Multi-Objective Optimisation For Environmentally Friendly Logistics Network

A thesis submitted in partial fulfilment

of the requirement for the degree of Doctor of Philosophy

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Declaration

This work has not previously been accepted in substance for any degree and is not concurrently submitted in candidature for any degree.

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To Kelvin, Thomas, Connor and Nikita
for their patience, love and support,
my mum Tamara and dad Alexander
who sadly did not see me complete my work.

Abstract

Traditionally, infrastructure modelling of logistics network design is driven by a need to reduce costs. However, many real-world cases may involve dealing with multiple and sometimes conflicting objectives, especially when climate change and environmental concerns have been increasingly discussed worldwide.
In this thesis we devise and investigate a multi-objective evolutionary optimization framework together with Lagrangian Relaxation to solve a large size Facility Location Problem (FLP) where 'green issues' (CO_2) and traditional objectives are solved simultaneously, offering the decision maker a choice of trade-off solutions. Lack of benchmark data for multi-objective FLP with environmental objectives created initial difficulties in our research. However, the opportunity to work with a leading UK supermarket supply chain provided a good basis for generating large artificial data sets and to test our techniques with a good range of parameter setting. The analysis of the research indicates that more facilities could be desirable to reduce the environmental impact and that it is possible to offer the decision maker good compromise solutions. Two variants of the FLP are considered during the investigation for building multi-objective decision tools: the uncapacitated and the capacitated.
Additionally, we investigate the optimization of a single source assignment problem as part of our collaborative work with industry. In this way we explore exact and heuristic approaches based on cost optimization as well as considering the environmental impact from vehicles in the two-objective approach on the models with realistic constraints. The trade-off solutions demonstrate to the decision maker how a small increase in cost could equate to a considerable decrease in the distance travelled by the vehicle.

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Notation

AHP	Analytic Hierarchy Process
$CFLP$	Capacitated Facility Location Problem
DC	Distribution Centre
$DEFRA$	Department for Environment, Food and Rural Affairs
EA	Evolutionary algorithm
FGP	Factory Gate Pricing
FLP	Facility Location Problem
FGP	Factory Gate pricing
GA	Genetic Algorithm
GAP	Generalized Assignment Problem
GHG	Greenhouse Gas
HGV	Heavy Goods Vehicles
LCA	life cycle analysis
LB	Lower Bound
LDV	Light Duty Vehicles
LR	Lagrangian Relaxation
$MCLP$	Maximal Covering location problem
MO	Multi-Objective
$MOEAs$	Multi-Objective Evolutionary Algorithms
$MOGA$	Multi-Objective Genetic Algorithm
MOO	Multi-Objective Optimization
$MPSS$	Multi-Period Single Sourcing Problem

$NAEI$	National Atmospheric Emissions Inventory
NP	Non Deterministic Polynomial Time Problems
NPV	Net Present Value
$NSGA-II$	Nondominated Sorting Genetic Algorithm II
OR	Operations Research
$PAES$	Pareto-Archived Evolution Strategy
$SCOR$	Supply Chain Operations Reference
$SEAMO2$	Simple Evolutionary Algorithm for Multi-objective Optimization 2
$SPEA$	Strength Pareto Evolutionary Algorithm
$SSFLP$	Single Source Capacitated Facility Location Problems
UB	Upper Bound
$UFLP$	Uncapacitated Facility Location Problem

Chapter 1

Introduction

1.1 Research motivation

Over the last few decades interest in man's impact on the environment has moved from the local to a global level of concern. There is a general consensus that rising temperature is contributing to disappearing glaciers and increasingly unstable weather patterns around the globe [129]. It is very likely that greenhouse gasses, such as carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) raise the temperature near the surface of our planet [129]. The greenhouse gasses produced by transport and other activities need to be reduced. Some companies are already trying to help the environment through the use of rail [19] and other methods, such as shipment consolidation. Under the Kyoto Protocol, the UK is now legally required to reduce greenhouse gas emissions by about 12.5% by 2012. The threat of climate change has been increasingly discussed at an international level, with greenhouse gas emissions from fossil energy sources being at the forefront of governmental concerns. Transportation, industrial processes and other commercial sectors have been linked to an increase in the greenhouse effect through their release of carbon dioxide, even though the influence of other gases should not be under estimated. The annual carbon dioxide (CO_2) emissions from all transport increased by 17 million tonnes of carbon in the UK during the period from 1970 to 2004 [34]. Although the growth rate has slowed down considerably since 1990, clearly the Government would like to see the trend reversed and emissions cut. Figure 1.1 shows a particular concern in the rise of CO_2 emissions over the past decade from heavy goods vehicles (HGV) and light duty vehicles (LDV), by 19% and 33% respectively [37].

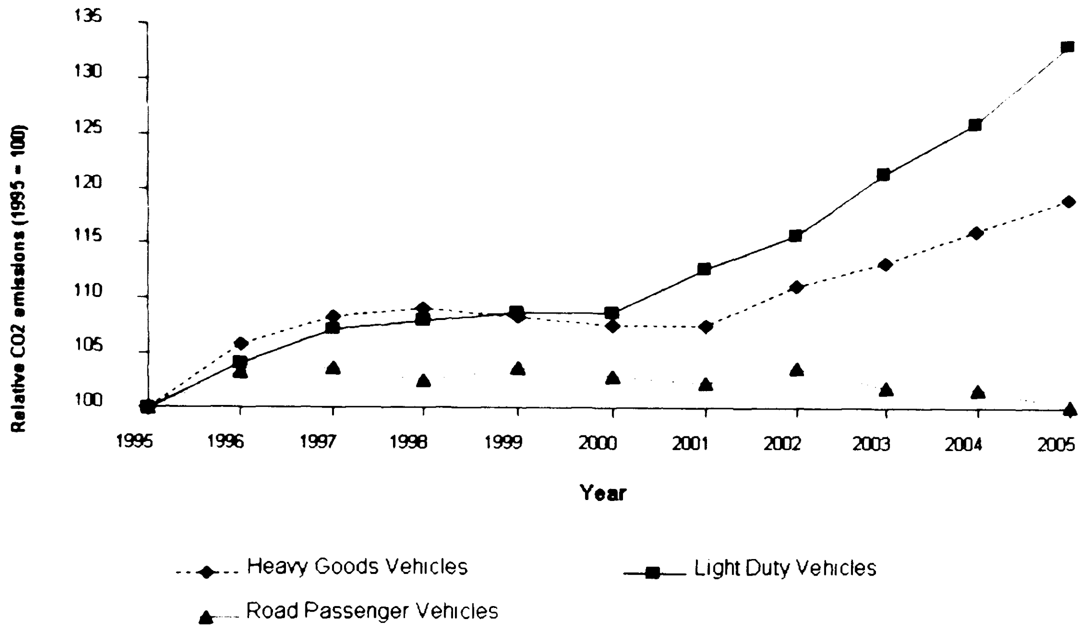
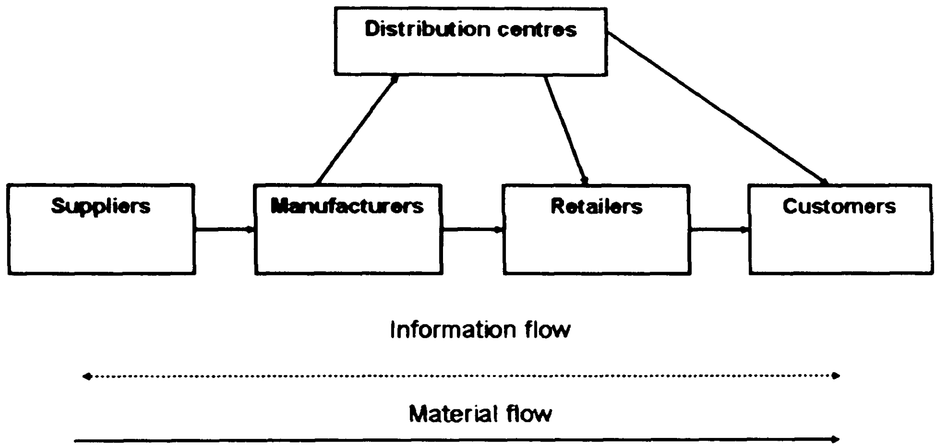


Figure 1.1: UK carbon dioxide emissions for road transport [37].

Historically a supply chain or logistics network has been defined as "a system whose constituent parts include material suppliers, production facilities, distribution services and customers linked together by the feed forward flow of materials and the feedback flow of information" [106]. The main features of a traditional supply chain [9] are illustrated in Figure 1.2, where the solid arrows represent material flows and the dotted arrow represents information flow. When designing a supply chain network, different levels of decisions need to be considered, from strategic through to operational. Strategic decisions typically have a planning period of many years and long lasting effects. The identification of the number, locations and capacities of serving facilities, such as distribution centres (DCs) and warehouses, in a supply chain network, would normally be regarded as strategic planning. Tactical decisions involve a shorter planning horizon, and they are usually revised monthly or quarterly. Tactical activities include the selection of suppliers, assignment of products to DCs, and determining the distribution channel and the type of transportation mode. Finally, operational decisions, such as scheduling and routing activities, consider the day-to-day flow of products through the network, the amount of the inventory to be



Based on Beamon [9].

Figure 1.2: Traditional logistics network.

held by the facilities and so on. These decisions can be modified easily within a short period of time, for instance on a daily or weekly basis.

In the last ten years several major companies have restructured their storage and distribution systems with a view to reducing their costs, and have subsequently reduced their CO_2 emissions as a result of those changes. The factory gate pricing (FGP) concept, where the retailer is responsible for transportation of the product from the supplier, has been analysed for the UK grocery [91] and the Dutch retail industry [67]. Both studies show that cost reductions have brought significant environmental benefits, such as reduced congestion and transport kilometres/miles. Potter *et al.* [91] analyse the Tesco supply chain and suggested that by implementing FGP with consolidation centres for inbound deliveries, a reduction of 28 per cent in vehicle-miles required to transport products to DCs could be achieved, equating to over 400,000 miles per week. Aronsson and Huge Brodin [3] describe three case studies, where companies had undergone changes in their distribution structures that had a positive effect not only on costs but also on the environment (reduced emissions). Among some of the typical changes made are new distribution structures with fewer nodes, larger warehouses, the introduction of new information systems, consolidation of flows, standardized vehicles and load carriers, and changes in transport mode. Therefore a reduced environmental impact frequently results as a by-product of a more efficient distribution system. In such cases improvements in environmental performance can be viewed as positive side effects of traditional methods, without having a fully integrated 'green' supply chain.

But not all infrastructure changes lead to a positive impact on the environment. Kohn [64] describes a case study of a manufacturer of submersible pumps and mixers where he analyses the effects of changing from a decentralised to a centralised network and reveals that lowering costs and improving service performance produce a negative impact on the environment. The overall analysis of direct effects from road transport indicates increase in both tonne-kilometres and CO_2 emissions. These findings correlate UK statistics, that the centralisation of warehousing which was done to reduce inventory has had a direct impact on transportation, increasing the average length of haul from 79 km in 1990 to 87 km in 2004 [33]. The same time, the analysis also opened new opportunities for the company to consider decisions that improve environmental performance of the network, such as shipment consolidation, change of transportation mode (e.g. from road transport to rail) and a reduction in emergency deliveries. Obviously, there are constraints and difficulties that can prevent companies from fully exploiting these opportunities. For example, a switch from road to rail would be difficult for many companies, as in the case discussed by Kohn and Huge Brodin [65], due to limitations imposed by the rail infrastructure of the European Union, but for other companies modal change may be more realistic and beneficial. Hence, the structural changes revealed an opportunity to make environmental improvements in its logistics operations. However, it is clear that environmental benefits are frequently a welcome result of an infrastructure redesign process aimed at reducing costs, this is not always the case as can be seen from the research discussed in Kohn [64]. For this reason, there is a need to address environmental objectives explicitly as part of the logistics design process by integrating economic and 'green' objectives.

1.1.1 Infrastructure modelling

Strategic design of a logistics network focuses on infrastructure modelling, which is not new to academia and has a very rich literature. It is the strategic decision processes which influence both tactical and operational level decisions for the long term efficient operation of a network. It determines the optimum number, capacity, location and allocation of facilities (such as warehouses, distribution centres and consolidation centres) to ensure efficient commodity flows from the service providers to the market. Infrastructure modelling techniques can be used with single or multiple objectives for simple single or multiple product networks. A large range of techniques have been applied to infrastructure modelling, from integer, dynamic, mixed-integer linear programming to heuristic methods and genetic algorithms. Coyle *et al.* [21] describe the principle modelling approaches such as mathematical optimization, simulation and heuristic models. Mathematical optimization aims to find optimum solutions based on precise mathematical procedures. Heuristic approaches, on the other hand, do not guarantee optimal solutions but can produce an acceptable solution in a reasonable amount of time. Simulation allows a user to test the effect of alternative locations on costs and service levels.

Traditionally, infrastructure modelling mainly focuses on a single objective function such as cost minimization or profit maximization, with all customer demands satisfied to a certain minimum level, and without exceeding the capacities of the facilities. With increasing environmental concerns and/or high levels of commercial competition, there is a need to deal with objectives to minimize the environmental impact and improve customer service simultaneously with that to reduce cost or maximize profits. When multiple objectives are involved then, conventionally, companies will try to adjust the various parameters under their control in order to simultaneously maximize profit (or minimize costs) and optimize customer service, for example. But, the two objectives are frequently in conflict and devising a single performance measure that weights the two objectives in a satisfactory way is a challenge. An added complication arises when we wish to incorporate appropriate quantifying environmental measures into the model.

There are several different approaches to dealing with multiple objectives. Traditional methods require user input to prioritize or weight the various objectives, in advance of any optimization. In recent years, however, a new approach has been developed that involves no such judgments and produces a set of viable alternatives (a Pareto set) from which a decision maker can make an informed selection at a later stage. This approach has the advantage that excellent solutions can be found that may be missed by the other methods. The disadvantage is that it may generate a large number of potential solutions.

1.1.2 OR and environmentally friendly network design

Operations Research (OR) uses mathematical or heuristic methods for analysis, optimization and decision making for solving real-world problems. A problem is normally formulated as a set of mathematical expressions with objective function(s) and constraints. The objective function, such as cost minimization, measures a system's performance whereas the constraints enforce realistic conditions, such as service level, to generate feasible solutions. Today the need for 'desirable' environmentally friendly networks is becoming ever more urgent. Bloemhof-Ruwaard *et al.* [14] address the need for an integrated assessment model to consider all aspects of the system, to identify the causes, measure the emissions, and assess the efficiency of transportation systems from a global perspective. They claim that the added value of OR consists of evaluation (efficiency) and improvement (effectiveness) of emission and waste reduction scenarios. It is also important to model environmental issues as objectives, rather than as constraints, because modelling them as objectives will generate more information regarding cost and implications of environmental impact [18].

This thesis focuses on three research areas: infrastructure modelling, environmentally friendly logistics design and multi-objective optimization. Infrastructure modelling at strategic and tactical level, is also known in OR as the ***Facility Location-Allocation Problem (FLP)*** and determines the optimum number, location and allocation of facilities. On the other hand, the ***Generalized Assignment Problem (GAP)*** focuses only on the allocation of customers to the facilities, and not the location of facilities. We are interested in both the FLP and the GAP in this thesis. Environmentally friendly logistics implies "an environmentally friendly and efficient transport distribution system" [95] and the focus of this thesis is on designing such a system that integrates transport operations into infrastructure modelling and minimises the environmental impact in terms of CO_2 emissions from running transport and depot operations.

1.2 Research Hypothesis and contribution

The physical infrastructure of logistics networks identifies the number, locations and allocation of open distribution centres, and has a direct effect on freight transport operations [72]. Traditionally, changes in logistics infrastructure have been driven by a need to reduce total costs and improve customer service levels and until recently environmental benefits have not been a major concern.

The aim of the current research is to investigate the feasibility of building multi-objective optimization (MOO) decision support tools for modelling the physical infrastructure of medium and large size logistics networks with special focus on the environment. Minimizing the environmental impact from the transport and depots is incorporated into the modelling as well as traditional objectives, such as minimizing cost and improving customer service level simultaneously. We explore a range of approaches to multi-objective optimization on different supply chain models from the traditional weight-based method, which transforms the problem into a single objective optimization and requires the user's input to prioritize or weight different objectives, to the latest multi-objective evolutionary algorithms (MOEA), which generate a large number of non-dominated solutions simultaneously. New approaches that are based on MOEAs, but constrain the size of the solution set will also be investigated. The method involves using empirical data in conjunction with industry. A particular interest of the project is the close integration of transport management within supply chains, with a view of having more control over the environmental impact of goods transport. Throughout the research presented in this thesis, we are trying to answer following two research questions:

- Is it possible to build multi-objective optimization decision tools for strategic modelling of large size traditional logistics networks where financial and environmental objectives are solved simultaneously?
- Is the optimum design of a particular logistics network based on cost the same as the optimum design based on CO_2 emissions?

We will come back to these research questions in the conclusion chapter and discuss how they have been addressed through out the research presented in this thesis.

The contribution of the research presented in this thesis summarised below:

1. We have demonstrated on a case study of a Pan-European automotive network based on Hammant [124] that the optimum design based on cost for infrastructure modelling (number of open depots) is sensitive to the vehicle utilization ratios when optimized by cost or CO_2 emissions.

2. We have undertaken a sensitivity analysis on Sainsbury's secondary distribution network into the effect of changes in various key variables on the allocation of the stores to depots. It allowed us to understand the relationship between transportation and warehousing factors and the allocation of stores. An environmental impact in terms of CO_2 emissions from total vehicle-km travelled was calculated for a cost-based and distance-based optimization as a single-objective function. The analysis showed that single-objective optimization based on cost or distance generate different results with different allocations therefore multi-objective approach could be applied to generate trade-off solutions.

3. A prototype of multi-objective optimization tool for assignment problem based on the Sainsbury case study was developed where two objectives are solved simultaneously: minimizing costs and minimizing environmental impact (CO_2 emissions).

4. We have developed new heuristic techniques for capacitated allocation of customers to open depots, focussing on large size problems with two capacity constraints. This technique is also utilized in the multi-objective capacitated facility location problem. The heuristic focuses on generating feasible solutions as part of an upper bound assignment procedure for single and multiple products.

5. Software for generating large size data instances has been developed for the capacitated allocation problem and the capacitated facility location problem. Our data sets reflect the real-life supply chain and based on Sainsbury's network model.

6. A prototype of the multi-objective optimization tool was developed for uncapacitated and capacitated facility location problems where economic costs and envir-

onmental (CO_2 emissions) impact from energy consumption in depots and transportation are considered. For the capacitated model, economic costs consist of transportation and depots components with time and distance based formulations are considered.

1.3 Note on Implementations

All algorithms used in this thesis are implemented in the Java programming language. This encapsulated using custom-based classes, functions as well as standard packages and classes with built in functions of the Java 2 platform, standard edition. Chapters 5, 6, 7 and 9 also use the CPLEX® optimization engine and associated packages with the Java programming language. This approach allowed the flexibility of implementing the user interface in Java and the power of CPLEX® for solving data instances to optimality.

As part of the evaluation throughout our research we compared the quality of our heuristically generated solutions either with previously published results in the literature or with the best known solutions produced by CPLEX® optimization software. Where it was not possible to do, the solution was compared to the known lower bound (LB) solution. As well as looking at the solution quality, the computational times for finding those solutions were analysed for some of the instances because current research focuses on large size data instances, and run time can easily become an issue. When applying MOO techniques for the capacitated facility location problem, each heuristic algorithm run 10 independent times for each data instance to obtain the final set of trade-off solution to ensure the reliability of the technique.

1.4 Thesis Structure

Chapter 1 describes the research motivations and hypothesis for the research presented in the thesis. The contribution and achievements of the research are documented and the structure of the thesis is presented.

Chapter 2 introduces the area of logistics modelling in an operational research context, and covers a literature review of the FLP and GAP. 'Green' formulations of supply chains are also presented in the chapter, together with different performance measures that are discussed in terms of their respective objective functions for green and traditional supply chains and their application in the literature.

Chapter 3 provides background information on the different approaches in the area of multi-objective optimization and its application within logistics design. The evolutionary multi-objective optimization algorithms NSGA-II and SEAMO2 are presented in the chapter in detail.

Chapter 4 documents the findings of the analysis on the simulation model of a Pan-European network from the automotive sector which is based on the case study by Hammant [124]. The research analyses the 'optimum design' based on costs and on CO_2 emissions from transportation and energy usage when decisions are made regarding the number of open depots and the vehicle utilization ratio.

Chapter 5 documents our investigation into the impact of changes to key variables, such as fuel price and labour costs, on the allocation of the stores to depots based on a Sainsbury's case study. A sensitivity analysis is performed using the CPLEX® optimization engine which uses techniques to solve problems to optimality.

Chapter 6 extends the single-objective Sainsbury's study to a two-objective optimization approach using classic weight based optimization. The distance based and cost functions are considered with different weights to give a decision maker a set of valuable trade-off solutions for the assignment problem (assigning stores to distribution centres).

Chapter 7 introduces two mathematical formulations of a Lagrangian Relaxation technique for a single and for multiple products where one capacity constraint is relaxed. The formulation for a single product is used for solving large size assignment instances. Large data sets were generated for assessing the effectiveness of the technique. The emphasis of this chapter is on finding feasible solutions as part of the assignment routine.

Chapter 8 documents an exploratory study into multi-objective uncapacitated facility location for environmentally friendly design. Customer service level, economic cost and green objectives are incorporated into the framework. The evolutionary multiobjective algorithms NSGA-II and SEAMO2 are compared in terms of execution times and solution quality for the strategic design of an uncapacitated network.

Chapter 9 presents the capacitated facility location problem where economic costs and CO_2 objectives are considered simultaneously using the evolutionary algorithm SEAMO2. The data instances used for the analysis are generated randomly and based on industrial data. Langrangian Relaxation technique discussed in Chapter 7 is utilized in assigning the customers to the depots after identifying which depots are open.

Chapter 10 summarises the research contributions and evaluation of the related findings across all research. Future direction with suggestions for extending the current work is also presented here.

Appendix A presents a mathematical formulation of a Lagrangian Relaxation procedure where two capacity constraints are relaxed (number of cases and number of stores).

1.5 Publications

The work in this thesis contributed to the following publications.

Journal Papers: Fully refereed

Harris I., Naim M., Palmer A., Potter A. and Mumford C., "Assessing the Impact of Cost Optimization Based on Infrastructure Modelling on CO_2 Emissions", International Journal of Production Economics, 2010, http://dx.doi.org/10.1016/j.ijpe.2010.03.005

Conference Papers

- Harris I., Mumford C., Naim M.(2009), "Multi-objective uncapacitated facility location model for Green Logistics", IEEE Congress on Evolutionary Computation (IEEE CEC 2009), Trondheim, Norway, May 18-2, pp. 2732-2739. (Fully refereed)

- Harris I., Naim M., Palmer A., Potter A. and Mumford C. (2008), "Assessing the Impact of Cost Optimization Based on Infrastructure Modelling on CO_2 Emissions", 15th International Working Seminar on Production Economics, Innsbruck, March 3-7, Pre-prints volume 3, pp 151-161.

- Harris I., Naim M. and Mumford C. (2007), "A review of infrastructure modelling for Green Logistics", Proceedings of the Logistics Research Network Annual Conference 2007, 5th - 7th September, pp 694-699.

Chapters in books

Harris I., Sanchez Rodriguez V., Naim M. and Mumford C.,"Restructuring of logistics systems and supply chains.", in "Green Logistics: Improving the Environmental Sustainability of Logistics ", eds. McKinnon A., Cullinane S, Browne M., Whiteing A., Kogan Page, 2010, ISBN: 0749456787.

Chapter 2

Logistics modelling

2.1 Introduction

This chapter reviews traditional and "green" logistics network design at both strategic and tactical levels, so that decisions regarding facility location and allocations of the customers to depots can be considered simultaneously, to allow an integrated approach to infrastructure modelling. Facility location problem formulations, appropriate traditional objective functions and cost structures are discussed for a traditional network design. In addition, the related generalized assignment problem is introduced, paying particular attention to single source formulation, in which each customer is assigned to only one facility. From a green perspective, the added activities to the close loop supply chain formulation are discussed as well as green performance measures (objectives) for logistics modelling. Finally, commercially available software packages for network design such as CAST [8] and IBM® ILOG LogicNet Plus® XE [68] are briefly discussed in terms of their functionality and relationship to the current research. The current chapter focuses on single objective optimization, whereas Chapter 3 describes a multi-objective optimization approach to network design.

2.2 Facility Location Problem

The Facility location problem (FLP) (also known as the location analysis problem) is a well-known problem in Operations Research and considers decisions concerned with determining the number of open facilities, their location, capacity, type of service/product they provide and could also consider which customers are assigned to which facilities to ensure that their demand is satisfied. It has wide application in both the private and the public sector where distribution centers, hospitals, retail points, fire stations, chemical plants etc. are under consideration. Depending on the application area, different objective functions and constraints are considered, varying from minimizing overall costs to maximizing the number of clients served. In the business environment and within a logistics context, minimizing overall cost is the most commonly used objective which would consist of the running cost of open serving facilities (fixed costs for operating facilities, production, storage, picking activities, etc) and a transportation element to deliver those goods to their customers. Facility location decisions could be strategic, if for example major long-term investment in new facilities is required. On the other hand, when the businesses are able to acquire or hire a facility for a shorter term, decisions could be deemed tactical rather than strategic. Some of the early models in location analysis date back to the last two centuries, and there is a rich literature of models and solution techniques. A detailed overview of facility location formulations and solution techniques is presented in Daskin [27], Drezner and Hamacher [28], Owen and Daskin [85]. Klose and Drexl [63] review some contributions to the current state-of-the art in facility location models for distribution system design, and below some of the classification types are considered as follow:

- *Discrete* vs *Continuous* location models. In the continuous models, it is feasible to locate the facilities anywhere of the plane whereas discrete models have an explicit sets of possible locations.
- The objective function in the problem formulation may be of the *minsum* or *minmax* type. Minsum models minimize the average distances while minmax models minimize maximum distance.
- *Uncapacitated* vs *Capacitated* models. The uncapacitated facility location problem (UFLP) assumes that facilities have unlimited capacity, whereas the capacitated facility location problem (CFLP) imposes capacity constraints on each facility.
- *Single-source* vs *Multiple-source*. Each customer will be assigned to just one facil-

ity in a single-source problem or to several facilities in a multiple-source problem.

- *Single* vs *Multiple* objective models. Single objective formulations dominate location analysis research (e.g. [50, 10, 57, 7, 4]), and involve the optimization of a single objective, such as minimizing cost or or maximizing profit. However, problems in real-world are frequently multi-objective nature, for example it may be desirable to simultaneously minimize cost and maximize customer service. Despite its relevance in the real world, published research on multi-objective location problems seems to be rather limited. Multi-objective optimization for facility location/allocation is a key topic of research in this thesis. An overview of the different approaches and their application to logistics design is described in detail in the Chapter 3

- *Single-stage* vs *Multi-stage* models. Multi-stage models consider the flow of goods from several hierarchical stages; whereas single-stage focuses on one stage explicitly, e.g. depot-customer.

- *Single* vs *Multiple* product. If the nature of the products are homogeneous they could be considered as a single product, e.g. chill product. On the other hand, if we have, for example, chill, ambient and frozen product types, the problem becomes a multiple-product formulation.

- *Static* vs *Dynamic* models. Static models consider a design over a single period of time, whereas dynamic models take account of variation over several time periods.

- *Deterministic* vs *Probabilistic* models. Deterministic models use averaged data based on past history or future forecasts, which is assumed to be exact and correct, whereas probabilistic models consider data under uncertainty.

- *Location-routing* problems combine location analysis with routing aspects of the design.

As can be seen from the classification, the field of FLP formulation and solution techniques is large. This thesis focuses on single source deterministic design models, and covers both uncapacitated and capacitated formulations with single and multiple product types. It also considers single and multiple objective variants, in which traditional economic and environmental objectives are balanced for a large size network. Chapter 3 provides a detailed overview of different techniques for multi-objective optimization and their application to facility location and allocation, and Section 2.4 in the present chapter gives background information on the green performance measures.

For most of this thesis we are concerned with the capacitated facility location problem, with single source customer assignment (CFLPSS). The CFLPSS is a combinatorial optimisation problem that belongs to the class of NP-hard problems [105], therefore solving it using exact algorithm poses difficulty for instances of a large size. Lagrangian relaxation techniques are leading methods for solving large CFLP and CFLPSS problems ([7], [10], [57], [4]. Other techniques, such as approximation algorithms and metaheuristic approaches are also applied to solving those model formulations.

In this thesis, we use the IBM® ILOG® CPLEX® (v12.1) optimization engine to solve a single objective formulation of the CFLPSS. CPLEX® formulates the CFLPSS as a mixed integer programming (MIP) problem which balances optimality and feasibility in its search using a dynamic search methodology [24]. The dynamic search algorithm consists of LP relaxation, branching, cuts, and heuristics to find an optimal solution. The problem formulation for CFLPSS is discussed in Chapter 9 where we use the results of CPLEX® optimization to compare the solutions produced by our multi-objective optimization algorithm.

2.3 Generalized Assignment Problem

An initial assignment of customers to serving facilities is carried as an integral part of solving a FLP formulation. However, the serving facilities usually remain in place for many years, in which circumstance may change. It is common therefore, to regularly re-optimise the allocation of customers to serving facilities, to take account of changes in demand and/or supply patterns etc. This assignment problem is known as the generalized assignment problem (GAP) and was first introduced by Ross and Soland [93]. Since then many papers have been published on the GAP in the literature. It has a wide application, including assigning workers to jobs, staff scheduling, assigning stores to the serving facilities and project assignment with the solution techniques are discussed in the survey by Cattrysse and Van Wassenhove [22] and Oncan [84]. The former authors discuss the solution algorithms and relaxations of them, and the latter author focuses on real-life applications and recent solution approaches.

The solution procedures for solving the GAP consists of exact and heuristic algorithms and new solution methods appear frequently in the literature. In logistics network design, examples are included in the following papers. Neebe and Rao [81] formulate a fixed charge model assigning the users to sources as a set partitioning problem with a solution based on linear programming relaxation. Foulds *at el* [44] present a mathematical model of an allocation problem arising in the New Zealand dairy industry with heuristic and integer programming techniques to solve it. Benjaafar *et al.* [12] consider the problem of allocating demand arising from multiple products to multiple production facilities with finite capacity and load-dependent lead times. They consider two types of demand allocation: in the first one they allow the demand for a product to be split among multiple facilities and in the second one demand from each product must be entirely satisfied by a single facility. Their solution procedures determine the optimal allocation of demand to facilities and the optimal inventory level for products at each facility. Freling *et al.* [45] consider the ***Multi-Period Single Sourcing Problem*** (MPSSP) where a set of customers is assigned to exactly one facility. They reformulate MPSSP as a GAP with a convex objective function and extend a branch-and-price algorithm to this problem.

Lagrangian relaxation is one of the techniques which could be used to solve the assignment problem, and is a relaxation technique where a hard constraint is moved into the objective function, thus imposing a (heavy) penalty if that constraint is not satisfied. The technique usually provides high-quality upper and lower bounds within a few iterations [48]. There are two possible constraints which can be relaxed as Lagrangian Relaxation bounds: the capacity constraint and the assignment constraint. For the capacitated facility location problem, several algorithms were developed based on Lagrangian Relaxation techniques ([10], [7], [53], [48], [134], [135]). Ghiani [48] provide an excellent description of the application of this technique to the capacitated plant/facility location problem

CFLP. Klincewicz and Luss [135] describe a LR heuristic algorithm for a single source FLP where the capacity constraint is relaxed. Their LR procedure uses ADD heuristic to find initial feasible solution for upper bound and a final adjustment heuristic technique to improve reassignment of customers to open facilities. In the ADD procedure, facilities are added one at a time to the set of open facilities and customers are assigned to the minimum assignment cost if capacity is not violated. If such assignment is not feasible, than another facility is open and reassignment is done again. In the final adjustment procedure, customers are reassigned to the lowest "true" cost from assignment based on augmented costs if there is a sufficient capacity at that particular facility. Darby-Dowman and Lewis [134] use the same Lagrangian relaxation to identify problems for which the optimum solution to the relaxed problem produces not feasible solution to the unrelaxed problem through establishing relationships between fixed and assignment costs. Fisher [39] illustrates Lagrangian Relaxation on the example of GAP because of its rich structure. Jornsten and Nasberg [136] propose a new LR approach based of a reformulation of GAP by introducing new substitution decision variables and new constraints. They show that the bounds from Lagrangian dual of their approach are at least as strong as the bounds from traditional LR approaches.

In this thesis, we present three mathematical formulations of a Lagrangian Relaxation technique which are discussed in Chapter 7 and in Appendix A for assigning customers to serving facilities. The formulations are for a single and multiple products with two constraints (cases and num of stores) and are based on the traditional lagrangian relaxation of the capacity constraints of GAP as stated in Jornsten and Nasberg [136]. We extend them to include extra constraints and multiple products which we discuss in more details in Chapter 7.

2.3.1 Transportation and warehousing costs in logistics modelling

In logistics network design, transportation and warehousing models should be considered as part of the process. The validity of the model formulations depend on the correctness of any assumptions made, and it is important to reflect the realistic cost structure of the particular business environment under consideration. Factors such as utilization of vehicles, cost per case and many other considerations will vary considerably depending on the particular scenario. Nevertheless, it is important to understand the fundamental components of transportation and warehousing costs. Sussams [104] provides an excellent overview of transportation and warehousing models which are briefly discussed here:

1. ***Transportation model*** is the first key component in logistics design and reflects all activities associated with the vehicle, and could consist of fixed costs per day, driver cost per hour, variable cost per mile and other factors. Therefore those costs could be differentiated into:
   - Time dependent costs, which may include drivers wages and related costs and depend on the vehicle type and working hours.
   - Distance dependent costs which may include fuel, tyres and depreciation.
   - Overhead costs which may include administration, management and supervision
2. ***Warehousing model*** is the second main cost component and is dictated by the specific purpose of the warehouse: as a transhipment point (no inventory costs) or as a storage place for goods. Typical costs associated with warehousing model would be as follows:
   - Warehouse operations costs associated with receiving, replenishing, picking orders depending on the throughput.
   - Warehouse administration include costs associated with order processing systems, accounts, wages and salaries of the management and other staff
   - Occupancy costs relate to costs associated with rent, insurance, maintenance.
   - Inventory carrying costs depends on the value of the stock, amount of the safety stock, lead times of the supplier.

The majority of formulations of the FLP at a strategic level do not consider decisions associated with replenishment of the inventory, and transportation costs are estimated by direct shipping [112]. Shen and Qi [112] argue that if inventory and transportation costs are not integrated together into the network design, it could produce sub-optimal solutions. For this reason some researchers have started incorporating those decisions into strategic modelling (Shen and Qi [112], Nozick and Turnquist. [80]). For example, Shen and Qi [112] model shipment costs from a DC to its customers using a vehicle routing model (routing costs approximation - non-linear routing costs) instead of direct linear costs and non-linear inventory costs. The authors also assessed the benefits of integrating routing decisions in different ways: a fully integrated approach (inventory, routing and location), a partially integrated approach (location and inventory with direct shipping costs) and a sequential approach (location first then inventory and routing decisions).

2.4 Performance measures for traditional supply chain

To ensure the efficient running of the supply chain, a range of performance measures have been developed over the years. Shepherd and Gunter [101] present a taxonomy and critical evaluation of performance measurements and various metrics of supply chains identified from 42 journal articles and books and from online resources published between 1990 and 2005. From their review, it is clear that different research studies classify the measures in very different ways. Nevertheless a taxonomy is presented according to the following classification:

- According to processes identified in the supply chain operations reference (SCOR) model which provides common metrics to analyse supply chain performance [128]: i.e., plan, source, make, deliver or return (customer satisfaction). This allows the identification of measures which are appropriate at the strategic, operational and tactical levels.
- Whether the measures used are cost, time, quality, flexibility or innovation. It is important to differentiate between cost and non-cost measures such as time, quality etc., because if a supply chain relies only on cost measures, it can produce a misleading picture of supply chain performance [20].
- Whether the measures are qualitative or quantitative. Qualitative measures, such as

customer satisfaction, reflect the happiness of the customers with the service and can not be measured using a single numeric measure. Quantitative measures, such as cost, flexibility or customer responsiveness can be directly described numerically.

Their review identified a total of 132 performance measures across different processes in the SCOR model. A very small proportion was related to the process of return or to customer satisfaction (5%), compared to other processes such as plan (30%), source (16%), make (26%) and deliver (20%). Regarding the cost classification, the major proportion focussed on cost (42%) over non-cost measures such as quality (28%), time (19%), flexibility (10%) and innovation (1%). The quantitative measures (82%) were dominating qualitative (18%). One of the main problems with the all metrics discussed is that they do not capture the performance of the supply chain as a whole.

Current *et al.* [18] classify the objectives specifically for facility location into four categories: cost minimization, demand oriented, profit maximization and environmental concerns. Environmental objectives such as air quality, risk to surrounding population, quality of life are included in their literature review. In this thesis we consider two traditional objectives: minimizing the cost and ensuring high customer service levels, which are relevant to any network design. Zhang and Huo [127] point out that there are various ways to define customer service and the perception would be different from a customer's and supplier's point of view. The infrastructure of the network has an enormous impact on the customer service level. They define the customer service for a facility location from a transportation and an inventory point of view:

- Transportation perspective - customers located near the serving facility receive their orders faster from those facilities, therefore elapsed time between placing an order and its shipment will be shorter for those customers. The customer service level in this case is represented by the distance between a serving facility and the customer.
- Inventory perspective - out of stock rate and order fulfillment rate would be examples of the measures used in this viewpoint. The out of stock rate relates to the percentage of the customers' orders that cannot be filled at the serving facility,

whereas the order fulfillment rate is the percentage of orders filled. A lower out of stock rate means a higher customer service level.

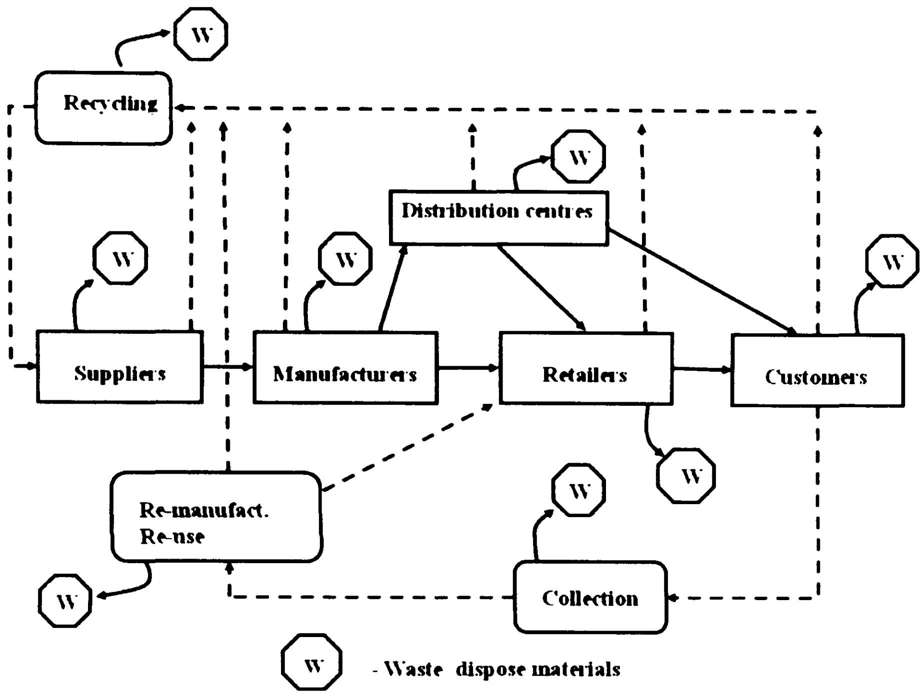
2.5 Green supply chains

The location analysis for facilities that produce hazardous materials, such us nuclear reactors and chemical plants, has been studied since the 1970's, when the environmental impact of airborne pollutants first became an issue. Today the need for 'desirable' environmentally friendly networks is becoming ever more urgent. Beamon [9] recognized the essential objective of the green or extended supply chain as the evaluation of the total direct and eventual environmental effects of all processes and all products. A fully integrated supply chain Figure 2.1) is described by Beamon as a supply chain having all the elements of the traditional configuration (Figure 1.2), but extended to incorporate product and packaging recycling as well as reuse and/or remanufacturing operations within a semi-closed loop. Consequently, it incorporates the elements of a reverse supply chain, reflecting the entire life cycle of the goods. Therefore, the main focus of a green supply chain is reducing energy consumption, emissions and waste, and increasing recycling and reuse.

To help deal with of the additional complexity of the extended supply chain, Beamon identified a new set of potential strategic and operational considerations including:

- the number and locations of facilities for product/packaging collection and re-use;
- the effects of traditional supply chain strategies (e.g., decentralised versus centralised functions, facility location decisions) on environmental performance;
- simultaneous operational and environmental supply chain optimization;
- incorporating environmental and operational goals into traditional analysis.

An example where a closed loop supply chain was considered could be seen from Paksoy *et al.* [89] who propose a multi-objective linear programming model to solve the green supply chain optimization problem with forward and reverse flows on a small size



Source: Based on Beamon [9].

Figure 2.1: The extended supply chain.

study with hypothetical data. The model aims to minimize total cost through minimising transportation costs in forward and reverse logistics, minimising total purchasing costs, penalty costs for extra CO_2 emissions and also total CO_2 emissions. They use LINDO 6.1 to obtain the optimal solution.

2.6 Green performance measures

A review of performance measurement systems and metrics under development for green supply chains is given by Hervani, Helms and Sarkis [52]. The selected list of metrics they identified range from atmospheric emissions to energy recovery. They include measures for on-site and off-site energy recovery, recycling and treatment; spill and leak prevention; and pollution prevention. Additional general measures include total energy use, total electricity use, total fuel use, other energy use, total water use, habitat improvements and damages due to enterprise operations, cost associated with environmental compliance, and others. Hervani, Helms and Sarkis point out that organizations may choose their environmental performance measurements specifically to meet new government regulations on emissions, energy consumption or the disposal of hazardous waste.

Bloemhof-Ruwaard *et al.* [14] address the increasing need to incorporate quantitative environmental measures into OR modelling. Beamon [9] outlines a range of sustainable performance measures, such as emissions, total energy consumed and others for green supply chains. Taplin *et al.* [107] propose a list of indicators for a sustainable metal production system for the simulation of production, transportation and recycling activities. More efficient use of energy and raw materials, reducing CO_2 emissions, scrap and waste and higher productivity made sustainable development practical and measurable.

From a logistics perspective, Aronsson and Huge Brodin [3], in their comprehensive literature review, identified the measurement of emissions as one of the most popular ways of assessing environmental impact. They noted, however, that even though the direct environmental impact can be assessed in terms of emissions, it is the root causes of these emissions that need to be addressed. Exactly what action to take needs to be determined by an appropriate analysis of the supply chain as a whole. Determining which sustainable measures to use and the difficulty of calculating them has been discussed by several researchers ([3], [52], [9]).

Potter *et al.* [90] propose a list of potential performance indicators for sustainable distribution which they refine using a quasi-delphi study. This study is a variation of the Delphi approach where a quasi Delphi group of people is brought together for a structured discussion [132]. In the research, presented by Potter *et al.* [90], a group of academic and industrial experts considered and refined performance measures and a questionnaire that has been developed for leading practitioners. As a result of this study, emission rate per item, amount of payload used (measure of vehicle utilization) and energy use per item are the top three ranked performance indicators. Khoo *et al.* [62] use low transport pollution with faster deliveries between plants, promotion of recycling of scrap metal and conservation of energy in the modelling of a supply chain concerned with the distribution of aluminum metal.

Some researchers have noted that an improved environmental impact sometimes follows a supply chain redesign exercise based on traditional performance measures, such as cost or customer service. However, as discussed in detail in Chapter 1, we note that this is not always the case. Therefore we strongly believe there is a need to consider environmental measures explicitly during the optimization process at the same time as traditional objectives. Khoo *et al.* [62] use a simulation approach to select plant locations that balance low total market costs and low transport pollution, fast deliveries between plants, promotion of recycling of scrap metal and conservation of energy, in a supply chain concerned with the distribution of raw aluminum metal. The simulation model was used to demonstrate the consequences of ignoring resource preservation and recycling activities as part of the network design. Paksoy [88] proposed multi-period supply chain design model which aims to minimize total transportation costs, CO_2 emissions from transportation and manufacturing, total penalty cost as a result of exceeding the emission limit. The model was validated using hypothetical data and solved using the LINGO package.

Other studies ([61]; [83]) use multi-objective optimization techniques for evaluating the trade-offs between different objectives. From our research we identify only a small number of papers which explicitly relate to multi-objective infrastructure modelling for Green Logistics with some of these specifically address hazardous network structures. Multi-objective optimization is discussed in more detail below and in the Chapter 3.

As well as using individual sustainability measures, there is an increasing need to incorporate these measures into an assessment framework/methodology that will include environmental measures alongside economic and social metrics. Singh *et al.* [103] provide an overview of various sustainability indices that have been included. In their paper, they consider sustainability in its broadest sense, covering aspects other than the environment, such as product-based sustainability and quality of life. In total, 70 indices were grouped under 12 categories, including the following environmental indices: Eco-system-based indices (Eco-Index Methodology, Living Planet Index, Ecological Footprint); Composite Sustainability Performance Indices for Industry (composite sustainable development index, ITT Flygt Sustainability Index, G Score method); Product-based Sustainability index (Life Cycle Index, Ford of Europe's Product Sustainability Index); Environmental

Indices for Industries (Eco-Points, Eco-compass, Eco-indicator 99); Social and Quality of Life-based Indices (e.g., Index for sustainable society) and others.

An example of a sustainable methodology use during supply chain design is described in Hugo and Pistikopoulos [61]. They present a generic mathematical programming model for assisting the strategic long-range planning and design of a bulk chemical network. Their multi-objective mixed integer programming problem is formulated to minimize the environmental impact resulting from the operations of the entire network whilst simultaneously maximizing the network's profitability. The method for impact assessment, the Eco-Indicator 99 method [111], is incorporated within the quantitative life cycle assessment model to formulate an appropriate environmental performance objective to guide strategic decision making. The Eco-Indicator 99 method attempts to model potential environmental impact on a European scale according to three categories: Human Health, Ecosystem Quality and Resource Depletion.

Another example involving the trading off of cost against environmental impact is described in Quariguasi Frota Neto *et al.* [83], where the reorganization of a European pulp and paper logistic network is described. The environmental impact was assessed using an environmental index proposed in Bloemhof *et al.* [123]. This index uses life cycle analysis (LCA) and considers the diverse emissions produced in the supply chain: namely global warming, human toxicity, ecotoxicity, photochemical oxidation, acidification, nitrification and solid waste.

To assess the environmental impact of supply chains, there is a pressing need for decision-making/support tools that incorporate green performance measurements. Hervani, Helms and Sarkis [52] point out that, although environmental performance measures are being incorporated into existing tools at an increasing rate, current availability is far from adequate. They discuss the various tools that are available, including the analytical hierarchy process, balanced scorecard, activity-based costing, design for environmental analysis and life cycle analysis. Some of the tools could be directly applied to aspects of green supply chain management and performance, while others require adjustments and extensions. The authors point out that on the whole there is no perfect tool for traditional or green performance measurement systems, and that their usage is greatly dependent on acceptance by organizations. However, introducing new tools or tools with an "unfamiliar feel" into a busy commercial environment can be challenging, if their adoption involves large capital investment, significant staff retraining or an unacceptable element of risk.

2.7 Environmental impact from transportation and facilities

Bloemhof-Ruwaard *et al.* [14] point out that the extent of environmental problems over the last few decades has shifted from the local and regional level to a continental and global level. The environmental changes expand from the air quality and health at the local level to climate change and depletion of the ozone layer on the global level. Greenhouse gases, such as carbon dioxide (CO_2), methane (CH_4) and nitrous oxide (N_2O) contribute to climate change and the temperature rise near the surface of our planet. Therefore the greenhouse gasses from transport and energy need to be addressed urgently.

Carbon dioxide emissions are produced by burning fossil fuel, and from a transportation point view are caused by different modes of transport such as road, rail, water and air. Different factors have an impact on the actual levels of emissions from road transportation and can be grouped under the following categories according to the National Research Council [82]:

- Travel-related factors - these depend on the trip taken and distances travelled and vary for different vehicle operating modes. The speed and acceleration of the vehicle and load on the engine over the distance of the trip also have impact.
- Driver behaviour, such as smoothness and consistency of vehicle speed.
- The physical highway network characteristics, such as long grades, signalized intersections and volumes of traffic entering the traffic flow.
- Vehicle characteristics such as fuels, engine size, vehicle condition.

Methodology	Reference
Carbon dioxide emissions per tonne-kilometre (The Network for Transport and Environment (Sweden))	Kohn [64]
Life cycle assessment model	Hugo and Pistikopoulos [61] Quariguasi Frota Neto *et al.* [83] Bojarski *et al.* [16]

Table 2.1: Examples of papers using different methods for calculating environmental impact.

There are different formulations available for calculating road related emissions. The National Atmospheric Emissions Inventory (NAEI) [78] provides a spreadsheet which contains a complete set of speed-emission factor coefficients for CO_2 and other greenhouse gases for different types and sizes of vehicles in the UK fleet travelling at average speeds. The Department for Environment, Food and Rural Affairs (DEFRA) [32] provides greenhouse gas (GHG) conversion factors to convert existing data sources, e.g. freight fuel consumption, electricity/gas consumption etc. into CO_2 equivalent data. Their carbon dioxide formulation also takes into account the diesel lorry type and percent of laden weight of the lorry (i.e. the maximum carrying capacity of the vehicle). Kohn [64] uses an equation from the The Network for Transport and Environment (Sweden), which allows the calculation of carbon dioxide emissions per tonne- kilometre for a particular vehicle type. Some researches calculate CO_2 emissions directly [64] and others use different methods for assessing the potential environmental damage. Table 2.1 shows examples of papers which include different methodologies for assessing environmental impact. Also the importance of monitoring green supply chain management practices with factors such as green purchasing, design of products for reduced consumption of material/energy and others is discussed by Zhu et al. [126]. For the present study we are using the DEFRA [32] formulation because it is widely used as a guideline to help UK businesses to calculate CO_2 emissions and thus identify and address their environmental impact. Our environmental model takes into account CO_2 emissions from both transportation and depots.

McKinnon [74] presents an analytical framework incorporating all the factors which influence traffic level and related energy consumption, to review the opportunities for the reduction of CO_2 emissions from the freight sector at a macro level. The framework links the weight of the goods produced/ consumed to CO_2 emissions from freight operations. Handling factor (no. of links in the supply chain), average length of haul, modal split, average load on laden trips, average % empty running, fuel efficiency and CO_2 intensity of energy source (fuel-specific) are seven critical key ratios which affect the overall CO_2 intensity of the freight sector. Determinants such as supply chain structure, choice of transportation mode, vehicle utilization on laden trips and others have a direct impact on the respective key ratios for reducing CO_2 emissions. The report illustrates the sensitivity to total CO_2 emissions from the freight sector when hypothetical changes have been applied to the key ratios. McKinnon [74] observes that modal split, average payload weight, the proportion of empty running and fuel efficiency have been moving in a direction which reduces CO_2 emissions per tonne-km over the period 1990-2004.

Depots have a very important role in logistics network design. They are used for stocking products or as an exchange point for transportation modes to service their stores or customers. Greenhouse gas emissions in buildings arise from the direct burning of fossil fuels to produce electricity and heat. The energy consumption of non-domestic buildings, such as depots or warehouses depends on the type of the product being stored. The storage of frozen and chilled goods would involve having a special storage space, which would involve higher energy consumption. DEFRA [32] provide UK conversion factors for different fuel types, such as electricity and natural gas to convert available energy data into CO_2 equivalent data. In the UK electricity is generated mainly by the burning fossil fuels such as coal, natural gas and oil; whereas in other countries the main supply could come from different sources. For example, nuclear power dominates electricity production in France. Therefore, different electricity conversion factors need to be applied to the available energy data.

2.8 Benchmarking data sets for multi-objective formulation with environmental objectives

In this thesis we consider traditional and environmental objectives simultaneously as part of the multi-objective facility location-allocation modelling. Our research confirmed findings published by Villegas *et al.* [109] that there are no multi-objective optimization problem instances available in the public domain, which created initial modelling challenges in our research. To address this problem, we generated random instances for bi-objective problems and the methodology for creating those instances is discussed in the Chapter 9. The generated problem instances reflect realistic features of the major retail supply chain in UK and considered as one of the contributions of this thesis to encourage further work in multi-objective design. In addition, we generated random data instances for the capacitated allocation problem which also have realistic characteristics and are suitable for bi-objective design. This methodology is discussed in Chapter 7.

2.9 Commercial Software for Strategic Modelling

Modelling of logistic networks at the strategic level is supported by specialized commercial computer tools such as CAST [8] and IBM® ILOG LogicNet Plus® XE [68].

CAST is a commercially available supply chain network design application available from Barloworld [8] for modelling global or regional logistics networks. The software is used by third party logistics, manufacturing, retail and consultancy sectors on a worldwide scale to model different supply chain strategies to improve service and reduce costs. CAST allows the user to "build mathematical models of their supply chain network infrastructure from the points of sourcing to the points of consumption (supply side to demand side)", and all elements are considered as a single integrated model. The functionality of the software includes running network strategy modelling, centre of gravity modelling, mixed integer programming optimisation, and provides a display map of the network, locations and roads in different countries across the world. CAST offer the additional functionally through a carbon emissions modelling for a particular network design in $CAST - CO_2$ module. This module allows comparisons of costs and CO_2 emissions across different scenarios taking into account different modes of transportation and warehouse operations by country. The software allows optimization by carbon footprint, carbon cost, supply chain cost or service level.

IBM® ILOG LogicNet Plus® XE [68] allows the modelling of manufacturing and distribution strategies: number and locations of plants, distribution centres, allocation of products to plants, assignment of customers to depots etc. The optimization focuses on the lowest total cost or the maximum total profit of the supply chain and calculates CO_2 emissions associated with supply chain activities or while adhering to the constraints on the carbon footprint. The software combines a graphical interface with advanced optimization software for modelling complex supply chains. The functionality of the software includes all-in-one network design and planning, flexible mapping, and integration with Microsoft Excel and Access tools.

On the whole, the commercially available software applications for supply chain network design are well accepted by different sectors of industry and provide efficient solutions to cover different formulations of network design. The use of the optimization technology based on the simulation approach and visualization of the network provide excellent support for decision makers. However, the real world consists of more complex (and not necessary standard) problem formulations, for which available commercial tools may not always be suitable. The main focus of this thesis is to formulate a framework for multi-objective optimization where financial cost and CO_2 emissions objectives are solved simultaneously to produce a set of trade-off solutions in one optimization run, for large size networks, and it seems that available software packages do not support such functionality.

2.10 Summary

This chapter provides a review of traditional and 'green' supply chain design where FLP and GAP problems are discussed together with the challenges of modelling environmental objectives as part of the design. Throughout the review, lack of benchmarking data for the environmentally friendly design of networks provides a challenge, and it is clear that the creation of this type of data is essential, in order to encourage future research into this area. Environmental objectives, such as emissions from transportation and energy consumption in the warehouse, have to be calculated using appropriate methodologies and this is further discussed in Chapter 4. Although, it seems that available commercial toolkits for network design have begun to incorporate 'green' objectives, this tends to be mostly as a byproduct of a cost based optimization process. This approach allows the decision maker to assess the impact of the particular design, but very many simulations will probably be needed to see the full picture of the trade-off solutions, using a traditional cost-based approach. This indicates a gap in commercially available software, for which we propose multi-objective optimization techniques, as this approach will better allow the decision maker to see important trade off solutions that are easily missed if traditional methods are used. Furthermore, the trade-off solutions can often be obtained much more quickly, in a single run, simultaneously considering all objectives as they are, without the need to assign relative importance or weights, or convert them to another unit, for example carbon cost. The next chapter describes in great detail the various approaches to multi-objective optimization and their current application in network design.

Chapter 3

Multi-objective Optimization

3.1 Introduction

There are many real-life situations where a decision-maker needs to consider more than one objective. It is often the case that a problem with multiple objectives will be converted into a single objective problem, by combining the objectives as a weighted sum which we explore in Chapter 6.1 on the problem of allocation of the stores to depots. We will use an alternative approach *in the facility location problem*, which allows a decision maker to evaluate a range of different trade-off solutions, for example between cost and distance, as a trade-off or (*Pareto front)*. When more than one objective is considered, the problem will have multiple distinct goals. The current research aims to minimize economic costs for one objective and minimize the environmental impact for the another objective, in the design of a logistics network.

Figure 3.1 illustrates a trade-off front in considering a range of hypothetical designs for a distribution network. If the company is interested only in obtaining the lowest possible cost solution, solution *6* would be chosen. On the other hand, if the emphasis is on minimizing the pollution into the environment, solution *1* would be considered. In either case, a particular single objective has high priority for the logistics modeller which does not reflect the multi-objective thinking process. In reality, the question we will have to ask is how can we design our network for a traditional supply chain where we can simultaneously minimize cost and minimize the impact on the environment? In our example, solutions *3* and *4* have lower impact by 50%-60%, yet the cost does not appear to be very much greater than solution *6*. Thus, solutions *3* and *4* could provide the decision maker

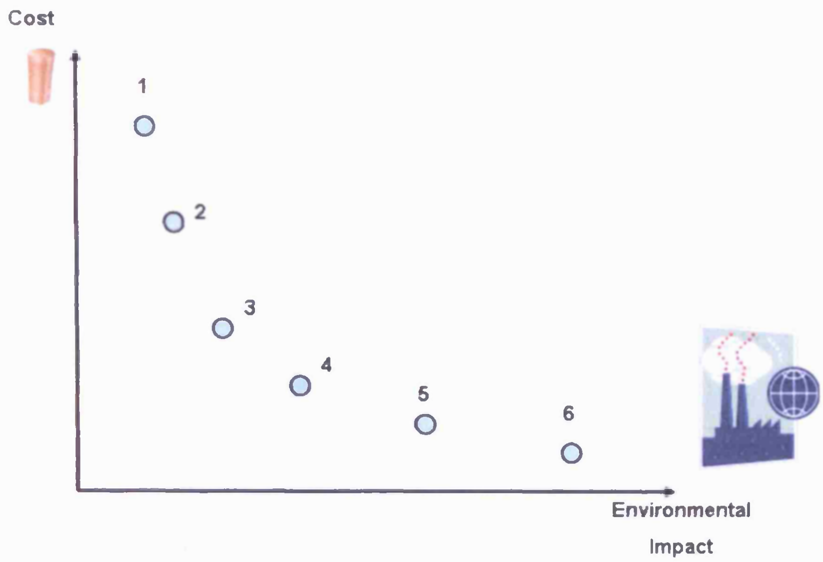
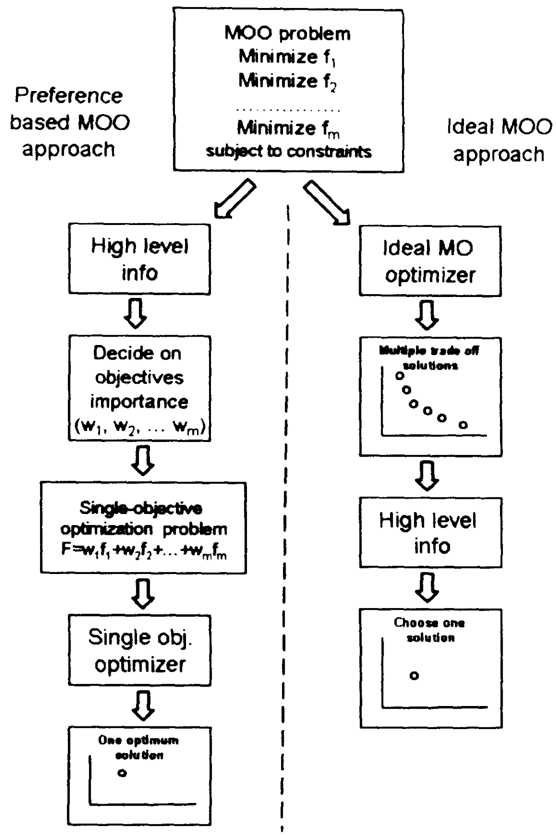


Figure 3.1: Trade off solutions to balance financial and environmental objectives.

with good compromise solutions.

Following the discussion above, there are noticeable fundamental differences between single and multi-objective optimization approaches. Firstly, there is more than one solution present in the final set of solutions when a multi-objective approach is used. This can be clearly seen in our example illustrated in Figure 3.1 where we have conflicting objectives. Secondly, there are three clear goals in multi-objective optimization (MOO) [117]: the solution set should be of good quality (as close as possible to the true Pareto front), broadly spread (with a wide range of solutions) and evenly spread over the Pareto front. Finally, there are different search spaces: objective space and decision space. This means that for each solution in the decision space there is a corresponding point in the objective space.

This chapter aims to introduce important concepts and techniques which are used in multi-objective optimization and will be applied to the operations research (OR) problems investigated in this thesis: facility location and allocation problems. Sections 3.2-3.4 are based on the book by Deb [31].



Based on Deb [31]

Figure 3.2: Different approaches to MOO.

3.2 Approaches to multi-objective optimization

There are different approaches to MOO and Deb [31] classifies them into two main categories: *preference-based* and *ideal* multi-objective optimization procedures. Figure 3.2 illustrates the differences between these approaches. In the *preference-based* MOO approach, firstly, based on the high-level information the decision-maker will need to determine a *relative importance vector* (w_M) for each of the objectives. For example, in Figure 3.1, solution 5 has a higher emphasis on reducing the cost compared to the environmental impact. On the other hand, in solution 2, minimizing environmental impact is more important compared to cost reduction. In the preference-based case, relative weighings will be assigned to the objectives which will form a composite objective function. This method is also known as a method of *scalarising* an objective vector into a single composite objective function. The composite objective function is then optimized using an appropriate technique and as a result of the optimization, one particular trade-off solution is produced. Sometimes, this procedure can be repeated with different preference vectors to obtain multiple trade-off solutions. It it important to point out that the solution obtained using this approach is very sensitive to the preference importance vector. One of the challenges with this approach is to decide on accurate preference vectors, and this tends to make this method somewhat subjective. On the other hand, in the *ideal* MOO procedure, all the objectives are treated equally and considered to have the same importance. Using an appropriate technique will generate a set of trade-off solutions simultaneously. After the solutions are found, the decision-maker chooses one of the generated solutions using high-level information. Treating all objectives equally, makes this approach less subjective because a user does not need to decide on the relative preference vector before the optimization. Choosing a particular solution available from a large pool of solutions could itself be considered a challenge, however. In practice, although the *ideal* MOO approach is generally preferable, as it is less subjective. On the other hand, if the decision-maker is confident regarding the weighting vector, then the *preference-based* approach could be adequate to find an acceptable solution.

3.3 Dominance and non-dominated solutions

Domination is an important concept in MOO. In particular, multi-objective algorithms aim to find *non-dominated solutions*, such that no solution can be considered better than another, yet all the solutions are different. MOO allows the comparison of any two given solutions in the objective space to whether one solution is 'better' compared to another solution. In this section, we define all the related terminology and notations for finding non-dominated solutions.

The concept of domination refers to the idea where two solutions are compared to each other on the basis of whether one solution dominates another solution or not. To demonstrate this concept, let us consider a problem with M objectives. *Definition 1* (below) is taken from Deb's book [31]. It is equally appropriate for minimization or maximization objective functions. The notion in the definition $i \lhd j$ implies that the solution i is better than solution j on a particular objective. Equally, the expression $i \rhd j$ means that the solution i is worse than solution j for a particular objective. One of the examples in this thesis considers two minimization functions: minimizing financial cost and minimizing environmental impact. Therefore in this case, the symbol $\lhd$ would mean the same as the operator "<".

Definition 1. A solution x_1 is said to dominate another solution x_2 ($x_1 \preceq x_2$), if both conditions 1 and 2 are true [31]:

1. The solution x_1 is no worse than x_2 in all objectives, or $f_j(x_1) \ntriangleright f_j(x_2)$ for all $j = \{1, 2, ..., M\}$
2. The solution x_1 is strictly better than x_2 in at least one objective, or $f_{\bar{j}}(x_1) \lhd f_{\bar{j}}(x_2)$ for at least one $\bar{j} = \{1, 2, ..., M\}$

If one of the conditions above is violated, then the solution x_1 does not dominate the solution x_2. If x_1 dominates the solution x_2 ($x_1 \preceq x_2$), this means that x_2 is dominated by x_1 and x_1 is non-dominated by x_2.

The following example of a two-objective minimisation optimization problem will allow us to illustrate the concept of dominance. Figure 3.3 shows six different solutions in an objective space with two objective functions, f_1 and f_2, which need to be minimized. Because we have more than one objective function in this example, it can be difficult to find a solution which is better with respect to both objectives, when those objectives conflict with each other. However, the definition of domination will allow us to make a decision on which solution is better when any two solutions are compared when we have a two-objective problem. Let us compare solutions 4 and 5. As can be seen from the Figure, solution 4 is better than solution 5 for both objectives. This means that solution 4 dominates solution 5. In the another example, let us take a look at solutions 2 and 4. In this case, solution 2 is better than solution 4 for the first objective function and solution 2 is not worse (they have the same value) than solution 4. Therefore, both conditions of *Definition 1* are satisfied, and solution 2 dominates solution 4. As a result of comparing

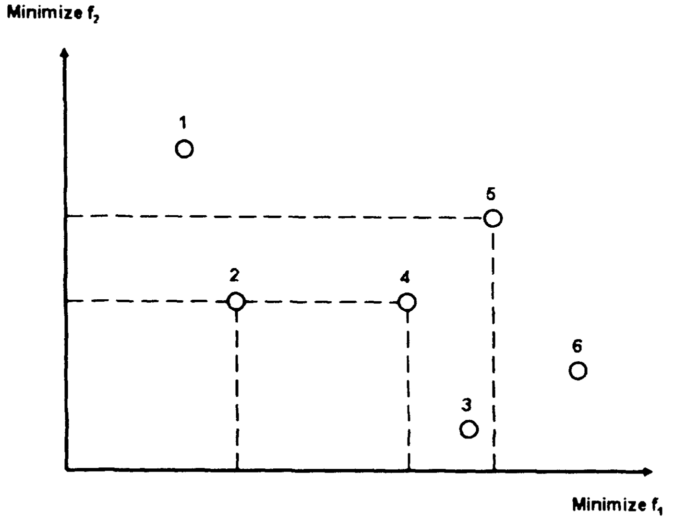


Figure 3.3: Example.

any two solutions x_1 and x_2 for dominance, there are three different outcomes can be concluded: solution x_1 dominates solution x_2, solution x_2 is dominated by solution x_1 or solutions $x1$ and x_2 are mutually non-dominating.

Another important notion of multi-objective optimization is a *non-dominated set*. To understand this concept better, let us take another look at our example in Figure 3.3. Consider solution 2 and solution 3, where solution 2 is better for objective one and worse for the second objective. Thus, the first condition in the *Definition 1* is not satisfied for these solutions. When two objectives are equally important, it is usually said that solutions 2 and 3 are *mutually non-dominating* with respect to each other. Therefore, we cannot say which solution is better or worse. Solutions 2 and 3 are part of the *non-dominated set* from the six solutions available. Identifying non-dominated sets allows the decision maker to consider a set of trade-off solutions. Therefore, for a given set of solutions, making all possible pair-wise comparisons, will allow us to identify the solutions which are of the non-dominated set. *Definition 2* and *Definition 3* also taken from Deb's book [31] and define a non-dominated set and the globally Pareto-optimal set:

Definition 2. Non-dominated set: Among a set of solutions P, the non-dominated set of solutions P' are those that are not dominated by any other members of the set P.

Definition 3. Globally Pareto-optimal set: The non-dominated set of the entire feasible search space S is the globally Pareto-optimal set.

Many multi-objective evolutionary algorithms (MOEAs) such as NSGA-II [30] and SEAMO2 [76], which are used in the current research and discussed later, need to identify non-dominated solutions in a particular population. SEAMO2 outputs a non-dominated set at the end of the algorithm, whereas NSGA-II algorithm sorts the population according to the different non-domination levels. The technique of identifying solutions which belong to a non-dominated set, involves comparing all possible pairs of solutions using a dominance operator, $\preceq$.

To summarise, the goal of the MOO algorithms is to find *Pareto optimal solutions* which are non-dominated solutions of the entire feasible space. Unfortunately, the complexity of some MO problems make the achievement of this goal almost impossible because of the problem size. In some cases for the combinatorial optimization problems this could be proven as computationally unachievable, therefore it is practical to analyse a set of the best known non-dominated solutions which is as close as possible to the Pareto optimal set. Therefore, the following conflicting goals need to be achieved in order to obtain a reasonable solution to a MOO problem [117]: 1)The solution set should converge to the Pareto optimal set which means that solutions should be of a good quality; 2) Solutions in the Pareto set should be evenly spread; and 3)The Pareto front should be widely spread to maximize the coverage.

3.4 Classical techniques for multi-objective optimization

The following two sections describe some of the popular approaches which are used in solving multi-objective optimization tasks, specifically focusing on techniques which are used in our current research and our reasons for choosing them behind it. There exist a variety of techniques for dealing with multiple objectives, where some of them are well established and others are fairly new in the research community.

The classical multi-objective optimization methods were developed in the last five decades. Different researchers refer to the the definition of classical methods to distinguish them from evolutionary methods. The majority of the methods convert a multi-objective problem into a single objective function by using user-defined parameters. Thus, the usefulness of the single solution obtained following the transformation, depends on making suitable choices of the parameters in the conversion model.

The *weight-based* method is the most popular classical method and converts a MOO problem into a single objective problem by using a weighted sum of the objectives, where the relative importance vector is defined by the user. This method is described below in the section 3.4.1. As mentioned earlier, the success of this approach depends on making appropriate choices for the weights. To generate multiple solutions simultaneously, on one run, Hajela and Lin [59] proposed a weight-based genetic algorithm for MOO where each solution in the population has a different weight vector in the calculation of the summed objective function. To promote diversity in the population of solutions, the weight vector could be adjusted as well. In contrast, the *ϵ-constraint* method converts all the objectives into constraints except for one objective. The user in this case would have to define the limits that constrain the objectives. The *goal programming* method, which was originally introduced for single-objective applications [23], suggests a way of transforming the multiple objectives into a single one before solving the problem using a single objective optimization algorithm. The method uses a goal value which needs to be achieved for each conflicting objective.

3.4.1 Classical weight-sum approach

One of the most popular approaches used in MOO due to its simplicity is called a weighted-sum method. The technique scalarizes a set of objectives by multiplying each objective with a user-defined weight [31]. This method is applied to solve an allocation problem with two-objective functions (min cost and min travelled distance) in Chapter 6 where we have the dilemma of balancing two objectives: minimizing overall costs and minimizing total distance for the allocation of stores to depots. The objectives have different units: $\pounds$ and km, with different numerical ranges, making it difficult to choose appropriate weights to control the relative contribution of each objective to the weighted total. To help matters, we normalize the objectives so that each one typically produces values between 0 and 1.

The formulation of the objective function can be seen as a sum of the weighed normalized objectives, which converts the problem into single-objective optimization problem. The following formulation was taken from [31]:

$$f(x) = \sum_{m=1}^{M} w_m f_m(x) \tag{3.1}$$

$$subject\ to:$$

$$g_j(x) \geq 0 \qquad j = 1, 2, ..., J$$

$$h_k(x) = 0 \qquad k = 1, 2, ..., K$$

$$x_i^{(L)} \leq x_i \leq x_i^{(U)} \qquad i = 1, 2, ..., n$$

Where the weight of the mth objective function in in the range $w_m \in [0, 1]$ and the weights are chosen in the such way that their sum is equal to 1, therefore $\sum_{m=1}^{M} w_m = 1$.

As mentioned earlier, this method is applied to solve an allocation problem with two-objective function, where $M = 2$. Having two weights w_1 and w_2, where only one weight is independent and the other one calculated by simple subtraction. Therefore, the sum of two weights added together is always equal to 1. The procedure for converting objective values to a single normalized value which is used for the objective function is discussed in Chapter 6.

3.5 Evolutionary multi-objective techniques

The inspiration from the principles of biological evolution and natural selection led to the development of a particular type of optimization technique in the 1950s and 1960s (Fraser [41], Rechenberg [119], Fogel[43]). John Holland in 1975 established the foundation for *genetic algorithms (GAs)* which was one of the starting points for a growing interest in the development of natural evolution inspired algorithms [60]. These days, a GA is a very popular metaheuristic method for solving many difficult problems in optimization, and belongs to the larger category of *evolutionary algorithms (EA)*. GAs are applied in many disciplines including engineering, manufacturing, physics, computational science and other areas.

Biological evolution and natural selection means that the fittest organisms survive within the natural environment. In the natural world, individuals compete for selecting companions for reproduction and it is believed that combining fitter individuals in terms of genetic material, will lead to fitter offspring. When this process occurs over several generations, the combination of fine characteristics from their predecessors could lead to the creation of an offspring which is better than its parents. This natural evolution allows the individuals to adapt better to the environment they are in. GAs operate in the similar way, on a *population* of solutions to a particular problem. The population consists of individuals which are usually represented as a population of strings and could be encoded as binary (1100101) or in other way. Those strings are also called *chromosomes*, and each have a *fitness* value associated it, which is derived from the objective function value for the solution that the particular chromosome represents. This value determines how fit a particular individual is to stay alive and breed. Individuals with better fitness are favoured in the mechanism for selection and reproduction and they could be selected several times during evolution. As a result of selecting better individuals through several generations, the final population of the solutions should converge to a near optimum solution or even the optimum solution.

The reproduction process of GAs involves taking two individuals which are recombined to create offspring. This is also knows as applying *crossover* operators on two individuals or *recombination*. The crossover is undertaken on the selected members of the population and a *selection probability* could be used to choose the proportion of the population selected. Recombination allows the offsprings to inherit good features from the parents. To ensure that the offspring adds the diversity to the population in exploring the search space, a random *mutation* operator may be applied to one or more positions of the offspring string following (or independently of) the crossover. This will alter the offspring chromosome, usually in a small way, which will introduce the randomness in the search space. The basic GA is outlined in Algorithm 3.1.

The selection of two individuals for the recombination could be done using different *se-*

Algorithm 3.1: ***Genetic Algorithm.***

Begin:

Initialization: Initialize a random population (chromosomes)

repeat

Evaluation: Evaluate the objective function of each chromosome

Selection: Select chromosomes (parents) from the population for reproduction. The selection criterion is based on the fitness assignment (or objective function) of the selected members of the population

Recombination: Apply crossover to selected chromosomes to produce offsprings, which will hopefully have better solutions. The probability rate could be used to select the chromosomes for the recombination.

Mutation: Apply mutation locally to some genes of the offsprings with a particular probability or alternatively to particular chromosomes.

Replacement: The new population is created by replacing selected members of the population with the offsprings

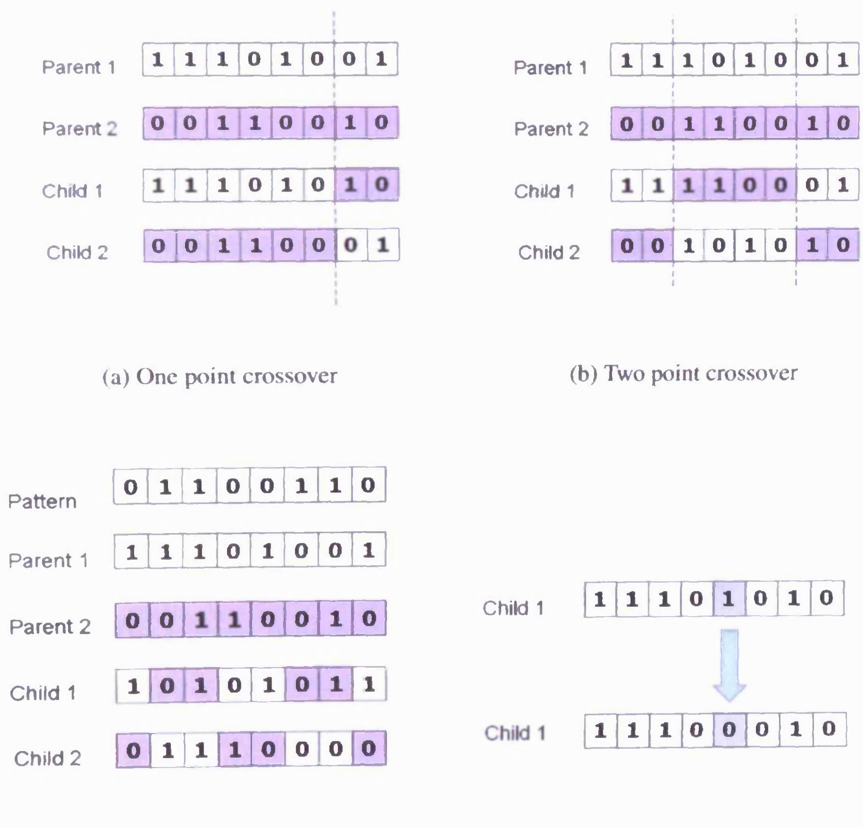
until terminating condition

End

lection methods. For example, *tournament selection* chooses two chromosomes at random which are used in the tournament against each other [31]. As a result of the tournament, the fittest chromosome is selected as a parent to be used in the recombination. This means that to choose n parent, n different tournaments will be undertaken. *Roulette-wheel selection* is similar to a roulette wheel in the casino. It assigns each individual in the population a fitness function value and then apportions that individual percentage based on the overall fitness [31]. The probability function is used to select chromosomes which will lead to chromosomes with better fitness tending to be selected more often.

Crossover and mutation operators are applied in traditional GAs to ensure that diversity is present within the solving environment [131]. Below, a few general crossover operators are described which are suitable for binary string representations. However, there are a great variety of crossovers in the literature, and the ones illustrated here would not be suitable for representations such as real numbered strings and permutation strings, for example. Also, bear in mind that there are many variants of genetic algorithms, using different selection methods and different criteria for replacing population members by offspring. In addition, crossover operators are sometimes used to produce just one off-spring each time, and sometimes two offspring are produced each time. There are very many choices.

The most popular crossover operators for simple binary strings are *one-point*, *two-point* and *uniform crossover*. In one-point crossover, the chromosome of the parents are cut at a randomly chosen point and the resulting parts of the chromosome are swapped [31]. In two-point crossover, the chromosome of the parents are cut at two points [31]. In uniform crossover, a random pattern is generated, in which each bit is provided by one of the two parents. A second pattern is automatically generated by exchanging the source of each bit [31]. Figure 3.4(a)-3.4(c) illustrates these three types of the crossover.



(a) One point crossover

(b) Two point crossover

(c) Uniform crossover

(d) Mutation

Figure 3.4: Crossover and mutation.

One of the most popular mutation operators is *point mutation* where the gene (one bit in the chromosome) in the solution is flipped either from 0 to 1 or 1 to 0. This is illustrated in the Figure 3.4(d). The mutation can be applied either to one gene or a number of genes across different solutions which are determined by the mutation probability (p_m).

Genetic Algorithms are very popular intelligent heuristic search methods, which are used as a foundation to many optimization evolutionary algorithms to solve problems with single and multiple objectives. One of the first multi-objective GAs, was the Vector Evaluated Genetic Algorithm (VEGA) proposed by Schaffer [120] in 1985. Following VEGA, the development of many new MOO techniques has taken place which includes the Multi-Objective Genetic Algorithm (MOGA) [121], elitist Nondominated Sorting Genetic Algorithm II (NSGA-II)[30], elitist Strength Pareto Evolutionary Algorithm 2 (SPEA2) [118], Pareto-Archived Evolution Strategy (PAES) [122], Simple Evolutionary Algorithm for Multi-objective Optimization 2 (SEAMO2) [76] and other techniques. Different techniques have adopted specific mechanisms to ensure that diversity is present in the final set of solutions, and this has often brought with it a cost of greater complexity when implementing these algorithms. For example, SPEA2 [118] uses a population and an archive with a fine-grained fitness assignment strategy. The algorithm preserves extreme points and the diversity mechanism is based on k-th nearest neighbour, whereas NSGA-II uses crowding distance and solution ranking is based on non-domination sorting.

As a result of our initial investigation of the different techniques, the following section introduces in detail two evolutionary multi-objective algorithms (MOEAs): *NSGA-II* and *SEAMO2* which are used in the current research for solving the multi-objective uncapacitated facility location problem (Chapter 8) and the capacitated facility location problem in Chapter 9. The reasons behind choosing those techniques are also explained in the following sections.

3.6 Multi-objective optimization formulation

Before discussion of the chosen evolutionary algorithms, we need to present formally the mathematical formulation of MOO in a general form. Let us consider a decision maker who requires to optimize M objective functions which can be minimized or maximized

[31]:

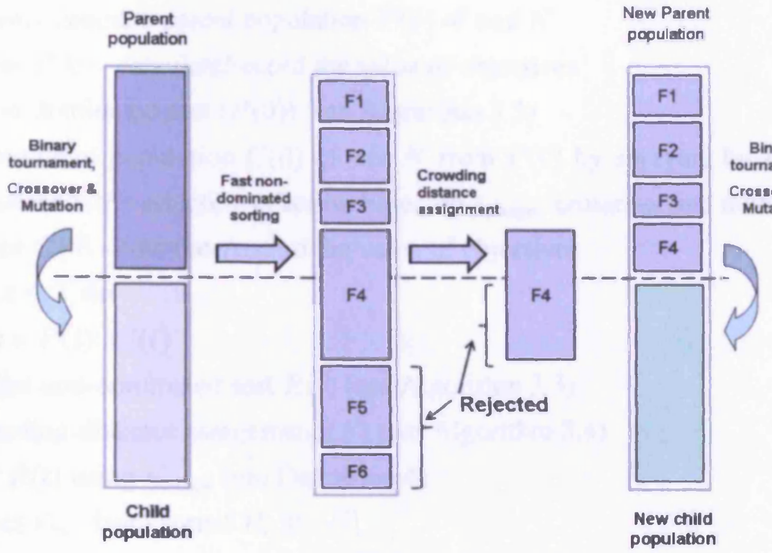
$$
\begin{aligned}
Minimize/Maximize\ f_m(x), \quad & m = 1,2,3....,M \qquad (3.2)\\
subject\ to: & \\
g(x) \geq 0 & \\
h(x) = 0 & \\
x_l \leq x \leq x_u &
\end{aligned}
$$

where $g(x)$ and $h(x)$ are inequality and equality constraints and a solution x is a vector of decision variables $x = (x_1, x_2, ...)$. A set of Pareto solutions is the solution of the above problem.

3.6.1 The Evolutionary Multi-Objective Algorithm: NSGA-II

The evolutionary Non-Dominated Sorting Genetic Algorithm-II (NSGA-II) [30] was chosen for implementation in the research for modelling the facility location problem because it has all the qualities which are needed to be taken into consideration when solving a multi-objective problem. It is a well tested algorithm in academia. It is elitist (preserving the best solutions) and uses a ranking procedure for fitness assignment of the solutions based on the fast nondominated sorting algorithm (see Algorithm 3.3). The algorithm uses a diversification mechanism, called a *crowding distance*, to ensure the solutions are widely and evenly distributed (see Algorithm 3.4).

The NSGA-II algorithm that we use for a multi-objective uncapacitated facility location problem (UFLP) is based on [109] and [30] and is illustrated in Figure 3.5 and outlined in *Algorithm 3.2*. Firstly, an initial parent population $P(0)$ of size N is created, at random. Each parent solution is encoded as a binary string. For each chromosome in $P(0)$, the objectives are evaluated by applying the assignment procedure. Then, a fast non-dominated sort is applied to $P(0)$ (see *Algorithm 3.3*), which assigns a "front number" to each solution which is equal to its non-dominated level, starting with 1 (1 is the best). Binary tournament selection in the parent population $P(0)$ is followed by crossover and mutation to generate the child population $C(0)$ of size N. Each child solution in $C(0)$ is then evaluated.



Based on Deb [31]

Figure 3.5: Overview of NSGA-II.

Next, the following elitist procedure for $t \geq 1$ described below is repeated for T generations. At the start of this, the parent and child populations are combined to form $R(t) = P(t) \cup C(t)$ of size $2 * N$ and a fast non-dominated sort is applied to $R(t)$. A new parent population, $P(t+1)$, is then formed from $R(t)$ by adding solutions beginning with the first front onward to make up a population of size N. Crowding distance is used to help make the last few selections, if addition of all individuals from a particular front would produce a population greater than N. Then, the child population $C(t+1)$ of size N is created from $P(t+1)$ by applying binary tournament selection, crossover and mutation. The overall complexity of the algorithm is $O(mN^2)$, where m is the number of objectives and N is the population size [30].

The fast non-dominated sort procedure (see *Algorithm 3.3*) uses the concept of domination (see *Definition 1*), where two chromosomes are compared on the basis of whether one chromosome p dominates another chromosome q or not. The algorithm works as follows. Initially, each chromosome p in population P is compared to each other and the algorithm calculates and stores two parameters for each chromosome: a set of solutions S_P that p dominates and the number of the solutions (n_p) that dominate the chromosome p. If no solutions in the population dominate p then it joins the first nondominated front

Algorithm 3.2: ***NSGA-II algorithm for MOFLP ([30], [109]).***

Begin:

Randomly generate parent population $P(0)$ of size N

Evaluate $P(0)$ - calculate/record the value of objectives

Fast non-dominated sort ($P(0)$) (see Algorithm 3.3)

Generate child population $C(0)$ of size N from $P(0)$ by applying binary tournament selection with the selection criterion based on $\leq_{nsga}$, crossover and mutation

Evaluate $C(0)$ - calculate/record the value of objectives

while $t \leq T$ **do**

  $R(t) = P(t) \cup C(t)$

  F=fast non-dominated sort $R(t)$ (see Algorithm 3.3)

  Crowding-distance assignment (F) (see Algorithm 3.4)

  Sort $R(t)$ using $\leq_{nsga}$ (see Definition 4)

  Select P_{t+1} from sorted R_t $[0 : N]$

  Generate child $C(t+1)$ of size N from $P(t+1)$ by applying binary tournament selection with the selection criterion based on $\leq_{nsga}$, crossover and mutation

  Evaluate $C(t+1)$ - calculate/record the value of objectives

  $t = t + 1$

Return all non-dominated solutions

which will have their domination count equal to zero. The first part of the algorithm finds the first nondominated front and the second part is repeated until all solutions are classified and assigned higher nondominated fronts. To do so, the algorithm iterates through each solution which has a domination count as zero ($p \in F_i$) and each member q of its set (S_p) and reduces chromosome's q domination count by one. When the domination count becomes equal to zero, then the solution q is added to a separate list H, which belong to the second nondominated front and so on. The procedure continues for each member of H to identify a third nondominated front until all solutions are assigned to nondominated fronts.

In Algorithm 3.2, the crowding comparison operator ($\leq_{nsga}$) compares two solutions and returns the fitter of the two as the "winner" (a binary tournament selection). It assumes that every solution i in the population has a non-domination rank r_i and a local crowding distance cd_i.

Definition 4. Crowded Tournament Selection Operator [31]: A solution i wins a tournament with another solution j if either of the two conditions below are true:

Algorithm 3.3: *Fast non-dominated sort (P) [30].*

Input Parameters: population (P), consisting of chromosomes, e.g. p, q

Begin:

for each $p \in P$ **do**
  for each $q \in P$ **do**
    if $(p \prec q)$ **then**
      $S_p = S_p \cup \{q\}$ {if p dominates q - save it in set of solutions S_p, which p dominates }
    else if $(q \prec p)$ **then**
      $n_p = n_p + 1$ {if q dominates p - keep the count of the solutions dominating p}
  if $n_p = 0$ **then**
    $F_1 = F_1 \cup \{p\}$ {if nobody dominates p then it joins the first front }
$i = 1$
while $F_i \neq 0$ **do**
  $H = 0$
  for each $p \in F_i$ **do**
    for each $q \in S_p$ **do**
      $n_q = n_q - 1$
      if $n_q = 0$ **then**
        $H = H \cup \{q\}$ {q joins list H}
  $i = i + 1$
  $F_i = H$ {form current front with members of H }
Return a list of non-dominated fronts F

1. If solution i has a better rank, that is, $r_i < r_j$

2. If they have the same rank but solution i has a better crowding distance than solution j, that is, $r_i = r_j$ and $cd_i > cd_j$

As can be seen in *Definition 4*, the NSGA-II algorithm integrates a density mechanism as part of crowded tournament selection operator. This technique calculates the crowding distance (see Algorithm 3.4) of each solution of the same front and serves as an indication of how widely spread the solutions are. It works on solutions in the objective space and measures a distance between the two nearest neighbours of a particular solution. At first, the algorithm initialises the distance values to zero for each point. The second *for* loop selects each objective function m at a time and sorts the population of the solutions according to the objective values. A very large value is assigned to the distance of the first and last position of the sorted population to ensure that those solutions are preserved. For all other solutions, the distance is calculated by adding the difference between two neighboring solutions on either side of the solution in question. The variables f_m^{max} and f_m^{min} represent the maximum and minimum values of the objective function m.

Algorithm 3.4: *Crowding-distance assignment (τ) [31].*

Input Parameters: solutions in set τ

Begin:

$l = |\tau|$ {number of solutions in front τ}

for each i **do**

    set $\tau[i]_{distance} = 0$ {initialize distance for each solution}

for each objective m **do**

    $\tau = sort(\tau, m)$ { sort using each objective value}

    $\tau[1]_{distance} = \tau[l]_{distance} = \infty$ {assign large values to the boundary solutions}

    for $i = 2$ **to** $(l-1)$ **do**

        $\tau[i]_{distance} = \tau[i]_{distance} + \frac{(f_m^{[i+1]} - f_m^{[i-1]})}{f_m^{max} - f_m^{min}}$

Return crowding distance of each point in set τ

Crossover. Binary tournament selection was used to choose the parents for crossover. Two individuals are randomly selected from the parent population $P(t)$ and the fitter one of the two is chosen as a parent, i.e., the one which wins the crowded tournament selection (see *Definition 4*). This means that the chromosome wins if it has higher non-domination level or if two chromosomes have the same non-domination level, then we choose the one that has a better crowding distance.

Mutation. For each solution (chromosome), a random mutation pattern is generated. A uniform random number between 0 and 1 is generated for each position in the solution. If this random number is less than mutation probability (p_m), the gene is flipped either from 0 to 1 or 1 to 0.

3.6.2 The Evolutionary Multi-Objective Algorithm: SEAMO2

A Simple Evolutionary Algorithm for Multi-objective Optimization (SEAMO2) [76], was chosen for implementation due its simplicity and good reported results in the earlier studies ([76], [77] and [113]). The algorithm uses uniform selection and does not use a fitness function to select the parents. The survival decision for the offspring is based on a straight comparison between the solution generated by a child with the solution produced by its parents or other members.

The SEAMO2 algorithm is described in Algorithm 3.5. The approach sequentially selects an individual from the population to be a parent once and pairs it with a second parent that is selected randomly (uniformly). A crossover is applied on those two parents which produces one offspring. Four different crossover patterns are compared for tuning the algorithm: one point, two point, uniform and no crossover. After the crossover, the mutation is applied to the offspring, where one random bit of the child solution mutates from 0 to 1 (or 1 to 0). The resulting child will either replace a member of the population or die, depending on the fitness of the child.

3.7 Applications of supply chain infrastructure techniques

The purpose of this section is to provide an overview of infrastructure modelling (facility location-allocation) research available in the literature in terms of multiple objectives and techniques. As can be seen from the Table 3.1, selected academic papers are presented in terms of the objective function and the techniques which are used to generate solution(s) which depend on the classical or evolutionary method. For example, the analytic hierarchy process (AHP) is a well known technique and has been used to assign different weightings to quantitative and qualitative measures for strategic modelling. Min and Melachrinoudis [71] use the AHP method to evaluate multiple objectives: minimization of relocating cost, quality of living, traffic accessibility, maximization of market opportunities, local incentives and site characteristics to relocate manufacturing/distribution facility.

Classical multi-objective optimization methods such as the ϵ-constraint have been used to transform a multi-objective problem into a single objective one, producing just one solution per simulation run. For example, Sabri and Beamon [102] develop an integrated multi-objective model involving strategic and operational planning under production, delivery and demand uncertainty. This method is used to consider cost, customer service

Algorithm 3.5: ***SEAMO2 algorithm [76].***

Begin:

Generate N random individuals

Evaluate the objective for each population member and store it

while stopping condition not satisfied **do**

  for each member of the population **do**

    This individual becomes a member

    Select a second parent at random

    Apply crossover to produce single offspring

    Apply single mutation to the offspring

    Evaluate each objective vector produced by the offspring

    if offspring harbors a new best-so-far Pareto component **then**

      a)it replaces a parent, if possible

      b) else it replaces another individual at random

    else if offspring dominates either parent **then**

      it replaces it

    else if offspring is neither dominated by nor dominates either parent **then**

      it replaces another individual that it dominates at random

    else

      otherwise it dies

Print all non-dominated solutions in the final population

End

level(fill rate) and flexibility (volume or delivery). Guillen *et al.* [49] use the ϵ -constraint method with a branch and bound technique to solve a multi-objective stochastic mixed integer linear programming model to determine an optimal supply chain configuration. The multiple objectives are the maximization of the net present value (NPV) and demand satisfaction, and the minimization of the financial risk. Hugo and Pistikopoulos [61] use a multi-objective optimization framework with the ϵ-constraint method for environmentally friendly network design with two objectives: maximising the NPV and minimizing impact that the network has on the environment.

Recently, an evolutionary approach to solving MOO problems, which is based on Pareto-optimal solutions, has been considered by a small number of researchers for infrastructure modelling. This method allows the decision makers to investigate trade-offs and select a

Model description	References								
	[71]	[102]	[80]	[62]	[61]	[49]	[100]	[109]	[2]
Traditional objectives									
Min costs	*	*	*	*			*	*	*
Service level		*	*			*	*	*	*
Max the net present value					*	*			
Min investment in opening facilities							*		
Min capacity utilization ratio									*
Min financial risk						*			
Fast deliveries between plants				*					
Quality of living	*								
Traffic access	*								
Market opportunity	*								
Local incentives	*								
Site characteristics	*								
Flexibility (volume or delivery)		*							
Green objectives									
Min transport pollution				*					
Promotion of recycling				*					
Conservation of energy				*					
Min impact on environment from entire sc (inc. transp. emissions)					*				
MOO techniques									
Classical	*	*	*	*	*	*	*		
Evolutionary								*	*

Table 3.1: Examples of multi-objective infrastructure modelling with techniques as applied to specific scenarios.

particular network design that best satisfies their compromise. For example, Altiparmak *et al.* [2] use a new approach based on genetic algorithms to design a supply chain for a product with three objectives: minimizing total costs, maximizing customer services and the maximization of capacity utilization balance for DCs for the producer of plastic products in Turkey.

Villegas *et al.* [109] present the bi-objective (minimizing overall cost and maximizing coverage) uncapacitated facility location problem to redesign a Colombian coffee network. They design an algorithm based on the Non-dominated Sorting Genetic Algorithm, an algorithm based on a Pareto Archive Evolution Strategy and an algorithm based on mathematical programming with one of the objectives treated as a constraint and they compare the two approaches for quality of their approximation to the Pareto frontier.

An example involving the trading off the cost against environmental impact is described in Quariguasi Frota Neto *et al.* [83], where the reorganization of a European pulp and paper logistic network is described. They use a techniques based on multi-objective programming to determine "optimal" configurations of the network.

From the literature review we identify the need to create environmentally friendly logistics systems where strategic decisions and the transport distribution system are considered together as part of the design. In our literature review we found only a small number of multi-objective infrastructure modelling in an environmentally friendly logistics context (e.g. [61], [62], [83], [92]). Pati *et al.* [92] uses goal programming to balance economic and environmental goals through increased wastepaper recovery for paper recycling logistics system. Khoo *et al.* [62] uses simulation approach in modelling of a supply chain concerned with the distribution of raw aluminum metal whereas Hugo and Pistikopoulos [61] use a classic multi-objective optimization method. Thus we highlight a fruitful area for our future research where environmental and economic concerns need to be modelled as explicit objectives to generate more information about cost and the implications on ecological impact. Multi-objective optimization techniques, such as evolutionary algorithms, are available to generate alternative solutions which allow the decision makers to investigate trade-offs between economic and environmental objectives. In practice, there is a wide range of algorithms that come under the Pareto-based category. Therefore there is a need to investigate these techniques for efficient infrastructure modelling for environmentally friendly networks.

3.8 Summary

The main aim of this chapter is to introduce the reader to the important concept of multi-objective optimization through different approaches which are based on the classical preference based methods or evolutionary techniques such as NSGA-II and SEAMO2. It also covers a brief literature review of previous work using multi-objective techniques for network design where environmental and economic costs are balanced.

The main conclusion of this section is that if environmental assessment is incorporated as part of infrastructure modelling then there is a possibility of achieving both economic and environmental savings. Every logistics design should include industry specific environmental assessment to prevent pollution and save the environment. Some tools and techniques are already available to researchers to help achieve this goal, but there is still much work to be done.

Chapter 4

Assessing the impact of cost optimization based on CO_2 emissions of infrastructure modelling

4.1 Introduction

This chapter describes a study where a simulation based approach of a European case study from the automotive industry by Hammant *et al.* [124] considers strategic and operational level decisions simultaneously for logistics network modelling. The study aims to assess the impact of the traditional cost optimization approach to strategic modelling on overall logistics costs and CO_2 emissions by taking into account the supply chain structure (number of depots) and different freight vehicle utilization ratios (90%, 75% and 60%). This data was previously evaluated from an economic perspective only and identified the optimum network design at two distribution centres. Taking an environmental perspective gives us new insights. We will consider the impact of strategic and operational level decisions simultaneously, focusing on inventory and transportation costs versus the environmental impact in terms of CO_2 emissions from transportation and non-domestic buildings such as depots. The calculation of CO_2 emissions from transportation considers vehicle type, utilization and vehicle speed. We use a supply chain network design application for our simulation with optimization based on costs alone. Attention is also paid to the sensitivity of our solutions when changes in key model parameters, such as vehicle utilization ratios (90%, 75% and 60%) and network structure (number of depots), occur.

4.2 Background information

The past 20–30 years have seen a significant restructuring of logistics networks for many companies, as they strive to reduce costs while improving customer service levels. In the context of transport, there has been a particular focus on improving vehicle fill and reducing the distance vehicles travel. While traditionally, attention has concentrated on outbound logistics, increasingly inbound distribution is also considered [25]. Not only do these changes bring about internal benefits to companies, but they also create wider benefits to society, leading to a reduction in external costs and its impact on the environment.

Since the 1980s, the development of supply chain management has resulted in managers becoming increasingly focused on the demands of their customers. Initiatives such as lean production have resulted in companies looking to deliver ever higher levels of customer service, while minimising the cost impact [108]. Logistics operations have been required to handle smaller and smaller shipments through their networks while maintaining efficiency. As a consequence of this, it has been necessary for companies to reconfigure their logistics operations. These have been categorised by McKinnon [75] into four main areas:

- Logistics structures - relates to the configuration of the distribution network and the choice of distribution channel. Control of this network also comes within this area.
- Pattern of trading links - determines the geographical spread of the logistics structure. Recently, moves towards outsourcing abroad have seen supply chains lengthen.
- Scheduling of product flow - affects the movement of products through the network and determines the size of the shipments to be made. Developments in this area include continuous replenishment and just-in-time deliveries.
- Management of transport resources - decides the actual transport requirements for particular shipments, and may include issues relating to modal choice.

All of the above decisions are likely to affect the transport requirements for an individual organisation, in terms of the distance, speed, frequency and timing of deliveries [26]. Traditionally, such changes would only influence the outbound logistics operations of a business [51], with inbound movements being viewed as the responsibility of the supplier. However, nowadays, there is a focus on this inbound network, as companies recognise the potential synergies that exist between them [25].

There are a number of examples within the published literature of how the efficiency of logistics operations can be improved, while also delivering environmental benefits. The consolidation of small shipments is a popular approach to reducing transport costs, and has particularly been used within the grocery industry in the UK [42]. Consequently, load consolidation has resulted in a reduction in the distance vehicles travel of around 20% [75].

4.3 Method

To explore the relationship between total logistics costs and environmental impact in terms of CO_2 emissions for strategic modelling in the logistics network, there is a need for an appropriate methodology for both assessments. The method and data we use for evaluating economic costs is based on the case first presented in Hammant *et al.* [124]. One of the objectives of that study was to describe the use of a simulation-based decision support system to establish the impact of restructuring the physical infrastructure of a Pan-European supply chain. The authors indicated the benefits of using a simulation approach for assessing network design. The optimum network design at two distribution centres was determined by minimizing the total overall logistics costs (transportation and inventory costs) while ensuring an appropriate customer service level. Subsequently Lalwani *et al.* [125] used the data to present a new method, which combines simulation and the Taguchi technique [130] to identify the factors that the structure of the distribution network is sensitive to. Their analyses indicated that the optimum design is highly at risk from the uncertainties associated with inventory holding stocks and delivery frequencies rather than customer demand and transport costs.

4.3.1 Modelling economic costs

To model our logistics networks, we use a commercially available supply chain design application CAST-dpm® (by Radical, nowadays known as CAST-NV by Barloworld Optimus). This is the same package that was used in the original study by Hammant *et al.* [124] and Lalwani *et al.* [125]. The software allows the decision-maker to evaluate different scenarios and aims to identify the optimum network infrastructure, such as the location and number of depots. It uses a heuristic approach to estimate the transportation costs of the network, distance run by the vehicles and the number of the vehicles needed for the particular output period. The Square Root Law [69] was used to estimate the costs of the inventory that is needed in the network.

As previously mentioned, the simulation model is based on the case study of an automotive aftermarket Pan-European distribution network. The network operates through different countries such as the UK, France, and Spain involving a large number of businesses and substantial operating distance. Throughout Europe, the company has around 550 suppliers and 10,000 customers. The case study is purely road transport based and does not take into account freight movements by sea or rail, although to join up with the UK road network a ferry or rail/tunnel journey would be used. All the input data, which was used in our model, was taken from the original case study, apart from the transportation data that we generated ourselves, as it was not available from the original source. Table 4.1 summarizes the input data used for the simulation model. Note that the total logistics costs derived from our design, are different to the original paper because we used different transportation data. The aim of the research presented in this chapter is not to replicate the precise data from the original case study but to reproduce general principles of the research by Hammant *et al.* [124] where the optimum network design based on costs was identified as two depots. This approach allowed us to analyse the trade-offs between the total costs and their environmental impact of a cost based optimization.

To analyse the relationship between total logistics costs and their environmental impact from transportation and depots, two different scenarios were considered for strategic modelling. For the first scenario, we used a centre of gravity method to determine the centroid locations of the distribution centres in the network. For the second scenario, the original

Customer (original data)	Name, location Annual demand by product group Number of deliveries per year Transportation mode
Supplier (original data)	Name, location Annual supply by product group Number of deliveries per year Transportation mode
Transport	Vehicle information: physical and distance constraints: Transportation costs functions
Warehouse locations (original data)	Location Throughput, total area (square meters)

Table 4.1: Simulation model's input data based on Hammant *et al.* [124].

locations from Hammant *et al.* [124] were used to derive network related costs and distances travelled. The original locations are the real physical depots existing in the network. In this scenario, we aimed to replicate the total logistics costs curve from the original case study, which identifies the optimum network design at two depots. The simulation model is not intended to find the optimum solution; it evaluates different options, which are input into the system.

4.3.2 Centre of gravity modelling

Two different scenarios were considered for the current study for strategic modelling: original published locations with an optimum design of two depots and a centre of gravity scenario. The centre of gravity approach is one of the well-known heuristic methods in facility location analysis. It indicates the centroid locations that minimize the total transportation cost. Traditionally, the transportation rate and the point of volume are the only location factors in this approach [6]. The method provides a good estimation to the least-cost solution. However, a certain amount of location flexibility has to be exercised by the decision-maker because of geographical obstacles, such as sea, mountains etc. The current network modelling software offers two alternatives for centre of gravity modelling: cost centre of gravity and volume centre of gravity. The cost centre of gravity model attempts to minimize the total cost; whereas the volume centre of gravity attempts to minimize the total tonne-kilometres travelled [96]. The cost based centre of gravity was used with a limited number of scenarios, but these generated similar results to the volume centre of gravity. Therefore, due to time constraints and the similarity of results the latter technique was used. As can been seen from Figure 4.1 the centroid depots locations for the UK and France have not moved too far from the original locations, due to the high supply and demand volumes in those areas. The other distribution centres have changed locations, which reflect current customer's and supplier's demands.

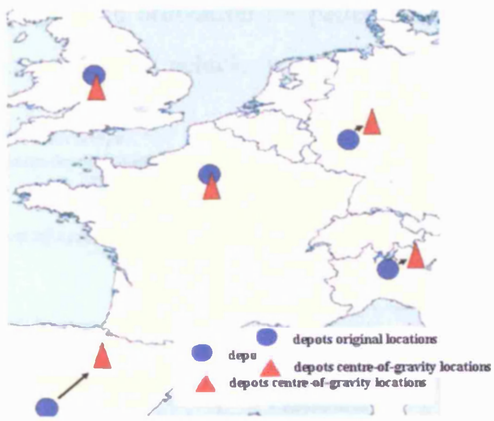


Figure 4.1: Depots locations (five depots scenario).

4.3.3 Modelling CO_2 emissions

After establishing the base design for each scenario, we used two determinants, supply chain structure and vehicle utilization factor as key decision variables for this research, to analyse the potential for reducing CO_2 emissions at the micro level. These factors and others impact on the respective key ratios identified by McKinnon [74] to influence CO_2 from freight transport. Supply chain structure has a direct impact on the two key ratios: handling factor and average length of haul. Handling factor is a crude measure of the number of the links in the supply chain, where the weight of the goods is converted into freight tonnes-lifted. Therefore, for our research, the supply chain structure was reflected in the reduction of the total number of depots in the network: from five depots down to one depot, decrementing in steps of one. Vehicle utilization has a direct impact on reducing vehicle traffic. Increasing vehicle utilization allows businesses to cut the number of vehicles on the road, which brings both economic and environmental advantages. The average weight-based utilization in 2006 in UK of rigid lorries on laden trips was 52% and for articulated vehicles 58% [35]. Average deck utilization of the vehicles, for pallet networks was 73%, for non-food 51%, and for food retail 53% [36]. Therefore, as the purpose of this research is to analyse the trade-offs between total costs and emissions, we used vehicle utilization factors at 90% (the 'ideal' vehicle utilization); at 75% (approximation from the average deck utilization for pallet network) and at 60% (average weight-based utilization for articulated vehicles).

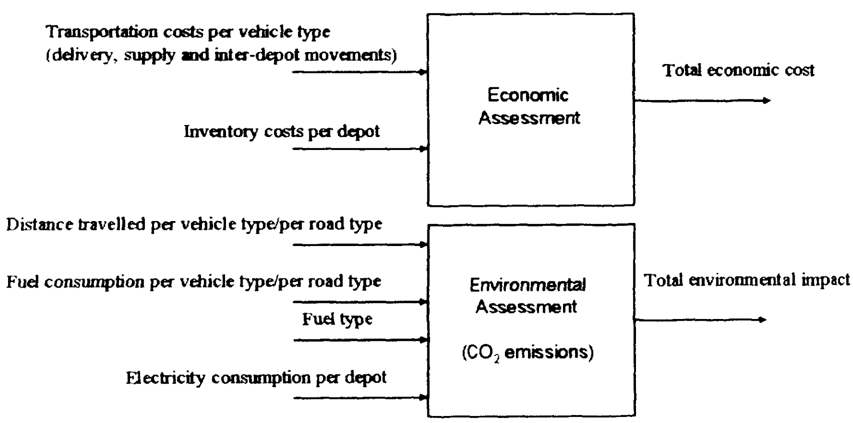


Figure 4.2: Input/Output diagram of the method for each scenario.

When using strategic modelling techniques to calculate CO_2 emissions from transportation and depots, it is important to establish boundaries for those estimations. To calculate CO_2 emissions from transportation, we will only consider the amount of goods being transported over the distance travelled. Our method does not take into account the life cycle assessment of the product from "the cradle to grave". For the present work, we use the outputs from the supply chain network design application, which runs over a particular period of time and establishes the network related costs and travelled distances for different vehicles types for a particular output period of 52 weeks. Hence, our estimates for CO_2 emissions cover the same period of 52 weeks. As mentioned previously, the case study described in this chapter is purely road transport based and does not take into account freight movements by sea or rail. Figure 4.2 represents the overall method with data requirements for each scenario.

Chapter 2, Section 2.7 provides an overview of the different formulations used for calculating road related emissions as part of logistics modelling. Due to the complexity of correctly estimating CO_2 emissions from transportation, some assumptions and simplifications have to be made with respect to driver's behaviour, volume of the traffic on the road, and so on. It is very difficult to take account of all the described factors which have an impact on fuel consumption to calculate CO_2 emissions from road freight. Assume that two types of diesel lorry are used for delivering goods across the network: a 5 tonne gross weight lorry and a 40 tonne gross weight lorry. To calculate CO_2 emissions from transportation for each vehicle type and vehicle payload, we used a distance-based formulation 4.1 from DEFRA [32] where the emissions from all road types (motorway, rural, urban and minor) are summed together:

$$\begin{aligned} Total\,CO_2(per\,vehicle\,type/payload) &= \sum_{road\,type} (total\,km\,travelled * LFPK * \\ &\quad * fuel\,conversion\,factor) \end{aligned} \tag{4.1}$$

where a fuel conversion factor of 2.63 kg/litre was used for diesel fuel; litres fuel per km (LFPK) is the fuel consumption (litres/km) of the vehicle.

Road type	Road traffic (HGV)	Average speed limit	% difference in fuel consumption compared to driving at 54mph
Motorway	42%	54 mph	0
Rural "A" roads	35%	45 mph	-5.53
Urban "A" roads	10%	36 mph	-5.55
Minor roads	13%	30 mph	-2.91

Table 4.2: Road traffic, speed and fuel consumption articulated for HGVs.

The fuel consumption (litres/km) figure for equation 4.1 is calculated depending on the vehicle speed (which is derived from the road type), vehicle type and vehicle payload. As can be seen from the Table 4.2, road types are classified as motorways (42 % of roads), rural "A" roads (35 %), urban "A" roads (10%) and minor roads (13 %) [37]. For each road type, for example a motorway, there is an average speed of 54 mph for HGV vehicles which was used establish the base case for our methodology. In addition, the table presents data for the road traffic which shows the allocation of the total distance travelled on the particular type of road, for example motorway road is 42 per cent of overall distance travelled. The following methodology was used to define fuel consumption accordingly:

1. Establish fuel consumption (litres/km) for the base case (for motorway road with average vehicle speed at 54 mph) for two types of vehicle and different vehicle payload. For a 40 tonne lorry we used data from Kohn [64], where a figure of 0.27 litres/km for fuel consumption unladen and 0.38 litres/km for fuel consumption with a full load was used for a vehicle speed of 54 mph. For a 5 tonne lorry we estimated that fuel consumption unladen is 0.157 litres/km and 0.275 litres/km for fuel consumption with a full load from the statistics of fuel consumption data by vehicle type from DfT [33]. Equation 4.2 presents a formulation which we used for calculating fuel consumption depending on the vehicle payload. Linear correlation between payload and fuel consumption correspond to the recent investigation by DfT [38].

$$\begin{aligned} LFPK &= LFPK(unladen) + (LFPK(full\,load) - LFPK(unladen)) * \\ &* (\%weight\,laden) \end{aligned} \tag{4.2}$$

   As the result of all calculations, Table 4.3 presents fuel consumption for the base case for our study for different vehicle types with different vehicle payload, where an average speed of 54 mph is used for motorway road.

2. Calculate fuel consumption (litres/km) from the base case (vehicle speed of 54 mph) to a vehicle speed of 45 mph (rural "A road"), 36 mph (urban "A" road) and 30 mph (minor roads) for different vehicles and payload. To calculate the percentage difference in fuel consumption between different vehicle speeds we used data from NAEI [78], where the user can estimate CO_2 emissions depending on the vehicle

Weight laden(%)	Fuel consumption (litres/km) 40 tonne lorry	Fuel consumption (litres/km) 5 tonne lorry
90%	0.369	0.263
75%	0.353	0.245
60%	0.336	0.228

Table 4.3: Fuel consumption (vehicle speed 54 mph).

	Weight laden (%)					
	40 tonne lorry			5 tonne lorry		
Road type	90%	75%	60%	90%	75%	60%
Motorway	0.369	0.353	0.336	0.263	0.245	0.228
Rural "A" road	0.349	0.333	0.317	0.249	0.232	0.215
Urban "A" road	0.349	0.333	0.317	0.249	0.232	0.215
Minor road	0.358	0.342	0.326	0.255	0.238	0.221

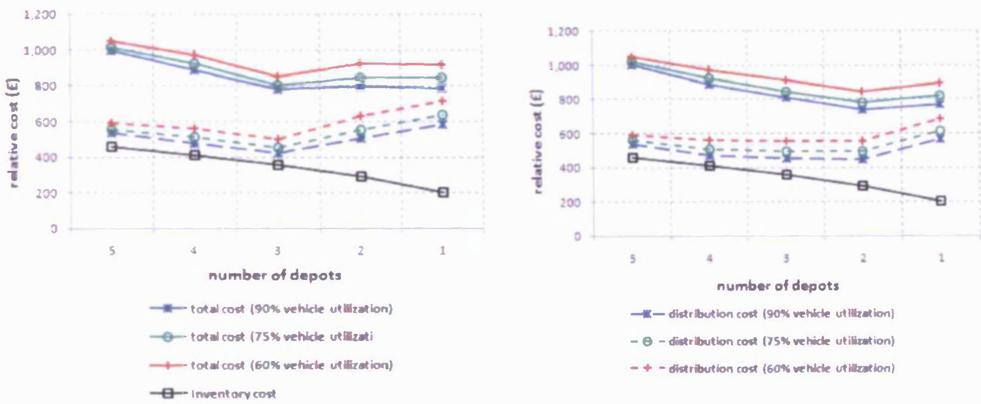
Table 4.4: Fuel consumption (litres/km) for different settings for vehicles, payload and road type.

type and the vehicle speed. The conversion of vehicle speeds from miles per hour to kilometres per hour was performed to calculate the emissions. For example, an articulated heavy goods vehicle with Euro II engine class produces around 5.53% less CO_2 emissions travelling at 45 mph compared to travelling at 54 mph. Because CO_2 emissions are determined mainly by fuel consumption [97], we assumed the same percentage difference for fuel consumption for each vehicle type in our model (Table 4.2). The same assumptions were applied to data generated for other countries. Therefore, the fuel consumption for each vehicle type, vehicle payload and road class was adjusted accordingly to the percentage of difference shown in Table 4.2 from the base case described in step 1. Table 4.4 represent resulting fuel consumption data for different vehicle types, weight laden and road types.

To calculate CO_2 emissions from electricity used at depots we need to estimate the average annual electricity consumption (kWh/m_2) per depot. In our automotive network, the product is of a nature that does not need a specialised storage environment requiring cooling or heating. The depot data was only available regarding the size of the buildings in m^2. Therefore, an average figure of $2\,kWh/m^2$ was used from the British Land Company PLC [15]. To convert energy data to CO_2 emissions for a UK-based depot, we used a conversion factor of 0.54 kg$C0_2$/kWh [32], which gives CO_2 emissions of 1.08 kg/m^2. For depots in other countries we used the following conversion factors: in France we used 0.083 kg$C0_2$/kWh, in Italy 0.525 kg$C0_2$/kWh, in Germany 0.539 kg$C0_2$/kWh,[40] in Spain 0.4556 kg$C0_2$/kWh [66].

4.4 Results

Identifying the optimum number of depots and their positions is of fundamental importance, if one is to lower total costs and ensure an appropriate level of customer service. In the current study, delivery, collections and inter depot movements are taken into account for calculating overall transportation costs and distances. There is a trade-off between inventory and transportation. Figure 4.3(a) and 4.3(b) show the results of Pan-European distribution network modelling and the effect that decreasing the number of the depots in the logistics network has on the transportation and inventory costs. The results in Figures 4.3, 4.4, 4.5 on costs, total distance traveled and CO_2 emissions are presented as relative values to make actual values anonymous. Transportation costs are a function of both distance and time related factors, which include fixed and operational (distance related) costs. We can observe that the transportation costs decrease as the number of facilities decreases due to the reduction in the inter-depot movements until it is reaches the point when it starts increasing again, due to the longer travel distances to the nearest depot. The inventory costs decline as the number of facilities decrease due to the lower levels of inventory. As you can see from Figure 4.3(a), the optimum number of depots for cost-based optimization in the centre of gravity scenario equated to three depots. Figure 4.3(b) shows the optimum number of depots for cost-based optimization equated to two in the original locations scenario, where depots are located at the real physical locations. By changing the vehicle utilization from 60% to 90% for the optimum design in the centre of gravity scenario we observe a decrease in total logistics costs of 8.9%. A slightly larger decrease of 12.9% is seen in the optimum design for original locations scenario. Unfortunately, 90% vehicle utilization is not a very realistic figure in the real world. By comparing the more practical levels of 60% and 75% vehicle utilization, we can see a 5.5% total logistics cost decrease for the centre of gravity locations scenario and 7.5% decrease for the original locations scenario.



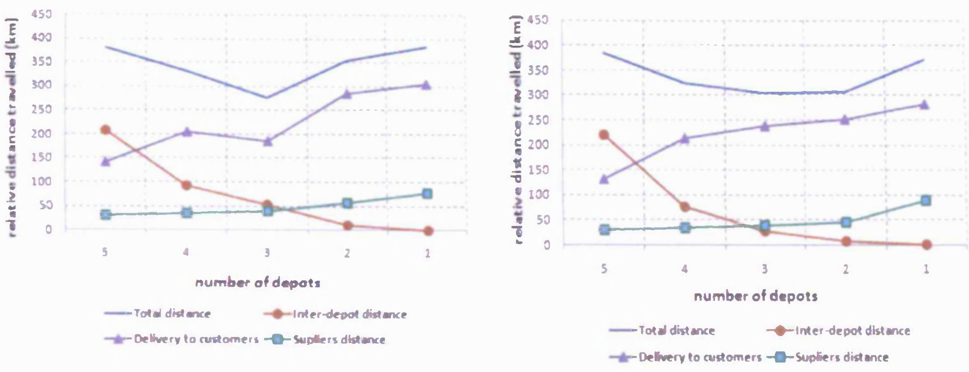
(a) Centre of gravity locations

(b) Original locations

Figure 4.3: Logistics costs related to number of depots and vehicle utilization parameters.

Figure 4.4(a) and 4.4(b) represent an overview of the transportation distances related to the number of depots and 90% vehicle utilization parameters for both case studies. The figure shows that inter depot distance is reducing as the number of depots decreases and the supplier and delivery distance travelled is increasing. Note that the optimum design based on travelled vehicle kilometres is three depots for both scenarios and all vehicle utilization parameters; while the optimum based on distribution costs is three depots for the centre of gravity locations scenario and two depots for the original locations scenario. Similar observations are produced for 75% and 60% vehicle utilization. Note, that in the original locations scenario, for 90% vehicle utilization the difference between the three and two depots design resulted in a reduction of total logistics costs by 8.8%; transport costs decreased by 1.4% and total vehicle kilometres travelled based on % laden weight of the vehicle increased by 0.67%, which is almost negligible.

Earlier we discussed the impact of vehicle utilization on total logistics costs. Now we will assess the impact of cost-based optimum network design on the total vehicle kilometres

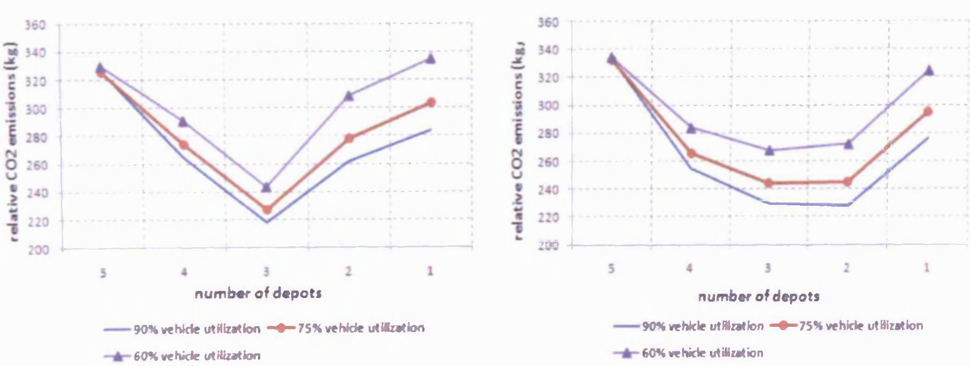


(a) Centre of gravity locations

(b) Original locations

Figure 4.4: Transportation distance breakdown related to number of depots and 90% vehicle utilization.

travelled based on % laden weight of the vehicle. For the centre of gravity locations scenario, changing from 60% to 90% in vehicle utilization show a decrease of 22% in distance travelled (km) and 27% for the original locations scenario. Changing the vehicle utilization from 60% to 75% has produced a reduction of 13.1% in distance travelled (km) for the centre of gravity scenario and 16.1% for original scenario.



(a) Centre of gravity locations

(b) Original locations

Figure 4.5: Total CO_2 emissions from transport and electricity related to number of depots and vehicle utilization parameters.

As discussed in Section 2.7, levels of emissions directly relate to different factors, including distances travelled, the load of the engine over the distance and the speed of the vehicle. The described factors are incorporated into our calculations of the emissions which give a more accurate figure for estimating the impact. As you can see from Figure 4.5(a) the optimum design based on total CO_2 emissions for the centre of gravity scenario is three depots for all vehicle utilization ratios. The increase in vehicle utilization from 60% and 90% shows a reduction of 10.6% in transport related CO_2 emissions and for vehicle utilization from 60% to 75% a reduction of 6.8% can be observed. Figure 4.5(b) shows that for the original locations scenario the optimum design based on CO_2 emissions is two depots for 90% vehicle utilization and three depots for 60% and 75 % vehicle utilization. Analysing the difference in CO_2 emissions between three and two depots for the original scenario, we can see that there is only a 0.57% decrease for 90% vehicle utilization, which is almost negligible. For 75% vehicle utilization, an increase of 0.55% in CO_2 emissions can be seen and for 60% vehicle utilization an increase of 1.62% can be observed. The analysis shows that for cost-based optimum design at two depots for the original locations scenario, the changes from 60% and 90% in vehicle utilization produce a reduction of 16.3% in transport related CO_2 emissions. For vehicle utilization from 60% to 75% there is a reduction of around 10%.

From our analysis we identified that environmental impact from electricity in depots in our case study was negligible and had little effect on the overall result of calculating CO_2 emissions. This was mainly due to the product not requiring any specific storage temperature.

4.5 Discussion of results and main conclusions

To analyse the relationship between total logistics costs and their environmental impact from transportation and depots we considered two different scenarios for strategic modelling: a centre of gravity locations scenario and a scenario using the original published locations with optimum network design consisting of two depots. The cost-based optimization for the centre of gravity scenario identifies the optimum number at three depots based on total logistics costs and CO_2 emissions. In the original locations scenario, the optimum design of cost-based optimization equated to two depots for total logistics costs and two/three depots for CO_2 emissions. The latter proved to be sensitive to the vehicle utilization ratios, even though there is a very small difference in transportation costs and vehicle kilometers between three and two depots. The methodology for calculating CO_2 emissions takes into consideration speed of the vehicle, vehicle type and vehicle utilization. The study based on original locations also indicates that increasing vehicle utilization by 15% could bring economic savings in total logistics costs (7.5%) as well as environmental benefits reflected in reduction of total vehicle kilometers traveled (16.1%) and reductions of transport related CO_2 emissions of around 10%. Therefore, due to the increasing environmental concerns, it is important to incorporate environmental objectives as part of logistics design and correctly estimate vehicle utilization ratio factors for emissions calculations, to allow the decision-maker to make an informed and objective decision regarding network design.

The current study has several limitations. Firstly, only one case study has been analysed. Secondly, the assumptions regarding transportation data also limits the study because in a realistic supply chain a wider variety of vehicles are used to transport commodities. Also, the lack of specific fuel consumption figures for transportation makes the study dependent on information available in the public domain.

This chapter considers a single objective optimization model based on costs where CO_2 emissions are calculated as a result of the cost-based optimization. This study allowed us to analyse factors that have an impact on the strategic model and demonstrated that the optimum solution based on costs is not necessarily the same for emissions which highlights the importance of incorporating environmental concerns as objectives. In Chapters 8 and 9 we are investigating the building of a multi-objective optimization decision support tool for strategic modelling (facility location-allocation problem), where traditional objectives, such as cost and improving service level and environmental impact are considered simultaneously. The approach will allow the decision maker to evaluate a set of viable alternatives, in contrast to traditional methods where environmental impact is calculated as a constraint or the user is required to prioritize objectives. The approach could potentially find excellent solutions which could be missed by other methods, but generating a large number of solutions could be also considered as a disadvantage. Therefore it is important to involve the decision-maker in the evaluation of the solutions according to the potential criteria needed.

4.6 Summary

This chapter aimed to analyse the relationship between total logistics costs and their environmental impact in terms of CO_2 emissions from transportation and electricity usage in depots when using a traditional cost-based optimization approach. Our simulation model was based on a Pan-European network from the automotive sector taken from an original study by Hammant *et al.* [124]. The present chapter describes a specific case study and does not attempt to generalize the results of the analysis. Nevertheless, we believe that our results highlight the following issue: the optimum solution for reducing costs does not necessarily equate to the optimum solution for reducing CO_2 emissions. Furthermore, our findings indicate the optimum design of a distribution network is highly sensitive to the level of vehicle utilization. Due to increasing global climate change, the chapter makes a case for considering environmental and economical objectives simultaneously.

Chapter 5

Cost-Based Optimization for a Capacitated Allocation Model Using Sainsbury's Data

5.1 Introduction

John Sainsbury PLC (Sainsbury's) is one of the five leading supermarket retailers in the UK. It began in 1869 as a butchers, and has now grown into a leading UK company with convenience stores and pharmacies as well as the supermarket chain. Sainsbury's also includes a thriving online business within its portfolio. In this study we focus on the logistics network concerned with delivering groceries from the distribution centers to the supermarket stores. The convenience stores, pharmacies and online store have separate distribution networks which are not considered here. The present chapter describes an investigation into the potential effect of changes in various key factors on the allocation of the stores to depots, undertaken with collaboration of Sainsbury's Central Strategy team during 2009. In addition, the environmental impact in terms of CO_2 emissions based on the total vehicle-km travelled of a cost-based optimization was undertaken to analyse the relationship of the changes in costs on the total distance of a particular allocation.

The study focuses on the secondary distribution network with three different goods movements: ambient, chill and produce, from the distribution centres to their stores. The constraints of the network include depot capacities for each product type in terms of 1) number of cases and 2) number of stores. The purpose is not to replicate Sainsbury's regular "best fit" exercise, but instead to explore a number of "what if?" scenarios and identify their potential impact on store allocation and total cost. Warehousing hourly rate, direct work rate and driver's hourly rate were analysed for sensitivity on five depots (out of ten depots): depot A, depot B, depot C, depot D and depot E. Due to the privacy issues the depots names in the network were made anonymous and abbreviated to a capital letter, e.g. depot A, B etc. The analysis examines behavioral changes to the number of stores assigned to the particular depot where changes to variables have been made; we look at the number of stores lost to that depot, the number of stores newly assigned to that depot and the total number of stores allocated differently across the entire network. The fuel rate scenario, analysing the potential effects of fuel price changes, has been undertaken across all ten depots simultaneously and looked at the total number of stores allocated differently from our baseline scenario.

5.2 Background information

Assigning customers to appropriate serving facilities is known as the Generalized Assignment Problem (GAP) and is an NP-hard combinatorial optimization problem. Within logistics network design, the GAP could be referred to as one of the tactical decisions which need to be re-evaluated every few months to ensure the continued competitiveness of the network. The variant of the GAP that interests us in the present study can be expressed as follows. For 1) a given set of customers with known demand for different types of products and 2) a given set of open facilities with volume constraints and limitations on the number customers that each facility is capable of servicing, the objective is to minimize the total cost of assigning the customers to the facilities. Furthermore, each customer's demand has to be satisfied by one facility only, which is referred to as a single source, and the constraints limiting each facility in terms of volume and numbers of customers must be adhered to.

The current Sainsbury's network used in this study consists of 10 depots and 520 stores. Each depot is characterized by a number of attributes, including its location and its capacity constraints, and each customer makes specific demands for the products supplied by the depots. The locations of depots and stores are fixed. The main depots under investigation are A, B, C, D and E; whereas other depots in the network are F, G, H, K and L. Each depot can supply three types of products: ambient, chill and produce, except for two depots: depot F, which serves only chill products, and depot G which serves ambient and produce (but not chill). Every depot has a certain capacity for each type of product in terms of the number of cases and the numbers of stores it is capable of serving. In reality the Sainsbury's logistic network also serves a small number of stores in Northern Ireland, which involves a sea crossing. While the demands of the Irish stores are taken into consideration in our model, for simplicity we do not include them in our cost and assignment analysis, which concentrates only on the mainland UK. We assign the volume requirement for the Irish stores to depot H, as this is Sainsbury's current practice.

Our allocation model consists of transportation and depot components. The transportation costs include distance-based (using stem-distances) and time-based formulations. In this chapter, we refer to stem distances, that is the distance travelled by the vehicle to deliver goods from a depot to a particular store and return back to the point of origin. The vehicle only visits one store at a time due to the high volume of the demand and does not undertake milk-round trips, visiting other stores as well. The depot component consists of the variables associated with running the depot, such as warehouse hourly rate and direct work rate. The fixed costs are not taken into account as part of the modelling.

Each store has a certain demand for each product type, and this is expressed as a weekly volume. The demand data we use was acquired in September 2009 and gives the number of cases per week per product averaged across a 26 week (6 month) period. The direct work rate (DWR) data for each depot is also averaged across this six month period (April - September). The scope of this study does not include inter-depot movements and excludes frozen and slow moving ambient products. In order to allow for some fluctuation in demand, we built some 'slack' into the system by capping all the depot capacities to 90% of the maximum number of cases quoted for each depot and product.

5.2.1 Objective

As mentioned previously, the objective is not to replicate Sainsbury's regular "best fit" exercise but to identify the impact of key factors on the allocation of the stores to depots. The different key elements (variables) come from the depot and transportation model. The overall labour costs of a depot, which include all staff functions, are represented by the warehouse hourly rate (WHR), and the work rate efficiency of staff directly associated with picking and loading is measured by the direct work rate (DWR). WHR and DWR reflect the depot model. The fuel costs, drivers' hourly rate and cases per load are the important components of the transportation model, and the transport hourly rate (THR) is the key measure that reflects all the time-related costs (e.g., drivers' pay).

An additional analysis was undertaken to understand the impact of the cost-based optimization on the distance based CO_2 emissions: e.g., does a decrease in cost equate to a reduction in vehicles-km and therefore to CO_2 emissions?

A sensitivity analysis was performed to analyse the impact of various changes to the key variables on the allocation model and overall cost structure. Five different "what if?" scenarios were performed (see below) where WHR, DWR and THR situations were analysed for sensitivity at five depots in turn: depot *A*, *B*, *C*, *D* and depot *E*. The fuel rate scenario was undertaken across all ten depots simultaneously and looked at the total number of stores allocated differently compared to our benchmarking scenario (described below).

The following five scenarios were considered for analysis:

1. The uniform scenario was used as a 'proof of concept', and the allocations and costs compared to the Company allocations in their latest "best fit" exercise. This scenario is characterised by uniform costs applied throughout the network. The average vehicle load is used at 1243 cases for each product with fuel related costs of £0.35 per km. The DWR for each depot was set at 97.6 for ambient, 100.13 for chill and 136.02 for produce. THR (£18.48) and WHR (£16.03) were the same across all depots.

2. The fuel related scenario is used to analyse the impact of changes in the fuel rate (fuel cost per km) across the network as a whole. We applied rates between £0.1 to £1.00 with a step size of £0.01; from £1 to £10 with a step size of £0.25; from £10 to £100 with a step size of £10 and at £500. We analysed a wide range of fuel rate values, from small to very large numbers on either side of the benchmarking rate value of £0.35 per km. This range is not necessarily very realistic in the current economic climate. The aim of this analysis is to understand the behavior of the network between different extreme points.

3. The direct work rate scenario investigates the impact of the work rate efficiency of pickers and loaders in the warehouses on the allocation of the stores to a particular depot. This is analysed by changing the DWR from 10% below the benchmarking rate for all products, and up to 15% above the benchmark with steps at 2.5%.

4. The transport hourly rate scenario investigates the impact of changes in drivers' hourly rate on the store allocation, with rate changes from £10 to £33 with a £1 step.

5. The warehouse hourly rate scenario investigates the impact of warehouse related costs, such as picking and loading, on the store allocation, with the rate changes from £10 to £33 with a £1 step.

In addition, we refer in our study to 'baseline' or 'benchmarking' scenario which uses the same data (September 2009) as we use for Scenarios 2-5. We also refer in our study to the Company allocation which is the model obtained in the most recent "best fit" exercise by the company. The Company allocation is carried out using stem distances only, and does not account for warehouse costs or transport hourly costs. We compare our allocation from Scenario 1 with the Company allocation where we equalize warehouse costs and transport hourly costs throughout the network, so that our allocation is based essentially on distance related costs only, similar to the Company model. On the other hand, the "benchmarking' allocation, or baseline, uses the original September 2009 data, maintaining the variable warehouse and transport costs which is used for Scenarios 2-5. It shows the allocation obtained using the CPLEX® optimization engine. In scenarios 2-5 we make "what if?"

changes to the original data, to observe their effects on the baseline network structure.

5.3 Methodology

The network was modelled using the Java programming language and the CPLEX® optimization engine. CPLEX® uses algorithms to solve given problems to optimality. This approach gave us great flexibility in implementation, utilizing the power of CPLEX® together with the visual display capabilities which we customized in the user interface using Java. Visualization of the results allows us to see beyond the numbers, and observe how the results are translated to geographical information. We have also implemented our own model based on Lagrangian relaxation (LR), which is faster but does not guarantee to obtain the optimum solution (although it is excellent in practice). Although we do not make use of LR in the present chapter, this approach is utilized extensively in the other chapters (7 and 9), when speed is more critical (e.g., for really large problems, multiple experiments and for considering multiple criteria simultaneously).

5.4 Mathematical formulation

For a given a set of stores with known demands for different type of product, and a set of open facilities (depots) with known volumes (numbers of cases) and maximum store allocation capacities, the objective is to minimize the cost of assigning the stores to the facilities. Each store's demand has to be satisfied by a single facility, e.g. one store is allocated to only one depot and the capacity of that facility must not be exceeded. The problem can be modelled using a complete directed graph, where the vertices in V_{DC} represent the facilities (distribution centres) and the vertices in V_C represent the stores (customers). The arcs are associated with the flow of goods between facilities and stores.

Glossary

V_{DC}	set of depots
V_C	set of stores

P	set of products
d_j^p	demand of store j for product p, $j \in V_C$, $p \in P$
v_j^p	number of vehicles needed to deliver demand of store j for product p, $j \in V_C$, $p \in P$
l^p	average vehicle load of product p, $p \in P$
f	distance related cost (fuel cost) per km
q_i^p	capacity of cases of facility i of product p, $i \in V_{DC}$, $p \in P$
n_i^p	number of stores assigned to facility i for product p, $i \in V_{DC}$, $p \in P$
tc_{ij}^p	transportation cost of total demand d_j^p for product p of store j from facility i
dc_{ij}^p	depot cost of total demand d_j^p of product p of store j from facility i
$dist_{ij}$	distance between store j from facility i
$time_{ij}$	time to travel between store j from facility i
THR_i	transport hourly rate (drivers cost) for facility i
WHR_i	warehouse hourly rate for facility i
DWR_i^p	direct work rate for facility i for product p
x_{ij}	is the decision variable for the problem, $x_{ij} = 1$, if store j is allocated to facility i, and 0 otherwise

Minimize

$$\sum_{i \in V_{DC}} \sum_{j \in V_C} \sum_{p \in P} (tc_{ij}^p + dc_{ij}^p) x_{ij} \tag{5.1}$$

subject to

$$\sum_{i \in V_{DC}} x_{ij} = 1, \forall j \in V_C \tag{5.2}$$

$$\sum_{j \in V_C} d_j^p x_{ij} \leq q_i^p, \forall i \in V_{DC}, \forall p \in P \tag{5.3}$$

$$\sum_{j \in V_C} x_{ij} \leq n_i^p, \forall j \in V_C, \forall p \in P \tag{5.4}$$

$$x_{ij} \in \{0,1\}, i \in V_{DC}, j \in V_C \tag{5.5}$$

by setting:

$$tc_{ij}^p = (dist_{ij} * f + time_{ij} * THR_i) * 2 * v_j^p, i \in V_{DC}, j \in V_C, p \in P \tag{5.6}$$

$$dc_{ij}^p = (d_j^p / DWR_i^p) * WHR_i, i \in V_{DC}, j \in V_C, p \in P \tag{5.7}$$

$$v_j^p = d_j^p / l^p, i \in V_{DC}, p \in P \tag{5.8}$$

where formulation (5.1) aims to minimize the transportation and depot costs of satisfying the total demand of all the stores, and constraints (5.2) with (5.5) guarantee that the demand for each store must be satisfied by one depot. Constraints (5.3) and (5.4) ensures that the capacity constraints for the facilities for each product type are not violated and (5.5) specifies that allocation is indivisible for the decision variable. Formulation (5.6) calculates the transportation costs of product p to satisfy the demand of store j, and (5.7) calculates the depot costs associated with demand of product p and store j. Finally, (5.8) calculates the number of vehicles needed to satisfy demand of store j and product p. We use a real number for the number of vehicles for each product type p as our business partners requested to ensure that exact values are used in the modelling without rounding those values. This is due to a large number of deliveries for each product type and if every value is an integer, the precision of the work will be lost as part of the calculations.

To calculate the total distance travelled (km), the following equation was used on the resulting allocation from the cost-based optimization based on the formulation 5.1:

$$totalDistance = \sum_{i \in V_{DC}} \sum_{j \in V_C} \sum_{p \in P} (dist_{ij} * 2 * v_j^p) x_{ij}, i \in V_{DC}, j \in V_C, p \in P \tag{5.9}$$

Product type	Ratio of total demand in cases / depot capacity	Ratio of total num of stores / depot capacity
Ambient	0.77	0.75
Chill	0.81	0.79
Produce	0.69	0.77

Table 5.1: Ratio of total demand versus available capacity across network .

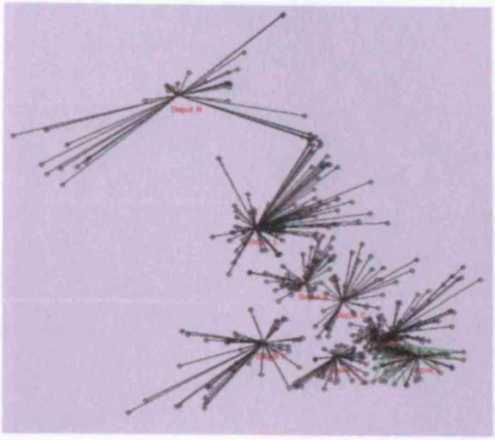
5.5 Analysis

5.5.1 Benchmarking allocation of stores to depots

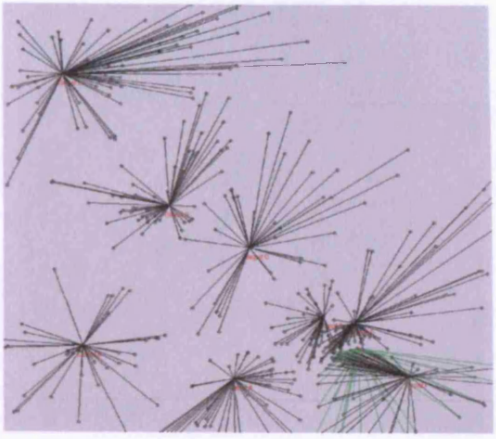
The benchmarking allocation uses the above mentioned September 2009 data, and the result of this allocation serves as our base or "benchmark" for comparing with the results from scenarios 2-5. The "best fit" Company's allocation model is based on stem distances (like our study), but we incorporate warehouse costs and time related transport costs which are not included in the Company "best-fit" study.

It is worth noting that the Sainsbury network has spare capacity, and is quite "loose" in terms of the capacity constraints in relation to total demand. Table 5.1 shows the ratio of total demand (in cases) to total capacity (second column) and the ratio of number of the stores versus total depot capacity (in number of stores) across the entire network. As you can see from the table, the chill product type is the tightest in terms of capacity. These ratios tell us that more than one feasible solution is likely to exist.

Figures 5.1(a) and 5.1(b) show screenshots of the optimum allocations produced by CPLEX® using the September 2009 data with fuel cost at £0.35 per km. The results of this allocation are used as our benchmark for comparison in the sensitivity analysis carried out in scenarios 2-5 (excluding the uniform scenario). As can be seen from Figure 5.2 and 5.3, the depot costs form the larger component (67.9%) and the transport costs account for only 32.1%. There are only 50 stores (9.6% of all stores) for which the transportation costs account for 50% or more.



(a) Overview



(b) Close up

Figure 5.1: Benchmarking scenario.

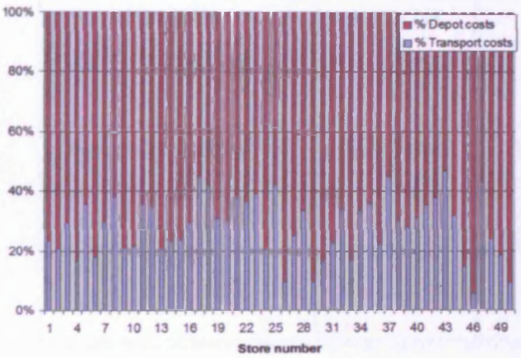


Figure 5.2: Proportion of depot and transport costs for benchmarking scenario for first 50 stores.

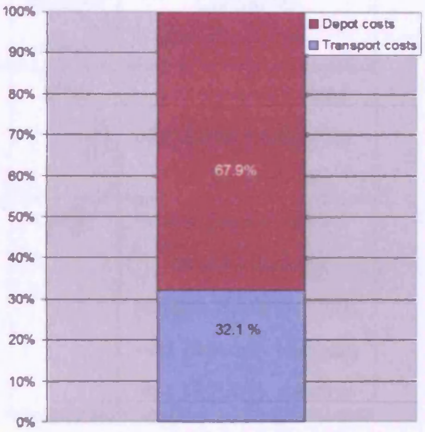
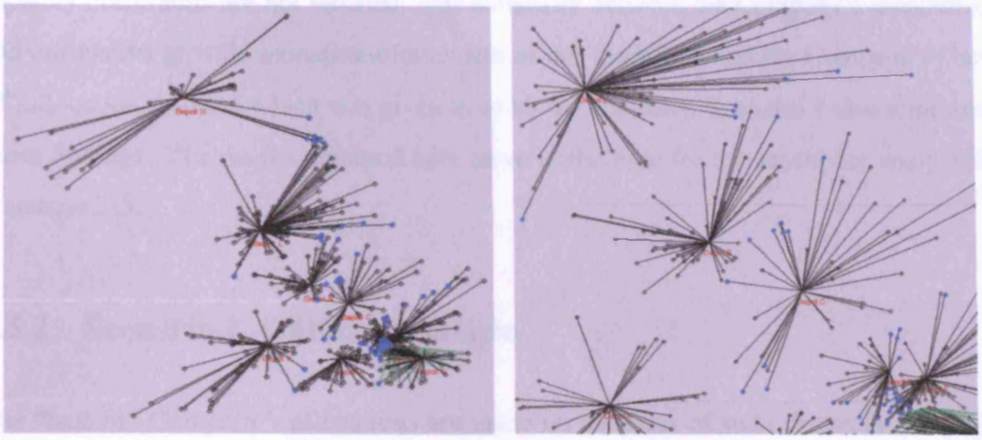


Figure 5.3: Overall depot and transport costs percentage for benchmarking scenario.

When our benchmarking allocation is compared to the "current" Company's allocation using the September 2009 data, the following results were produced. Table 5.2 compares the solution produced by CPLEX® with the actual allocation used by company using September 2009 data. The difference in solution is very small at 0.72%. Only 61 stores (11.7% of all stores) have different allocations, and this can be seen from the screenshots in Figures 5.4(a) and 5.4(b). The blue dots represent those stores which are allocated



(a) Overview (b) Close up

Figure 5.4: Comparison of benchmarking scenario to company's allocation.

Solution using SEPTEMBER 2009 data	Total cost (£)	% transport costs	% depot costs	Comments
CPLEX®	3,006,318	32.1	67.9	->61 stores allocated differently comparing to Company's.
Company's allocation	3,027,945	32.1	67.9	->not feasible if 90% volume capacity; feasible-if full capacity; ->0.72% diff. compare to CPLEX® solution

Table 5.2: Results for the September 2009 data .

differently, compared with the Company allocation. As can be seen from the figures, all the stores which are assigned differently, are located on the edges of the intersection with other depots which could easily win those stores with a slightly lower cost. Table 5.2 illustrates that the cost value between both scenarios is very similar and this is because we applied our cost model to the "best fit" allocation to calculate the actual company's allocation cost. The company provided us with the "best fit" allocation (not the cost solution but the allocation of stores) where they use an iterative approach in Microsoft Excel and in each iteration they reallocate stores manually to the minimum cost to ensure that the capacity constraints are not violated. The similarity between the Company's assignment and our results provide mutual reinforcement of our modelling and the Company's "best fit" allocation solution which was given to us by the company. Scenario 1 also reinforces those findings. The results obtained here serve as the base for the sensitivity analysis in scenarios 2-5.

5.5.2 Scenario 1. Uniform scenario

The "best fit" Company's allocations are made on the basis of stem distances (i.e., fuel related costs) only. On the other hand, our benchmarking allocations 5.5.1 depend on warehouse costs and time related transport costs as well as stem distances. While it is encouraging to note the similarities between the allocations made by the Company and by us using CPLEX® in Part 5.5.1, on the September 2009 data, we are clearly not comparing "like with like". For this reason we include Scenario 1 which effectively allocates stores to depot on the basis of fuel related costs (i.e., stem distances) alone, to make a fairer comparison with the Company allocation, and thus provide mutual validation for the models.

The uniform scenario uses the same costs and productivity rates across all depots and vehicles, and the result of this allocation is also used for comparison with the results from "best fit" company's allocations. The following average data was used: the average vehicle load (ambient=chill=produce=1243) with fuel related costs of £0.35 per km. DWR for each depot was set at 97.6 for ambient product, 100.13 for chill and 136.02 for produce. WHR was equal to £16.03 and THR was equal to £18.48 for each depot. Because the same rates are applied across all the depots, the depot related costs will be the same for each store, and so will the time dependent transport costs (e.g., drivers' pay), regardless of its allocation. Thus, just like the Company exercise, our optimization will be based on stem distances only, as these are the only variables used in the modelling.

The cost structure of the optimal solution for the uniform scenario consists of depot and transport costs. As expected, the majority of the costs in the solution are (once again) allocated to depot costs. Furthermore, there are only 44 stores for which the transporta-

Solution using SEPTEMBER 2009 data	Total cost (£)	% transport costs	% depot costs	Comments
CPLEX®	2,912,529	30.4	69.6	->54 stores allocated differently comparing to Company's.
Company's allocation	2,925,700	30.4	69.6	->not feasible if 90% volume capacity; feasible-if full capacity. ->0.45% diff. compare to CPLEX® solution.

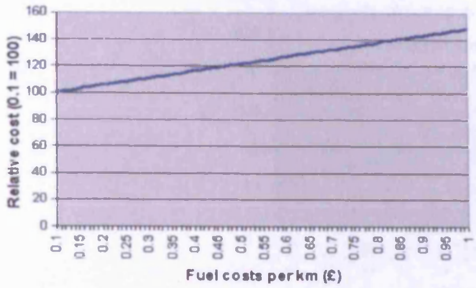
Table 5.3: Results for average scenario.

tion costs matches or exceeds 50% among the 520 stores. The overall percentage of depot costs is 69.6% with transport costs responsible for just 30.4%. Table 5.3 compares the optimum solution produced by CPLEX® with the actual allocation used by the Company. The difference in solution cost equates to 0.45%, which is almost negligible. These figures reinforce our confidence in our modelling approach and our solution quality. There are only 54 stores (10.4% of all stores) allocated differently compare to the Company's allocation.

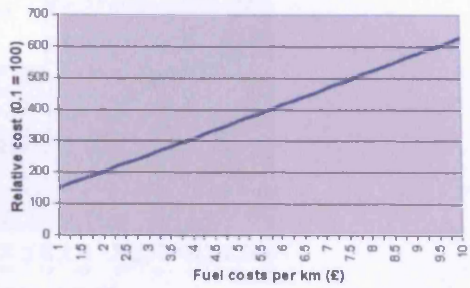
5.5.3 Scenario 2. Fuel scenario

The fuel-related scenario is used to understand the impact of fuel-related costs on the allocation of the stores to depots. Several experiments were performed with different values and steps: from £0.10 to £1.00 with step £0.01; from £1 to £10 with step £0.25; from £10 to £100 and with step £10 and at £500 for the final experiment. The analysis was performed on the September 2009 data. Only the fuel-related costs were changed and these were varied across all depots simultaneously.

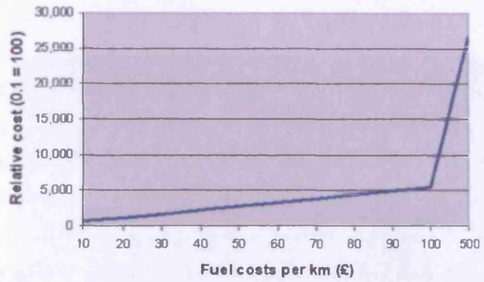
As expected, the overall costs increase linearly, because the fuel cost increases across all the depots at the same rate. Figures 5.5(a)-5.5(c) clearly show the increase in overall costs across all depots as the fuel rate increases for all the experimental ranges. In this chapter, we use a notation of a relative cost or distance on the y-axes to make the actual values



(a) Fuel related cost, range £0.10 - £1 (step 0.01)



(b) Fuel related cost, range £1 - £10 (step 10)



(c) Fuel related cost, range £10 - £100 (step 0.25) and £500

Figure 5.5: Total cost for a range of fuel related cost .

anonymous. For example, in Figure 5.5(a), on the y-axes we assign the relative total cost of the particular network configuration to 100 units of financial cost when the value of the fuel cost per km as equal to £0.10.

As the fuel rate increases, therefore the proportion of transport costs increases relative to the depot costs, which can be seen from Figure 5.6. When the fuel rate equated to £0.10, around 24% of the overall cost were allocated to transportation costs. As the fuel rate increased, the break point for cost allocation was at around £1.11 per km with transportation and warehousing costs equating to around 50% at this point.

For this scenario, the total distance travelled by the vehicles (Figure 5.7) decreases as the overall cost increase as a result of the changes in the fuel costs. This happens due to the increase in the proportion in the overall transport costs, therefore the optimized allocation

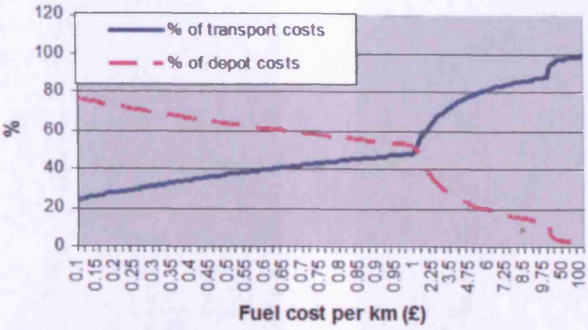


Figure 5.6: Proportion of transport costs to depot costs, fuel cost range from £0.1 - £500, overview.

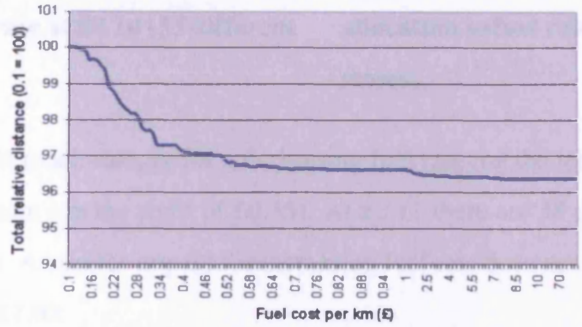


Figure 5.7: Total distance (km) travelled for fuel cost from £0.1 - £500 .

becomes slowly influenced mostly by the transportation costs as fuel costs increase. When the fuel rate is low, stores may be assigned to depots that are a long way away, if the depot cost is low enough. Figures 5.8 and 5.9 visualize the screenshots of the allocations where you can see the impact of the lowest and the highest fuel rates on the distribution pattern.

The purpose of the experiments in this scenario is to understand the impact of the fuel rate changes on the number of stores which are assigned differently compared to the benchmarking scenario of £0.35 per km. As a result of the analysis, it seems that a decrease in rate has a larger proportional impact on the number of stores allocated differently compared to an increase in the fuel rate, which can be observed from Figure 5.10. Although the number of stores which are differently allocated is relatively marginal, reaching a maximum of only 43 stores (8.3% of overall stores) at a fuel rate of £500.0 per km, the

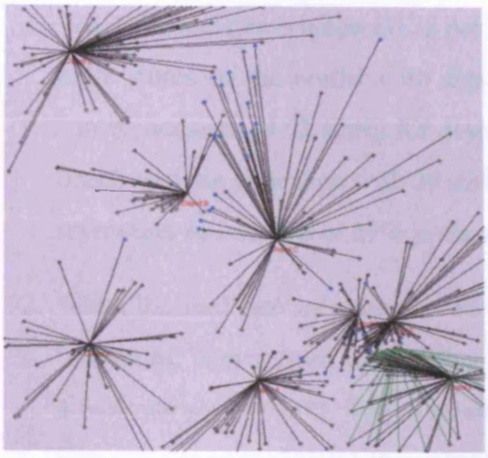


Figure 5.8: Comparison of benchmarking allocation to fuel rate at £0.10 (33 different stores).

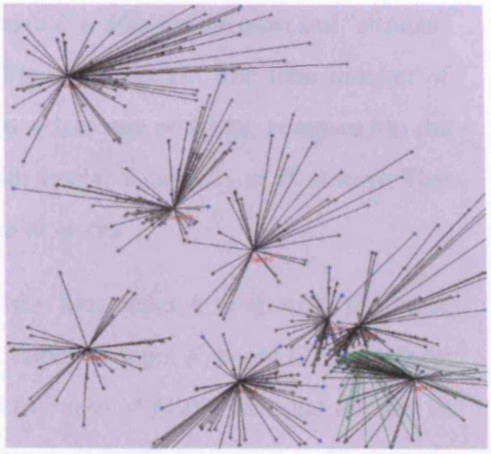


Figure 5.9: Comparison of benchmarking allocation to fuel rate at £1.00 (22 different stores).

slope of the curve is much steeper for a decreasing fuel rate (on the left of £0.35) than for an increasing fuel rate (on the right of £0.35). At £0.11 there are 38 different allocations (7.3% of all stores). A similar impact for increasing fuel rate does not occur until the cost has reached about £7.00.

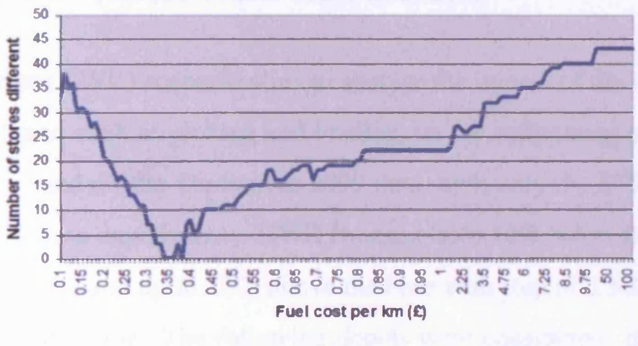


Figure 5.10: Number of stores differently allocated compared to the basic scenarios at £ 0.35 per km for fuel cost from £0.1 - £500.

An interesting observation comes from analyzing which depots have attracted more stores as a result of the fuel rate change. Figures 5.8 and 5.9 show the stores in blue colour which are allocated differently compared to the benchmarking allocation:

1. When the fuel rate is below £0.35 per km, depot C is affected the most and "attracts" extra stores on the border with depot B (Figures 5.8-5.9). The total number of stores increases to 62 stores for depot C for a fuel rate of £0.10, compared to the benchmarking allocation with 39 stores (with available capacity of 80 stores). That represents an increase of 59% in the number of stores.

2. When the fuel rate increases above £0.35 per km, depot A is affected the most, "attracting" stores from surrounding areas: depot E, depot F, depot G, depot D for a fuel rate up to £1.00. For example, at a fuel rate of £1.00, the total number of stores assigned to depot A increased to 57 stores (18.8%) compared to 48 stores in the benchmarking scenario.

As can be seen from our analysis, although the total number of stores allocated differently is quite small compared to the store total, the impact of even relatively small changes in the fuel rate could have a significant impact on particular depots as described above. Figures 5.8 and 5.9 visualize the impact of the changes.

5.5.4 Scenario 3 - Direct Work Rate scenario

The direct work rate (DWR) scenario aims to analyse the impact of the labour associated costs within the depot, such as picking and loading, on the assignment of the stores. The analysis was performed on the September 2009 data, with only the DWR rate changing for the purposes of these experiments. DWR changes from 10% below the benchmarking data across all products and up to 15% above the base with step of 2.5%, on five depots, taking one depot at the time. The following depots were considered: depot A, depot B, depot C, depot D and depot E. The analysis involved investigating the impact according to the following criteria:

- Number of stores assigned to a particular depot
- Number of stores 'lost' to the particular depot
- Number of stores newly assigned to the particular depot

- Total number of stores differently allocated across the entire network

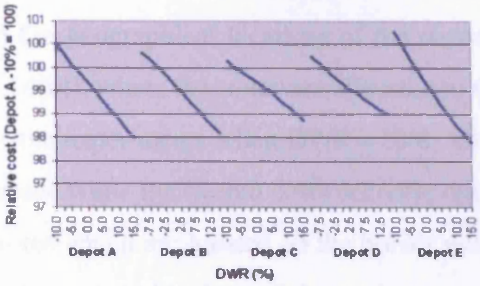


Figure 5.11: Total cost for five depots, DWR changes from -10% to +15%, step 2.5% for each depot.

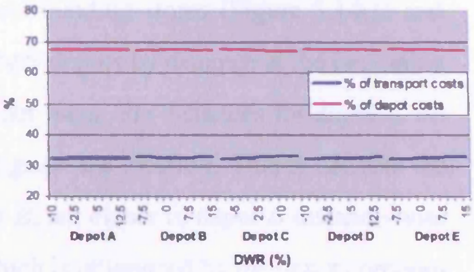


Figure 5.12: Proportion of transport costs to depot costs, DWR changes from -10% to +15%, step 2.5% for each depot.

The impact of the changes of DWR on the overall costs can be seen from Figure 5.11 for each depot. As expected, when the DWR rate increases, the overall costs decrease across all experiments. Around 67% of the costs in the optimum solution were allocated to the depot costs (see Figure 5.12).

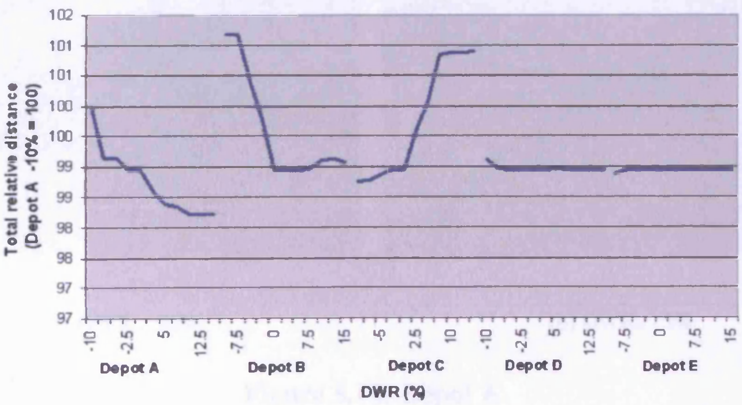
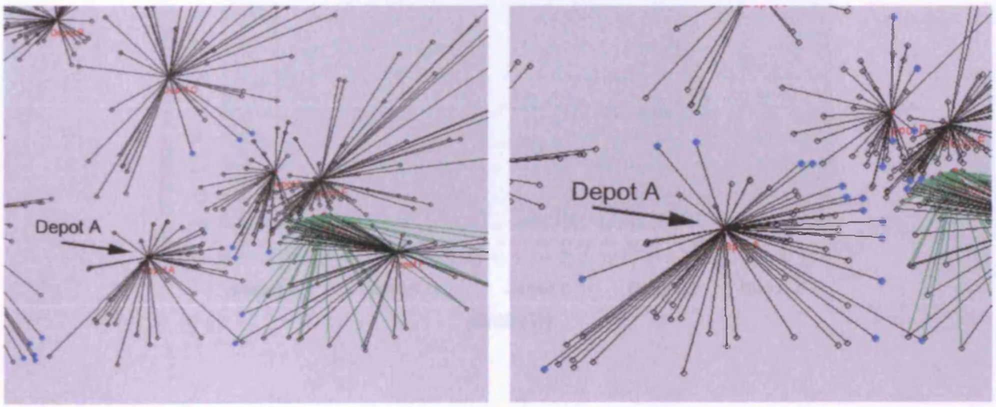


Figure 5.13: Total distance (km) travelled for a particular depot, DWR changes from -10% to +15% with step 2.5% in relation to benchmarking data.

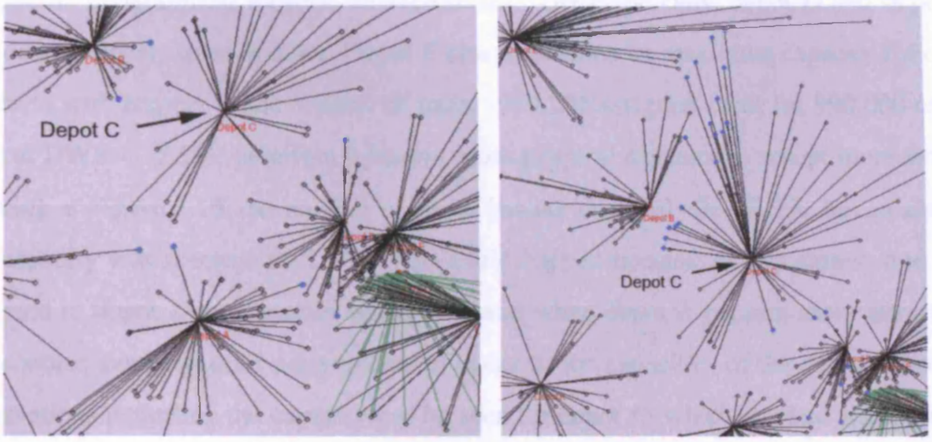
The changes in total distance travelled by the vehicles is different depending on the geographical locations of depots under investigation (see Figure 5.13) where the costs are influenced by the depot cost component. The distances for depot *A* and depot *B* decrease as well as the overall costs when DWR rate increase. The reassignment of the stores located on the border with other depots is balanced by lower costs and lower distances due to the geographical locations of the depots and surrounding stores (Figure 5.14(a) and 5.14(b)), where the stores are allocated to the furthers depots by distance at the beginning of the experiments when DWR=-10%. On the other hand, the distances for depot *C* increase while the overall costs decrease due its geographical location. This is because the stores which are located on the border with depot *B*, are closer to depot *B* distance-wise and reassigned to depot *C* due to the lower cost which is influenced by the depot component (Figure 5.15(a) and 5.15(b)). As can be seen from the Figure 5.13, the overall travelled distances for depots *D* and *E* do not change when the rate changes in DWR experiments. Strategically, both depots are located closely to each other and almost at full capacity, therefore any changes in rate will bring them to the full capacity very rapidly which is discussed in more detail below.



(a) DWR=-10% (b) DWR=15%

Figure 5.14: Depot A.

The number of stores assigned to a depot steadily increases in the cases of depot *A* and depot *B* (Figure 5.16). This pattern would be expected throughout the entire network because as DWR increases at particular depot, that facility becomes more attractive (cheaper) for surrounding stores. For depot *A*, the number increases to 63 stores when DWR=15% above compared to 48 stores for to the base rate at DWR=0% above, representing an increase of 31% in the number of stores. For depot *B*, the increase in number of stores



(a) DWR=-10% (b) DWR=15%

Figure 5.15: Depot C.

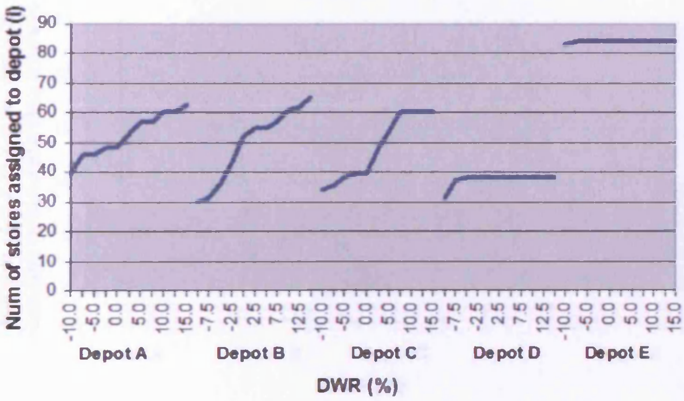


Figure 5.16: Number of stores assigned to a particular depot, DWR changes from -10% to +15% with step 2.5% in relation to benchmarking data.

equates to 25%, from 52 to 65 stores. A different pattern can be observed for depot *C*, depot *D* and depot *E* (Figure 5.16). For depot *C*, changes in the number of stores only had an impact up to DWR=7.5%. For depot *D*, changes in the number of stores did not occur after DWR= - 5% (i.e., 5% below benchmark rate), whereas for depot *E* changes in the number of stores did not happen after DWR= - 7.5%. There could be two possible explanations why store allocations fail to increase as the depot becomes more cost effective: the overall physical capacity of the depot was reached for one of the products or because of specific geographical location characteristics. Geographically, depot *D* and depot *E* are located closely to each other. Depot *E* almost reached its maximum capacity for chill products with respect to the number of cases (989,205 assigned cases for 990,000 capacity) at DWR= - 2.5%, therefore it has no more physical capacity to accept more stores. Looking at Figure 5.16, the number of stores has not changed after -7.5%, but an almost full capacity was reached at -2.5%. This could happen because of two stores: one was assigned to depot *D* and another to depot *E* and when depot *E* became more attractive, those stores swapped their assignments to better fit the capacities of the depots. Similar observations regarding the capacity can be seen for depot *D*, which reached almost maximum capacity for the number of cases and the number of stores assigned at DWR=-5% (38 assigned stores to capacity of 40 stores; 448,320 cases assigned to capacity of 450,000 cases for chill products).

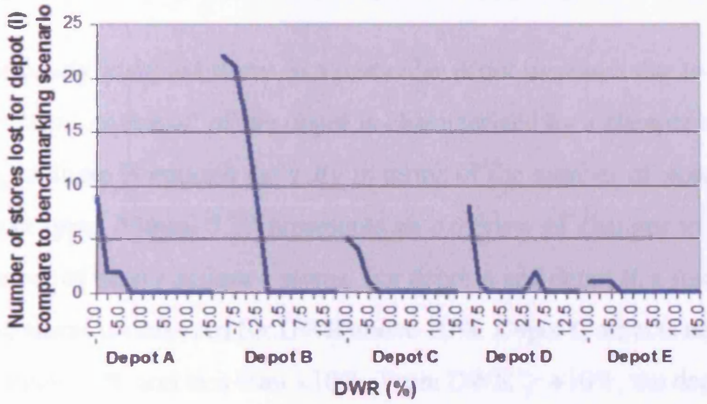


Figure 5.17: Number of stores lost at particular depot as a result of DWR changes from -10% to +15% with step 2.5% in relation to benchmarking data.

The number of stores lost at a particular depot decreases as expected (see Figure 5.17), because each depot becomes more attractive for stores due to DWR increasing. Depot *A*, depot *C* and depot *E* stop losing stores at DWR=-2.5%; depot *B* at 0% and depot *D* at -5%. All depots follow the same pattern except depot *D*, which has 2 lost stores when DWR= +5%. As well as loosing two stores to depot *E*, at the same time depot *D* acquired two new stores from depot *E* (Figure 5.18). As the stores are located on the border between both depots, their allocation is likely to be highly sensitive to small changes in costs or productivity.

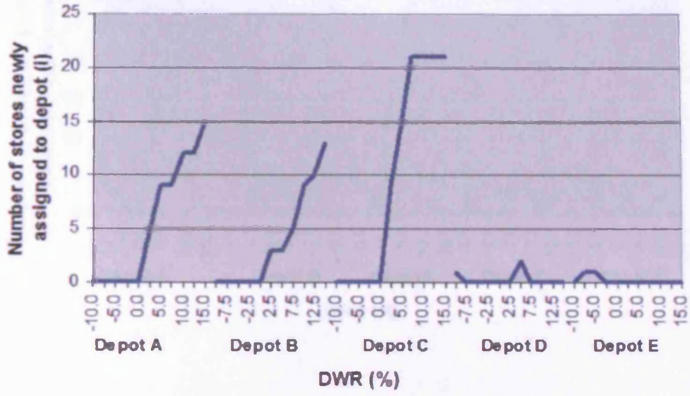


Figure 5.18: Number of stores newly assigned to a particular depot as a result of DWR changes from -10% to +15% with step 2.5% in relation to benchmarking data.

The number of newly assigned stores to a particular depot increases due to the depot cost decrease. The "attractiveness" of the depot is characterized by a cheaper assignment for stores, as long as there is enough capacity in terms of the number of stores and volume for each product type. Figure 5.18 represents an overview of changes to DWR and the associated number of newly assigned stores. For depot *A* and depot *B*, a steady increase in newly assigned stores is observed for DWR above +0%. Depot *C* attracts new stores when DWR is more than + 0% and less than +10%. From DWR $\geq$ +10%, the depot reaches the demand needed for ambient products of 584,149 cases with capacity of 585,000. Depot *D* has one new store when DWR= -10% and two new stores when DWR= +5%. The former comes from the store which is located on the border with depot *E* and is a result of optimization. As discussed above, depot *D* and depot *E* do not have enough spare capacity to attract new stores, therefore the pattern of newly assigned stores for them is different compared to other depots.

The total number of stores differently allocated compare to the benchmarking scenario can be seen in Figure 5.19. Depot *A* and depot *B* produce V-shaped curves, where the depot is either too "expensive" or more "attractive" to other stores; also a small impact of those changes can be observed for the other depots. Depot *C* has a similar curve shape with a slight difference: it reaches almost maximum capacity at DWR=+7.5% and after

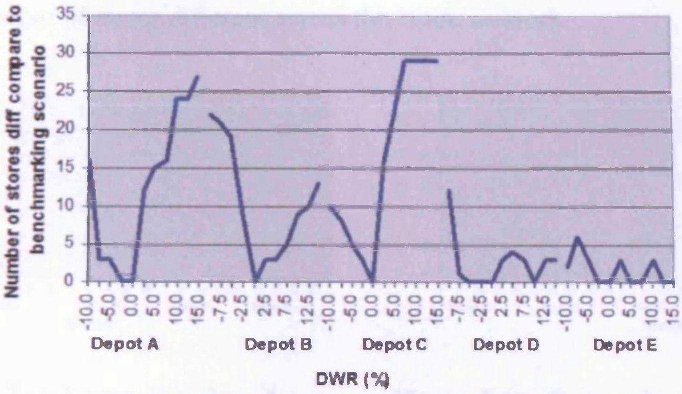


Figure 5.19: Total number of stores different as a result of DWR changes from -10% to +15% with step 2.5% in relation to benchmarking data.

that cannot physically attract more stores. Thus the changes in rate have no impact on other depots. The changes in DWR have no significant impact on depot *D* and depot *E*. Although a slight fluctuation can be observed in the number of stores assigned differently for both depots, this can be mostly attributed to the stores located on the border between the depots *H* and *L*. The only significant difference in the number of stores was for Depot *D* when DWR= -10%, which resulted in stores located on the border with other depots to be allocated to a much "cheaper" option.

5.5.5 Scenario 4. Warehouse Hourly Rate scenario

The warehouse hourly rate (WHR) scenario aims to analyse the impact of changes to the warehouse associated costs within a depot. The analysis was performed on specific depots with WHR changing from £10 - £33 with step of £1. The following depots were considered: depot *A*, depot *B*, depot *C*, depot *D* and depot *E*. The investigation was performed according to the following criteria:

- Number of stores assigned to a particular depot
- Number of stores lost at the particular depot
- Number of stores newly assigned to the particular depot

- Total number of stores different across the entire network

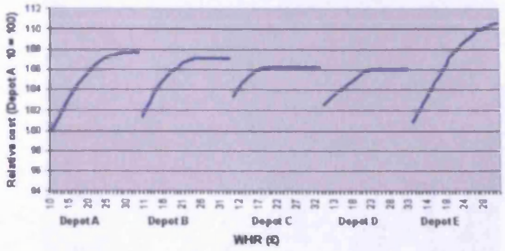


Figure 5.20: Total cost for five depots, WHR changes from £10 - £33, step 1 for each depot.

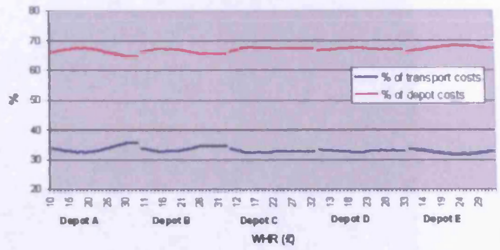


Figure 5.21: Proportion of transport costs to depot costs, WHR changes from £10 - £33, step 1.

The impact of the changes to WHR on the overall costs can be seen from Figure 5.20 for each depot. As expected, when the WHR rate increases for a particular depot, the overall costs increase across all experiments. Looking at the curve in Figure 5.20, we can see that for the four depots at some point, the curve stabilizes, which indicates that at a particular rate that depot becomes too expensive to be part of the network and is not presented as part of the final optimum solution. For more discussion regarding breakpoints please see below. Around 64-67% of the costs in the optimum solution were allocated to the depot costs (see Figure 5.21).

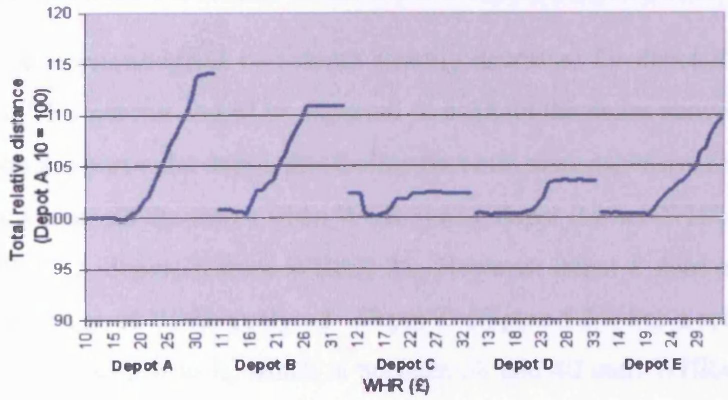
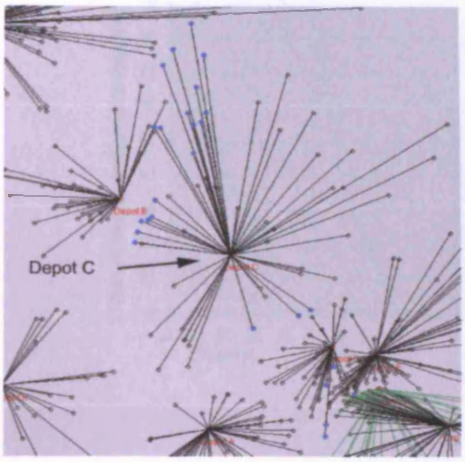
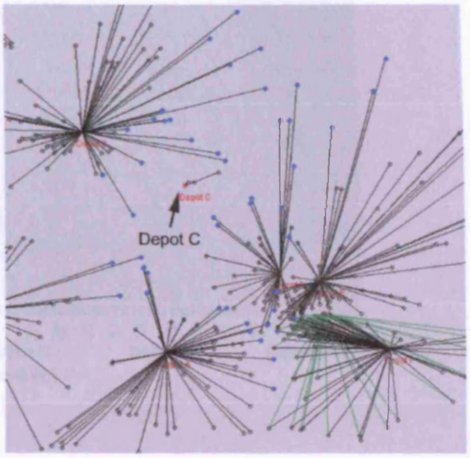


Figure 5.22: Total distance (km) travelled for a particular depot when WHR changes from £11 - £33, step 1 for each depot.



(a) WHR=£11



(b) WHR=£21

Figure 5.23: Depot *C*.

As overall costs increase for all depots, the total vehicle-km also increase but with a different gradient of increase (see Figure 5.22). WHR rate is the depot-based component of the cost therefore as it increase, the depots under investigation will be less attractive to the stores located on the edges, which could change their assignment to depots further away in terms of vehicle-kilometers. Some of the depots will "disappear" from the network due to the high WHR rate and their stores will be reassigned to the other depots. Figure 5.23(a)-5.23(b) visualize this impact on the depot *C* depot changes in rate.

The number of stores assigned to a depot steadily decreases for depot *B* and depot *C* (Figure 5.24). This pattern would be expected throughout the entire network because as WHR increases at a particular depot, that facility becomes more expensive for surrounding stores. Depot *A* loses all the stores when WHR $\geq$ £32, depot *B* when WHR $\geq$ £26, depot *C* when WHR$\geq$ 22, depot *D* when WHR$\geq$ 25. However, depot *E* does not lose all its stores over the range of WHR analysed. Depot *D* (Figure 5.24) has a relatively stable number of stores assigned to it, which is between 38 and 40 until WHR=£20. This is because the depot is serving up to full capacity for chill product (for example, assigned demand is 448,320 cases with available capacity of 450,000). Depot *E* also has a stable number of assigned stores of 83-84 until WHR=£19 due to the capacity constraints on chill products.

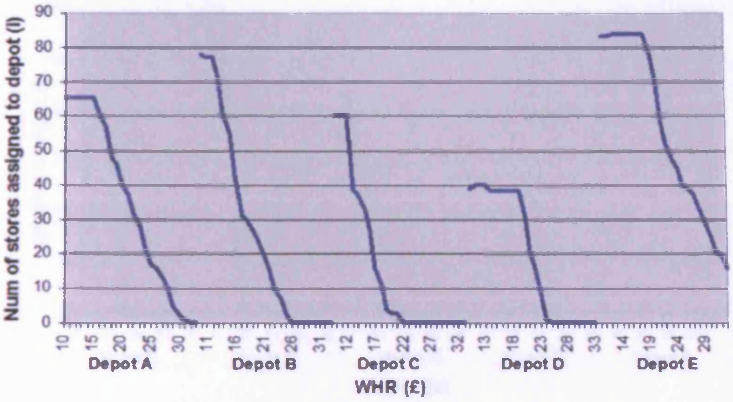


Figure 5.24: Number of stores assigned to a particular depot when WHR changes from £10 - £33, step 1 for each depot.

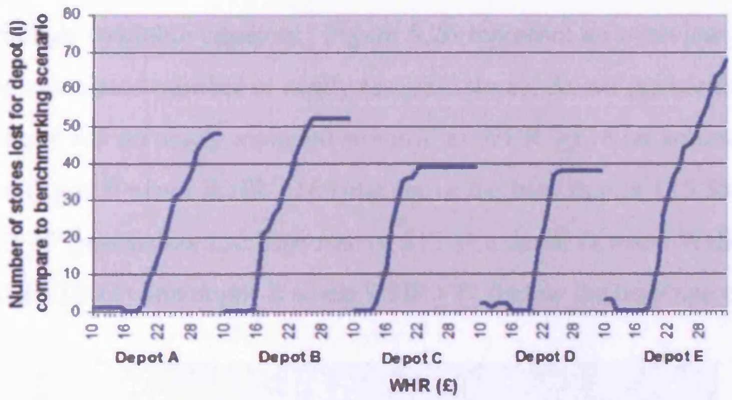


Figure 5.25: Number of stores lost to a particular depot, WHR changes from £10 to £33, step £1.

The number of stores lost at a particular depot increases as expected (see Figure 5.25), because each depot becomes less attractive for stores due to WHR increasing. As discussed earlier, we can see from Figure 5.25, depot A loses all the stores when WHR $\geq$ £32, depot B when WHR $\geq$ £26, depot C when WHR$\geq$ 22, depot D when WHR$\geq$ 25. On the other hand, depot E still has not lost all its stores when WHR=£33.

The number of stores newly assigned to a particular depot is characterized by the "attractiveness" of that depot where the cheapest assignment is used to allocate the stores, as long

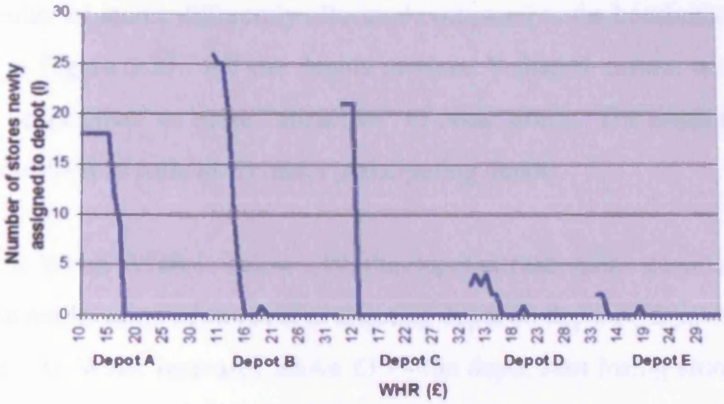


Figure 5.26: Number of stores newly assigned to a particular depot, WHR changes from £10 to 33 with step £1.

as there is enough available capacity. Figure 5.26 represent an overview of changes in WHR and the associated number of newly assigned stores. As we can see from the figure, for depot A, there are no newly assigned stores after WHR $\geq$£18 (at around the base rate of 17.98), for depot B when WHR$\geq$16 (just above the base rate of £15.56), for depot C when WHR $\geq$13 (just below the base rate of £13.11), depot D when WHR $\geq$16 (below the base rate of £18.11) and depot E when WHR$\geq$12 (below the base rate of £16.55).

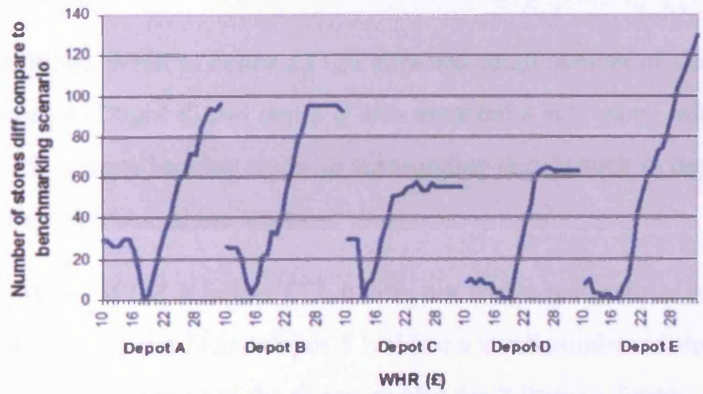


Figure 5.27: Total number of stores different , WHR changes from £10 to 33 with step £1.

The total number of stores differently allocated compared to the benchmarking scenario can be seen in Figure 5.27. All the depots produce V-shaped curves, where the depot is either too "expensive" or more "attractive" to other stores. The impact of the WHR increase is described as follows for each participating depot:

- Depot A. When WHR is below £19, the depot attracts more stores surrounding it, with the reallocation of stores also affecting depot D, depot E, depot L, depot G and depot F. As WHR increases above £19 - the depot start losing stores to depot K, depot L, depot C, depot D, depot E and depot B following the reallocation of the stores.

- Depot B. When WHR is below £15, the depot attracts surrounding stores from depot L and depot C. As WHR increases above £15, it starts loosing stores to depot C (majority of the lost stores), depot K, depot L, depot A, depot D, depot E and depot G/depot F until it loses all the stores.

- Depot C. When WHR is below £13, the depot attracts surrounding stores from depot B (majority of the lost stores) and depot D. When WHR is above £15, the depot start losing stores to depot D, depot B, depot A, depot K, depot E, depot L and depot F/depot G until it loses all the store.

- Depot D. When WHR is below £15, it attracted small number of stores from depot E and depot A. Depot C and depot E also attracted a very small number of stores. Above £15, it starts loosing stores to surrounding depots such as depot A, depot C, depot E until it loses all the stores.

- Depot E. When WHR is below £17, it does not attract many stores but other depots such as depot C, depot D and depot A had lost a small number of stores to depot E. Above £17, reassignment of the stores involve the following depots: depot D, depot A, depot C, depot B, depot K, depot L and depot G/depot F.

5.5.6 Scenario 5. Transport Hourly Rate Scenario

The Transport hourly rate (THR) scenario aims to analyse the impact of changes in transport associated costs for each participating depot. The analysis was performed on specific depots with THR changing from £10 - £33 with a step of £1. The following depots were considered: depot *A*, depot *B*, depot *C*, depot *D* and depot *E*. The investigation was performed according to the following criteria:

- Number of stores assigned to a particular depot
- Number of stores lost at the particular depot
- Number of stores newly assigned to the particular depot
- Total number of stores different across the entire network

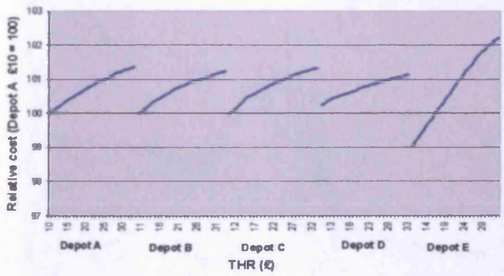


Figure 5.28: Total cost for five depots, THR changes from £10 - £33, step 1 for each depot.

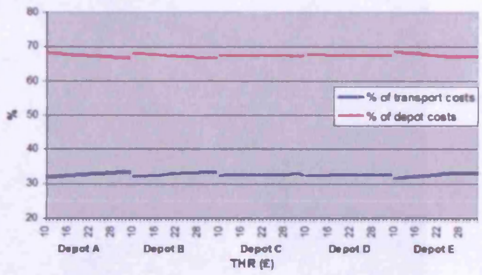


Figure 5.29: Proportion of transport costs to depot costs, THR changes from £10 - £33, step 1.

The impact of the changes of THR on the overall costs can be seen from Figure 5.28 for each depot. As expected, as the THR rate increases for a particular depot, the overall costs increase across all experiments. Looking at the curve in Figure 5.28, we can see that for all depots the curve is still growing as THR increases and does not stabilize, which indicates that there is still available capacity to assign to the stores. Around 32-33% of the overall costs in the optimum solution are allocated to transport costs (Figure 5.29).

The impact of the increasing overall cost has a different influence on the total vehicle-km depending on the geographical location, surrounding demand and available capacity

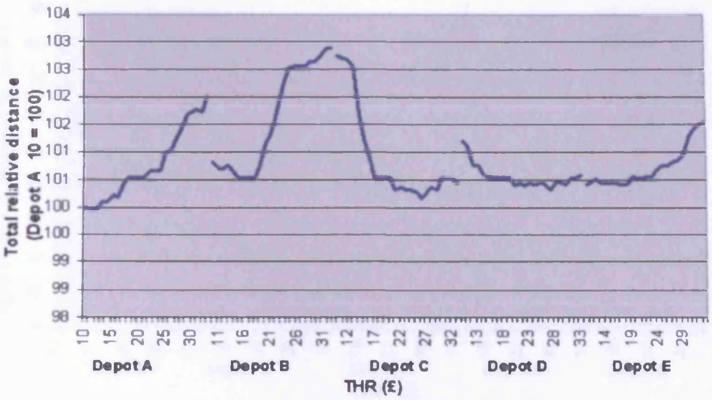
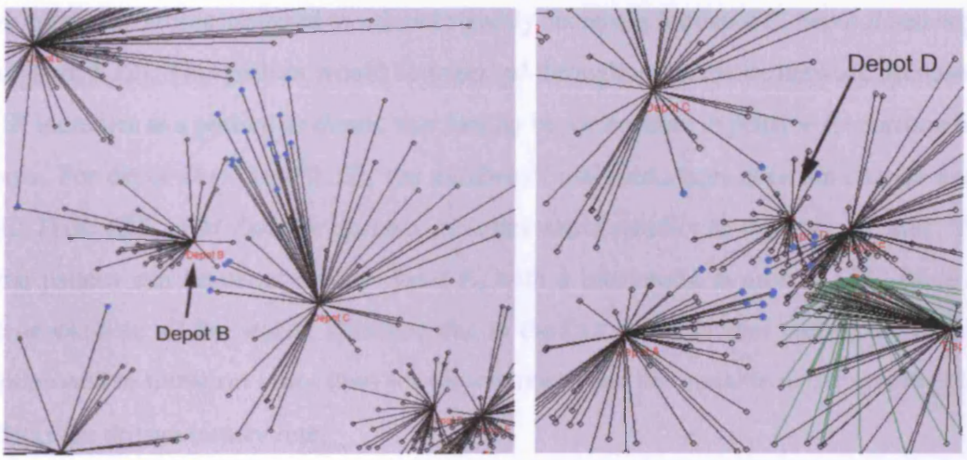


Figure 5.30: Total distance (km) travelled for a particular depot when THR changes from £11 - £33, step 1 for each depot.



(a) Depot B, THR=£27 (b) Depot D, THR=£31

Figure 5.31: Impact on allocation of THR changes.

(Figure 5.30). For depot A and depot B, the overall distance and associated costs increase as the THR rate goes up. Again, we can see a different picture for depot C, where the distance decreases as cost increases due to its geographical location. The reason for the decrease is that the stores surrounding depot B are re-assigned to depot B, with lower distances when the rate increases at depot C. Depot D and depot E do not have such sharp decreases/increases in overall distance as other depots. Figures 5.31(a)-5.31(b) visualize the impact on the some of the depots.

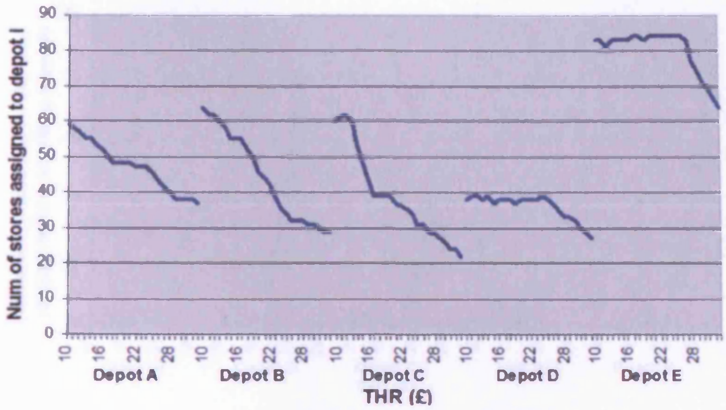


Figure 5.32: Number of stores assigned to a particular depot when THR changes from £10 - £33, step 1 for each depot.

The number of stores assigned to a depot steadily decreases for depot *C*, depot *B* and depot *C* (Figure 5.32). This pattern would be expected throughout the entire network because as THR increases at a particular depot, that facility becomes more expensive for surrounding stores. For depot *D* (Figure 5.32), the number of assigned stores does not change much until THR=£27, after that the number of stores starts steadily to decrease as well. The same pattern can be observed for depot *E*, with a breakpoint at around £28. None of the depots lose all the stores, which is due to the fact that a smaller part of the costs is apportioned to transport costs than warehouse costs, and the variable under investigation reflects the drivers hourly rate.

The number of stores lost at a particular depot increases as expected for depot *A*, depot *B* and depot *C* (Figures 5.33), because each depot becomes less attractive for stores due to THR increasing. Depot *D* and depot *E* have a V-shaped curve where each side represents that depot loses the stores to the surrounding depots. One of the explanations why this is happening is because of the geographical location of these two depots, which are located closely to each other. As well as attracting new stores, those depots also lose some of the stores to the surrounding depots, which could be due to the capacity constraints. The breakpoint for depot *D* would be at around £16-£18 and for depot *E* at £20-£23.

The number of stores newly assigned to a particular depot can be seen in Figure 5.34. The number of newly assigned stores decreases for depot *A*, depot *B* and depot *C*, as THR

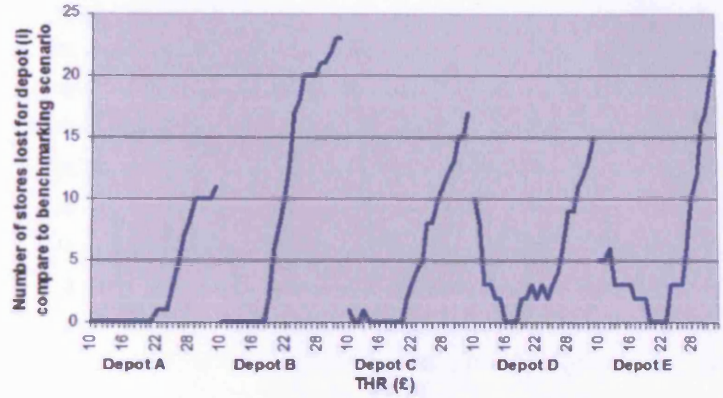


Figure 5.33: Number of stores lost to a particular depot, THR changes from £10 to £33, step £1.

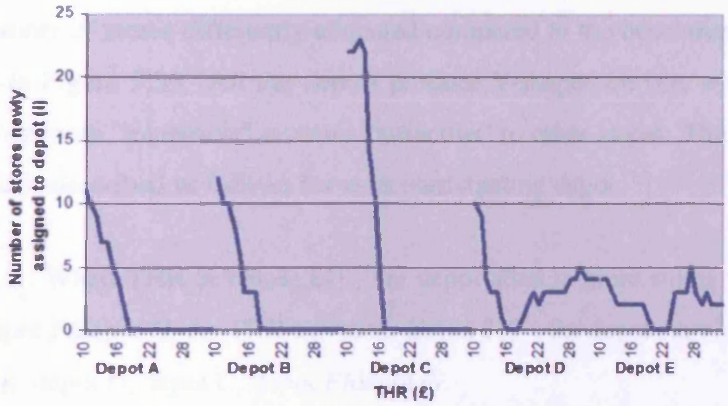


Figure 5.34: Number of stores newly assigned to a particular depot, THR changes from £10 to 33 with step £1.

increases. For depot D and depot E, the curve resembles a V-shape, with a breakpoint at which no new stores are assigned, and on both sides of that point there are newly assigned stores. We would expect the number of new stores to increase when THR is cheaper, but at the same time an increase in the number of new stores when THR is more expensive. This could happen because the stores are located closely to each other and as well as attracting new stores, those depots also lose some of the stores to the surrounding depots, perhaps due to the capacity constraints. The breakpoint for depot D would be between £16 and £18 and for depot E between £20 and £24.

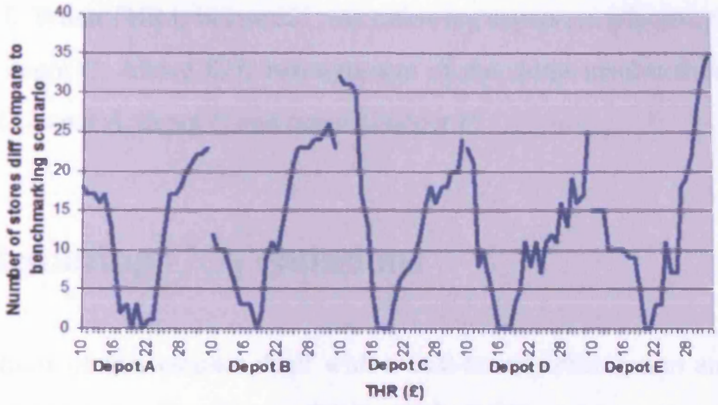


Figure 5.35: Total number of stores different , THR changes from £10 to 33 with step £1.

The total number of stores differently allocated compared to the benchmarking scenario can be seen in Figure 5.35. All the depots produce V-shaped curves, where the depot becomes either more "expensive" or more "attractive" to other stores. The impact of the THR increase is described as follows for each participating depot:

- Depot *A*. When THR is below £19, the depot attracts more stores from depot *D* and depot *F*/depot *G*. As THR increases above £19 - the depot start losing stores to depot *K*, depot *D*, depot *C*, depot *F*/depot *G*.

- Depot *B*. When THR is below around £18, the depot attracts surrounding stores from depot *L*. As THR increases above £18, it starts losing stores to depot *C* (majority of the lost stores) and depot *K*.

- Depot *C*. When THR is below £17, the depot attracts surrounding stores from depot *B* (majority of the lost stores) and depot *D*. When THR is above £17, the depot start losing stores to depot *A*, depot *D*, depot *B*, depot *E*, depot *K*, depot *L* and depot *F*/depot *G*.

- Depot *D*. When THR is below £17, it attracts stores from depot *E* and depot *B*. Above £17, it starts losing stores to surrounding depots such as depot *E*, depot *A*, depot *B*, depot *L* and depot *G*/depot *F*.

- Depot E. When THR is below £21, the following depots are affected: depot D, depot A and depot C. Above £21, reassignment of the stores involve following depots: depot D, depot A, depot C and depot G/depot F.

5.6 Calculating CO_2 emissions

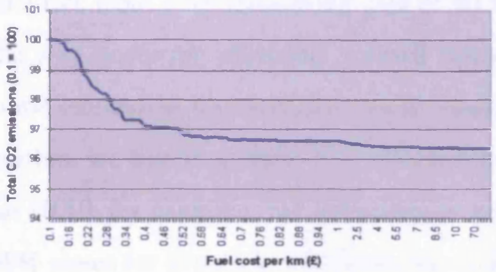
Previous sections of this chapter dealt with a cost-based optimization and the impacts of the changes in various key factors (DWR, THR, WHR and fuel-related costs) on the allocation, overall cost and distance. To calculate CO_2 emissions from the network, only the vehicle based emissions were considered. The emissions from running the depots were not taken into account because they would be considered as fixed emissions for each depot, which is the same as depots fixed costs which has not been taken into account either during modelling. Adding the fixed cost will not have an impact on the final allocation solution. The purpose of the current investigation is not to calculate total emissions from the network but to evaluate relevant factors which have an impact on the solution.

In this chapter we use a formulation to calculate CO_2 emissions of the particular network configuration from DEFRA [32] following the company's usual practice. We do not consider vehicle speed as part of calculations as we did in the Chapter 4 because the DEFRA formulation is an accepted guideline for UK business to calculate carbon dioxide emissions and it does take into account the diesel lorry type and percent of laden weight of the lorry.

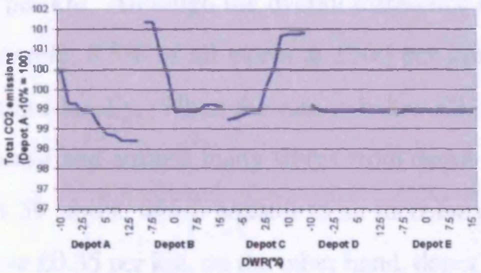
$$CO_2 = \sum_{i \in V_{DC}} \sum_{j \in V_C} \sum_{p \in P} (totDist_{ij} * fuelConsump * fuelFactor) x_{ij}, i \in V_{DC}, j \in V_C, p \in P \tag{5.10}$$

where $fuelConsump$ is the figure which was provided by the Company and is an average fuel consumption of a truck, $fuelFactor$ is a fuel conversion factor for a particular fuel type and $totDist_{ij}$ is the total distance travelled by the vehicles to depot i to satisfy a particular demand of a customer j which is a stem distance multiplied by 2. We used a figure of 0.33 litres per km for $fuelConsump$, which is the average figure over 12 months

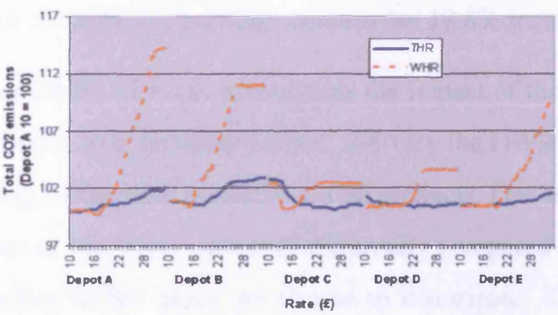
(April 09 - March 10) across all depots. For the $fuelFactor$ parameter, a fuel conversion factor of 2.63 kg/litre was used for diesel fuel. Figures 5.36(a)-5.36(c) visualize CO_2 emission results for all scenarios, where the shape of the curve is the same as the curve for the total vehicles-km travelled.



(a) Fuel scenario



(b) DWR scenario



(c) THR and WHR scenarios

Figure 5.36: CO_2 emissions for all scenarios.

5.7 Discussion of results and main conclusions

The main conclusion of the analysis is that the current network configuration is robust to small fluctuations in the rates investigated due to spare capacity in the depots. Furthermore, the findings show that the effects of making changes to various costs or productivity parameters vary considerably and depend on the geographical location and available capacity of the depots involved as well as on the local topology of the network. In some cases relatively large changes to variables associated with an individual depot have very little impact, even locally, whereas in other situations similar changes when applied to a different depot may result in a large number of reallocations, throughout the network.

The fuel-based scenario looks at the impact of changes to fuel-related costs on the allocation of the stores. Our analysis investigates the impact of increasing and decreasing the fuel rate from a benchmarking cost of £0.35 per km. Although the overall difference in the way stores are allocated is small (for example, 8.3% of all stores at £500 per km), some interesting observations can be made more locally. When the rate is below £0.35 per km, we find that depot *C* is affected the most and attracts many stores from depot *B* (at £0.10, for example, the depot has an extra 39 stores, which equates to an increase of 59% stores for depot *C*). When the rate is above £0.35 per km, on the other hand, depot *A* is affected the most, followed by depot *E*, depot *F*/depot *G*, and depot *D*. For example, at a fuel rate of £1.00, the total number of stores assigned to depot *A* increased to 57 stores compared to 48 stores in the benchmarking scenario (an 18.8% increase).

The direct work rate (DWR) scenario investigates the impact of the work rate efficiency on the allocation of stores to a particular depot. We vary the DWR from -10% to +15% from the benchmarking values with step 2.5% for all products. Our analysis shows that the results for the number of the stores allocated differently compared to the benchmarking scenario differ according to the depot we choose to investigate. Depot *A* and depot *B* demonstrate predictable patterns in terms of the number of stores assigned: the higher DWR (i.e., productivity)- the more attractive a particular depot becomes for surrounding stores. Depot *C* follows a similar trend but reaches its maximum capacity early at DWR= +7.5%, and at this point it cannot physically accept any more stores. For depot *D* and depot *E*, only slight variations in store allocations result from the imposed changes to DWR with one exception: if DWR is decreased by 10% for depot *D* - this makes the depot too expensive, and as a result it 'loses' some of its stores.

The warehouse hourly rate (WHR) scenario analyses the impact of the changes in the warehouse associated costs within a particular depot under investigation. An increase in WHR would imply that as the rate increases at a particular depot, that facility will become less attractive/ more expensive for the surrounding stores. Indeed this pattern of steadily decreasing numbers of stores assigned to a depot is observed for almost all depots, until a particular rate when the depot looses all the stores. Depot A loses all its stores after WHR $\geq$ £32, depot B when WHR $\geq$ £26, depot C when WHR$\geq$ 22, depot D when WHR$\geq$ 25. On the other hand, depot E maintains some stores over the complete range of WHR analysed in our study. Our findings suggest that in situations where depot costs are high in comparison to transport costs, depots tend to 'lose' all their stores more readily as WHR increases. On the other hand, the total number of stores allocated differently compared to the benchmarking scenario produces a V-shaped curve for each depot under the investigation. In the centre of the 'V' we have the benchmarking case that gives the expected number of stores allocated to a depot, thus representing zero deviation from expectation. To the left of this, stores are gained by a depot as WHR is decreasing making the depot cheaper and more "attractive". To the right, WHR is increasing, making a depot more "expensive". In this case, stores will be lost from the depot (Figure 5.24).

The transport hourly rate (THR) scenario analyses the impact of changes in the transport associated costs within a particular depot under investigation. As THR increases for a particular depot, it not surprisingly becomes less attractive for the surrounding stores. However, changes in THR do not seem to have quite such a drastic effect as we observed when studying the WHR scenario. As mentioned above, depot costs generally tend to be higher than transport costs. Thus it is perhaps not surprising to note that the network is more sensitive to changes in warehouse costs than it is to changes of similar magnitudes to transport costs. Nevertheless, in THR scenario, as THR increases, the number of stores assigned steadily decreases for depot A, depot B and depot C depots. For depot D, the number of stores does not change a great deal until THR=£27, after which the number of stores starts steadily to decrease as well. The same pattern can be observed for depot E, with a breakpoint at around £28. The total number of stores allocated differently compared to the benchmarking scenario produces similar V-shaped curves to those observed for changes in WHR, where the depot either 'loses' stores as the depot becomes too expensive, or gains stores more as it becomes more cost effective.

The investigation into allocation allowed us to cover different scenarios for network design. In some cases the depot costs dominated the cost function and in other the transportation element was the greatest. Having real store and depot locations allowed an insight of real-world design. The geographical locations have a drastic impact of the behavior on the allocations, where the overall cost of the allocation could increase as the changes in various key factors occur and at the same time the overall distance travelled may decrease. Without detailed analysis of the network, it would be very hard to predict that behaviour.

In an attempt to generalise results presented in this study, the findings show that effects vary considerably and depend on the geographical location and available capacity of the depots involved as well as on the local topology of the network. In some cases relatively large changes to variables associated with an individual depot have very little impact, even locally, whereas in other situations similar changes applied to a different depot may result in a large number of reallocations, throughout the network. The decision maker needs to be aware that changes in one facility will have a ripple impact on other serving facilities as stores which are located mainly on boundaries will be reallocated. Also, the proximity of depots need to be considered because it will have a direct impact on which depots are likely to be affected. The results presented in this chapter confirmed findings by Lalwani *et al.* [125] where authors undertaken the sensitivity analysis on the strategic network design (locatio-allocation) that the optimum design is overall less sensitive to transport cost changes due to its smaller proportion contribution to the overall logistic costs. On the other hand, in this research we also show that changes in transport related costs could have a significant impact on the allocation of individual depots.

5.8 Summary

This chapter presents a case study based on Sainsbury's data with multiple products, where the impact of changes of key variables such as fuel costs, transport and warehouse associated costs is analysed based on the allocation of the stores to depots. The main conclusion of the analysis is that the current network configuration is robust to small fluctuations in the rates investigated due to spare capacity in the depots. This seems to reflect current practices company deploy to ensure that there is enough spare capacity to deal with uncertainty.

Chapter 6

Optimizing Dual Objectives for a Capacitated Allocation Model Using Sainsbury's Data

6.1 Introduction and Motivation

In the previous chapter a cost-based capacitated allocation model based on Sainsbury's data was investigated, for which the optimization was undertaken using a single objective function based on cost. The present chapter extends the study to include distance-based optimization. Given current concerns about the environment, we consider that a reduction in the total distance travelled in a distribution network will be likely to equate with a reduction in greenhouse gas emissions, such as CO_2, thus providing a simple way to explore the trade-off between "cost versus carbon emissions".

We begin with a simple distance only optimization of the network, and compare these results with the "cost only" model from the previous chapter, observing any differences between the distribution networks optimized on the two different criteria: cost and distance/environmental impact. We wish to ascertain whether cost and distance based optimization on our data set will produce the same (or similar) solutions in terms both of the allocation and the objective values for the capacitated allocation problem.

Next we try a slightly more sophisticated approach: we combine the two objectives, cost and distance, in a simple weighted sum, and by varying the weights we produce a set of trade-off solutions that balance economic and environmental objectives. In this way we are able to solve the allocation problem and offer the decision-makers alternative solutions from which to make a choice. We also apply multi-objective evolutionary algorithms in Chapters 8 and 9 for multi-objective facility location problems.

Although there is clearly a close relationship between cost and distance, insofar as travelling greater distances requires more fuel, and more fuel costs more money, the relationship is not as straightforward as it may seem, because there are many additional elements included in the cost model, that may make it cost effective to travel further in some circumstances. The cost-based formulation includes both a transportation and a warehousing component, and staff related costs normally form a significant part, associated with picking and loading the products, as well as driving the delivery vehicles. Some warehouses are more efficient than others, and staff costs vary according to the location, for example they are higher in the South East of England than elsewhere.

The work in this chapter was undertaken only on the benchmarking values to solve the allocation problem at a fuel-related cost of $\pounds 0.35$, which was discussed in more detail in Chapter 5. The data we use is the September 2009 data used in the previous chapter. Recall that Sainsbury's secondary distribution network consists of 10 depots and 520 stores, where each store has a certain demand for three different product types, and the depots have capacity constraints on the number of cases and also the maximum number of stores which it can serve. The data for the demand is averaged across a 6 month period and reflects the market situation during 2009.

6.2 Distance-based optimization

The cost-based allocation model described in Chapter 5 was modified for the distance-based formulation as follows:

Mathematical formulation

We are given a set of stores with known demands for different types of product. We are also given a set of open facilities with capacity constraints for maximum volumes (numbers of cases) and maximum numbers of store allocation capacities, for each product type at each facility. The objective is to minimize the total distance travelled each week, assigning each of the stores to exactly one facility, adhering to all the capacity constraints. This is a single source problem, in which each store obtains all of its product from a single facility.

Glossary

V_{DC}	set of depots
V_C	set of stores
P	set of products
d_j^p	demand of store j of product p, $j \in V_C$, $p \in P$
v_j^p	number of vehicles needed to deliver demand of store j of product p, $j \in V_C$, $p \in P$
l^p	average vehicle load of product p, $p \in P$
f	distance related cost (fuel cost) per km
q_i^p	capacity of cases of facility i of product p, $i \in V_{DC}$, $p \in P$
n_i^p	number of stores assigned of facility i of product p, $i \in V_{DC}$, $p \in P$
tc_{ij}^p	transportation cost of total demand d_j^p of product p of store j from facility i
dc_{ij}^p	depot cost of total demand d_j^p of product p of store j from facility i
$dist_{ij}$	distance between store j from facility i
$time_{ij}$	time to travel between store j from facility i
THR_i	transport hourly rate (drivers cost) for facility i
WHR_i	warehouse hourly rate for facility i
DWR_i^p	direct work rate for facility i for product p
x_{ij}	is the decision variable for the problem, $x_{ij} = 1$, if store j is allocated to facility i, and 0 otherwise

Minimize

$$\sum_{i \in V_{DC}} \sum_{j \in V_C} \sum_{p \in P} (dist_{ij} * 2 * v_j^p)\ x_{ij} \tag{6.1}$$

subject to

$$\sum_{i \in V_{DC}} x_{ij} = 1, \forall j \in V_C \tag{6.2}$$

$$\sum_{j \in V_C} d_j^p x_{ij} \leq q_i^p, \forall i \in V_{DC}, \forall p \in P \tag{6.3}$$

$$\sum_{j \in V_C} x_{ij} \leq n_i^p, \forall i \in V_{DC}, \forall p \in P \tag{6.4}$$

$$x_{ij} \in \{0, 1\}, i \in V_{DC}, j \in V_C \tag{6.5}$$

by setting:

$$v_j^p = d_j^p / l^p, i \in V_{DC}, p \in P \tag{6.6}$$

where formulation (6.1) aims to minimize the total distance to satisfy the total demand of all the stores, and constraints (6.2) with (6.5) guarantee that the demand for each store must be satisfied by one depot. Constraints (6.3) and (6.4) ensure that the capacity constraints for the facilities for each product type are not violated and (6.5) specifies that allocation is indivisible for the decision variable. Formulation (6.6) calculates the number of vehicles needed to satisfy demand of store j and product p.

To calculate the total cost, the following equation was used on the resulted allocation from distance-based optimization in formulation 6.1:

$$\sum_{i \in V_{DC}} \sum_{j \in V_C} \sum_{p \in P} (tc_{ij}^p + dc_{ij}^p) x_{ij} \tag{6.7}$$

by setting:

$$tc_{ij}^p = (dist_{ij} * f + time_{ij} * THR_i) * 2 * v_j^p, i \in V_{DC}, j \in V_C, p \in P \tag{6.8}$$

$$dc_{ij}^p = (d_j^p / DWR_i^p) * WHR_i, i \in V_{DC}, j \in V_C, p \in P \tag{6.9}$$

	Optimization based on costs	Optimization based on distance
Cost (£)	3,006,318.25	3,019,889.81
Distance (km)	1,418,938.40	1,405,138.95
Difference in cost	0	13,571.56
% Difference in cost	0	0.45
Difference in distance	0	-13,799.44
% Difference in distance	0	-0.97

Table 6.1: Results for the benchmarking scenario with fuel-related costs at £ 0.35 .

6.2.1 Results and analysis

The optimum allocation of the stores to the depots is based on minimizing overall distance. This involves calculating the total distance travelled by vehicles to satisfy the particular demand of the store and this is multiplied by two because the network uses stem distances and the vehicle needs to get back to the original depot. The total distance travelled is calculated for all stores and added together. Therefore, the distance-based optimization is purely based on distances and any cost fluctuations in the market will have no impact on the optimum solution.

Table 6.1 shows the results of the optimization by cost and by distance when optimized separately. The cost value is slightly increased in the distance-based formulation compared to the cost approach by 0.45%, which equates to a monetary value of £ 13,571.56. On the other hand, the distance is actually decreased by 13,799.44 km (0.97%) as was expected for the optimization based on distance. Another important point to take from this analysis is that if we consider the percentages when looking at the difference between the cost and distance based formulations, the difference seems insignificant at 0.97% decrease in total km travelled. In reality however, this would nevertheless equate to 13,799.44 km saved per week which is the significant amount of the CO_2 emissions from the transportation per week and across the year.

A comparison between the allocations of stores to distribution centres for the cost and the distance-based optimizations show that different allocations are made and different totals for cost and distance are produced. This is not surprising as there is an obvious difference between two formulations for this particular case study. The economic model has transportation and warehousing models combined together, and the transportation model has distance-related and time-related elements. Recall from the previous chapter that the cost structure for this particular case study is heavily influenced by the warehousing component which contributes a higher proportion of the overall costs than the transportation component. Additionally, the transport costs include driver wages which are time related. On the other hand, the optimization based on the distance only considers a transportation model with distance-related costs only. This study supports the idea that it is possible to balance economic and environmental objectives using multi-objective optimization techniques and one of those approaches, a simple weight-based technique is discussed in the next section.

6.3 Multi-objective weighted sum approach

To allow a decision maker to evaluate the different trade-off solutions between cost and distance based optimization in our allocation problem, we used a multi-objective (MO) optimization approach. When there is more than one objective considered, the problem will have multiple distinct goals, in our case this involves minimizing costs for one objective and minimizing distance for the other objective:

$$\begin{aligned} Minimize\ f_1 &= \sum_{i \in V_{DC}} \sum_{j \in V_C} \sum_{p \in P} (tc^p_{ij} + dc^p_{ij}) x_{ij} \\ Minimize\ f_2 &= \sum_{i \in V_{DC}} \sum_{j \in V_C} \sum_{p \in P} (dist_{ij} * 2 * v^p_j)\ x_{ij} \end{aligned} \tag{6.10}$$

$$subject\ to:$$

$$\sum_{i \in V_{DC}} x_{ij} = 1, \forall j \in V_C$$

$$\sum_{j \in V_C} d_j^p x_{ij} \leq q_i^p, \forall i \in V_{DC}, \forall p \in P$$

$$\sum_{j \in V_C} x_{ij} \leq n_i^p, \forall i \in V_{DC}, \forall p \in P$$

$$x_{ij} \in \{0, 1\}, i \in V_{DC}, j \in V_C$$

There exist many different methods for dealing with multiple objectives which are discussed in detail in Chapter 3. The approach we use here is a simple weighted-sum method. This is a widely used approach in multi-objective optimization due to its simplicity [31]. The technique scalarizes a set of objectives by multiplying each objective with a user-supplied weight [31] and the formulation of the technique is presented in the equation (6.11). In our allocation problem, we have the dilemma of balancing two objectives: minimizing overall costs and minimizing total distance for the allocation of stores to depots. The objectives have different units: £ and km, with different numerical ranges, making it difficult to choose appropriate weights to control the relative contribution of each objective to the weighted total. Therefore, we normalize the objectives to bring them so that each one typically produces values between 0 and 1. The formulation of the objective function can be seen as a sum of the weighed normalized objectives, which converts the problem into single-objective optimization problem:

$$F = w_1 f_1^* + w_2 f_2^* \tag{6.11}$$

The two weights w_1 and w_2 are chosen in such way where one weight is independent and the other one is calculated by simple subtraction. Therefore, the sum of the weights is equal to 1, where $\sum_{m=1}^{M} w_m = 1$. To convert each objective f_1 and f_2 to a single normalized value which is used for the composite objective function (6.11), the following procedure was used for each function, which generates a number between 0 and 1:

1. Find $minVal$ value which represent the lowest number in the data set.

2. Find $maxVal$ value which represent the highest number in the data set.

3. Calculate the normalized objective value for each store to each depot: $normValue = abs((aValue - minVal)/(maxVal - minVal))$, where $aValue$ is the corresponding value used for the optimization function, either total cost for all products from a store to the depot or the overall distance.

As a result of the above procedure after applying weights to the normilized values, a matrix of the normalized values is produced, for which each element of the matrix correspond to the value of the assignment between each store and each depot.

The experiments were conducted using Java 2 and CPLEX® optimization engine, on a PC with Intel Xeon E5345 and 4 GB RAM.

6.3.1 Results and analysis

The modelling and evaluation of the results was carried out on the allocation of stores to depots with fuel-related costs at £ 0.35. As discussed in the previous section, the cost and distance values were normalized to the same units between 0 and 1, then multiplied by the appropriate weight and added together to get the total objective value.

Table 6.2 displays the results of the optimization based on different weights for cost (w_c) and distance (w_d). As we can see from this table, the results are compared to the optimization purely based on costs only. Weighting both objectives allowed us to generate trade-off solutions and understand the relationship between cost and distance to effectively minimize environmental impact from the distance related emissions without a detrimental impact on the financial objective. For example, balancing cost and distance objectives with weights $w_c = 0.5$ and $w_d = 0.5$ allowed us to reduce total vehicle-km travelled by around 10,207 km by increasing cost by only £ 1,576 a week compare to the optimization based only on costs. This equate to around 530,764 km a year (52 weeks), which contributes to a significant amount of CO_2 emissions in a year. If the decision maker only considers the percentage increase, the reduction of 0.72% in km travelled appears insignificant compared to the number of km saved of 10,207. Therefore, looking at only % increase/decrease in the trade-off solutions may hide valuable information on the impact

Weight for cost w_c	Weight for distance w_d	Obj. (Normal. value)	Cost (£)	Difference in cost	% diff. for cost	Distance (km)	Difference in distance	% diff. for distance
0	1	17.62	3,019,889.81	13,571.56	0.45	1,405,138.95	-13,799.44	-0.97
0.1	0.9	21.20	3,014,219.61	7,901.36	0.26	1,405,622.87	-13,315.53	-0.94
0.2	0.8	24.77	3,011,532.45	5,214.20	0.17	1,406,213.11	-12,725.29	-0.90
0.3	0.7	28.34	3,009,836.96	3,518.72	0.12	1,406,991.03	-11,947.37	-0.84
0.4	0.6	31.90	3,009,178.52	2,860.27	0.10	1,407,438.55	-11,499.85	-0.81
0.5	0.5	35.46	3,007,894.59	1,576.35	0.05	1,408,731.10	-10,207.29	-0.72
0.6	0.4	39.02	3,007,894.59	1,576.35	0.05	1,408,731.10	-10,207.29	-0.72
0.7	0.3	42.58	3,007,467.07	1,148.83	0.04	1,410,159.03	-8,779.37	-0.62
0.8	0.2	46.13	3,007,102.90	784.65	0.03	1,411,761.46	-7,176.94	-0.51
0.9	0.1	49.68	3,006,607.62	289.37	0.01	1,415,033.40	-3,904.99	-0.28
1	0	53.22	3,006,318.25	0	0.00	1,418,938.40	0	0

Table 6.2: Results of the optimization with different weights for w_c and w_d which compared to the optimization purely based on costs (w_c =1, w_d=0) .

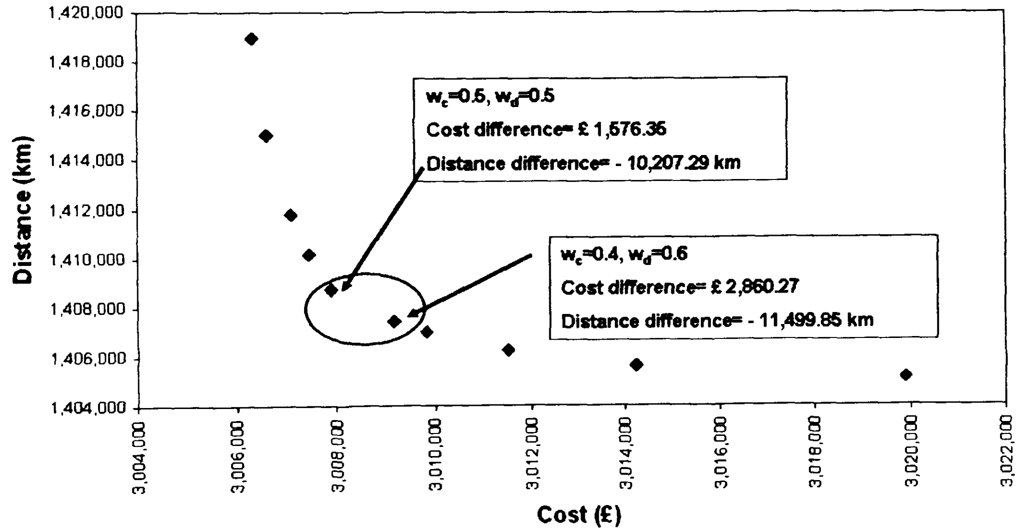


Figure 6.1: Trade-off solutions for the allocation problem.

on the overall vehicle-km. Figure 6.1 visualizes the different choices for the allocation which are available to the decision-maker.

Figure 6.1 visualizes the non-dominated solutions obtained by the simple weighted - based approach which are presented in the Table 6.2 and discussed earlier. This allows the decision-maker to see straight away good compromise solutions. For example, solutions which are circled in the figure have relatively low distance travelled just before the curve steepens towards high overall distance travelled. The good compromise solutions were already discussed earlier where a large number of the kilometers could be saved for a relatively small economic cost.

To conclude, the analysis shows that single objective optimizations based either on cost or distance, generate different results with different allocations, costs and total distances. Even though these differences were relatively small percentagewise, we note that nevertheless significant savings can be made in reducing the total distance travelled, at a very small cost. A simple weighted sum approach gave the desirable trade-off solutions between both objectives. When comparing the trade-off solutions to the cost-based optimization, we can see that even a small decrease in distance (0.72%) equates to around 10,000 km a week. This decrease in vehicle-km also equate to the small increase in costs at around £ 1,500. If we only used the analysis based on percentages, there would have been a tendency to underestimate the real impact on the reduction in distance. This approach allows us to generate a good set of compromised solutions and analyse their impact.

6.4 Summary

In this chapter we extend our Sainsbury's case study to focus on balancing the economic costs versus the environmental impact of transport, using a simple distance-based model as a rough assessment of the environmental impact. Firstly, we optimized on distance alone and compared our results with those obtained previously optimizing economic cost. In addition, a weighted sum two-objective model was presented to produce a trade-off front for overall costs and distances. Java was used as our implementation language, together with the CPLEX® optimization engine. The weighted sum approach for the multi-objective optimization produced a set of non-dominated solutions by re-running the optimization several time, each time applying different weights to the cost and distance values. The approach allowed to produce good trade-off solutions that can be found using this technique. An investigation into the impact of the trade-off solutions is discussed in relation to absolute values, in terms of £s and km, as well as percentages.

Chapter 7

Lagrangian relaxation for a single source facility allocation problem

7.1 Introduction

Logistics network design involves making decisions at different levels to ensure the continued competitiveness and responsiveness of the network. At the *strategic level* an optimum number, location and capacity for the depots is determined and may be evaluated every few years. At the *tactical level* decisions, such as customer assignment (to depots), supplier selection and transportation mode may be reconsidered every few months, as demand patterns or other factors change. In this chapter we are concerned with the process of assigning customers to appropriate depots which extends our allocation study in Chapter 5 and 6 to large size instances. We consider the strategic problem of capacitated facility location in Chapter 9. Assigning customers to the most appropriate depot is also performed as a sub-routine of the the strategic facility location problem, because it is not possible to assess the quality of a particular facility location problem (i.e., with certain facilities identified as "open", and others as "closed"), without carrying out a full customer allocation to evaluate the costs of serving the customers and satisfying their demand. Recently issues, such as fuel prices and climate change and its impact on the network design, have recently been discussed in the press. Gilmore [47] reports the finding of Dr. David Simchi-Levi who volunteered to take a look at the impact of rising oil prices using data from a real consumer goods company. He reported that the optimum network design stays the same with five distribution centres for this particular case until the price reached $150, then the optimal network changed to seven depots, where one depot is closed and three new facilities are open. These findings show the importance of periodically evaluating the network design when any factors that may influence the design change. Clearly, the optimum number of facilities and their location is very sensitive to many variable factors. In practice, however, building new warehouses, or closing or resizing them is a very costly business and will be considered only occasionally. On the other hand, it could be possible to make significant savings simply by re-allocating customers to fixed facilities, on a regular basis, to reflect changes in circumstances.

This chapter describes two Lagrangian Relaxation (LR) approaches to solving the single source facility allocation problem for a single and multiple products. The motivation for this research is to apply a Lagrangian Relaxation technique as part of a multi-objective capacitated facility location problem, where economic costs and environmental impact are solved simultaneously. Multi-objective optimization techniques allow the decision maker to evaluate different trade-off solutions of the design. Obviously optimization software packages, such as CPLEX® exist, to calculate the optimum assignment. Even though developers can embed CPLEX® optimizers into Java applications to solve complex optimization problems, for our research we need an efficient technique to produce a good solution within a reasonable amount of time as part of the development of our multi-objective optimization tool for strategic modelling. Many thousands evaluations are typically required for multi-objective optimization and CPLEX® would be too slow.

Our single source mathematical model for the allocation of customers to facilities involves two capacity constraints: 1) number of cases and 2) number of customers assigned to a particular depot. We use a notation of an average case in the problem formulation which consists of a number of items packed together in one box. Inheritably, different products have different weights and different numbers of items packed in the case, therefore it is common to use the notation of the average case. Single source terminology implies that a customer is assigned to just one serving facility. The objective of this study is to develop an efficient heuristic procedure, which provides an effective solution for large-scale data instances for a single-echelon assignment problem, consisting of number facilities and customers. Although the technique does not guarantee to obtain the optimum solution, it has proven excellent in practice, and is very useful when speed is critical (e.g., for really large problems, multiple experiments and for considering multiple criteria simultaneously). We apply Lagrangian Relaxation with relaxed capacity constraints to obtain a lower bound solution, and a set of multipliers is used to ensure that if the facility has spare capacity, then it is more attractive for assignment in the next iteration. The current technique was adapted from Ghiani [48] where it was applied to CFLP. The formulations presented in this chapter are based on the traditional lagrangian relaxation of the capacity constraints of GAP [136] and extended to incorporate extra constraints and multiple products. The heuristic algorithm for solving each relaxed formulation is discussed as part of the development of the LR technique in this chapter and focuses on obtaining a feasible solution for upper bound for an assignment problem and not for the facility location problem as presented in the study by Klincewicz and Luss[135] where fixed costs are present in the relaxed formulation.

Section 7.2 describes a LR approach based on a traditional formulation for a single product where the capacity (number of cases) constraint is relaxed. We also present in the Appendix A, a new LR procedure where two capacity constraints are incorporated into relaxed formulation. The results of the technique in the Section 7.2 are compared to the optimum solutions produced by CPLEX® in terms of the quality of solution and execution time on the benchmarking data available in the public domain (ORLIB [11]) and also on some large-size problem instances which were randomly generated by ourselves. The benchmarking data available from ORLIB was used for testing the technique on the model with one capacity (cases) constraint and the data created by us was used to assess the performance of the solution technique on the model with two capacity constraints: number of cases and number of stores.

Section 7.3 introduces multiple product formulation with LR solution technique based on Sainsbury's data. The emphasis of the discussion in this section is on finding feasible solutions in the multiple product formulation. The LR technique for multiple products will need to have further investigation on randomly generated data sets which are outside the scope of this project and addressed in the future work chapter 10.

7.2 Problem Definition for a single product

For a given a set of customers with known demand for a product and a set of open facilities with known capacities, the objective is to minimize the cost of assigning the customers to the facilities. The customers' demand has to be satisfied by a single facility, and capacity constraints have to be adhered to. The problem can be modelled by a complete directed graph, G, where the vertices in V_{DC} represent the facilities and the vertices in V_C represent the customers. The arcs are associated with the flow of goods between facilities and customers.

Glossary

V_{DC} set of facilities;

V_C set of customers;

d_j demand (cases) of customer j, $j \in V_C$;

q_i capacity (cases) of facility i, $i \in V_{DC}$;

n_i capacity (number of customers) of facility i, $i \in V_{DC}$;

c_{ij} is the cost of satisfying the total demand of customer j, d_j, from facility i;

x_{ij} is the decision variable for the problem, $x_{ij} = 1$, if customer j is allocated to facility i, and 0 otherwise.

Mathematical formulation

Minimize

$$\sum_{i \in V_{DC}} \sum_{j \in V_C} c_{ij} x_{ij} \tag{7.1}$$

subject to

$$\sum_{i \in V_{DC}} x_{ij} = 1, \forall j \in V_C \tag{7.2}$$

$$\sum_{j \in V_C} d_j x_{ij} \leq q_i, \forall i \in V_{DC} \tag{7.3}$$

$$\sum_{j \in V_C} x_{ij} \leq n_i, \forall i \in V_{DC} \tag{7.4}$$

$$x_{ij} \in \{0, 1\}, i \in V_{DC}, j \in V_C \tag{7.5}$$

(7.1) aims to minimize the costs of satisfying the total demand of all the customers, and constraint (7.2) specifies that the demand of each customer must be satisfied by a single facility. (7.3) and (7.4) ensure that the capacity constraints for demand (cases) and number of stores for the facilities are not violated and finally, (7.5) specifies that allocation y for the decision variable x_{ij} and the demand is satisfied by one facility.

7.2.1 Solution Formulation 1 for relaxing one constraint: number of cases

The first solution formulation of LR techniques relaxes only one constraint: the number of cases. This is a simple approach, yet we found the solution quality and efficiency comparable to a more complex two constraint LR model, which we developed later and further work will be needed to test the approach on our own generated data sets. We also present the two constraint model in the Appendix A. Please note that by relaxing only one constraint initially we were making the assumption that the number of cases is a harder constraint compare to the number of stores constraint, and this seemed appropriate on close examination of the data. Nevertheless, the feasibility of the LB and UB solutions was checked for violation of both constraints to ensure only feasible results were produced.

The main step in the Lagrangian relaxation is the determination of a lower bound obtained by relaxing the capacity (cases) satisfaction constraint using Lagrangian multipliers. Let $\lambda_i \in \mathbb{R}, \forall i \in V_{DC}$.

Minimize

$$\sum_{i \in V_{DC}} \sum_{j \in V_C} c_{ij} x_{ij} + \sum_{i \in V_{DC}} \lambda_i \Big(\sum_{j \in V_C} d_j x_{ij} - q_i \Big) \tag{7.6}$$

subject to

$$\sum_{i \in V_{DC}} x_{ij} = 1, \forall j \in V_C \tag{7.7}$$

$$\sum_{j \in V_C} x_{ij} \leq n_i, \forall i \in V_{DC} \tag{7.8}$$

$$x_{ij} \in \{0, 1\}, i \in V_{DC}, j \in V_C \tag{7.9}$$

In (7.6) the term in brackets on the right, $(\sum_{j \in V_C} d_j x_{ij} - q_i)$, calculates the difference between the total demand on a facility i imposed by the relaxed formulation, and its ability to meet that demand (i.e., its capacity(cases), q_i). If the capacity is violated, or underutilized, the value of total cost in (7.6) will change, depending on the value of λ_i.

One issue that needs to be considered regarding the right-hand side of formula (7.6), is that normally a Lagrangian Relaxation technique will make adjustments to the cost only when a constraint is violated. Thus, in the case of (7.6) we would expect the term $(\sum_{j \in V_C} d_j x_{ij} - q_i)$ to equal zero, for any facility for which its capacity has not been exceeded. However, this is not the case, as underutilized capacities will produce non-zero values. Later on in this chapter we will make some suggestions as to how the Lagrangian scheme can be adapted to cope with this issue, by constraining the λ_i values: if $\lambda_i = 0$, it follows that $\sum_{i \in V_{DC}} \lambda_i (\sum_{j \in V_C} d_j x_{ij} - q_i)$ also equals zero.

Problem (7.6) - (7.9) can be decomposed into $|V_C|$ subproblems. For a given set of multipliers, $\lambda_i \in \mathbb{R}$, the optimal lower bound of the problem (7.6) - (7.9), $LB(\lambda)$, can be found by solving the following subproblem for each customer $j \in V_C$.

Minimize

$$\sum_{i \in V_{DC}} (c_{ij} + d_j \lambda_i) x_{ij} \tag{7.10}$$

subject to

$$\sum_{i \in V_{DC}} x_{ij} = 1, \forall j \in V_C \tag{7.11}$$

$$\sum_{j \in V_C} x_{ij} \leq n_i, \forall i \in V_{DC} \tag{7.12}$$

$$x_{ij} \in \{0, 1\}, i \in V_{DC}, j \in V_C \tag{7.13}$$

and then by setting

$$LB(\lambda) = \sum_{j \in V_C} LB^j(\lambda) - \sum_{i \in V_{DC}} \lambda_i q_i \tag{7.14}$$

(7.10) is easily solved for the relaxed problem simply by applying a greedy algorithm to allocate each customer along the lowest cost arc, according to the augmented costs, $c_{ij} + d_j\lambda_i$. By suitably modifying the Lagrangian multipliers, it is possible to obtain a feasible solution to the original capacity constrained problem. To provide a good updating formula for the Lagrangian multipliers, we will need an upper bound, in addition to the lower bound in (7.14).

For an upper bound (UB) we will use a feasible solution obtained on the basis of the allocations of customers to facilities discovered in the evaluation of $LB(\lambda)$. However, it is likely that the allocation made for the lower bound calculation will produce some capacity violations. In order to obtain the best possible upper bound (i.e., with the lowest cost), we need to establish a good way of reallocating customers when facilities are over-subscribed. For an upper bound, it is best to allocate customers with high demand first, to try to ensure that individual depots have sufficient unused capacity. One possible way of doing this is to sort customers in non-increasing order of demand level (highest demand first), then work through the list, assigning customers in the same way as the LB, whenever possible. When capacity constraints are violated for LB assignment, we try to assign to the next lowest augmented cost depot without violating the capacity constraints etc. If all facilities are overcapacity, then we assign to the lowest available cost value (non-augmented cost).

Updating the Lagrangian multipliers

For each facility at time step, k

$$s_i^k = \sum_{j \in V_C} x_{ij}^k d_j - q_i \tag{7.15}$$

where x_{ij}^k is the solution of the Lagrangian relaxation (7.6) - (7.9) using $\lambda_i^k \in \mathbb{R}, \forall i \in V_{DC}$ as the Lagrangian multipliers. Now set

$$\lambda_i^{k+1} = \begin{cases} \lambda_i^k + \beta^k s_i^k & if\ s_i^k > 0 \\ 0 & otherwise \end{cases} \tag{7.16}$$

where β^k is a suitable scalar coefficient. We will start the procedure by seeding all the Lagrangian multipliers to zero. Formula (7.16) can be explained in the following way. If for a certain facility i, s_i^k is positive, it means that demand outstrips supply for that facility, and thus the corresponding value of λ_i should be increased to increase the cost of assigning customers to that facility in the next round. Similarly, if s_i^k is negative, it means that there is spare capacity, so λ_i should be reduced to make that facility more attractive for assignment in the next iteration. However, as we pointed out earlier, it may not be appropriate to make adjustments to the multipliers when the capacity has not been violated for a facility. Formula (7.16) ensures that the λ_i^k are always positive.

Tuning of the Lagrangian heuristic technique

To ensure that the algorithm is robust and performs efficiently, several experiments were performed in order to tune the β^k coefficient and also to determine how many iterations to perform between updates for the constant α. The coefficient β^k was tested with two different settings (7.17) and (7.18), where the difference between two formulations was insignificant therefore the equation (7.17) was chosen and incorporated into our final algorithm.

$$\beta^k = \frac{\alpha(UB - LB(\lambda^k))}{\sum_{i \in V_{DC}} (s_i^k)^2} \tag{7.17}$$

$$\beta^k = \frac{\alpha(UB(\lambda^k) - LB(\lambda^k))}{\sum_{i \in V_{DC}} (s_i^k)^2} \tag{7.18}$$

Parameter α is a constant in the interval $(0, 2]$ [48]. Here, α is used starting with 2 and halved whenever the iteration's feasible upper bound failed to improve on the best known feasible upper bound for n iterations. Parameter n was tested in the range from [1,100] with step 1 for all benchmarking problems to identify the best value for n. As a result, a value of 100 was used for n in the smaller problems (Beasley data sets and our own data sets with 10 depots) and a value of 70 for larger sized problems. Those values of n were chosen because our algorithm produced its best solutions (or very close) for most of the instances tested with these values.

The total number of iterations was tested at: 500, 1000 and 2000. We discovered no difference in the final results, so a value of 500 was used for the total number of the iterations to minimize the computational time. Finally, the algorithm for the Lagrangian relaxation is described in Algorithm 7.1.

7.2.2 Test instances

Benchmarking data

The quality of the solution produced by our Lagrangian heuristic was tested on the benchmark data instances available in the literature for single source capacitated facility location problems (SSFLP) for single product available from the OR-library [11]. The benchmarking data sets available do not necessary reflect all real-life situations, however, where additional constraints may apply. For example, in our model described above, we have an extra capacity constraint in terms of a maximum number of stores that can be served by a particular depot (Equations (7.4), (7.8) and (7.12)). Thus, the "number of stores" constraint did not apply and was dropped from our model for the OR-library benchmark sets.

Algorithm 7.1: ***Lagrangian heuristic algorithm for a single source capacitated allocation problem, single product.***

Begin
(*initialization*)
Select a tolerance level $\epsilon \geq 0$
Set $difference = +\infty$, $LB = -\infty$, $UB = +\infty$, $k = 1$ and $\lambda_i^k = 0, i \in V_{DC}$
while $(difference \geq \epsilon)$ OR (k $\leq$number of iterations) **do**
  (*Computation of a new lower bound*)
  Solve the Lagrangian relaxation (7.6) - (7.9) using $\lambda_i^k \in \mathbb{R}, \forall i \in V_{DC}$ multipliers (Greedy algorithm with on uncapacitated version based on augmented costs). Let $LB(\lambda^k)$ be its cost.
  if $LB(\lambda^k)$ solution is feasible **then**
    STOP algorithm and return cost $LB(\lambda^k)$
  else if $LB(\lambda^k) > LB$ **then**
    set $LB = LB(\lambda^k)$
    (*Computation of a new upper bound*)
    Determine the corresponding upper bound (modified greedy algorithm, as described in the text). Let $UB(\lambda^k)$ be its cost.
    if $UB(\lambda^k) < UB$ **then**
      set $UB = UB(\lambda^k)$
    Calculate $difference = (UB - LB)/LB$
    Update parameters s_i^k, β^k and compute Lagrangian multipliers λ_i^{k+1} (7.15)-(7.17), $\forall i \in V_{DC}$
    Update k=k+1
Return cost of the UB feasible solution
End

Recall that this chapter covers facility allocation only (and not facility location) and is focussed on comparing run times and solution quality for CPLEX® versus LR, the goal begin to develop a fast routine that we can incorporate into our multi-objective approaches, where it will be necessary to repeat the allocation procedure many times. The chosen benchmarks however, are all facility location/allocation problems. Thus it was necessary to begin with solutions to the location problems, with the open facilities defined at the start, before we attempted to solve the allocation problems, and carry out the timings and solution quality comparisons for CPLEX® and LR. Unfortunately, although optimum solutions in terms of cost are given for small and medium instances, for large problems which were are used for testing there was no available cost solutions, only information on LB. Also, there is no information available to identify which depots are open and which closed in the optimum solutions. To determine which depots are open and which are closed in the optimum solutions as well as their solutions, all instances were solved by us to optimality using CPLEX®.

Because the Lagrangian relaxation was developed for a large-size instances, only the 12 largest sets of the Beasley ([10],[11]) instances were used for comparing the CPLEX® solution to the Lagrangian relaxation. The description of the data instances from Beasley, which is suitable for a single source problem formulation are presented in the Table 7.1. For example, instances from the set A (capa1.txt, capa2.txt, capa3.txt, capa4.txt) were randomly generated by Beasley and consist of 100 potential facilities and 1,000 customers. The locations of the customers/warehouses were generated within an 1000 by 1000 Euclidean square. The cost was calculated per unit of demand supplied as proportional to the Euclidean distance between the customer and the warehouse, multiplied by a real random number in range [1.00,1.25]. The demand for each customer was generated as a random integer the range [1,100]. The capacity of the serving facility is equal to 8000 for capa1.txt, 10000 for capa2.txt an so on. For more information how the data was generated, please refer to the original paper [10].

Problem set	Number of facilities	Number of customers	Facility capacity	Fixed cost per facility
A (e.g. capa1.txt)	100	1000	8000/10000/12000/14000	Random
B (e.g. capb1.txt)	100	1000	5000/6000/7000/8000	Random
C (e.g. capc1.txt)	100	1000	5000/5750/6500/7250	Random

Table 7.1: Data sets from Beasley [10] for a single source problem formulation.

Our Data

As discussed earlier, the benchmarking data sets do not match our case study data, where constraints also exist limiting the number of stores for each serving depot. To reflect this situation, an extra constraint (7.4) is used in the model formulation (7.1) for the allocation problem.

Because there is no available published data sets for the model formulation with the two constraints in the public domain, we generated some new data. The methodology for generating data was inspired by Sainsbury's case study (see Chapter 5), where the cost structure has transportation and warehousing models. The transportation model consists of distance and time related costs and the warehousing (depot) model of the labour costs of picking and loading goods and the picking productivity rate. The single commodity version of the algorithm for creating data sets is described in the Algorithm 7.2, where randomly generated parameters are uniformly distributed. A variety of problem sizes were generated with different numbers of depots, stores, and different ratios of total available demand to total available capacity in terms of the number of cases and also in terms of the number of stores.

The purpose of developing a Lagrangian relaxation technique is to develop a heuristic technique capable of producing "good" solutions to large-sized instances within a reasonable amount of time: faster than linear programs tools such as CPLEX®. Therefore, two type of test instances were generated: $set1$ and $set2$. $Set1$ instances have a ratio of overall demand to capacity of 0.9 for the number of cases and number of stores and $set2$ instances have a ratio of 0.8. We used large sized networks for our instances, which we based on our observations of major national/international supply chain networks, where the number of customers can be 5,000 or more. For each type of problem (i.e., with ratio 0.8, and 0.9), we generated instances with 10 and 50 depots, and for each ratio-depot configuration, a following numbers of stores was generated: 100, 500, 1000, 2000,...10000, e.g. $set1\_10\_1000\_r0.9.txt$, $set2\_10\_1000r0.8.txt$. The locations for stores and depots were randomly generated within a square of 700 by 700 units. The following parameters for each depot which were used for calculating the transportation depot cost structure were generated: THR rate in the range of [1.7,2]; WHR rate in the range of [1.3,1.8] and DWR rate in the range of [7,12]. The value of 0.04 was used for the cost of the fuel per unit of distance and the average truck load was equal to 130. A value of 40 was used as travel speed to calculate travelling times between two points. The latter values were used in calculation of the transportation costs. We generated the demand for each store in the range of [30, 3000] and the capacity for each depot was calculated taking into account the overall demand in the network (see Algorithm 7.2).

In total, 24 instances were created covering different possibilities of size, cost structure and the ratio of the overall demand over available capacity for cases and number of stores.

All the experiments were conducted on a PC with an Intel(R) Pentium(R) D CPU 3.4 GHz and 2 GB RAM.

7.2.3 Results

To make a fair comparison between CPLEX® and our LR approach, the stopping condition for the termination of techniques was set to different values for the tolerance gaps which indicates how close a solution should be to optimality. To set the stopping criterion for the tolerance gap, the following parameters were changed in the solution techniques. In CPLEX, a relative MIP gap tolerance, which is an important criterion for the termination, was changed to the values below. A relative MIP gap tolerance is commonly used as a stopping criterion and indicates that CPLEX® should stop when the integer feasible solution has been proved to be within a particular distance from optimality. The default gap tolerance value is $1e10^{-4}$ [24]. In the Lagrangian relaxation technique, the difference between the UB and LB solution was used as a tolerance gaps for our stopping criteria. The tolerance level in the experiments was set at following values 0.05, 0.04, 0.03, 0.02 and 0.0001. The information regarding the solution for each setting of the stopping criteria gave us an opportunity to compare our heuristic to the optimum solution in each run as a percentage difference which was rounded to 2 decimal places.

Tables 7.2 and 7.3 show the summary of the test results of the Beasley benchmarking instances: the solution quality and the execution times of both techniques. For all tolerance levels, the LR technique proved to find good quality solutions, which came at

Algorithm 7.2: ***Algorithm for generating data sets for the allocation problem, single product, single source.***

Begin:

SET parameters:

number of depots ($NumDep$)

number of stores($NumStores$)

min coordinates X and $Y(MinCoordX, MinCoordY)$

max coordinates X and Y ($MaxCoordX$,$MaxCoordY$)

min and max demand for stores ($MinDemand$, $MaxDemand$)

$multiplyerCapacityCases$ value which controls the ratio of the overall demand over the available capacity for all depots to make network "looser" or "tighter"

$multiplyerCapacityNumOfSt$ is similar to $multiplyerCapacityCases$ with the difference that it controls the ratio of overall number of stores over available capacity for all depots

$travelSpeed$ is the speed with with a vehicle travels one Euclidian unit of distance

$avgTruckLoad$ is the average number of cases per truck

$fuelCost$ is the fuel related cost for one Euclidian unit of distance

Generate random X and Y coordinates for each depot in the range [$MinCoordX$, $MaxCoordX$], [$MinCoordY$, $MaxCoordY$]

Generate random X and Y coordinates for each store in the range [$MinCoordX$, $MaxCoordX$], [$MinCoordY$, $MaxCoordY$]

Calculate Euclidian distance between each store i to depot j: $distance[i][j] = \sqrt{(xStore_i - xDepot_j)^2 + (yStore_i - yDepot_j)^2}$

Calculate travelling time for an Euclidian distance between each store i and depot j: $time[i][j] = distance[i][j]/travelSpeed$

Generate random $demand$ in cases for each store, range [$MinDemand$, $MaxDemand$]

Calculate capacity (cases) for each depot j: $depotCapacityCases[j]$=($totalDemand/NumDep$) * $multiplyerCapacityCases$

Calculate capacity (num of stores) for each depot j: $depotCapacityNumOfStores[j]$= ($NumStores/NumDep$)*$multiplyerCapacityNumOfSt$

Generate random THR rate for each depot in the range [$MinTHR$, $MaxTHR$]

Generate random WHR rate for each depot in the range [$MinWHR$, $MaxWHR$]

Generate random DWR rate for each depot in the range [$MinDWR$, $MaxDWR$]

Calculate transport cost between each store i and depot j: $transportCost[i][j] = (distance[i][j] * fuelCost + time[i][j] * THR[j]) * 2 * (demand[i]/avgTruckLoad)$

Calculate depot cost between each store i and depot j: $depotCost[i][j] = (demand[i]/DWR[j]) * WHR[j]$

Calculate total connection cost between each store i and depot j: $totalCost[i][j] = transpotCost[i][j] + depotCost[i][j]$

End

around 0-1.99% difference from the solution found by the CPLEX® optimization engine. The LR technique also proved to be more efficient in computational times. Table 7.3 presents an analysis of how fast the LR technique finds the solutions withing the set tolerance level. It is expressed as the % of time needed to solve a data sets using LR compared to the CPLEX® solving times. For example, for test instance $capa2.txt$, CPLEX® takes 797 ms and LR takes 47 ms to solve the data, which is equated to 5.9% of CPLEX® running time. In fact LR performs better for all tolerance levels except for 0.01%, when the algorithm tries to find the true "optimum". Out of 48 experiments (12 data sets and four tolerance levels: 0.05, 0.04, 0.03, 0.02), only 8 were above 10% of CPLEX® running times for the LR heuristic. This gives us a very promising results, indicating that LR could be useful on large data sets.

The next step of the analysis was to compare the solution quality and running times for large data sets which were created by us and described earlier in the chapter. To recap, we created two types of the data: ratio of 0.9 ($set1$) and ratio 0.8 ($set2$) of overall demand (cases/number of stores) to the available capacity in the depots. As a result of the initial testing, ratio 0.9 is proved to be too tight for the LR technique to find feasible solutions. From the experience working with a real data sets from Sainsbury's, in their network, the tightest ratio was 0.81 for chill products (see Chapter 5), therefore it seems to be more sensible to use ratio of 0.8 for all experiments. This information also gave us an insight into the limitations of the heuristic, where a very high ratio prevents the algorithm from finding feasible solutions.

Tables 7.4, 7.5, 7.6 and 7.7 present the solution quality and execution times for $set2$ with 10 and 50 depots, where the ratio is equal to 0.8. Looking at the CPLEX® solutions, we can see that there is no information available for some of the data sets, which was due to the limitations in technology: CPLEX® ran out of memory for those data sets. On the other hand, the LR approach found feasible solutions to all data sets within a reasonable amount of time. For the tolerance levels of 5%, 4% and 3%, 2% and 1%, the solutions produced by the technique were less than 5.01% different compared to the solutions found by CPLEX® and the majority were less than 2.5%. For a tolerance level of 0.01%, again LR produced excellent quality solutions compared to CPLEX and on the

Test instance	Tolerance level								
	5%			4%			3%		
	CPLEX	Lagr.	% diff	CPLEX	Lagr.	% diff	CPLEX	Lagr.	% diff
capa1.txt	19,242,450.15	19,501,820.92	1.35%	19,242,450.15	19,501,820.92	1.35%	19,242,450.15	19,430,478.52	0.98%
capa2.txt	18,451,166.16	18,702,263.38	1.36%	18,451,166.16	18,702,263.38	1.36%	18,451,166.16	18,600,101.24	0.81%
capa3.txt	17,765,201.95	17,765,201.95	0.00%	17,765,201.95	17,765,201.95	0.00%	17,765,201.95	17,765,201.95	0.00%
capa4.txt	17,160,815.54	17,275,620.41	0.67%	17,160,815.54	17,275,620.41	0.67%	17,160,815.54	17,275,620.41	0.67%
capb1.txt	13,668,638.07	13,886,676.18	1.60%	13,668,638.07	13,886,676.18	1.60%	13,668,638.07	13,839,765.34	1.25%
capb2.txt	13,385,143.32	13,616,417.76	1.73%	13,385,143.32	13,616,417.76	1.73%	13,385,143.32	13,543,257.11	1.18%
capb3.txt	13,255,296.35	13,470,793.28	1.63%	13,255,296.35	13,470,793.28	1.63%	13,255,296.35	13,393,444.42	1.04%
capb4.txt	13,086,451.48	13,347,108.06	1.99%	13,086,451.48	13,347,108.06	1.99%	13,086,451.48	13,166,476.57	0.61%
capc1.txt	11,709,354.71	11,756,281.83	0.40%	11,709,354.71	11,756,281.83	0.40%	11,709,354.71	11,756,281.83	0.40%
capc2.txt	11,570,437.68	11,591,895.27	0.19%	11,570,437.68	11,591,895.27	0.19%	11,570,437.68	11,591,895.27	0.19%
capc3.txt	11,536,854.31	11,660,534.33	1.07%	11,536,854.31	11,660,534.33	1.07%	11,536,854.31	11,660,534.33	1.07%
capc4.txt	11,516,656.16	11,520,064.91	0.03%	11,516,656.16	11,520,064.91	0.03%	11,516,656.16	11,520,064.91	0.03%

Test instance	Tolerance level								
	2%			1%			0.01%		
	CPLEX	Lagr.	% diff	CPLEX	Lagr.	% diff	CPLEX	Lagr.	% diff
capa1.txt	19,242,450.15	19,367,902.86	0.65%	19,242,450.15	19,298,972.84	0.29%	19,241,057.80	19,249,966.47	0.05%
capa2.txt	18,451,166.16	18,600,101.24	0.81%	18,451,166.16	18,527,533.53	0.41%	18,438,329.78	18,439,832.15	0.01%
capa3.txt	17,765,201.95	17,765,201.95	0.00%	17,765,201.95	17,765,201.95	0.00%	17,765,201.95	17,765,201.95	0.00%
capa4.txt	17,160,815.54	17,275,620.41	0.67%	17,160,815.54	17,203,715.43	0.25%	17,160,815.54	17,161,398.06	0.00%
capb1.txt	13,668,638.07	13,753,023.75	0.62%	13,668,638.07	13,718,399.01	0.36%	13,657,482.15	13,686,246.45	0.21%
capb2.txt	13,385,143.32	13,462,352.07	0.58%	13,385,143.32	13,430,862.85	0.34%	13,363,068.68	13,388,609.35	0.19%
capb3.txt	13,255,296.35	13,260,432.75	0.04%	13,255,296.35	13,260,432.75	0.04%	13,199,420.27	13,213,813.63	0.11%
capb4.txt	13,086,451.48	13,166,476.57	0.61%	13,086,451.48	13,155,211.49	0.53%	13,083,451.13	13,091,906.73	0.06%
capc1.txt	11,709,354.71	11,756,281.83	0.40%	11,709,354.71	11,705,838.65	-0.03% *	11,647,531.06	11,654,099.05	0.06%
capc2.txt	11,570,437.68	11,591,895.27	0.19%	11,570,437.68	11,591,895.27	0.19%	11,570,437.68	11,570,437.68	0.00%
capc3.txt	11,536,854.31	11,567,564.46	0.27%	11,536,854.31	11,567,564.46	0.27%	11,519,413.38	11,520,815.85	0.01%
capc4.txt	11,516,656.16	11,520,064.91	0.03%	11,516,656.16	11,520,064.91	0.03%	11,505,861.86	11,505,861.86	0.00%

Table 7.2: Beasley data sets, cost solution.

** due to the tolerance levels associated with both solutions*

Test instance	Tolerance level								
	5%			4%			3%		
	CPLEX	Lagr.	Lagr % of CPLEX	CPLEX	Lagr.	Lagr % of CPLEX	CPLEX	Lagr.	Lagr % of CPLEX
capa1.txt	984	906	92.07%	968	875	90.39%	953	890	93.39%
capa2.txt	797	47	5.90%	781	47	6.02%	812	63	7.76%
capa3.txt	703	47	6.69%	703	47	6.69%	704	47	6.68%
capa4.txt	609	47	7.72%	625	47	7.52%	609	63	10.34%
capb1.txt	1469	47	3.20%	1422	47	3.31%	1422	63	4.43%
capb2.txt	1250	47	3.76%	1219	62	5.09%	1219	891	73.09%
capb3.txt	1063	47	4.42%	1093	47	4.30%	1047	62	5.92%
capb4.txt	953	62	6.51%	953	62	6.51%	937	78	8.32%
capc1.txt	1531	47	3.07%	1563	47	3.01%	1703	47	2.76%
capc2.txt	1421	47	3.31%	1391	47	3.38%	1578	47	2.98%
capc3.txt	1344	47	3.50%	1359	47	3.46%	1484	47	3.17%
capc4.txt	1359	47	3.46%	1328	63	4.74%	1547	47	3.04%

Test instance	Tolerance level								
	2%			1%			0.01%		
	CPLEX	Lagr.	Lagr % of CPLEX	CPLEX	Lagr.	Lagr % of CPLEX	CPLEX	Lagr.	Lagr % of CPLEX
capa1.txt	1109	875	78.90%	985	953	96.75%	1109	3984	359.24%
capa2.txt	890	63	7.08%	828	63	7.61%	922	3969	430.48%
capa3.txt	797	47	5.90%	703	47	6.69%	703	47	6.69%
capa4.txt	609	47	7.72%	609	63	10.34%	610	297	48.69%
capb1.txt	1422	921	64.77%	1469	953	64.87%	1828	4125	225.66%
capb2.txt	1266	922	72.83%	1234	954	77.31%	1625	4078	250.95%
capb3.txt	1094	94	8.59%	1078	94	8.72%	1625	4016	247.14%
capb4.txt	1000	78	7.80%	953	93	9.76%	1375	4078	296.58%
capc1.txt	1547	46	2.97%	1547	62	4.01%	1984	4125	207.91%
capc2.txt	1406	47	3.34%	1453	47	3.23%	1391	94	6.76%
capc3.txt	1344	62	4.61%	1422	63	4.43%	1469	4015	273.32%
capc4.txt	1344	47	3.50%	1390	47	3.38%	1438	375	26.08%

Table 7.3: Beasley data sets, execution time (ms).

larger data sets it was impossible to compare solutions due to CPLEX® limitations. Regarding execution times, again for the tolerance levels between 5% and 1%, LR found good solutions within a fraction of CPLEX® running time for the majority of the data sets and on the larger data sets, the technique found the solutions within a reasonable time whereas CPLEX® ran out of memory. The results obtained from these experiments, gave us confidence in the LR approach for large data sets, and provided the way forward for solving allocation for multiple products and to be used as a subroutine in CFLP where the optimum assignment needs to be determined for open facilities.

7.3 Lagrangian relaxation for a multi-commodity allocation problem

Previous sections presented a model formulation and a solution technique for a single source, single commodity large-size assignment problem with very promising results for solution quality and computational times using LR technique which relaxes a capacity constraint. In this section, an investigation is carried out into multi-commodity assignment problem based on Sainsbury's data by extending the LR technique to the multi-commodity variant of the assignment model. The simplified version of the single source, multi-commodity model formulation is presented in this chapter where constraints for capacities for cases and number of stores are taken into consideration. Due to the multiple products nature of the requirements, the discussion of the approach put a great emphasis on finding feasible solutions. Therefore different settings were tested to ensure that technique finds feasible UB solutions. Because there is no available benchmarking data in the public literature and due to the time constraints of the project, the LR technique is only tested on Sainsbury's data and will need further assessment on randomly generated large data sets for this particular model formulation. The current study provides very promising results which could be very efficient, especially for large-size data sets.

Test instance	Tolerance level								
	5%			4%			3%		
	CPLEX	Langr.	% diff	CPLEX	Langr.	% diff	CPLEX	Langr.	% diff
set2_10_100.txt	46,524.15	46,822.07	0.64%	46,524.15	46,822.07	0.64%	46,524.15	46,822.07	0.64%
set2_10_500.txt	220,224.89	226,584.95	2.89%	220,224.89	225,466.25	2.38%	220,224.89	224,529.89	1.95%
set2_10_1000.txt	479,074.37	489,183.23	2.11%	479,074.37	489,183.23	2.11%	479,074.37	489,183.23	2.11%
set2_10_2000.txt	892,089.76	913,079.05	2.35%	892,089.76	913,079.05	2.35%	892,089.76	913,079.05	2.35%
set2_10_3000.txt	1,328,056.13	1,359,290.89	2.35%	1,328,056.13	1,359,290.89	2.35%	1,328,056.13	1,359,290.89	2.35%
set2_10_4000.txt	1,824,948.12	1,898,723.42	4.04%	1,824,948.12	1,876,980.40	2.85%	1,824,948.12	1,871,286.23	2.54%
set2_10_5000.txt	2,429,097.51	2,490,337.79	2.52%	2,429,097.51	2,490,337.79	2.52%	2,429,097.51	2,455,507.51	1.09%
set2_10_6000.txt	2,980,322.13	3,043,027.75	2.10%	2,980,322.13	3,043,027.75	2.10%	2,980,322.13	3,014,317.33	1.14%
set2_10_7000.txt	3,286,944.54	3,321,522.91	1.05%	3,286,944.54	3,321,522.91	1.05%	3,286,944.54	3,321,522.91	1.05%
set2_10_8000.txt	3,723,125.46	3,852,259.43	3.47%	3,723,125.46	3,795,938.78	1.96%	3,723,125.46	3,795,938.78	1.96%
set2_10_9000.txt	4,391,346.93	4,544,417.09	3.49%	4,391,346.93	4,515,789.68	2.83%	4,391,346.93	4,486,984.47	2.18%
set2_10_10000.txt	4,491,745.86	4,491,745.94	0.00%	4,491,745.86	4,491,745.94	0.00%	4,491,745.86	4,491,745.94	0.00%
set2_50_100.txt	41,479.32	42,772.61	3.12%	41,479.32	42,772.61	3.12%	41,479.32	42,772.61	3.12%
set2_50_500.txt	180,748.70	181,853.18	0.61%	180,748.70	181,469.19	0.40%	180,748.70	180,731.79	-0.01%*
set2_50_1000.txt	351,997.41	359,841.79	2.23%	351,997.41	357,032.47	1.43%	351,997.41	355,600.41	1.02%
set2_50_2000.txt	727,648.28	743,725.13	2.21%	727,648.28	743,725.13	2.21%	727,648.28	738,984.36	1.56%
set2_50_3000.txt	1,103,652.63	1,136,542.75	2.98%	1,103,652.63	1,125,551.29	1.98%	1,103,652.63	1,124,034.69	1.85%
set2_50_4000.txt	1,408,338.74	1,454,711.67	3.29%	1,408,338.74	1,448,162.59	2.83%	1,408,338.74	1,439,706.80	2.23%
set2_50_5000.txt	1,831,451.55	1,887,660.70	3.07%	1,831,451.55	1,877,437.09	2.51%	1,831,451.55	1,866,449.06	1.91%
set2_50_6000.txt	2,154,781.36	2,237,995.46	3.86%	2,154,781.36	2,237,995.46	3.86%	2,154,781.36	2,237,995.46	3.86%
set2_50_7000.txt	N/A	2,504,221.68	N/A	N/A	2,484,323.62	N/A	N/A	2,477,674.21	N/A
set2_50_8000.txt	N/A	3,090,608.12	N/A	N/A	3,071,766.41	N/A	N/A	3,056,062.35	N/A
set2_50_9000.txt	N/A	3,378,361.33	N/A	N/A	3,361,194.31	N/A	N/A	3,340,116.48	N/A
set2_50_10000.txt	N/A	3,867,393.77	N/A	N/A	3,846,414.95	N/A	N/A	3,817,126.58	N/A

Table 7.4: Our data sets, cost solution.

* *due to the tolerance levels associated with both solutions*

Test instance	Tolerance level								
	2%			1%			0.01%		
	CPLEX	Langr.	% diff	CPLEX	Langr.	% diff	CPLEX	Langr.	% diff
set2_10_100.txt	46,524.15	46,496.20	-0.06%*	45,843.42	46,164.98	0.70%	45,805.58	46,164.98	0.78%
set2_10_500.txt	220,224.89	221,333.50	0.50%	220,224.89	221,102.54	0.40%	218,854.20	221,102.54	1.03%
set2_10_1000.txt	479,074.37	484,905.92	1.22%	479,074.37	483,220.77	0.87%	478,894.53	479,238.74	0.07%
set2_10_2000.txt	892,089.76	903,142.45	1.24%	892,089.76	898,327.91	0.70%	891,882.17	892,512.80	0.07%
set2_10_3000.txt	1,328,056.13	1,344,953.56	1.27%	1,328,056.13	1,337,577.29	0.72%	1,327,894.85	1,328,510.50	0.05%
set2_10_4000.txt	1,824,948.12	1,855,723.00	1.69%	1,824,948.12	1,841,194.04	0.89%	1,824,543.51	1,826,182.54	0.09%
set2_10_5000.txt	2,429,097.51	2,455,507.51	1.09%	2,429,097.51	2,447,757.09	0.77%	2,428,823.47	2,429,205.15	0.02%
set2_10_6000.txt	2,980,322.13	3,014,317.33	1.14%	2,980,322.13	3,001,659.07	0.72%	2,980,322.13	2,983,707.72	0.11%
set2_10_7000.txt	3,286,944.54	3,321,522.91	1.05%	3,286,944.54	3,311,550.37	0.75%	3,286,525.85	3,291,404.22	0.15%
set2_10_8000.txt	3,723,125.46	3,782,589.61	1.60%	3,723,125.46	3,753,535.65	0.82%	3,722,635.87	3,724,247.28	0.04%
set2_10_9000.txt	4,391,346.93	4,464,858.49	1.67%	4,391,346.93	4,429,306.27	0.86%	4,390,885.28	4,392,558.85	0.04%
set2_10_10000.txt	4,491,745.86	4,491,745.94	0.00%	4,491,745.86	4,491,745.94	0.00%	4,491,745.86	4,491,745.94	0.00%
set2_50_100.txt	40,996.44	42,772.61	4.33%	40,730.67	42,772.61	5.01%	40,646.71	42,772.61	5.23%
set2_50_500.txt	177,094.67	180,731.79	2.05%	177,094.67	180,731.79	2.05%	176,861.96	180,731.79	2.19%
set2_50_1000.txt	351,997.41	352,704.67	0.20%	348,125.90	352,671.24	1.31%	N/A	352,671.24	N/A
set2_50_2000.txt	727,648.28	733,804.68	0.85%	727,648.28	733,804.68	0.85%	N/A	733,804.68	N/A
set2_50_3000.txt	1,103,652.63	1,119,391.12	1.43%	1,103,652.63	1,113,577.70	0.90%	N/A	1,113,577.70	N/A
set2_50_4000.txt	1,408,338.74	1,430,353.09	1.56%	1,408,338.74	1,423,726.57	1.09%	N/A	1,423,726.57	N/A
set2_50_5000.txt	1,831,451.55	1,864,618.96	1.81%	1,831,451.55	1,864,618.96	1.81%	N/A	1,864,618.96	N/A
set2_50_6000.txt	2,154,781.36	2,237,995.46	3.86%	2,154,781.36	2,237,995.46	3.86%	N/A	2,237,995.46	N/A
set2_50_7000.txt	N/A	2,462,018.33	N/A	N/A	2,449,246.83	N/A	N/A	2,449,246.83	N/A
set2_50_8000.txt	N/A	3,044,801.36	N/A	N/A	3,044,801.36	N/A	N/A	3,044,801.36	N/A
set2_50_9000.txt	N/A	3,322,044.53	N/A	N/A	3,318,690.60	N/A	N/A	3,318,690.60	N/A
set2_50_10000.txt	N/A	3,816,505.59	N/A	N/A	3,816,505.59	N/A	N/A	3,816,505.59	N/A

Table 7.5: Our data sets, cost solution.

* *due to the tolerance levels associated with both solutions*

Test instance	Tolerance level								
	5%			4%			3%		
	CPLEX	Langr.	Lagr % of CPLEX	CPLEX	Langr.	Lagr % of CPLEX	CPLEX	Langr.	Lagr % of CPLEX
set2_10_100.txt	62	16	25.81%	62	0	0.00%	62	0	0.00%
set2_10_500.txt	203	16	7.88%	203	0	0.00%	203	16	7.88%
set2_10_1000.txt	375	46	12.27%	359	31	8.64%	359	47	13.09%
set2_10_2000.txt	547	0	0.00%	578	0	0.00%	578	16	2.77%
set2_10_3000.txt	844	16	1.90%	953	0	0.00%	953	16	1.68%
set2_10_4000.txt	1,734	15	0.87%	1,609	31	1.93%	1,609	796	49.47%
set2_10_5000.txt	1,796	16	0.89%	1,891	15	0.79%	1,891	16	0.85%
set2_10_6000.txt	2,312	16	0.69%	3,469	0	0.00%	3,469	15	0.43%
set2_10_7000.txt	2,610	15	0.57%	3,422	47	1.37%	3,422	31	0.91%
set2_10_8000.txt	2,984	16	0.54%	3,204	78	2.43%	3,204	31	0.97%
set2_10_9000.txt	3,922	16	0.41%	4,797	47	0.98%	4,797	47	0.98%
set2_10_10000.txt	3,125	0	0.00%	3,672	16	0.44%	3,672	16	0.44%
set2_50_100.txt	735	250	34.01%	625	250	40.00%	610	234	38.36%
set2_50_500.txt	1,438	171	11.89%	1,187	282	23.76%	1,141	1,000	87.64%
set2_50_1000.txt	5,016	453	9.03%	4,360	500	11.47%	3,500	1,234	35.26%
set2_50_2000.txt	9,766	1,422	14.56%	6,922	2,203	31.83%	7,203	2,328	32.32%
set2_50_3000.txt	9,203	1,594	17.32%	8,250	1,781	21.59%	7,000	3,156	45.09%
set2_50_4000.txt	12,765	94	0.74%	10,250	1,453	14.18%	9,782	1,390	14.21%
set2_50_5000.txt	19,813	4,969	25.08%	14,796	4,656	31.47%	17,219	4,250	24.68%
set2_50_6000.txt	23,344	14,141	60.58%	17,703	14,843	83.84%	17,922	15,609	87.09%
set2_50_7000.txt	N/A	125	N/A	N/A	2,422	N/A	N/A	3,047	N/A
set2_50_8000.txt	N/A	6,297	N/A	N/A	6,219	N/A	N/A	9,343	N/A
set2_50_9000.txt	N/A	7,266	N/A	N/A	8,250	N/A	N/A	7,938	N/A
set2_50_10000.txt	N/A	7,922	N/A	N/A	8,625	N/A	N/A	10,375	N/A

Table 7.6: Our data sets, execution time (ms).

Test instance	Tolerance level								
	2%			1%			0.01%		
	CPLEX	Langr.	Lagr % of CPLEX	CPLEX	Langr.	Lagr % of CPLEX	CPLEX	Langr.	Lagr % of CPLEX
set2_10_100.txt	47	47	100.00%	109	94	86.24%	187	94	50.27%
set2_10_500.txt	234	16	6.84%	312	250	80.13%	563	281	49.91%
set2_10_1000.txt	500	47	9.40%	516	47	9.11%	859	594	69.15%
set2_10_2000.txt	718	16	2.23%	734	16	2.18%	1,125	1,313	116.71%
set2_10_3000.txt	1,109	0	0.00%	1,109	31	2.80%	2,141	2,188	102.20%
set2_10_4000.txt	1,984	781	39.36%	1,593	828	51.98%	4,703	3,234	68.76%
set2_10_5000.txt	2,000	16	0.80%	2,188	31	1.42%	3,500	3,765	107.57%
set2_10_6000.txt	2,906	63	2.17%	3,031	31	1.02%	3,125	4,625	148.00%
set2_10_7000.txt	3,125	32	1.02%	3,281	125	3.81%	8,188	5,265	64.30%
set2_10_8000.txt	3,844	47	1.22%	3,954	125	3.16%	6,094	6,172	101.28%
set2_10_9000.txt	3,922	62	1.58%	4,953	172	3.47%	7,516	6,985	92.94%
set2_10_10000.txt	3,437	0	0.00%	3,218	0	0.00%	3,984	62	1.56%
set2_50_100.txt	703	234	33.29%	718	250	34.82%	1,250	250	20.00%
set2_50_500.txt	3,453	1,032	29.89%	3,328	1,031	30.98%	599,718	1,031	0.17%
set2_50_1000.txt	3,812	1,875	49.19%	6,703	2,563	38.24%	N/A	2,656	N/A
set2_50_2000.txt	7,031	2,594	36.89%	8,609	4,687	54.44%	N/A	5,063	N/A
set2_50_3000.txt	7,172	4,703	65.57%	6,828	6,828	100.00%	N/A	7,297	N/A
set2_50_4000.txt	10,640	4,000	37.59%	9,344	8,907	95.32%	N/A	8,875	N/A
set2_50_5000.txt	16,031	11,188	69.79%	15,063	12,703	84.33%	N/A	12,234	N/A
set2_50_6000.txt	18,438	16,469	89.32%	18,062	15,078	83.48%	N/A	14,843	N/A
set2_50_7000.txt	N/A	3,625	N/A	N/A	14,015	N/A	N/A	14,468	N/A
set2_50_8000.txt	N/A	17,938	N/A	N/A	17,031	N/A	N/A	17,968	N/A
set2_50_9000.txt	N/A	11,531	N/A	N/A	19,172	N/A	N/A	19,766	N/A
set2_50_10000.txt	N/A	23,594	N/A	N/A	23,000	N/A	N/A	24,344	N/A

Table 7.7: Our data sets, execution time (ms).

7.3.1 Problem Definition

The full problem formulation for a multi-commodity assignment problem is described in Chapter 5, Section 5.4. In this section, for simplicity we assume that connection cost c_{ij}^p is equal to overall transport and depot costs $(tc_{ij}^p + dc_{ij}^p)$ for each customer j to each facility i of product p, $i \in V_{DC}$, $j \in V_C$, $p \in P$

Glossary

V_{DC}	set of depots
V_C	set of customers
P	set of products
d_j^p	demand of customers j of product p, $j \in V_C$, $p \in P$
q_i^p	capacity of cases of facility i of product p, $i \in V_{DC}$, $p \in P$
n_i^p	number of stores assigned of facility i of product p, $i \in V_{DC}$, $p \in P$
c_{ij}^p	is the connection cost consisting of transporting and depot cost function based on the total demand of product p of customer j, d_j^p from facility i
x_{ij}	is the decision variable for the problem, $x_{ij} = 1$, if store j is allocated to facility i, and 0 otherwise

Mathematical formulation

Minimize

$$\sum_{i \in V_{DC}} \sum_{j \in V_C} \sum_{p \in V_P} c_{ij}^p x_{ij} \tag{7.19}$$

subject to

$$\sum_{i \in V_{DC}} x_{ij} = 1, \forall j \in V_C \tag{7.20}$$

$$\sum_{j \in V_C} d_j^p x_{ij} \leq q_i^p, \forall i \in V_{DC}, \forall p \in V_P \tag{7.21}$$

$$\sum_{j \in V_C} x_{ij} \leq n_i^p, \forall j \in V_C, \forall p \in P \tag{7.22}$$

$$x_{ij} \in \{0,1\}, i \in V_{DC}, j \in V_C \tag{7.23}$$

where formulation (7.19) aims to minimize the total connection cost of satisfying the total demand of all the stores, and constraints (7.20) with (7.23) guarantee that the demand for each store must be satisfied by one depot. Constraints (7.21) and (7.22) ensures that the capacity constraints for the facilities for each product type are not violated and (7.23) specifies that allocation is indivisible for the decision variable.

7.3.2 Solution Formulation

The solution formulation presented in this section is based on the LR formulation 1 in section 7.2.1 where only a single constraint is relaxed and lagrangian multipliers are used to determine a lower bound value as a main step in the Lagrangian relaxation. Let $\lambda_i^p \in \mathbb{R}, \forall i \in V_{DC}, \forall p \in V_P$. The relaxed model formulation is presented as follow:
Minimize

$$\sum_{i \in V_{DC}} \sum_{j \in V_C} \sum_{p \in V_P} c_{ij}^p x_{ij} + \sum_{i \in V_{DC}} \sum_{p \in V_P} \lambda_i^p (\sum_{j \in V_C} d_j^p x_{ij} - q_i^p) \tag{7.24}$$

subject to

$$\sum_{i \in V_{DC}} x_{ij} = 1, \forall j \in V_C \tag{7.25}$$

$$\sum_{j \in V_C} x_{ij} \leq n_i^p, \forall j \in V_C, \forall p \in P \tag{7.26}$$

$$x_{ij} \in \{0,1\}, i \in V_{DC}, j \in V_C \tag{7.27}$$

The relaxed formulation computes the difference between the total demand assigned to the facility i for a particular product p and its capacity of that product q_i^p on the right side of the formulation. The value of the total cost in the equation (7.24) will change if the capacity of facilities is violated or has not been exceeded. As part of the solution procedure, we only want to make adjustment to costs if a capacity is violated , therefore we would expect the term $(\sum_{j \in V_C} d_j^p x_{ij} - q_i^p)$ to equal zero, for any facility which has a spare capacity. Therefore, we constraint λ_i^p values to 0 if the capacity is not violated for a facility i.

To obtain the optimum lower bound to the relaxed formulation (LB^λ), we decompose problem (7.24) - (7.27) into $|V_C|$ subproblems. For a given set of multipliers, $\lambda_i^p \in \mathbb{R}$, we solve subproblem ($LB^j(\lambda^p)$) for each customer $j \in V_C$. This will assign each customer to the facility with minimal (augmented) connection cost, summed over all products, $p \in P$.

For each customer j, minimize

$$\sum_{i \in V_{DC}} (\sum_{p \in P} (c_{ij}^p + d_j^p \lambda_i^p)) x_{ij} \tag{7.28}$$

subject to

$$\sum_{i \in V_{DC}} x_{ij} = 1, \forall j \in V_C \tag{7.29}$$

$$\sum_{j \in V_C} x_{ij} \leq n_i^p, \forall j \in V_C, \forall p \in P \tag{7.30}$$

$$x_{ij} \in \{0, 1\}, i \in V_{DC}, j \in V_C \tag{7.31}$$

Then by setting:

$$LB(\lambda) = \sum_{j \in V_C} LB(\lambda)^j - \sum_{i \in V_{DC}} \sum_{p \in P} \lambda_i^p q_i^p \tag{7.32}$$

The relaxed problem (7.28) is solved by applying a greedy heuristic where each customer is allocated to the lowest total augmented cost $\sum_{p \in P}(c_{ij}^p + d_j^p \lambda_i^p)$ and the lower bound is calculated in (7.32) for a particular iteration of λ values. The allocation made for $LB(\lambda)$ is used to obtain a feasible upper bound $UB(\lambda)$, where those two values are used in the formula to update the Lagrangian multipliers. It is unlikely that the allocation used for the lower bound will be feasible, especially in a multi-product problem, such as we have here. Therefore, we need to construct a procedure of reallocating customers when facilities are over capacity.

For a single product problem, the customers were sorted in non-increasing order according to their demand, and then assigned to facilities based on the lower bound assignment without violating capacity constraints. When capacity constraints are violated for LB assignment, the customer is assigned to the next lowest augmented cost etc. If all facilities are overcapacity then the customer is allocated to the lowest true (non-augmented cost). However, the current model formulation have multiple products where each customer will have a certain demand for each product type, and each DC with have a maximum capacity for each product. Therefore, different settings were devised to ensure that allocating the customers produces feasible solutions when calculating UB assignment. Following settings were tested for finding feasible UB assignment:

1. (*Sorting using normalized demand.*) The customers are sorted in non-increasing sequence of the sum of the normalized demand over all products. The customers with highest value are assigned first to the appropriate depot. Thus, for customer j:
$$normalizedDemand_j = \sum_{p \in P} \frac{d_j^p}{\sum_{j \in V_C} d_j^p} \tag{7.33}$$

2. (*Sorting using highest fraction of normalized demand across all products.*) The highest fraction of normalized demand across all product types for each customer is chosen for sorting customers in non-increasing order. The customers with highest value are assigned first. Thus, for customer j:
$$highFractNormDemand_j = max\{for\ each\ p \in P : \frac{d_j^p}{\sum_{j \in V_C} d_j^p}\} \tag{7.34}$$

3. (*Sorting using normalized demand and the depot load ratio.*) The depot load ratio per product type is defined as the ratio of total demand of all customers per product type and total capacity of all depots for that product type. Thus, for each product p:
$$ratio^p = \frac{\sum_{j \in V_C} d_j^p}{\sum_{i \in V_{DC}} q_i^p} \tag{7.35}$$
The highest fraction of normalized demand across all product types (Setting 2) per customer is multiplied by the load ratio of that particular product. This value is used to sort customers in non-increasing order, where the customer with highest value is assigned first for UB assignment.

4. (*Sorting using highest demand.*) The highest demand across all product types per customer is used to sort customers in non-increasing order, where the customer with highest value is assigned first.

5. (*Sorting using highest demand and the depot load ratio.*) The highest demand across all product types per customer is multiplied by the load ratio of that particular product. This is used to sort customers in non-increasing order, where the customer with highest value is assigned first for UB assignment.

Updating the Lagrangian multipliers

To update a set of Lagrangian multipliers for each product, following formulations are used for each facility and each product at time step, k

$$s_i^{p,(k)} = \sum_{j \in V_C} x_{ij}^k d_j^p - q_i^p \tag{7.36}$$

where x_{ij}^k is the solution of the Lagrangian relaxation (7.24) - (7.27) using $\lambda_{ip}^k \in \mathbb{R}, \forall i \in V_{DC}, \forall p \in V_P$ as the Lagrangian multipliers. Now set

$$\lambda_i^{p,(k+1)} = \begin{cases} \lambda_i^{p,(k)} + \beta^k s_i^{p,(k)} & if\ s_i^{p,(k)} > 0 \\ 0 & otherwise \end{cases} \tag{7.37}$$

where β^k is a suitable scalar coefficient. The procedure will start by setting all the Lagrangian multipliers to zero. Formula (7.36) demonstrates that if a certain facility i and product p, $s_i^{p,(k)}$ is positive, it means that demand is higher than available capacity for that facility, and thus the corresponding value of λ_i^p will be increased to increase the cost of assigning customers to that facility in the next time, step $k+1$. If $s_i^{p,(k)}$ is a negative number, it means that there is spare capacity, therefore λ_i^p should be reduced to make that facility more attractive for assignment in the next iteration. This means that there is no need to adjust the multipliers if the facility is under capacity, therefore formula (7.37) ensures that the $\lambda_i^{p,(k)}$ are always positive.

Formula (7.38) is used to update proportionality coefficient $\beta^{p(k)}$, which is used for updating each set of λ_i^p values; where α could be a constant in the interval $(0, 2]$. In this research, α starts with 2 and after 20 or 30 iterations α value is halved if UB cost does not decrease.

$$\beta^{p(k)} = \frac{\alpha(UB(\lambda) - LB(\lambda))}{\sum_{i \in V_{DC}} (s_i^{p(k)})^2} \tag{7.38}$$

The algorithm for the Lagrangian relaxation for multiple products is very similar to the Algorithm 7.1 described for a single product in section 7.2. Therefore it would be very easy to adopt the algorithm to the description of the solution formulation for multiple products described in this chapter.

7.3.3 Discussion of the results

The main aim of this section is to analyse five different settings described in section 7.3.2 which are used for testing the allocation routine to determine a feasible solution for UB assignment. Determining a feasible UB assignment for a multi-product problem formulation is the key issue to ensure that a feasible solution is found to the stated problem. Because of the multi-product nature of the problem, sorting customers according to the demand of a single product does not incorporate characteristics of all other products, therefore more advanced approaches were introduced. The technique is tested on the Sainsbury's data and the results of the different settings are presented in Table 7.8. The solutions of different settings are not presented in terms of the execution times or compared to optimum solutions with different tolerance levels as in Section 7.2.3 for a single product because the aim of this section is to identify the best setting for the allocation of the customers in the multi-product setting. The results are compared among each other according to the difference between global lower and upper bounds, which provides a good indication to the quality of the solution.
The setting 1, where the customers are assigned in with the highest value of the normalized demand first, for the current case study produced a feasible solution with the highest gap difference between LB and UB solution of 3.19%. The reason could be that normalized demand is not a true representation of the real demand from the customers due to the multiple product configuration. Settings 2-5 produced feasible solutions (α=20) with the gap difference between LB and UB solution less than 0.25%. By analysing the solutions quality in the table, we can see that Settings 2-5 produce similar

Setting	Number of iterations for α	Solution cost	$(UB - LB)/LB$ (%)	Is solution feasible
1	20	3,103,028.32	3.19	yes
2	20	3,011,984.71	0.20	yes
2	30	3,010,055.49	0.13	yes
3	20	3,011,984.71	0.20	yes
3	30	3,011,984.71	0.20	yes
4	20	3,013,512.88	0.25	yes
5	20	3,012,745.33	0.22	yes
5	30	3,007,899.02	0.06	yes

Table 7.8: Multi-commodity allocation problem - results of different settings for UB assignment.

solutions among those settings, with difference of around 0.20%. Furthermore, after further tuning, where the parameter α was updated after 30 iteration in setting 5, the quality of the solution improved by 0.16% for setting 5 which is a good improvement on the solution quality.

As can be seen from the Table 7.8, the initial results are very encouraging and the technique produces good quality solutions, specially for setting 5. It needs further testing on the randomly generated test data for multiple products. Due to lack of benchmarking data in the literature for multiple products and the time constraints, further work will need to be done to better assess the quality of the generated solutions. In the current research our aim is to explore multi-objective optimization for facility location-allocation and allocation problem, for a single product where economic and environmental objectives are balanced.

7.4 Summary

This chapter presents two new Lagrangian Relaxation solution techniques to solve the capacitated allocation of customers to serving facilities with different relaxation procedures for single and multiple products, single source.

In the first LR procedure, the relaxation is done on the number of cases for a single product. The quality of the solutions and execution times of the LR were compared to the solutions found by CPLEX® for a number of the tolerance levels on the benchmarking instances and some new randomly generated larger data sets. As a result of the above analysis, the first LR approach finds good quality solutions within a reasonable amount of time. Where CPLEX® failed to find the solutions to the large data sets, the LR heuristic had no problems at all. The results show that LR is an efficient technique, which will be used for developing a multi-objective capacitated facility location problem tool, where the assignment is a sub-routine of the approach and presented in Chapter 9.

The second LR solution technique considers a situation where multiple products are available to the customers and relaxes the capacity (number of cases) constraint. The approach was tested on Sainsbury's data and will need further investigation on the randomly generated large size benchmarking instances to explore a wide range of situations.

Chapter 8

Multi-Objective Uncapacitated Facility Location Problem

8.1 Introduction

Previous chapters described model formulations and solutions techniques for the allocation of customers to distribution centres. Assignment is an essential part of the periodical re-evaluation needed to maintain the continued economic viability of a distribution network. In the present chapter we consider the facility location problem (FLP), which has a much longer planning horizon. The goal is to identify the optimum number and locations of depots or warehouses, in a distribution network, in which deliveries are made to local customers and/or goods are collected from local suppliers (see Figure 8.1). In addition, each customer (or supplier) can be assigned to exactly one depot. In practice there are many variations of the FLP, for example, storing inventory before it is transported to customers, or including transshipment points, where the goods are reloaded from the supplier to be forwarded to the retail stores. Also the FLP is easily adapted to identify the optimum number of recycling or collection facilities in a network. The facility location problem is not new to academia and has a very rich literature. For example, [70] describes the role of facility location models within a supply chain context as "an extremely interesting and fruitful application area domain".

Typical formulations for the FLP aim to minimize cost as a single objective. In this approach, the total cost is frequently expressed as a sum of various component expenses, most simply as transportation and fixed costs. However, in many practical situations, the

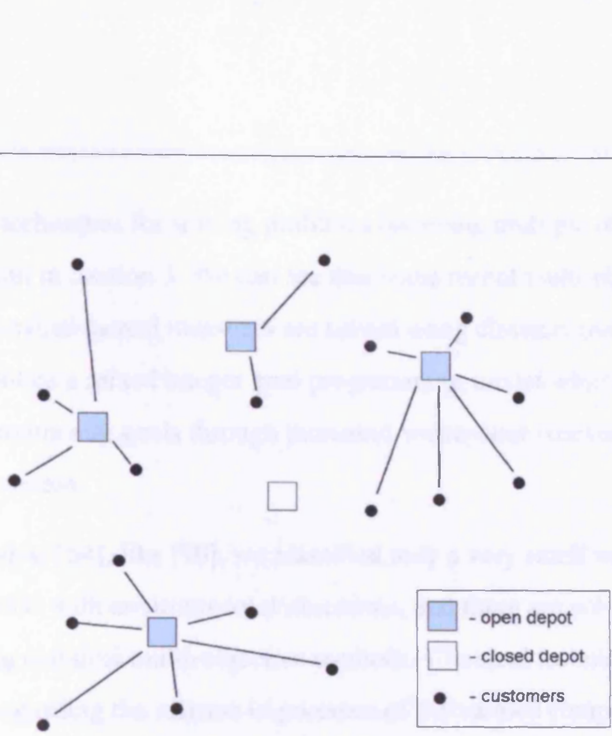


Figure 8.1: The Uncapacitated Facility Location Problem.

optimum design may involve dealing with multiple and sometimes conflicting objectives. In a recent survey [70], 98 articles published in the last decade were categorized: 75% had a single cost minimization objective, 16% had a single profit maximization objective and only 9% were modelled with multiple and conflicting objectives. The multiple objectives mentioned include resource utilization and customer responsiveness, in addition to the standard economic objectives.

Recent concerns regarding climate change, however, have shifted the focus of modelling to incorporate environmental objectives. For example, [61] present a generic mathematical programming model for assisting the strategic long range planning and design of a bulk chemical network. Their multi-objective mixed-integer programming problem is solved using an approach which applies the ϵ -constraint method [99] as part of their multi-objective solution. The model minimizes the environmental impact resulting from the operations of the entire network, and simultaneously maximizes the profitability of the network. Another example, [83] develop a framework for the design and evaluation of sustainable logistics networks, in which profitability and environmental impacts are balanced. The re-organization of a European pulp and paper logistic network is used to illustrate proposed methodology.

There are different techniques for solving problems involving multiple objectives which are discussed in detail in Section 3. We can see that some recent multi-objective models which incorporate environmental measures are solved using classical methods. For example, [92] formulate a mixed integer goal programming model which captures economic and environmental goals through increased wastepaper recovery for a paper recycling logistics system.

From our recent review [54], like [70], we identified only a very small number of multi-objective models with environmental objectives, and these are solved predominantly using classical multi-objective methods. Classical techniques rely on *a priori* judgements regarding the relative importance of the various component objectives. In contrast, there are other approaches that do not rely on such assumptions and treat all objectives equally. Such techniques will generate a set of solutions, with the objectives traded off in different ways, instead of a single optimum with respect to a predefined (perhaps arbitrary) trade-off situation. In this way it is possible to provide a decision maker with sufficient choices to make an informed judgement when trading off the relative merits of the conflicting objectives. In this research we explore elitist multi-objective evolutionary algorithms for the strategic modelling of a logistics network, where economic and environmental objectives are considered simultaneously.

In this chapter, two evolutionary algorithms are explored on a multi-objective uncapacitated facility location problem (MOUFLP):

- Non-Dominated Sorting Genetic Algorithm (NSGA-II) [30] (Algorithm[3.2]). It is a well tested algorithm, where elitism is preserved and has a diversity mechanism in terms of calculating crowding distance. It is much more complex to implement than other evolutionary algorithms like SEAMO2.
- Simple Evolutionary Algorithm for Multi-Objective Optimization (SEAMO2) [76] (Algorithm[(3.5]). This algorithm is simple to implement and includes a specific mechanism to improve the quality and range of the solutions set. Known weakness: there is no specific mechanism to ensure an even spread of solutions across the Pareto front.

Both algorithms are described in full details in Chapter 3, section 3.6.

The uncapacitated facility location problem (UFLP) is the simplest form of FLP, and involves identifying which depots to open, assigning the customers to open depots, and has no constraints regarding the capacity of the facilities (Figure 8.1). Our multi-objective model has two different settings: two-objectives (min cost - min environmental impact) and three-objectives (min cost, min environmental impact and min uncovered demand). For this simple model, our environmental objective is formulated in a similar way to our objective measuring economic cost, and is made up of two components: depot costs and transportation costs. However, we weight these components differently for assessing the environmental impact, working under the assumption that the environmental cost of transport is large in comparison to the impact involved in operating distribution centres or warehouses (in terms of CO_2 emissions, for example). We further conjecture that the full impact on the environment is not reflected in the costs incurred by logistics operators. Based on these ideas, we investigate a number of "what if ?" scenarios, by varying the relative weighting of the impact of transport versus depots on the environment to provide sets of non-dominated solutions to some test instances. This is an exploratory study aimed at investigating the potential of multi-objective optimization techniques for the FLP.

8.2 Our Multi-objective optimization model

The main drivers in traditional logistics network design are to reduce total costs and improve customer service levels. Due to recent concerns regarding climate change, minimizing the environmental impact from depots and transport needs to be addressed as well. Our proposed multi-objective uncapacitated facility location problem incorporates those three goals. Therefore, the problem definition in this chapter is a mixture of three mathematical programming formulations: the uncapacitated facility location problem, a revised UFLP with environmental weightings, and the maximal covering location problem (MCLP). The MCLP involves first asserting a global (ideal) maximum distance between customer and serving depot. Once customers have been assigned to depots, covered demand can be measured as the percentage of customer demand met within the given distance radius. Customers assigned to depots that are further away than this maximum, represent uncovered demand which is equivalent to (100 - percentage of covered demand)%.

The UFLP and MCLP models we use have been adapted from Villegas *et al.* [109] who originally based their formulation on [94]. Villegas *et al.* [109] present a bi-objective UFLP (min cost - max coverage) in their paper. To solve the problem, they designed and implemented three different algorithms to obtain a good approximation of the Pareto frontier. The algorithms are based on the Non-Dominated Sorting Genetic Algorithm, the Pareto Archive Evolution Strategy and on mathematical programming.

8.2.1 Problem formulation and objective functions

We will assume that the customers each have a certain demand and that transportation costs and fixed costs for the open depots. We further assume that at least one depot from a set of depots will be open, and that each depot will serve its customers directly. The model does not include inter-depot movements of transport to ensure the goods flow between depots. The problem is to determine how many depots to locate, where to locate them and which depot serves which customer, in order to satisfy the two or three objectives: minimize cost, minimize environmental impact and minimize uncovered demand. Solving this problem requires two main routines: one to determine which depots to open, and the other to assign the customers to the open depots (the assignment rule), where each customer is assigned to exactly one depot. The values of the two or three objectives can be computed once a network configuration has been defined.

The following notation is used in the formulation of the model:

$V_{DC} = \{1...i\}$	set of potential depots;
$V_C = \{1...j\}$	set of customers;
c_{ij}	transportation cost of attending demand from customer j to depot i;
f_i	fixed cost for operating a depot i;
d_j	demand of customer j that could not be attended within D_{max}

	by particular depot i;
h_{ij}	distance between depot i and customer j;
D_{max}	maximal covering distance - the customers within this distance to an open depot are considered well served;
V_{unc}	set of depots that could not attend customer i within the maximal covering distance D_{max};

The decision variables are:

x_{ij} equals 1 if the whole demand of customer j is attended by depot i and 0 otherwise;

y_i equals 1 if depot is chosen to operate and 0 otherwise;

The following objectives functions are considered simultaneously as part of the location design:

- *Minimising costs.* The objective is to find the best number and location of depots that minimizes total transportation and fixed costs. The first term represents the cost of attending demand of customers by the open depots and the second term represents the fixed facility cost of operating depots.

$$minimize[\sum_{i \in V_{DC}} \sum_{j \in V_C} c_{ij} x_{ij} + \sum_{i \in V_{DC}} f_i y_i] \tag{8.1}$$

- *Minimising uncovered demand.*

  The objective measures total uncovered demand as a sum of the demand of customers which could not be attended by depot within maximal covering distance.

$$minimize[\sum_{j \in V_C} d_j \sum_{i \in V_{unc}} x_{ij}] \tag{8.2}$$

- *Minimizing the environmental impact from transport and depots.* The objective is to find the best number and location of facilities that minimizes the total environmental impact from transportation and depots. This is essentially the same formulation as we use to minimize economic costs, but we introduce W_T and W_F to weight the transport and fixed costs, respectively, for environmental impact. In this model, higher values of W_T imply worse pollution from transport.

$$minimize[\sum_{i \in V_{DC}} \sum_{j \in V_C} c_{ij} * W_T * x_{ij} + \sum_{i \in V_{DC}} f_i * W_F * y_i] \tag{8.3}$$

  where W_T is the factor which derives the environmental impact from transport in relation to transportation costs and W_F is the factor which derives the environmental impact from depots in relation to fixed costs. For the present study we used following values: $W_F = 1$ and $W_T \in [1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24]$. Those particular values for W_T were chosen to explore different "what if ?" scenarios.

- *Subject to following constraints:*

$$\sum_{i \in V_{DC}} x_{ij} = 1, j \in V_C \tag{8.4}$$

$$x_{ij} \leq y_i, j \in V_C, i \in V_{DC} \tag{8.5}$$

$$x_{ij} \in \{0, 1\}, j \in V_C, i \in V_{DC} \tag{8.6}$$

$$y_i \in \{0, 1\}, i \in V_{DC} \tag{8.7}$$

Constraints 8.4 and 8.6 ensure that each customer is attended by only one depot. Constraint 8.5 assigns the customers to open depots. Constraints 8.6 and 8.7 define decision variables as binary.

For our analysis we looked at two different objective settings:

1. for the two-objective UFLP: minimizing costs and minimizing environmental impact;
2. for the three-objective UFLP: minimizing cost, minimizing environmental impact and minimizing uncovered demand.

8.2.2 Test data.

From our research, we confirmed findings published in Villegas *et al.* [109]: that there are no available MOUFLP test instances in the public domain for benchmarking. Especially for our research, we needed instances which consider environmental information. To begin our research, we obtained bi-objective problem instances (min cost and max coverage) from [109] of two different types: instance A and instance B. The difference between the instances is that the locations have been generated in two different ways: uniformly distributed locations within a square (instance A) or depot locations chosen from customer locations (instance B). A and B instances come in three different sizes: 10 depots-25 customers, 30 depots-75 customers and 50 depots-150 customers. Each size also differs in its fixed depot cost structure (uniformly distributed (C1-C3) or the same fixed costs for all depots (C4-C6)). For example, instance A10–25C3 is an instance of type A with 10 available depots, 25 customers and a uniformly distributed fixed depot cost structure. In total, 26 different problem instances are provided. For our analysis, we have chosen a representative sample consisting of one instance of each type, size and cost structure - in total 12 test instances. For example, the following instances of type A are used for analysis: A10–25C3, A10–25C6, A30–75C3, A30–75C6 etc.

The data sets described above model economic costs and coverage but not environmental costs. To include an environmental objective, we used the simple weighted model described above, and applied it to the fixed costs and transportation costs of the "standard" UFLP. We know that the environmental impact from transport is closely related to fuel consumption. However, there are other factors that have an impact on the actual levels of emissions from transportation, such as the speed and acceleration of the vehicle; the load on the engine over the distance traveled; the type of fuel used, vehicle condition, engine size etc. [82]. Of course, all of these factors will also impact on economic cost. On the other hand, other expenses, such as vehicle maintenance, road tax, training costs, drivers' wages etc., will not directly impact on the environment in the same way. Regarding depot costs, the environmental impact from depots comes from the electricity and gas consumption by the buildings. Economic costs for depots also include rent/rates and staffing costs. In this chapter we will assume that transport has a relatively greater impact on the environment than depots, relative to economic costs, and we will use our simple model to explore various scenarios related to different weightings of the environmental impact of transport.

8.3 Solution encoding and assignment procedure

Solution encoding. Each solution for MOUFLP is encoded as a binary string of length equal to the total number of (potential) depots, where each bit indicates whether depot is open (value of 1) or closed (value of 0), e.g. 1101100100 represents 10 potential depots with depot 1,2,3,4,5 and 8 open. However, a solution also involves the assignment of customers to depots at the minimum transportation costs.

Assignment procedure. In a location models it is very important to decide how the customers are assigned to the particular facilities. In some circumstances, the assignment depends on the distance or travel time, in other cases it could depend on the range or quality of the products dispatched or collected. Our model incorporates a customer service level objective, therefore we used the assignment procedure described in [109], which tries to minimize cost without impacting on coverage. Provided a customer is located within a given maximum distance radius, D_{max}, then that customer is assigned to the depot at the minimum transportation cost. If a customer cannot be covered (i.e., the nearest depot is further than D_{max}) then it is assigned to the depot with the smallest transportation cost, regardless of its distance. Ties are broken on a first-come-first-served basis.

8.4 Comparing and tuning evolutionary algorithms

The two evolutionary algorithms NSGA-II (Algorithm 3.2) and SEAMO2 (Algorithm 3.5) are implemented in Java and adopted for MOUFLP to obtain a good approximation of the Pareto frontier. Chapter 3 discusses both evolutionary algorithms in details. Two test problems from Villegas *et al.* [109] (A30-75C3 and A30-75C6) are used to compare the algorithms on the quality of the non-dominated solutions using the *S* metric [114, 115]. The difference between the C3 and C6 problems is that the latter has the same fixed costs for all open depots and the former has different costs. The *S* metric or hypervolume measure is a measure of the volume of the dominated space determined by the number of objectives which are enclosed by the nondominated points and the point of origin. This is a measure of the quality of solution set for a given nondominated set A. *Definition 5* defines the hypervolume metric and taken from Zitzler's thesis [115]:

Definition 5. (Size of the dominated space) Let $A = (x_1, x_2, ..., x_l) \subseteq X$ be a set of l decision vectors. The function $S(A)$ gives the volume enclosed by the union of the polytopes $p_1, p_2, ...p_l$, where each p_i is formed by the intersections of the following hyperplanes arising out of x_i , along with the axes: for each axis in the objective space, there exists a hyperplane perpendicular to the axis and passing through the point $(f_1(x_i), f_2(x_i), ..., f_k(x_i))$. In the two-dimensional case, each p_i represents a rectangle defined by the points (0, 0) and $(f_1(x_i), f_2(x_i))$.

Definition 5 assumes a maximazation problem and since then other models were developed for the problem where all objectives are minimized [31, 115, 133]. For example, a reference point (vector of a worst objective function values) could be used to calculate a hypercube for each solution which is after used to calculate a hypervolume metric [31]. In this thesis, we use Zitzler's definition and convert our model to a maximization problem to calculate the *S* metric with following units: maximizing 'unspent' cost, covered demand and 'unspent' environmental impact. Maximizing 'unspent' cost represents maximizing the difference between upper bound limit for costs and overall costs for a particular open depot and lowest assignment combination. The upper bound limit is calculated as a result of assigning stores to the furthest depot while all depots are open and consist of fixed and connection costs. Similar procedure was done to convert minimizing environmental impact to maximizing 'unspent' environmental impact, where we maximize the difference between upper bound limit for impact and overall impact from the open depots/lowest assignment combination. Covered demand represents the percentage of the total demand which has been covered within given distance D_{max}. The maximazation units discussed above are used in the Figures 8.3, 8.4, 8.5.

Before comparing evolutionary algorithms, a fine tuning of the algorithms was carried out involving an exploration of a range of settings such as different crossovers, crossover probabilities, population sizes etc. Twenty independent runs of each test problem and setting are conducted for each algorithm and the final result (per problem instance and algorithm) is the approximate Pareto frontier obtained by aggregating the fronts of the 20 independent runs. The two algorithms were compared under the same number of iterations (10,000) and under equal execution time (10,000 ms) to ensure fairness in comparing algorithms.

Tuning the individual evolutionary algorithm. Before comparing the NSGA-II and SEAMO2 algorithms for MOUFLP, each algorithm was tuned to its best performance. In order to make comparisons fair, we used the same number of evaluations (10,000 iterations) for each algorithm. For the experiments, two different combinations for population size and numbers of generations are used: smaller population/bigger iterations (pop=40 gen=250) and bigger population/smaller iterations (pop=250 gen=40).

Figure 8.2 illustrates a flow chart of all experiments which were undertaken for fine tuning and comparing different settings for both algorithms. In the step 1, each algorithm is run 20 times for different types of crossover: one-point, two-point, uniform and no crossover on two instances with population size of 40 and number of generations of 250. The mutation was applied after the crossover was performed. After the best crossover was chosen, the number of generations was changed to 40 and the population size to 250 and different population and generation sizes were compared. The step 2 in Figure 8.2 illustrates a flow of different experiments when NSGA-II and SEAMO2 algorithms are compared under equal number of the evaluations and equal number of execution times. The next subsection describes this process in more detail. One-way analysis of variance

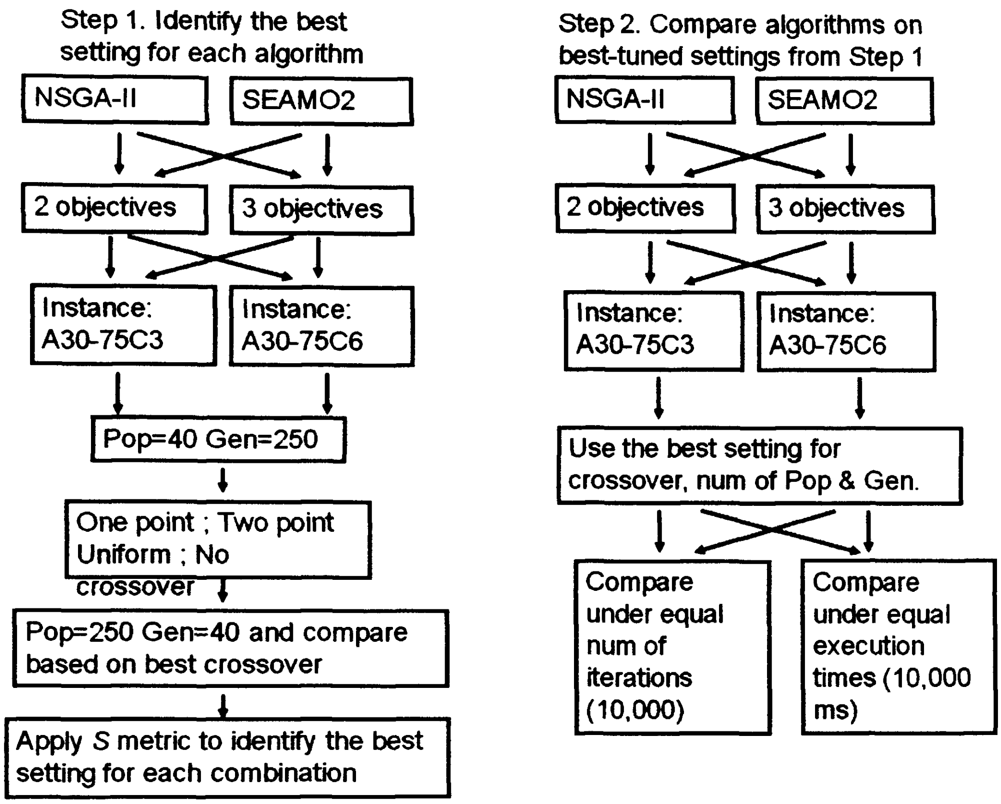


Figure 8.2: Tunable settings for MOEA.

on the S metrics was used to compare different crossovers and the best-tuned settings, depending on the number of objectives, which are statistically significant, are presented in the Table 8.1.

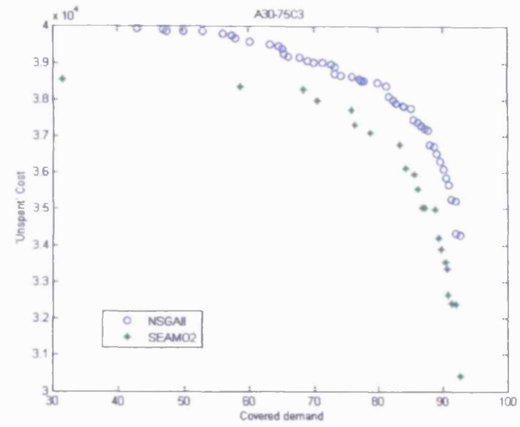
Comparing NSGA-II and SEAMO2. Tuned to the best performance, NSGA-II and SEAMO2 algorithms are compared (maximization of two/three objectives) under two requirements: 1)the same number of evaluations of 10,000 and 2) equal execution time of 10,000 ms for each run. In Chapter 3, we discuss a *selection probability* for parameters of evolutionary algorithms where the crossover probability (p_c) is used to determine the number of chromosomes participating in the crossover and the mutation probability (p_m) determines the number of bits that will be mutated. When the value of the probability is equal to 0% then no crossover or mutation has been performed and offspring has the same properties as it's parent. After initial experiments, $p_c = 0.7$ and $p_m = 0.06$ were used as final parameters in NSGA-II.

Algorithm	Number of obj.	Best-tuned settings for algorithms
NSGA-II	2 obj.	Two-point crossover pop=40, gen=250
	3 obj.	No crossover pop=250, gen=40
SEAMO2	2 obj.	Uniform crossover pop=40, gen=250
	3 obj.	Uniform crossover pop=40, gen=250

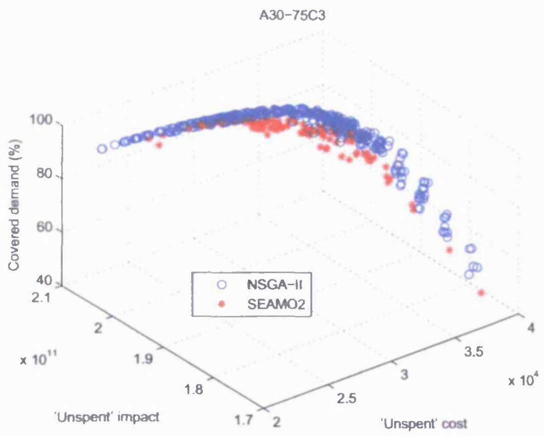
Table 8.1: Best-tuned settings for NSGA-II and SEAMO2.

Figure 8.3(a) and 8.3(b) show the approximate Pareto frontier for two and three objectives under equal numbers of evaluations obtained by both algorithms. Table 8.2 shows the execution time and the number of non-dominated solutions for each algorithm. In both cases NSGA-II outperformed SEAMO2 by obtaining statistically significant results as a result of undertaking one-way analysis of variance on the S metrics. However, SEAMO2 algorithm was much faster than NSGA-II. SEAMO2 is an algorithm which is much simpler to implement compare to NSGA-II and preserves best solutions during the execution time. Therefore, by running SEAMO2 and NSGA-II algorithms under the same execution time, the SEAMO2 algorithm could find an approximate Pareto frontier of the same (perhaps better) quality as the NSGA-II algorithm faster.

Figure 8.4(a) and 8.5(a) show the approximate Pareto frontier for two and three objectives under equal execution time of 10,000 ms for each run obtained by both algorithms. The final approximate Pareto frontiers visually indicate very little difference for two and for three objectives. The reason could be that SEAMO2 has a simple and fast search strategy that obtains a good quality approximate frontier quickly. But the interesting results you can see in Figure 8.4(b) and 8.5(b) which show box plots for the S metric for the two algorithms. Each plot represent the distribution of the non-dominated space for 20 independent runs and results are statistically significant in favor of



(a) Two objectives



(b) Three objectives

Figure 8.3: Non-dominated solutions from 20 runs on A30-75C3, NSGA-II and SEAMO2 under equal number of evaluations of 10,000 for each run.

NSGA-II. Furthermore, the gap between two algorithms is bigger for two objectives.

Also, Table 8.3 shows that NSGA-II produces a larger number of non-dominated solutions compared to SEAMO2.

To summarize, we observed from the pilot experiments, that the NSGA-II algorithm performed generally better than the SEAMO2 algorithm in terms of quality of the

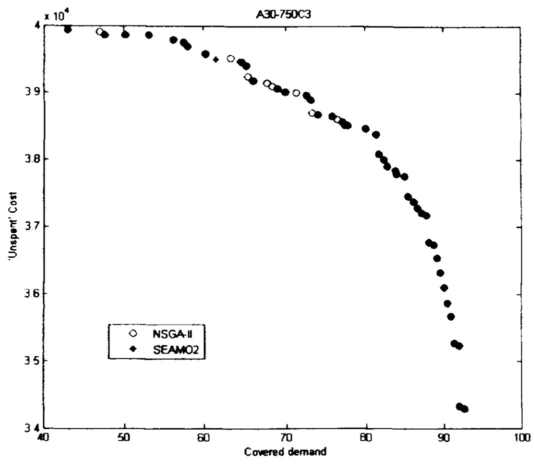
	NSGA-II		SEAMO2	
	time (ms)	Non-dominated solutions	time(ms)	Non-dominated solutions
Two objectives: cost, coverage				
A30-75C3	53,909	50	532	22
A30-75C6	53,951	73	606	29
Three objectives: cost, impact coverage				
A30-75C3	190,859	397	564	111
A30-75C6	187,965	120	622	33

Table 8.2: NSGA-II and SEAMO2 under equal number of evaluations of 10,000 for each run, 20 runs.

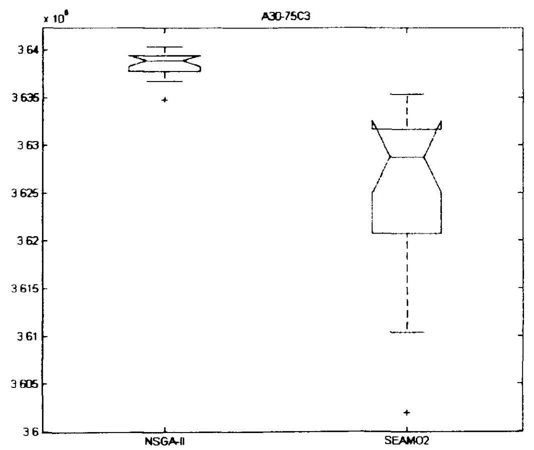
	NSGA-II		SEAMO2	
	time (ms)	Non-dominated solutions	time(ms)	Non-dominated solutions
Two objectives: cost, coverage				
A30-75C3	200,000	52	200,000	44
A30-75C6	200,000	75	200,000	63
Three objectives: cost, impact coverage				
A30-75C3	200,000	443	200,000	254
A30-75C6	200,000	116	200,000	72

Table 8.3: NSGA-II and SEAMO2 under equal execution time for 20 runs .

approximation of Pareto front. However, the SEAMO2 algorithm was very efficient in terms of execution time. This means that for large size data sets, SEAMO2 would be able to find non-dominated solutions more quickly and provide the decision-maker with an initial set of solutions, which can be explored further if desired. The SEAMO2 algorithm is used in Chapter 9 for modelling capacitated facility location problem for large size data sets, where execution time could be challenging. However, for the current data sets, NSGA-II was chosen for the experimental analysis described in the rest of this



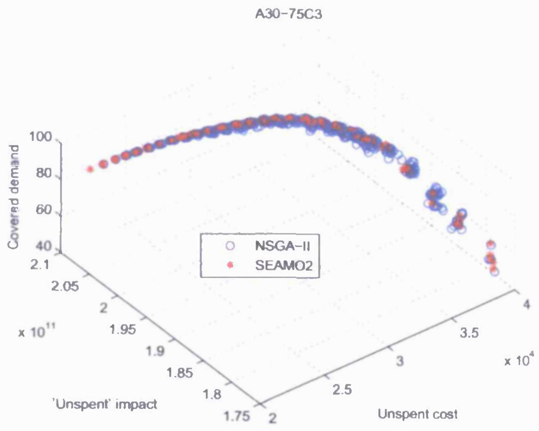
(a) Non-dominated solutions from 20 runs



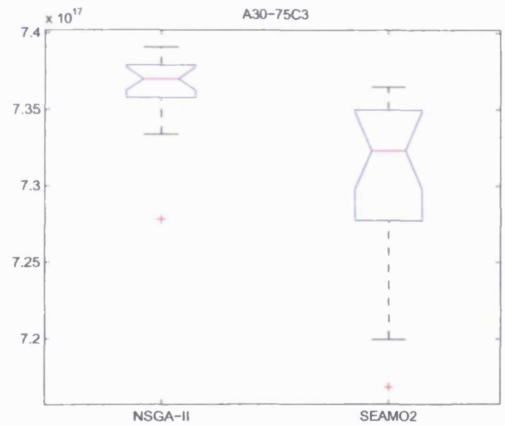
(b) Dominated space for 20 runs

Figure 8.4: Instance A30-75C3, comparison of NSGA-II and SEAMO2 under equal execution time of 10,000 ms for each run (two objectives).

chapter where two-point crossover was used for the two-objective problem and no crossover for the three objective problem.



(a) Non-dominated solutions from 20 runs



(b) Dominated space for 20 runs

Figure 8.5: Instance A30-75C3, comparison of NSGA-II and SEAMO2 under equal execution time of 10,000 ms for each run (three objectives).

8.5 Experimental method

The purpose of our experiments is to answer the following questions for each MOUFLP, the two-objective problem (min cost and min environmental impact) and the three-objective problem (min cost, min uncovered demand and min environmental impact):

1. Does this approach hold promise – do we obtain a reasonable trade-off front?
2. What happens to the solution set as we explore scenarios in which the environmental impact of transport increases disproportionately to its cost?
3. How do we select suitable trade-off solutions from the approximate Pareto front?

As previously mentioned, six instances of type A and six similar instances of type B are selected from the test data taken from [109]. Recall that these instances have data for a two-objective problem (economic cost, coverage), and we applied our environmental weightings, W_T and W_F, to transport and fixed costs, respectively. The plan is to assess the environmental impact for a range of "what if?" scenarios, in which $W_F = 1$ in all cases, and $W_T \in [1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24]$. We assume that the environmental impact of transport outweighs the environmental cost of maintaining depots. For each problem instance we performed 12 experiments, one for each of the above environmental factors for transport (W_T). In each run, the initial solution was created randomly at the start.

The NSGA-II algorithm creates a child population from its parent population using fast non-dominated sort, crossover and mutation. The parent population (P_t) and child population (C_t) is set to the same size N=40 for our experiments, and for each run, on every instance, the number of generations was 250. After the initial experiments, we settled on the crossover probability $p_c = 0.7$ for all the tests. Two-point crossover was used for the two-objective problem and no crossover for the three objective problem, due to results which are discussed in Section 8.4. A mutation probability of $p_m = 0.06$ was used across all the settings and all the test instances. Experiments are conducted using Java 2, on a PC with an Intel Pentium D CPU 3.4 GHz and 2 GB RAM.

8.6 Results

In this section we shall attempt to answer the questions posed in the previous section. Consider a situation in which $W_T = W_F = 1$. This corresponds to identical objectives for environmental impact and economic cost. For this special case, there is only one global optimum for the economic cost versus environmental impact model, because the problem reduces to a single objective. On the other hand, as the value of W_T is allowed to increase, one would expect to obtain sets of non-dominated solutions in which high transport impact favours higher numbers of open depots than are cost effective when considered from the point of view of economic cost.

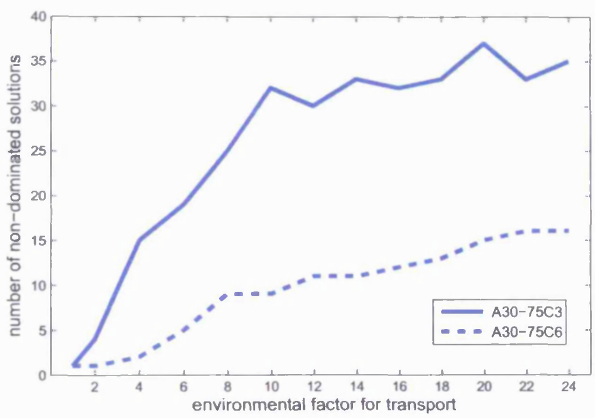


Figure 8.6: Total number of non-dominated solutions for different environmental factors from transport for two-objective problem (cost-impact).

Figure 8.6 shows how the number of non-dominated solutions obtained by NSGA-II changes as the environmental factor increases from 1 to 24. This diagram illustrates the situation for two of our instances (A30-75C3 and A30-75C6). However, the pattern is similar for the other 10 instances. We can see that when $W_T = 1$, there is a single solution, as expected. As W_T increases, however, so does the size of the non-dominated set. In the case of A30-75C6, the size of the solution set stabilizes at about 15, while A30-75C3 settles at about 35. Due to the exploratory nature of the current research, the curves of the Pareto frontier represent single runs for each W_T setting; hence, their lack of smoothness. The solutions depend on the scale - more solutions are found as the environmental impact from transport increases. Visual representation of the approximate Pareto fronts can be seen in Figure 8.7 and Figure 8.8 where a transport factor of 6 was used in the former and a transport factor of 16 in the latter case. As expected, we observe that more depots need to be opened to mitigate the environmental impact than is desirable from the point of view of economic cost. For example, in Figure 8.7 ($W_T = 6$) the extreme solutions require 2 depots for minimizing cost, and 5 depots for minimizing the environmental impact. In Figure 8.8 ($W_T = 16$) even more depots (8) are required to mitigate the environmental cost of transport.

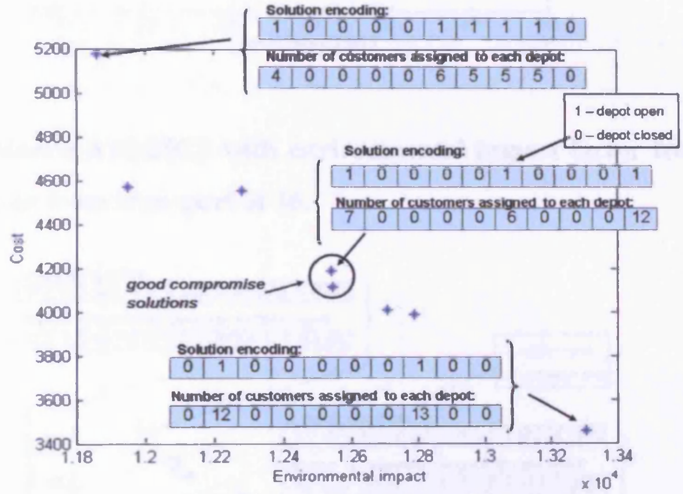


Figure 8.7: Instance A10-25C3 with environmental impact factor from depot of 1 and impact factor from transport of 6.

Now we are going take a more detailed look at how increases in the environmental factor from transport impacts on the required number of the open depots, for the two-objective and three-objective problems. For each of the 12 settings for W_T we will examine the extreme solution that minimizes environmental impact at the expense of economic cost (top left of Figures 8.7 and 8.8). A similar pattern can be seen in Figure 8.9 for three objective problem. Figure 8.10 shows how the number of depots increase with increasing W_T for two-objective problem, and Figure 8.11 shows similar findings for the three-objective problem.

Now, to answer the three questions posed in Section 8.5.

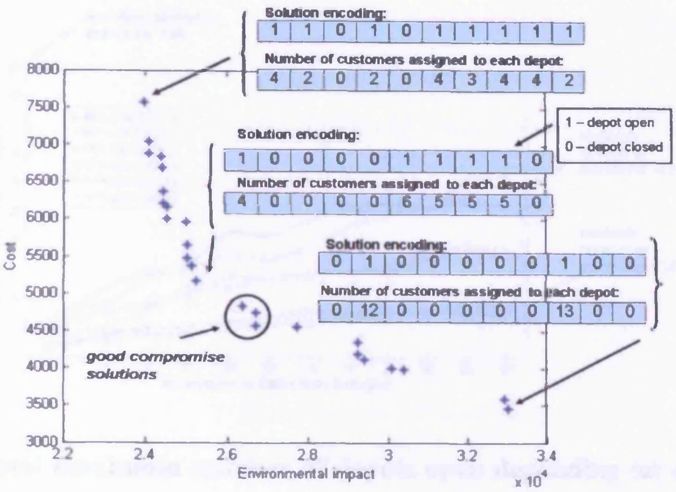


Figure 8.8: Instance A10-25C3 with environmental impact factor from depot of 1 and impact factor from transport of 16.

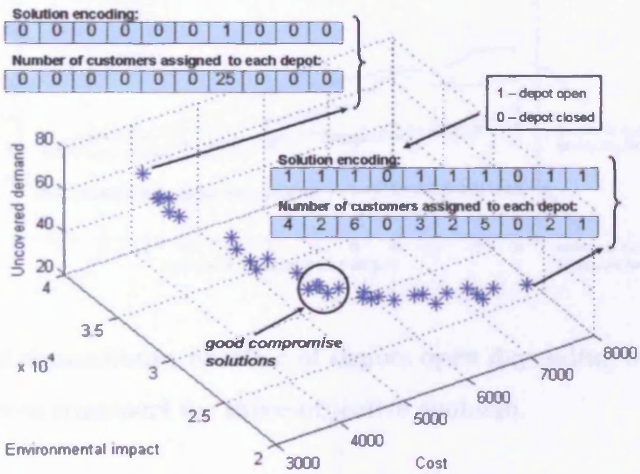


Figure 8.9: Instance A10-25C3 with environmental impact factor from depot of 1 and impact factor from transport of 16.

1. We do obtain a reasonable trade-off front with a range of solutions, indicating that this approach is worth pursuing further, until such time that environmental cost is fully absorbed into the economic costs incurred by the stakeholders.

2. As the environmental impact of transport increases disproportionately with the

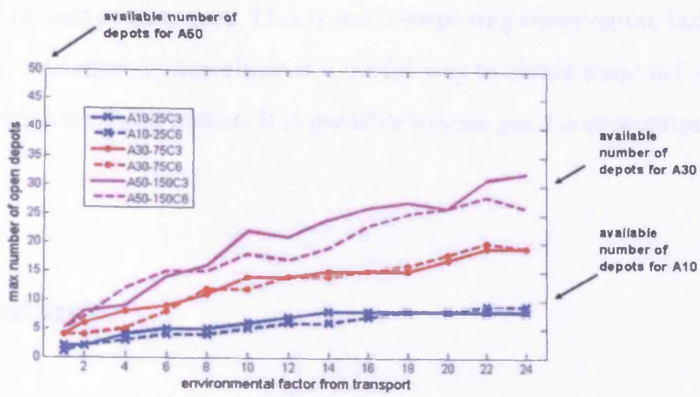


Figure 8.10: Total maximum number of depots open depending on environmental impact factor from transport for two-objective problem.

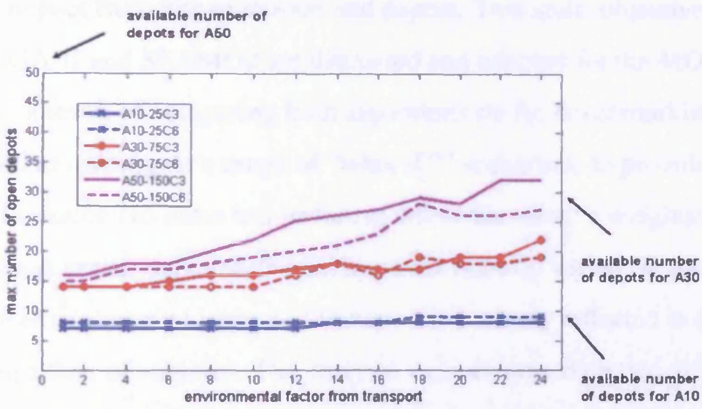


Figure 8.11: Total maximum number of depots open depending on environmental impact factor from transport for three-objective problem.

cost of operating depots, the environmentally friendly solution will require more open depots than is cost effective from an economic point of view.

3. We can spot good compromise solutions, for example as indicated in Figures 8.7, 8.8, and 8.9. We can select solutions with relatively low environmental impact, just before the curve steepens towards very high economic costs. At this stage there are only very small environmental gains to be made at very high economic cost.

To conclude, from the two and three objective studies it may be desirable to open more depots than may be optimal from a cost only perspective, in order to reduce the environmental impact of transport. This is not a surprising observation, but our studies indicate that an evolutionary algorithm is a useful way to obtain trade-off solutions to present to a human decision maker. It is possible to spot good compromise solutions in this way.

8.7 Summary

This chapter describes a MOUFLP with an environmental objective. The model includes traditional objectives: minimizing cost and improving customer service levels (minimizing uncovered demand) and an environmental objective: minimizing the environmental impact from transportation and depots. Two multi-objective evolutionary algorithms, NSGA-II and SEAMO2 are discussed and adopted for the MOUFLP. Furthermore, as a result of comparing both algorithms on the benchmarking data, NSGA-II is used to investigate a range of "what if ?" scenarious to provide a set of non-dominated solutions to some test instances where the relative weighting of the impact of transport versus depot on the environment is being varied. In this study we assume that the environmental impact of transport is not truly reflected in the economic costs of running a fleet of vehicles. The analysis was performed on two different settings: a two-objective model (min cost - min environmental impact) and a three-objective model (min cost, min environmental impact and min uncovered demand). The investigation also included the evaluation of the impact of the different scenarios on the number of open depots.

The next chapter extends our exploratory study to a capacitated FLP and a more realistic model using some randomly generated data instances based on Sainsbury's data. In the new study, the environmental impact from transportation and depot management is properly evaluated using a relevant carbon footprint methodology and the environmental information for the generated data instances is based on real figures from the industry.

Chapter 9

Multi-Objective Capacitated Facility Location Problem

9.1 Introduction

In this chapter we extend the UFLP formulation to the two-objective *Capacitated Facility Location Problem* (CFLP). The CFLP model formulation described in this chapter is based on Sainsbury's logistics network and has two capacity constraints: one for the number of cases and one for the number of stores. Recall that the UFLP has no constraints on the capacity at all, and was based on data sets available from the public domain adapted by adding a component for environmental impact. The two-objective CFLP model aims to balance the financial cost and the environmental cost, taking into account activities such as moving, picking, and loading the goods as well as transporting them and opening the depots needed to serve the customers' needs. The environmental impact is extracted from running the logistics network in terms of CO_2 emissions from transportation and the emissions caused by energy use for the day-to-day running the depots. Because of the lack of multi-objective data sets with environmental aspects for the CFLP in the public domain, we have generated our own large size data problems, based on data from industry. Locations, capacities and levels of demand etc. have been randomly generated, but within reasonable upper and lower bounds observed from our real-world data sets. We have used Government sources to obtain correct environmental information regarding energy consumption.

To solve the multi-objective CFLP and generate a set of trade-off solutions for our benchmark data, we use a solution technique which is based on the elitist evolutionary multi-objective algorithm SEAMO2 and adapted it for capacitated formulation. We utilized the Lagrangian Relaxation technique which was discussed in Chapter 7 and allows the assignment of customers to open depots for each individual in the population of the solutions.

9.2 Problem formulation and objective functions

We assume that customers have a certain demand in cases and associated transportation and warehousing costs for a particular depot. Each depot has a given capacity in cases and the number of stores it able to serve. The customers are served directly by a depot, and transportation costs are based on stem distances and reflect time and distance based components. The warehouse costs reflect any associated costs with picking and loading the goods.

The problem is to determine how many facilities to open in order to satisfy all customers demand while solving both objectives simultaneously: minimise the environmental impact from operating depots and transport in terms of CO_2 emissions and minimise the overall financial cost. As in UFLP, the capacitated version of this problem is also divided into two sub-problems: determine which depots to open and assign customers to the open facilities without violating the number of cases or the number of customers capacity constraints.

The following notation is used in the formulation of the model:

Glossary

$V_{DC} = \{1...i\}$	set of potential depots;
$V_C = \{1...j\}$	set of customers;
c_{ij}	cost of attending demand from customer j to depot i consisting of overall transportation and depot costs;
f_i	fixed cost for operating a depot i;
d_j	demand of customer j;

q_i capacity (cases) of facility i, $i \in V_{DC}$;

n_i capacity (number of customers) of facility i, $i \in V_{DC}$;

e_t_{ij} CO_2 emissions from transport between depot i and customer j to satisfy customer demand d_j;

e_g_i CO_2 emissions from gas consumption for each depot i, $i \in V_{DC}$;

e_e_i CO_2 emissions from electricity consumption for each depot i, $i \in V_{DC}$;

The decision variables are:

x_{ij} is the decision variable for the problem, $x_{ij} = 1$, if customer j is allocated to facility i, and 0 otherwise;

y_i equals 1 if depot is chosen to operate and 0 otherwise;

The following economic and environmental objective functions are considered simultaneously as part of the network design:

- *Minimising costs.* This financial objective finds the best combination of open depots that allows cost minimization of overall cost of the network. It consists of overall cost (associated transport and depots) of servicing a demand of customers by open depots and fixed cost of operating depots

$$minimize[\sum_{i \in V_{DC}} \sum_{j \in V_C} c_{ij} x_{ij} + \sum_{i \in V_{DC}} f_i y_i] \tag{9.1}$$

- *Minimizing the CO_2 emissions from transport and running depots.* The environmental objective is expressed as CO_2 emissions and aims to find the best number of open facilities that minimizes the total CO_2 emissions from transportation and energy consumption for running facilities. The first term of the formulation represents the emissions from transport to attend the demand of customers by the open depots, and the second term represents the total emissions from the electricity and gas usage of open depots.

$$minimize[\sum_{i \in V_{DC}} \sum_{j \in V_C} e\_t_{ij} x_{ij} + \sum_{i \in V_{DC}} (e\_g_i + e\_e_i) y_i] \tag{9.2}$$

- *Subject to following constraints:*

$$\sum_{i \in V_{DC}} x_{ij} = 1, j \in V_C \tag{9.3}$$

$$x_{ij} \leq y_i, j \in V_C, i \in V_{DC} \tag{9.4}$$

$$\sum_{j \in V_C} d_j x_{ij} \leq q_i, \forall i \in V_{DC} \tag{9.5}$$

$$\sum_{j \in V_C} x_{ij} \leq n_i, \forall i \in V_{DC} \tag{9.6}$$

$$x_{ij} \in \{0, 1\}, i \in V_{DC}, j \in V_C \tag{9.7}$$

$$y_i \in \{0, 1\}, i \in V_{DC} \tag{9.8}$$

Constraints 9.3 and 9.7 ensure that each customer is attended by only one depot and the demand is satisfied by that facility. Constraint 9.4 assigns the customers to open depots only. (9.5) and (9.6) ensure that the capacity constraints for demand (cases) and number of stores for the depots are not violated and finally, constraints 9.7 and 9.8 define decision variables as binary.

9.3 Test data

Due to the lack of environmental test data for MO CFLP in the public domain, we generated randomly data sets based on the company data, which we used in Chapter 5 to investigate the impact of key variables on the allocation of stores to depots.

The ranges for all values, such as the demand (weekly volume), productivity and costs are based on the industry data. To generate the environmental data for each depot, one way of dealing with it is to base the figures on average consumption of electricity and gas across some real depots. Having that information from the company allowed us to generate this environmental data which was absolutely invaluable for our research. This information allowed us to derive a formula for calculating energy consumption in kWH for a particular capacity of a depot. We use a value of 0.0933 kWh per a case of demand (weekly) for electricity consumption and a value of 0.0045 kWh for gas. Depending on the available capacity of the serving facilities, each of those values is multiplied by the total capacity to calculate a total energy consumption in kWh which were converted to CO_2 emissions using conversion factors from DEFRA[32] (0.54 $kgCO_2$ per kWh for electricity and 0.19 $kgCO_2$ for gas). To calculate CO_2 emissions from the transport we calculated the total distance travelled by a vehicle to satisfy a demand of a particular customer to a depot, which was multiplied by a fuel conversion factor (2.63) and then multiplied by fuel consumption (of 0.35 litres per km) [32].

The financial costs consist of both transportation and depot related costs, where transportation costs have distance and time related components as per Sainsbury's data. We assume that $c_{ij} = (tc_{ij} + dc_{ij})$, where tc_{ij} is the transportation costs and dc_{ij} is the related depot costs between customer j and depot i. Also, to reflecting the fact that costs can vary, depending on geographical locations (e.g., labour costs tend to be higher in London and the South East), each depot has its own rates for transport and warehousing components.

The capacity of each depot was calculated taking into account the overall demand across all depots. Therefore, to calculate a capacity for each depot, the total demand was multiplied by a capacity ratio value and then divided by a total number of depots. For the initial experiments, the following capacity ratio values were tested, when data sets were first generated: 2, 3, 4, 5, 6, 7, 8, 9 and 10. A similar procedure was used to calculate a capacity for the maximum number of stores which each depot could serve. As a result, all depots have the same capacity for cases and number of stores. The fixed costs for each depot were calculated taking into account the capacity of that particular depot. The various fixed cost ratio values of 0.5, 0.75, 1.25 and 1.5 were used for generating fixed costs for all depots in each scenario. For example in the scenario where the ratio value is 0.5, this number was applied across all depots and it was used to multiply the depot capacity.

The different ratios for capacities and fixed costs were initially tested to determine the effect it has on the number of non-dominated solutions when the MOO technique was applied. The SEAMO2 algorithm was used to explore non-dominated solutions for CFLP. As a result of those initial tests the capacity ratio values of 4 and 8 and the fixed cost ratios values of 0.75 and 1.5 were used to generate larger data instances for CFLP. The instances had 10 depots and five different settings for the following number of customers: 2000, 4000, 6000, 8000 and 10000. The name given to each data problem reflects different values generated for that particular problem. For example, problem *set1_10_2000_r4.0_fc1.25* has 10 depots, 2000 customers, a capacity ratio of 4 and fixed costs ratio of 1.25. In total, 20 different test instances were generated for analysis of the MO CFLP where financial and environmental objectives are solved simultaneously.

9.4 Solution encoding and assignment procedure

The *solution encoding* procedure was the same as for MO UFLP and encoded as a binary strings, where *1* represents an open depot, and *0* a closed depot, e.g. 0011011011 indicated that depots 3,4,6,7,9 and 10 are open, and the others are closed.

The *assignment procedure* for the capacitated model is an extremely important procedure which ensures that assigned customers do not violate capacities in those depots in terms of cases and numbers of stores. Here, we utilize the Lagrangian Relaxation technique for assigning the customers to open depots without violating either capacity constraints. The assignment procedure could be undertaken based on cost or environmental impact. In this research, we are solving the allocation problem using LR technique based on the financial cost because it is still a common practice among practitioners to use a cost function as a main objective during modelling and analysis. Figure 9.1 illustrates the assignment procedure. As a result of applying this procedure, the customers are assigned to the minimum possible cost objective function, depending on the capacity constraints. The LR technique is discussed in detail in Chapter 7.

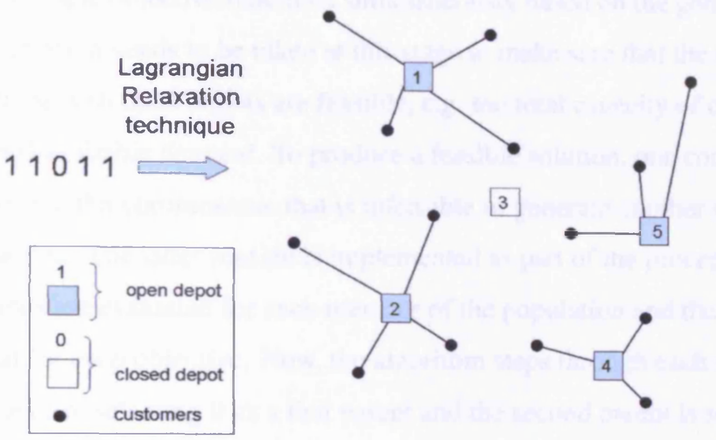


Figure 9.1: The assignment for Capacitated Facility Location Problem.

9.5 Multi-Objective Evolutionary Algorithm for CFLP

The evolutionary MO algorithm SEAMO2 was used for modelling the CFLP and it obtained a good approximation of the Pareto front. We found that this algorithm is considerably faster in computational times compared to the better known NSGA-II algorithm, as was discussed in Chapter 8 for the UFLP. It is very important for larger data instances to provide the decision maker with an initial set of trade-off solutions which can be investigated further if needed. Another reason for choosing SEAMO2 is that the quality of the approximated Pareto frontier seems to be of similar quality in comparison to NSGA-II for UFLP. The uncapacitated version of the SEAMO2 algorithm was adapted to the capacitated version where the open facilities and the customer assignment produced by Lagrangian Relaxation technique was used to calculate both objective functions.

Figure 9.2 presents an evolutionary multi-objective optimization framework where the

SEAMO2 algorithm and LR approach are integrated together for the CFLP. Initially, a population of solutions (binary strings) is randomly generated and for each solution we apply LR heuristics to allocate stores to open depots and as a result of the allocation, we could evaluate multiple objective functions simultaneously based on the generated solution. Consideration needs to be taken at this stage to make sure that the randomly generated solutions with open depots are feasible, e.g. the total capacity of open depots is larger than total available demand. To produce a feasible solution, one could undertake a repair procedure to the chromosome that is infeasible or generate another random string that is feasible. The latter routine is implemented as part of the procedure. The objective functions are evaluated for each member of the population and the best-so far values are stored for each objective. Now, the algorithm steps through each member of the population in turn, selecting it as a first parent and the second parent is selected at random for a crossover and mutation operation to produce a single offspring. The SEAMO2 algorithm is steady state where an offspring is considered to enter the population depending on a number of comparisons. If an offspring produces best-so far for either objective with appropriate best-so-far objective updated, then it replaces the parent if possible otherwise it replaces an individual at random that it dominates. If the offspring is not a duplicate in the current population, it dies if the match is found to preserve the diversity of the population. If the offspring survives, then we check if it dominates either parent and the parent will be replaced if it is being dominated by an offspring. Finally, if there is no decision made so far regarding an offspring, then the last condition checks if it has a mutually non-dominating relationship with both of its parents and will enter the population if a suitable member has been identified which is dominated by the new offspring. The algorithm will repeat until the stopping condition is satisfied and at the end, it will produce a set of non-dominated solutions from the final population.

Before running the experiments on the generated data sets, the algorithm was tuned to its best performance on three data instances: *set1_10_2000_r4.0_fc0.75*, *set1_10_2000_r4.0_fc1.25* and *set1_10_8000_r4.0_fc1.25*. The minimization problem was converted to a maximization problem for both objectives ('unspent' cost and 'unspent' impact) to compare different types of crossover on the quality of the non-dominated solutions using the S metric. A population size of 40 was used and run

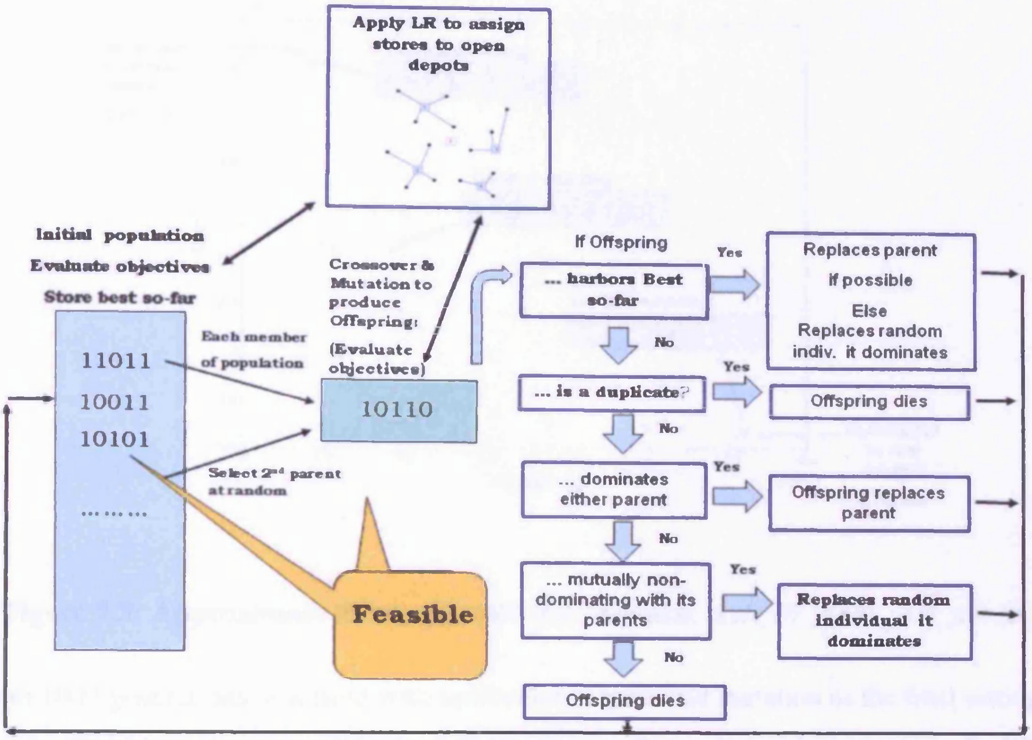


Figure 9.2: Multi-Objective evolutionary framework combining SEAMO2 and LR.

for 250 generations for tuning purposes. Three different combinations of crossover and mutation were tested: no crossover/no mutation, one-point crossover/mutation, two-point crossover/mutation, and uniform crossover/mutation. In total, 12 different experiments were undertaken for tuning purposes. The *S* metric and final approximate Pareto frontier were obtained from 20 independent runs for each data instance and settings for mutation/crossover. As a result of undertaking one-way analysis of variance on the *S* metrics for different set of experiments, results indicate no statistical significance among all settings. Nevertheless, we choose *uniform crossover with mutation* to bring the diversity into the population of solutions. This crossover operator is the same as it was used when SEAMO2 was tested for modelling the UFLP with two objectives. As a final step before undertaking experiments on randomly generated data sets, the size of the population and number of generations were increased to ensure that the algorithm runs a sufficient amount of time to find good quality solutions. Therefore, a population of 100,

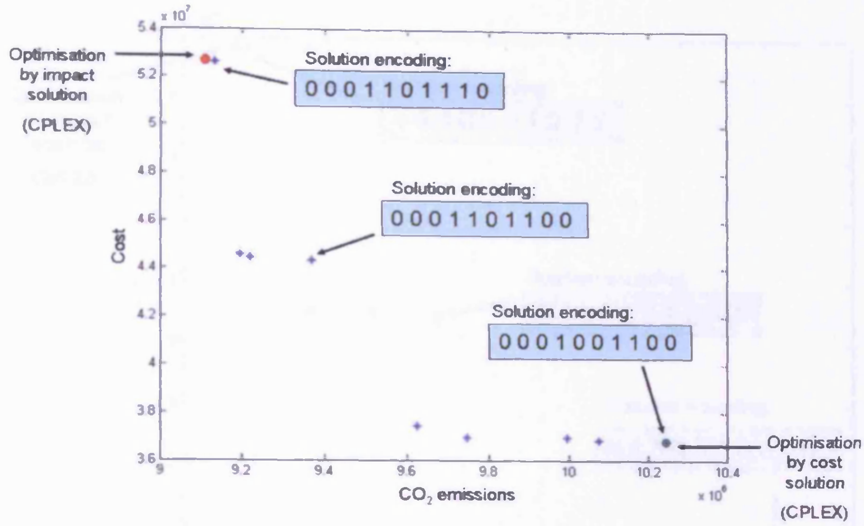


Figure 9.3: Approximate Pareto frontier for instance: *set1_10_2000_r4.0_fc0.75.*

with 1000 generations was used with uniform crossover and mutation as the final settings for the SEAMO2 algorithm. The final Pareto frontier for each data instance was obtained by aggregating the fronts of 10 independent runs for the MO algorithm (see Figures 9.3, 9.4).

9.6 Discussion of the results

This section considers the analysis of the solution set produced by SEAMO2 for an environmentally friendly network design. To compare the quality of solutions located an the edges produced by the MOO technique, we attempted to solve each of the instances as two separate single objective problem (min cost and min impact) using CPLEX® to acquire the best known solution for each of the individual objective functions. Unfortunately, due to the large size of the data instances, it was impossible to determine the best solution using CPLEX® for all instances. In the optimization by cost, all instances with 2,000 customers were solved which took between 10 and 30 hours to solve each set; and only one instance with 4,000 customers was solved. Some of instances with 4,000 customers were taking more than 2 days. This means that it was

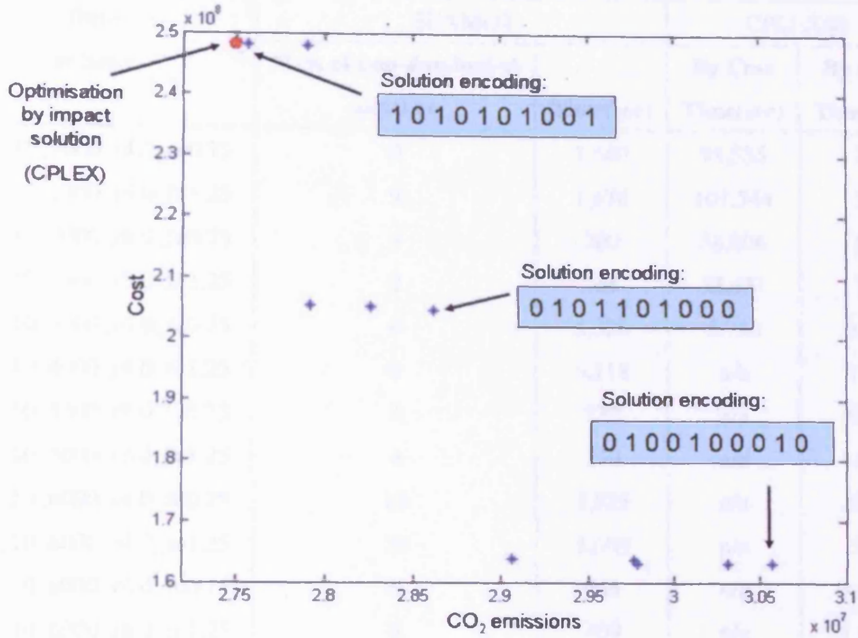


Figure 9.4: Approximate Pareto frontier for instance: ***set1_10_6000_r4.0_fc1.25.***

unreasonable to try and solve all the data sets by cost using CPLEX®. The complexity of the cost function, which consists of transport and depot costs and fixed costs for operating depots, makes the cost problem much more complicated than the CO_2 emission problem, for which CPLEX®, solved all the instances within a reasonable amount of time. For example, one of data set with 10,000 customers took only around 2 hours to solve for CO_2 emissions. Thus, all instances were solved for CO_2 emissions, but only one set with 4,000 customers and all instances with 2,000 customers or less were solved by CPLEX® for cost.

Figure 9.3 illustrates the approximate Pareto frontier obtained by SEAMO2 for instance *set1_10_2000_r4.0_fc0.75*. The technique found very good solutions for both edge points of the front. The solution which was found by CPLEX® for the optimization by cost alone, is identical to the solution found by SEAMO2, and the solution found in the CPLEX® optimization by CO_2 is very close to the other extreme solution on the edge of the Pareto front. The trade-off solutions do not appear to be very evenly spread across the approximated Pareto set, which could be due to the problem configuration. It could

Data instance	SEAMO2		CPLEX®	
	Num of non-dominated solutions	Time(sec)	By Cost Time(sec)	By CO_2 Time(sec)
set1_10_2000_r4.0_fc0.75	9	1,640	94,535	26
set1_10_2000_r4.0_fc1.25	9	1,676	107,544	24
set1_10_2000_r8.0_fc0.75	5	200	58,606	65
set1_10_2000_r8.0_fc1.25	5	166	33,451	77
set1_10_4000_r4.0_fc0.75	6	5,326	5,730	149
set1_10_4000_r4.0_fc1.25	6	5,118	n/a	157
set1_10_4000_r8.0_fc0.75	4	232	n/a	962
set1_10_4000_r8.0_fc1.25	4	250	n/a	1,066
set1_10_6000_r4.0_fc0.75	10	5,525	n/a	505
set1_10_6000_r4.0_fc1.25	10	5,309	n/a	533
set1_10_6000_r8.0_fc0.75	6	444	n/a	1,674
set1_10_6000_r8.0_fc1.25	6	409	n/a	1,671
set1_10_8000_r4.0_fc0.75	7	9,702	n/a	630
set1_10_8000_r4.0_fc1.25	7	9,840	n/a	564
set1_10_8000_r8.0_fc0.75	4	819	n/a	3,640
set1_10_8000_r8.0_fc1.25	4	817	n/a	3,662
set1_10_10000_r4.0_fc0.75	10	9,957	n/a	1,831
set1_10_10000_r4.0_fc1.25	10	9,641	n/a	3,277
set1_10_10000_r8.0_fc0.75	4	402	n/a	8,093
set1_10_10000_r8.0_fc1.25	4	440	n/a	8,063

Table 9.1: Comparison between SEAMO2 and CPLEX® optimization .

be that the two capacity constraints (for cases and numbers of stores) combine to reduce the number of solutions in the search space. A similar pattern can be observed for all the data sets. Figure 9.4 illustrates trade off solutions for problem *set1_10_6000_r4.0_fc1.25*. For some of the instances, as for example for problem *set1_10_2000_r4.0_fc0.75* in Figure 9.3 it is not so straight forward for the decision-maker to spot good compromise solutions which should be located in the middle of the approximate Pareto front. But at the same time the solutions nearest to the middle of the frontier could offer that choice to the decision maker.

Analysing the trade-off solutions for the current data sets which are based on the data from the industry, and what it means for the number of open depots across approximated Pareto front allowed us to confirm the findings which were made for UFLP. The low cost solution produces highest CO_2 emissions with smaller number of depots open, where as the best solution for environmental impact needs more depots to be open.

Table 9.1 shows execution times and the total number of non-dominated solutions in the approximate Pareto frontier for all instances using the SEAMO2 multi-objective algorithm, and also execution times when instances were solved by a single objective function based on costs or emissions using the CPLEX® optimization software. As can be seen from the table, the MOO algorithm finds a set of non-dominated solutions much faster than CPLEX® , which solves a particular problem to optimality based on single-objective function. These results show us that in terms of execution times, SEAMO2 (combined with LR for allocation of customers to stores) is computationally very fast, and can provide a good set of trade-off solutions to a decision maker.

To conclude, through our the analysis on our large data sets it was demonstrated that the MOO technique is able to find efficient trade-off solutions balancing cost and CO_2 emissions for network design very quickly. For some data instances, it was easy to spot compromise solutions and in other cases it was not so straight forward. This is probably due to having two capacity constraints which make it harder for the technique to find feasible solutions to balance both objectives.

9.7 Summary

This chapter presents the a MO CFLP problem which balances two objectives: financial cost and CO_2 emissions. The emissions from transportation and the energy consumption from running depots allowed us to model a sophisticated network based on real industrial data. We generated random data sets based on real-life data to use in experiments. In addition, an evolutionary multi-objective optimization algorithm is discussed in this chapter to solve MO CFLP, which uses a Lagrangian relaxation technique to find the best assignment of stores to open depots for any particular individual of the population. Thus the MOO algorithm determines which depots are open, and the LR takes care for the allocation of customers to depots. The results are compared to the single-objective optimization based on cost or environmental impact using CPLEX® optimization software whenever possible. The analysis of the findings confirmed the results from our earlier study on UFLP which are discussed in the previous chapter. It illustrates again that it could be more desirable to open more depots for more environmentally friendly network design.

Chapter 10

Conclusions

This chapter summarises our research findings in answer to the research questions posed in Chapter 1 and establishes some possible directions for future research in the area of environmentally friendly logistics design, focusing on the facility location-allocation and generalized assignment problems. The chapter aims to critically evaluate our contribution and explore the limitations of the presented research study. There are three sections in this chapter. Section 10.1 presents a summary of the research findings presented in this thesis. Section 10.2 summarises how research questions set out in this thesis have been addressed. Those questions were:

- Is it possible to build multi-objective optimization decision tools for strategic modelling of large size traditional logistics networks where financial and environmental objectives are solved simultaneously?
- Is the optimum design of a particular logistics network based on cost the same as the optimum design based on CO_2 emissions?

Finally, Section 10.3 presents suggestions for future research.

10.1 Research Summary

The main focus of current research has been to build a foundation for future research in the area of multi-objective optimization for the strategic and tactical design of traditional supply chains in times of increased environmental concerns. We do not envisage trying to force industry to change its day to day running operations in network design. Their primary focus will remain financial costs, as they have to stay in business. Instead we suggest that extending the use of multi-objective optimization techniques can offer industry a straightforward way to generate a range of the trade-off solutions, which will frequently offer a decision maker an opportunity to select an option that will considerably reduce emissions or pollution, yet will do so at a very modest financial cost. To achieve Government targets on reducing CO_2 emissions will mean that future legislations could force industry into higher savings for different pollutants including carbon dioxide emissions. The popularity of multi-objective optimization techniques is on the increase at present, and together with the research presented in this thesis should develop a foundation for incorporating those techniques into commercial software for strategic and tactical modelling of logistics networks.

At the initial stages into our research, we aimed to understand the relationship between total logistics costs and CO_2 emissions from transportation and energy usage for a single-optimization approach. We created a simulation model based on a Pan-European network from automotive sector based on the case study by Hammant [124]. During modelling attention is paid to the sensitivity of our solutions when changes in supply chain structure (number of depots) and vehicle utilization ratios (90%, 75%, 60%) occur. The limitations of the study is that only one case study was analysed and also we had to rely on the data from the public domain with assumptions regarding transportation data. Nevertheless, the study allowed us to see that vehicle utilization ratio could be one of the factors that has an impact on the optimum supply chain structure, when optimized by cost or CO_2 emissions. This research was published in the International Journal of Production Economics [56].

In our research, we focused on the development of a simple multi-objective optimization framework that could be easily understood to solve large size problem instances. In recent years there has been an explosion of academic interest into the development of MOO algorithms due to the advances into the computational power and clear need for the decision maker to have a choice among solutions. There exist a number of different approaches to MOO and in this thesis we explored an evolutionary approach to solving MOO problems where objectives are solved simultaneously. Evolutionary algorithms are becoming more complex which bring the challenge of replicating them to analyse their performance. In our research, we analysed two MOEA's: NSGA-II (widely used algorithm in academia) and SEAMO2 (due to it's simplicity) from computational complexity and quality of solutions as part of MOO framework for solving CFLP. Because the focus of the research is on the large data sets, we need an algorithm which obtains a good quality approximate frontier quickly. Therefore, SEAMO2 was chosen to solve CFLP formulation where the algorithm runs in the population of the decision variables to obtain a set of open facilities. The presented framework in Chapter 9 has two levels of decisions: identifying which facilities to open and how to assign stores to open depots where we used LR technique to solve the assignment problem. As a result of our experiments, we have developed and tested a prototype for multi-objective optimization algorithm for capacitated FLP which produces trade-off solutions between economic and CO_2 emissions. The analysis of the findings on more realistic data confirmed the findings from the study on UFLP which indicated that more facilities may be needed to balance economic and environmental objectives.

The single source assignment problem was also investigated separately from CFLP formulation when we have engaged in collaborative research with an industrial partner from a major UK food supply chain from financial and environmental aspects. This allowed us to understand the impact of different variables such as fuel and depot associated costs on their current network configuration (allocation of stores to depots). Our sensitivity analysis shows the effect of those changes depends on the geographical location of the depots under investigation and that their current structure is robust because it has enough current capacity to deal with rate fluctuations. A prototype of the software was developed which uses a single objective function where overall transportation and depot associated costs are minimized which was extended further to focus on balancing financial and environmental impact from transport using a simple distance-based approach when greater distance indicates higher fuel usage and more CO_2 emissions. Firstly, we compared the results from the optimization based on distance (CO_2 emissions) to those obtained previously from optimisation by cost which generated different results with different allocations. Secondly, we produced a weighted sum two-objective allocation model to produce trade off solutions for costs and distances.

The trade-off solutions allowed us to illustrate to the decision maker how small increase in cost could equate to a considerable decrease in the distance travelled by the vehicles, thus reducing the environmental impact.

The collaborative research with industry helped us to develop the LR technique which is used in MOO framework for strategic design. The lagrangian heuristic was developed for two problem formulations to solve the single product capacitated assignment problem with two capacity constraints for the number of cases and number of stores. The approaches differ according to how the user treats the constraints. If one of the constraints such as the number of stores is not as highly regarded as another, e.g. capacity constraint for demand, the approach relaxes only one constraint (the capacity for demand). This is could be computationally more efficient compared to relaxing both constraints. This approach was tested on our benchmark data and the results are compared to the optimization by CPLEX® in Chapter 7. The quality of solutions and executional times shows the effectiveness of our LR approach. The second formulation of the LR approach where both capacity constraints are relaxed is presented in Appendix A as a mathematical formulation and future work will be needed to analyse the technique. As an extension of the problem formulation, the LR approach was extended to the multiple product formulation with relaxing demand capacity constraints and was tested on the Sainsbury's data with great emphasis on the discussion of different approaches to finding feasible solutions. We have generated large size data instances for a single product, single source assignment problem because there are no available problem instances in public domain. It allowed the comparison of the solution technique between CPLEX® optimization engine and our LR technique.

One of the challenges we encounted during the investigation into environmentally friendly design was a lack of multi-objective data instances with environmental data in literature. As a result, initially, we have undertaken an exploratory study to investigate the potential of multi-objective optimization techniques for a simple model of UFLP where we derive the environmental impact from economic costs by varying the relative weighting. A prototype was developed to consider traditional objectives: minimizing cost and improving customer service level (minimizing uncovered demand) and an environmental objective: minimizing the environmental impact from transportation and depots. Despite the limitations of the study, this research gave us an insight into generated trade-off solutions and also suggested that it could be more desirable to open more depots for an environmentally friendly design. This research was published in the IEEE Congress on Evolutionary Computation [55]. Limitations of the availability of the data sets were addressed when the exploratory study into MO UFLP was extended to a more sophisticated supply chain model (CFLP) with two capacity constraints. We have written the software to randomly generate test data based on the data from industry which considers networks with depot and transportation elements of the logistics modelling which aims to encourage future research in multi-objective optimization in the academic environment.

10.2 Evaluation of Research Questions

In this section, we discuss each of the research questions presented at the beginning of this chapter and how they have been addressed through out the research presented in this thesis.

Research question 1 addresses the feasibility of building MOO decision tools for a large size traditional logistics network where financial and environmental objectives are solved simultaneously. In this thesis, we present an evolutionary MOO framework for solving large size CFLP where we aim to generate a good quality set of solutions for the decision maker within a reasonable amount of time. We consider CO_2 emissions from transport and serving facilities as environmental objectives and one of the challenges we encountered is to identify an appropriate methodology for calculating carbon footprint and the way of estimating emissions from a particular design because there was no available appropriate data instances in the public domain. Guidelines for company reporting on greenhouse gas emissions from Defra [32] were used as a way of calculating carbon dioxide emissions and collaborative work with industry allowed us to estimate energy consumption figures which are used in our data sets. The presented MOO framework could be extended to different model formulations as well as using different approaches to the solution representation which are discussed in the next Section 10.3.

Research question 2 considers if the network design based on costs and on CO_2 emissions is the same. This question was analysed for the assignment and capacitated facility location allocation problems separately. In both cases, the answer to this question is "no" - the design based on cost is different to the network design based on CO_2 emissions. For the assignment problem, Chapter 6 considers a case study from industry where the analysis shows that single objective optimization based either on cost or distance (CO_2 emissions related to the distance traveled) produces different results with different customer allocations, costs and distances. In the strategic design, where CFLP model formulation was considered as part of the multi-objective optimization, we demonstrated in Chapter 9 that the low cost solution needs less depots to be open and produces highest CO_2 emissions where as the best solution for carbon dioxide emissions needs more depots to be open at a higher cost.

10.3 Future Work

This section presents future work suggestions, to extend the research presented in the thesis:

10.3.1 Extension of Lagrangian Relaxation technique

The Appendix A presents a mathematical formulation of the LR technique where two capacity constraints (number of cases and number of stores) are relaxed. The technique needs to be investigated further and tested on large size data instances for single and multiple products to ensure that the formulation is correct and finds feasible solutions. Extending the formulation to multiple products would reflect real-life scenarios where the businesses have demand for multiple products. In Chapter 7 we tested multiple product formulation on Sainsbury's data, but further work is needed to analyse the technique on appropriate benchmark data sets for multiple products to encourage further research.

10.3.2 Modelling location and assignment decisions simultaneously in MOO framework.

In this thesis, we consider the assignment of stores to depots based on cost only as a subroutine of the facility location problem where objectives are solved simultaneously. In Chapter 6 we show that optimization of the allocation problem based on costs produce different solutions compared to optimization based on the environmental impact and have higher distance travelled for cost based optimization and vice verse. This means that if we use cost based assignment optimization in FLP formulation, then our solutions in the multi-objective framework are skewed towards the financial objective while both objectives are solved simultaneously. To avoid this bias towards one objective, the location and allocation decision parts could be combined together in one solution encoding and multiple objectives solved simultaneously. One way of dealing with it is to represent the solution encoding as a two-part string where the first part would present which depots are open and the second part will display assignment of each store to a depot. For example if we have three depots and five stores, then in the integer representation, 01123323, first three numbers illustrate that depots 2 and 3 are open and other five numbers represent allocation where the first store is assigned to depot 2, second store to depot 3 etc. Due to the challenge of modeling large data sets, this would bring an issue of the computational complexity, especially when we have a population of long solution encodings over a number of iterations where the length of our string could be a size of 10,000 bits long. Also, when applying crossover and mutation operators, a repair mechanism would be implemented to avoid stores being assigned to closed facilities. Another way of avoiding bias towards one objective is to solve the allocation routine as a weight based approach for multiple objectives but in this case, suitable weights will have to be considered as part of the decision making process. Both approaches would need to be investigated further as part of the future work from computational time and the quality of the solutions perspectives.

10.3.3 Multi-objective network design under uncertainty

We consider deterministic models in this thesis where the average data is used as part of the model formulation. Those models do not include any stochastic elements which reflect different factors of uncertainty in the supply chain. Different sources of uncertainty, such as delivery operations, demand uncertainty, or customer's behavior could be analysed by undertaking the sensitivity analysis of the design of those stochastic elements and will allow to deal with uncertainty to an extend as we did in Chapter 5. Another way of dealing with the design under uncertainty is to incorporated those factors into the strategic design, therefore appropriate solution techniques need to be developed for multi-objective optimization framework to ensure the robustness of the solutions at the strategic level.

10.3.4 Improving MOO for supply chain design

The analysis of the allocation of the customers to depots based on the financial cost or environmental impact for MO CFLP will need to be investigated in terms of the impact on the trade-off solutions. Another extension would be to improve the prototype for our multi-objective optimization algorithms incorporating other traditional and green objective functions such as capacity utilization ratio and traffic access. We would also plan to explore the opportunities for integrating our MOO approach with commercially available software for strategic modelling like CAST.

10.3.5 Extension of MOO to other supply chain problems

Different types of logistics network design problems are discussed in Chapter 2 which extend our current static single echelon network design. The approach taken would depend on which of the alternative facility location models was selected. Models with multi-echelon structures consider suppliers and manufacturers as part of the design. Dynamic location models, in contrast to static models, reflect modelling data changing over different planning periods. On the other hand, probabilistic models have elements of uncertainty based on forecasting. Location routing problems, which reflect the position of the facilities depending on the routing choices, would also make an interesting field for future research.

10.3.6 Extension of MOO to closed loop supply chains

Current research focuses on the traditional supply chain where the physical flow of the goods in the supply chain stops at the consumers end. Extending the traditional network to semi-closed loops will also allow us to incorporate recycling facilities as part of the MOO network design. This will need different objective functions which will consider recycling and re-manufacturing objectives. In this model the locations of the recycling facilities as well as serving facilities could be considered as part of the design.

To conclude, the research presented in this thesis contributes to the area on MO facility location-allocation analysis where economic and environmental objectives are considered simultaneously. The ideas and techniques presented here could be extended further and integrated within logistics modelling to give a decision-maker a scientific choice which is expressed as a set of trade-off solutions. It also shows a knowledge transfer and positive collaboration between industry and academia.

Appendix A: Solution formulation for relaxing two constraints: number of cases and number of stores

This appendix presents a new LR solution technique where two capacity constraints (number of cases and number of stores) are relaxed as part of the formulation to determine a lower bound solution using Lagrangian multipliers. Let $\lambda_i \in \mathbb{R}, \forall i \in V_{DC}$ and $\psi_i \in \mathbb{R}, \forall i \in V_{DC}$.

Minimize

$$\sum_{i \in V_{DC}} \sum_{j \in V_C} c_{ij} x_{ij} + \sum_{i \in V_{DC}} \lambda_i \Big(\sum_{j \in V_C} d_j x_{ij} - q_i\Big) + \sum_{i \in V_{DC}} \psi_i \Big(\sum_{j \in V_C} x_{ij} - n_i\Big) \tag{10.1}$$

subject to

$$\sum_{i \in V_{DC}} x_{ij} = 1, \forall j \in V_C \tag{10.2}$$

$$x_{ij} \in \{0, 1\}, i \in V_{DC}, j \in V_C \tag{10.3}$$

In (10.1) the term in brackets in the middle, $(\sum_{j \in V_C} d_j x_{ij} - q_i)$, calculates the difference between the total demand on a facility i and capacity(cases), q_i. The term in brackets in the right, $(\sum_{j \in V_C} x_{ij} - n_i)$, calculates the difference between the number of stores allocated to a facility i imposed by the relaxed formulation, and its ability to meet that demand (i.e., its capacity(number of stores), n_i). If the capacity is violated, the value of total cost in (10.1) will change, depending on the value of λ_i and ψ_i.

One issue that needs to be considered in the formula (10.1), is that normally a Lagrangian Relaxation technique will make adjustments to the cost only when both constraints are violated. Formula (10.1) will make adjustments to the costs when one of the constraints is violated, therefore if the capacity has not been exceeded then the term $(\sum_{j\in V_C} d_j x_{ij} - q_i)$ will be equal to zero for any facility. The same will apply for the other term $(\sum_{j\in V_C} x_{ij} - n_i)$ as well. On the other hand, when the facility is underutilized, then this formulation will also produce non-zero value. To ensure that this is not the case, we constraint the λ_i and ψ_i values: if $\lambda_i = 0$, it follows that $\sum_{i\in V_{DC}} \lambda_i(\sum_{j\in V_C} d_j x_{ij} - q_i)$ also equals zero; if $\psi_i = 0$, it then $\sum_{i\in V_{DC}} \lambda_i(\sum_{j\in V_C} x_{ij} - n_i)$ is also equals to zero.

Problem (10.1) - (10.3) can be decomposed into $|V_C|$ subproblems. For a given set of multipliers, $\lambda_i \in \mathbb{R}$, $\psi_i \in \mathbb{R}$, the optimal lower bound of the problem (10.1) - (10.3), $LB(\lambda\psi)$, can be found by solving the following subproblem for each customer $j \in V_C$. Minimize

$$\sum_{i\in V_{DC}} (c_{ij} + d_j\lambda_i + \psi_i)x_{ij} \tag{10.4}$$

subject to

$$\sum_{i\in V_{DC}} x_{ij} = 1, \forall j \in V_C \tag{10.5}$$

$$x_{ij} \in \{0,1\}, i \in V_{DC}, j \in V_C \tag{10.6}$$

and then by setting

$$LB(\lambda\psi) = \sum_{j\in V_C} LB^j(\lambda\psi) - \sum_{i\in V_{DC}} \lambda_i q_i - \sum_{i\in V_{DC}} \psi_i n_i \tag{10.7}$$

(10.4) is easily solved for by applying a greedy algorithm to allocate each customer along the lowest cost according to the augmented costs, $c_{ij} + d_j\lambda_i + \psi_i$. By suitably modifying the Lagrangian multipliers, it is possible to obtain a feasible solution to the original capacity constrained problem. To provide a good updating formula for the Lagrangian multipliers, we will need an upper bound, in addition to the lower bound in (10.7).

The UB will represent a feasible solution obtained on the basis of the evaluation of the $LB(\lambda\psi)$ solution. To obtain the best possible upper bound (i.e., with the lowest cost), we could allocate customers with high demand first to ensure that individual depots have sufficient unused capacity. This could be done by sorting customers in decreasing order of demand (highest demand first), then assign customers in the same way as the LB, whenever possible. When capacity constraints are violated for LB assignment, we could assign the customer to the next lowest augmented cost depot without violating the capacity constraints etc. If all facilities are overcapacity, then we assign to the lowest available cost value (non-augmented cost).

Updating the Lagrangian multipliers

For each facility at time step, k

$$s_i^k = \sum_{j \in V_C} x_{ij}^k d_j - q_i \tag{10.8}$$

$$r_i^k = \sum_{j \in V_C} x_{ij}^k - n_i \tag{10.9}$$

where x_{ij}^k is the solution of the Lagrangian relaxation (10.1) - (10.3) using $\lambda_i^k \in \mathbb{R}$ and $\psi_i^k \in \mathbb{R}, \forall i \in V_{DC}$ as the Lagrangian multipliers. Now set

$$\lambda_i^{k+1} = \begin{cases} \lambda_i^k + \beta^k s_i^k & if\ s_i^k > 0 \\ 0 & otherwise \end{cases} \tag{10.10}$$

$$\psi_i^{k+1} = \begin{cases} \psi_i^k + \gamma^k r_i^k & if\ r_i^k > 0 \\ 0 & otherwise \end{cases} \tag{10.11}$$

where β^k and γ^k are suitable scalar coefficients.

The procedure will start by seeding all the Lagrangian multipliers to zero. Formula (10.10) shows that if for a certain facility i, s_i^k is positive, it means that demand outstrips supply for that facility, and thus the corresponding value of λ_i should be increased to increase the cost of assigning customers to that facility in the next round. Similarly, if s_i^k is negative, it means that there is spare capacity, so λ_i should be reduced to make that facility more attractive for assignment in the next iteration. However, the adjustments to the multipliers when the capacity has not been violated for a facility do not need to be done and formula (10.10) ensures that the λ_i^k are always positive. Formula (10.11) could be explained in the same way as above.

We will use the following proportionality coefficients β^k and γ^k in equations 10.12 and 10.13, where α is a constant in the interval $(0, 2]$. Here, α could used used starting with 2 and halved whenever the iteration's lower bound failed to improve on the best known lower bound for every n iterations.

$$\beta^k = \frac{\alpha(UB - LB(\lambda^k))}{\sum_{i \in V_{DC}} (s_i^k)^2} \tag{10.12}$$

$$\gamma^k = \frac{\alpha(UB - LB(\psi^k))}{\sum_{i \in V_{DC}} (r_i^k)^2} \tag{10.13}$$

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