, so the column vector of b_{pj} is unique and can be determined by taking the inverse matrix of the coefficient matrix (Chen 1980). Those factor loadings of a subset of assets can then be extended to the entire sample by mathematical programming software.

In Chen (1983), the data used in the test is the simulated daily stock returns from 1963 to 1978 provided by the Center for Research in Security Prices (CRSP). The sample period is divided into four subperiods: 1963-1966, 1967-1970, 1971-1974 and 1975-1978.

The regression being used for the APT in this test is:

$$R_i = \lambda_0 + \lambda_1 \hat{b}_{i1} + \cdots \lambda_k \hat{b}_{ik} + \varepsilon_i \tag{1.20}$$

Where $\hat{b}_{ij}$ is the estimate of factor loadings by Chen's method (treated as independent variables here). In contrast, a regression for the CAPM is also adopted

$$R_i = \lambda_0 + \lambda_1 \hat{\beta}_i + \eta_i \tag{1.21}$$

similarly, the $\hat{\beta}_i$ is also taken as the independent variable (Chen 1983).

The results show that the estimated λ_1 for each subperiod is negative, so one would expect the factor loading of factor one to be negative as well. Based on Chen's study (1980), the factor loadings of factor one are highly correlated with β and are all significantly negative, which is consistent with the result here. Notably, the estimates of λ are only statistically significant in subperiod one and four, which is consistent with the results of the CAPM regression in that it only prices significantly in those two periods.

Then, a joint test on the null hypothesis that $\lambda_1=\lambda_2=\lambda_3=\lambda_4=\lambda_5=0$ is applied to see if the APT has enough explanatory power in cross-sectional. The result shows that the F statistic is significant at a 10% level for each period and the null hypothesis is rejected.

Test of the own variance effect on the APT

A large amount of evidence shows that there is correlation between stock returns and its own variance. For instance, Fama and Macbeth (1973), Roll and Ross (1980), and Rhaiem, Ammou and Mabrouk (2007), all find that own variance had explanatory power as an independent variable when it comes to explaining asset returns. The relationship between stock returns and their own variances is defined as the own variance effect.

To test the own variance effect in the APT, Chen (1983) firstly computed the variance of each asset in the sample. The variance series was not then put into the APT as an independent variable due to the possible correlation between factor loadings and variances. Alternatively, all the assets were divided into two groups based on their variances, one group consisted of the assets that had small variance (less than the median value of the variance), while the other group consisted of high variance assets. Then, a programming technique was adopted to form two portfolios, one from each group, which assured both portfolios possessed some factor loadings. So the null hypothesis of the test is if the own variance effect does appear in the APT, then those two portfolios should yield relatively the same return. The alternative hypothesis would be:

$$R_i = \lambda_0 + \lambda_1 b_{i1} + \cdots \lambda_k b_{ik} + f(\sigma_i^2) \quad (1.22)$$

where $f(\sigma^2)$ is a function of variances, treated as an independent variable. The result shows that the mean difference between two portfolios is not significant and there is no autocorrelation between each period. Therefore, it can be concluded from the test that the own variance effect does not appear in the APT.

Test of the firm size effect on the APT

The firm size effect has become a benchmark of whether an asset pricing model is misspecified for more than thirty years. Banz (1981) found that small firms tended to have higher risk adjusted average returns than large firms. Moreover, Reinganum (1981) found that firm size effect appeared in the APT. Up to now, nobody could explain whether the firm size is an unobservable variable affecting asset returns or just a proxy for other variables not uncovered.

To test the firm size effect in the APT again, Chen (1983) used the data partially from

Reinganum's study and the rest from CRSP. The methodology is similar to that employed in testing for the own variance effect. The data were divided into two groups based upon the firm market value (large firms and small firms), then two portfolios with the same factor loadings were formed. If the firm size does not have explanatory power in the APT, those two portfolio should obtain the same returns, otherwise, the firm size effect does appear in the APT.

The results of the test show that after correcting for autocorrelation, the mean difference for each sample period is not statistically significant. Also, for equal weighted portfolios and top to bottom decile firms, the mean differences are still insignificant after adjusting for factor loadings. So the conclusion of the test is that firm size does not have explanatory power in the APT. More recently, Mei (1993) find that the size effect can be captured by the APT by using a quasi-differencing approach to eliminate unobservable factors in the model. While Funga and Leug (2000), Fernald and Rogers (2002), Fan et al. (2009) and Abdullahi et al. (2011) find that size effect is not significant in emerging stock market by using the APT.

Tests of other risk factors on the APT

Apart from the empirical tests described above, some other macroeconomic variables are also tested in the APT to see whether they could affect asset returns. Chen et al (1986) took long and short interest rates, expected and unexpected inflation, industrial production, market returns, aggregate consumption and the oil price index as six risk factors, and regressed them on asset returns. Although no more than five factors are in the APT's favor, six factors is still not over the top. The results show that industrial production, changes in market premium, and changes in interest rates have a significant influence on asset returns; expected and unexpected inflation has a somewhat weakened effect on asset returns; finally, aggregate consumption and changes in oil price do not have a significant influence on asset returns. Moreover, Fama and French (1992) tested whether the book to market ratio (B/M) and earning/price ratio (E/P) could have an impact on stock prices. Their results show that stock returns have a positive relationship with B/M and E/P. Similar studies are numerous, and more and more variables which could affect asset returns are being uncovered by researchers.

1.5.3 Empirical comparison between the APT and the CAPM

To assess the relative quality of the APT or the CAPM, an empirical comparison is necessary. The easiest way would be to compare the differences between the asset returns priced by the APT and the CAPM. However, due to the high correlation between b_{ij} and β, putting them in a regression as independent variables could cause multicollinearity. Therefore, an alternative regression by Davidson and Mackinnon (1981) is adopted

$$r_i = \alpha \hat{R}_{i,APT} + (1-\alpha)\hat{R}_{i,CAPM} + e_i , \qquad (1.23)$$

Where r_i is the actual return of the asset i, $\hat{R}_{i,APT}$ and $\hat{R}_{i,CAPM}$ are the expected returns of asset i generated by the APT and the CAPM, respectively. If the APT performance is better than the CAPM, α is expected to be close to 1. The results indicate that although in some cases the α is not statistically significant, the point estimates of α are supportive to the APT rather than the CAPM (Chen 1983).

Moreover, according to Chen's study, the performance measurement of the APT is tested. If the APT could correctly price the assets relative to the CAPM, how much information the APT captures in the error term of the CAPM is of interest. Hence, one can regress the error term of the CAPM (given by $\hat{\eta}$) on the APT to test the performance of the APT

$$\hat{\eta}_i = \lambda_0 + \lambda_1 \hat{b}_{i1} + \lambda_2 \hat{b}_{i2} + \lambda_3 \hat{b}_{i3} + \lambda_4 \hat{b}_{i4} + \lambda_5 \hat{b}_{i5} + \xi_i \,. \tag{1.24}$$

The error term of the APT (given by $\hat{\varepsilon}$) is also regressed on the CAPM for the sake of comparison

$$\hat{\varepsilon}_i = \lambda_0 + \lambda_1 \hat{\beta}_i + \omega_i \,. \tag{1.25}$$

The result of both regressions indicates that the APT is superior.

1.6 Return anomalies

Since the 1970s, return anomalies in equity market have become a key area of cross-sectional asset pricing research. As a number of studies have documented, most return anomalies demonstrate that asset returns do not follow the classic financial theory but are related to variables which are based on certain financial characteristics. Also, quite a number of return anomalies are related to cross-sectional risk and asset returns, therefore, it is sensible to conduct a broad review and summary. Following Subrahmanyam's (2010) and Goyal's (2012) studies, return anomalies can be attributed to return-based ratios, price-based ratios and accounting-based ratios.

1.6.1 Return-based ratios

Lo and Mackinlay (1988) find a positive serial correlation among weekly returns of aggregate portfolios, and Fama and French (1988) show a negative correlation on the market portfolio over a three to five years span. Jegadeesh (1990) shows that winner stocks tend to have low returns in the following month, and Lehmann (1990) extends the results to weekly frequency data. And investors' reaction to news can also cause return anomalies. De Bondt and Thaler (1986, 1987) propose the over-reaction hypothesis that loser stocks tend to perform better than winner stocks in the following 3 to 5 years, and it is attributed to the investors' over-reaction to recent information and neglect the importance to past news. Contradictory to the over-reaction hypothesis, Jegadeesh and Titman (1993, 2001) propose the under-reaction hypothesis that stock returns in the past 12 months possesses a strong predictive power to the return in the following year. It is attributed to that the investors would slowly adapt recent news and incorporate them into stock prices. Chen et al. (1996) also support the under-reaction hypothesis.

Moreover, Rouwenhorst (1998), Griffin et al. (2003), Chui et al. (2010) and Fama and French (2012) all document momentum effects directly related to return anomalies, especially in international markets. Based on Jegadeesh's (1990) study, Heston and Sadka (2008) point out that loser stocks, on average, will outperform winner stocks in the next twenty weeks. More recently, Asness et al. (2009) and Moskowitz et al. (2012)

find that momentum effects also appear in government and corporate bonds, commodity and currency markets. Furthermore, Hong et al. (2000) show that momentum profits are much stronger in small stocks in the US market, and Doukas and Mcknight (2005) find that Hong’s result is also valid in the European markets. Cooper et al. (2004) show that momentum effects tend to follow positive market returns rather than the negative ones. Hvidkjaer (2006) point out that for those listed production firms, momentum profit is related to the number of orders they receive. Avramov et al. (2007) state that low credit quality firms are more likely to earn momentum profits. However, Goyal and Wahal (2011) argue that Avramov et al.’s results are very much location limited.

1.6.2 Price-based ratios

Miller and Scholes (1972) find that stocks with a relatively low price will earn higher returns. According to Basu’s (1977) and Ball’s (1978) studies, firms with low price-earnings ratios tend to have high returns. Stattman (1980) and Rosenberg et al. (1985) point out that stock returns are positively related to book-to-market ratio. Banz (1981) finds that small capitalization firms earn higher returns than large ones. In Bhandari’s (1988) study, he shows that firms with higher leverage tend to earn higher returns. Moreover, Fama and French (1992) find that the stock returns are more related to price-based ratios rather than classic beta estimates and market portfolio, and they even go so far as to predict the death of beta. Their results are further proved in Fama and French (1996).

1.6.3 Accounting-based ratios

Back in the 1960s, Ball and Brown (1968) found that stock prices were more volatile after a firm's earnings were announced. Lakonishok et al. (1994) point out that there is a negative relationship between long term returns and a firm's total sales and earnings. La Porta (1996) finds a negative relationship between the stocks' future returns and a firm's earning growth. According to Haugen and Baker's (1996) and Cohen et al.'s (2002) studies, more profitable firms tend to earn higher returns than less profitable ones.

Moreover, Chan et al. (2001) find that firms with high research and development expenditure have higher future returns than those with low research and development expenditure. However, they also point out that there is no significant difference on future returns between firms doing research and development and ones not doing so. Titman et al. (2004) find a negative relationship between a firm's investment and stock returns. Cooper et al. (2008) also discover a negative relationship between a firm's asset growth and stock returns. Additionally, Titman et al. (2010) and Watanabe et al. (2011) all show that Cooper et al.'s (2008) findings exist not only in the US market, but also in a number of international markets, except Japan.

Furthermore, Ikenberry et al. (1995) show that, on average, there will be positive stock returns after repurchases. Solan (1996) finds a negative relationship between accounting accruals and stock returns. According to Daniel and Titman's (2006) and Pontiff and Woodgate's (2008) studies, there is a negative relationship between long term stock returns and stock issues.

1.7 The literature on downside risk, time-varying beta

1.7.1 Downside risk

The idea of downside risk is motivated by treating risk asymmetrically in asset pricing. According to the classic portfolio theory, risk factors symmetrically affect returns, while the innovation of downside risk opens a new area of the asymmetry of risks.[6] The downside risk is defined as the risk investors will face in a relatively falling market. There are quite a number of studies which document the asymmetry of risks, specifically to downside risk. Roy (1952), Markowitz (1959), Quirk and Saposnik (1962), and Mao (1970) find that rational investors care more about downside losses rather than upside gains by using semi-variance as a measure of risk. Moreover, Bawa (1975) proposed lower partial moment to measure the downside risk. Fishburn (1977), Bawa and Lindenberg (1977), Kahneman and Tversky (1979), Gul (1991), and Sing and Ong (1993) all focus on modifying the CAPM to treat the risk asymmetrically.

Since the concept of downside risk was proposed, the arguments about the relationship between downside risk and asset returns began. According to Jahankani (1976) and Harlow and Rao (1989), downside risk is not significantly priced in the

[6] Different from prospect theory, here it is assumed that all investors are rational and risk averse.

CAPM. Ang and Chen (2002), and Ang et al. (2006) show that there is a positive relationship between downside risk and stock returns, and the downside risk premium is estimated to be 6% per annum. More recently, Huang and Hueng (2008) argue that in a downside market, there is a significant and negative risk-return relationship.

1.7.2 Time-varying beta

Allowing beta to be time-varying aims to rescue the beta estimate from the constant beta assumption of the CAPM and reveal the risk-relationship at each point in time. There are a vast number of studies using time-varying beta instead of constant beta. For instance, Fama and Macbeth (1973), and Ang et al. (2002) use a moving window approach to obtain time-varying beta. Hardel et al. (1985), Hardle (1992), Wand and Jones (1995), Ang and Kristensen (2011), and Li and Yang (2011) use simulation to generate time-varying beta. Moreover, Engle (2002), Andersen et al. (2002), Choudhry and Wu (2009) and Nieto et al. (2011) compute time-varying beta based on the generalized autoregressive conditional heteroskedasticity (GARCH) model. However, data are treated equally in all the above studies, and none of them allows beta to be time-varying with flexible adjustment to data.

Although there is a comprehensive amount of literature available on downside risk and time-varying beta, only a few studies focus on both of them. Ang et al. (2002, 2006) obtain the time-varying downside beta estimates by using the OLS moving window approach. They point out that there is a positive relationship between downside beta and stock returns. While Huang and Hueng (2008) find an inverse relationship by using the adaptive least square (ALS) moving window approach to generate the time-varying downside beta estimates. Aside from the above studies, it is hard to find a study which focuses on the downside risk in a time-varying context.

The contribution of this thesis to the literature is that it aims to reveal the cross-sectional risk-return relationship in both the upside and downside market in a time-varying context. Specifically, using the cubic piecewise polynomial function (CPPF) and Fourier Flexible Form (FFF) model to flexibly adjust the data and allowing for the beta estimates to be time-varying is an innovation to the literature.

1.8 Conclusion

As reviewed, cross-sectional asset pricing is such a wide area to explore. It begins with factor selection and model selection. According to different factor selection approaches, different models are finally built. The basic methodology of cross-sectional asset pricing is a two-stage approach, starting with a time-series regression to obtain factor loading estimates, and then a cross-sectional regression is applied. In this regard, the Fama-Macbeth regression methodology is the most widely used approach.

The empirical evidence of the CAPM and the APT are reviewed in detail. The former model is a milestone asset pricing model with a single market factor, and the latter one is well-known by proposing the idea of a multi-factor model. There are also quite a number of asset pricing models that have a pervasive influence in the finance literature, such as Fama-French's three factor model, and Carhart's four-factor model. There is never a lack of argument about which model outperforms the other, and quite a lot of empirical studies are not in favour of the CAPM.

The innovation of downside risk challenges the classic symmetric risk portfolio theory, and allowing beta to be time-varying relaxes the assumption of a constant beta in the CAPM. There are only a few studies which focus on downside risk in a time-varying context, and this thesis builds on these studies by innovatively using non-linear models to measure beta.

Chapter 2

The cross-sectional determinants of US stock returns: The impact of downside risk

2.1 Introduction

This chapter investigates the relationship between the downside risk and US stock returns, with a comparison to UK data. Since the CAPM was first proposed, it has been widely believed that the expected excess return on a stock is constantly proportional to its market beta, irrespective of how the market excess return has fluctuated. However, through exploration of market movements and stock returns, researchers have found that stock returns did not react symmetrically to market movements. Some stocks tend to move upward in a rising market more than falling in a dropping market, while some stock returns tend to move downward in a falling market much more than moving upward in a rising market. It follows that such stocks are not as desirable as others since the average payoffs would be low. In this chapter, it will be demonstrated that the positive impact of downside risk is reflected in the cross-section of stock returns, while when beta is controlled, a downside negative impact on stock returns will appear on future stock returns. This chapter is arranged in the following order: section 2.2 introduces the literature on downside risk, section 2.3 contains an outline of a downside risk model, section 2.4 describes the data, section 2.5 presents the empirical results of the US data,[7] section 2.6 and 2.7 present the cross-sectional relationship between downside beta and other factors, section 2.8 examines the predictive power of downside beta, and the final section concludes the chapter.

[7]The empirical results of the UK data are presented when a comparison is necessary.

2.2 Literature review

There has been a long history of using downside risk measures in portfolio analysis, specifically, treating risk asymmetrically represents a vast improvement over traditional portfolio theory. The most commonly accepted downside risk measures are semi-variance and the lower partial moment (Nawrocki 1999). There are a number of studies that focus on the asymmetry of risks, particularly on downside losses rather than upsides gains. Roy (1952) pointed out that investors care more about downside risk than upside gains. Markowitz (1959) was the first to propose that semi-variance can be adopted as a risk measure instead of variance, since semi-variance draws more attention to downside losses. Consequently, a large number of studies in exploring the theoretical application of semi-variance have appeared. For instance, the theoretical superiority of semi-variance compared to variance is illustrated in Quirk and Saposnik (1962). Mao (1970) shows that investors would be more interested in downside risk rather than upside gains, and semi-variance is the appropriate tool to measure the downside risk. In the mid-1970s, another measure of downside risk, the lower partial moment was proposed. According to Bawa (1975) and Fishburn (1977), the lower partial moment can liberate investors from risk seeking to risk neutral and finally to risk aversion. Bawa (1975), firstly defined the lower partial moment as a below target semi-variance, which connects the lower partial moment with the semi-variance measure.

Following the introduction of these two measures of downside risk, a large number of empirical tests have been conducted. Among them, quite a number of researchers have attempted to combine the measures of downside risk with the original asset pricing model and the investors' utility function. This, in turn, has led to new measures of downside risk. Bawa and Lindenberg (1977) suggested a modified CAPM that treats risk asymmetrically with respect to downside and upside sensitivities. Under the framework of behavioural finance, Kahneman and Tversky (1979) proposed the loss aversion preference. Gul (1991) advanced the disappointment aversion preference theory. According to disappointment aversion preference theory, risk averse investors would demand a premium to compensate the downside risk they are bearing when the market is falling. It is even more obvious in the disappointment aversion utility function that more weights are put on downside losses. Sing and Ong (1993) proposed the co-lower partial moment, and extended it to the classic CAPM. Ang and Chen (2002) proposed downside conditional correlations as a new measure of downside risk. Moreover, Nielsen et al. (2008) proposed downside realized semivariance as a new measure of downside risk, which is constructed using high frequency data.

Even though the perception of downside risk began as early as the 1950s, there have been few empirical studies that focus on downside risk as being a factor rather than a factor loading. Early studies discovered little evidence of how downside risk is priced and the downside risk premium, since researchers did not pay much attention to evaluating the premium cross-sectionally. For instance, Jahankani (1976) failed to discover any enhancement on the standard CAPM with normal betas replaced by downside betas, however, the portfolios used in his study were sorted only by classic

CAPM betas. Similarly, Harlow and Rao (1989) failed to estimate the downside risk premium, instead, they measured the downside risk under the maximum likelihood framework, with only the consistency of returns to risk-free assets tested across all portfolios. Neither of these studies illustrated how stocks that closely co-vary with a declining market would obtain a corresponding risk premium.

However, Ang et al. (2006) successfully demonstrated that stocks with higher downside risk obtain higher average returns and that downside risk is a significant risk factor affecting stock returns. Moreover, they estimated the downside risk premium at 6% per annum in cross section. While Huang and Hueng (2008) argue that in a downside market, there is a significant and negative risk-return relationship. This chapter follows Ang et al.'s (2006) study by using US data, but innovates in a number of ways. Firstly, this chapter examines whether individual stocks that have higher downside beta obtain, on average, higher returns during both the current period and the next period. By contrast, Ang et al. (2006) show that higher returns are obtained only in the same period. Secondly, this chapter shows that the time-varying downside beta is a significant attribute to contemporaneous stock returns by using OLS regression with a moving window approach in a cross-sectional fashion. Thirdly, the downside risk premium is estimated by controlling for other coefficients such as coskewness. The downside beta is then employed as a risk factor in cross-sectional regressions, and its predictive power is tested.

2.3 A model of downside risk

In order to price downside risk, on a theoretical basis, a disappointment aversion (DA) utility function firstly proposed by Gul (1991) is adopted. Use of the DA utility function effectively assumes that investors care differently about downside losses and upside gains, specifically caring more about the former. There are a number of other models existing which focus on investors' aversion to losses, Shumway (1997) advanced a behavioural model according to the level of investors' loss aversion, and Barberis and Huang (2001) developed a cross-sectional equilibrium model based on a risk averse utility function, with a mental accounting factor which formulates investors' loss aversion. More recently, some improvements and constraints have been placed on utility functions. For instance, Chen et al. (2001) added short sale constraints, and Kyle and Xiong (2001) constructed wealth constraints. However, neither of these models directly relate the measurement of downside risk to the cross-sectional stock returns in a perfect market.

Instead of adding too many constraints and behavioural conditions, taking the rational disappointment aversion utility function as a basis to treat risk asymmetrically is the most reasonable way to measure the downside risk in a cross-sectional fashion. The advantage of it is that as the DA function is universally concave, portfolio allocation problems, especially optimal finite portfolio allocation problems are solvable (Ang et al., 2006). The difference between the DA utility function being taken in this chapter and the one in Gul's (1991) study is that the utility function in this chapter is under a rational representative agent framework while the one is Gul's (1991) study only aims to solve the problem in an aggregate market, specifically in a consumption setting (Routledge and Zin, 1993). In this study, wealth is measured by the market portfolio, and all assumptions comply with the CAPM.

Gul's (1991) disappointment aversion utility function is shown below

$$U(\mu_W) = \frac{1}{K}\left(\int_{-\infty}^{\mu_W} U(W)dF(W) + A\int_{\mu_W}^{\infty} U(W)dF(W)\right), \tag{2.1}$$

where *U(W)* is the utility function of wealth *W* by the end of the period, *F(x)* is the cumulative density function of wealth *W*, and μ_W is a certain level of wealth. According to Gul (1991), *U(W)* is set to be a power utility function, that is,

$$U(W) = W^{(1-\gamma)} / (1-\gamma). \tag{2.2}$$

The parameter *A* in equation (2.1) is the disappointment aversion coefficient given $0<A\leq 1$, and *K* is a scalar which is given by

$$K = \Pr(W \leq \mu_W) + A\Pr(W > \mu_W). \tag{2.3}$$

Therefore, if the result is below the μ_W, it is called a disappointing outcome. The reason *A* is between 0 and 1 is that, the disappointing outcomes would take relatively more weight than the contrary outcomes. In other words, the disappointment averse investors care more about downside risk than upside risk. On the other hand, when $A=1$, the disappointment aversion utility function will become the mean-variance utility function (Ang et al., 2006).

As a key component of the mean–variance utility function, the regular beta is given by

$$\beta = \frac{\text{cov}(xR_i, xR_M)}{\text{var}(xR_M)}, \tag{2.4}$$

where xR_i is asset i's excess return, xR_M is the market excess return. Beta could be a powerful parameter to explain and describe the risk-return relationship of each asset, since it is true that each asset's expected return will increase in a rising market and decrease in a declining market when the beta is high. However, since investors would pay more attention to downside risk, the disappointment aversion utility function risk coefficient does not have enough explanatory power on downside risk. To overcome this issue, the downside beta, denoted by β^-, as a measurement of downside risk is introduced by Bawa and Lindenberg (1977).[8] Mathematically, β^- is given by

$$\beta^- = \frac{\text{cov}(xR_i, xR_M \mid xR_M < \overline{xR_M})}{\text{var}(xR_M \mid xR_M < \overline{xR_M})} \quad , \tag{2.5}$$

where $\overline{xR_M}$ is the average market excess return over the sample period.

On the other hand, a DA investor would like to hold a stock with high upside potential payoffs at a relative discount. Compared to downside risk, high upside potential stocks could bring more wealth to investors when the investors' wealth is already high, so that these stocks are not as attractive as the ones which deliver a payoff when the market is declining. Therefore, stocks with high upside potential would not require a high expected return to make investors hold them, which is the reason for the discount. In order to measure the upside risk, an upside beta is introduced, denoted by β^+ (Bawa and Lindenberg 1977). Similarly to downside beta, upside beta is given by

[8] Downside beta measures the co-movement between stock return and return of market portfolio in a falling market. To a stock, the larger downside beta is, and more losses will be suffered in a downside market, and vice versa.

$$\beta^{+} = \frac{\mathrm{cov}(xR_i, xR_M \mid xR_M \geq \overline{xR_M})}{\mathrm{var}(xR_M \mid xR_M \geq \overline{xR_M})} \quad , \tag{2.6}$$

where all notations are consistent with downside beta.

Apart from downside and upside betas, another two statistics; coskewness and cokurtosis (Harvey and Siddique, 2000) are introduced to explain the risk-return relationship. Harvey and Siddique (2000), point out that there is a negative relationship between return and coskewness. Mathematically, coskewness and cokurtosis are given by

$$coskewness = \frac{E[(xR_i - \overline{xR_i})(xR_M - \overline{xR_M})^2]}{\sqrt{\mathrm{var}(xR_i)}\ \mathrm{var}(xR_M)} \quad , \tag{2.7}$$

and

$$cokurtosis = \frac{E[(xR_i - \overline{xR_i})(xR_M - \overline{xR_M})^3]}{\sqrt{\mathrm{var}(xR_i)}\ \mathrm{var}(xR_M)^{3/2}} , \tag{2.8}$$

where $\overline{xR_i}$ is the average excess return of asset *i*, and other notations are consistent with downside beta.[9]

Since the regular beta, upside beta and downside beta are not independent of each other, in order to distinguish the effects among them, two more statistics have been introduced by Ang et al. (2006): the relative upside beta, denoted by $(\beta^{+}-\beta)$ and relative downside beta denoted by $(\beta^{-}-\beta)$. In the subsequent analysis, comparison among the relations between regular beta, upside beta, downside beta, relative upside beta, relative downside beta and coskewness and realized return are summarized.

[9] Coskewness measures the asymmetry of stock return's probability distribution in relation to the distribution of market return, while cokurtosis measures the degree of peak. Since stock return is the main variable to investigate, it always has the order of 1 in the formula.

2.4 Data and data transformations

The US data used in this chapter are taken from the CRSP database. This chapter focuses on the ordinary common stocks listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and NASDAQ measured on a monthly frequency from January 1960 to December 2010.[10] American depositary receipts (ADR), real estate investment trust (REIT), closed-end funds, foreign firms and other securities which do not have a CRSP share code of 10 or 11 are excluded from the sample. Each stock is required to have at least 5 years of consecutive monthly adjusted return observations with at most 5 missing observations. The return of each stock is adjusted for stock splits, mergers and acquisitions, and dividends (dividends are subtracted from stock prices for adjustment), giving 13557 stocks. The value-weighted return of all listed stocks is taken as a measure of the market portfolio, and the one month Treasury bill rate represents the risk free rate.[11]

The UK data used in this chapter are taken from Datastream. With the same requirement as the US data, monthly stock prices of FTSE All Share firms between December 1979 and December 2010, the UK three-month T-bill middle rate, and FTSE All Share price index over the same period are used. For the monthly stock prices, all are adjusted for dividend payments and stock splits, and there are 564 stocks in total.

[10] The NASDAQ data are only available from January 1972.
[11] Using the same criteria Ang et al. (2007) used and no filtering out outliers aims to follow their study as close as possible. Notably, using this criteria could cause survival bias.

In order to conduct the analysis, essential data transformations were prepared on the original data. Firstly, continuously compounded returns are derived from stock prices to estimate and calculate beta, downside beta, upside beta and all other parameters. The transformation is as follows:

$$R_{i,t} = \ln(p_{i,t}) - \ln(p_{i,t-1}), \quad (2.9)$$

where $R_{i,t}$ is the continuously compounded return of stock i at time t, $p_{i,t}$ is the adjusted price of stock i at time t (on monthly frequency), and ln is the natural logarithm. The same transformation applies to the FTSE All Share price index to calculate the return of the market portfolio.

Secondly, the excess return of each stock and the market return are derived by taking the difference of the continuously compounded return and the risk free rate as follows

$$xR_{i,t} = R_{i,t} - R_{f,t}, \quad (2.10)$$

where $xR_{i,t}$ is the excess return of stock i at time t, and $R_{f,t}$ is the risk free rate at time t.

Moreover, $R_{f,t}$ is derived from the UK 3-month T-bill middle rate. Since the original data is in annual percentage terms to each corresponding month, we therefore calculate $R_{f,t}$ as follows:

$$R_{f,t} = {r_{f,t}}/{1200}, \quad (2.11)$$

where $r_{f,t}$ is the annualized return.

A general summary of statistics is shown in the Table 2.1. It can be seen from the table that, overall, the UK stocks yield a lower return and a higher standard deviation than the US stocks. For the UK stocks, the average annualized return is 3.61% with a standard deviation of 22.21%. For the US stocks, generally, over the whole sample, the average annualized return is 12.08% with a standard deviation of 17.06%. Stocks that have been listed in all three stock exchanges yielded highest average annualized return at 14.61%, while stocks that have been listed on both NASDAQ and AMEX are most volatile with a standard deviation of 20.17%.

Table 2.1 A summary of Both UK and US stocks[12]

	Stock Exchange	Number of Stocks	Percentage to whole Sample	Average Annualized Return	Standard Deviation
UK DATA	FTSE ALL SHARES	564	100%	3.61%	22.21%
US DATA	NYSE	2198	16.21%	10.6%	10.99%
	NASDAQ	7636	56.33%	12.32%	20.13%
	AMEX	1105	8.15%	10.99%	16.92%
	NYSE & NASDAQ	1031	7.60%	14.12%	13.55%
	NYSE & AMEX	556	4.10%	14.10%	13.72%
	NASDAQ & AMEX	829	6.11%	11.24%	20.17%
	NYSE & NASDAQ & AMEX	202	1.49%	14.61%	16.15%
	Total Sample	13557	100%	12.08%	17.06%

2.5 Empirical results[13]

To demonstrate the relationship between annual realized returns of stocks and various types of betas, results are summarized in subsequent tables. All the realized betas, realized downside betas, and realized upside betas are estimated by OLS, while

[12] The average annual risk free rates are 5.43% and 7.11% for US and UK, respectively.
[13] All data transformation, computation and empirical work are done by Stata 11.2 MP

realized relative downside betas, realized relative upside betas and realized downside beta less upside betas are generated subsequently.

When betas are calculated, a moving window method is adopted. For both the US and the UK data, a 3 year window is employed to calculate each beta for individual stocks, so when the first beta of that stock is calculated, the next beta of that stock is calculated by moving the window forward by one month. When all types of betas are calculated, each type is sorted into five portfolios according to their values each month. Specifically, all stocks are cross-sectionally sorted into five quintiles according to different types of corresponding beta measurements at each point of time. The low beta portfolios contain the stocks which have the lowest 20% betas among all stocks each month, and the other four portfolios contain the stocks falling into 20% - 40%, 40% - 60%, 60% - 80% and 80% - 100% beta measurement intervals, respectively. After the portfolios have been constructed, the equally weighted average beta for the portfolios are calculated and assigned to the beta of the portfolio. To demonstrate the sample period impact and predictive power of betas, both the same period and the following year average annual excess return of each portfolio are calculated.

2.5.1 Empirical results: US data

For the US stocks, it can be seen from Table 2.2 Panel 1 that when stocks are sorted by β, average annual stock excess returns increase along with the increase of β.

Table 2.2 US Stocks Sorted By Factor Loadings With Average Market Excess Return As A Benchmark

This table presents the relationship between excess stock returns and factor loading of stock market portfolio with average market excess return as a benchmark. The column labeled "return" reports the average annualized stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β^-	β^+
1 Low	5.44%	0.33	0.58	0.01
2	7.90%	0.78	0.99	0.54
3	9.09%	1.07	1.24	0.84
4	10.88%	1.37	1.46	1.20
5 High	15.03%	2.06	1.92	2.17
High - Low	9.59%	1.74	1.34	2.15

Panel 2 Stocks Sorted by β^-

Portfolio	Return	β	β^-	β^+
1 Low	7.28%	0.59	0.24	0.54
2	9.12%	0.88	0.86	0.74
3	10.09%	1.11	1.18	0.98
4	10.35%	1.33	1.54	1.11
5 High	11.49%	1.71	2.37	1.38
High - Low	4.21%	1.11	2.13	0.84

Panel 3 Stocks Sorted by β^+

Portfolio	Return	β	β^-	β^+
1 Low	6.57%	0.68	1.09	-0.61
2	9.03%	0.81	1.01	0.42
3	9.92%	1.03	1.16	0.87
4	10.44%	1.28	1.31	1.37
5 High	12.38%	1.81	1.61	2.71
High - Low	5.81%	1.12	0.52	3.32

Panel 4 Stocks Sorted by relative β^-

Portfolio	Return	β	β^-	β^+
1 Low	12.12%	1.45	0.73	1.70
2	10.57%	1.14	1.01	1.10
3	9.71%	1.02	1.12	0.87
4	9.15%	0.99	1.33	0.69
5 High	6.78%	1.01	1.99	0.40
High - Low	-5.34%	-0.45	1.26	-1.29

Panel 5 Stocks Sorted by relative β^+

Portfolio	Return	β	β^-	β^+
1 Low	8.24%	1.1	1.48	-0.39
2	10.39%	1.02	1.22	0.55
3	10.17%	1.00	1.12	0.87
4	9.57%	1.06	1.09	1.23
5 High	9.96%	1.43	1.27	2.49
High - Low	1.72%	0.32	-0.21	2.88

Panel 6 Stocks Sorted by $(\beta^- - \beta^+)$

Portfolio	Return	β	β^-	β^+
1 Low	10.92%	1.47	0.97	2.39
2	10.09%	1.09	1.02	1.22
3	9.87%	1.01	1.12	0.88
4	10.12%	1.00	1.30	0.57
5 High	7.33%	1.03	1.77	-0.31
High - Low	-3.59%	-0.44	0.8	-2.70

The span of returns is from 5.44% to 15.03% with a spread of 9.59%. At the same time, β^- and β^+ present the same pattern as β, the span of β^- and β^+ are from 0.58 to 1.92 and 0.01 to 2.17 with a spread of 1.34 and 2.15, respectively.

When stocks are sorted by β^-, it can be seen from Panel 2 that the average excess return, β, and β^+ all exhibit a very similar increasing pattern as in Panel 1. The average excess return increases from 7.28% in portfolio 1 to 11.49% in portfolio 5. Notably, although Panel 2 shows a similar pattern to Panel 1, the spread of average excess returns in Panel 2 is 4.21%, much less than that in Panel 1 (9.59%). Similarly, in Panel 3, when stocks are sorted by β^+, an increasing pattern appears for each portfolio, the average excess return grows from 6.57% to 12.38% with a spread of 5.81%. Noticeably, the spread of β^- decreases dramatically when stocks are sorted by β^+, it is only 0.52 in Panel 3, while the spread of β^+ also decreases when stocks are sorted by β^-, for instance in Panel 2, the spread of β^+ is only 0.84. At this stage, β^+ shows a positive relationship with β, when stocks are sorted by both estimates, the returns exhibit a very similar pattern with a high spread. Regarding β^-, further research needs to be done, since stock returns show a similar pattern but have a lower spread. Additionally, it suggests that β^- and β^+ exhibit a negative relationship since the stock returns show reverse patterns when sorted by each of them.

In order to examine the unique properties of β^- and β^+, stocks are sorted by relative β^-, denoted by $(\beta^- - \beta)$ and relative β^+, denoted by $(\beta^+ - \beta)$, both of them are controlled for the effect of β. It can be seen from Panel 4 that when stocks are sorted by $(\beta^- - \beta)$, only

β^- shows an increasing pattern from 0.73 in portfolio 1 to 1.99 in portfolio 5, while both average excess returns and β^+ exhibit reverse patterns from portfolio 1 to portfolio 5. Moreover, β exhibits a generally decreasing trend from portfolio 1 to portfolio 4 but with a subtle increase in portfolio 5. By contrast, when stocks are sorted by $(\beta^+ - \beta)$, it is clear in Panel 5 that β^+ increases from -0.39 in portfolio 1 to 2.49 in portfolio 5. Generally, in Panel 5, average excess returns and β increase and β^- shows a decreasing trend, all are reverse versions of Panel 4. Finally, when stocks are sorted by $(\beta^- - \beta^+)$, patterns similar to Panel 4 appear. It can be seen from Panel 6 that only β^- shows an increasing pattern from 0.97 in portfolio 1 to 1.77 in portfolio 5, while both average excess returns and β^+ exhibit decreasing patterns from portfolio 1 to portfolio 5. And β exhibits a generally decreasing trend from portfolio 1 to portfolio 4 but with a slight increase in portfolio 5.

It can be concluded from Table 2.2 that the average excess returns are consistent with the classic high beta high return relationship when stocks are sorted by β. Similar relationships still hold when stocks are sorted by β^- and β^+, and all of these results are consistent with Ang et al’s (2006) findings. However, there are some results which contradict Ang et al’s (2006) findings: the spread of average excess returns drops when stocks are sorted by β^- while it increased in Ang et al’s (2006) study. When controlling for β, the unique properties of β^- and β^+ become more obvious. Clearly from Panel 4, Panel 5 and Panel 6, β^- exhibits a negative relationship with average excess return, while β^+ has a positive relationship which is consistent with β. Whereas in Ang et al’s (2006) study, a totally reversed result is found.

In order to explore the downside risk component at a deeper level, inspired by Ang et al.'s (2006) study, rather than using average market excess return as a benchmark to compute the downside and upside beta, two more benchmarks are employed. The motivation for changing the benchmark is to examine the sensitivity of stock returns to beta, downside beta and upside beta. First, the risk free rate is employed instead of the average market excess return, therefore the corresponding downside and upside beta measures can be written as

$$\beta_{Rf}^{-} = \frac{\operatorname{cov}(xR_i, xR_M \mid xR_M < R_f)}{\operatorname{var}(xR_M \mid xR_M < R_f)} \quad , \tag{2.12}$$

and

$$\beta_{Rf}^{+} = \frac{\operatorname{cov}(xR_i, xR_M \mid xR_M \geq R_f)}{\operatorname{var}(xR_M \mid xR_M \geq R_f)} \quad . \tag{2.13}$$

The second new benchmark employed assumes that the market excess return equals zero, then the corresponding downside and upside beta can be written as

$$\beta_{0}^{-} = \frac{\operatorname{cov}(xR_i, xR_M \mid xR_M < 0)}{\operatorname{var}(xR_M \mid xR_M < 0)} \quad , \tag{2.14}$$

and

$$\beta_{0}^{+} = \frac{\operatorname{cov}(xR_i, xR_M \mid xR_M \geq 0)}{\operatorname{var}(xR_M \mid xR_M \geq 0)} \quad , \tag{2.15}$$

where all previous notations remain the same.

Following the same method of Table 2.2, Table 2.3 and Table 2.4 present the risk-return relationship with β_{Rf}^{-} and β_{Rf}^{+} , and β_{0}^{-} and β_{0}^{+} employed, [14]

[14] Moving window approach is employed.

respectively. Surprisingly, although the benchmarks have changed, Table 2.3 and Table 2.4 exhibit almost identical patterns as in each panel in Table 2.2. It can be concluded from Table 2.2 to Table 2.4 that three types of downside (and upside) betas have the same impact on stock returns, with the average excess return always consistent with the classic high beta high return relationship when stocks are sorted by β. This relationship still holds when stocks are sorted by any of the downside beta and upside beta measures, but the spread of average excess returns drops when stocks are sorted by downside beta. When controlling for β, from Panel 4, Panel 5 and Panel 6 of Table 2.2 to Table 2.4, downside betas exhibit a negative relationship with average excess return, while upside beta has a positive relationship regardless of the benchmark used. To sum up, the risk-return relationship does not change with the measure of beta. Compared to Ang et al's (2006) study, results are consistent when stocks are sorted by beta, downside beta and upside beta. However, when beta is controlled, the unique effects of downside beta and upside beta on stock returns are presented. Unlike Ang et al's (2006) results, in this study, relative downside and upside beta have a negative and a positive impact on stock returns, respectively.

Table 2.3 US Stocks Sorted By Factor Loadings With Average Risk-Free Rate As Benchmark

This table presents the relationship between excess stock returns and factor loading of stock market portfolio with average risk-free rate as a benchmark. The column labeled "return" reports the average annual stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β_{Rf}^-	β_{Rf}^+
1 Low	5.44%	0.33	0.63	0.05
2	7.90%	0.78	1.01	0.55
3	9.09%	1.07	1.26	0.85
4	10.88%	1.37	1.48	1.22
5 High	15.03%	2.06	1.97	2.20
High - Low	9.59%	1.74	1.33	2.15

Panel 2 Stocks Sorted by β_{Rf}^-

Portfolio	Return	β	β_{Rf}^-	β_{Rf}^+
1 Low	7.23%	0.59	0.23	0.56
2	9.41%	0.89	0.87	0.78
3	9.79%	1.12	1.20	0.99
4	10.50%	1.33	1.57	1.12
5 High	11.42%	1.68	2.49	1.41
High - Low	4.19%	1.09	2.26	0.85

Panel 3 Stocks Sorted by β_{Rf}^+

Portfolio	Return	β	β_{Rf}^-	β_{Rf}^+
1 Low	6.37%	0.66	1.14	-0.53
2	9.19%	0.82	1.04	0.43
3	10.12%	1.04	1.18	0.88
4	10.43%	1.28	1.34	1.38
5 High	12.21%	1.81	1.66	2.72
High - Low	5.84%	1.15	0.52	3.25

Panel 4 Stocks Sorted by Relative β_{Rf}^-

Portfolio	Return	β	β_{Rf}^-	β_{Rf}^+
1 Low	12.07%	1.45	0.72	1.72
2	10.41%	1.12	1.00	1.08
3	9.84%	1.03	1.13	0.87
4	8.90%	1.00	1.36	0.71
5 High	7.11%	1.01	2.13	0.49
High - Low	-4.96%	-0.44	1.41	-1.24

Panel 5 Stocks Sorted by Relative β_{Rf}^+

Portfolio	Return	β	β_{Rf}^-	β_{Rf}^+
1 Low	8.35%	1.11	1.56	-0.28
2	10.23%	1.01	1.23	0.55
3	10.35%	0.99	1.12	0.87
4	9.59%	1.07	1.12	1.24
5 High	9.81%	1.43	1.32	2.50
High - Low	1.46%	0.32	-0.24	2.78

Panel 6 Stocks Sorted by $(\beta_{Rf}^- - \beta_{Rf}^+)$

Portfolio	Return	β	β_{Rf}^-	β_{Rf}^+
1 Low	10.92%	1.46	0.94	2.37
2	10.08%	1.10	1.04	1.23
3	9.85%	1.01	1.14	0.89
4	9.85%	1.01	1.32	0.58
5 High	7.64%	1.03	1.91	-0.20
High - Low	-3.28%	-0.44	0.97	-2.56

Table 2.4 US Stocks Sorted By Factor Loadings With Zero Return As Benchmark

This table presents the relationship between excess stock returns and factor loading of stock market portfolio with zero return as a benchmark. The column labeled "return" reports the average annual stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β_0^-	β_0^+
1 Low	5.44%	0.33	0.62	0.05
2	7.90%	0.78	1.00	0.55
3	9.09%	1.07	1.26	0.86
4	10.88%	1.37	1.47	1.22
5 High	15.03%	2.06	1.94	2.18
High - Low	9.59%	1.74	1.32	2.14

Panel 2 Stocks Sorted by β_0^-

Portfolio	Return	β	β_0^-	β_0^+
1 Low	6.99%	0.62	0.19	0.57
2	9.70%	0.90	0.87	0.80
3	9.84%	1.11	1.19	0.97
4	10.53%	1.33	1.57	1.13
5 High	11.27%	1.66	2.49	1.38
High - Low	4.28%	1.04	2.30	0.80

Panel 3 Stocks Sorted by β_0^+

Portfolio	Return	β	β_0^-	β_0^+
1 Low	5.86%	0.64	1.12	-0.48
2	9.23%	0.81	1.04	0.44
3	10.08%	1.05	1.18	0.89
4	10.21%	1.29	1.33	1.36
5 High	12.96%	1.83	1.63	2.64
High - Low	7.10%	1.19	0.51	3.12

Panel 4 Stocks Sorted by Relativeβ_0^+

Portfolio	Return	β	β_0^-	β_0^+
1 Low	11.76%	1.46	0.67	1.68
2	10.21%	1.13	1.00	1.09
3	9.98%	1.04	1.14	0.89
4	9.12%	0.99	1.35	0.71
5 High	7.27%	1.00	2.14	0.48
High - Low	-4.49%	-0.45	1.47	-1.20

Panel 5 Stocks Sorted by Relative β_0^+

Portfolio	Return	β	β_0^-	β_0^+
1 Low	7.91%	1.09	1.55	-0.23
2	9.81%	1.03	1.25	0.59
3	10.56%	0.99	1.12	0.87
4	9.74%	1.07	1.11	1.22
5 High	10.33%	1.43	1.28	2.40
High - Low	2.42%	0.34	-0.27	2.64

Panel 6 Stocks Sorted by (β_0^- - β_0^+)

Portfolio	Return	β	β_0^-	β_0^+
1 Low	11.20%	1.47	0.88	2.27
2	9.91%	1.10	1.02	1.22
3	10.02%	1.02	1.13	0.89
4	9.77%	1.00	1.33	0.60
5 High	7.44%	1.02	1.94	-0.12
High - Low	-3.76%	-0.45	1.06	-2.40

2.5.2 Empirical results: UK data

Whole sample analysis

In order to expand Ang et al's (2006) study and to provide a comparison, the relationship between beta measures and average annual excess returns using UK data is examined. Dimson et al (2003) and Gregory (2011) focus on UK stock market risk premium but not particularly considering downside market. Unlike the US data, the UK sample is shorter with a number of bear market periods such as the 1989 market crash, the Dot com bubble and the sub-prime crisis. Overall, the UK sample is mostly characterized by bear markets, therefore unusual results are expected, and the time sensitivity of downside and upside beta is observed.

It can be seen from Table 2.5 that when the largest data sample is used, from January 1980 to December 2010, some unusual results between realized return and beta measurements occur. According to the conventional definition of the risk–return relationship, high beta should generate high returns, and vice versa. However, Table 2.5 illustrates the opposite result.

When stocks are sorted by conventional beta, average beta estimates for the lowest beta portfolio is 0.32, while the one for the highest beta portfolio is 1.79, the average beta estimates go up at relatively stable intervals from low to high, and the spread between the highest beta estimate and lowest one is 1.47. Although these beta estimates are smaller than expected, they are still within a reasonable range.

Table 2.5 UK Stocks Sorted By Factor Loadings (Jan 1980-Dec 2010)

This table presents the relationship between excess stock returns and factor loading of stock market portfolio with average market excess return as a benchmark. The sample uses FTSE All Shares from January 1980 to December 2010. The column labeled "return" reports the average annual stock returns over three month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β^-	β^+
1 Low	1.94%	0.32	0.51	0.36
2	0.66%	0.75	1.07	1.05
3	0.39%	1.00	1.39	1.45
4	0.35%	1.24	1.71	1.82
5 High	-2.82%	1.79	2.42	2.71
High - Low	-4.76%	1.47	1.91	2.36

Panel 2 Stocks Sorted by β^-

Portfolio	Return	β	β^-	β^+
1 Low	1.24%	0.36	0.43	0.62
2	-0.06%	0.77	1.04	1.14
3	0.57%	1.01	1.39	1.47
4	-0.06%	1.23	1.73	1.75
5 High	-1.19%	1.73	2.50	2.41
High - Low	-2.43%	1.37	2.07	1.79

Panel 3 Stocks Sorted by β^+

Portfolio	Return	β	β^-	β^+
1 Low	3.80%	0.41	0.81	0.19
2	0.60%	0.78	1.15	1.00
3	0.34%	1.00	1.38	1.45
4	-0.51%	1.21	1.62	1.87
5 High	-3.72%	1.70	2.14	2.88
High - Low	-7.52%	1.29	1.33	2.69

Panel 4 Stocks Sorted by Relative β^-

Portfolio	Return	β	β^-	β^+
1 Low	0.30%	0.66	0.65	1.41
2	-1.19%	0.86	1.11	1.37
3	-0.31%	1.01	1.39	1.49
4	1.04%	1.15	1.67	1.56
5 High	0.65%	1.41	2.28	1.55
High - Low	0.35%	0.75	1.63	0.14

Panel 5 Stocks Sorted by Relative β^+

Portfolio	Return	β	β^-	β^+
1 Low	3.65%	0.66	1.28	0.34
2	1.69%	0.83	1.25	1.03
3	0.61%	0.99	1.37	1.44
4	-1.14%	1.14	1.47	1.83
5 High	-4.28%	1.48	1.71	2.74
High - Low	-7.93%	0.82	0.43	2.39

Panel 6 Stocks Sorted by $(\beta^- - \beta^+)$

Portfolio	Return	β	β^-	β^+
1 Low	-3.62%	1.24	1.34	2.45
2	-0.91%	1.05	1.34	1.72
3	0.73%	0.98	1.35	1.43
4	1.40%	0.91	1.38	1.14
5 High	2.90%	0.91	1.67	0.65
High - Low	6.53%	-0.33	0.33	-1.79

The average downside beta estimates and upside beta estimates of these 5 portfolios follow the direction of the change of conventional beta estimates from low to high. Surprisingly, when it comes to the realized returns, the average realized returns for each portfolio are in the reverse order of the corresponding beta estimates. The lowest beta portfolio generates a rate of return of 1.94% per annum, while the highest beta portfolio suffered a loss of 2.82% per annual (the spread from high to low is -4.76%). The realized returns show a decreasing trend from low beta portfolio to high beta portfolio.

The same phenomenon appears again when stocks are sorted by downside beta, upside beta and relative upside beta. When stocks are sorted by relative downside beta, the returns exhibit a U-shaped pattern from the low beta portfolio to high beta portfolio. The annual return for the lowest relative downside beta portfolio is 0.3%, and then drops below zero along with the increase of the relative downside beta. It then comes back above zero and finally yields at 0.65% per annum. Although there is a tiny fall between the annual returns of the second highest relative downside beta portfolio and the highest one, a U-shaped pattern of their returns is still clearly observed.

An upward trend of annual returns from low beta to high beta portfolio is finally apparent when stocks are sorted by downside beta less upside beta $(\beta^- - \beta^+)$. The annual return for the low $(\beta^- - \beta^+)$ portfolio is -3.62% and increasing along with the growth of $(\beta^- - \beta^+)$, it closes at 2.9% for the high $(\beta^- - \beta^+)$ portfolio with a spread between high to low at 6.53%. Overall, Table 2.5 illustrates a surprising risk-return relationship, the pattern of returns is totally contradictory to the conventional risk–return theory, and a non-linear relationship occurs when stocks are sorted by relative downside beta. In addition, the values of returns are smaller than expected when the sample from January 1980 to December 2010 is chosen.[15]

Sub-period analysis

In order to examine in more detail the risk–return relationship and time sensitivity of downside risk (especially when a relatively small sample contains a number of crises is used), a number of shorter and reshaped samples are chosen. Table 2.6 gives results pertaining to January 1980 to December 2007. The reason for analyzing this period is the sub-prime crisis. Taking out the data from January 2008 to December 2010 from the original data set aims to remove the influence of the global financial crisis. It can be seen from Table 2.6 that after shortening the sample, the realized return series in each panel of Table 2.6 becomes more realistic, with the average highest annual rate of return across all the panels at around 6%, rather than close to zero as in Table 2.5. However, focusing on the risk–return relationship, the results do not follow the classic portfolio theory, but exhibit similar patterns as in Table 2.5. When stocks are sorted by relative downside beta, the returns demonstrate a U-shaped pattern from the low beta portfolio to high beta portfolio. The annual rate of return of the lowest relative downside beta portfolio is 4.19%, and then falls dramatically to 1.9% when the relative downside beta increases to the next level. Then, the rate of return starts going

[15] Average annual return of each portfolio is expected to be similar to the US one.

up from the median beta portfolio at 2.36% and finally closes at 2.53%. Although there is a 0.9% drop between the annual returns of the second highest relative downside beta portfolio and the highest one, a U-shaped pattern in their returns is still clearly observed.

Overall, Table 2.6 shows a similar risk and return relationship as Table 2.5, the pattern of returns is still contradictory to the conventional risk–return theory, and a U-shaped pattern of returns occurs when stocks are sorted by relative downside beta. Moreover, in Table 2.6, the values of returns are much greater than the ones in Table 2.5 and are quite close to what excess returns are expected, this can be attributed to the removal of the sub-prime crisis period.

Comparing Table 2.5 to Table 2.6, it is clear that although shortening the sample size still cannot give an expected result, the annual rate of returns appears more normal. To further analyze the risk–return relationship in light of Table 2.5 and Table 2.6, all financial crises are excluded and a number of subsamples are considered.

Table 2.6 UK Stocks Sorted By Factor Loadings (Jan 1980-Dec 2007)

This table presents the relationship between excess stock returns and factor loading of stock market portfolio with average market excess return as a benchmark. The sample uses FTSE All Shares from January 1980 to December 2007. The column labeled "return" reports the average annual stock returns over three month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β^-	β^+
1 Low	5.95%	0.30	0.49	0.31
2	3.91%	0.73	1.05	0.98
3	3.00%	0.98	1.37	1.38
4	2.49%	1.20	1.66	1.75
5 High	-0.93%	1.74	2.36	2.64
High - Low	-6.88%	1.44	1.87	2.33

Panel 2 Stocks Sorted by β^-

Portfolio	Return	β	β^-	β^+
1 Low	5.43%	0.35	0.41	0.56
2	3.11%	0.74	1.02	1.07
3	3.01%	0.98	1.36	1.41
4	2.34%	1.19	1.68	1.67
5 High	0.51%	1.68	2.45	2.34
High - Low	-4.92%	1.33	2.04	1.78

Panel 3 Stocks Sorted by β^+

Portfolio	Return	β	β^-	β^+
1 Low	7.38%	0.39	0.78	0.15
2	3.64%	0.75	1.12	0.94
3	3.01%	0.97	1.35	1.38
4	1.97%	1.18	1.58	1.80
5 High	-1.59%	1.66	2.09	2.78
High - Low	-8.98%	1.27	1.31	2.64

Panel 4 Stocks Sorted by Relative β^-

Portfolio	Return	β	β^-	β^+
1 Low	4.19%	0.64	0.62	1.33
2	1.90%	0.84	1.09	1.32
3	2.36%	0.99	1.37	1.43
4	3.41%	1.12	1.63	1.50
5 High	2.53%	1.36	2.22	1.47
High - Low	-1.67%	0.72	1.60	0.14

Panel 5 Stocks Sorted by Relative β^+

Portfolio	Return	β	β^-	β^+
1 Low	6.58%	0.62	1.25	0.30
2	4.50%	0.79	1.21	0.97
3	3.32%	0.96	1.33	1.37
4	1.56%	1.11	1.44	1.75
5 High	-1.55%	1.46	1.69	2.65
High - Low	-8.12%	0.84	0.44	2.35

Panel 6 Stocks Sorted by $(\beta^- - \beta^+)$

Portfolio	Return	β	β^-	β^+
1 Low	-0.66%	1.22	1.32	2.36
2	2.13%	1.03	1.32	1.66
3	3.35%	0.95	1.32	1.37
4	4.12%	0.87	1.34	1.07
5 High	5.46%	0.87	1.63	0.60
High - Low	6.11%	-0.35	0.32	-1.76

Table 2.7 provides results pertaining to the risk–return relationship of sorted portfolios. The data used are consistent with the ones used in Table 2.6, but exclude the sub-prime crisis (January 2008 to December 2010), the October 1989 stock market crash, and the Dot com bubble crash (March 2000 to October 2002). Bredin et al. (2007), Gregoriou et al. (2009) and Nneji et al. (2011) also focus on the crash periods on UK equity market, however neither of their studies specifically relates to the downside risk. When the stocks are sorted by beta, there is a downward pattern on the returns along with an increase of beta. The downward pattern is similar to the ones in Table 2.5 and Table 2.6, however, the spread of returns between the highest beta portfolio and the lowest beta portfolio is much narrower than those in previous two tables (it is only -2.98%, compared to -4.76% in Table 2.5 and -6.88% in Table 2.6).

Comparing the return of the lowest beta portfolio in panel 1 of Table 2.7 with the one in Table 2.6, the difference is very small. However, the return of the highest beta portfolio in panel 1 in both tables are obviously far away from each other (the one for Table 2.7 is 2.98% per annum, and the one for Table 2.6 is only -0.93% per annum), the reason for the spread becoming narrower is the increase in the return of the highest beta portfolio. Moreover, the returns of the three middle beta portfolios in Panel 1 of Table 2.7 are all relatively higher than those in Table 2.6. Though not as high as expected, the increase in the return of the highest beta portfolio after discarding abnormal stock price movements, does tend to change the return pattern. Apart from Panel 1, the other panels in Table 2.7 show a fairly similar pattern in the risk–return relationship as in Table 2.6.

Table 2.7 UK Stocks Sorted By Factor Loadings (Excluding Financial Crises)

This table presents the relationship between excess stock returns and factor loading of stock market portfolio with average market excess return as a benchmark. The sample uses FTSE All Shares January 1980 to December 2010 excluding the market crash (October 1989), the Dot com bubble (March 2000 to October 2002) and sub-prime crisis (January 2008 to December 2010). The column labeled "return" reports the annual average stock returns over three month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β^-	β^+
1 Low	5.55%	0.31	0.53	0.31
2	5.25%	0.73	1.08	0.99
3	4.19%	0.98	1.40	1.38
4	3.10%	1.21	1.71	1.73
5 High	2.57%	1.71	2.40	2.53
High - Low	-2.98%	1.40	1.87	2.23

Panel 2 Stocks Sorted by β^-

Portfolio	Return	β	β^-	β^+
1 Low	5.10%	0.36	0.45	0.56
2	4.43%	0.76	1.05	1.08
3	4.17%	0.98	1.40	1.39
4	3.41%	1.20	1.73	1.68
5 High	3.54%	1.65	2.49	2.22
High - Low	-1.56%	1.29	2.05	1.66

Panel 3 Stocks Sorted by β^+

Portfolio	Return	β	β^-	β^+
1 Low	7.19%	0.39	0.82	0.17
2	5.01%	0.75	1.16	0.95
3	3.99%	0.98	1.40	1.37
4	3.00%	1.19	1.63	1.78
5 High	1.47%	1.64	2.12	2.67
High - Low	-5.73%	1.25	1.30	2.50

Panel 4 Stocks Sorted by Relative β^-

Portfolio	Return	β	β^-	β^+
1 Low	4.32%	0.65	0.65	1.28
2	3.43%	0.84	1.11	1.28
3	3.24%	0.99	1.40	1.42
4	4.54%	1.12	1.67	1.50
5 High	5.11%	1.35	2.29	1.46
High - Low	0.79%	0.70	1.64	0.18

Panel 5 Stocks Sorted by Relative β^+

Portfolio	Return	β	β^-	β^+
1 Low	6.98%	0.63	1.32	0.33
2	5.55%	0.80	1.26	0.98
3	4.08%	0.96	1.37	1.36
4	2.64%	1.11	1.48	1.73
5 High	1.41%	1.43	1.69	2.54
High - Low	-5.57%	0.80	0.37	2.20

Panel 6 Stocks Sorted by $(\beta^- - \beta^+)$

Portfolio	Return	β	β^-	β^+
1 Low	1.94%	1.18	1.29	2.22
2	2.93%	1.02	1.33	1.62
3	4.43%	0.96	1.36	1.36
4	5.35%	0.89	1.40	1.09
5 High	6.00%	0.90	1.74	0.66
High - Low	4.06%	-0.28	0.45	-1.57

Moreover, when stocks are sorted by relative downside beta, there is an obvious U-shaped return pattern without the small drop which appears in both Table 2.5 and Table 2.6 for the highest relative beta portfolio. Furthermore, when stocks are sorted by *(β^{-} -β^{+})*, the upward pattern of returns appear as in Table 2.5 and Table 2.6. However, the spreads between the return of the highest beta portfolio and the lowest beta portfolio in each panel in Table 2.7 are much less than the corresponding ones in Table 2.6 except when stocks are sorted by *(β^{-} -β^{+})*. However, the returns for the lowest beta measurement portfolio in both tables are quite close to each other. In other words, the narrowing of the spreads is due to the increase in returns to the high beta measurement portfolio. The only change made on the data set is discarding the crisis period, therefore, it can be concluded that bear market periods do have more impact on high beta stocks than low ones.

To explore the impact of crisis period stock price movements on the risk–return relationship, an analysis specific to the Dot com bubble over the period from March 2000 to October 2002 is conducted. The results of this analysis are shown in Table 2.8. Not surprisingly, over this period, on average, stocks suffered huge losses. When stocks are sorted by beta, downside beta, upside beta, relative downside beta and relative upside beta, none of the portfolios generate a positive rate of return, and the spreads between high beta and low beta measurements are quite wide, from -18.77% to -29.78%. In these 5 panels, it can be seen that the highest beta measurement portfolio generates the lowest rate of return. When stocks are sorted by conventional beta, the highest beta portfolio generates the lowest rate of return among all portfolios.

Table 2.8 UK Stocks Sorted By Factor Loadings (Mar 2000-Oct 2002)

This table presents the relationship between excess stock returns and factor loading of stock market portfolio with average market excess return as a benchmark. The sample uses FTSE All Shares during the Dot com bubble (March 2000-October 2002). The column labeled "return" reports the annual average stock returns over three month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β^-	β^+
1 Low	-1.64%	0.11	0.23	-0.05
2	-13.97%	0.60	0.88	0.82
3	-17.47%	0.94	1.30	1.44
4	-19.04%	1.27	1.71	2.06
5 High	-31.44%	2.04	2.64	3.59
High - Low	-29.79%	1.93	2.41	3.64

Panel 2 Stocks Sorted by β^-

Portfolio	Return	β	β^-	β^+
1 Low	-2.39%	0.14	0.19	0.23
2	-13.58%	0.61	0.86	0.89
3	-17.19%	0.95	1.30	1.47
4	-21.62%	1.25	1.73	1.94
5 High	-28.79%	2.01	2.69	3.32
High - Low	-26.40%	1.86	2.50	3.09

Panel 3 Stocks Sorted by β^+

Portfolio	Return	β	β^-	β^+
1 Low	-4.13%	0.19	0.49	-0.23
2	-13.21%	0.62	0.93	0.79
3	-16.67%	0.94	1.29	1.46
4	-18.86%	1.24	1.62	2.12
5 High	-30.69%	1.97	2.43	3.72
High - Low	-26.56%	1.78	1.94	3.95

Panel 4 Stocks Sorted by Relative β^-

Portfolio	Return	β	β^-	β^+
1 Low	-4.73%	0.48	0.43	1.26
2	-15.91%	0.74	0.97	1.28
3	-17.57%	0.95	1.30	1.49
4	-20.65%	1.21	1.68	1.83
5 High	-24.73%	1.58	2.38	2.00
High - Low	-20.00%	1.10	1.95	0.74

Panel 5 Stocks Sorted by Relative β^+

Portfolio	Return	β	β^-	β^+
1 Low	-9.70%	0.39	0.85	-0.10
2	-12.81%	0.64	0.97	0.80
3	-13.94%	0.93	1.27	1.45
4	-18.62%	1.20	1.54	2.10
5 High	-28.47%	1.80	2.14	3.61
High - Low	-18.77%	1.41	1.29	3.72

Panel 6 Stocks Sorted by $(\beta^- - \beta^+)$

Portfolio	Return	β	β^-	β^+
1 Low	-24.94%	1.54	1.74	3.28
2	-15.44%	1.15	1.47	2.04
3	-15.12%	0.91	1.24	1.43
4	-14.38%	0.72	1.08	0.89
5 High	-13.69%	0.65	1.24	0.21
High - Low	11.24%	-0.89	-0.50	-3.07

Focusing on the pattern on returns, in the first 5 panels, a downward pattern appears when the corresponding beta measurement is increasing. In addition, the previously mentioned U-shaped pattern of returns when stocks are sorted by relative downside beta disappears, replaced by a downward pattern. As in previous tables, when stocks are sorted by $(\beta^{-}-\beta^{+})$, the upward pattern of returns appeared again with all negative values. Overall, it can be seen from Table 2.8 that the abnormal stock price movements do not change the return pattern and risk–return relationship so much. However, during the Dot com bubble, it is clear that stocks suffered huge losses, especially on the high beta measurement portfolios. It shows again that high beta stocks are influenced the most when downward stock price movements occur.

After analyzing the Dot com bubble, a sub-period analysis is conducted and the time sensitivity of downside risk is shown more obviously. The original data are divided into three sub-periods, January 1980 to December 1989, January 1990 to December 1999 and January 2000 to December 2010. The results of the analysis are illustrated in Table 2.9, Table 2.10 and Table 2.11, respectively. It can be seen from these three tables that the results for each sub-period are quite different from each other in terms of the risk-return relationship.

Firstly, there appears to be no steady relationship between return and corresponding beta measurements in Table 2.9 which covers the 1989 market crash. The pattern of returns is non-linear in each panel with U-shaped patterns present and the highest beta measurement portfolio always generating a negative rate of return. Secondly, a different pattern of returns is shown in Table 2.10. When stocks are sorted by conventional beta, returns exhibit a downward pattern from the low to the high beta portfolio. The same patterns appear again when stocks are sorted by downside beta, upside beta and relative upside beta. When stocks are sorted by relative downside beta, the returns demonstrate a U-shaped pattern from the low beta portfolio to the high beta portfolio.

Results become more interesting when turning to Table 2.11, when the January 2000 to December 2010 period is analyzed. Consistent with Ang et al's (2006) findings, when stocks are sorted by conventional beta, the rate of return series exhibits an upward pattern along with the increase of beta estimates. When stocks are sorted by downside beta, relative downside beta, relative upside beta and downside beta less upside beta, the upward pattern of returns corresponding to the increase in corresponding beta measure becomes more obvious. Finally, when stocks are sorted by upside beta, a clear downward pattern of returns is exhibited. Overall, this sub-period analysis shows three different risk–return relationships depending on the sample period used.

To summarize, different types of portfolio return patterns appear when they are sorted with respect to corresponding beta measurements. The UK data present evidence of sensitivity to the time period used, especially when financial crises happen and rates of return are low. When the longest sample size is adopted, a reversed pattern of returns appears, contrary to conventional portfolio theory.

Table 2.9 UK Stocks Sorted By Factor Loadings (Jan 1980-Dec 1989)

This table presents the relationship between excess stock returns and factor loading of stock market portfolio with average market excess return as a benchmark. The sample uses FTSE All Shares from January 1980 to December 1989. The column labeled "return" reports the annual average stock returns over three month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β^-	β^+
1 Low	2.14%	0.41	0.61	0.53
2	1.07%	0.78	1.06	1.15
3	2.51%	0.96	1.30	1.42
4	2.13%	1.11	1.50	1.65
5 High	-2.73%	1.42	1.94	2.08
High - Low	-4.87%	1.01	1.33	1.55

Panel 2 Stocks Sorted by β^-

Portfolio	Return	β	β^-	β^+
1 Low	2.40%	0.45	0.56	0.74
2	0.97%	0.79	1.05	1.20
3	1.87%	0.96	1.30	1.45
4	1.11%	1.10	1.52	1.58
5 High	-1.26%	1.38	2.00	1.86
High - Low	-3.66%	0.93	1.44	1.12

Panel 3 Stocks Sorted by β^+

Portfolio	Return	β	β^-	β^+
1 Low	2.78%	0.48	0.82	0.39
2	0.94%	0.81	1.16	1.10
3	1.73%	0.96	1.31	1.41
4	2.04%	1.09	1.44	1.69
5 High	-2.37%	1.34	1.69	2.23
High - Low	-5.15%	0.86	0.87	1.84

Panel 4 Stocks Sorted by Relative β^-

Portfolio	Return	β	β^-	β^+
1 Low	2.54%	0.66	0.71	1.31
2	1.13%	0.84	1.08	1.36
3	1.78%	0.96	1.29	1.42
4	1.41%	1.03	1.47	1.40
5 High	-1.76%	1.19	1.86	1.33
High - Low	-4.31%	0.52	1.15	0.01

Panel 5 Stocks Sorted by Relative β^+

Portfolio	Return	β	β^-	β^+
1 Low	1.83%	0.65	1.15	0.50
2	1.46%	0.87	1.26	1.13
3	1.43%	0.96	1.31	1.41
4	0.91%	1.03	1.32	1.65
5 High	-0.55%	1.18	1.39	2.14
High - Low	-2.38%	0.53	0.25	1.64

Panel 6 Stocks Sorted by $(\beta^- - \beta^+)$

Portfolio	Return	β	β^-	β^+
1 Low	0.52%	1.01	1.13	1.93
2	1.35%	0.97	1.23	1.58
3	1.97%	0.95	1.29	1.41
4	0.58%	0.90	1.32	1.17
5 High	0.66%	0.85	1.46	0.74
High - Low	0.14%	-0.16	0.33	-1.19

Table 2.10 UK Stocks Sorted By Factor Loadings (Jan 1990-Dec 1999)

This table presents the relationship between excess stock returns and factor loading of stock market portfolio with average market excess return as a benchmark. The sample uses FTSE All Shares from January 1990 to December 1999. The column labeled "return" reports the average annual stock returns over three month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β^-	β^+
1 Low	2.59%	0.21	0.37	0.16
2	-1.08%	0.68	1.00	0.89
3	-3.03%	0.95	1.35	1.33
4	-3.53%	1.20	1.67	1.75
5 High	-8.02%	1.78	2.47	2.70
High - Low	-10.61%	1.58	2.10	2.53

Panel 2 Stocks Sorted by β^-

Portfolio	Return	β	β^-	β^+
1 Low	1.43%	0.27	0.28	0.45
2	-1.41%	0.70	0.96	0.99
3	-2.90%	0.96	1.34	1.37
4	-3.64%	1.18	1.70	1.65
5 High	-6.55%	1.72	2.57	2.37
High - Low	-7.98%	1.45	2.29	1.92

Panel 3 Stocks Sorted by β^+

Portfolio	Return	β	β^-	β^+
1 Low	3.66%	0.29	0.70	0.01
2	-1.74%	0.69	1.05	0.86
3	-2.33%	0.94	1.33	1.34
4	-3.79%	1.18	1.59	1.79
5 High	-8.87%	1.71	2.18	2.83
High - Low	-12.53%	1.42	1.48	2.81

Panel 4 Stocks Sorted by Relative β^-

Portfolio	Return	β	β^-	β^+
1 Low	-0.11%	0.56	0.49	1.21
2	-3.44%	0.80	1.04	1.26
3	-3.14%	0.95	1.34	1.37
4	-2.54%	1.11	1.64	1.50
5 High	-3.85%	1.38	2.34	1.50
High - Low	-3.74%	0.82	1.86	0.30

Panel 5 Stocks Sorted by Relative β^+

Portfolio	Return	β	β^-	β^+
1 Low	1.93%	0.55	1.23	0.19
2	-0.41%	0.74	1.16	0.89
3	-1.28%	0.93	1.31	1.33
4	-4.34%	1.11	1.44	1.75
5 High	-8.95%	1.48	1.70	2.68
High - Low	-10.88%	0.93	0.47	2.49

Panel 6 Stocks Sorted by $(\beta^- - \beta^+)$

Portfolio	Return	β	β^-	β^+
1 Low	-7.33%	1.19	1.25	2.32
2	-3.45%	1.02	1.31	1.64
3	-1.72%	0.92	1.29	1.32
4	-0.57%	0.84	1.32	1.02
5 High	-0.01%	0.84	1.68	0.54
High - Low	7.32%	-0.35	0.43	-1.78

Table 2.11 UK Stocks Sorted By Factor Loadings (Jan 2000-Dec 2010)

This table presents the relationship between excess stock returns and factor loading of stock market portfolio with average market excess return as a benchmark. The sample uses FTSE All Shares from January 2000 to December 2010. The column labeled "return" reports the average annual stock returns over three month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β^-	β^+
1 Low	1.11%	0.37	0.58	0.45
2	2.25%	0.81	1.15	1.14
3	2.66%	1.09	1.50	1.59
4	3.31%	1.38	1.88	2.01
5 High	2.67%	2.03	2.68	3.15
High - Low	1.56%	1.66	2.10	2.70

Panel 2 Stocks Sorted by β^-

Portfolio	Return	β	β^-	β^+
1 Low	0.28%	0.42	0.51	0.71
2	0.72%	0.83	1.13	1.25
3	3.42%	1.09	1.50	1.60
4	2.99%	1.37	1.89	1.96
5 High	4.59%	1.98	2.76	2.82
High - Low	4.31%	1.56	2.25	2.10

Panel 3 Stocks Sorted by β^+

Portfolio	Return	β	β^-	β^+
1 Low	4.62%	0.49	0.91	0.24
2	2.87%	0.84	1.24	1.08
3	2.29%	1.08	1.48	1.58
4	1.33%	1.34	1.78	2.08
5 High	0.89%	1.93	2.38	3.36
High - Low	-3.73%	1.43	1.47	3.12

Panel 4 Stocks Sorted by Relative β^-

Portfolio	Return	β	β^-	β^+
1 Low	-0.73%	0.77	0.77	1.69
2	-0.31%	0.92	1.19	1.50
3	1.34%	1.11	1.51	1.66
4	4.63%	1.28	1.82	1.73
5 High	7.07%	1.60	2.49	1.75
High - Low	7.80%	0.83	1.71	0.06

Panel 5 Stocks Sorted by Relative β^+

Portfolio	Return	β	β^-	β^+
1 Low	-2.74%	1.43	1.56	2.91
2	-0.03%	1.13	1.44	1.89
3	2.19%	1.05	1.45	1.56
4	3.68%	0.99	1.48	1.22
5 High	7.14%	1.02	1.80	0.71
High - Low	9.54%	-0.43	0.22	-2.22

Panel 6 Stocks Sorted by $(\beta^- - \beta^+)$

Portfolio	Return	β	β^-	β^+
1 Low	-2.39%	1.45	1.58	2.92
2	0.32%	1.14	1.45	1.90
3	2.54%	1.07	1.46	1.57
4	4.03%	1.00	1.49	1.23
5 High	7.49%	1.03	1.81	0.72
High - Low	9.89%	-0.42	0.23	-2.20

When the sample size is shortened, the reversed pattern still exists, however, the spreads between the highest beta measurement portfolio and the lowest one became much narrower. When notable financial crises are excluded from the data set, an unexpected U-shaped pattern appears in returns across beta portfolios. When a sub-period analysis is conducted, three totally different risk–return relationships are obtained. The period of January 2000 to December 2010 exhibits a risk-return relationship which is closest to the conventional portfolio theory, when this sample is characterized by a long bull market. The reasons for this are difficult to explain, but are most likely to be the booms before and after the slump, which offset the negative impact of the bull market. Clearly, all crises within the UK data exist in the US data, but the results of both samples are different. Since the measurements of beta are the same, the reason for the different results can be attributed to the sample used. The US data has a much longer sample size which could alleviate the effect of downside markets. However, for the UK data, since the sample size is not long enough (too many missing value for a longer sample) and most of the sample is in the bear market, the downside market dominates the sample, and unexpected results appear.

2.6 Fama-Macbeth regressions

In order to further illustrate the impact of beta measures on US stock returns, as introduced in the literature review, the widely used cross-sectional approach of Fama and Macbeth (1973) is adopted. This methodology consists of regressing excess stock returns upon beta measures and pertinent independent variables. The essence of the

Fama-Macbeth regression is that, in stage one, a time-series regression is conducted to obtain beta estimates. In stage two, estimates from stage one are treated as independent variables, and a cross-sectional regression of excess stock returns upon their beta estimates are run at each point in time. The mean of the coefficient estimates for each variable are calculated as the final estimates of impact. The Fama-Macbeth regression aims to treat estimates as factors rather than factor loadings, and highlight their impact on stock returns in a cross-sectional fashion. Newey-West's (1987) heteroscdastic robust standard errors with 12 lags are employed to calculate the t-statistics.

Apart from the above beta measures, other measures of risk: the standard deviation (*sd*), coskewness and cokurtosis of each stock at each point of time, and β_L which is the liquidity beta estimate of each stock are employed.[16] Specifically, β_L is based on Pastor and Stambaugh (2003), and is estimated as follows:

$$xR_i = \alpha_i + \beta_L \cdot L + \beta_M \cdot xR_M + \beta_{SMB} \cdot SMB + \beta_{HML} \cdot HML + \varepsilon_i \quad , \qquad (2.16)$$

where α_i is the intercept, L is the innovation in aggregate liquidity (calculated by dividing daily stock returns by the volume) collected from CRSP, and *SMB* and *HML* are the Fama-French (1993) firm size and book to market factors also collected from CRSP.

To investigate multicollinearity, the correlation coefficients between the variables are presented in Table 2.12.

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16 A moving window approach is adopted for computing all estimates.

Table 2.12 Correlations Of Factor Loadings

This table reports the correlation coefficients between all factor loadings. To avoid repeating, only lower triangle of the matrix is shown.

	β	β^-	β^+	β_{Rf}^-	β_{Rf}^+	β_0^-	β_0^+	sd	β_L	Coskewness	Cokurtosis
β	1.000										
β^-	0.599	1.000									
β^+	0.628	0.218	1.000								
β_{Rf}^-	0.527	0.905	0.174	1.000							
β_{Rf}^+	0.648	0.183	0.962	0.199	1.000						
β_0^-	0.522	0.893	0.179	0.938	0.193	1.000					
β_0^+	0.670	0.159	0.951	0.146	0.976	0.196	1.000				
sd	0.437	0.347	0.155	0.351	0.190	0.321	0.184	1.000			
β_L	-0.125	0.004	-0.228	0.009	-0.227	0.040	-0.206	-0.051	1.000		
Coskewness	0.013	-0.334	0.274	-0.314	0.281	-0.329	0.282	0.077	-0.144	1.000	
Cokurtosis	0.228	0.265	0.197	0.230	0.190	0.250	0.185	-0.290	-0.202	-0.594	1.000

It is obvious from Table 2.12 that β is highly correlated with all other downside and upside betas, with the correlation coefficients between β and each downside or upside beta all above 0.5. Moreover, higher correlations appear among the three types of downside betas, and the same phenomenon also appears among the three types of upside betas. The most highly correlated variables are β_{Rf}^{+} and β_{0}^{+} with a correlation coefficient of 0.976. Besides that, β_L appears to be moderately correlated with β with a correlation coefficient of 0.437, and the remaining variables only exhibit weak correlation with each other. Since there is no clear benchmark to identify multicollinearity in econometric theory, here we define the variables with a correlation coefficient above 0.5 or below -0.5 as highly correlated, and to avoid multicollinearity, variables which are highly correlated are not to be employed in the same regression as independent variables. Therefore, β is not employed in the same regressions with all other downside and upside betas, and each type of downside beta is only employed in the regression in pairs with the same type of upside beta.

The results of the Fama-Macbeth regressions are summarized in Table 2.13 to Table 2.16. It can be seen from Table 2.13 that when β is employed as an independent variable, it is highly significant at the 1% significance level among all five regressions with different combinations of the independent variables. Coskewness and cokurtosis did not perform significantly in the regressions, coskewness is never significant even at the 10% significance level and cokurtosis is only significant in regression 5. Standard deviation and liquidity beta are always highly significant at the 1% level.

Table 2.13 Fama-Macbeth Regression Of Factor Loadings

This table reports the result of the Fama-Macbeth regression of factor loadings on excess stock returns. The t-statistics in the square brackets are calculated by using the Newey-West (1987) heteroscedastic robust standard error with 12 lags. * denotes significance at the 10% level, ** denotes significance at the 5% level and ***denotes significance at the 1% level.

	1	2	3	4	5
β	0.0566***	0.0575***	0.0582***	0.0523***	0.0498***
	[24.59]	[23.90]	[24.10]	[20.32]	[16.90]
Coskewness		-0.00906	-0.00444		0.00699
		[-1.14]	[-0.55]		[0.84]
Cokurtosis		-0.00234	-0.00174		0.00348*
		[-1.30]	[-0.96]		[1.66]
β_L			0.0137***	0.0139***	0.0145***
			[3.28}	[3.41]	[3.48]
sd				0.0901***	0.109***
				[4.75]	[4.95]
Intercept	0.0332***	0.0341***	0.0334***	0.0228***	0.0179***
	[11.14]	[9.56]	[9.36]	[6.40]	[3.77]
Adjusted R^2	0.043	0.043	0.044	0.045	0.045
No. of Obs	1910051	1910051	1910051	1910051	1910051

Among the five regressions in Table 2.13, regression 4 presents the highest R^2 with all independent variables highly significant.[17] When β^- and β^+ are employed as independent variables, it can be seen from Table 2.14 that coefficients of β^- and β^+ are highly significant at the 1% significance level in regression 1 to 3. Moreover standard deviation and liquidity beta are always highly significant at the 1% level in all regressions.

[17] The adjusted R^2 is employed.

Table 2.14 Fama-Macbeth Regression Of Factor Loadings With Average Market Excess Return As Benchmark

This table reports the result of the Fama-Macbeth regression of factor loadings on excess stock returns. The downside and upside beta are calculated taking average market excess return as a benchmark. The t-statistics in the square brackets are calculated by using the Newey-West (1987) heteroscedastic robust standard error with 12 lags. * denotes significance at the 10% level, ** denotes significance at the 5% level and ***denotes significance at the 1% level.

	1	2	3	4	5
β^-	0.0136***	0.0121***	0.0120***	0.0157	0.00601
	[7.22]	[5.84]	[5.79]	[0.70]	[3.03]
β^+	0.0155***	0.0168***	0.0176***	0.0137***	0.0154***
	[12.94]	[11.57]	[11.93]	[9.17]	[12.55]
Coskewness		-0.0170*	-0.0144	0.00321	
		[-1.75]	[-1.47]	[0.33]	
Cokurtosis		-0.00195	-0.00165	0.00976***	
		[-0.99]	[-0.84]	[4.50]	
sd			0.0139***	0.0163***	0.0161***
			[3.26]	[3.86]	[3.82]
β_L				0.257***	0.205***
				[12.43]	[11.13]
Intercept	0.0652***	0.0650***	0.0647***	0.0217***	0.0400***
	[23.37]	[19.32]	[19.22]	[4.51]	[11.23]
Adjusted R^2	0.020	0.020	0.020	0.031	0.029
No. of Obs	1910051	1910051	1910051	1910051	1910051

Table 2.15 Fama-Macbeth Regression Of Factor Loadings With Average Risk-Free Rate As Benchmark

This table reports the result of the Fama-Macbeth regression of factor loadings on excess stock returns. The downside and upside beta are calculated taking the average risk-free rate as a benchmark. The t-statistics in the square brackets are calculated by using the Newey-West (1987) heteroscedastic robust standard error with 12 lags. * denotes significance at the 10% level, ** denotes significance at the 5% level and ***denotes significance at the 1% level.

	1	2	3	4	5
β_{Rf}^{-}	0.0108***	0.00904***	0.00893***	0.00426	0.000446
	[6.47]	[4.94]	[4.88]	[2.41]	[0.23]
β_{Rf}^{+}	0.0167***	0.0186***	0.0193***	0.0161***	0.0143***
	[13.58]	[12.38]	[12.74]	[12.67]	[9.12]
Cokewness		-0.0229**	-0.0203**		0.00137
		[-2.36]	[-2.08]		[0.14]
Cokurtosis		-0.00216	-0.00185		0.00976***
		[-1.09]	[-0.94]		[4.46]
β_L			0.0140***	0.0161	0.0161***
			[3.30]	[3.82]	[3.81]
sd				0.200***	0.254***
				[10.82]	[12.16]
Intercept	0.0666***	0.0654***	0.0650***	0.0417***	0.0224***
	[24.85]	[19.70]	[19.57]	[11.94]	[4.64]
Adjusted R^2	0.020	0.020	0.021	0.029	0.031
No. of Obs	1910051	1910051	1910051	1910051	1910051

Table 2.16 Fama-Macbeth Regression Of Factor Loadings With Zero Return As Benchmark

This table reports the result of the Fama-Macbeth regression of factor loadings on excess stock returns. The downside and upside beta are calculated taking zero return as a benchmark. The t-statistics in the square brackets are calculated by using the Newey-West (1987) heteroscedastic robust standard error with 12 lags. * denotes significance at the 10% level, ** denotes significance at the 5% level and ***denotes significance at the 1% level.

	1	2	3	4	5
β_0^-	0.0130***	0.0109***	0.0107***	0.00722***	0.00359*
	[7.80]	[5.96]	[5.81]	[4.14]	[1.85]
β_0^+	0.0201***	0.0229***	0.0235***	0.0194***	0.0190***
	[15.69]	[14.74]	[15.03]	[14.79]	[11.75]
Coskewness		-0.0312***	-0.0286***		0.00917
		[-3.22]	[-2.94]		[0.93]
Cokurtosis		-0.00442**	-0.00407**		0.00622***
		[-2.26]	[-2.08]		[2.87]
β_L			0.0131***	0.0159	0.0159***
			[3.11]	[3.80]	[3.78]
sd				0.186***	0.224***
				[10.21]	[10.90]
Intercept	0.0608***	0.0610***	0.0608***	0.0372***	0.0227***
	[22.73]	[18.55]	[18.48]	[10.60]	[4.73]
Adjusted R^2	0.026	0.027	0.028	0.034	0.036
No. of Obs	1910051	1910051	1910051	1910051	1910051

Coskewness is only significant at the 10% level in regression 2, but not significant in regression 3 and 5. And cokurtosis is significant at the 1% level in regression 5 and not significant in other regressions. Among the five regressions in Table 2.14, regression 4 presents the highest R^2, though β^- and coskewness are not significant. Furthermore, β_{Rf}^- and β_{Rf}^+ are employed as independent variables in all five regressions in Table 2.15. β_{Rf}^{-} is significant at the 1% level in regressions 1 to 3, at the 5% level in regression 4 and is not significant in regression 5, while β_{Rf}^{+} is highly significant at the 1% level in all regressions. Standard deviation and liquidity beta are always highly significant at the 1% level in all regressions in which they are employed. Coskewness is significant at the 5% level in both regression 2 and 3, but is not significant in regression 5. Cokurtosis is significant at the 1% level in regression 5, but not significant in other regressions. Similar to Table 2.14, among the five regressions in Table 2.15, regression 5 presents the highest R^2 value but with independent variables β_{Rf}^{-} and coskewness as not significant. Finally in Table 2.16, β_0^{-} and β_0^{+} are employed. Coskewness is more significant in regression 2 and 3 at the 5% level but not significant in regression 5, while cokurtosis is significant at least at the 5% level in all regressions in which it is employed. β_0^{-}, β_0^{+}, *sd* and β_L are all significant at the 1% level except β_0^{-} is significant at the 10% level in regression 5. In Table 2.16, regression 5 which employs all variables shows the highest R^2, but with non-significant coskewness and β_0^{-} is not as highly significant as the remaining variables. In regression 4, coskewness and cokurtosis are not employed, but the remaining variables are highly significant, it presents the second highest R^2 value.

It can be concluded from Table 2.13 to Table 2.16 that β, three types of upside beta, standard deviation and liquidity are always highly significant in a cross-sectional regression context, with the three types of downside beta highly significant except when cokurtosis is significant. The significance of coskewness is identical to cokurtosis when standard deviation is not employed. When standard deviation is employed, coskewness is never significant. Notably, when all variables are employed in the regression, both types of downside beta and coskewness are not significant. From R^2 values, we can conclude that β or three types of downside and upside beta along with standard deviation and liquidity beta do make an impact on stock returns, and fit the regression better than when coskewness and cokurtosis are employed.

Overall, consistent with Ang et al. (2006), without controlling for beta, downside betas have a positive impact on stock returns in a cross-sectional context, and downside risk is priced at 1.36% per annum, while upside betas have a positive impact on stock returns which does not appear in Ang et al's (2006) finding. However, downside betas are not always highly significant as expected in each circumstance.

2.7 Cross-sectional relationship between downside beta and coskewness

As mentioned above, downside beta measures the co-movement between stock returns and a relatively falling market, while coskewness measures the distribution of stock returns to its mean relative to the skewness of the market portfolio. Both downside beta and coskewness are statistics which show the relationship between stock returns and the market portfolio when it is not symmetric about the mean. Therefore, downside beta estimates could have a potential relationship with coskewness. Harvey and Siddique (2000) they found a negative relationship between stock returns and coskewness. Therefore, it is essential to examine the relationship between downside beta and coskewness to uncover the unique property on each of them. The relationship among annualized stock excess returns, three types of downside beta and coskewness are presented in Table 2.17 to Table 2.19. Each table contains two panels, in Panel 1, stocks are sorted into five quintiles based on their coskewness. Then for each quintile, stocks are further sorted into 5 portfolios based on the corresponding downside beta. Therefore, there are 25 portfolios in total, and the average excess return of each portfolio is presented. The steps are reversed in Panel 2, stocks are firstly sorted based on the corresponding downside beta, and further sorted by coskewness. This gives 25 portfolios with the average excess return of each portfolio presented.

It can be seen from Table 2.17 that when stocks are firstly sorted by coskewness, only the lowest coskewness portfolios present a generally decreasing trend from 9.11% to 6.15% with a slight increase in portfolio 2 in average excess returns along with the increase of β^{-}. The remaining four groups of portfolios all illustrate an increasing trend on average excess returns when they are further sorted by β^{-}. The last column labelled "Average" presents the relationship between average excess returns of portfolios and β^{-} controlling for coskewness. It is clear that when controlling for coskewness, on average, excess returns are increasing along with the increase of β^{-}. Comparatively, in Panel 2, when stocks are sorted by coskewness controlling for β^{-}, no clear pattern appears in either group and on average, the excess return of each portfolio is fluctuating along with the increase of coskewnes. Therefore, coskewness does not appear to have an obvious impact on stock returns when controlling for β^{-}. Unsurprisingly, Table 2.18 and Table 2.19 present similar patterns as in Table 2.17.

Table 2.17 The Relationship Between β^{-} And Coskewness

This table reports the relationship between downside beta and coskewness. Panel 1 reports the average annual excess returns of each portfolio when controlling for coskewness. The top row represents the coskewness quintiles from the lowest 20% to the highest 20%, while the left column represents the 5 downside beta quintiles when controlling for coskewness. Panel 2 reports the average annual excess returns of each portfolio when controlling for downside beta. The top row represents the downside beta quintiles from the lowest 20% to the highest 20%, while the left column represents the 5 coskewness quintiles when controlling for the downside beta. The downside betas are computed with average market excess return used as a benchmark.

Panel 1 Average annual returns sorted by β^{-} controlling for coskewness

Coskewness Quintiles

	1Low	2	3	4	5High	Average
1Low	9.11%	7.57%	8.41%	7.76%	5.82%	7.73%
2	9.94%	9.55%	9.79%	8.75%	7.13%	9.03%
3	9.30%	10.26%	11.06%	9.13%	7.99%	9.55%
4	8.82%	11.01%	10.90%	10.91%	9.51%	10.23%
5High	6.15%	11.69%	14.32%	15.61%	11.22%	11.80%
High-Low	-2.96%	4.12%	5.91%	7.85%	5.40%	4.06%

Panel 2 Average annual returns sorted by coskewness controlling for β^{-}

β^{-} Quintiles

	1Low	2	3	4	5High	Average
1Low	7.45%	9.35%	9.86%	9.61%	5.22%	8.30%
2	7.74%	9.58%	9.41%	9.01%	10.47%	9.24%
3	8.28%	9.71%	10.21%	11.85%	13.66%	10.74%
4	7.03%	9.23%	11.34%	10.27%	14.07%	10.39%
5High	5.88%	7.75%	9.63%	11.02%	14.06%	9.67%
High-Low	-1.57%	-1.60%	-0.23%	1.41%	8.84%	1.37%

Table 2.18 The Relationship Between downside Beta And Coskewness

This table reports the relationship between downside beta and coskewness. Panel 1 reports the average annual excess returns of each portfolio when controlling for coskewness. The top row represents the coskewness quintiles from the lowest 20% to the highest 20%, while the left column represents the 5 downside beta quintiles when controlling for coskewness. Panel 2 reports the average annual excess returns of each portfolio when controlling for downside beta. The top row represents the downside beta quintiles from the lowest 20% to the highest 20%, while the left column represents the 5 coskewness quintiles when controlling for the downside beta. The downside betas are computed with average risk-free rate used as a benchmark.

Panel 1 Average annual returns sorted by β_{Rf}^{-} controlling for coskewness

	Coskewness Quintiles					
	1Low	2	3	4	5High	Average
1Low	9.25%	7.89%	8.38%	7.44%	5.52%	7.70%
2	9.72%	10.25%	10.51%	9.20%	6.86%	9.31%
3	9.59%	9.14%	10.13%	9.62%	8.18%	9.33%
4	8.81%	10.70%	11.61%	10.64%	9.97%	10.35%
5High	5.94%	12.09%	13.86%	15.28%	11.14%	11.66%
High-Low	-3.31%	4.20%	5.48%	7.84%	5.62%	3.97%

Panel 2 Average annual returns sorted by coskewness controlling for β_{Rf}^{-}

	β_{Rf}^{-} Quintiles					
	1Low	2	3	4	5High	Average
1Low	7.89%	8.99%	9.81%	8.66%	5.73%	8.22%
2	7.87%	9.58%	9.71%	9.09%	10.37%	9.32%
3	7.95%	9.98%	10.29%	11.87%	13.16%	10.65%
4	6.67%	10.02%	9.90%	11.83%	13.67%	10.42%
5High	5.76%	8.46%	9.22%	11.04%	14.18%	9.73%
High-Low	-2.13%	-0.53%	-0.59%	2.38%	8.45%	1.52%

Table 2.19 The Relationship Between Downside Beta And Coskewness

This table reports the relationship between downside beta and coskewness. Panel 1 reports the average annual excess returns of each portfolio when controlling for coskewness. The top row represents the coskewness quintiles from the lowest 20% to the highest 20%, while the left column represents the 5 downside beta quintiles when controlling for coskewness. Panel 2 reports the average annual excess returns of each portfolio when controlling for downside beta. The top row represents the downside beta quintiles from the lowest 20% to the highest 20%, while the left column represents the 5 coskewness quintiles when controlling for the downside beta. The downside betas are computed with zero return used as a benchmark.

Panel 1 Average annual returns sorted by β_0^- controlling for coskewness

	Coskewness Quintiles					
	1Low	2	3	4	5High	Average
1Low	9.13%	7.84%	8.57%	7.30%	5.08%	7.58%
2	9.56%	8.91%	10.15%	9.33%	6.82%	8.95%
3	9.12%	10.18%	10.68%	9.25%	8.82%	9.61%
4	9.26%	10.80%	11.47%	11.01%	9.92%	10.49%
5High	6.23%	12.35%	13.62%	15.29%	11.03%	11.70%
High-Low	-2.90%	4.51%	5.05%	7.99%	5.95%	4.12%

Panel 2 Average annual returns sorted by coskewness controlling for β_0^-

	β_0^- Quintiles					
	1Low	2	3	4	5High	Average
1Low	7.37%	9.41%	9.13%	9.73%	5.10%	8.15%
2	8.29%	10.09%	9.34%	9.54%	10.34%	9.52%
3	7.23%	10.17%	10.32%	11.05%	14.07%	10.57%
4	6.72%	9.99%	10.31%	11.85%	13.29%	10.43%
5High	5.37%	8.84%	10.10%	10.50%	13.58%	9.68%
High-Low	-2.00%	-0.57%	0.97%	0.77%	8.48%	1.53%

It can be seen from Panel 1 and Panel 2 of both Table 2.18 and Table 2.19 that when controlling for coskewness, excess returns increase along with the increase of β_{Rf}^- and β_0^-, while coskewness does not show an obvious impact on stock returns when controlling for β_{Rf}^- and β_0^-. Therefore, we can conclude that three types of downside beta do have a negative impact on stock returns even when controlling for coskewness, and this is consistent with the previous findings when downside beta is controlled for beta. However, in Ang et al's (2006) study, a reversed result is presented. While the impact of coskewness on stock returns when controlling for any type of downside beta are not traceable from the data used in this chapter.

2.8 Predictive power of downside beta

2.8.1 Predictive power of downside beta: US stocks

To investigate the predictive power of beta and the three types of downside beta, all stocks are sorted into five portfolios cross-sectionally based on the value of beta and three types of downside beta. The previously described methodology is adopted, except now the relationship between the risk and one year future excess returns are calculated. Results pertaining to US data are presented in Table 2.20. It can be seen from Panel 1 that when stocks are sorted by β, β^-, β_{Rf}^-, β_0^-, future average excess returns present a totally different reversed U-shaped pattern. It specifically increases from 10.13% in portfolio 1 to 21.9% in portfolio 3, and then starts dropping, ending up with 9.92% in portfolio 5. A similar relationship appears when stocks are sorted by β^-, β_{Rf}^- and β_0^- and the results are shown in Panel 2 to Panel 4.

Table 2.20 US Stocks Sorted By Downside Factors With Future Excess Returns

This table presents the relationship between future excess stock returns and downside factor loadings. The following year's returns are taken as the future excess returns. The column labeled "return" reports the annual average future stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β

Portfolio	Return	β	β^-	β_{Rf}^-	β_0^-
1 Low	10.13%	0.32	0.56	0.63	0.62
2	15.46%	0.78	0.99	1.01	1.01
3	21.90%	1.08	1.23	1.26	1.25
4	16.43%	1.39	1.47	1.50	1.50
5 High	9.92%	2.11	1.94	2.00	1.97
High - Low	-0.21%	1.79	1.37	1.37	1.36

Panel 2 Stocks Sorted by β^-

Portfolio	Return	β	β^-	β_{Rf}^-	β_0^-
1 Low	10.97%	0.60	0.21	0.27	0.26
2	20.96%	0.88	0.86	0.89	0.89
3	21.49%	1.12	1.18	1.21	1.21
4	17.17%	1.34	1.54	1.57	1.57
5 High	3.25%	1.74	2.40	2.46	2.42
High - Low	-7.72%	1.15	2.19	2.19	2.16

Panel 3 Stocks Sorted by β_{Rf}^-

Portfolio	Return	β	β^-	β_{Rf}^-	β_0^-
1 Low	11.57%	0.60	0.26	0.20	0.20
2	21.22%	0.90	0.88	0.87	0.88
3	22.84%	1.12	1.19	1.21	1.20
4	17.75%	1.35	1.54	1.58	1.56
5 High	0.45%	1.72	2.32	2.55	2.50
High - Low	-11.12%	1.12	2.06	2.36	2.30

Panel 4 Stocks Sorted by β_0^-

Portfolio	Return	β	β^-	β_{Rf}^-	β_0^-
1 Low	11.07%	0.62	0.30	0.25	0.15
2	21.06%	0.90	0.87	0.89	0.86
3	23.17%	1.12	1.19	1.21	1.20
4	18.30%	1.35	1.55	1.56	1.58
5 High	0.23%	1.69	2.28	2.50	2.56
High - Low	-10.84%	1.07	1.99	2.25	2.41

It can be seen from Table 2.20 that the peak of future excess returns always appears in the medium valued β, β^-, β_{Rf}^- and β_0^- portfolio, and the lowest future excess return constantly appears in the highest valued β, β^-, β_{Rf}^- and β_0^- portfolio, especially when stocks are sorted by the three types of downside beta (the lowest portfolio return is much lower than the one when stocks are sorted by β). Therefore, it can be concluded from Table 2.20 that β, β^-, β_{Rf}^- and β_0^- do have predictive power on future returns. To investors, portfolios with high β, β^-, β_{Rf}^- and β_0^- are expected to have low future returns especially for the three types of downside beta. A medium value beta measures would predict a possibly high future return.

2.8.2 Predictive power of downside beta: UK stocks

As a comparison and for the sake of completeness, the predictive power of downside beta based on UK data is examined.[18]

Table 2.21 UK Stocks Sorted By Factor Loadings With Future Excess Return

This table presents the relationship future excess stock returns and factor loading of stock market portfolio with average market excess return as benchmark. The sample uses FTSE All Shares from January 1980 to December 2007, the following year's excess returns are taken as the future excess return. The column labeled "return" reports the average annual future stock returns over three month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by β				
Portfolio	Return	β	β^-	β^+
1 Low	3.44%	0.30	0.49	0.31
2	4.01%	0.73	1.05	0.98
3	2.95%	0.98	1.37	1.38
4	1.62%	1.20	1.66	1.75
5 High	-3.43%	1.74	2.36	2.64
High - Low	-6.87%	1.44	1.87	2.33

Panel 2 Stocks Sorted by β^-				
Portfolio	Return	β	β^-	β^+
1 Low	2.88%	0.35	0.41	0.56
2	4.49%	0.74	1.02	1.07
3	3.25%	0.98	1.36	1.41
4	1.40%	1.19	1.68	1.67
5 High	-3.43%	1.68	2.45	2.34
High - Low	-6.31%	1.33	2.04	1.78

The results in Table 2.21 show that, when stocks are sorted by conventional beta, downside beta, a U-shaped pattern of returns appears over beta space. Notably, the second lowest beta portfolio always generates the highest rate of return, and the highest beta portfolio always suffers a loss. Overall, the predictive power of the UK downside beta is weak, but the medium valued betas are always a positive signal of

[18] The patterns of upside beta are presented for the sake of completeness.

future returns to investors.

2.9 Conclusion

Compared to the results based on US data, the UK stock returns exhibit unusual patterns on β, β^- and β^+. When the whole sample is used (which covers quite a few financial crises), a reversed pattern of portfolio returns exists—a result which is against the classic literature. In the sub-period analysis, it is still hard to conclude a constant pattern between risk and return, and the predictive power of β^- is weak according to the results. Overall, the UK data presents obvious time sensitivity especially in regards to financial crises.

However, as a main focus, the US data presents a much more sensible pattern. It can be concluded from this chapter that β and three types of downside and upside beta (β^- and β^+, β_{Rf}^- and β_{Rf}^+, and β_0^- and β_0^+) do have a significant relationship with portfolio returns in a cross-sectional fashion. Portfolio returns are consistent with the classic high beta high return positive relationship when stocks are sorted by β. Consistent with Ang et al (2006), this relationship still holds when stocks are sorted by any one of the downside beta and upside beta measures, but the spread of average excess returns drops when stocks are sorted by downside beta. The downside risk premium is priced at 1.36% per annum in a Fama-Macbeth regression context.[19] However, when controlling for β, the unique property of downside and upside beta are

[19] The downside risk premium is priced at 6% per annum in Ang et al's (2006) study.

revealed, and results are contrary to Ang et al's (2006) study. Downside betas exhibit a negative relationship with average excess return, while upside beta presents a positive relationship regardless of the benchmark used. Regarding the additional variables, liquidity beta and standard deviation also have a significant positive impact on stock returns. It has been further shown that three types of downside beta do have a negative impact on stock returns even when controlling for coskewness, while the impact of coskewness on stock returns when controlling for any type of downside beta is not found. Moreover, high β, β^-, β_{Rf}^- and β_0^- values have a negative impact on future returns especially when the three types of downside beta are considered. The significance of β, β^- and β^+ in a cross-sectional fashion are found, however, in order to improve the goodness of fit of the model, other econometric methods rather than classic OLS regression could be employed as alternative methods to examine the relationship.

Chapter 3

The cross-sectional determinants of US stock returns: The impact of time-varying downside risk

3.1 Introduction

Most studies of downside risk follow the classic approach, employing the linear market model to estimate beta. In this chapter, two non-linear models, the cubic piecewise polynomial function (CPPF) model and the Fourier Flexible Form (FFF) model are employed to model portfolio returns in order to examine the significance of beta, downside beta and upside beta estimates.[20] Both models take flexible approaches, yet are parsimonious, allowing beta estimates to be time-varying. Innovatively, various numbers of knots and orders are applied on the CPPF model and the FFF model respectively, to smooth the sample. Also, the Akaike Information Criteria (AIC) is adopted to determine the most appropriate number of knots and order for the sample. With the AIC, the best fitted estimates of beta, upside beta and downside beta for both models are generated. These estimates are sorted into portfolios to examine the risk-return relationship, and Fama-Macbeth regressions are then performed to discover the significance of the estimates in a cross-sectional framework.

Taking the CPPF and the FFF approach is motivated by their flexibility, with both approaches allowing the beta estimates to vary over time. The CPPF approach is analogous to cubic spline approach but with no constraints of intercept columns, and the estimates at each point in time are the product of a vector of initial estimates and a

[20] Non-linear refers to the CPPF and FFF approach in terms of trend, the regression is still based the classic OLS.

piecewise polynomial matrix. For the FFF approach, sine and cosine functions are adopted to construct a matrix which creates a nonlinear pattern bounded between -1 and 1. The pattern is finally presented on the estimates at each point in time to allow time-variation. The importance of time-varying estimates is that estimates can present the true relationship between variables at each point of time, which allows us to discover the variation of co-movements among variables rather than a single estimate over the whole sample. Compared to the moving window approach used in previous chapters, the advantage of the CPPF model and the FFF model is that the whole sample is considered, while the moving window approach is limited to past data and the length of the window used.

We found the estimates of beta, downside beta and upside beta estimates of both models to be highly significant in relation to stock returns, and to have an impact on driving stock returns. The beta estimates positively drive stock returns. The downside and upside beta estimates demonstrate reversed impacts on stock returns, the downside beta has a negative impact on stock returns, while the upside beta, consistent with beta estimates, has a positive impact. This chapter is arranged as follows: section 3.2 provides literature reviews of both models, followed by section 3.3 which is a description of the data. Then, section 3.4 explains the econometric models and methods applied in this chapter, section 3.5 and 3.6 provide the empirical results, results of the Fama-Macbeth regressions, and section 3.7 is the conclusion.

3.2 Literature review

There is a long history of literature that has argued that the classic CAPM proposed by Sharpe (1964) and Lintner (1965) is inadequate to explain the risk-return relationship due to the assumption of constant beta. To solve this drawback, relaxing the constant beta assumption and allowing time-varying beta is one possible method. There are a number of approaches to obtain time-varying betas, for instance, in Fama and Macbeth's (1973) study, moving window OLS regression is applied to the market factor model to obtain time-varying beta. According to Härdle et al (1985), Härdle (1992), Wand and Jones (1995), Ang and Kristensen (2011), and Li and Yang (2011), a nonparametric approach which is based on simulation is another method to obtain the time-varying beta. Moreover, the time-varying beta can also can be obtained by the multivariate GARCH based model proposed by Engle (2002), Andersen et al. (2002) and Nieto et al. (2011).

Instead of regressing the stock return upon the market portfolio return in a linear fashion, two alternative methods have been employed in this chapter to estimate the corresponding time-varying beta coefficients. The first method adopted is the CPPF regression, and the second one is the FFF regression, both methods adopt various degrees of flexibility.

Cubic spline approach

In order to introduce CPPF approach, a review of cubic spline is essential. The cubic spline method was originally used in mathematics and engineering (Ferguson 1963). Mathematically, as a third order piecewise polynomial function, the cubic spline is used to smooth discrete points into a continuous curve. According to Rorres and Anton (1984), a cubic spline can be expressed mathematically in the following form

$$S(x) = \begin{cases} s_1(x) \ \text{if} \ x_1 \le x \le x_2, \\ s_2(x) \ \text{if} \ x_2 \le x \le x_3, \\ \vdots \\ s_{n-1}(x) \ \text{if} \ x_{n-1} \le x \le x_n, \end{cases} \tag{3.1}$$

where it is assumed that s_i is the third order polynomial function defined by

$$s_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i \, , \tag{3.2}$$

for i = 1, 2, 3…….n-1.

The first and second order derivative of equation (3.1) defines the fundamentals of the process. These derivatives are given by

$$s_i'(x) = 3a_i(x - x_i)^2 + 2b_i(x - x_i) + c_i \, , \tag{3.3}$$

$$s_i''(x) = 6a_i(x - x_i) + 2b_i \, , \tag{3.4}$$

for i = 1, 2, 3…….n-1.

The piecewise polynomial function has the following properties:

1. The piecewise polynomial function interpolates all data points.
2. The $S(x)$ function is continuous in the interval $[x_1, x_n]$.
3. The first derivative of the $S(x)$ function is continuous in the interval $[x_1, x_n]$.
4. The second derivative of the $S(x)$ function is continuous in the interval $[x_1, x_n]$.

There are a number of studies which employ the cubic spline approach in financial modeling, mainly focusing on estimation of the term structure, autoregressive conditional duration (ACD) models and volatility of high frequency data. Vasicek and Fong (1981) and Jarrow and Ruppert (2004) employ the cubic spline approach in estimating interest rate term structure. Engle and Russell (1998) proposed an ACD model which treats the time between transactions as a stochastic process. Within the ACD model, a daily seasonal factor is modelled by a cubic spline series. Then in Zhang et al. (2001), a threshold autoregressive conditional duration (TACD) model is proposed and shown to be superior to the classic ACD model.[21] More recently, Taylor (2004a, b) and Giot (2005) both adopted cubic splines in their studies in the context of modelling the volatility of high frequency data via the ACD model. Aside from the above studies, there are a few studies that have also adopted cubic splines as a modelling tool, such as Engle and Rivera (1991) who estimated the density factor by using cubic splines in an autoregressive conditional heterosecdasticity (ARCH) context. Yu and Ruppert (2002) introduced the cubic spline approach into the estimation of the single index model. Evans and Speight (2010) employed the cubic spline approach to model intraday exchange rate volatility. The advantages of the cubic spline approach are, firstly, data can be flexibly adjusted without considering the sample size. Secondly, for research in different international markets with time differences (in different time zones), it allows different cubic splines to be estimated among various selected knots. Thirdly, apart from the spline elements, the nature of the original data is retained and there are no extra functions or patterns to be put into the model.

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[21] The cubic spline approach was particularly used to approximate seasonal factors within the model.

Competitive basis of cubic spline

According to Eilers and Marx (2004), there are mainly two approaches used in cubic spline regression: the B-spline basis and truncated power functions basis.

For the B-spline basis approach, Eilers and Marx (2004) use equally-spaced knots and spline function *B*. Mathematically, the B-spline model can be written as

$$E(y) = \mu = B\alpha, \tag{3.5}$$

and the objective function to be minimized is

$$Q_B = |y - B\alpha|^2 + \lambda |D_d \alpha|^2. \tag{3.6}$$

Which λ is a non-negative parameter, and D_d is the *d*-th difference of α, it can be written as

$$D_d = \Delta^d \alpha, \tag{3.7}$$

and

$$\Delta\alpha_j = \alpha_j - \alpha_{j-1}, \tag{3.8}$$

$$\Delta^2\alpha_j = \alpha_j - 2\alpha_{j-1} + \alpha_{j-2}, \tag{3.9}$$

and so on for higher orders. So the objective function of Q_B leads the B-spline model to

$$(B'B + \lambda D_d' D_d)\hat{\alpha} = B'y. \tag{3.10}$$

It can be seen from equation (3.10) that when λ=0, it becomes the classic equation of linear regression.

For the truncated power functions basis, according to Ruppert et al. (2003), for a given asset *i*, column *j* and degree *p*, the truncated power function of *F* is written as

$$F_{ij} = (x_i - t_j)^P I(x_i > t_j), \tag{3.11}$$

where *I(u)* is an indicator function, it is 0 when *u*<0 and 1 otherwise. The vector *t* contains the knots, and the knots are placed as quantiles of *x*. Consequently, the model for *E(y)* can be written as

$$E(y_i) = \sum_{k=0}^{P} \beta_k x_i^k + \sum_{j=1}^{n-1} F_{ij} b_j \ . \tag{3.12}$$

And the objective function to be minimized is given by

$$Q_F = |y - \beta x - Fb|^2 + \kappa |b|^2 . \tag{3.13}$$

The increasing of κ will increase the smoothness.

Eliers and Marx (2004) point out that both bases allow a mixed model approach, and the B-spline basis can be derived from the truncated power basis. They also show that the truncated power basis has bad numerical properties, and could cause discontinuities in estimation, while the B-spline basis approach has no such issue. However, according to Taylor (2004), the truncated power basis is employed in the spline-based periodical GARCH model on high frequency commodity future return data, which produces excellent smooth estimates. Therefore, in light of Taylor's (2004) study, the truncated power basis is employed in the CPPF approach in this thesis, and the detail of the piecewise polynomial matrix used will be introduced in section 3.4.[22]

[22] The CPPF approach is related to the cubic spline approach described above. However, we do not refer to CPPF as a spline because we allow for discontinuities at each knot.

The FFF approach

An alternative approach is the FFF which was first proposed and refined by Gallant (1981, 1982 and 1984). This mathematical function, based on a Fourier series, was initially used to approximate the utility function and derive an appropriate expenditure system for the whole economy. Mathematically, it can be written as

$$\sum_{\alpha=1}^{A}\sum_{j=-J}^{J} a_{j\alpha} e^{ijk'_\alpha x} = \sum_{\alpha=1}^{A}\left\{ u_{0\alpha} + 2\sum_{j=1}^{J}[u_{j\alpha}\cos(jk'_\alpha x) - \upsilon_{j\alpha}\sin(jk'_\alpha x)] \right\} \quad , \qquad (3.14)$$

where

$$a_{j\alpha} = u_{j\alpha} + i\upsilon_{j\alpha}, \qquad \alpha = 1,2,3...A, \qquad j = 0, \pm1, \pm2... \pm J.$$

Whereas i is defined as the imaginary unit, k is the order of the expansion, a_j is the coefficient given by

$$a_j = u_j + i\upsilon_j \quad . \qquad (3.15)$$

Recently, the FFF was widely applied in two aspects of economics and finance: estimation of production and banking efficiency, and modeling high frequency volatility periodicity.

In the former aspect, Chung et al. (2001) and Huang and Wang (2001) both applied the FFF in estimating the scale and scope of the Asian banking industry. Huang and Wang (2004) expanded the FFF and applied it to panel data to estimate multiproduct banking efficiency. Featherstone and Cader (2005) employed the FFF in a Bayesian econometrics context to evaluate agricultural production. And Yu et al. (2007) adapted the FFF to estimate agricultural banking efficiency.

Within a volatility context, Andersen and Bollerslev (1998) introduced the FFF into high frequency data volatility modelling. In their study, under GARCH framework, the FFF was used to estimate an intraday periodicity component in order to capture volatility reactions to macroeconomic announcements. The FFF within their study has been simplified as

$$f(\theta,t,n) = \mu_0 + \sum_{k=1}^{D} \lambda_k \cdot I_k(t,n) + \sum_{p=1}^{P} (\delta_{c,p} \cdot \cos\frac{p2\pi}{N} n + \delta_{s,p} \cdot \sin\frac{p2\pi}{N} n) \quad , \qquad (3.16)$$

where $I_k(t,n)$ is the indicator of event k during time interval n on day t, θ is the parameter vector to be estimated, and μ_0, λ_k, $\delta_{c,p}$ and $\delta_{s,p}$ are the fixed coefficients to be estimated (Andersen and Bollerslev, 1998). Moreover, Andersen et al. (2000) applied the FFF in the Japanese stock market, while Bollerslev et al. (2000) employed the FFF in analyzing the US bond market. More recently, Evans and Speight (2010) further adopted the FFF in the foreign exchange market.

The advantages of the FFF approach are: firstly, in the context of high frequency data, the macroeconomic news announcement effect has been filtered by the periodic pattern of the FFF, so there is no need to model the macroeconomic news announcement effect; secondly, the FFF approach creates a smooth pattern for volatility dynamics and changes; thirdly, the FFF approach is based on sound mathematics and the fit of the periodicity of financial data is widely agreed.

Although the cubic spline approach and FFF approach are widely used in the financial literature, the majority of studies use high frequency data in a financial derivatives market, banking industry or foreign exchange market. There is hard to find any study using both approaches to estimate the downside and upside components of risk in stock markets. This chapter employs the CPPF and FFF as tools, with various numbers of knots and the AIC used to uncover the best fit of beta, downside beta and upside beta estimates of monthly data with a long span in the US stock market and to improve the goodness of fit of asset pricing models.

3.3 Data

The US data used in this chapter are the same as those used in Chapter 2.[23]

3.4 Econometrics models and methods

All econometric models in this chapter are based on the well-known market model proposed by Sharpe (1964). It can be written as

$$R_{it} - R_{ft} = \alpha_{it} + \beta_{it} \cdot (R_{Mt} - R_{ft}) + \varepsilon_{it} \quad , \tag{3.17}$$

where R_{it} is the rate of return of stock i at time t, R_{ft} is the risk free rate at time t, α_{it} is the constant at time t, β_{it} is the coefficient to be estimated and represents the co-movement between stock i and the market at time t, R_{Mt} is the rate of return of market portfolio at time t, $(R_{Mt} - R_{ft})$ is the excess return of the market portfolio, and ε_{it} is the error term of stock i at time t. It is this equation that will be estimated using the CPPF and FFF models.

[23] For a summary of data, see Table 2.1.

For convenience, we define

$$xR_{it} = R_{it} - R_{ft}\text{,} \tag{3.18}$$

and

$$xR_{Mt} = R_{Mt} - R_{ft}\text{,} \tag{3.19}$$

where xR_{it} is the excess rate of return of stock i at time t, and xR_{Mt} is the excess rate of return of the market portfolio at time t.

3.4.1 The CPPF model

In this section, the CPPF (with knots) model is described. By using the CPPF model, the excess rate of return on the market portfolio xR_M will be divided into different numbers of series depending on the number of knots selected, thus allowing the betas to vary over time.

Deciding the number of knots to use is an interesting tradeoff (Stone, 1986). If a small number of knots are chosen, the estimates will be over-smooth with less variability, and could also be biased. By contrast, if a high number of knots is selected, the bias can be avoided, however, it will also lead to a high variability of estimates in the fit and could result in overfitting. Eilers and Marx (1996) discovered that up to 4 to 5 knots is most appropriate for most applications, therefore, the number of knots selected for the CPPF model will vary from 0 to 5.

Placement of knots follows the quintile method proposed by Stone (1986). In his study, he found that placing knots according to the quintile point with respect to the total number of observations results in less bias than placing knots according to a fixed number of observations. Therefore, the knots are placed at the quintile points as follows:

Table 3.1 Placement points of knots

Number of knots	0	1	2	3	4	5
Placement points		50%	33.3%	25%	20%	16.6%
			66.6%	50%	40%	33.30%
				75%	60%	50%
					80%	66.6%
						83.3%

The econometric models used here take advantage of the CPPF approach, and apply it to the classic market model. To estimate the beta coefficient of each stock, the model, in matrix terms, can be written as

$$\mathbf{XR_i = \alpha_i} + \mathbf{(XR_M} \odot \mathbf{S_N)\cdot B_i} + \boldsymbol{\varepsilon}_\mathbf{i} \qquad N = 0, 1, 2, 3, 4, 5, \qquad (3.20)$$

where $\mathbf{XR_i}$ is a $(t\times1)$ column vector of excess returns of stock *i*, $\boldsymbol{\alpha}_\mathbf{i}$ is a $t\times1$ column vector, $\mathbf{XR_M}$ is a $(t\times1)$ column vector of excess returns of the market portfolio, $\mathbf{S_N}$ is a $(t\times n)$ piecewise polynomial matrix with *N* representing the number of knots, $\odot$ is the element to element multiplication sign which results in ($\mathbf{XR_M} \odot \mathbf{S_N}$) becoming a $(t\times n)$ matrix,[24] $\mathbf{B_i}$ is the $(n\times1)$ estimated beta column vector, and $\boldsymbol{\varepsilon}_\mathbf{i}$ is the $(t\times1)$ column vector error term.

[24] $\odot$ is conventionally used as an element to element multiplication sign when two matrices are in the same rank, we borrow it here for different rank matrices for the sake of simplicity.

Specifically, the piecewise polynomial matrix, $\mathbf{S_N}$, varies along with the number of knots selected. When the CPPF has no knots, $\mathbf{S_0}$ can be expressed as

$$S_0 = \begin{bmatrix} 1^0 & 1 & 1^2 & 1^3 \\ 2^0 & 2 & 2^2 & 2^3 \\ 3^0 & 3 & 3^2 & 3^3 \\ 4^0 & 4 & 4^2 & 4^3 \\ \vdots & \vdots & \vdots & \vdots \\ t^0 & t & t^2 & t^3 \end{bmatrix} . \tag{3.21}$$

Moreover, when one knot is selected, the knot will be placed at the 50% point of observations, with the $\mathbf{S_0}$ elements remaining in $\mathbf{S_1}$, plus new elements added in with elements valued 0 above the knot, therefore $\mathbf{S_1}$ can be written as

$$S_1 = \begin{bmatrix} 1^0 & 1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 \\ 2^0 & 2 & 2^2 & 2^3 & 0 & 0 & 0 & 0 \\ 3^0 & 3 & 3^2 & 3^3 & 0 & 0 & 0 & 0 \\ 4^0 & 4 & 4^2 & 4^3 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \left(\frac{t}{2}\right)^0 & \left(\frac{t}{2}\right) & \left(\frac{t}{2}\right)^2 & \left(\frac{t}{2}\right)^3 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 \\ \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 \\ \vdots & \vdots & \vdots & \vdots & 3^0 & 3^1 & 3^2 & 3^3 \\ \vdots & \vdots & \vdots & \vdots & 4^0 & 4^1 & 4^2 & 4^4 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ t^0 & t & t^2 & t^3 & \left(\frac{t}{2}\right)^0 & \left(\frac{t}{2}\right) & \left(\frac{t}{2}\right)^2 & \left(\frac{t}{2}\right)^3 \end{bmatrix} , \tag{3.22}$$

The expression for $\mathbf{S_N}$ with two knots (N=2), three knots (N=3), four knots (N=4) and five knots (N=5) can be found in the Appendix. It can be seen from the expression for $\mathbf{S_0}$ and $\mathbf{S_1}$ that as the number of knots increases, the number of columns in $\mathbf{S_N}$ will increase. More precisely, for every one extra knot placed, the number of columns in $\mathbf{S_N}$ will increase by 4, so the dimensions of $\mathbf{S_0}$, $\mathbf{S_1}$, $\mathbf{S_2}$, $\mathbf{S_3}$, $\mathbf{S_4}$ and $\mathbf{S_5}$ will be ($t\times4$), ($t\times8$),

$(t \times 12)$, $(t \times 16)$, $(t \times 20)$ and $(t \times 24)$ respectively.

The OLS regression is applied to each stock to get the vector of beta estimates. Then, beta estimates for each stock at each point in time ($\mathbf{B_S}$ in vector form) can be calculated as follows:

$$\mathbf{B_S} = \mathbf{S_N} \cdot \mathbf{B_i}, \tag{3.23}$$

It can be seen from equation (3.14) that $\mathbf{B_S}$ is a product of a $(t \times n)$ matrix $\mathbf{S_N}$ and a $(n \times 1)$ estimated beta vector $\mathbf{B_i}$, therefore regardless of the number of knots placed in the function, the rank of $\mathbf{B_S}$ will always be $t \times 1$. Since the number of knots varies from 0 to 5, there will be 6 possible $\mathbf{B_i}$ vectors for each stock corresponding to the number of knots used. In order to find the best fit for each stock, we follow Eilers and Marx's (1996) study, and use the AIC.[25] The AIC can be expressed as

$$AIC = -2\ln(L) + 2k \quad , \tag{3.24}$$

where L is the maximium value of the likelihood function, and k is the number of parameters within the model. It can be seen from Figure 1 in the Appendix that there are discontinuities at knots points, while the fitted values are smooth between each knot. These discontinuities are due to the column of ones in the piecewise polynomial matrix, and in these cases, we let the data to decide the appropriate value of the estimates. As a comparison, sample plots between the CPPF and cubic spline estimates can be found in Figure 2 in the Appendix.

[25] The Schwarz Information Criteria (SIC) can also be used to determine the appropriate number of knots, this chapter chooses AIC instead of SIC since the SIC shows less tolerance when the number of parameters in the model is high, according to Eilers and Marx's study (1996).

In order to calculate the downside and upside beta estimates by using the CPPF model, the same logic is used with equation (3.11) modified. Referring to Ang et al. (2006), the downside beta and upside beta are calculated as

$$\beta^{-} = \frac{\operatorname{cov}(xR_i, xR_M \mid xR_M < \overline{xR_M})}{\operatorname{var}(xR_M \mid xR_M < \overline{xR_M})} \quad , \tag{3.25}$$

and

$$\beta^{+} = \frac{\operatorname{cov}(xR_i, xR_M \mid xR_M \geq \overline{xR_M})}{\operatorname{var}(xR_M \mid xR_M \geq \overline{xR_M})} \quad , \tag{3.26}$$

where $\overline{xR_M}$ is the average market excess return over the sample period of the stock, and previous notations hold. In light of Ang et al. (2006), dummy variables (vectors) D_{1i} and D_{2i} are created and employed for each stock. D_1 and D_2 (with time subscript t) can be expressed as

$$D_{1i} = 1 \text{ and } D_{2i} = 0 \text{ if } \quad xR_{M,t} < \overline{xR_M} \quad , \tag{3.27}$$

and

$$D_{1i} = 0 \text{ and } D_{2i} = 1 \text{ if } \quad xR_{M,t} \geq \overline{xR_M} \quad , \tag{3.28}$$

It can be seen from equation (3.18) and (3.19) that $D_{1i} = 1$ and $D_{2i} = 0$ if the market excess return at time t is below the average market excess return, while $D_{1i} = 0$ and $D_{2i} = 1$ if the market excess return at time t is above the average market excess return. Then two more variables are created as follows:

$$D_{1i}xR_M = D_{1i} \odot xR_M \quad , \tag{3.29}$$

$$D_{2i}xR_M = D_{2i} \odot xR_M \quad . \tag{3.30}$$

It can be seen from equations (3.20) and (3.21) that two new variables $D_{1i}xR_M$ and $D_{2i}xR_M$ are the element to element products of dummy variables of stock i and the corresponding excess market return over the sample period of the stock. For the former, observations are excess market returns if they are below the average excess market return over the sample period, and 0 otherwise. For the latter, observations are excess market returns if they are above the average excess market return over the sample period, and 0 otherwise.

The econometric model used to estimate downside and upside betas for each stock, in matrix form, can be written as

$$\mathbf{XR_i = D_{1i} + D_{2i} + (D_{1i}XR_M \odot S_N)\cdot B_i^- + (D_{2i}XR_M \odot S_N)\cdot B_i^+ + \varepsilon_i}\ , \tag{3.31}$$

where N = {0, 1, 2, 3, 4, 5}, $\mathbf{B_i^-}$ and $\mathbf{B_i^+}$ are the (n×1) estimated downside and upside beta estimate column vectors. Since the number of knots varies from 0 to 5, there will be 6 pairs of $\mathbf{B_i^-}$ and $\mathbf{B_i^+}$ vectors. The best downside and upside beta estimates as determined by the AIC for a stock at each point of time, ($\mathbf{B_S^{-*}}$ and $\mathbf{B_S^{+*}}$ in vector form) can be conducted as follows

$$\mathbf{B_S^{-*}} = \mathbf{S_N} \cdot \mathbf{B_i^-}, \tag{3.32}$$

and

$$\mathbf{B_S^{+*}} = \mathbf{S_N} \cdot \mathbf{B_i^+}. \tag{3.33}$$

As mentioned in the previous paragraph, regardless of the number of knots placed in the function, the dimensions of $\mathbf{B_S^{-*}}$ and $\mathbf{B_S^{+*}}$ will always be t×1. Both downside and upside betas can be interpreted in an analogous manner to classic beta regarding to downside and upside market.

3.4.2 The FFF model

In this section, the FFF model is described in detail. In light of Andersen and Bollerslev (1998), Andersen et al. (2000), Bollerslev et al. (2000), and Evans and Speight (2010), the econometric model with the FFF specification employed in this chapter is defined as

$$xR_{it} = \alpha_{it} + \sum_{p=1}^{P}[\beta_{cos,p} \cdot (\cos\frac{p2\pi}{N} n \cdot xR_{Mt}) + \beta_{\sin,p} \cdot (\sin\frac{p2\pi}{N} n \cdot xR_{Mt})] + \varepsilon_{it}, \quad (3.34)$$

where α_{it} is the constant, $\beta_{cos,p}$ and $\beta_{sin,p}$ are the coefficients to be estimated for stock *i*, *N* is the total number of observations of stock *i*, *n* is the order of observations with *n* = {1, 2, 3…t} , ε_{it} is the error term of stock *i* at time *t*, and *p* is the order of the FFF. The order of the FFF can vary from 1 to infinity. However, in order to provide efficient and unbiased estimates, according to previous studies, we chose up to 4.[26] In this chapter, the order from 1 to 4 is selected to examine and discover the best fit of the estimates.

The OLS regression is applied to each stock to get the $\beta_{cos,p}$ and $\beta_{sin,p}$ estimates. The AIC will then be used for each regression. Since an order of 1 to 4 is examined, there are 4 AICs for each stock. Taking advantage of the nature of the AIC, the regression that produces the least AIC will indicate the optimal fit. To calculate the best estimate for a stock at each point of time, the AIC supported estimates of $\beta_{cos,p}$ and $\beta_{sin,p}$ for each stock are used to get the best estimates at each point of time, specifically,

[26] See Andersen and Bollerslev (1998), and Evans and Speight (2010) for similar assumptions.

$$\beta_F^* = \sum_{p=1}^{P} (\beta_{cos,p} \cdot \cos\frac{p2\pi}{N} n + \beta_{\sin,p} \cdot \sin\frac{p2\pi}{N} n) \quad . \qquad (3.35)$$

In order to calculate the downside and upside beta estimates using the above FFF model, the same logic is followed as in the CPPF case. The same variables created in equation (3.20) and (3.21) are created and employed for each stock in the new FFF model and the new market model is given by

$$xR_{it} = D_{1i} + D_{2i} + \sum_{p=1}^{P} [\beta_{\cos,p}^{-} \cdot (\cos\frac{p2\pi}{N} n \cdot D_{1i} xR_{Mt}) + \beta_{\sin,p}^{-} \cdot (\sin\frac{p2\pi}{N} n \cdot D_{1i} xR_{Mt})]$$

$$+ \sum_{p=1}^{P} [\beta_{\cos,p}^{+} \cdot (\cos\frac{p2\pi}{N} n \cdot D_{2i} xR_{Mt}) + \beta_{\sin,p}^{+} \cdot (\sin\frac{p2\pi}{N} n \cdot D_{2i} xR_{Mt})] + \varepsilon_{it} , \qquad (3.36)$$

where $\beta_{cos,p}^{-}$ and $\beta_{sin,p}^{-}$ are the downside market coefficients to be estimated for stock i, and $\beta_{cos,p}^{+}$ and $\beta_{sin,p}^{+}$ are the upside market coefficients to be estimated for stock i, and previous notations hold. As in equation (3.22), there is no conventional constant term in the model, rather, full set of dummy variables are instead used. Since the order of the FFF examined varies from 1 to 4, there will be 4 groups of $\beta_{cos,p}^{-}$, $\beta_{sin,p}^{-}$, $\beta_{cos,p}^{+}$ and $\beta_{sin,p}^{+}$ for each stock. For each group an AIC value is calculated and the lowest value indicates the best fit group of $\beta_{cos,p}^{-}$, $\beta_{sin,p}^{-}$, $\beta_{cos,p}^{+}$ and $\beta_{sin,p}^{+}$.

Furthermore, the best downside and upside beta estimates for each point in time are given by

$$\beta_F^{-*} = \sum_{p=1}^{P} (\beta_{\cos,p}^{-} \cdot \cos\frac{p2\pi}{N} n + \beta_{\cos,p}^{-} \cdot \sin\frac{p2\pi}{N} n) \quad , \qquad (3.37)$$

and

$$\beta_F^{+*} = \sum_{p=1}^{P} (\beta_{\cos,p}^{+} \cdot \cos\frac{p2\pi}{N} n + \beta_{\cos,p}^{+} \cdot \sin\frac{p2\pi}{N} n) \quad , \qquad (3.38)$$

and all previous notations hold. In the next section, the empirical results will be demonstrated and analyzed in detail. Figure 3 in Appendix shows the plots of FFF estimates

3.5 Empirical results

As the method explained in the previous section, the best fits for both the CPPF and FFF models are obtained. In order to illustrate the results in a clearer way, having the best fitted estimates β_S^*, β_S^{-*} and β_S^{+*} for the CPPF model and β_F^*, β_F^{-*} and β_F^{+*} for the FFF model, the number of stocks with corresponding numbers of knots or orders and the percentage of the whole sample are shown in Table 3.2 and Table 3.3 respectively.

Table 3.2 Stocks With Corresponding Knots To Construct $\boldsymbol{\beta_S^*}$, $\boldsymbol{\beta_S^{-*}}$ And $\boldsymbol{\beta_S^{+*}}$

This table reports the number and percentage of stocks with different knots to construct the best fit estimates of CPPF model.

Knots		0	1	2	3	4	5
β_S^*	Number of Stocks	9409	1455	861	655	614	563
	Percentage to Whole sample	69.40%	10.73%	6.35%	4.83%	4.53%	4.15%
$\beta_S^{-*}$and β_S^{+*}	Number of Stocks	9399	903	483	416	729	1627
	Percentage to Whole sample	69.33%	6.66%	3.56%	3.07%	5.38%	12.00%

For the CPPF model, it can be seen from Table 3.2 that for 9409 stocks (69.4%),[27] best estimates are obtained when no knots are used. For other knot values, the number of stocks decreases. Typically when 5 knots are used, just 563 stocks (4.15%) produced the best estimates. Similar results are obtained when the downside and upside beta estimates are constructed.

For the FFF model, it is clear from Table 3.3 that to construct β_F^*, 6204 stocks (45.76%) have an order of 1. Orders 2, 3 and 4 are generally selected less often. This pattern is even more obvious when constructing β_F^{-*} and β_F^{+*}, 8377 stocks (61.79%) produce the best estimates with order 1, 2293 stocks (16.91%) with order 2, and the number of stocks with order 3 and 4 are 1429 (10.54%) and 1458 (10.75%), respectively.

Table 3.3 Stocks With Corresponding Orders To Construct $\boldsymbol{\beta_F^*}$, $\boldsymbol{\beta_F^{-*}}$ And $\boldsymbol{\beta_F^{+*}}$

This table reports the number and percentage of stocks in different orders to construct the best fit estimates of the FFF model.

Order		1	2	3	4
β_F^*	Number of Stocks	6204	2746	2099	2508
	Percentage to Whole sample	45.76%	20.26%	15.48%	18.50%
$\beta_F^{-*}$and β_F^{+*}	Number of Stocks	8377	2293	1429	1458
	Percentage to Whole sample	61.79%	16.91%	10.54%	10.75%

[27] In the brackets are the percentage to the whole sample.

Furthermore, the relationships among stock returns and corresponding beta, downside beta and upside beta estimates for the CPPF model and the FFF models are examined. In order to uncover the relationship in a cross-sectional fashion, following the methodology used in Chapters 2, stocks at each point of time are cross-sectionally assigned into five portfolios according to the value of the estimate. Since the beta, upside beta and downside beta estimates for both models are not independent of each other, to distinguish the effects among them, the relative upside beta, denoted by $(\beta^+-\beta)$ and relative downside beta denoted by $(\beta^--\beta)$ are considered. To sort the portfolio, at each point of time, all stocks are sorted into five quintiles according to the value of the beta estimate. Therefore, portfolio 1 contains stocks with the lowest 20% of estimates, portfolio 2 contains stocks with the second lowest 20% of estimates, and so on. When stocks are sorted into 5 portfolios at each point of time,[28] the equally weighted average of the estimate for each portfolio and the corresponding average annualized stock returns are calculated. The results of both models are summarized in Table 3.4 and Table 3.5, respectively.

It can be seen from the CPPF model results in Table 3.4 that when sorting by β_S^*, portfolio 1 has an average β_S^* of -0.24 while on the other hand, portfolio 5 has an average β_S^* of 2.68. Consistent with the literature, the average annualized realized rates of return of each portfolio show an ascending order as the average β_S^* increases, portfolio 1 yields a return of 1.53% while portfolio 5 shows a return of 24.84%. The

[28] Since monthly data are used in this chapter, and the whole sample is from January 1960 to December 2010, there are 612 time points.

average β_S^{-*} and β_S^{+*} values of each portfolio follow the same trend as β_S^*, an average β_S^{-*} is 0.47 in portfolio 1 and increases to 2.18 in portfolio 5. Similarly, the average β_S^{+*} is -0.21 in portfolio 1 and increases to 2.4 in portfolio 5.

Interestingly, a different pattern in returns was demonstrated when stocks are sorted by β_S^{-*}. It is clear that average returns demonstrate a reversed trend while average β_S^* shows the same ascending trend from portfolio 1 to portfolio 5 along with the increase of β_S^{-*}. β_S^{-*} is -7.2 in portfolio 1 with an average β_S^* of 0.48 and an average return of 25%, while in portfolio 5, β_S^{-*} grows to 10.01 with an average β_S^* increasing to 1.86 and the average return drops to -2.67%. The pattern of β_S^{+*} is generally increasing but with a subtle variation in that, it drops from 0.85 to 0.62 from portfolio 1 to portfolio 2, and then keeps growing to portfolio 5 ending up with a value of 1.23. Notably, although β_S^* and β_S^{+*} still have increasing trends in this panel, the difference between values for portfolio 1 and portfolio 5 (1.38 and 0.39 respectively) are narrower than the ones in Panel 1 (2.92 and 2.62 respectively).

When stocks are sorted by β_S^{+*}, a similar pattern appears to those in Panel 1. It can be seen from Panel 3 that, β_S^{+*} is -3.87 in portfolio 1 with an average β_S^* of 0.42 and an average return of -11.93%, and in portfolio 5, β_S^{+*} grows to 5.94 with average β_S^* increasing to 2.11 and average returns increasing to 35.58%. The pattern of β_S^{+*} is also generally increasing but with a sudden drop from 1.32 to 0.91 between portfolio 1 to portfolio 2, and then keeps increasing to portfolio 5 and ends up with a value of

Table 3.4 Relationships Between Stock Returns And CPPF Factor Loadings

This table presents the relationship between excess stock returns and factor loading of the CPPF model. The column labeled "return" reports the average stock returns over a one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1. Notably, in a perfect market, the average value of β_S^* is assumed to be 1.

Panel 1 Stocks Sorted by β_S^*

Portfolio	Return	β_S^*	β_S^{-*}	β_S^{+*}
1 Low	1.53%	-0.24	0.47	-0.21
2	6.76%	0.64	0.80	0.32
3	8.95%	0.99	1.23	0.84
4	11.59%	1.41	1.54	1.25
5 High	24.84%	2.68	2.18	2.4
High - Low	23.30%	2.92	1.72	2.62

Panel 2 Stocks Sorted by β_S^{-*}

Portfolio	Return	β_S^*	β_S^{-*}	β_S^{+*}
1 Low	25.00%	0.48	-7.2	0.85
2	11.08%	0.77	0.68	0.68
3	10.53%	1.03	1.1	0.83
4	9.73%	1.33	1.61	0.99
5 High	-2.67%	1.86	10.01	1.23
High - Low	-27.67%	1.38	17.21	0.39

Panel 3 Stocks Sorted by β_S^{+*}

Portfolio	Return	β_S^*	β_S^{-*}	β_S^{+*}
1 Low	-11.93%	0.42	1.32	-3.87
2	6.67%	0.71	0.91	0.34
3	9.85%	0.96	1.21	0.81
4	13.43%	1.28	1.35	1.37
5 High	35.58%	2.11	1.42	5.94
High - Low	47.51%	1.69	0.1	9.81

Panel 4 Stocks Sorted by $(\beta_S^{-*}-\beta_S^*)$

Portfolio	Return	β_S^*	β_S^{-*}	β_S^{+*}
1 Low	35.87%	1.64	-6.6	2.03
2	14.74%	1.11	0.88	1.09
3	9.72%	0.96	1.05	0.72
4	5.72%	0.95	1.38	0.6
5 High	-12.35%	0.83	9.49	0.16
High - Low	-48.22%	-0.81	16.09	-1.87

Panel 5 Stocks Sorted by $(\beta_S^{+*}-\beta_S^*)$

Portfolio	Return	β_S^*	β_S^{-*}	β_S^{+*}
1 Low	-6.16%	1.35	1.88	-3.38
2	7.24%	1.03	1.37	0.52
3	9.94%	0.96	1.16	0.84
4	13.12%	1	1.1	1.21
5 High	29.51%	1.14	0.71	5.4
High - Low	35.67%	-0.2	-1.18	8.78

Panel 6 Stocks Sorted by $(\beta_S^{-*}-\beta_S^{+*})$

Portfolio	Return	β_S^*	β_S^{-*}	β_S^{+*}
1 Low	36.79%	1.43	-6.32	5.09
2	14.15%	1.05	0.89	1.2
3	10.07%	0.96	1.05	0.83
4	6.43%	0.98	1.35	0.53
5 High	-13.76%	1.05	9.23	-3.06
High - Low	-50.55%	-0.38	15.55	-8.15

1.42. Compared to Panel 1, the spread of β_S^*, and β_S^{-*} between portfolio 5 and portfolio 1 is less, however the spread of β_S^{+*} and average return is much higher (9.81 and 47.51% in Panel 3, while 2.62 and 23.3% in Panel 1). It is clear that β_S^{+*} has the same positive impact on portfolio returns as β_S^*.

In order to examine how β_S^{-*} is driving the return not considering the impact of β_S^*, a new estimate (β_S^{-*}- β_S^*) is employed in the analysis. Using this estimate to sort portfolios could discover the unique property of β_S^{-*} after controlling for β_S^*. When stocks are sorted by (β_S^{-*}- β_S^*), an unfamiliar pattern appears in Panel 4. From portfolio 1 to portfolio 5, all average returns, β_S^* and β_S^{+*} are in descending order while only β_S^{-*} increases from -6.6 to 9.49. Although in Panel 1, Panel 2 and Panel 3, β_S^{-*} are also in ascending order, the spread of average returns in Panel 4 is the highest and reaches -48.22%. So controlling for β_S^*, it can be seen that β_S^{-*} shows a negative relationship with portfolio returns and β_S^*.

As with (β_S^{-*}- β_S^*), (β_S^{+*}- β_S^*) is employed to uncover the unique property of β_S^{+*} after controlling for β_S^*. It can be seen from Panel 5 that from portfolio 1 to portfolio 5, both average return and β_S^{+*} are in ascending order, starting at -6.16% and -3.38, increasing to 29.51% and 5.4, respectively. β_S^{-*} exhibits a descending trend for the first time within these panels. It drops from -1.88 to 0.71. A U-shaped pattern in β_S^* is apparent, it starts at 1.35 in portfolio 1 and drops to 0.96 in portfolio 3, but restores to 1.14 in portfolio 5.

In Panel 6, (β_S^{-*}- β_S^{+*}) is adopted to sort the portfolio in order to control β_S^{+*} from β_S^{-*} and for the sake of precision. In can be seen from Panel 6 that both average returns and β_S^{+*} are in descending orders, starting at 36.79% and 5.09 and dropping to 13.76% and -3.06 respectively, while β_S^{-*} exhibits an ascending trend increasing from -6.32 to 9.23. As in Panel 5, a U-shaped pattern appears in β_S^{*}, starting at 1.43 in portfolio 1 dropping to 0.96 in portfolio 3 and recovering to 1.05 in portfolio 5. Notably, the spread of returns in Panel 6 is the highest among all 6 panels at -50.55%.

Regarding the FFF model, it can be seen from Table 3.5 that when sorting by β_F^{*}, portfolio 1 has an average β_F^{*} of -1.19 while on the other hand, portfolio 5 shows an average β_F^{*} of 1.22. Again, consistent with the literature, the average annualized rates of return to each portfolio are presented in an ascending order with the β_F^{*}. Portfolio 1 yields a return of 3.54%, while portfolio 5 has a return of 22.26%. Average β_F^{-*} and β_F^{+*} for each portfolio follow the same trend as β_F^{*}, with an average β_F^{-*} of -0.74 in portfolio 1 and 0.76 in portfolio 5. Similarly, the average β_F^{+*} is -0.87 in portfolio 1 and increases to 0.95 in portfolio 5. A different pattern was demonstrated when stocks are sorted by β_F^{-*}. It is clear that average returns exhibit a reversed trend while average β_F^{*} shows the same ascending trend from portfolio 1 to portfolio 5 along with the increase of β_F^{-*}. β_F^{-*} is -1.32 in portfolio 1 with an average β_F^{*} of -0.67 and average return of 21.88%, while in portfolio 5, β_F^{-*} grows to 1.33, average β_F^{*} increases to 0.69 and the average return drops to -0.03%. The pattern of β_F^{+*} is consistent with β_F^{-*}, it starts at -0.31 in portfolio 1, and then keeps growing to

Table 3.5 Relationships Between Stock Returns And The FFF Factor Loadings

This table presents the relationship between excess stock returns and factor loading of the FFF model. The column labeled "return" reports the average stock returns over one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1. Notably, in a perfect market, the average value of β_F^* depends on the average value of the intercept.

Panel 1 Stocks Sorted by β_F^*

Portfolio	Return	β_F^*	β_F^{-*}	β_F^{+*}
1 Low	3.54%	-1.19	-0.74	-0.87
2	7.51%	-0.37	-0.26	-0.28
3	9.11%	-0.01	-0.01	-0.03
4	11.24%	0.36	0.24	0.23
5 High	22.26%	1.22	0.76	0.95
High - Low	18.72%	2.41	1.49	1.82

Panel 2 Stocks Sorted by β_F^{-*}

Portfolio	Return	β_F^*	β_F^{-*}	β_F^{+*}
1 Low	21.88%	-0.67	-1.32	-0.31
2	13.09%	-0.24	-0.38	-0.12
3	10.46%	-0.01	-0.01	-0.01
4	8.22%	0.23	0.37	0.11
5 High	-0.03%	0.69	1.33	0.33
High - Low	-21.91%	1.36	2.65	0.64

Panel 3 Stocks Sorted by β_F^{+*}

Portfolio	Return	β_F^*	β_F^{-*}	β_F^{+*}
1 Low	-7.72%	-0.81	-0.29	-1.28
2	5.02%	-0.27	-0.13	-0.36
3	9.44%	-0.02	-0.01	-0.02
4	14.58%	0.24	0.13	0.33
5 High	32.33%	0.86	0.3	1.33
High - Low	40.05%	1.67	0.59	2.61

Panel 4 Stocks Sorted by $(\beta_F^{-*}-\beta_F^*)$

Portfolio	Return	β_F^*	β_F^{-*}	β_F^{+*}
1 Low	33.18%	0.52	-0.71	0.65
2	16.29%	0.11	-0.18	0.19
3	10.34%	-0.02	-0.02	-0.01
4	4.66%	-0.14	0.17	-0.21
5 High	-10.84%	-0.48	0.74	-0.61
High - Low	-44.02%	-1	1.45	-1.27

Panel 5 Stocks Sorted by $(\beta_F^{+*}-\beta_F^*)$

Portfolio	Return	β_F^*	β_F^{-*}	β_F^{+*}
1 Low	-4.27%	0.36	0.57	-0.63
2	5.48%	0.12	0.21	-0.14
3	9.74%	0	-0.01	-0.01
4	14.99%	-0.14	-0.23	0.12
5 High	27.69%	-0.34	-0.54	0.66
High - Low	31.95%	-0.7	-1.1	1.29

Panel 6 Stocks Sorted by $(\beta_F^{-*}-\beta_F^{+*})$

Portfolio	Return	β_F^*	β_F^{-*}	β_F^{+*}
1 Low	36.12%	0.17	-0.81	0.85
2	16.44%	-0.02	-0.24	0.17
3	10.15%	-0.02	-0.01	-0.02
4	4.34%	-0.02	0.22	-0.2
5 High	-13.41%	-0.12	0.85	-0.81
High - Low	-49.53%	-0.3	1.66	-1.66

portfolio 5 ending up with a value of 0.33. Notably, although β_F^* and β_F^{+*} still have an increasing trend in this panel, the difference between values in portfolio 1 and portfolio 5 (1.36 and 0.64, respectively) are narrower than the ones in Panel 1 (2.41 and 1.82, respectively)

When sorting by β_F^{+*}, a similar pattern to that in Panel 1 appears. It can be seen from Panel 3 that β_F^{+*} is -1.28 in portfolio 1 with an average β_F^* of -0.81 and an average return of -7.72%, and in portfolio 5, β_F^{+*} grows to 1.33 with an average β_F^* increasing to 0.86 and an average return increasing to 32.33%. The pattern of β_F^{-*} is consistent with β_F^{+*}, it starts at -0.29 in portfolio 1, and then keeps growing to portfolio 5 ending up with a value of 0.3. Compared to Panel 1, the spread of β_F^*, and β_F^{-*} between portfolio 5 and portfolio 1 are less. However, the spread of β_F^{+*} and average returns is much higher (2.61 and 40.05% in Panel 3, and 1.82 and 18.72% in Panel 1, respectively).

As with the CPPF model, we consider (β_F^{-*}- β_F^*) in the analysis. Using this estimate to sort portfolios could uncover further properties of β_F^{-*} after controlling for β_F^*. When stocks are sorted by (β_F^{-*}-β_F^*), an unfamiliar pattern appears in Panel 4. From portfolio 1 to portfolio 5, average returns, β_F^* and β_F^{+*} all decrease only with β_F^{-*} increasing from -0.71 to 0.74. As in Panel 1, Panel 2 and Panel 3, β_F^{-*} are also in ascending order, the spread of average returns in Panel 4 is highest and reaches -44.02%. Referring back to when stocks are sorted by (β_S^{-*}-β_S^*), a similar pattern appears. Therefore, it can be concluded that when controlling for the effect of beta, the relative downside beta estimates of both models have a negative relationship with portfolio returns, which can be interpreted as stocks tend to suffer a loss if they have large downside betas.

When (β_F^{+*}- β_F^*) is employed (to uncover the unique property of β_F^{+*} after controlling for β_F^*), it can be seen from Panel 5 that from portfolio 1 to portfolio 5, both average returns and β_F^{+*} increase, starting at -6.16% and -3.38% and increasing to 29.51% and 5.4, respectively, while both β_F^* and β_F^{-*} decrease from 0.36 and 0.57 to -0.34 and -0.54, respectively.

Also in Panel 6, (β_F^{-*}- β_F^{+*}) is adopted to sort the portfolio to control β_F^{+*} from β_F^{-*} and for the sake of precision. It can be seen from this panel that the same pattern as in Panel 4 appears. From portfolio 1 to portfolio 5, average returns, β_F^* and β_F^{+*} all decrease while only β_F^{-*} increases from -0.81 to 0.85. Notably, the spread of returns in Panel 6 is the highest among all 6 panels at -49.53%.

To sum up, the results of the CPPF and the FFF models, β_S^* and β_F^* as classic risk estimates, still have an obvious impact on driving stock returns. Specifically, when stocks are sorted by β_S^* and β_F^*, average returns follow exactly the same increasing trend with the β_S^* and β_F^* presented, even the portfolio return is inversely related to the market return . More importantly, it can be seen from these panels that β_S^{-*}, β_S^{+*},

β_F^{-*} and β_F^{+*} do have an impact on stock returns. When stocks are sorted by β_S^{-*}, (β_S^{-*}- β_S^{*}), β_F^{-*} and (β_F^{-*}- β_F^{*}), clearly, downside related estimates have a negative relationship with the realized returns, this is even more obvious in Panel 6 when stocks are sorted by (β_S^{-*}- β_S^{+*}) and (β_F^{-*}- β_F^{+*}), while when stocks are sorted by β_S^{+*}, (β_S^{+*}- β_S^{*}), β_F^{+*} and (β_F^{+*}- β_F^{*}), positive relationships appear between upside related estimates and realized returns. Moreover, the classic estimates β_S^{*} and β_F^{*} appear to have a similar impact as the upside related estimates β_S^{+*}, (β_S^{+*}- β_S^{*}), β_F^{+*} and (β_F^{+*}- β_F^{*}). To rationalize that, when downside beta is calculated, the return of the market portfolio is below the average, and very likely to be negative. The stock expected excess return is the product of beta and excess returns to the market portfolio, so when stocks are sorted by downside beta into portfolios, the larger the downside beta, the lower the return. In addition to that, the panic on the falling market of investors' could also be a reason for aggravating the negative returns.

Compared to the empirical results in the previous chapter,[29] these results are quite similar except when stocks are sorted by downside beta. In the previous chapter, when stocks are sorted by downside beta, the excess return of the portfolio increases with the increase of downside beta. However, in this chapter, it can be seen from Table 3.5 and Table 3.6 that for the CPPF and the FFF model, a reversed pattern exhibits. However, when beta is controlled and stocks are sorted by relative downside, both chapters present similar results. Therefore, it can be concluded that downside beta

[29] Table 2.2 in chapter 2.

clearly has a negative relationship with stock returns when beta is taken into account. The reversed results are attributed to different approaches used in these two chapters, in the previous chapter, a simple moving window estimation is used, while in this chapter, nonlinear time-varying approaches are used.

3.6 Fama-Macbeth regression

In this section, in order to uncover the impact of β_S^*, β_S^{-*}, β_S^{+*}, β_F^*, β_F^{-*} and β_F^{+*} on stock returns from a regression point of view, following the previous chapter, a series of Fama-Macbeth regressions are performed which employ different combinations of the above estimates as independent variables.

For the purposes of comparison and to demonstrate the importance of placing an appropriate number of knots for the CPPF model and choosing an appropriate order for the FFF model, additional variables β_{S0}, β_{S0}^-, β_{S0}^+, β_{F1}, β_{F1}^- and β_{F1}^+ are introduced. For β_{S0}, β_{S0}^- and β_{S0}^+, they are the beta estimate, downside beta estimate and upside beta estimate, respectively, for each stock at each point in time estimated with the CPPF model without placing a knot. However for β_{F1}, β_{F1}^- and β_{F1}^+, they are the beta estimate, downside beta estimate and upside beta estimate, respectively, for each stock at each point in time estimated with the FFF model with order 1.

Table 3.6 Correlations Of Factor Loadings Without Knot And In Order One

This table reports the correlation coefficients between factor loadings of both the CPPF model with zero knots and the FFF model in order one. To avoid unnecessary repetition, only the lower triangle of the matrix is shown.

	β_{S0}	β_{S0}^{-}	β_{S0}^{+}	β_{F1}	β_{F1}^{-}	β_{F1}^{+}
β_{S0}	1.0000					
β_{S0}^{-}	0.3363	1.0000				
β_{S0}^{+}	0.4326	0.0365	1.0000			
β_{F1}	0.4397	0.1345	0.1770	1.0000		
β_{F1}^{-}	0.3209	0.2611	0.0458	0.7331	1.0000	
β_{F1}^{+}	0.4620	0.0336	0.2737	0.8093	0.2886	1.0000

In order to avoid multicollinearity in the Fama-Macbeth regression, the correlation coefficient matrices for groups of variables are presented in Table 3.6 and Table 3.7. It can be seen from Table 3.6 that β_{F1} is highly correlated with both β_{F1}^{-} and β_{F1}^{+} (the correlation coefficients are 0.7331 and 0.8093, respectively). The other pairs of variables are correlated with each other to some extent, but not as highly as the two mentioned pairs (above 0.5), for instance, β_{S0} and β_{F1}^{+} exhibits the highest correlation with a coefficient of 0.462 after the two peak values. Therefore, in the following Fama-Macbeth regression, β_{F1} will not appear in the same regression with β_{F1}^{-} or β_{F1}^{+}, and the other variables will form different combinations of independent variables.

It is clear from Table 3.7 that, as in Table 3.6, high correlations appear between the FFF based estimates, with β_F^* highly correlated with β_F^{-*} and β_F^{+*} (with correlation coefficients of 0.5239 and 0.6689, respectively). The remaining variables exhibit a weaker correlation with each other. Thus in the following Fama-Macbeth regression, β_F^* will not appear in the same regression with β_F^{-*} or β_F^{+*}, and the other variables will form different combinations of independent variables. Notably, in Table 3.8, for both the CPPF and the FFF models, the upside beta estimates β_S^{+*} and β_F^{+*} are negatively correlated with the downside beta estimates of the CPPF model β_S^{-*} with correlation coefficients of -0.0001 and -0.0018, respectively.

Table 3.7 Correlations Of Factor Loading With Corresponding Knots And Order

This table reports the correlation coefficients between factor loadings of both the CPPF model with appropriate number of knots and the FFF model in appropriate order, to avoid unnecessary repetition, only the lower triangle of the matrix is shown.

	β_S^*	β_S^{-*}	β_S^{+*}	β_F^*	β_F^{-*}	β_F^{+*}
β_S^*	1.0000					
β_S^{-*}	0.0130	1.0000				
β_S^{+*}	0.0237	-0.0001	1.0000			
β_F^*	0.3154	0.0029	0.0085	1.0000		
β_F^{-*}	0.2609	0.0082	0.0025	0.5239	1.0000	
β_F^{+*}	0.3099	-0.0018	0.0098	0.6689	0.1889	1.0000

This phenomenon potentially shows that the downside beta estimates do have an opposite impact on stock returns compared to the upside beta estimates which complies with the conclusion made in the previous sections.

After deciding on the possible combinations of variables, the Fama-Macbeth regressions are performed on both groups of variables which are demonstrated in Table 3.6 and Table 3.7. Since the data are at monthly frequency from January 1960 to December 2010, there are 612 cross-sectional time points and 2,398,103 observations of each regression. Newey-West (1987) heterosecdasticity robust standard errors with 12 lags are employed to calculate the t-statistics and the adjusted R^2 values obtained from the cross-sectional regressions are provided in Table 3.8 and Table 3.9.[30]

As concluded in the previous paragraph, β_{F1} cannot not appear in the same regression with β_{F1}^{-} or β_{F1}^{+}, therefore, there are 11 possible combinations among β_{S0}, β_{S0}^{-}, β_{S0}^{+}, β_{F1}, β_{F1}^{-} and β_{F1}^{+} as independent variables. It can be seen from Table 3.8 that regression 1, 2 and 3 examine the impact of estimates of the CPPF model without a knot. Generally, these three regressions exhibit poor fit with the intercept in regression 1 and coefficients of β_{S0}^{-} and β_{S0}^{+} in regression 2 not significant at the 5% significance level and with adjusted R^2 values of 0.06 and 0.055, respectively. In regression 3, the coefficient on β_{S0}^{-} and the intercept are not significant even at the 10% significance level with an adjusted R^2 value of 0.098.

[30] Factor size could affect the significance of the estimates.

Regression 4 and 5 examine the impact of estimates of the FFF model with order 1. Estimates of regression 4 and 5 are all significant at the 1% significance level, however both regressions present low adjusted R^2 values of 0.017 and 0.029, respectively.

The remaining regressions 6 to 11 employ variables from both the CPPF model and the FFF model to examine the impact of these variables on stock returns. It is clear from Table 3.8 that with the exception of the coefficient on β_{S0}^{-} and coefficients on β_{S0}^{-} and β_{S0}^{+} in regression 10, remaining estimates are all significant at the 5% level. Among the regressions in Table 3.8, regression 11 shows the highest adjusted R^2 value at 0.106 and it also contains the most variables. Notably, consistent with the literature, the estimated coefficients of β_{S0} and β_{F1} are always positive among regressions, and illustrate that the beta estimates for both models have a positive impact on stock returns. Moreover, the estimated coefficients on β_{F1}^{-} and β_{F1}^{+} are always significant at the 1% significance level, and their signs are constantly negative and positive, and show that the downside and upside risk estimates of the FFF model have negative and positive impacts on stock returns, respectively. However, the significance and sign of the estimated coefficients on β_{S0}^{-} and β_{S0}^{+} vary across regressions in Table 3.9, therefore it is difficult to provide a definitive conclusion.

Table 3.9 shows the 11 possible combinations among β_{S}^{*}, β_{S}^{-*}, β_{S}^{+*}, β_{F}^{*}, β_{F}^{-*} and β_{F}^{+*}. It can be concluded from Table 3.9 that, similar to Table 3.8, regressions 1 to 3 examine the impact of estimates of the CPPF model with appropriate numbers of knots according to the AIC. Moreover, regressions 4 and 5 examine the impact of best estimates of the FFF model (according to the AIC). The remaining regressions 6 to 11 employ variables from both the best CPPF and FFF models to examine the impact of these variables on stock returns. Unlike Table 3.8, all estimated coefficients except the intercept in regression 1 are significant at the 1% significance level. These best fit estimates are all highly significant, and the beta coefficients are always positive, which is consistent with the classic literature (as in Table 3.8). Moreover, the estimated coefficients of downside and upside beta estimates show negative and positive signs, respectively, over all regressions, which is consistent with the conclusions made regarding Table 3.4 and Table 3.5. Furthermore, among all 11 regressions in Table 3.9, regression 11 exhibits the highest adjusted R^2 value at 0.153.

Table 3.8 Fama-Macbeth Regressions of Factor Loadings Restricted Estimates

This table reports the results of the Fama-Macbeth regression of factor loadings without knot and order. The t-statistics in the square brackets are calculated by using Newey-West (1987) heteroscedastic robust standard error with 12 lags. * denotes significance at the 10% level, ** denotes significance at the 5% level and ***denotes significance at the 1% level.

	1	2	3	4	5	6	7	8	9	10	11
Intercept	0.00155	0.00743***	0.00154	0.00852***	0.00907***	0.00212	0.00705***	0.00213*	0.00345***	0.00801***	0.00251**
	[1.14]	[3.82]	[1.21]	[3.48]	[3.90]	[1.57]	[3.88]	[1.70]	[2.89]	[4.06]	[2.24]
β_{S0}	0.00663***		0.00824***			0.00614***		0.00768***	0.00525***		0.00626***
	[3.25]		[3.68]			[3.02]		[3.46]	[2.73]		[2.98]
β_{S0}^{-}		0.0000637	-0.000721				0.00187***	-0.000678	-0.0119***	-0.000103	0.00147***
		[0.08]	[-1.28]				[2.99]	[-1.20]	[-13.04]	[-0.15]	[2.95]
β_{S0}^{+}		0.000937*	-0.000977***				-0.000884**	-0.000983**	0.0115***	0.000520	-0.00205***
		[1.90]	[-2.13]				[-2.21]	[-2.14]	[14.41]	[1.14]	[-4.72]
β_{F1}				0.00542***		0.00132**		0.00141***		0.00509***	
				[3.35]		[2.50]		[2.66]		[3.77]	
β_{F1}^{-}					-0.0113***		-0.0126***				-0.0131***
					[-9.43]		[-12.50]				[-15.27]
β_{F1}^{+}					0.0148***		0.0156***				0.0127***
					[10.83]		[12.60]				[17.27]
Number of obs	2398103	2398103	2398103	2398103	2398103	2398103	2398103	2398103	2398103	2398103	2398103
Average R^2	0.055	0.060	0.098	0.017	0.029	0.056	0.077	0.100	0.065	0.069	0.106

Table 3.9 Fama-Macbeth Regressions of Factor Loadings with Appropriate Knots and Orders

This table reports the results of the Fama-Macbeth regression of factor loadings with appropriate knots and orders on stock excess returns. The t-statistics in the square brackets are calculated by using the Newey-West (1987) heteroscedastic robust standard error with 12 lags. * denotes significance at the 10% level, ** denotes significance at the 5% level and ***denotes significance at the 1% level.

	1	2	3	4	5	6	7	8	9	10	11
Intercept	0.00186	0.00898***	0.00538***	0.00826***	0.00943***	0.00279**	0.00965***	0.00618***	0.00369***	0.00947***	0.00623***
	[1.52]	[7.05]	[5.04]	[3.56]	[4.42]	[2.19]	[7.54]	[5.76]	[3.38]	[7.14]	[6.23]
β_S^*	0.00610***		0.00448***			0.00481***		0.00383***	0.00490***		0.00384***
	[3.42]		[3.82]			[3.00]		[3.54]	[3.20]		[3.42]
β_S^{-*}		-0.0120***	-0.00948***				-0.0106***	-0.00944***	-0.00994***		-0.00850***
		[-9.82]	[-11.19]				[-10.25]	[-11.19]	[-12.94]		[-11.30]
β_S^{+*}		0.0134***	0.00911***				0.0117***	0.00890***	0.0107***		0.00823***
		[14.47]	[14.41]				[15.25]	[14.63]	[15.74]		[14.44]
β_F^*				0.00673***		0.00334***		0.00238***		0.00342***	
				[4.16]		[5.67]		[4.73]		[3.91]	
β_F^{-*}					-0.0113***		-0.00419***			-0.0111***	-0.00485***
					[-9.13]		[-8.21]			[-10.53]	[-13.74]
β_F^{+*}					0.0155***		0.00647***			0.0121***	0.00586***
					[13.00]		[11.34]			[15.64]	[13.51]
Number of obs	2398103	2398103	2398103	2398103	2398103	2398103	2398103	2398103	2398103	2398103	2398103
Adjusted R^2	0.092	0.13	0.147	0.039	0.055	0.096	0.138	0.150	0.11	0.137	0.153

However, since β_F^* cannot appear in the same regression with β_F^{-*} or β_F^{+*}, checking the alterative regression (regression 8) shows that employing β_F^* instead of β_F^{-*} and β_F^{+*}, generates the second highest adjusted R^2 among all regressions at 0.15. Therefore, it can be concluded from Table 3.9 that the variables produce a much higher adjusted R^2 value than the variables used in Table 3.8, thus indicating that placing appropriate numbers of knots in the CPPF model and selecting the appropriate order in the FFF model produces better beta estimates. Also, although β_F^* cannot appear in the same regression with β_F^{-*} or β_F^{+*}, all the variables in regression 8 and regression 11 do have significant effects on excess stock returns, while the regression that employs β_F^{-*} and β_F^{+*} outperforming the one that employs β_F^*.

Compared to the results in the previous chapter,[31] using the CPPF and the FFF approach and allowing the beta estimates to be time-varying delivers statistically significant improvements in fit of the cross-sectional asset pricing model. With all coefficients significant, the highest adjusted R^2 value is only 0.045 in chapter 2,[32] while the highest adjusted R^2 value is 0.153 in this chapter. It can be concluded that the nonlinear time-varying estimates have more significant explanatory power than the moving window estimates considered in the previous chapter.

[31] Table 2.13 and Table 2.14 in Chapter 2.
[32] Regression 4 in Table 2.13 in Chapter 2.

3.7 Conclusion

It can be concluded from this chapter that the beta, upside beta and downside beta estimates produced by the CPPF model and the FFF model do have a significant impact on cross-sectional stock returns. The beta estimates, whose role has been doubted in the literature for several decades, are significant in driving the stock returns for both models. Moreover, the downside and upside beta estimates of both models demonstrate reversed impacts on stock returns. The reason for that is when downside beta is calculated, the return of the market portfolio is below the average, and very likely to be negative. The expected excess stock return is the product of beta and excess returns to the market portfolio, so when stocks are sorted by downside beta into portfolios, the larger downside beta, the lower the return, and vice versa. The former ones show negative impacts on stock returns, while the latter ones, consistent with the beta estimates, have positive effects (both are significant). For stocks with negative downside beta, they are inversely related with downside risk and more desirable in a downside market, therefore positive returns are rewarded.

Moreover, placing the appropriate number of knots in the CPPF model and selecting the correct order of the FFF model are crucial procedures to generate the best fit estimates according to the AIC. It has been shown in this chapter that estimates with the appropriate number of knots (or order) deliver more significant impacts on stock returns within the cross-sectional return regressions with respect to those based on non-optimal knots or orders. Furthermore, in order to avoid potential multicollinearity, beta estimates based on the FFF model β_F^* can be treated as an alternative variable of downside and upside beta estimates (β_F^{-*} and β_F^{+*}, respectively). However, employing β_F^{-*} and β_F^{+*} in the regression produces higher adjusted R^2 values than employing β_F^*.

Finally, compared to the previous chapter, it can be concluded that downside beta clearly has a negative relationship with stock returns when beta is controlled for. The reversed results of stock returns after sorting by downside beta (decreasing with downside beta in this chapter and reversed in the previous chapter) are attributed to different approaches used in these two chapters. In terms of fit, allowing more flexible on the data, taking time-varying estimates (generated by the CPPF and FFF approaches) as factors delivers much higher adjusted R^2 values than classic market model, therefore, the CPPF and the FFF model outperform the classic market model.

Chapter 4

The cross-sectional determinants of US stock returns: The impact of commodity market and business conditions

4.1 Introduction

In this chapter, two multi-factor non-linear models are considered: the cubic piecewise polynomial function (CPPF) model and Fourier Flexible Form (FFF) model with various knots and orders employed to examine the significance of classic, downside and upside risks. With reference to the previous chapter, using the CPPF and FFF models allows the beta estimates to be time-varying, in order to present the true relationship between variables at each point in time. We will extend the idea in this chapter, and consider a market portfolio, a commodity price index and the Aruoba-Diebold-Scotti (ADS) real business index simultaneously, as the risk factors. The AIC is adapted to uncover the most appropriate number of knots and orders for the sample. With the AIC, the best fit estimates of the classic, downside and upside risks for both models are generated. These estimates are sorted into portfolios to examine the risk-return relationship. Fama-Macbeth regressions are then performed to discover the significance of the estimates cross-sectionally. We find that all three factors have a significant impact on individual stock returns. Moreover, downside and upside estimates provide more explanatory power than classic estimates. However, the predictive power of all estimates is found to be poor. This chapter is arranged as follows: section 4.2 contains a literature review, followed by section 4.3 which is a description of the data, then section 4.4 explains the econometric models and methods applied in this chapter, sections 4.5 to 4.7 provide the empirical results, and section 4.8 is the conclusion.

4.2 Literature review

There has been a long history of literature suggesting that the use of the market factor is not enough to explain the risk-return relationship of stocks. Rose (1951) pointed out that economic news and information can be quantified and treated as an additional risk factor for stock returns. Moreover, the APT model assumes asset returns follow a multi-factor return generating process. (Ross 1976)

Among the large number of multi-factor asset pricing studies, macroeconomic variables are the most popular ones to be employed within this context. There are quite a number of studies that investigate the relationship between stock returns and inflation. For instance, in the studies by Bodie (1976), Jaffe and Mandelker (1976), Nelson (1976), Fama and Schwert (1977), Fama (1981), and Fama and Gibbons's (1982), inflation is employed as a common factor, and all of them found a negative relationship between stock returns and inflation with strong evidence provided. Apart from inflation, other macroeconomic variables have been employed as a risk factor. Chen et al. (1986) show that industrial production, the spread between long and short interest rates, expected and unexpected inflation and the spread between high and low grade bonds are significantly priced. They find that neither aggregate consumption nor oil price differences are priced separately.

Fama (1990) used the economic growth rate to proxy the shock to cash flows, and he showed that the variance of stock returns is well explained by the economic growth rate. In Chen's (1991) later study, more variables are employed. The results indicate that the lagged production growth rate and the market dividend to price ratio are significantly priced, and they are positively correlated with future market excess returns. Bilson et al (1999) attempted to use macroeconomic variables to proxy local risk factors in emerging markets to explain the volatility of stock returns, with moderate evidence found to support the hypothesis. Flannery and Protopapadakis (2002) employed seventeen macroeconomic factors as independent variables in a GARCH model, and found that the consumer price index (CPI), producer price index (PPI), monetary aggregate, balance of trade, unemployment rate and housing starts are significantly priced. Duca (2007), Granger causality has been tested between the stock market excess return and GDP, (GDP is a component of the ADS index) with the results showing strong evidence that GDP Granger causes excess market returns. Also, Gay (2008) failed to find any significant influence of exchange rates on stock returns in emerging markets.

Moreover, Gan et al. (2006), Tursoy et al. (2008) and Singh et al. (2011) employed various macroeconomic variables to explain the movement of their local stock markets (New Zealand, Turkey and Taiwan, respectively), with only weak evidence found to support their proposed ideas. Although there are a large number of studies employing macroeconomic variables in asset pricing models, few studies employ a macroeconomic factor which can measure the real economy (business) from all aspects (GDP is a good indicator, however, it fails to measure the employment rate and other important aspects). The reason for that is obvious, to measure all aspects of the economy, there are too many variables to be employed, and there are few variables available which are capable of measuring all aspects.

The innovation in this chapter is that it employs the ADS business conditions index, which measures the real economy from most aspects. The index itself is measured over a daily frequency but is computed using a number of macroeconomic variables with different frequencies. The constituents of the ADS index have been extended and modified ever since it was proposed, and the components and computing method were last fixed in 2011.[33] There are six macro components of the ADS index. At a weekly frequency, there are initial jobless claims; at a monthly frequency, there are payroll employment, industrial production, personal income less transfer payments and manufacturing and trade sales; and at a quarterly frequency, real GDP (adjusted for inflation and deflation) is employed. It would be possible to employ individual macroeconomic factors rather than the ADS index. However, since each individual factor is measured over a different frequency, using the ADS index is clearly a better choice.

Apart from macroeconomic variables, commodity prices as an indicator of the price level of essential goods have been employed in a few asset pricing studies. For instance, Hirshleifer (1989) found that the variability of stock market returns would

[33] The ADS index was set to start on 29th February 1960, and was re-estimated on 18th August 2011 due to the full release of manufacturing and trade sales in US.

increase the premium of hedging in the commodity market. Buyuksahin et al. (2010) failed to find any evidence to support the co-movement between a commodity index and stock returns. Buyuksahin and Robe (2011) pointed out that the commodity index and stock prices are correlated more closely when hedge funds perform actively in the market, while the correlation is much lower during a financial crisis. Hong and Yogo (2012) argue that commodity future prices are a good predictor of commodity returns. However, there is only weak evidence that commodity prices are a significant factor in the stock market.

Moreover, as energy, especially crude oil becomes more valuable, oil prices become more of a focus in asset pricing. There are quite a few studies that focus on the relationship between the oil price and stock returns. Sadorsky (1999) finds a significant negative relationship between oil price shocks and the US stock market. Papapetrou (2001) pointed out that oil prices can affect both stock returns and the real economy, while stock returns only appear to have a weak influence on oil prices and the real economy. Miller and Ratti (2009) state that stock returns and oil prices are cointegrated, however, they failed to explain why stock returns and oil prices grew apart during several sub-periods. Kilian and Parker (2009) proposed that changes in stock prices differ significantly depending on whether the change of oil price is driven by supply or demand. They found that the change in stock prices is always consistent with the change in oil prices when it was driven by a drop in demand. However, when the change in oil price is driven by supply, stock prices move randomly and are difficult to predict. Notably, as one of the key macroeconomic factors, inflation, is not employed by the ADS index. This chapter therefore also employs a commodity price index as a risk factor to represent the whole commodity market and as a measure of inflation.

4.3 Data and data transformation

The data used in this chapter aims to be consistent with the previous chapter as much as possible. The sample size is from March 1960 to December 2010.[34] The ADS index is collected from the Federal Reserve Bank of Philadelphia website. It is a dynamic daily index starting from 1st March 1960 to date. This index is derived from and updated by the above mentioned six macroeconomic variables to track the real business conditions of the US with a mean of zero.[35] Therefore, if the value of the ADS index is below zero, it means that at that point in time, the business conditions are worse than average, and vice versa. Since monthly data are used in this chapter, the last observation of each month of the ADS index is used.

The commodity price index is provided by the Commodity Research Bureau and collected from Datastream. This index is a commodity spot price index measured over a monthly frequency and has an average value of zero in year 1967. In order to conduct the analysis, continuously compounded returns of the commodity price index

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[34] Stock returns are in monthly frequency, and a summary table can be seen in Table A in Appendix.
[35] Subject to the availability of the variables when updating.

are derived as follows:

$$CR_t = \ln(CP_t) - \ln(CP_{t-1}) \quad , \tag{4.1}$$

whereas CR_t is the continuously compounded return of the commodity price index at time t, CP_t is the commodity price index at time t, and ln is the natural logarithm.

The stock prices and risk-free rate used are the same as those used in the previous chapter.[36]

4.4 Econometrics models and methods

4.4.1 The CPPF model

Supported by the evidence in the previous chapter, in this section, a CPPF model is to be presented. By using the CPPF model, all risk factors (excess return on the market portfolio xR_M, excess return on the commodity market xCR,[37] and the ADS business index *ADS)* will be divided into a different number of series depends on the numbers of knots selected.

The rule for deciding the number of knots placed follows chapter 3. Moreover, as in chapter 3, the placement of knots follows the quintile method proposed by Stone (1986).[38]

[36] To comply with the ADS index, the sample starts from 1st March 1960.
[37] We define $xCR = CR - R_f$.
[38] See Table 3.1.

The models used in this chapter take full advantage of the CPPF approach, building on the classic market model. To estimate the coefficients of the risk factors for each stock, the augmented market model can be written as

$$xR_i = \alpha_i + b_i \cdot (xR_M \odot S_N) + c_i \cdot (xCR \odot S_N) + d_i \cdot (ADS \odot S_N) + \varepsilon_i$$

$$N = 0, 1, 2, 3, 4, 5, \qquad (4.2)$$

where ($xR_M \odot S_N$), ($xCR \odot S_N$) and ($ADS \odot S_N$) are all in dimension of (t×n), b_i, c_i and d_i are the OLS coefficient estimates of market factor, commodity factor and ADS factor, respectively, measuring the co-movement between the risk factor and stock returns.

The OLS regression is then applied to each stock to get the vectors of estimate coefficients, with each coefficient vector having the dimension n×1. Then, using the cubic piecewise polynomial matrix multiplied by the vectors of coefficients estimates, thetime-varying coefficient estimates for each risk factor of a stock b_s, c_s, and d_s ($\mathbf{B_s}$, $\mathbf{C_s}$ and $\mathbf{D_s}$ in vector form respectively) can be obtained, as follows:

$$\mathbf{B_{s,i}} = \mathbf{S_N} \cdot \mathbf{B_i}, \qquad (4.3)$$

$$\mathbf{C_{s,i}} = \mathbf{S_N} \cdot \mathbf{C_i}, \qquad (4.4)$$

$$\mathbf{D_{s,i}} = \mathbf{S_N} \cdot \mathbf{D_i}, \qquad (4.5)$$

where $\mathbf{B_i}$ $\mathbf{C_i}$ and $\mathbf{D_i}$ are vector forms of b_i, c_i and d_i, respectively . It can be seen from equations (4.3), (4.4) and (4.5) that $\mathbf{B_{s,i}}$, $\mathbf{C_{s,i}}$ and $\mathbf{D_{s,i}}$ are products of the piecewise polynomial matrix $\mathbf{S_N}$ with the dimension t×n and the coefficient vector with the dimension n×1. Therefore, regardless of the number of knots placed in the function, the dimension of $\mathbf{B_{s,i}}$, $\mathbf{C_{s,i}}$ and $\mathbf{D_{s,i}}$ will always be t×1. In other words, the coefficient estimates are always time-varying. Since the number of knots varies from 0 to 5, there will be 6 groups of b_i, c_i and d_i for each stock, one for each corresponding number of knots. In order to find the best estimates of b_S^*, c_S^* and d_S^* for each stock, AIC is an appropriate indicator to decide the best fit of b_i, c_i and d_i.

To calculate the downside and upside estimates by using the CPPF model, the same approach to conducting b_S^*, c_S^* and d_S^* is followed, with equation (4.2) is modified accordingly. As in Ang et al. (2006), the downside beta and upside estimates in this chapter are calculated as

$$b_i^- = \frac{\text{cov}(xR_i, xR_M \mid xR_M < \overline{xR_M})}{\text{var}(xR_M \mid xR_M < \overline{xR_M})}, \tag{4.6}$$

$$b_i^+ = \frac{\text{cov}(xR_i, xR_M \mid xR_M \geq \overline{xR_M})}{\text{var}(xR_M \mid xR_M \geq \overline{xR_M})}, \tag{4.7}$$

$$c_i^- = \frac{\text{cov}(xR_i, xCR \mid xCR < \overline{xCR})}{\text{var}(xCR \mid xCR < \overline{xCR})}, \tag{4.8}$$

$$c_i^+ = \frac{\text{cov}(xR_i, xCR \mid xCR \geq \overline{xCR})}{\text{var}(xCR \mid xCR \geq \overline{xCR})}, \tag{4.9}$$

$$d_i^- = \frac{\text{cov}(xR_i, ADS \mid ADS < \overline{ADS})}{\text{var}(ADS \mid ADS < \overline{ADS})}, \tag{4.10}$$

$$d_i^+ = \frac{\text{cov}(xR_i, ADS \mid ADS \geq \overline{ADS})}{\text{var}(ADS \mid ADS \geq \overline{ADS})}, \tag{4.11}$$

where $\overline{xR_M}$, $\overline{xCR}$ and $\overline{ADS}$ are the average market excess return, average commodity market excess return and average ADS business index value, respectively, over the sample period, and all other notation remains the same. In light of Ang et al. (2006), dummy variables (vectors) $D_{1,xRM}$, $D_{2,xRM}$, $D_{1,xCR}$, $D_{2,xCR}$, $D_{1,ADS}$ and $D_{2,ADS}$, are created and employed for each stock. These dummy variables can be expressed as (time subscript t is used)

$$D_{1,xRM} = 1 \text{ and } D_{2,xRM} = 0 \text{ if } \quad xR_{M,t} < \overline{xR_M}\,, \tag{4.12}$$

$$D_{1,xCR} = 1 \text{ and } D_{2,xCR} = 0 \text{ if } \quad xCR_t < \overline{xCR}, \tag{4.13}$$

$$D_{1,ADS} = 1 \text{ and } D_{2,ADS} = 0 \text{ if } \quad ADS_t < \overline{ADS}, \tag{4.14}$$

and

$$D_{1,xRM} = 0 \text{ and } D_{2,xRM} = 1 \text{ if } \quad xR_{M,t} \geq \overline{xR_M}\,, \tag{4.15}$$

$$D_{1,xCR} = 0 \text{ and } D_{2,xCR} = 1 \text{ if } \quad xCR_t \geq \overline{xCR}, \tag{4.16}$$

$$D_{1,ADS} = 0 \text{ and } D_{2,ADS} = 1 \text{ if } \quad ADS_t \geq \overline{ADS}. \tag{4.17}$$

It can be seen from equations (4.12) to (4.17) that $D_{1,xRM}$, $D_{1,xCR}$ and $D_{1,ADS}$ represent the downside stock market, commodity market and real business condition dummies, respectively, while $D_{2,xRM}$, $D_{2,xCR}$ and $D_{2,ADS}$ represent the upside ones, respectively.

The CPPF augmented market model can be written as

$$xR_i = b_i^- \cdot (D_{1,xRM} \odot xR_M \odot S_N) + b_i^+ \cdot (D_{2,xRM} \odot xR_M \odot S_N) + c_i^- \cdot (D_{1,xCR} \odot xCR \odot S_N)$$
$$+ c_i^+ \cdot (D_{2,xCR} \odot xCR \odot S_N) + d_i^- \cdot (D_{1,ADS} \odot ADS \odot S_N) + d_i^+ \cdot (D_{2,ADS} \odot ADS \odot S_N) + \varepsilon_i$$
$$N = 0, 1, 2, 3, 4, 5. \tag{4.18}$$

It can be seen from equation (4.18) that in order to avoid multi-collinearity, there is no constant term. The value of $D_{1,xRM} \odot xR_M$ is xR_M if the value of xR_M is below the mean, and zero otherwise. On the other hand, the value of $D_{2,xRM} \odot xR_M$ is xR_M if the value of

xR_M is equal or above the mean, and zero otherwise.

The parameters b_i^-, c_i^- and d_i^- are the downside risk estimate coefficients while b_i^+, c_i^+ and d_i^+ are the upside risk estimate coefficients associated with stock i. In terms of the matrices, all of the estimates are column vectors with a dimension of $n \times 1$. Since the number of knots varies from 0 to 5, there will be 6 pairs of downside and upside vectors for each stock, with each pair of vectors having an associated AIC value. Among the 6 AICs, the lowest one indicates the best fitting pair of estimates, in addition, the associated best fitting time-varying downside and upside estimate coefficients for stock i, $b_{S,i}^{-*}$, $c_{S,i}^{-*}$, $d_{S,i}^{-*}$ and $b_{S,i}^{+*}$, $c_{S,i}^{+*}$, $d_{S,i}^{+*}$ ($\mathbf{B}_{\mathbf{S,i}}^{-*}$, $\mathbf{C}_{\mathbf{S,i}}^{-*}$, $\mathbf{D}_{\mathbf{S,i}}^{-*}$ and $\mathbf{B}_{\mathbf{S,i}}^{+*}$, $\mathbf{C}_{\mathbf{S,i}}^{+*}$, $\mathbf{D}_{\mathbf{S,i}}^{+*}$ in vector form) can be calculated as follows:

$$\mathbf{B}_{\mathbf{S,i}}^{-*} = \mathbf{S}_{\mathbf{N}} \cdot \mathbf{B}_{\mathbf{S,i}}^{-}, \quad (4.19)$$

$$\mathbf{C}_{\mathbf{S,i}}^{-*} = \mathbf{S}_{\mathbf{N}} \cdot \mathbf{C}_{\mathbf{S,i}}^{-}, \quad (4.20)$$

$$\mathbf{D}_{\mathbf{S,i}}^{-*} = \mathbf{S}_{\mathbf{N}} \cdot \mathbf{D}_{\mathbf{S,i}}^{-}, \quad (4.21)$$

$$\mathbf{B}_{\mathbf{S,i}}^{+*} = \mathbf{S}_{\mathbf{N}} \cdot \mathbf{B}_{\mathbf{S,i}}^{+}, \quad (4.22)$$

$$\mathbf{C}_{\mathbf{S,i}}^{+*} = \mathbf{S}_{\mathbf{N}} \cdot \mathbf{C}_{\mathbf{S,i}}^{+}, \quad (4.23)$$

$$\mathbf{D}_{\mathbf{S,i}}^{+*} = \mathbf{S}_{\mathbf{N}} \cdot \mathbf{D}_{\mathbf{S,i}}^{+}, \quad (4.24)$$

where $\mathbf{B}_{\mathbf{S,i}}^{-}$, $\mathbf{C}_{\mathbf{S,i}}^{-}$, $\mathbf{D}_{\mathbf{S,i}}^{-}$ and $\mathbf{B}_{\mathbf{S,i}}^{+}$, $\mathbf{C}_{\mathbf{S,i}}^{+}$, $\mathbf{D}_{\mathbf{S,i}}^{+}$ are the vector forms of b_i^-, c_i^-, d_i^- and b_i^+, c_i^+, d_i^+. As mentioned in the previous paragraph, regardless of the number of knots placed in the function, the dimension of $\mathbf{B}_{\mathbf{S,i}}^{-*}$, $\mathbf{C}_{\mathbf{S,i}}^{-*}$, $\mathbf{D}_{\mathbf{S,i}}^{-*}$ and $\mathbf{B}_{\mathbf{S,i}}^{+*}$, $\mathbf{C}_{\mathbf{S,i}}^{+*}$, $\mathbf{D}_{\mathbf{S,i}}^{+*}$ will always be $t \times 1$.

4.4.2 The FFF model

In this section, as an alternative way of generating time-varying risk estimate coefficients, the FFF model is presented. By using the FFF model, all risk factors will be divided into a different number of series depending on the order number.

In light of Andersen and Bollerslev (1998), Andersen et al. (2000), Bollerslev et al. (2000), and Evans and Speight (2010), and following chapter 3, the FFF market model employed in this chapter is given by

$$xR_i = \alpha_i + \sum_{p=1}^{P}[b_{cos,p,i} \cdot (\cos\frac{p2\pi}{N} n \cdot xR_M) + b_{\sin,p,i} \cdot (\sin\frac{p2\pi}{N} n \cdot xR_M)]$$

$$+\sum_{p=1}^{P}[c_{cos,p,i} \cdot (\cos\frac{p2\pi}{N} n \cdot xCR) + c_{\sin,p,i} \cdot (\sin\frac{p2\pi}{N} n \cdot xCR)]$$

$$+\sum_{p=1}^{P}[d_{cos,p,i} \cdot (\cos\frac{p2\pi}{N} n \cdot ADS) + d_{\sin,p,i} \cdot (\sin\frac{p2\pi}{N} n \cdot ADS)] + \varepsilon_i, \qquad (4.25)$$

whereas α_i is the constant term, $b_{cos,p,i}$, $b_{sin,p,i}$, $c_{cos,p,i}$, $c_{sin,p,i}$, $d_{cos,p,i}$ and $d_{sin,p,i}$,are the coefficients to be estimated of each factor for stock *i*, *N* is the total number of observations of stock *i*, *n* is the order of observations with $n = \{1, 2, 3\ldots T\}$, *p* is the order of the FFF model and the remaining notation remains the same. Following chapter 3, according to Andersen and Bollerslev (1998), the order of the FFF could vary from 1 to infinity. However, in order to improve the efficiency of the estimates, we follow their lead and chose 4 as the appropriate order. In this chapter, orders from 1 to 4 are considered.

The OLS regression is then applied to each stock to obtain the estimated risk factor coefficient vector produced from equation (4.25). The AIC is then computed for each regression. Since the orders of 1 to 4 are considered, there are 4 AICs for each stock. Taking advantage of the nature of the AIC, the regression that produces the lowest AIC gives the best fit. To calculate the best fitting time-varying coefficients $b^*_{F,i}$, $c^*_{F,i}$ and $d^*_{F,i}$ for each stock, the minimum AIC estimated vector for stock i are calculated as follows:

$$b^*_{F,i} = \sum_{p=1}^{P}(b_{cos,p,i} \cdot \cos\frac{p2\pi}{N}n + b_{\sin,p,i} \cdot \sin\frac{p2\pi}{N}n)\,, \tag{4.26}$$

$$c^*_{F,i} = \sum_{p=1}^{P}(c_{cos,p,i} \cdot \cos\frac{p2\pi}{N}n + c_{\sin,p,i} \cdot \sin\frac{p2\pi}{N}n)\,, \tag{4.27}$$

$$d^*_{F,i} = \sum_{p=1}^{P}(d_{cos,p,i} \cdot \cos\frac{p2\pi}{N}n + d_{\sin,p,i} \cdot \sin\frac{p2\pi}{N}n)\,. \tag{4.28}$$

In order to calculate the downside and upside estimates by using the above FFF model, the same procedure used above is followed. The dummy variables $D_{1,xRM}$, $D_{2,xRM}$, $D_{1,xCR}$, $D_{2,xCR}$, $D_{1,ADS}$ and $D_{2,ADS}$ used in equation (4.18) are created and employed again for each stock in the new FFF model. The new model is defined as

$$\begin{aligned} xR_i = &\sum_{p=1}^{P}[b^-_{\cos,p,i} \cdot (\cos\frac{p2\pi}{N}n \cdot xR_M \odot D_{1,xRM}) + b^-_{\sin,p,i} \cdot (\sin\frac{p2\pi}{N}n \cdot xR_M \odot D_{1,xRM})] \\ &+\sum_{p=1}^{P}[b^+_{\cos,p,i} \cdot (\cos\frac{p2\pi}{N}n \cdot xR_M \odot D_{2,xRM}) + b^+_{\sin,p,i} \cdot (\sin\frac{p2\pi}{N}n \cdot xR_M \odot D_{2,xRM})] \\ &+\sum_{p=1}^{P}[c^-_{\cos,p,i} \cdot (\cos\frac{p2\pi}{N}n \cdot xCR \odot D_{1,xCR}) + c^-_{\sin,p,i} \cdot (\sin\frac{p2\pi}{N}n \cdot xCR \odot D_{1,xCR})] \\ &+\sum_{p=1}^{P}[c^+_{\cos,p,i} \cdot (\cos\frac{p2\pi}{N}n \cdot xCR \odot D_{2,xCR}) + c^+_{\sin,p,i} \cdot (\sin\frac{p2\pi}{N}n \cdot xCR \odot D_{2,xCR})] \end{aligned}$$

$$+\sum_{p=1}^{P}[d^{-}_{\cos,p,i}\cdot(\cos\frac{p2\pi}{N}n\cdot ADS\odot D_{1,ADS})+d^{-}_{\sin,p,i}\cdot(\sin\frac{p2\pi}{N}n\cdot ADS\odot D_{1,ADS})]$$

$$+\sum_{p=1}^{P}[d^{+}_{\cos,p,i}\cdot(\cos\frac{p2\pi}{N}n\cdot ADS\odot D_{2,ADS})+d^{+}_{\sin,p,i}\cdot(\sin\frac{p2\pi}{N}n\cdot ADS\odot D_{2,ADS})]+\varepsilon_i\,.$$

(4.29)

Whereas $b^{-}_{cos,p,i}$, $b^{-}_{sin,p,i}$, $c^{-}_{cos,p,i}$, $c^{-}_{sin,p,i}$, $d^{-}_{cos,p,i}$, and $d^{-}_{sin,p,i}$, are the downside coefficients to be estimated for stock i, while $b^{+}_{cos,p,i}$, $b^{+}_{sin,p,i}$, $c^{+}_{cos,p,i}$, $c^{+}_{sin,p,i}$, and $d^{+}_{cos,p,i}$, and $d^{+}_{sin,p,i}$ are the upside coefficients to be estimated for stock i. For the same reason as for equation (4.24), there is no conventional constant term in the model to avoid multi-collinearity. Since the order of the FFF examined varies from 1 to 4, there will be 4 groups of estimated risk factor coefficient vectors for each stock. The best fit time-varying downside and upside estimates for stock i, $b^{-*}_{F,i}$, $b^{+*}_{F,i}$, $c^{-*}_{F,i}$, $c^{+*}_{F,i}$, $d^{-*}_{F,i}$ and $d^{+*}_{F,i}$ can be calculated as follows

$$b^{-*}_{F,i}=\sum_{p=1}^{P}(b^{-}_{\cos,p,i}\cdot\cos\frac{p2\pi}{N}n+b^{-}_{\cos,p,i}\cdot\sin\frac{p2\pi}{N}n)\,, \tag{4.30}$$

$$b^{+*}_{F,i}=\sum_{p=1}^{P}(b^{+}_{\cos,p,i}\cdot\cos\frac{p2\pi}{N}n+b^{+}_{\cos,p,i}\cdot\sin\frac{p2\pi}{N}n)\,, \tag{4.31}$$

$$c^{-*}_{F,i}=\sum_{p=1}^{P}(c^{-}_{\cos,p,i}\cdot\cos\frac{p2\pi}{N}n+c^{-}_{\cos,p,i}\cdot\sin\frac{p2\pi}{N}n)\,, \tag{4.32}$$

$$c^{+*}_{F,i}=\sum_{p=1}^{P}(c^{+}_{\cos,p,i}\cdot\cos\frac{p2\pi}{N}n+c^{+}_{\cos,p,i}\cdot\sin\frac{p2\pi}{N}n)\,, \tag{4.33}$$

$$d^{-*}_{F,i}=\sum_{p=1}^{P}(d^{-}_{\cos,p,i}\cdot\cos\frac{p2\pi}{N}n+d^{-}_{\cos,p,i}\cdot\sin\frac{p2\pi}{N}n)\,, \tag{4.34}$$

$$d^{+*}_{F,i}=\sum_{p=1}^{P}(d^{+}_{\cos,p,i}\cdot\cos\frac{p2\pi}{N}n+d^{+}_{\cos,p,i}\cdot\sin\frac{p2\pi}{N}n)\,. \tag{4.35}$$

It can be seen from equation (4.30) to (4.35) that regardless the order of the model,

$b_{F,i}^{-*}$, $b_{F,i}^{+*}$, $c_{F,i}^{-*}$, $c_{F,i}^{+*}$, $d_{F,i}^{-*}$ and $d_{F,i}^{+*}$ always have the dimension $t \times 1$.

4.5 Empirical results

Based on the methods explained in the previous section, the best estimates for both the CPPF model and the FFF model are obtained. To summarize the estimation details, distribution of the best fitting knots and orders are presented in Table 4.1 and Table 4.2.

Table 4.1 Knots of CPPF Model Selected to Construct Best Fit Estimates

This table reports the number and percentage of stocks with different knots to construct the best fit estimates of the CPPF model

Knots		0	1	2	3	4	5
Classic estimates	Number of Stocks	7307	587	288	328	824	4223
	Percentage of Whole sample	53.90%	4.33%	2.12%	2.42%	6.08%	31.15%
Downside and upside estimates	Number of Stocks	3923	350	494	909	1981	5900
	Percentage of Whole sample	28.94%	2.58%	3.64%	6.71%	14.61%	43.52%

For the CPPF model, it can be seen from Table 4.1 that 7307 stocks (53.90% of the sample) construct b_S^*, c_S^* and d_S^*, when no knots are placed, when the number of knots varies from 1 to 4, a much lower number of best fit estimates are produced. However, 4223 stocks obtain the best estimates with 5 knots placed (31.15% of the sample). On the other hand, to construct downside and upside risk factor coefficients, unlike the classic risk factor estimate coefficients, no knots are used for only 3923 stocks (28.94% of the sample). Similar to classic risk estimates, a much lower number of best fit estimates are produced when 1 to 4 knots are placed. Surprisingly, 5900 stocks obtain best fit estimates when 5 knots are placed (more than 40% of the sample). Compared with the previous chapter, more stocks produce best fit downside and upside risk coefficients when 5 knots are placed. It shows that in a multi-factor model, it is possible that more knots are needed.[39]

Table 4.2 The order of the FFF model selected to construct best fit estimates

This table reports the number and percentage of stocks in different order to construct the best fit estimates of the FFF model

Order		1	2	3	4
Classic estimates	Number of Stocks	8999	2164	1205	1189
	Percentage of Whole sample	66.38%	15.96%	8.89%	8.77%
Downside and upside estimates	Number of Stocks	9079	1414	742	2322
	Percentage of Whole sample	66.97%	10.43%	5.47%	17.13%

For the FFF model, it is clear from Table 4.2 that to construct b_F^*, c_F^* and d_F^*, 8999 stocks used order 1 (66.38% of the sample). Stocks with orders 2, 3 and 4, however, produce a lower number of best estimates. It is more obvious when constructing downside and upside estimates, 9079 stocks obtain the best estimates with order 1 (66.97% of the sample), 1414 stocks with order 2 (10.43% of the sample), and 742

[39] Only 12% of stocks produced best fit downside and upside estimate coefficients when 5 knots are placed in the chapter 3.

and 2322 stocks with orders 3 and 4 produced the best estimates (5.47% and 17.13% of the sample, respectively).

Furthermore, the relations among stock returns and classic, downside and upside estimates of both the CPPF model and the FFF model betas are examined. In order to present the relationship in a cross-sectional fashion, following the methodology used in the previous chapter, stocks at each point in time are cross-sectionally assigned to five portfolios according to the value of the risk estimates. Since the classic, downside and upside beta estimates are not independent of each other due to the nature of the calculation, to distinguish the effects among them, more statistics are introduced. Specifically, we consider, for the CPPF model, the relative estimates denoted by (b_S^{-*}- b_S^*), (c_S^{-*}- c_S^*) and (d_S^{-*}- d_S^*) for the downside market, and (b_S^{+*}- b_S^*), (c_S^{+*}- c_S^*) and (d_S^{+*}- d_S^*) for the upside market. Similarly, for the FFF model, (b_F^{-*}- b_F^*), (c_F^{-*}- c_F^*) and (d_F^{-*}- d_F^*) for the downside market, and (b_F^{+*}- b_F^*), (c_F^{+*}- c_F^*) and (d_F^{+*}- d_F^*) for the upside market are computed. Introducing these statistics aims to illustrate the impact of downside and upside estimates after controlling for classic estimates.

To sort the portfolio, at each point in time, all stocks are sorted into five quintiles according to the value of the target estimate. When stocks are sorted into 5 portfolios at each point of time (since monthly data are used in this chapter, and the whole sample is from March 1960 to December 2010, so there should be 610 time points), the equally weighted average of the estimate for each portfolio and the corresponding same period average annualized stock returns and average values of the risk estimates are calculated. The results of both models are summarized in Table 4.3 to Table 4.8.

4.5.1 Empirical results: the CPPF model

For the CPPF model, Table 4.3 presents the results pertaining to the relationship between annualized excess stock returns and estimates of market beta. It can be seen from Panel 1 that when stocks are sorted by b_S^*, portfolio 1 has an average b_S^* value of -0.40, while on the other hand, portfolio 5 shows an average b_S^* value of 2.15. Consistent with the classic literature, the average annualized return of each portfolio increases with b_S^*, portfolio 1 yields a return of 4.40% while portfolio 5 yields a return of 20.74%. The average b_S^{-*} and b_S^{+*} values of each portfolio follow the same trend as b_S^*, with average b_S^{-*} equaling -0.91 in portfolio 1 and increasing to 1.81 in portfolio 5. Similarly, average b_S^{+*} is -0.92 in portfolio 1 and increases to 2.01 in portfolio 5.

When stocks are sorted by b_S^{-*}, it can be seen from Panel 2 that the average returns generally drop from 14.02% to 8.19%, however from portfolio 2 to portfolio 4, returns present a U-shaped pattern. When stocks are sorted by b_S^{+*}, both returns and b_S^* increase dramatically from portfolio 1 to portfolio 5, with a negligible drop in b_S^* in portfolio 2. b_S^{-*} slumps from 1.82 to -1.41 along with the increase of b_S^{+*}.

Table 4.3 Excess Stock Returns Sorted by Stock Market Factor Loadings of CPPF Model

This table presents the relationship between excess stock returns and stock market factor loadings associated with the CPPF model. The column labeled "return" reports the annual average stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by b_S^*

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	4.40%	-0.40	-0.91	-0.92
2	7.62%	0.62	0.91	0.55
3	9.41%	0.99	1.29	1.06
4	11.64%	1.44	1.53	1.51
5 High	20.74%	2.15	1.81	2.01
High-Low	16.34%	2.55	2.72	2.93

Panel 2 Stocks Sorted by b_S^{-*}

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	14.02%	1.06	-1.03	1.62
2	9.62%	0.72	0.22	0.96
3	10.40%	1.01	1.02	0.98
4	11.56%	1.40	1.99	0.92
5 High	8.19%	1.61	2.15	-1.94
High-Low	-5.83%	0.55	3.18	-3.56

Panel 3 Stocks Sorted by b_S^{+*}

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	5.23%	0.66	1.82	-0.39
2	7.54%	0.63	1.13	0.10
3	10.05%	0.95	0.97	0.91
4	12.64%	1.43	1.03	1.85
5 High	18.34%	2.13	-1.41	2.30
High-Low	13.11%	1.47	-3.23	2.68

Panel 4 Stocks Sorted by (b_S^{-*} - b_S^*)

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	14.39%	1.98	-1.74	1.84
2	13.54%	1.17	0.48	1.54
3	9.88%	0.94	0.98	0.90
4	8.71%	0.98	1.77	0.43
5 High	7.27%	-0.26	2.46	-1.39
High-Low	-7.13%	-2.24	4.20	-3.24

Panel 5 Stocks Sorted by (b_S^{+*} - b_S^*)

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	8.23%	1.70	1.93	-2.06
2	9.15%	1.07	1.61	0.31
3	10.05%	0.94	0.97	0.91
4	11.86%	1.08	0.64	1.66
5 High	14.50%	-0.46	-1.47	2.24
High-Low	6.28%	-2.16	-3.40	4.30

Panel 6 Stocks Sorted by (b_S^{-*} - b_S^{+*})

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	14.76%	1.64	-1.79	2.19
2	12.88%	1.15	0.50	1.63
3	9.94%	0.97	0.98	0.91
4	8.85%	0.99	1.76	0.35
5 High	7.36%	-1.06	2.11	-1.94
High-Low	-7.40%	-2.70	3.90	-4.13

When controlling for b_S^*, returns drop from 14.39% to 7.27% when stocks are sorted by (b_S^{-*}- b_S^*). In contrast, it is clear from Panel 5 that returns increase gradually from 8.23% to 14.5% when stocks are sorted by (b_S^{+*}- b_S^*). And it can be seen from Panel 6 that only b_S^{-*} increases from -1.79 to 2.11, while returns drop from 14.76% to 7.36%.

Table 4.4 presents the relationship between excess stock returns and estimates of commodity market risk. It can be seen from Panel 1 that when stocks are sorted by c_S^*, portfolio 1 has an average c_S^* value of -1.82. On the other hand, portfolio 5 shows an average c_S^* value of 1.78. Unlike Panel 1 in Table 4.4, the average annualized return of each portfolio declines along with the increase of c_S^*, portfolio 1 yields a return of 15.31% while portfolio 5 yields a return of 7.85%. The average c_S^{-*} and c_S^{+*} of each portfolio follows the same trend as c_S^*, the average c_S^{-*} is -1.43 in portfolio 1 and increases to 1.8 in portfolio 5. Similarly, the average c_S^{+*} is -1.52 in portfolio 1 and increases to 2.22 in portfolio 5.

When stocks are sorted by c_S^{-*}, it can be seen from Panel 2 that the average returns decrease from 16.66% to 5.82%. c_S^* presents a reversed U-shaped pattern along with the increase of c_S^{-*}, starting at -1.36 in portfolio 1 and finishing at 0.26 in portfolio 5, reaching a peak at 0.56 in portfolio 4. Moreover, c_S^{+*} drops dramatically from 1.71 to -1.94. When stocks are sorted by c_S^{+*}, returns increase dramatically from portfolio 1 to portfolio 5 and c_S^* increases gradually from portfolio 1 to portfolio 4, with a drop in portfolio 5. While c_S^{-*} slumps from 1.43 to -0.64 along with the increase of c_S^{+*}.

Table 4.4 Excess Stock Returns Sorted by Commodity Market Factor Loadings of CPPF Model

This table presents the relationship between excess stock returns and commodity market factor loadings associated with the CPPF model. The column labeled "return" reports the annual average stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by c_S^*

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	15.31%	-1.82	-1.43	-1.52
2	11.29%	-0.34	-1.29	-1.28
3	10.17%	0.10	-0.57	-1.01
4	9.18%	0.60	0.27	1.95
5 High	7.85%	1.78	1.80	2.22
High-Low	-7.46%	3.60	3.23	3.75

Panel 2 Stocks Sorted by c_S^{-*}

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	16.66%	-1.36	-1.45	1.71
2	12.49%	-0.19	-1.30	0.49
3	10.00%	0.07	0.05	0.15
4	8.81%	0.56	1.61	-0.35
5 High	5.82%	0.26	2.17	-1.94
High-Low	-10.84%	1.62	3.62	-3.65

Panel 3 Stocks Sorted by c_S^{+*}

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	7.87%	-1.03	1.43	-1.41
2	8.74%	-0.25	0.84	-1.06
3	10.02%	0.07	-0.04	0.09
4	12.45%	0.35	-0.48	1.72
5 High	14.71%	0.19	-0.64	1.92
High-Low	6.84%	1.22	-2.07	3.33

Panel 4 Stocks Sorted by $(c_S^{-*} - c_S^*)$

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	11.14%	1.83	-1.46	1.35
2	10.80%	0.34	0.67	1.19
3	10.17%	0.10	1.18	1.09
4	10.14%	-0.09	1.50	0.15
5 High	11.53%	-1.83	1.94	-0.78
High-Low	0.39%	-3.67	3.40	-2.12

Panel 5 Stocks Sorted by $(c_S^{+*} - c_S^*)$

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	7.96%	1.58	1.61	-1.94
2	7.56%	0.26	0.67	-1.46
3	10.27%	0.07	-0.34	0.10
4	13.60%	0.02	-0.87	1.56
5 High	14.39%	-1.58	-1.25	1.95
High-Low	6.44%	-3.16	-2.86	3.89

Panel 6 Stocks Sorted by $(c_S^{-*} - c_S^{+*})$

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	14.36%	-0.85	-1.63	1.78
2	13.48%	0.13	-1.06	1.48
3	10.26%	0.05	0.02	0.10
4	8.02%	0.15	1.38	-1.35
5 High	7.67%	-0.14	1.97	-1.71
High-Low	-6.69%	0.71	3.61	-3.49

To control for c_S^*, when stocks are sorted by (c_S^{-*}- c_S^*), returns present a U-shaped pattern but with little change in its value, starting at 11.14% and finishing at 11.53%. In contrast, it is clear from Panel 5 that returns increase steadily from 7.96% to 14.39% with a negligible drop in portfolio 2 when stocks are sorted by (c_S^{+*}- c_S^*). In Panel 6, (c_S^{-*}- c_S^{+*}) is employed for the same reason mentioned in Table 4.4. It can be seen from Panel 6 that only c_S^{-*} increases obviously from -1.63 to 1.97, while returns and c_S^{+*} are decreasing and it is difficult to trace the pattern of c_S^*.

Table 4.5 presents the relationship between excess stock returns and estimates of ADS risk. It can be seen from Panel 1 that when stocks are sorted by d_S^*, portfolio 1 has an average d_S^* value of -0.89 and portfolio 5 shows an average d_S^* value of 0.89. The average annualized return of each portfolio shows a U-shaped pattern along with the increase of d_S^*. Portfolio 1 yields a return of 10.6% while portfolio 5 is at a peak of 12.11%, and there is a slight drop in portfolio 2. The average d_S^{-*} and d_S^{+*} values of each portfolio follows the same trend as d_S^*, average d_S^{-*} is -0.79 in portfolio 1 and increases to 0.82 in portfolio 5. Also, average d_S^{+*} is -0.53 in portfolio 1 and increases to 1.23 in portfolio 5.

When stocks are sorted by d_S^{-*}, it can be seen from Panel 2 that average returns decrease from 16.93% to 4.83%. d_S^* and d_S^{+*} both increase gradually along with the increase of d_S^{-*}. When stocks are sorted by d_S^{+*}, both returns and d_S^* increase steadily from portfolio 1 to portfolio 5.

Table 4.5 Excess Stock Returns Sorted by Business Conditions Factor Loadings of CPPF Model

This table presents the relationship between excess stock returns and business conditions factor loadings associated with the CPPF model. The column labeled "return" reports the annual average stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by d_S^*

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	10.60%	-0.89	-0.79	-0.53
2	10.15%	-0.02	-0.13	-0.20
3	10.28%	0.00	-0.03	0.57
4	10.66%	0.01	0.48	0.77
5 High	12.11%	0.89	0.82	1.23
High-Low	1.51%	1.78	1.62	1.76

Panel 2 Stocks Sorted by d_S^{-*}

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	16.93%	-0.04	-0.73	-0.34
2	13.32%	-0.01	-0.12	-0.07
3	10.15%	0.00	0.01	0.13
4	8.55%	0.01	0.10	0.29
5 High	4.83%	0.05	1.11	1.45
High-Low	-12.11%	0.09	1.84	1.79

Panel 3 Stocks Sorted by d_S^{+*}

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	4.54%	-0.06	1.60	-0.36
2	8.95%	-0.01	0.85	-0.13
3	10.36%	0.00	0.25	0.01
4	12.24%	0.00	0.10	0.12
5 High	17.70%	0.07	-1.45	1.53
High-Low	13.15%	0.12	-3.05	1.89

Panel 4 Stocks Sorted by (d_S^{-*} - d_S^*)

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	14.52%	0.47	-0.59	1.61
2	13.70%	0.01	-0.11	1.26
3	10.47%	0.00	-0.01	1.19
4	8.21%	-0.01	0.10	0.24
5 High	6.88%	-0.46	1.80	0.05
High-Low	-7.64%	-0.93	2.39	-1.56

Panel 5 Stocks Sorted by (d_S^{+*} - d_S^*)

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	8.02%	0.61	-1.02	-1.23
2	9.25%	0.00	0.59	-0.13
3	10.28%	0.00	1.30	0.01
4	12.02%	-0.01	0.36	1.12
5 High	14.21%	-0.60	-1.07	1.61
High-Low	6.19%	-1.21	-0.05	2.85

Panel 6 Stocks Sorted by (d_S^{-*} - d_S^{+*})

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	12.81%	0.01	-1.73	1.71
2	13.75%	-0.01	-0.11	1.29
3	10.39%	0.00	-0.01	0.01
4	8.48%	0.00	1.09	-0.12
5 High	8.34%	0.00	1.71	-1.55
High-Low	-4.47%	-0.01	3.45	-3.26

However, d_S^{-*} slumps from 1.6 to -1.45 along with the increase of d_S^{+*}. When stocks are sorted by (d_S^{-*}- d_S^{*}), all returns, d_S^{*} and d_S^{+*} drop gradually while d_S^{-*} increases substantially. In contrast, it is clear from Panel 5 that returns increase steadily from 8.02% to 14.21% when stocks are sorted by (d_S^{+*}- d_S^{*}). Meanwhile, d_S^{*} decreases along with the increase of d_S^{+*}, while d_S^{-*} presents a reversed U-shaped pattern. In Panel 6, (d_S^{-*}- d_S^{+*}) is employed to sort stocks. It can be seen from this panel that only d_S^{-*} increases obviously from -1.73 to 1.71, while returns and d_S^{+*} decrease in general and a U-shaped pattern is presented in d_S^{*}.

Overall, the results are consistent with previous chapters, for stock market risk and ADS index risk, beta and upside beta have a positive impact on stock returns, while downside beta shows a negative impact. For commodity market risk, the beta shows a reverse impact on stock returns compared to the other two risk factors, while downside and upside beta follow the same impact on stock returns as the other two factors.

4.5.2 Empirical result of the FFF model

A similar approach applies to the FFF-based estimates. Table 4.6 to Table 4.8 presents the risk-return relationship between annualized excess stock returns and estimates of stock market risk, commodity market risk and ADS index risk, respectively.

Table 4.6 Excess Stock Returns Sorted by Stock Market Factor Loadings of the FFF Model

This table presents the relationship between excess stock returns and stock market factor loadings associated with the FFF model. The column labeled "return" reports the annual average stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by b_F^{*}

Portfolio	Return	b_F^{*}	b_F^{-*}	b_F^{+*}
1 Low	5.24%	-1.09	-0.78	-0.87
2	8.22%	-0.33	-0.26	-0.27
3	9.28%	-0.01	0.00	-0.03
4	10.98%	0.32	0.27	0.22
5 High	20.08%	1.11	0.79	0.94
High-Low	14.84%	2.20	1.57	1.81

Panel 2 Stocks Sorted by b_F^{-*}

Portfolio	Return	b_F^{*}	b_F^{-*}	b_F^{+*}
1 Low	13.83%	-0.50	-2.08	0.36
2	10.58%	-0.20	-0.43	-0.05
3	9.92%	0.00	0.00	0.00
4	9.86%	0.20	0.43	0.05
5 High	9.59%	0.51	2.08	-0.36
High-Low	-4.23%	1.01	4.16	-0.72

Panel 3 Stocks Sorted by b_F^{+*}

Portfolio	Return	b_F^{*}	b_F^{-*}	b_F^{+*}
1 Low	4.05%	-0.58	0.42	-1.93
2	8.31%	-0.22	-0.05	-0.39
3	10.20%	-0.02	0.00	-0.01
4	11.75%	0.20	0.04	0.38
5 High	19.46%	0.62	-0.41	1.96
High-Low	15.40%	1.20	-0.83	3.89

Panel 4 Stocks Sorted by $(b_F^{-*} - b_F^{*})$

Portfolio	Return	b_F^{*}	b_F^{-*}	b_F^{+*}
1 Low	18.92%	0.27	-1.68	1.07
2	12.82%	0.03	-0.30	0.22
3	10.33%	-0.01	-0.01	-0.01
4	8.21%	-0.05	0.29	-0.24
5 High	3.51%	-0.24	1.70	-1.04
High-Low	-15.41%	-0.50	3.38	-2.11

Panel 5 Stocks Sorted by $(b_F^{+*} - b_F^{*})$

Portfolio	Return	b_F^{*}	b_F^{-*}	b_F^{+*}
1 Low	8.17%	0.17	1.08	-1.53
2	9.49%	0.08	0.27	-0.23
3	9.66%	0.00	-0.01	-0.01
4	11.74%	-0.09	-0.28	0.23
5 High	14.73%	-0.15	-1.06	1.54
High-Low	6.56%	-0.32	-2.14	3.07

Panel 6 Stocks Sorted by $(b_F^{-*} - b_F^{+*})$

Portfolio	Return	b_F^{*}	b_F^{-*}	b_F^{+*}
1 Low	17.52%	0.08	-1.63	1.50
2	12.81%	-0.01	-0.30	0.23
3	10.01%	-0.01	-0.01	-0.01
4	8.56%	0.00	0.30	-0.24
5 High	4.88%	-0.05	1.65	-1.48
High-Low	-12.64%	-0.13	3.28	-2.98

Table 4.7 Excess Stock Returns Sorted by Commodity Market Factor Loadings of the FFF Model

This table presents the relationship between excess stock returns and the commodity market factor loadings associated with the FFF model. The column labeled "return" reports the annual average stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by c_F^*

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	14.96%	-1.57	-1.36	-1.03
2	11.37%	-0.38	-0.37	-0.26
3	10.06%	0.00	-0.01	0.00
4	8.93%	0.37	0.36	0.22
5 High	8.48%	1.58	1.38	1.08
High-Low	-6.48%	3.15	2.73	2.11

Panel 2 Stocks Sorted by c_F^{-*}

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	16.32%	-0.76	-3.51	1.00
2	11.89%	-0.22	-0.60	0.15
3	10.29%	0.00	-0.01	0.02
4	9.53%	0.22	0.59	-0.16
5 High	5.75%	0.75	3.53	-1.00
High-Low	-10.57%	1.51	7.04	-2.00

Panel 3 Stocks Sorted by c_F^{+*}

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	6.88%	-0.54	0.97	-4.06
2	9.80%	-0.16	0.12	-0.59
3	10.54%	0.00	0.00	0.00
4	11.39%	0.15	-0.15	0.57
5 High	15.17%	0.55	-0.94	4.07
High-Low	8.29%	1.10	-1.91	8.13

Panel 4 Stocks Sorted by $(c_F^{-*} - c_F^*)$

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	14.24%	0.21	-3.01	1.85
2	11.17%	0.01	-0.49	0.41
3	10.38%	0.00	-0.01	0.02
4	10.11%	-0.01	0.48	-0.40
5 High	7.87%	-0.19	3.03	-1.87
High-Low	-6.38%	-0.40	6.04	-3.73

Panel 5 Stocks Sorted by $(c_F^{+*} - c_F^*)$

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	5.23%	0.35	1.77	-3.61
2	9.15%	0.12	0.41	-0.45
3	10.29%	-0.01	-0.02	0.00
4	12.21%	-0.13	-0.42	0.44
5 High	16.90%	-0.34	-1.75	3.62
High-Low	11.67%	-0.69	-3.52	7.23

Panel 6 Stocks Sorted by $(c_F^{-*} - c_F^{+*})$

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	16.34%	-0.09	-2.77	3.43
2	12.01%	-0.06	-0.50	0.44
3	10.47%	-0.01	-0.01	0.00
4	9.47%	0.05	0.48	-0.46
5 High	5.49%	0.11	2.81	-3.41
High-Low	-10.85%	0.20	5.59	-6.84

Table 4.8 Excess Stock Returns Sorted by Business Condition Factor Loadings of The FFF Model

This table presents the relationship between excess stock returns and business conditions factor loadings associated with the FFF model. The column labeled "return" reports the annual average stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by d_F^*

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	11.77%	-0.06	0.00	-0.04
2	10.37%	-0.01	-0.01	-0.01
3	9.63%	0.00	-0.02	0.00
4	10.26%	0.01	-0.01	0.01
5 High	11.76%	0.06	0.05	0.04
High-Low	-0.01%	0.12	0.05	0.08

Panel 2 Stocks Sorted by d_F^{-*}

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	16.72%	-0.02	-1.05	0.02
2	11.80%	-0.01	-0.02	0.01
3	10.40%	0.00	0.00	0.00
4	9.44%	0.01	0.02	-0.01
5 High	5.44%	0.02	1.05	-0.02
High-Low	-11.28%	0.04	2.10	-0.04

Panel 3 Stocks Sorted by d_F^{+*}

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	6.13%	-0.02	-0.12	-0.18
2	9.03%	-0.01	0.00	-0.02
3	10.06%	0.00	0.00	0.00
4	11.50%	0.00	-0.01	0.02
5 High	17.06%	0.02	0.13	0.18
High-Low	10.93%	0.04	0.25	0.36

Panel 4 Stocks Sorted by $(d_F^{-*} - d_F^*)$

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	15.62%	0.01	-1.03	0.04
2	11.16%	0.00	-0.01	0.01
3	10.13%	0.00	0.00	0.00
4	9.83%	0.00	0.01	-0.01
5 High	7.05%	-0.01	1.03	-0.04
High-Low	-8.56%	-0.02	2.07	-0.09

Panel 5 Stocks Sorted by $(d_F^{+*} - d_F^*)$

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	7.79%	0.01	-0.07	-0.16
2	9.26%	0.00	0.00	-0.02
3	10.10%	0.00	0.00	0.00
4	11.26%	0.00	-0.02	0.02
5 High	15.37%	-0.01	0.09	0.16
High-Low	7.58%	-0.02	0.15	0.33

Panel 6 Stocks Sorted by $(d_F^{-*} - d_F^{+*})$

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	16.13%	0.00	-1.02	0.14
2	12.06%	0.00	-0.01	0.02
3	10.30%	0.00	0.00	0.00
4	8.76%	0.00	0.01	-0.02
5 High	6.54%	0.00	1.02	-0.13
High-Low	-9.59%	0.00	2.04	-0.27

To sum up, although some patterns are not identical between both models, the relationship between returns and risk estimates are quite similar in general. It can be concluded from Table 4.3 to Table 4.8 that for both models, consistent with previous chapters, the conventional estimates of the stock market risk measures b_S^* and b_F^* do have a positive influence on stock returns, and are consistent with the classic literature of "high beta high return". However, the classic estimates of the commodity market risk measures c_S^* and c_F^* appear to have a negative impact on stock returns. The reason for that is most likely that risk in the stock market and commodity market are inversely related while ADS index risk measures d_S^* and d_F^* did not exhibit an obvious impact on stock returns. Furthermore, for the downside estimates, except (c_S^{-*}- c_S^*), all have strong negative effects on stock returns. When downside risk estimates increase, stock returns decrease dramatically. There is no clear evidence that (c_S^{-*}-c_S^*) has an impact on stock returns. Moreover, it is shown in Table 4.4 to Table 4.9 that all the upside estimates (even when controlling for the classic estimates) have a strong positive impact on stock returns. When upside estimates increase, stock returns also increase substantially.

With these findings, the roles of downside and upside estimates are not simply components of classic estimates, but are new risk measures. Therefore, it is worthwhile examining the importance of downside and upside estimates as factors rather than factor loadings.

4.6 Fama-Macbeth regressions

In this section, in order to illustrate the impact of estimates of both models on driving stock returns from a cross-sectional regression point of view, a series of Fama-Macbeth regressions are performed which employ different combinations of the above estimates as independent variables.

In order to investigate possible multicollinearity, the correlation coefficient matrix of all estimates is presented in Table 4.9. It can be seen from Table 4.9 that none of the estimates are highly correlated with one another.[40] Between these estimates, the most correlated pair is b_S^* and d_S^* with a correlation coefficient at 0.43, followed by b_F^* and b_F^{+*}, and b_S^{-*} and c_S^{+*} at 0.38 and 0.33 respectively. Since none of the estimates is highly correlated with another, econometrically, all of them can be employed in Fama-Macbeth regression methodology.[41]

Fama-Macbeth regressions are performed on different combinations of estimates. The estimated coefficients are shown in Table 4.10 to Table 4.12 with Newey-West (1987) heteroscedastic robust standard errors with 12 lags employed to calculate the t-statistics and the R^2 values presented in the tables are adjusted R^2 values.

[40] Here we define high correlation as a correlation coefficient greater than 0.5 or less than -0.5.
[41] Only bivariate correlations are tested in this thesis, multivariate correlations should have tested due to limited space, and multicollinearity is unlikely to change the result.

Table 4.9 Correlation Coefficients Between Factor Loadings of Both Models'

This table reports the correlation coefficients between all factor loadings of the CPPF and the FFF models. To avoid repetition, only the lower triangle of the matrix is shown.

	b_S^*	b_S^{-*}	b_S^{+*}	b_F^*	b_F^{-*}	b_F^{+*}	c_S^*	c_S^{-*}	c_S^{+*}	c_F^*	c_F^{-*}	c_F^{+*}	d_S^*	d_S^{-*}	d_S^{+*}	d_F^*	d_F^{-*}	d_F^{+*}
b_S^*	1.0000																	
b_S^{-*}	0.0004	1.0000																
b_S^{+*}	-0.0013	-0.1297	1.0000															
b_F^*	0.0073	0.0012	0.0025	1.0000														
b_F^{-*}	0.0026	0.0077	-0.0035	0.2997	1.0000													
b_F^{+*}	0.0035	-0.0041	0.0121	**0.3774**	-0.2454	1.0000												
c_S^*	-0.2451	-0.0101	0.0169	-0.0037	-0.0121	0.0060	1.0000											
c_S^{-*}	-0.0003	-0.0116	0.0191	0.0005	-0.0004	0.0012	0.0010	1.0000										
c_S^{+*}	-0.0013	**0.3318**	-0.0030	-0.003	0.0006	-0.0026	-0.0023	-0.0259	1.0000									
c_F^*	-0.0011	-0.0012	-0.0028	-0.0139	-0.0317	0.0111	0.0090	0.0013	0.0025	1.0000								
c_F^{-*}	-0.006	-0.0044	-0.0024	0.0258	-0.1546	0.2540	0.0169	0.0009	0.0026	0.3038	1.0000							
c_F^{+*}	0.0001	-0.0022	0.0016	-0.0444	0.0932	-0.1733	-0.0036	0.0002	0.0066	0.1358	-0.1648	1.0000						
d_S^*	**0.4324**	0.0022	-0.0045	-0.0001	0.0016	-0.0016	-0.2643	-0.0005	-0.0020	-0.0022	-0.0062	-0.0000	1.0000					
d_S^{-*}	0.0001	0.0013	0.0004	-0.0002	-0.0027	0.0011	-0.0002	0.0007	0.0031	-0.0001	-0.0003	0.0003	0.0002	1.0000				
d_S^{+*}	-0.003	0.0043	0.0525	0.0021	0.0009	0.0000	0.0058	0.0053	-0.0009	0.0014	-0.0003	0.0010	-0.0014	-0.0000	1.0000			
d_F^*	-0.0026	0.0026	0.0028	-0.0413	0.0017	-0.0346	-0.0035	-0.0009	0.0019	-0.1584	-0.0650	-0.0118	0.0076	0.0025	-0.0025	1.0000		
d_F^{-*}	-0.0039	-0.0003	0.0004	0.0228	-0.0029	0.0258	0.0041	-0.0001	-0.0000	-0.0086	-0.0388	-0.0090	0.0123	0.0000	0.0000	0.0008	1.0000	
d_F^{+*}	-0.001	0.0012	-0.0025	-0.008	0.0814	-0.0858	-0.0020	0.0004	0.0023	-0.0182	0.0649	-0.0371	0.0029	-0.0006	-0.0058	0.1207	0.0080	1.000

Table 4.10 Fama-Macbeth Regression of CPPF Model Factor Loadings

This table reports the result of the Fama-Macbeth regression of the CPPF model factor loadings on excess stock returns. The t-statistics in the square brackets are calculated by using Newey-West (1987) heteroscedastic robust standard error with 12 lags. * denotes significance at the 10% level, ** denotes significance at the 5% level and ***denotes significance at the 1% level.

	1	2	3	4	5	6	7	8
b_S^*	0.00336***		0.000435***			0.00063 1*	0.00136***	
	[3.38]		[3.09]			[1.87]	[2.79]	
c_S^*	-0.000246			0.00000587		-0.0000503		0.0000986
	[-0.30]			[0.12]		[-0.36]		[0.43]
d_S^*	-0.0329				-0.00173		-0.0104	0.000218
	[-0.63]				[-0.39]		[-0.66]	[0.02]
b_S^{-*}		-0.00814***		-0.000390***	-0.000264*			0.00000105
		[-6.55]		[-3.63]	[-1.80]			[0.04]
b_S^{+*}		0.0110***		0.00103***	0.000438***			0.0000540***
		[10.69]		[4.05]	[3.24]			[2.81]
c_S^{+*}		-0.00599***	-0.000286***		-0.000131***		-0.0000179*	
		[-6.34]	[-4.57]		[-3.65]		[-1.68]	
c_S^{-*}		0.00485***	0.000397***		0.000234***		0.0000244**	
		[6.43]	[3.35]		[2.60]		[2.46]	
d_S^{+*}		-0.251***	-0.0137***	-0.0206**		-0.00425		
		[-3.81]	[-2.81]	[-2.02]		[-1.22]		
d_S^{-*}		0.217***	0.0199***	0.0235***		0.00291**		
		[6.24]	[4.00]	[3.91]		[2.00]		
Cons	0.00417**	0.00286*	0.00795***	0.00760***	0.00834***	0.00795***	0.00692***	0.00842***
	[2.47]	[1.72]	[3.05]	[3.00]	[3.26]	[3.25]	[2.90]	[3.24]
No. of Obs	2396262	2396262	2396262	2396262	2396262	2396262	2396262	2396262
Adjusted R^2	0.129	0.296	0.035	0.037	0.032	0.032	0.038	0.032

It can be seen from Table 4.10 that estimates of the CPPF-based cross-sectional model are employed in different possible combinations to examine the sensitivity of risk factor coefficients to stock returns. Among these eight regressions, regression 2 produces the highest adjusted R^2 value at 0.30, with all estimates highly significant at the 1% significance level. Regression 2 employs all the downside and upside estimates of the CPPF model to explain the movement of stock returns without considering the classic estimates. Among the independent variables in regression 2, d_S^{-*} and d_S^{+*} have coefficients of 0.251 and 0.217, respectively. Regression 1 aims to employ all the classic estimates to explain the movement of stock returns regardless of the downside and upside estimates. It produces the second highest R^2 value among the eight regressions, however, c_S^* and d_S^* are not significant even at the 10% significance level. Regression 3 employs downside and upside estimates of commodity market and ADS index risk to explain stock returns. All of the independent variables are significant at the 1% significance level. However, it produces a much lower R^2 value than regression 2 at 0.04.

It can be concluded that classic estimates do not have enough explanatory power on stock returns, and when dividing the market risk into downside and upside risk, downside and upside estimates have more explanatory power than classic ones. Regarding the importance of market risk, although commodity market and business risk do have a relationship with the stock market, market risk is still an essential element relating to stock returns.

It can be seen from Table 4.11 that estimates of the FFF model are employed in different combinations. Among these eight regressions, regression 2 produces the best fit with an adjusted R^2 value of 0.14, with all estimates significant at the 1% level.

Table 4.11 Fama-Macbeth Regression of the FFF Model Factor Loadings

This table reports the result of the Fama-Macbeth regression of the FFF model factor loadings on excess stock returns. The t-statistics in the square brackets are calculated by using Newey-West (1987) heteroscedastic robust standard error with 12 lags. * denotes significance at the 10% level, ** denotes significance at the 5% level and ***denotes significance at the 1% level.

	1	2	3	4	5	6	7	8
b_F^*	0.00567***		0.00608***			0.00597***	0.00578***	
	[3.52]		[4.38]			[4.08]	[3.97]	
c_F^*	-0.00156			0.00144		-0.000992		-0.00143
	[-1.28]			[-1.47]		[-0.95]		[-1.34]
d_F^*	-0.0301				-0.0104		-0.00434	-0.0259
	[-0.37]				[-0.15]		[-0.06]	[-0.35]
b_F^{-*}		-0.0110***		-0.00371***	-0.00322***			-0.00134***
		[-8.17]		[-5.35]	[-5.04]			[-2.99]
b_F^{+*}		0.0153***		0.00702***	0.00582***			0.00316***
		[13.07]		[8.45]	[8.07]			[5.58]
c_F^{-*}		-0.00791***	-0.00253***		-0.00271***		-0.00117***	
		[-7.95]	[-6.83]		[-5.64]		[-4.38]	
c_F^{+*}		0.00661***	0.00267***		0.00276***		0.00142***	
		[7.35]	[6.33]		[5.70]		[5.57]	
d_F^{-*}		-0.277***	-0.114***	-0.143***		-0.0856***		
		[-4.16]	[-4.52]	[-4.21]		[-4.13]		
d_F^{+*}		0.242***	0.120***	0.109***		0.0745***		
		[6.80]	[5.59]	[5.60]		[5.07]		
Con	0.00929***	0.00716***	0.00721***	0.00688***	0.00928***	0.00728***	0.00917***	0.00909***
	[4.45]	[3.85]	[3.12]	[3.18]	[4.40]	[3.27]	[4.13]	[4.23]
No. of Obs	2396262	2396262	2396262	2396262	2396262	2396262	2396262	2396262
Adjusted R^2	0.071	0.135	0.095	0.094	0.099	0.075	0.077	0.079

It employs all the downside and upside risk estimated coefficients of the FFF model to explain the movement of stock returns without considering the classic betas. Among the independent variables in regression 2, d_F^{-*} and d_F^{+*} have coefficients of 0.277 and 0.242, respectively. Regression 3 produced the second best fit with an adjusted R^2

of 0.1. It employs the classic stock market beta with downside and upside commodity market and ADS index risk to explain stock returns, with all variables significant at the 1% level significance.

It can be concluded from Table 4.10 and Table 4.11 that when the estimates are separately employed in the Fama-Macbeth regression based on their original models, the downside and upside risk estimates of both models of all three risk factors are significantly priced and produce the best fit, while the classic estimates did not perform as well as downside and upside ones. Among the downside and upside risk estimates, the ones that employ ADS index risk explain stock returns the most. Downside and upside risk estimates of the CPPF model, employed as independent variables in Fama-Macbeth regression produced the best fit among all regressions in both Table 4.10 and Table 4.11.

For the sake of completeness, rather than dividing estimates into two groups based on their original models, all available estimates are employed in different combinations to perform Fama-Macbeth regressions, in order to examine whether putting estimates from both models together could enhance the cross-sectional explanatory power. The results of these exercises are shown in Table 4.12. It is obvious that when all estimates are employed, regression 7 produces the highest R^2 value of 0.38 among all regressions from Table 4.10 to Table 4.12. However the best fit does not make all estimates significant, particularly, d_S^* c_S^*, c_F^* and d_F^* which are not even significant at the 10% significance level. Regression 2 employs all the downside and upside estimates of both models and produces the second best fit with an adjusted R^2 value of 0.35. All estimates of regression 2 are highly significant at the 1% significance level. The remaining regressions in Table 4.12 produce low adjusted R^2 values with certain independent variables being not significant.

Notably, the classic estimates of the commodity market and ADS index risk of both models, c_S^*, d_S^*, c_F^* and d_F^*, have never been significant in any regression shown in Table 4.10 to Table 4.12. In contrast, the downside and upside estimates of all three factors associated with both models are almost always significant. It can be concluded from Table 4.10 to Table 4.12 that from a cross-sectional point of view, downside and upside estimates are not only components of classic estimates, but also produce better explanatory power than classic estimates. The importance of downside and upside estimates show that explaining the movement of stock returns can be more precisely achieved by examining the downside and upside of risk factors individually rather than treating risk factors as a whole. Moreover, it also can be summarized that apart from stock market risk itself, commodity market and ADS index risk do have significant relations with stock returns. The downside risk estimates have a negative relationship with stock returns, while upside estimates show a positive one. Furthermore, between the CPPF model and the FFF model, with all estimates significant, the former one does produce a slightly better fit than the latter one.

Table 4.12 Fama-Macbeth Regression of the both models' Factor Loadings

This table reports the result of the Fama-Macbeth regression of both the CPPF model and the FFF model factor loadings on excess stock returns. The t-statistics in the square brackets are calculated by using Newey-West (1987) heteroscedastic robust standard error with 12 lags. * denotes significance at the 10% level, ** denotes significance at the 5% level and ***denotes significance at the 1% level.

	1	2	3	4	5	6	7
b_S^*	0.00292***		0.00125**		0.00255***		0.000892*
	[3.56]		[2.08]		[3.61]		[1.93]
c_S^*	-0.0000123		0.000106		-0.0000623		0.000122
	[-0.02]		[0.25]		[-0.10]		[0.36]
d_S^*	-0.0208		0.00230		-0.0154		0.00457
	[-0.49]		[0.22]		[-0.43]		[0.57]
b_F^*	0.00312***			0.00257***		0.00263**	0.00128**
	[2.62]			[3.53]		[2.45]	[2.56]
c_F^*	-0.00150**			-0.00105**		-0.000579	-0.000327
	[2.01]			[-2.21]		[-0.92]	[-1.39]
d_F^*	-0.00837			-0.0021		0.00967	0.0127
	[-0.17]			[-0.09]		[0.46]	[1.50]
b_S^{-*}		-0.00789***	-0.00799***			-0.00796***	-0.00778***
		[-6.49]	[-6.46]			[-6.54]	[-6.42]
b_S^{+*}		0.0106***	0.0107***			0.0107***	0.0105***
		[10.73]	[10.57]			[10.73]	[10.64]
c_S^{-*}		-0.00580***	-0.00586***			-0.00587***	-0.00572***
		[-6.30]	[-6.28]			[-6.32]	[-6.25]
c_S^{+*}		0.00469***	0.00473***			0.00475***	0.00462***
		[6.45]	[6.44]			[6.45]	[6.45]
d_S^{-*}		-0.244***	-0.247***			-0.245***	-0.241***
		[-3.80]	[3.79]			[-3.81]	[-3.79]
d_S^{+*}		0.211***	0.212***			0.214***	0.208***
		[6.23]	[6.25]			[6.23]	[6.24]
b_F^{-*}		-0.00486***		-0.00998***	-0.00885***		-0.00388***
		[-6.17]		[-9.10]	[-7.99]		[-6.24]
b_F^{+*}		0.00668***		0.0128***	0.0111***		0.00434***
		[8.85]		[13.35]	[12.73]		[8.30]
c_F^{-*}		-0.00305***		-0.00667***	-0.00606***		-0.00212***
		[-7.64]		[-8.25]	[8.08]		[-7.19]
c_F^{+*}		0.00262***		0.00599***	0.00477***		0.00179***
		[6.50]		[7.48]	[8.11]		[6.91]

Table 4.12 -continued

	1	2	3	4	5	6	7
d_F^{-*}		-0.0660***		-0.235***	-0.198***		-0.0473***
		[-4.80]		[-4.30]	[-4.35]		[-4.58]
d_F^{+*}		0.0687***		0.209***	0.177***		0.0420***
		[5.97]		[6.86]	[6.83]		[5.79]
Cons	0.00567***	0.00304**	0.00160	0.00769***	0.00468***	0.00349**	0.00261**
	[3.59]	[2.30]	[1.39]	[4.23]	[3.31]	[2.44]	[2.56]
No. of Obs	2396262	2396262	2396262	2396262	2396262	2396262	2396262
Adjusted R^2	0.160	0.135	0.347	0.094	0.099	0.075	0.380

For the sake of completeness, cross-sectional regressions employing all possible variables are exercised. Although there is no sign of multicollinearity between all available variables econometrically (bivariate correlations), the implication of employing risk estimates of the same risk factors from both models is still questionable. Nevertheless, it is clear that employing downside and upside estimates of both models produces a much higher adjusted R^2 value with all estimates significant. It is most likely that the CPPF model and the FFF model can complement each other, and the downside and upside risk estimated coefficients could capture something that the one of the other models could not.

Finally, consistent with the results of chapter 3,[42] the downside and upside estimates of market risk are highly significant, and have negative and positive relations with stock returns, respectively. More importantly, employing commodity market and ADS index risk in the regressions leads to a dramatic increase in adjusted R^2 values. In the

[42] See Table 3.9 in Chapter 3.

previous chapter, with all estimates significant, the highest adjusted R^2 value is 0.153,[43] while it increases to 0.35 (regression 2 table 4.12). It is obvious that commodity market risk and real business risk do have strong explanatory power on stock returns. While there could be other factors significantly driving stock prices, the above three factors are preferred because they measure the whole economy in a more comprehensive way.

4.7 The predictability of risk factor estimate coefficients

After revealing the relationship between realized stock returns and estimates of both models, the predictability of the risk factor coefficients is examined. As in earlier sections, the relative estimates associated with the CPPF model, denoted by (b_S^{-*}- b_S^*), (c_S^{-*}- c_S^*) and (d_S^{-*}- d_S^*) for the downside market, and (b_S^{+*}- b_S^*), (c_S^{+*}- c_S^*) and (d_S^{+*}- d_S^*) for the upside market, and repetitive measures associated with the FFF model. Moreover, the annualized average excess return of each stock are computed based on the following year's data. Furthermore, all stocks in the sample are assigned into five portfolios based on the mean of the target estimate. Finally, the equally weighted average of estimates and future one year excess returns for each portfolio are computed. The results are shown in Tables 4.13 to 4.18.

[43] See regression 11 in Table 3.9 in Chapter 3.

Table 4.13 Future Excess Stock Returns Sorted by Stock Market Factor Loadings of the CPPF Model

This table presents the relationship between future excess stock returns and the stock market factor loadings associated with the CPPF model. The column labeled "return" reports the annual average future stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by b_S^*

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	8.69%	-1.01	-0.48	0.11
2	17.44%	0.65	0.31	-0.13
3	21.64%	1.06	1.11	0.58
4	18.31%	1.49	1.64	2.00
5 High	7.76%	2.10	1.92	1.24
High-Low	-0.93%	3.11	2.41	1.12

Panel 2 Stocks Sorted by b_S^{-*}

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	7.45%	0.99	-1.37	1.61
2	20.86%	1.05	-0.72	1.13
3	23.40%	1.09	1.08	0.98
4	16.94%	1.06	1.75	0.77
5 High	5.19%	1.12	1.92	-1.69
High-Low	-2.26%	0.13	3.30	-3.30

Panel 3 Stocks Sorted by b_S^{+*}

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	6.68%	0.60	1.96	-0.54
2	12.42%	1.05	1.55	-0.53
3	21.47%	1.16	1.04	0.92
4	20.60%	1.31	0.81	1.14
5 High	12.66%	1.19	-1.85	1.81
High-Low	5.98%	0.59	-3.81	2.35

Panel 4 Stocks Sorted by $(b_S^{-*} - b_S^*)$

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	8.82%	1.79	-0.36	1.36
2	20.84%	1.33	0.55	1.15
3	22.13%	1.03	1.06	0.98
4	15.78%	0.82	1.58	0.41
5 High	6.27%	-1.67	1.93	-1.71
High-Low	-2.55%	-3.46	2.30	-3.07

Panel 5 Stocks Sorted by $(b_S^{+*} - b_S^*)$

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	6.86%	1.42	1.71	-1.50
2	11.98%	1.19	1.43	-0.38
3	22.23%	1.00	0.95	0.94
4	21.85%	0.92	1.25	1.01
5 High	10.93%	-1.84	-1.20	1.73
High-Low	4.07%	-3.26	-2.91	3.23

Panel 6 Stocks Sorted by $(b_S^{-*} - b_S^{+*})$

Portfolio	Return	b_S^*	b_S^{-*}	b_S^{+*}
1 Low	10.62%	1.42	-1.21	1.61
2	22.66%	1.32	0.53	1.64
3	21.18%	0.97	1.02	0.91
4	12.94%	0.90	1.34	-0.34
5 High	6.43%	0.69	1.84	-1.04
High-Low	-4.19%	-0.73	3.06	-2.65

Table 4.14 Future Excess Stock Returns Sorted by Commodity Market Factor Loadings of the CPPF Model

This table presents the relationship between future excess stock returns and the commodity market factor loadings associated with the CPPF model. The column labeled "return" reports the annual average future stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by c_S^*

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	5.94%	-1.33	-0.01	-2.57
2	19.74%	-0.30	-1.93	-2.00
3	19.77%	1.11	0.79	0.60
4	19.45%	1.58	-0.87	1.94
5 High	8.94%	1.82	2.16	1.30
High-Low	3.00%	3.14	2.18	3.87

Panel 2 Stocks Sorted by c_S^{-*}

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	7.35%	-0.43	-0.58	1.67
2	18.81%	-0.78	1.07	1.84
3	20.88%	1.02	1.46	0.46
4	17.76%	-0.07	1.52	-1.57
5 High	9.05%	1.63	2.10	-1.12
High-Low	1.70%	2.06	2.68	-2.79

Panel 3 Stocks Sorted by c_S^{+*}

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	6.54%	-1.07	1.97	-1.18
2	19.90%	0.01	1.33	-0.55
3	21.53%	1.11	-0.11	0.14
4	19.22%	-0.44	0.36	1.44
5 High	6.65%	-0.24	-1.41	1.76
High-Low	0.11%	0.83	-3.38	2.94

Panel 4 Stocks Sorted by $(c_S^{-*} - c_S^*)$

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	8.76%	0.81	-0.53	0.82
2	18.31%	1.49	-0.16	1.56
3	21.77%	0.11	0.06	0.27
4	15.82%	-0.28	1.05	0.51
5 High	9.17%	-1.77	1.69	-1.90
High-Low	0.41%	-2.58	2.22	-2.72

Panel 5 Stocks Sorted by $(c_S^{+*} - c_S^*)$

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	6.70%	0.97	1.77	-0.45
2	18.18%	0.53	1.76	-0.17
3	23.83%	0.14	-0.08	0.18
4	18.50%	-0.29	0.18	1.33
5 High	6.62%	-1.99	-1.49	1.51
High-Low	-0.08%	-2.96	-3.26	1.96

Panel 6 Stocks Sorted by $(c_S^{-*} - c_S^{+*})$

Portfolio	Return	c_S^*	c_S^{-*}	c_S^{+*}
1 Low	7.74%	-1.78	-0.17	1.78
2	18.21%	1.32	0.48	1.17
3	23.54%	1.16	1.00	0.05
4	17.55%	-1.20	1.20	-1.13
5 High	6.80%	1.50	1.60	-1.27
High-Low	-0.94%	3.28	1.77	-3.05

Table 4.15 Future Excess Stock Returns Sorted by Business Conditions Factor Loadings of the CPPF Model

This table presents the relationship between future excess stock returns and the business conditions factor loadings associated with the CPPF model. The column labeled "return" reports the annual average future stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by d_S^*

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	11.42%	-0.69	-1.57	-1.07
2	18.47%	-0.02	-1.51	-0.69
3	23.10%	0.00	0.75	0.53
4	16.24%	1.01	-0.17	0.40
5 High	4.61%	1.58	-1.17	-1.22
High-Low	-6.81%	2.27	0.40	-0.15

Panel 2 Stocks Sorted by d_S^{-*}

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	9.31%	-0.07	-1.04	-2.10
2	18.05%	-0.02	-0.87	1.20
3	23.09%	0.00	-0.01	0.03
4	18.25%	1.03	0.74	-0.27
5 High	5.14%	1.14	1.68	-0.50
High-Low	-4.17%	1.21	2.72	1.60

Panel 3 Stocks Sorted by d_S^{+*}

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	8.07%	-0.19	-0.30	-1.79
2	15.46%	1.03	-0.87	-0.47
3	22.68%	-0.01	1.08	1.01
4	15.99%	1.31	1.72	1.40
5 High	11.64%	-0.04	-0.30	1.81
High-Low	3.57%	0.15	0.00	3.60

Panel 4 Stocks Sorted by (d_S^{-*} - d_S^*)

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	10.15%	1.13	-0.91	-1.16
2	17.54%	1.06	-0.84	-0.58
3	20.92%	-1.00	1.01	1.12
4	20.62%	-1.04	1.43	-0.86
5 High	4.61%	-1.26	1.86	-1.56
High-Low	-5.54%	-2.39	2.78	-0.40

Panel 5 Stocks Sorted by (d_S^{+*} - d_S^*)

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	8.08%	1.20	-0.50	-0.68
2	13.76%	1.05	-0.02	-0.47
3	23.44%	-1.00	1.14	1.00
4	17.44%	-1.05	0.95	1.40
5 High	11.13%	-1.32	-1.24	1.72
High-Low	3.05%	-2.52	-0.74	2.40

Panel 6 Stocks Sorted by (d_S^{-*} - d_S^{+*})

Portfolio	Return	d_S^*	d_S^{-*}	d_S^{+*}
1 Low	10.88%	-1.06	-1.01	1.84
2	18.11%	-1.03	-1.01	0.49
3	22.84%	-0.03	1.01	0.01
4	17.10%	1.10	1.94	-0.53
5 High	4.91%	-1.04	2.52	-1.87
High-Low	-5.97%	0.02	3.54	-3.72

For the CPPF model, it can be seen from Table 4.13 that when stocks are sorted by b_S^*, b_S^{-*}, b_S^{+*}, (b_S^{-*}- b_S^*) and (b_S^{+*}- b_S^*), the highest future returns all appear in portfolio 3, and returns present a reversed U-shaped pattern. When stocks are sorted by (b_S^{-*}- b_S^{+*}), the reversed U-shaped pattern still exists. When stocks are sorted by estimates of the commodity market and ADS index risk, it is even more obvious from Table 4.14 and Table 4.15 that the reversed U-shaped pattern of future returns is present, and with portfolio 3 of each group producing the highest future return.

For the FFF model, it can be seen from Table 4.16 to Table 4.18 that the reversed U-shaped pattern on future returns on all groups of portfolios exists except when stocks are sorted by b_F^{+*} and (b_F^{+*}- b_F^*). For the remaining groups, portfolio 1 or portfolio 5 never produces the highest future return but constantly has the lowest future return.

It can be concluded from the results that the medium value estimates of the commodity market and ASD index risk lead to a high future return, while the top and bottom value estimates constantly lead to a low future return. Moreover, for estimates of stock market risk, there is very weak evidence that low upside estimates indicate a high future return on the FFF model. However, the estimates of the CPPF model do not support the evidence, the remaining estimates of stock market risk appear to be consistent with the estimates of the commodity market and ADS index risk.

Table 4.16 Future Excess Stock Returns Sorted by Stock Market Factor Loadings of the FFF Model

This table presents the relationship between future excess stock returns and stock market factor loadings associated with the FFF model. The column labeled "return" reports the annual average future stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by b_F^*

Portfolio	Return	b_F^*	b_F^{-*}	b_F^{+*}
1 Low	19.19%	0.82	0.83	0.84
2	19.48%	0.96	0.97	0.98
3	19.66%	1.00	1.00	1.00
4	16.99%	1.03	1.02	1.03
5 High	-1.48%	1.20	1.14	1.19
High-Low	-20.67%	0.38	0.31	0.35

Panel 2 Stocks Sorted by b_F^{-*}

Portfolio	Return	b_F^*	b_F^{-*}	b_F^{+*}
1 Low	-3.61%	0.93	0.50	1.16
2	13.40%	0.98	0.94	1.00
3	20.08%	1.00	1.00	1.00
4	24.21%	1.02	1.04	1.00
5 High	19.77%	1.09	1.48	0.87
High-Low	23.38%	0.16	0.98	-0.29

Panel 3 Stocks Sorted by b_F^{+*}

Portfolio	Return	b_F^*	b_F^{-*}	b_F^{+*}
1 Low	29.28%	0.91	1.13	0.57
2	21.82%	0.98	1.00	0.96
3	21.32%	1.00	1.00	1.00
4	10.25%	1.02	0.99	1.04
5 High	-8.84%	1.10	0.83	1.46
High-Low	-38.12%	0.18	-0.30	0.89

Panel 4 Stocks Sorted by (b_F^{-*} - b_F^*)

Portfolio	Return	b_F^*	b_F^{-*}	b_F^{+*}
1 Low	-9.21%	1.04	0.55	1.27
2	6.64%	0.99	0.95	1.02
3	20.98%	1.00	1.00	1.00
4	25.84%	0.99	1.03	0.97
5 High	29.61%	0.99	1.42	0.77
High-Low	38.82%	-0.05	0.87	-0.50

Panel 5 Stocks Sorted by (b_F^{+*} - b_F^*)

Portfolio	Return	b_F^*	b_F^{-*}	b_F^{+*}
1 Low	23.47%	1.02	1.25	0.63
2	23.44%	1.00	1.02	0.97
3	21.27%	1.00	0.99	1.00
4	10.98%	0.99	0.96	1.02
5 High	-5.33%	1.00	0.72	1.41
High-Low	-28.80%	-0.03	-0.53	0.78

Panel 6 Stocks Sorted by (b_F^{-*} - b_F^{+*})

Portfolio	Return	b_F^*	b_F^{-*}	b_F^{+*}
1 Low	-9.57%	1.02	0.58	1.38
2	8.23%	0.99	0.95	1.02
3	20.00%	1.00	0.99	1.00
4	26.19%	1.00	1.03	0.97
5 High	29.00%	1.00	1.39	0.66
High-Low	38.57%	-0.02	0.81	-0.72

Table 4.17 Future Excess Stock Returns Sorted by Commodity Market Factor Loadings of the FFF Model

This table presents the relationship between future excess stock returns and commodity market factor loadings associated with the FFF model. The column labeled "return" reports the annual average future stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by c_F^*

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	-0.18%	-1.31	-1.33	-1.28
2	19.79%	-1.04	-1.04	-1.03
3	20.56%	0.90	-1.00	-1.01
4	21.96%	1.04	1.02	1.03
5 High	11.71%	1.30	1.30	1.13
High-Low	11.89%	2.62	2.63	2.41

Panel 2 Stocks Sorted by c_F^{-*}

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	2.80%	-1.14	-1.96	1.23
2	15.89%	-1.03	-1.08	1.00
3	19.39%	-1.01	-1.01	-1.01
4	20.35%	1.02	1.06	-1.02
5 High	15.42%	1.14	1.94	-1.35
High-Low	12.62%	2.28	3.90	-2.59

Panel 3 Stocks Sorted by c_F^{+*}

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	15.40%	0.90	1.25	-0.16
2	22.85%	0.97	0.99	0.93
3	22.08%	1.00	0.99	0.99
4	13.80%	1.02	0.97	1.06
5 High	-0.29%	1.09	0.74	2.03
High-Low	-15.69%	0.19	-0.50	2.18

Panel 4 Stocks Sorted by $(c_F^{-*} - c_F^*)$

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	5.36%	1.01	0.12	1.39
2	15.92%	0.99	0.93	1.04
3	20.40%	1.00	0.99	1.00
4	18.29%	1.00	1.05	0.96
5 High	13.88%	0.98	1.86	0.45
High-Low	8.52%	-0.03	1.74	-0.94

Panel 5 Stocks Sorted by $(c_F^{+*} - c_F^*)$

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	19.70%	1.05	1.38	-0.08
2	23.17%	1.00	1.04	0.94
3	22.16%	1.00	0.99	0.99
4	13.30%	0.99	0.94	1.04
5 High	-4.49%	0.94	0.59	1.95
High-Low	-24.19%	-0.11	-0.79	2.03

Panel 6 Stocks Sorted by $(c_F^{-*} - c_F^{+*})$

Portfolio	Return	c_F^*	c_F^{-*}	c_F^{+*}
1 Low	-1.55%	0.97	0.23	1.87
2	15.03%	0.99	0.94	1.05
3	20.38%	0.99	0.99	0.99
4	21.22%	1.00	1.05	0.94
5 High	18.77%	1.02	1.74	-0.01
High-Low	20.32%	0.04	1.51	-1.87

Table 4.18 Future Excess Stock Returns Sorted by Business Condition Factor Loadings of the FFF Model

This table presents the relationship between future excess stock returns and the business conditions factor loadings associated with the FFF model. The column labeled "return" reports the annual average future stock returns over the one month T-bill rate. "High-Low" reports the difference between portfolio 5 and portfolio 1.

Panel 1 Stocks Sorted by d_F^*

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	-0.91%	0.99	0.63	0.99
2	17.59%	1.00	0.85	1.00
3	23.82%	1.00	1.30	1.00
4	23.05%	1.00	1.40	1.40
5 High	10.29%	1.01	1.43	1.41
High-Low	11.20%	0.03	0.80	0.42

Panel 2 Stocks Sorted by d_F^{-*}

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	-3.70%	1.00	-0.68	1.00
2	16.25%	1.00	1.00	1.00
3	22.31%	1.00	1.00	1.00
4	20.83%	1.00	1.00	1.00
5 High	18.16%	1.00	1.56	0.99
High-Low	21.86%	0.01	2.24	-0.01

Panel 3 Stocks Sorted by d_F^{+*}

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	16.08%	1.00	0.34	0.94
2	15.88%	1.00	0.85	1.00
3	20.54%	1.00	0.96	1.00
4	19.19%	1.00	1.01	1.00
5 High	2.16%	1.00	0.72	1.05
High-Low	-13.92%	0.01	0.37	0.10

Panel 4 Stocks Sorted by (d_F^{-*} - d_F^*)

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	1.28%	1.00	-0.68	1.00
2	18.61%	1.00	1.00	1.00
3	20.29%	1.00	1.00	1.00
4	17.82%	1.00	1.00	1.00
5 High	15.85%	1.00	1.56	0.99
High-Low	14.57%	0.00	2.24	-0.02

Panel 5 Stocks Sorted by (d_F^{+*} - d_F^*)

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	14.57%	1.00	0.25	0.95
2	17.45%	1.00	0.99	1.00
3	22.26%	1.00	0.91	1.00
4	18.84%	1.00	0.98	1.00
5 High	0.72%	1.00	0.75	1.05
High-Low	-13.85%	0.00	0.50	0.10

Panel 6 Stocks Sorted by (d_F^{-*} - d_F^{+*})

Portfolio	Return	d_F^*	d_F^{-*}	d_F^{+*}
1 Low	-2.45%	1.00	-0.68	1.03
2	18.30%	1.00	1.00	1.00
3	21.17%	1.00	1.00	1.00
4	18.35%	1.00	1.00	1.00
5 High	18.48%	1.00	1.55	0.96
High-Low	20.93%	0.00	2.23	-0.08

4.8 Conclusion

From the cross-sectional point of view, the time-varying conventional estimates of stock market risk play important roles in determining stock returns. Specifically, b_S^* and b_F^* are found have a positive influence on stock returns – a result that is consistent with the classic literature (the high beta high return, low beta low return theory still holds) and previous chapters. However, the classic estimates of commodity market risk c_S^* and c_F^* appear to have a negative impact on stock returns, while classic estimates of ADS index risk d_S^* and d_F^* did not show an obvious impact on stock returns. Furthermore, for all the downside estimates of both models, even when controlling for the classic estimates and upside estimates, there are strong negative impacts on stock returns. When downside estimates increase, stock returns decrease dramatically. There is no clear evidence that (c_S^{-*}- c_S^*) has an impact on stock returns. Moreover, it is found that all the upside estimates even when controlling for the classic estimates, have strong positive impacts on stock returns. When upside estimates increase, stock returns also increase substantially.

When estimates are treated as factors rather than factor loadings in the Fama-Macbeth regression methodology, this chapter finds that downside and upside estimates are not only components of classic estimates, but also produce better explanatory power than classic estimates. The evidence from downside and upside estimates shows that explaining the movement of stock returns can be enhanced by examining the downside and upside of risk factors individually rather than treating the risk factor as a whole. Moreover, it can be summarized that apart from stock market risk, the commodity market and ADS index risks are found to have a significant impact on stock returns. The downside estimates have a negative relationship with stock returns, while upside estimates have a positive impact. Furthermore, between the CPPF model and the FFF model, with all estimates significant, the former one produces a better fit than the latter one. However, it is found that employing downside and upside estimates of both models can produce a much higher adjusted R^2 value with all estimates significant. This could be due to the complementary property of both models.

Finally, the predictive power of all classic, downside and upside estimates of both models is found to be poor. There is very weak evidence that low upside estimates of stock market risk indicate a higher future return when the FFF model is employed.[44] However the estimates of the CPPF model do not support the evidence. Regarding the remaining estimates, only medium value estimates lead to higher future returns, while the top and bottom portfolios continuously lead to lower future returns.

[44] See Panel 3, Table 4.16.

Conclusion

Consistent results are obtained from Chapter 2 to Chapter 4. From a cross-sectional point of view, when the single market factor model is adopted, stock returns accord with the classic high beta high return positive relationship when stocks are sorted by conventional beta. Similar results are obtained when stocks are sorted by downside beta. When controlling for beta, downside betas exhibit a negative relationship with average excess returns, while upside betas present a positive relationship regardless of the benchmark used to compute them.

When the CPPF model and the FFF model are adopted, and time-varying betas are generated. All results are consistent with Chapter 2 (with moving window approach) except for downside beta. The beta is positively significant in driving stock returns for both models. The downside beta shows a negative impact on stock returns, while the upside beta, consistent with beta estimates, shows a positive impact. The different results obtained are most likely due to the approaches used. Chapter 2 adopted the single market factor model with a moving window approach, while Chapter 3 and 4 adopted more flexible time-varying models. Specifically, the estimated coefficients generated by the CPPF and the FFF model produce a much better cross-sectional fit than the single market factor model.

When commodity market risk and real business risk are employed, the conventional commodity market risk appears to have a negative impact on stock returns. Moreover, all the downside risk estimates, even when controlling for the classic and upside risk estimates, exhibit strong negative relationships with stock returns. When downside estimates increase, stock returns decrease dramatically. The reverse patterns appear on the remaining conventional beta and upside beta estimates.

Overall, downside and upside betas deliver more explanatory power. The risk-return relationship shown in this thesis can be attributed to the fact that when downside beta is calculated, the return of the market portfolio is below the average, and very likely to be negative. The expected stock excess return is the product of beta and the excess return of the market portfolio, so when stocks are sorted by downside beta into portfolios, the larger the downside beta, the lower the return, and vice versa. As for the commodity market and ADS index risks, all downside and upside risk estimates exhibit similar effects to those of the market portfolio. While the conventional commodity market risk estimates exhibit a reversed impact on stock returns as the classic beta does. There is no clear evidence that conventional ADS index risk has an impact on stock returns. It suggests that, on both downside and upside, the stock market risk is not the only cross-sectional determinant of US stocks, the commodity market and ADS index risks are also important ones.

In terms of fit, non-linear time-varying estimates deliver much higher adjusted R^2 values in cross-sectional regressions than conventional estimates. It is more obvious when commodity market risk and ADS index risk are involved. Clearly the CPPF and the FFF model outperform the classic market model. It is also found that regressing stock returns on estimates generated from both the CPPF and the FFF models delivers the highest adjusted R^2 values.

Regarding predictive power, downside and upside estimates of both models are found to be poor. There is very weak evidence that low upside estimates of stock market risk indicate a high future return on the FFF model. However the estimates of the CPPF model do not support the evidence. Regarding the remaining estimates, only medium value betas lead to high future returns, while the top and bottom portfolios continuously lead to low future returns.

There are certainly some limitations in this study. First, this study only broadly divides markets into downside and upside, but there could be other criteria to classify the market, for instance, downside and upside of emerging and developed markets. Second, the ADS index is the most updated indicator to describe real business conditions, but there might be other indicators to represent the whole economy in more specific aspects (such as exchange rate, surplus or deficit). Third, although employing all risk estimates of both the CPPF and the FFF model produced the highest adjusted R^2 value, the implication of risk estimated coefficients of the same risk factors in different models is still to be explored. Fourth, alternative nonlinear approaches such as FFF with no periodic constraint can be tested. Fifth, this study mainly focuses on the US stock market. However, emerging markets such as the Chinese stock market could also be the target of further research.

Appendix

$$
S_2 = \begin{bmatrix}
1^0 & 1^1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2^0 & 2^1 & 2^2 & 2^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3^0 & 3^1 & 3^2 & 3^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4^0 & 4^1 & 4^2 & 4^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{3}\right)^0 & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{2t}{3}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
t^0 & t^1 & t^2 & t^3 & \left(\frac{2t}{3}\right)^0 & \left(\frac{2t}{3}\right)^1 & \left(\frac{2t}{3}\right)^2 & \left(\frac{2t}{3}\right)^3 & \left(\frac{t}{3}\right)^0 & \left(\frac{t}{3}\right)^1 & \left(\frac{t}{3}\right)^2 & \left(\frac{t}{3}\right)^3
\end{bmatrix}
$$

Piecewise polynomial matrix with 2 knots

$$
S_3 = \begin{bmatrix}
1^0 & 1^1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2^0 & 2^1 & 2^2 & 2^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3^0 & 3^1 & 3^2 & 3^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{4}\right)^0 & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{2}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{3t}{4}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
t^0 & t^1 & t^2 & t^3 & \left(\frac{3t}{4}\right)^0 & \left(\frac{3t}{4}\right)^1 & \left(\frac{3t}{4}\right)^2 & \left(\frac{3t}{4}\right)^3 & \left(\frac{t}{2}\right)^0 & \left(\frac{t}{2}\right)^1 & \left(\frac{t}{2}\right)^2 & \left(\frac{t}{2}\right)^3 & \left(\frac{t}{4}\right)^0 & \left(\frac{t}{4}\right)^1 & \left(\frac{t}{4}\right)^2 & \left(\frac{t}{4}\right)^3
\end{bmatrix}
$$

Piecewise polynomial matrix with 3 knots

$$
S_4 = \begin{bmatrix}
1^0 & 1^1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2^0 & 2^1 & 2^2 & 2^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3^0 & 3^1 & 3^2 & 3^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{5}\right)^0 & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{2t}{5}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{3t}{5}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{4t}{5}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
t^0 & t^1 & t^2 & t^3 & \left(\frac{4t}{5}\right)^0 & \left(\frac{4t}{5}\right)^1 & \left(\frac{4t}{5}\right)^2 & \left(\frac{4t}{5}\right)^3 & \left(\frac{3t}{5}\right)^0 & \left(\frac{3t}{5}\right)^1 & \left(\frac{3t}{5}\right)^2 & \left(\frac{3t}{5}\right)^3 & \left(\frac{2t}{5}\right)^0 & \left(\frac{2t}{5}\right)^1 & \left(\frac{2t}{5}\right)^2 & \left(\frac{2t}{5}\right)^3 & \left(\frac{t}{5}\right)^0 & \left(\frac{t}{5}\right)^1 & \left(\frac{t}{5}\right)^2 & \left(\frac{t}{5}\right)^3
\end{bmatrix}
$$

Piecewise polynomial matrix with 4 knots

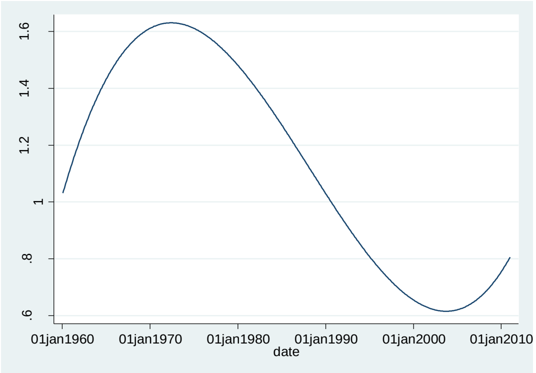
$$
S_5 = \left[\begin{array}{cccccccccccccccccccccccc}
1^0 & 1^1 & 1^2 & 1^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2^0 & 2^1 & 2^2 & 2^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3^0 & 3^1 & 3^2 & 3^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{6}\right)^0 & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{3}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{t}{2}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{2t}{3}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 & \vdots & \vdots & \vdots & \vdots \\
\left(\frac{5t}{6}\right)^0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 1^0 & 1^1 & 1^2 & 1^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 2^0 & 2^1 & 2^2 & 2^3 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
t^0 & t^1 & t^2 & t^3 & \left(\frac{5t}{6}\right)^0 & \left(\frac{5t}{6}\right)^1 & \left(\frac{5t}{6}\right)^2 & \left(\frac{5t}{6}\right)^3 & \left(\frac{2t}{3}\right)^0 & \left(\frac{2t}{3}\right)^1 & \left(\frac{2t}{3}\right)^2 & \left(\frac{2t}{3}\right)^3 & \left(\frac{t}{2}\right)^0 & \left(\frac{t}{2}\right)^1 & \left(\frac{t}{2}\right)^2 & \left(\frac{t}{2}\right)^3 & \left(\frac{t}{3}\right)^0 & \left(\frac{t}{3}\right)^1 & \left(\frac{t}{3}\right)^2 & \left(\frac{t}{3}\right)^3 & \left(\frac{t}{6}\right)^0 & \left(\frac{t}{6}\right)^1 & \left(\frac{t}{6}\right)^2 & \left(\frac{t}{6}\right)^3
\end{array}\right]
$$

Piecewise polynomial matrix with 5 knots

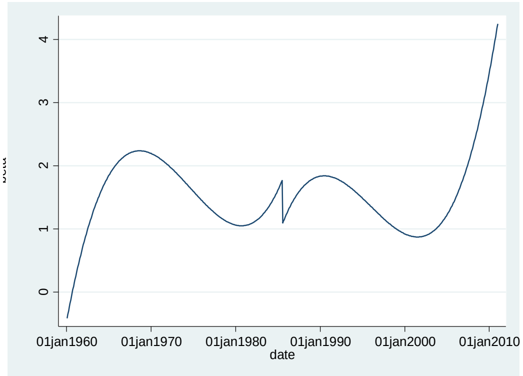
Figure 1 Plots of the CPPF Estimates

This figure shows the plots of the CPPF estimates with different numbers of knots. Panel 1 is based on Boeing Company returns, Panel 2 is based on 21st Century Fox returns, Panel 3 is based on AT&T Inc. returns, Panel 4 is based on HCP Inc. returns, Panel 5 is based on SAIC returns, and Panel 6 is based on PPG Industries returns.

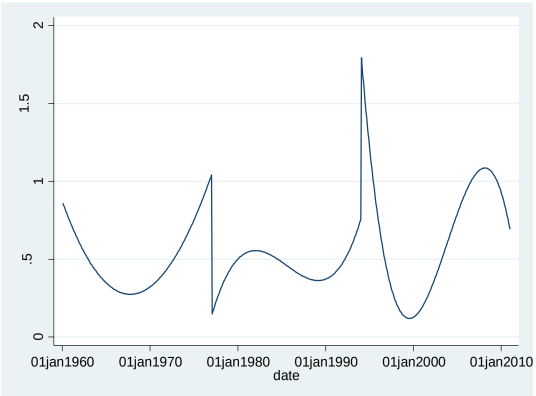
Panel 1 Estimates without knot



Panel 2 Estimates with one knot

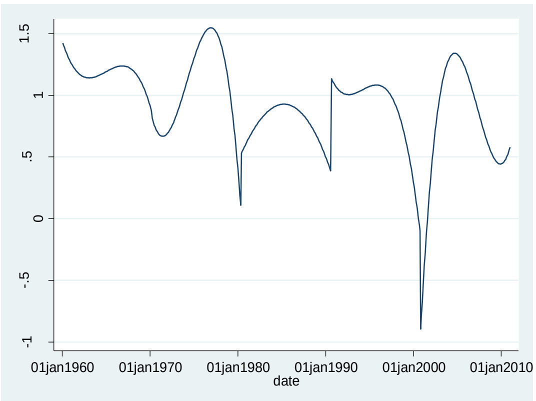


Panel 3 Estimates with two knots



Panel 4 Estimates with three knots

Panel 5 Estimates with four knots



Panel 6 Estimates with five knots

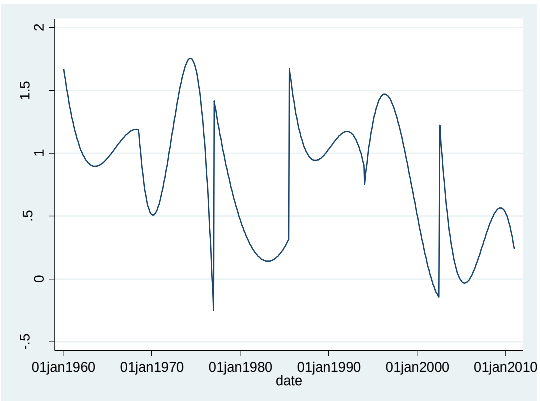


Figure 2 Plots of the CPPF and Cubic Spline Estimates

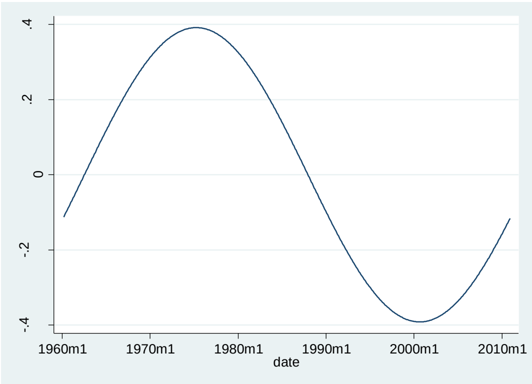
This figure shows the sample plots of the CPPF and cubic spline estimates with five knots. These plots are based on Praxair Inc. returns.

Panel 1 The CPPF estimates | Panel 2 The cubic spline estimates

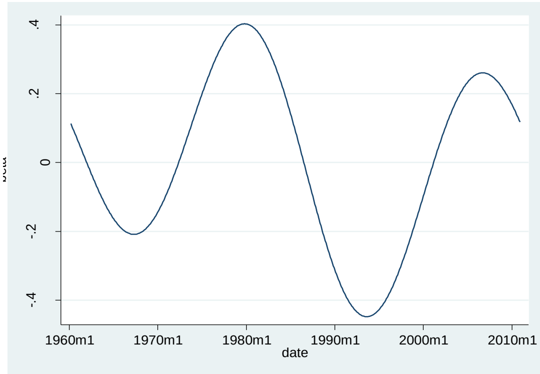
Figure 3 Plots of The FFF Estimates

This figure shows the plots of the FFF estimates with different orders. Panel 1 is based on Nucor Corp. returns, Panel 2 is based on ONEOK returns, Panel 3 is based on Republic Service Inc. returns, and Panel 4 is based on Xilinx Inc. returns.

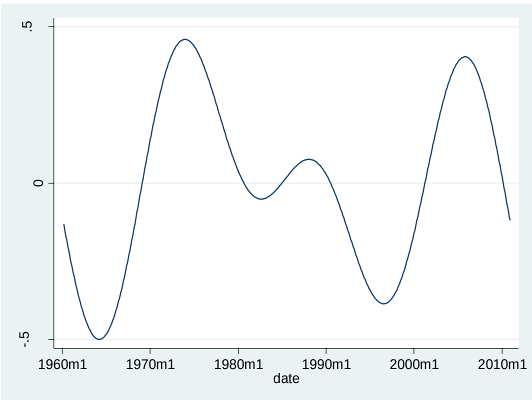
Panel 1 Estimates on order one



Panel 2 Estimates on order two



Panel 3 Estimates on order three



Panel 4 Estimates on order 4

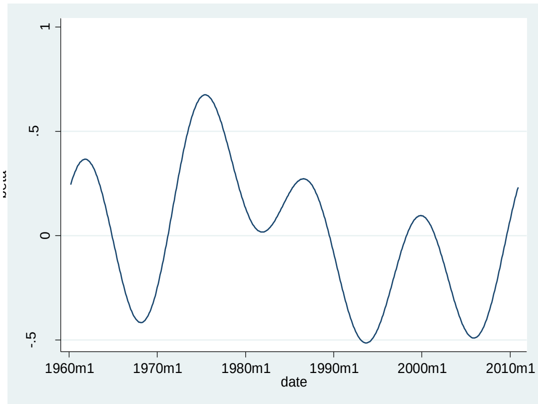


Table A Summary of Both US stocks

This table summarizes the constituents of US stocks, average annual returns and volatility used in Chapter 4, the sample size is from March 1960 to December 2010.

Stock Exchange	Number of Stocks	Percentage to whole Sample	Average Annualized Return	Standard Deviation
NYSE	2198	16.21%	10.59%	10.99%
NASDAQ	7636	56.33%	12.32%	20.13%
AMEX	1105	8.15%	10.98%	16.91%
NYSE & NASDAQ	1031	7.60%	14.12%	13.55%
NYSE & AMEX	556	4.10%	14.10%	13.72%
NASDAQ & AMEX	829	6.11%	11.24%	20.17%
NYSE & NASDAQ & AMEX	202	1.49%	14.61%	16.15%
Total Sample	13557	100%	12.08%	17.06%

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