Modelling Nominal Rigidities in General Dynamic Equilibrium Framework

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A Thesis Submitted in Fulfilment of the Requirements for the Degree of
Doctor of Philosophy of Cardiff University

Cardiff Business School

Cardiff University

February 2008

UMI Number: U584225

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Dissertation Publishing

ProQuest®

ProQuest LLC
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106-1346

Acknowledgement

Abstract

The widely-used `New Keynesian' model assumes that there is no price flexibility, but prices and wages are extremely `sticky'. In such a model, it is also usual to assume some scheme of lagged indexation which increases the stickiness of inflation. Theoretically, however, we have found that indexation not to lagged actual inflation but to lagged expected inflation (rational indexation) turns out to be the best way to carry out indexation. One major implication of applying this as the indexation formula is that the New Keynesian model behaves in a more `classical' manner, with very little price stickiness, though still stickiness in real wages and in price mark-ups. Also, given the rational indexation scheme, the optimal monetary policy turns out to be price-level targeting, because such a rule would ensure price stability and minimise the distortions to relative prices due to price shocks. However, whether this socially optimal indexation scheme is feasible in practice is determined by a new empirical testing method suggested by Minford, Theodoridis and Meenagh (2007). From this test, we find that all models with a Calvo contracts framework with different indexation schemes are comprehensively rejected, including the case with theoretically optimal indexation, and so is a 'New Classical' model version, with flexible prices and wages and a one quarter information lag. There is no evidence that indexation of any sort existed. However, the New Classical model does not perform worse than the simple Calvo contract model, suggesting that when the model is improved sufficiently to pass this test, price rigidity will not necessarily feature in the specification.

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Introduction

In this work the object is to determine what sort of nominal rigidity can be justified both theoretically and empirically in macroeconomic models.

Theoretically it is usually assumed that there are some constraints on the flexibility of prices (Blinder, Canetti, Lebow and Rudd, 1998; Zimmermann, 2003; Akerlof and Yellen, 1985; Mankiw, 1985; Parkin, 1985; Eichenbaum and Fischer, 2003; Dotsey, King and Wolman, 1999, Taylor, 1979; and Calvo, 1983). Thus in Calvo's model of contracts it is assumed that people cannot change prices except stochastically because of menu costs of price change. Nevertheless it is usually assumed that such costs would not apply to an additional general indexation scheme (e.g. Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2003, 2007); so within this supposed constraining framework it is reasonable to ask what sort of indexing scheme would be best.

An indexation scheme is an arrangement whereby price-setters all agree that their prices will go up automatically by some formula in addition to whatever discretionary changes they may make; it could emerge in a variety of ways. It could be negotiated by leading firms or unions as a coordinated move. Or it could come about by the actions of individuals setting their own prices using their own chosen formula; by an evolutionary process the best such formula, from the viewpoint of general welfare, is eventually chosen by all. Or it could be imposed by government acting on behalf of all. We do not say more here about the process by which the best indexation arrangement is chosen; but we assume that the arrangement which maximizes social welfare will be chosen some way or another (Gray, 1976; Henin and Zylberberg, 1986). Hence our work discusses which of the various potential indexation arrangements is socially optimal.

Of course just as we do not know exactly what obstructs price flexibility in practice, so too we do not know exactly either what might obstruct indexation arrangements of different sorts in practice. Therefore while we can discover optimal arrangements, they may not be possible in practice, just as perhaps flexible prices may not be (and are generally believed not to be). It follows that empirical evidence on the capacity of different models embodying different indexation of contracts to explain the data is judge and jury. Economists have been willing to embrace Calvo contracts with lagged indexation for example because they have thought the models with this feature could explain the data better than those without (Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2003; Woodford, 2003; Gali and Gertler, 1999; Ireland, 2000; Collard and Dellas, 2006).

So in the final part of this thesis we look at the explanatory power of the different models of indexation using the testing method in Minford, Theodoridis and Meenagh (2007). If we found for example that a model which optimises welfare cannot explain the data, we would conclude that some unknown costs must constrain agents from adopting this model. Of course if we found that there was coincidence between the optimizing model and the one best fitting the data we could conclude

that there was no such cost. Thus the empirical evidence is our effective guide to determining the existence of cost constraints we cannot observe directly.

The plan of the thesis is as follows. In the first chapter we conduct a positive analysis to the Calvo contract model with a given monetary policy (an interest rate setting rule structured in such models) to find out the optimal indexation that can maximise the welfare level of a representative agent. In the second chapter we investigate the links between optimal indexation and different monetary policy to work: that is, we ask how different policies affect optimal indexation and also how welfare varies with these policies and their associated indexation optimum. In the third chapter we examine the empirical evidence. In the last chapter we draw some conclusions.

Chapter 1

Optimal Indexation in General Equilibrium

1.1 Literature Review

The Lucas critique (1976) states that if expectations are formed rationally, then unless the estimated equations used in a model to evaluate the consequences of alternative government policies are genuinely structural or behavioural, the implications of such simulations or evaluations may be seriously flawed. Thus, in order to avoid this, macroeconomic modelling has adopted the dynamic general equilibrium approach with optimising agents. After a long period of focusing on non-monetary factors in the business cycle (Kydland and Prescott, 1982; and Long and Plosser, 1983), economists turned to the question of how to conduct monetary policy following empirical work (Romer and Romer, 1989; Bernanke and Blinder, 1992; Gali, 1992; Bernanke and Mihov, 1997a; Christiano, Eichenbaum and Evans, 1996, 1998; Leeper, Sims and Zha, 1996) making the case for the short-run effects of monetary policy on the real economy.

1.1.1 Long run relationship

Almost all economists accept that the long-run effects of money fall entirely or almost entirely on prices, with little impact on real variables. In the long run, McCandless and Weber (1995) look at data for a 30-year period for 110 countries using different definitions of money and examine average rates of inflation, output growth, and different money growth rates. They show that in all countries the correlation between inflation and the growth rate of money supply is almost 1 depending on the money supply definition. This result is consistent with studies based on smaller samples for different countries and time periods (Lucas, 1980b; Geweke, 1986) and with the quantity theory of money:

a change in the growth rate of money induces an equal change in inflation (Lucas, 1980b) but they find no correlation between either inflation or money growth and the growth rate of real output. Using a sub-sample of OECD countries, they also report a positive relation between output growth and money growth. However, Kormendi and Meguire (1985) for a sample of almost 50 countries and Geweke (1986) for the United States argue that the data reveal no long-run effect of money growth on real output growth.

The relationship between interest rates, inflation and money has also been studied. The Fisher equation shows that if real returns are independent of inflation, then nominal interest rates should be positively related to expected inflation. In terms of long-run correlation, nominal interest rates should be positively correlated with average rates of inflation, which in turn are positively correlated with average money supply, so that nominal rates and money growth rates are positively related. For example, Monnet and Weber (2001) examine annual average interest rates and money growth rates over 1961-1998 for a sample of 31 countries and find a correlation of 0.87 between these variables in the long-run.

1.1.2 Short run relationships

Among studies of real effects of monetary disturbances in the short run, Walsh (2003) plots the quarterly data of the detrended log of real GDP and the log of M2 from 1967:1 to 2000:4 for the U.S. and finds that M2 is positively correlated at lags but negatively correlated at leads: the movements in money lead movements in output. This pattern is a replication of that of Friedman and Schwartz (1963b) who provide important empirical evidence that money matters for business fluctuations. They use U.S. data for the period from 1876 to 1960 to show that money growth rate changes lead changes in real economic activity. Faster money growth tends to be followed by increases in output above trend, and a slow-down in money growth tends to be followed by a decline in output. A similar relationship between money and real variables is also revealed in the early 1980s. Romer and Romer (1989) use the historical record of the post-war era to identify six monetary shocks when the Federal Reserve created recession to reduce inflation, and find that this anti-inflationary policy led, on average, to a rise in the unemployment rate of two percentage points. The effect is statistically significant and robust to a variety of changes in specification. Also they find that in the interwar era monetary disturbances have large real effects. All this evidence is based on timing patterns, but the simple correlation may not indicate the true causal role of money (Walsh, 2003).

Many studies indicate that money supply is caused by real economic activity. Tobin (1970) models formally the positive correlation between money and output, but he shows that the correlation in Friedman and Schwartz (1963b) can be interpreted as output causing money. He uses a deterministic model with the Keynesian idea that money and real activity respond to aggregate demand so that

money is endogenous. On the other hand, King and Plosser (1984) using a stochastic neo-classical growth model in which money and real activity are assumed to respond to variations in real opportunities (e.g. tax) show that the real sector actually drives the monetary sector. Inside money (e.g. M1 or M2) is more highly correlated with output movements in the U.S. than is outside money because it reacts to other non-policy shocks. In addition, Coleman (1996) estimates a model with endogenous money and finds that money is more highly correlated with lagged output rather than with future output, which means that output leads the money supply.

However, if the monetary authority sets a short-term interest rate, then the changes in money stock are endogenous. Walsh (2003) shows that interest rates typically increase prior to economic downturns, but this does not mean monetary policy causes cyclical fluctuations. Friedman and Meiselman (1963) find that monetary policy is important in the determination of nominal income. To establish the causality of this relationship, Sims (1972) uses log levels of U.S. nominal GNP and money to test causality. He finds evidence that money Granger-caused GNP- the past behaviour of money helps to predict future GNP. However, using the index of industrial production to measure real output, Sims (1980) finds that the fraction of output variation explained by money is reduced when the nominal interest rate is included. In the U.S, a short-term interest rate provides a better measure of monetary policy actions than the money supply. Stock and Watson (1989) provide a systematic treatment of the trend specification in testing whether money Granger causes real output. They conclude that money does predict future output even when prices and an interest rate are included.

Studies of the relationship between monetary policy and real economic activity have been also carried out in a Vector Autoregression (VAR) framework. The consensus has emerged that an economy responds to monetary policy shocks. A variety of VARs estimated for a number of countries all indicate that, in response to a policy shock, output follows a hump-shaped pattern. However, despite this consensus on the real effects of monetary shocks, there is still disagreement on how these shocks are identified in a VAR, since the disturbances to the equations are in general a linear combination of all the shocks in the system. One method of identification is to assume that policy shocks affect output with a lag (Sims, 1972, and Bernanke and Blinder, 1992). Christiano, Eichenbaum and Evans (1999) assume that the time t variables in the Fed's information set do not respond to time t realisations of the monetary policy shock. The other method is to impose restrictions on the long-run effects of the disturbances on the observed variables, for example, Blanchard and Quah (1989) and Gali (1992). Sims (1992) summarises the VAR evidence on money and output from France, Germany, Japan, the U.K. and the U.S. He estimates separate VARs for each country using their industrial production, consumer prices, a short-term interest rate, a measure of the money supply, an exchange rate index and a commodity price index. He identifies the shocks so that monetary shocks potentially affect the other variables contemporaneously, while the interest rate is not affected

contemporaneously by innovations in any of the other variables; and he finds that in all countries, the negative output effects of a contractionary shock build to a peak after several months and then gradually die out. One of the most prominent empirical results on the real effects of monetary shocks is produced by Christiano, Eichenbaum and Evans (2005), using the identification scheme under which interest rates respond to a monetary shock in time t and other variables do not respond contemporaneously to interest rates. They find a set of stylised facts that the subsequent literature takes for granted. There is a hump-shaped response of output, consumption and investment to a monetary policy shock, with the peak effect occurring after about 1.5 years, a hump-shaped response in inflation, with a peak response about 2 years, a fall in the interest rate for roughly one year, a rise in profits, real wages and labour productivity and an immediate rise in the growth rate of money.

1.1.3 Characteristics of New Keynesian models

This empirical work above created an agenda of research to find the answers to some central questions of monetary policy. For example, what caused the increased inflation experienced by many countries in the 1970s?; what sort of monetary policies and institutions would reduce the likelihood of it happening again?; how should the Federal Reserve respond to shocks that impact the economy? These investigations are possible due to considerable improvements in the underlying theoretical frameworks allowing a more realistic account of the real effects of monetary disturbances. This approach is called the New Neo-Keynesian Synthesis (NNS), or often New Keynesian for short.

Borrowing from the RBC literature, the NNS has a strong microeconomic foundation with optimising agents. Borrowing from the New Keynesian economics literature, all NNS models explicitly include price-setting firms as monopolistic competitors who have differentiated goods and maybe also monopolistic labour market with differentiated labour. The early authors of the New Keynesian theory, Samuelson, Modigliani and Tobin, believed that the classical models of Smith and Marshall describe the equilibrium towards which the economy gradually evolves, but the New Keynesian approach allows a description of the time horizon when prices are adjusting slowly. So to understand the slow adjustment in wages and prices in the short run, one's model set-up must consist of agents who set the price, which serves as the microfoundations for Keynesian features. Dixon (2007) states that if all agents are price-takers, price can only be explained if there is a shadowy Walrasian main auctioneer who acts as market maker and who would adjust prices gradually in response to excess demand or supply, but this argument is theoretically weak. Given the explicit slow adjustment in nominal variables, monetary policy can raise money supply and lower short-run interest rates with resulting expansions in the aggregate economy. The RBC research (Kydland and Prescott, 1982; and Long and Plosser, 1983) puts forward the radical idea that nominal wage and price behaviour is irrelevant for understanding macroeconomic dynamics: changes in output and employment are

solely driven by real shocks and the intertemporal substitution between consumption and leisure, all prices adjust instantly to clear the market. If combined with the cash-in-advance assumption RBC models give a role to money growth via the effect of the inflation tax on labour supply and so output; this is distinct from Lucas' (1972) islands model in which the money surprise affects labour supply and so output through a information lag. The NNS model obtains the effect of money on output via long lasting nominal rigidities due to wage/ price contracts. Clearly then differences of approach lead to very different predictions of the effects of monetary policy. In this thesis my main focus is on NNS models with the Calvo contract set up, though I shall retain as a benchmark alternative the Lucas islands surprise model. The NNS suggests some conclusions about the role of monetary policy. First, NNS suggests that monetary policy actions are important to the real economy, persisting over some period of time, due to the gradual adjustment of individual prices and the general price level. Second, while in the short-run there is this nominal rigidity resulting from the price-setting behaviour, in the long-run there is neutrality.

The basic idea of introducing the short-run effects of money on output is as follows. Given nominal prices, usually when the money supply increases, this leads to a decrease in the nominal interest rate and, due to nominal price rigidities, to a decrease of the real interest rate. This decrease in real interest rates means an increase in aggregate demand at a given price, but the higher capacity utilisation increases marginal costs. Thus, this gives price-setters a reason to increase their relative prices. If prices are set continuously, then the attempt of each price-setter to increase his relative price would fail, and all prices and also the aggregate price would rise until the inflation rate is equal to the change in money supply, and aggregate demand and output go back to initial equilibrium. However, individual prices do not adjust continuously in the real world (Zimmermann, 2003); at the microeconomic level, price-setting is carried out at intervals. This leads to fluctuations in output at the aggregate level and also macroeconomic inefficiencies. Blanchard (2000) metaphorically, explains this as the effect of staggered price adjustments of individual prices on the slow adjustment of the aggregate price level in reaction to monetary shocks as follows: a chain gang moves slowly, because it can coordinate its movements only very imprecisely; the shorter the length between two members of the chain gang, and the greater of number of members, the slower will be the movement of the whole chain. During the adjustment process after a positive monetary shock, real money supply and aggregate demand are higher than initial values whereas real interest rates are lower than their initial values. In the long run, inflation equals the rate of money supply growth, so that aggregate demand, output and relative prices return to initial values and monetary policy is neutral. However, in the short run, given an imperfectly competitive market with price-setters, the higher aggregate demand goes together with higher output.

How do agents set their price or wage and what determines the length of adjustment intervals?

Rotemberg (1982) studies the consequences of firms slowly adjusting prices on the aggregate output. He constructs the rational expectations equilibrium of an economy with many such firms and showed that nominal shocks have a persistent effect on aggregate output. Subsequently, Akerlof and Yellen (1985), Mankiw (1985) and Parkin (1986) introduce the new idea of menu costs to explain this discontinuity in price adjustment: costs to changing a price or costs of implementing a price change. For example, Mankiw (1985) presented this assumption of a monopoly firm's pricing decisions- the firm sets its price in advance and changes that price ex post only by incurring a small menu cost, and showed this creates an economy-wide equilibrium that is below the social optimum. This is more costly to society than to firms, who therefore have no incentive to return the economy to the equilibrium. This idea was used in many models with monopolistic competition in either goods or labour markets to explain that even smaller menu costs give rise to nominal rigidity. For example, Ball and Romer (1990) argued that if there were some real rigidity in the economy, it would react with the nominal rigidity of prices, reducing the size of menu costs required to induce nominal rigidity. The real rigidity might take the form of an efficiency wage model for example, where the equilibrium determined the real wage which was not sensitive to the level of economic activity.

On the empirical side, Ball, Mankiw and Romer (1988) argued that menu cost theory had a clear prediction for the relation between inflation and the inflation-output trade-off. If the steady-state inflation was higher, for a given level of menu cost, firms will change prices more frequently, so there is less nominal rigidity and changes in nominal demand would have less effect on output when inflation is higher.

It is, however, difficult to believe that the menu costs can rationalise sluggish price-setting and be the reason for the effectiveness of the monetary shocks. Walsh (2003) argues that adjusting production is costly too and it is difficult to see how closing down a plant can be cheaper than the cost of printing some menus especially nowadays the technology has made those price processes much quicker and cheaper. The menu cost argument can be used in the model with only steady state (Dixon, 2007), but in the dynamic setting, there are different approaches: time-dependent models and state-dependent pricing models (Eichenbaum and Fischer, 2003). The latter assumes the number of firms changing prices in any given period is determined endogenously, depending on the state of the economy. Individual firms discretely adjust their prices at infrequent intervals and firms are more likely to adjust prices when there are large shocks to their markets. Dotsey, King and Wolman (1997) model this by assuming that firms pay a fixed cost when they change their price. In contrast, Burstein (2006) assumes that firms pay a fixed cost for changing price plans. Once they pay this cost, firms can choose not only their current price, but also a plan specifying an entire sequence of future prices. A key property is that small and large monetary policy changes have qualitatively different effects on the real economy. However, it is difficult to work with such state-dependent

models. Caplin and Leahy (1997) discuss the difficulties of introducing the state-dependent price adjustment into complete macroeconomic models: it is possible only with some extreme restrictions on the rules of the central bank, on behaviour of consumers and the nature of money demand. Also, both pricing set-ups generate similar results for many policy experiments. For example, Burstein (2006) shows that for moderate changes in the growth rate of money (less than or equal 5% on a quarterly basis), the time dependent models are a good approximation of state-dependent models. Eichenbaum and Fischer (2003) state that these are the reasons why the time-dependant models of pricings (Taylor 1979 and Calvo 1983) are still largely in use. They assume that the number of firms/households that change prices/wages in any given period is specified exogenously.

Taylor (1979) developed a model in terms of nominal wage-setting behaviour. He assumed an economy in which all wage contracts are periods long and a constant fraction of all firms determine their wage contracts in any given time period. A contract specifies a fixed nominal wage rate which will apply for the duration of the contract; employment is then determined by fluctuations in the demand for labour, given this nominal wage during the contract period. The contracts overlap each other. At the time that a given wage contract is in the process of being set, there will still be contracts set in the last periods which will be in effect during part of the current contract period. Moreover, during the next periods, contracts will be written which will also be in effect during part of the current contract period. Wage rates set in the current period reflect the wage rates set in these previous and future contracts. Therefore, each contract is written relative to other contracts, and this causes shocks to be passed on from one contract to another (contract multiplier). This contract formation generates inertia of wages and persistence in unemployment. The model shows that staggered wage contracts as short as 1 year can generate the type of unemployment persistence which has been observed during the post-war business cycle in the United States. A contract multiplier causes business cycles to persist beyond the length of the longest contract. Persistence of inflation is also generated by the contract. This original setup can be used for price setting too.

An alternative model of staggered price adjustment is by Calvo (1983). It is based on a constant hazard rate model: each period, the agent faces a given probability of re-setting its price/wage. The expected duration of the price/wage when it is set is the reciprocal of the reset probability. When the agent sets its price it looks into the infinite horizon, and takes into account the future price/wage with the probability that the current price being set will still be in force when the higher marginal costs and marginal product of labour materialise. The implication is that the deviations of price/wage inflation from their long-run equilibrium depend on current and future expected changes in real marginal costs/marginal productivity of labour. If the reset probability is 1/4 per quarter, there will be 25% of firms resetting price in any one quarter. When it sets its price, each firm expects that the price it is setting will last for 4 quarters, but there is ever diminishing probability

that the price might last forever longer. Firms choose an optimal price in a dynamic setting, but the fundamental probability of resetting the price is not explained. This price setting behaviour can also be applied to households, who reset their wages. The price/wage setting means that there is a distribution of firms' prices/households' wages since some firms/households have more chances to change their price and some may never have a chance, which will be in the tails of the distribution. The bigger the distribution, the more dispersion, the more inefficiency there is in the economy.

The staggered price creates inefficiency in household consumption decisions, and staggered wage setting leads to inefficiency in firm hiring decisions. Assuming there are two groups of firms, following a nominal shock one firm has a 'high price' and the other has a 'low price' due to price rigidity, then consumption moves away from 'high price' firms to 'low price' firms. This means that output in the 'high price' firms decreases but rises in the 'low price' firms. Therefore, 'high price' firms will employ less labour, while 'low price' firms increase their labour demand. Thus, labour moves from 'high price' firms to 'low price' firms. Due to diminishing returns, average labour productivity is lower than it would be if all firms faced the same level of demand. It means that at the same level of employment as in the flex-price economy, firms produce less output at the aggregate level. Or equivalently, for a given output level, the economy with price rigidity needs to employ more labour input to meet the demand; thus it incurs higher total costs of production compared with an economy with no price rigidity. On the other side, the wage rigidity can create groups of 'high wage' and 'low wage' households, and firms would naturally want to hire low cost labour, so that the demand for labour shifts from the 'high wage' households to 'low wage' ones. Due to increasing marginal disutility of labour supply the same amount of labour would cause total costs to rise compared to a flex-price economy. To keep total costs unchanged, the economy would demand less labour and thus produce less output. Therefore, the model with nominal rigidities produces an inefficiently low output level. The more nominal rigidity, the more inefficiency in the economy.

To close the NNS model monetary policy is represented by a rule for setting the nominal rate of interest. The nominal quantity of money is endogenously determined to achieve the desired nominal interest rate. Walsh (2003) comments that central banks implement monetary policy via a short-term nominal interest rate. This policy can stabilise the economy and rule out explosive solutions and ensure a unique rational equilibrium path if it satisfies the Taylor principle, which states that if nominal inflation rises the central bank should raise the nominal interest rate by more, so that the real interest rate rises. Monetary policy should also be designed to stabilise the economy in response to real shocks, the intrinsic uncertainty facing the economy (Clarida et al, 1999).

As Lane (2001) points out, New Keynesian Models also offers other attractions. The presentation of explicit utility and profit maximization problems allows the researcher to conduct welfare analysis, thereby laying the ground for policy evaluation. Allowing for nominal rigidities and market

imperfections alters the transmission mechanism for shocks and also makes monetary policy effective on aggregate activity. Therefore, thanks to the improvement in both theoretical foundations and empirical argument, the model has been intensively applied in discussion of monetary policy in both the closed and open economy.

1.1.4 Performance of NNS models

The aim of NNS models is to fit the "dynamic facts" of the economy, in particular those concerning the impacts of monetary shocks to the real economy. These facts are summarised in Christiano et al. (2005), we saw above: after an expansionary monetary policy shock there is 1) a hump-shaped response of output, consumption and investment, with the peak effect occurring after about 1.5 years, 2) a hump-shaped response in inflation, with a peak response after about 2 years, 3) a fall in the interest rate for roughly one year, 4) a rise in profits, real wages and labour productivity and 5) an immediate rise in the growth rate of money.

At the beginning of the research into NNS models most models considered only the case of nominal price rigidity. This, however, does not give an empirically realistic degree of persistence, assuming an empirical realistic average interval between price changes (Chari, Kehoe and McGrattan, 2000). They study a quantitative general equilibrium model with rational price-setting firms and Taylor (1980) staggered price-setting and ask if the model can generate a monetary business cycle. They use a variant of a standard sticky price model in which imperfectly competitive firms set staggered nominal prices and real money balances enter the consumer's utility function; there is also capital accumulation. They conclude that this model with a short period of exogenous price stickiness by itself can not generate business cycles driven by monetary shocks. The effect of staggered price-setting on the persistence of output is measured by the contract multiplier- the ratio of the half-life of output deviations after a monetary shock with staggered price-setting to the corresponding half-life with synchronised price setting. The half-life of output deviations with synchronised price-setting is roughly one-half the length of exogenous price stickiness. The half-life of output deviations in the data is 10 quarters. Thus if the period of exogenous price stickiness is about one quarter, then the model with staggered price-setting must produce a contract multiplier of about 20 in order to match the data. However, the calibration's contract multiplier is only 1. Thus, the benchmark model with only staggered price setting does not generate persistence because the elasticity of the notional short-run aggregate supply curve is quite large, so that monetary shocks induce firms to adjust to raise their prices even more than they would in flexible-price equilibrium. Ascari (2000) reaches a similar conclusion in a model that is similar to the framework in Chari, Kehoe and McGrattan (2000) but uses wage stickiness. Woodford (2003) describes this as prices are strategic substitutes by which he means that an increase in other prices makes it optimal for a firm to reduce the price

of its own good. Mankiw and Reis (2002) also argue that it cannot produce plausible inflation and output dynamics following a monetary shock, because although the price level is sticky in the model, the inflation rate can change quickly.

The empirical evidence indicates that inflation responds sluggishly to economic shocks (Walsh, 2003). However, based on a theory of sluggish price adjustment proposed by Calvo or Taylor, it implies that inflation is a purely forward-looking variable and can jump immediately in response to changes in output or expected inflation. The model of sluggish price adjustment does not give a chance to the inflation process to be persistent; any dynamics exhibited by inflation simply reflect the dynamic process that characterises the output gap which is exhibited in the NNS Phillips Curve. Nelson (1998) investigates the ability of the optimising models of price adjustment to match U.S. inflation data, but he concludes that inflation in the data is much more persistent than predicted by these models.

Moreover, Eichenbaum and Fischer (2003) find that using post-war U.S. time-series data, there is strong evidence against the standard Calvo model. However, if they allow for a lag between the time that firms re-optimise and the time that they implement their new plans or allow for measurement error in inflation, the Calvo model is no longer rejected. They use the econometric strategy of Hansen (1982) and Hansen and Singleton (1982) to exploit the fact that in any model incorporating Calvo pricing, certain restrictions must hold; they then test these restrictions without making assumptions about other aspects of the economy.

Beside the studies of price sluggishness in New Keynesian models, there is also attention to stickiness of nominal wages. Some researchers emphasise that wage rigidity is more important than price rigidity in explaining the stylised facts (e.g. Christiano et al., 2005; Dellas and Tavlas, 2005). Keynes (1936) explains the real effects of monetary policy when wage adjustment is slow. On the one hand, pure wage stickiness means that real wages have countercyclical movements, a monetary policy expansion raises the price level, and the resulting decline in real wages induces firms to increase employment and output, but Dixon and Kara (2006) shows that the nominal wage rigidity of the Calvo contract type itself does not generate hump-shaped inflation responses to monetary disturbances. On the other hand, the empirical evidence for the U.S. by Christiano et al. (1999) using VAR analysis show that the real wage is mildly pro-cyclical in response to identified monetary shocks. Given that these responses are uncorrelated to the change in productivity, this means that the real wage fails to decline sharply at the time of the economic expansion and so is inconsistent with a sticky wage model. Despite the criticism of a pure sticky wage model, Woodford (2003) says that allowing both wage and price stickiness can help to explain some failures of the pure sticky price model. Christiano et al. (1999) criticise sticky price models, because as a result of a negative monetary shock, the real wage declines so sharply that producers' profits ought to actually increase,

despite their reduced sales. Rotemberg and Woodford (1997) suggest correcting this by altering the preference parameters to give a more elastic labour supply. However, the alternative correction would include a slow adjustment of wages to changes in labour demand.

Nominal rigidities in both wage and price are now commonly used (Erceg et al., 1999, Canzoneri et al. 2004, etc, Christiano et al., 2005, Smets and Wouters, 2003). Huang and Li (1998) argue that wage stickiness is more important than price stickiness for generating output persistence. Christiano et al. (2005) estimate a model with both types of stickiness using U.S. data and report that wage rigidity and not price rigidity is the key to accounting for the observed dynamics of inflation and output. In contrast, Goodfriend and King (2001) argue that though nominal wages are sticky, the long-term nature of employment relationships means that nominal wage rigidity has little implication for real resource allocation: the labour market is characterised by long-term relationships where there is opportunity and reason for firms and workers to neutralise the allocative effect of temporarily sticky nominal wages; the product market is characterised by spot transactions where there is less opportunity for the effects of sticky nominal prices to be privately neutralised. Therefore, the consequences of temporary nominal wage rigidity are likely to be minor, but temporarily sticky nominal product prices can influence the average mark-up significantly over time. Woodford (2003) came to a similar conclusion about the necessity for including wage stickiness in the NNS model. He says that if researchers are only interested in constructing a positive model of the co-movement of inflation and output, and then given the way that both can be affected by monetary policy, wage stickiness does not matter, because it only flattens the short run Phillips curve further compared with price stickiness; the same effects in wage stickiness can be achieved by manipulating the values of other coefficients under the flexible wage model. Also, it leads to an additional way in which real disturbances may shift the Phillips curve, but Woodford (2003) says that similar consequences for the inflation and output dynamics can be obtained by postulating an exogenous cost-push shock. So he agrees that the wage rigidity can be neglected[1].

Nevertheless, Collard and Dellas (2006) state that the standard version of the New Keynesian model is unable to produce plausible inflation and output dynamics following a monetary shock-the delayed hump-shaped response of inflation documented by Christiano, Eichenbaum and Evans (2005). They also show that despite the introduction of price and/or wage rigidity and various real

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[1] If researchers are interested in monetary stabilisation policy from the welfare point of view, then the inclusion of wage rigidity does matter. King and Wolman (1999) show that strict price inflation targeting achieves the constrained optimum in a model with price inertia, thereby providing a rationale for inflation targeting within the class of NNS models. However, Erceg et al. (1999) find that this monetary regime induces substantial welfare costs when staggered wage setting is added to the model, since strict price inflation targeting forces all adjustments in real wages to occur through nominal wage rates, but the latter respond only to the output gap. Therefore, compared with the optimal policy rule, such a strategy induces excessive variation in output and nominal wages. The central bank should also respond to movements in the nominal wage. When wage inertia is the only inertia in the model, then the optimum can be achieved by strict wage inflation targeting. In the same vein, Canzoneri et al. (2004) show that wage inflation targeting is attractive and its good performance is robust across a variety of models; it strongly dominates price inflation targeting and appears to be as good as the best hybrid rule reacting to both wages and prices.

rigidities, the New Keynesian model still fails to replicate the dynamics.

There are other modifications to improve the New Keynesian model's performance. The models of sticky information of Mankiw and Reis (2002), incomplete information of Collard and Dellas (2004) and predetermined expenditure of Rotemberg and Woodford (1997) offer some solutions, while keeping to strict rationality. Another branch of literature- Christiano et al. (2005), Gali and Gertler (1999), Ireland (2000) Smets and Wouters (2003) - typically contains several sources of costly real adjustments alongside the nominal rigidities. Of these, the performance of the models of Christiano et al. (2005), and Smets and Wouters (2003) with lagged indexation offer support to the view that they are the most empirically successful version of the NNS models. In the most recent studies, Smets and Wouters (2007) using a Bayesian likelihood approach estimate a DSGE model with indexation and many nominal and real rigidities; they include seven types of structural shocks for the US economy using seven macroeconomic time series; and they find that price and wage stickiness are equally important in the set-up. Christiano et al. (2005), Smets and Wouters (2003), and Giannoni and Woodford (2003b) assume partial and full indexation of this kind for both wages and prices and argue that this extension of the Calvo pricing model improves the empirical fit of their models. This is because the allowance for backward-looking indexation generates the New Keynesian Phillips curve where the inflation rate is still a forward looking function of the expected path of the output gap, but also depends on the past period's inflation rate. This extension implies, therefore, inflation inertia to an extent that is greater as the indexation parameter is larger. Christiano et al. (2005) argue that a model with full indexation better fits their estimated impulse responses than the standard non-index model. Smets and Wouters (2002) treat the indexation parameters for price and wage as free ones and conclude that their model fits the data well when there are only 64% and 42% of price and wage being indexed to lagged inflation. These results are considered to be the most empirically successful variant of the New Keynesian model. Giannoni and Woodford (2003b) also treat this indexation parameter as a free parameter, but find that full indexation is the best scheme. These authors find that a model with staggered wage-setting as well as staggered price-setting and automatic indexation of both wages and prices to recent past inflation can account fairly well for the joint dynamics of wages, prices and real activity. Also, Collard and Dellas (2006) using a model where some firms index their prices to aggregate inflation show that the model can generate inertia in output and a liquidity effect, investment adjustment costs play a important role in producing this result too. However, Smets and Wouters (2007) estimate a model based on Christiano et. al (2005), on US data covering the period 1966 Q1- 2004 Q4. They find that backward inflation indexation is relatively unimportant in both goods and labour markets. The marginal likelihood of the estimation improves with very low values of the price indexation, this means it would be better to leave this friction out. Moreover, leaving out either wage and price indexation does not make any impact on

the other parameters.

Indexation helps empirically; but why in principle should one include it? The argument here only focuses on the Calvo contract models, but it applies equally to other types of nominal rigidities models too. In the Calvo contract nominal rigidity can last indefinitely in the sense that there is limited chance for wage- or price-setters to change their setting in any period. Hence once a price or wage is 'out of line' with its equilibrium there is a chance it will continue for ever. However, it is not an optimal and realistic assumption. It has been recognised (Christiano et al., 2005, Smets and Wouters, 2003, Yun, 1996, and Woodford, 2003) that the uncompromising nominal rigidity in Calvo (1983) ought to be modified to allow for some indexing process whereby general inflation is passed through by wage/price-setters. The argument has been that the chances of changing prices identified in the Calvo model relate to the changing of a relative price. Thus the 'menus' outside the restaurants are all updated for general inflation, some individual menus are then raised more or less than that according to micro shocks. The indexation thus allows all agents to enjoy a "nominal protection" that is protecting price- and wage-setters against movements in the general level of prices or wages. The basic Calvo model has this element but only for those who are able to change their prices. For wages the same argument would indicate that wages should additionally be updated automatically for general rises in wages, so that a general wage index would be used for them.

Two specific ways have been widely pursued for doing this indexing to: 'core' inflation and to lagged inflation. Yun (1996) assumes that prices are automatically increased at some rate between occasions on which they are reconsidered, where this inflation rate is the actual long-run average rate of inflation in the economy and unlike the classic Calvo model, this assumption results in a vertical long-run Phillips curve. In practice, Woodford (2003) says that indexation schemes are generally based on a measure of inflation over some relatively short recent time interval, because there is no presumption that inflation can always be expected to remain near some nonzero steady-state value. It is more plausible to assume automatic indexation of price commitments (or wages) to the change in the overall price index over some recent past period. It would be ideal to assume the current price index, but it is not a realistic or possible assumption. It is more plausible to imagine a policy of automatic indexation of one's price to the change in an overall price index over some past time interval.

Thus it seems that, assuming there are nominal rigidities or menu costs in the world, indexation must be the way to deal with them, allowing all agents to keep up with the general trend. There are many ways that one can index prices and wages. However, what is optimal indexation? This question is addressed next. Of course what is optimal may not be feasible (just as flexible wages and prices may not be) and therefore may not occur. Since we cannot determine feasibility from direct observation, the ultimate test of it will lie in whether different indexation models fit the dynamic

facts. This issue we return to in the third chapter of this thesis.

1.1.5 Indexation- previous work

There has been a vast amount of research on the appropriate rate of indexation in contracts. The idea is that money can affect the economy populated with rational agents through the presence of long-lived nominal contracts (Fischer, 1977, and Phelps and Taylor, 1977). Unanticipated inflation in this economy would produce substantial income distribution effects and the indexation of nominal contracts in general and of wage contracts can mitigate these effects. However, indexation in turn can affect the stability of the economy (Karni, 1983).

Gray (1976) and Fischer (1977) suggest labour contracts are insurance for workers against unexpected movements in the price level (e.g. Azariadis, 1975, and Baily, 1974). They argue that wages would not be fully indexed because it was not feasible to draw up a full-contingent contract expressed throughout in real terms. The indexation parameter is set to respond to a shock only partially. That is, if the shock raises the consumer price by 1%, the wages may not rise by the same amount since there may be an increase in other costs of production that the employer also has to pay. The optimal indexation parameter reflects the average equilibrium rise in wages in response to the typical shock, as compared with the same shock's effect on consumer price. Gray (1976) determines this optimal indexation of wage to the price level from a loss function given the mean squared deviation of the aggregate level of output from the output levels that would be obtained under complete information about the realisation of the random disturbances before the wage rate was set. Fischer (1977) demonstrates the existence of an indexation scheme for long-term wage contracts that replicates the equilibrium that would be obtained under non-indexed short-term wage contracts. The aim is to analyse the effectiveness of monetary policy to achieve stability rather than to analyse welfare implications of alternative indexation arrangements. He shows that monetary policy is effective even in a model with rational expectations. The consensus from these studies is that while full indexation of the nominal wage rate to the price level is an effective means of insulating real variables from the impact of monetary shocks, it exacerbates the impacts on the real variables of real disturbances: real wages are more responsive to shocks under nominal contracts, nominal wages more responsive under indexed contracts. Therefore, in response to real shocks wage contracts should be nominal and in response to nominal shocks they should be indexed. Minford, Nowell and Webb (2003) state that in aggregate supply and aggregate demand models, partial indexation maximises the social objective functions. The work by Mourmouras (1997) has explored social benefits of indexation further. He was motivated by the difference in theoretical idea about wage indexation and the policy in the early 1970s. From the theoretical point of view (e.g. Devereux (1987) and Fischer and Summers (1989)) if the natural rate of output is less than the socially optimal level, wage indexation by mak-

ing the Phillips curve steeper, reduces the incentives of a government to create surprise inflation. However, the policymakers did not like the wage indexation because it shows that the government is tolerant with inflation and not committed to fight it (Emerson, 1983; Simonsen, 1983). He uses a game-theoretic model of monetary policy with Gray-Fischer imperfectly indexed wage contracts to show that wage indexation leads an optimising government with objectives over both inflation and employment to adopt a more inflationary monetary policy. Thus, he supports the view of the policymakers that wage indexation weakens the will of a government to fight inflation. This approach concludes that indexation is only appropriate when the shocks are large.

These studies, however, lack a microeconomic foundation. Nominal contracts have been introduced into dynamic general equilibrium models like the ones in the RBC literature. Benassy (1995), Cho (1993), Cho and Cooley (1995) and King (1993) model economies with nominal contracts to study the importance of monetary shocks, propagated by contracts in aggregate fluctuations. They find that nominal contracts are important in general equilibrium models and show that only a little rigidity is necessary for monetary shocks to have substantial real effects. Still welfare costs of nominal wage contracts are missing in these analyses. Cho, Cooley and Phaneuf (1997) consider the welfare cost of different indexation and contract lengths. They estimate the welfare cost of wage contracts relative to an economy where wages are perfectly flexible. They find that the costs of having nominal contracts can be high as the rate of indexation rises or the labour supply elasticity falls. However, they do not model the benefits, and therefore the analysis of optimality is incomplete.

More recently, Minford et al. (2003) revised this approach, using a fully micro founded model with overlapping contracts and the representative agents maximising their welfare, restricted by available technologies. They choose the optimum wage contract from linear combinations of three: auction wages, a pre-set nominal wage where employment is set by the firm and a pre-set real wage converted into nominal payment by indexation. In the complete market, the pure auction contracts can be accompanied with insurance, then it can be optimal, but no financial market operation to reduce risk is allowed in this model. The wage contract, therefore, has to do both marginal reallocation and insurance that biases it away from the auction outcome and its equivalent fully-indexed version. They investigate whether an approach based index imperfection can be applied to a macro economy with plausible results. It was found that there is lag between an indexation payment and the time it can be spent (cash-in-advance), indexation is only useful only if there are permanent shocks to the price level. Indexation, however, destabilise consumption in the face of temporary shocks to price, because by the time indexation payment is spent, the shocks no longer exist. Empirically, it is not the size of monetary shocks but their persistence that determines indexation. This implies that monetary policy should target the level of money and not the rate of change in money, because this would reduce persistence and the degree of indexation; this in turn affects aggregate supply

and demand responses in the economy, flattening the Phillips Curve. This stabilises unemployment and output in the face of supply shocks; and provided money shocks can be kept low, there will be stability in the economy.

These papers establish that the degree of optimal nominal protection will vary with the characteristics of the monetary regime. At the one extreme where the regime is volatile and creates persistent shocks protection is high, at the other extreme where there is no monetary volatility at all protection is likely to be low and its exact nature depends on how real shocks behave. In assessing optimality these papers have usually considered a cooperative equilibrium strategy. This is a natural choice since agents might be deterred from changing prices or wages automatically in response to come general measure of inflation unless they were sure others were doing the same.

In the New Keynesian framework prices and wages are often assumed to be indexed to some general inflation process. The rationale for this as follows. Based on qualitative measures of indexation and wage contract length produced by Bruno and Sachs (1985) and estimates of wage indexation produced by Layard, Nickell and Jackman (1991), Minford et al. (2003) show that there was a substantial degree of wage protection in the mid-1970's when inflation was extremely high. However, they want to compare it with the 90s for which they have no direct estimates of the degree of indexation or its equivalent, because there was no explicit general indexation, so they have to use a technique suggested by Layard et al. (1991), where one could think that nominal wage contracts imply that real wages will be disturbed from their planned real level by inflation surprises, in a moving average process of length equal to that of the longest contract. Hence, they estimate an equation for real wage growth separately for the 70s and the 90s for some major OECD countries. From the regression, the sum of the coefficients on the inflation surprises will reflect the extent of nominal contracts, and they do show widespread indexation in these economies, even without the explicit official indexation.

There is no analogous test available for prices. We have to rely on testing of models, whether at the micro or macro level. At the micro level, some survey evidence shows that many (though not all) individual prices remain unchanged in money terms for several months, or even longer and they provide one of the main arguments for supposing that prices are not continuously re-optimised. For example, Alvarez et al. (2006) find that firms in the euro area change their prices infrequently, on average around once a year, these price durations are significantly longer than in the US. Bils and Klenow (2004) develop microeconomic evidence on the frequency of price adjustment and report that in the US firm level prices have a median duration of 4.3 months. However, without detailed modelling of individual firms and markets, it is simply not clear why this should be evidence of forced discontinuity in repricing, rather than voluntary setting of price guides (with unobserved adjustment for particular sales), including indexation to changes in general inflation in some way.

Of course chapter 3 is an attempt to test for the existence of indexation and a high degree of price flexibility at the macro level. As chapter 3 also shows the rigid-price model fares no better than the ones with more price/wage flexibility.

In short there is no knock-down empirical reason to ignore the theoretical arguments in favour of arrangements to introduce price and wage flexibility to some degree. As the theoretical parts of the thesis attempt to show there is a welfare argument for such arrangements and in particular for expected inflation as the index when Calvo pricing is unavoidable for relative prices.

1.1.6 Criticism of NNS models

In the context of the NNS models, lagged and rule of thumb indexation have been suggested as the solution, but are they optimal?

Indexation is an important part of designing an economic model since it acts as insurance against the fluctuations in general price and wage levels. There must be an optimal level of indexation that rational agents choose to maximise their welfare. However, in the NNS literature the indexation is introduced fully or partially just as an element that helps to bring the empirical fitness to the model, disregarding the welfare implications. Most of the time this indexation takes a form of lagged indexation (e.g. Christiano et al., 2005; Smets and Wouters, 2003), but there are problems with lagged indexation. This choice is not consistent with optimising behaviour because it means agents do not use all the information available at the time they set prices and wages. It is not a theoretically optimising behaviour and it effectively makes the Phillips Curves be exploitable because Minford and Peel (2003) argue that there is no micro foundation for such an assumption to make agents vulnerable to known disturbances. That is, the natural rate property does not hold, the monetary policy can influence output in the future after which all nominal contracts will be renegotiated. Using this non-exploitability as a specification test for the Phillips Curve, Minford and Peel (2003) find that similarly to the arguments of Phelps (1970) and Friedman (1968), if rational agents set contracts to optimise their welfare, then the Phillips Curve with lagged indexation is exploitable because the expected exogenous processes including monetary policy can cause the expected real wage to diverge from the planned optimal or equilibrium real wage. This is avoided if welfare optimising agents index rationally to the general price level. The Phillips Curve is non-exploitable in the sense that expected exogenous processes including monetary policy cannot cause expected output to diverge from its equilibrium level.

It is an important argument. McCallum (2007) remarked that the natural-rate hypothesis is regarded as a fundamental concept of monetary macroeconomics, it convinced even Keynesian economists by 1980. This agreement, however, was implicitly overturned by the introduction of the Calvo adjustment mechanism, where a positive inflation rate can attain a permanent positive output gap,

and it creates an opportunity for a monetary policy maker to exploit the Phillips Curve's relationship. One might say that this type of exploitable Phillips Curve creates no problem, since in the Calvo framework monetary policy does not want to destabilise the average mark-up from its flexible price level because the average mark-up level in the Calvo set-up acts like a tax on the labour and so the inflation rate is kept at zero, and if they aim to achieve this objective in all periods, it implies that they do not want to exploit the Phillip Curve, and the output is always at its potential level. The inflation and output gap trade-off is eliminated completely by the monetary authority. This applies to the case of the Calvo contract with lagged indexation in exactly the same way: if a central bank can commit to deliver an inflation rate of 0, then the Phillips Curve is not exploited. However, if a model includes both price and wage rigidities, there is a trade-off between the output gap and inflation; Goodfriend and King (2001) give an example of how New Keynesian models react to an adverse productivity shock: a temporary adverse productivity shock causes a reduction in hours worked accompanied by a low real wage, but if the nominal wage is sticky, the real wage cannot fall, then a monetary policy which aims to stabilise the mark-up and price must steer output below its potential to raise the marginal product of labour sufficiently to keep the mark-up and price constant.

Goodfriend and King (1997) questioned the inflation target as proposed by the New Keynesian literature. They argue that if the policy is credible, then one can make low-inflation target lawful mandate; but whether central bank can commit to a such policy depends on how forgiving price setters are to mistakes made by the policy-maker, because due to imperfect information, mistakes are unavoidable, and if the central bank cannot correct them in time, they would accummulate and inflation would move higher, and the forward looking behaviour in the Calvo framework would be the source of destabilisation. And since in the New Keynesian model the central bank has an incentive to cheat because its action can reduce the mark-up and boost employment, they may exploit this position. However, the rational agents would never let this happen. McCallum (2007) said that he was surprised that this Calvo Phillips curve would be used so frequently in today's analysis.

In what it follows in this chapter, we construct a normative analysis of how the Calvo contract framework ought to be altered. We address theoretically only the question of how indexation should be best carried out in the NNS framework, taking the optimality as if feasibility is not a problem. Though this issue has been addressed in other frameworks (Minford and Peel, 2004), it has never been carried out in a DSGE model with Calvo contracts. Assuming that nominal protection takes the form of 100% indexation, as cited in Christiano et al. (2005), and also assuming a standard monetary regime with a nominal interest rate rule, then should that index be lagged inflation or should it be lagged expected inflation?

1.2 The Model

The considered model contains formal descriptions of the behaviour of private agents and policy marker, and their interactions in markets of goods and labour. The main idea is that households consume goods. When deciding on their current level of consumption, and their level of borrowing and savings, they want to keep their lifetime consumption smooth. To do this, they can borrow and save using the bonds. On the other hand, firms seek to maximise profits by hiring labour and buying capital in order to produce output. Firms and workers bargain over the wages and given this, firms choose any level of labour at on going wage rate so that the costs of any extra workers are compensated for by the higher revenues they generate. Firms determine their level of capital so that the cost of capital is equal to the return to extra investment. To close the model, the monetary authority has the job of anchoring the nominal side of the economy using an nominal target rule. To create the effect of monetary policy on the real economy, prices and wages are assumed to be rigid, so that the central bank by changing this monetary policy, nominal interest rate, has the ability to influence real interest rates and that of the real economy. Lower real interest rates encourage consumers to spend and invest more now, since it is lowering financial costs. This effect is to push up domestic demand. To meet that demand, firms will demand more of factors used in the production. This increases the costs of labour and capital.

1.2.1 Firms' price setting behaviour

We assume there is a continuum of monopolistically competitive firms indexed by f on the unit interval. They produce differentiated goods. Instead of assuming that the households have the problem of choosing the optimal quantity of each differentiated good $Y_t(f)$ for $f \in [0, 1]$, we assume, as in Chari, Kehoe and McGrattan (2000), the artifice of a competitive bundler: the bundler combines these firms' goods $Y_t(f)$ at the prices $P_t(f)$ into a single product. The bundler acquires the goods in the same proportions as households and the government would choose, and then sells this single product to households and the government, as either a consumption good or capital good. Therefore, the bundler's demand for each differentiated good f is equal to the total demand.

The combined output Y_t is assembled using a constant returns to scale technology of the Dixit and Stiglitz (1977) form:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{(\phi_p - 1)}{\phi_p}} \, \partial f \right]^{\frac{\phi_p}{(\phi_p - 1)}}, \tag{1.1}$$

where $\phi_p > 1$ is the constant elasticity of substitution.

The output bundler chooses the bundle of goods that minimizes the cost of producing a given quantity of output index Y_t, taking the prices $P_t(f)$ of the goods $Y_t(f)$ as given. The bundler sells

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units of the output index at their unit cost P_t (aggregate price index):

$$P_t = \left[\int_0^1 P_t(f)^{(1-\phi_p)} \partial f \right]^{\frac{1}{(1-\phi_p)}} \tag{1.2}$$

The bundler's demand for each good $Y_t(f)$ — or equivalently total household demand for this good — is given by

$$Y_t^d = \left[\frac{P_t}{P_t(f)} \right]^{\phi_p} Y_t \tag{1.3}$$

Each differentiated good is produced by a single firm that hires capital $K_{t-1}(f)$ [2] and labour $N_t(f)$ at the rental rate R_t and wage rate W_t respectively. Every firm faces the same Cobb-Douglas production function, with an identical level of total factor productivity Z_t:

$$Y_t(f) = Z_t K_{t-1}(f)^\nu N_t(f)^{\nu-1}, \tag{1.4}$$

where $0 < \nu < 1$. Here, the productivity Z_t is assumed to follow a simple auto-regressive process: $\log Z_t = \rho \log Z_{t-1} + \varepsilon_{p,t}$. The firm chooses an optimal bundle of capital stock and labour services in order to minimise its cost:

$$\frac{R_t}{W_t} = \frac{\nu}{1-\nu} \frac{N_t(f)}{K_{t-1}(f)} \tag{1.5}$$

and the firm's marginal cost can be expressed as a function of total productivity, the rental rate and the wage index:

$$MC_t(f) = \frac{1}{\nu^\nu (1-\nu)^{(1-\nu)}} \frac{R_t^\nu W_t^{1-\nu}}{Z_t} \tag{1.6}$$

To introduce nominal price stickiness in to the model, we assume that firms set prices according to Calvo (1983) but subject to the ability to change all prices in line with an indexing formula, $\widetilde{P}_t$. The price-setters operate under imperfect competition where if prices were flexible they would be continuously set as a mark-up on marginal cost. However, prices are rigid. That is, forward-looking firms are allowed only periodically reoptimize their prices, so they incorporate higher future expected real marginal costs into their reset prices in order to maximize the stream of profits. They do this, because they may not be able to raise prices when the higher marginal costs come. Therefore, the setting is as following. In each period, a firm faces a constant probability, $1 - \alpha$, of being able to reset its price level. This provides the average duration of a price contracts is $(1-\alpha)^{-1}$ periods. Whenever the firm is not allowed to reset its price level, it sticks to its old one. For simplicity, it assumes that the firm's ability to reoptimise its price is independent across firms and time. Therefore, a constant fraction $(1 - \alpha)$ of firms are allowed to reset their contracts prices each period. However, all prices additionally rise with the general price index formula, $\widetilde{P}_t = \left(\alpha' P_{t-1} + \left(1 - \alpha' \right) E_{t-1} \log P_t \right)^{\alpha''}$.

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[2] $K_{t-1}(f)$ is the firm's demand for capital in period t. The aggregate capital stock is predetermined at the beginning of the period t.

If firm f gets to reset a new contract in period t, it chooses a new price $P_t^*(f)$ to maximize the value of its profit stream over states of nature in which the new price is expected to hold (note that the price prevailing at each period j will be $P_j^*(f)\widetilde{P}_j$):

$$E_t\Sigma_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j\left[P_t^*(f)\widetilde{P}_jY_j(f) - TC_j(f)\right],\qquad(1.7)$$

where $TC(f)$ is the firm's total cost, β is the household's discount factor , and λ_j is the households' marginal utility of nominal wealth. The first-order condition for a price-setting firm is:

$$P_t^* = \mu_p\frac{PB_t}{PA_t},\qquad(1.8)$$

where $\mu_p = \frac{\phi_p}{\phi_p-1}$ is a monopoly mark-up factor and

$$PB_t = E_t\Sigma_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j\frac{MC_j(f)}{P_j}\left(\frac{P_j}{\widetilde{P}_j}\right)^{\phi_p+1}\widetilde{P}_jY_j = \alpha\beta E_tPB_{t+1}+\lambda_j\frac{MC_j(f)}{P_j}\left(\frac{P_j}{\widetilde{P}_j}\right)^{\phi_p+1}\widetilde{P}_jY_j\quad(1.9)$$

$$PA_t = E_t\Sigma_{j=t}^{\infty}(\alpha\beta)^{j-t}\lambda_j\left(\frac{P_j}{\widetilde{P}_j}\right)^{\phi_p}\widetilde{P}_jY_j = \alpha\beta E_tPA_{t+1}+\lambda_j\left(\frac{P_j}{\widetilde{P}_j}\right)^{\phi_p}\widetilde{P}_jY_j\qquad(1.10)$$

Before the simplification, this first order condition can be read that the firm sets its price so that the sum of its expected discounted real revenue is equal to the price markup factor multiplied by the sum of discounted real costs. In the special case of flexible prices (where no indexing is necessary), all firms set their prices every period ($\alpha \to 0$), then $P_t^*(f) \to \mu_pMC_j(f)$, which is the standard imperfect competition outcome that the maginal product of labour equal to the real wage multiplied by the price markup. Since $\mu_p > 1$, output will be inefficiently low because the monopolistic competition exists in the market. The expectations operator $E_tx_{t+i} = E(x_{t+i} \mid \Phi_{t-1})$ where Φ_t is information (macro and micro) for period t. Notice that the expressions for PB_t and PA_t involve solely real variables and relative prices, viz. $\lambda_j\widetilde{P}_jY_j = (\frac{P_jY_j}{P_jC_j})$, $\frac{MC_j(f)}{P_j}$ and $(\frac{P_j}{\widetilde{P}_j})$.

1.2.2 Households' wage setting behaviour and capital accumulation

We assume that a continuum of monopolistically competitive households, indexed by h on the unit interval, who supply differentiated labour services to the production sector. Firms regard each household's labour services $L_t(h)$, $h \in [0,1]$, as an imperfect substitute for the labour services of other households: thus the labour market has a form of monopolistic competition. Again, we assume the artifice of a competitive bundler, who assembles all households' labour supplies $L_t(h)$ at the wages $W_t(h)$ in the same proportions as firms would choose. Thus, the bundler's demand for each household's labour is equal to the sum of firms' demands. The labour combination N_t has the

Dixit-Stiglitz form:

$$N_t = \left[\int_0^1 L_t(h)^{\frac{(\phi_w - 1)}{\phi_w}} \partial h \right]^{\frac{\phi_w}{\phi_w - 1}}, \quad \phi_w > 1 \qquad (1.11)$$

The labour bundler is given each household's wage rate $W_t(h)$ and has to choose an optimal amount of labour service so that it minimizes its total cost, and then sells units of the combined labour to the production sector at their unit cost W_t (aggregate wage index):

$$W_t = \left[\int_0^1 W_t(h)^{1 - \phi_w} \partial h \right]^{\frac{1}{1 - \phi_w}} \qquad (1.12)$$

The bundler's demand for the labour hours of household h (total demand for this household's labour by all goods firms) is given as:

$$L_t^d(h) = \left(\frac{W_t}{W_t(h)} \right)^{\phi_w} N_t \qquad (1.13)$$

The utility of the household is:

$$\max U_t(h) = E_t \Sigma_{\tau=t}^{\infty} \beta^{\tau - t} \left[\frac{1}{1 - \theta} C_\tau(h)^{1 - \theta} - \frac{1}{1 + \chi} AL_\tau \right], \qquad (1.14)$$

where $C_\tau(h)$ is the household's consumption of Y_t, and the second term on the right hand side of the equation reflects the disutility of work[3]. θ is the coefficient of relative risk aversion. Lucas (2003) focused on this parameter, arguing that the welfare cost of fluctuations in consumption are negligible unless θ is very high. However, here the model assumes a log utility function, where $\theta = 1$. The average disutility of work is $AL_t = \int_0^1 L_t(h)^{1 + \chi} \partial h$. If wages are flexible ($\omega = 0$), then $W_t(h) = W_t$ and firms hire the same amount of work from each household $AL_t = \int_0^1 L_t(h)^{1 + \chi} \partial h = L_t(h)^{1 + \chi}$. In this case, households are identical, and welfare is $U_t = U_t(h)$. If wafes are sticky ($\omega > 0$), then there is a dispersion of wages that makes firms hire different amount from each household. This creates inefficiency similar to the one created by price disperion- the composite labour serivce used by firm $N_t = \left[\int_o^1 L_t(h)^{\frac{\phi_w - 1}{\phi_w}} \partial h \right]^{\frac{\phi_w}{\phi_w - 1}}$ will not be maximised from given aggregate labour input $\int_o^1 L_t(h) \partial h$. The aggregate disutility of work is

$$AL_t = N_t^{1 + \chi} DW_t$$
$$DW_t = (1 - \omega) \left(\frac{W_t}{W_t^* \tilde{W}_t} \right)^{\phi_w(1 + \chi)} + \omega \left(\frac{W_t}{W_{t-1}} \right)^{\phi_w(1 + \chi)} \left(\frac{\tilde{W}_{t-1}}{\tilde{W}_t} \right)^{\phi_w(1 + \chi)} DW_{t-1}$$

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[3] The utility function and budget constraint should include a term in real money balances, but following much of NNS literature in assuming that this term is negligible. Since an interest rate rule is specified for monetary policy, there is no real need to model money explicitly (Canzoneri et al., 2007).

$1/\chi$ is the Frisch elasticity of labour supply. The household h maximizes (14) subject to its budget constraint, its labour demand curve (13), and its capital accumulation constraint. The households aim to maximise their lifetime utility subject to their constraints.

Household's budget constraint in period t states that consumption expenditures plus asset accumulation must equal disposable income:

$$E_t\left[\Delta_{t,t+1}B_{t+1}\left(h\right)\right] + P_t\left[C_t(h)\right] + I_t\left(h\right) + T_t] = B_t\left(h\right) + W_t\left(h\right)L_t^d\left(h\right) + R_t K_{t-1}\left(h\right) + D_t\left(h\right) \quad (1.15)$$

where the first term on the LHS is a portfolio of state-contingent claims (The number of (period $t+1$) dollar claims in the portfolio contingent on a given state's occurring, times the stochastic discount factor, $\Delta_{t,t+1}$-the price of a dollar claim divided by the probability of the state); I_t, is the household's investment in capital, T_t is a lump sum tax (used in the government budget constraint to balance every period, so that total lump sum transfers are equal to seinorage revenue), and the last three terms on the RHS are the household's wage, rental and dividend income. The household's capital accumulation is given by:

$$K_t\left(h\right) = (1-\delta)K_{t-1}\left(h\right) + I_t(h) - \frac{1}{2}\psi\left[\frac{I_t(h)}{K_{t-1}\left(h\right)} - \delta\right]^2 K_{t-1}\left(h\right) \quad (1.16)$$

where δ is the depreciation rate, and the last term of this equation is the cost of adjusting the capital stock.

Households set wages in staggered contracts, under assumptions symmetric to those stated earlier for price contracts. In any given period, each household gets a probability $(1-\omega)$ to reset their wage contract but again subject to the indexation formula, $\widetilde{W}_t = \left(\omega' W_{t-1} + \left(1 - \omega'\right) E_{t-1}W_t\right)^{\omega''}$; in this case the formula relates to wage behaviour, so that indexing occurs in relation to information about real wage behaviour as well as prices. Whenever the household is not allowed to reset its wage contract, the old contract wage remains in force apart from the indexing formula. The average duration of the wage contract is $(1-\omega)^{-1}$ periods. The probability of reoptimisation of wage contracts is independent across firms and time, so that every period there is a constant number $(1 - \omega)$ of households, who are allowed to reset their wage contracts. If household h gets to announce a new contract in period t, it chooses the new wage $W_t^*\left(h\right)$ to maximize its stream of lifetime welfare :

$$W_t^{*(1+\phi_w\chi)} = \mu_w\frac{WB_t}{WA_t} \quad (1.17)$$

where $\mu_w = \frac{\phi_w}{\phi_w - 1}$ is a monopoly markup factor, and

$$WB_t = E_t \Sigma_{j=t}^{\infty} (\omega\beta)^{j-t} N_j^{1+\chi} \left(\frac{W_j}{\widetilde{W}_j}\right)^{\phi_w(1+\chi)} = \omega\beta E_t WB_{t+1} + N_t^{1+\chi} \left(\frac{W_t}{\widetilde{W}_t}\right)^{\phi_w(1+\chi)} \qquad (1.18)$$

$$WA_t = E_t \Sigma_{j=t}^{\infty} (\omega\beta)^{j-t} \lambda_j \tilde{P}_j N_j \left(\frac{W_j}{\widetilde{W}_j}\right)^{\phi_w} = \omega\beta E_t WA_{t+1} + \lambda_t \widetilde{W}_t N_t \left(\frac{W_t}{\widetilde{W}_t}\right)^{\phi_w} \qquad (1.19)$$

where λ is the household's marginal utility of nominal wealth. In the limiting case where $\omega \to 0$, all households are allowed to reset their wages each period, that is a flexible wage model (again with no indexing required). Then $W_t^*(h) = \mu_w \frac{N_t^{\chi}}{\lambda_t W_t}$, that is, the wage is a markup over the marginal disutility of work. Or the real wage equals to the marginal rate of substitution of labour for consumption multiplied by the wage mark-up. Since the markup is greater than 1, the labour supply will be inefficiently low in the flexible wage solution. When wages are sticky ($\omega > 0$), wage rates will differ across households, and firms will demand more labour from households charging lower wages. Canzoneri et al.(2004) set this model so that there are heterogeneous agents, but assume complete contingent claims markets, that means that households are identical in terms of their consumption and investment decisions[4]. So, in equilibrium, aggregate consumption will be equal each household's consumption and to per capita consumption: $C_t = \int_0^1 C_t(h)\partial h = C_t(h) \int_0^1 h = C_t(h)$. The same is true for aggregate capital stock.

So, we can write the equilibrium versions of the households' first order conditions for consumption and investment in terms of aggregate values (where $\theta = 1$, making the welfare function into a log-linear function):

$$(C_t) \qquad \lambda_t = \frac{1}{P_t C_t^{\theta}} \qquad (1.20)$$

$$(B_{t+1}) \qquad E_t \frac{\lambda_{t+1}}{\lambda_t} = E_t \Delta_{t,t+1} = \frac{1}{1+i} \qquad (1.21)$$

$$(I_t) \qquad \lambda_t P_t = \xi_t - \xi_t \psi \left[\frac{I_t}{K_{t-1}} - \delta\right] \qquad (1.22)$$

$$(K_t) \quad \xi_t = \beta E_t \left\{ \lambda_{t+1} R_{t+1} + \xi_{t+1} \left([1-\delta] - \frac{1}{2}\psi \left[\frac{I_{t+1}}{K_t} - \delta\right]^2 + \psi \left[\frac{I_{t+1}}{K_t} - \delta\right] \frac{I_{t+1}}{K_t} \right) \right\}, \quad (1.23)$$

where λ_t and ξ_t are the Lagrangian multipliers for the households' budget and capital accumulation constraints, and i_t is the return on a 'risk free' bond.

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[4] The FOC for $B_{t+1}(h)$ is: $\Delta_{t,t+1} = \frac{\lambda_{t+1}(h)}{\lambda_t(h)}$, where $\lambda_t(h)$ is the marginal utility of wealth. All households face the same discount factor, $\Delta_{t,t+1}$; so if all households have the same initial wealth, $\lambda_t(h) = \lambda_t$, for all h. First order condition for $C_t(h), I_t(h)$ and $K_t(h)$ are identical for all h.

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1.2.3 The aggregate price and wage levels, aggregate employment and aggregate output

The aggregate price level is:

$$P_t = \left[\int_{j=0}^{\infty} P_t(f)^{1-\phi_p} \partial f \right]^{\frac{1}{1-\phi_p}} = \left[\sum_{j=0}^{\infty} (1-\alpha)\alpha^j \left(P_{t-j}^*(f)\widetilde{P}_t \right)^{1-\phi_p} \right]^{\frac{1}{1-\phi_p}}, \qquad (1.24)$$

since the law of large numbers implies that $(1-\alpha)\alpha^j$ is the fraction of firms that set their prices $t-j$ periods ago, and have not got to reset them since.

Equation (1.24) for the general price level can be converted into an inflation equation

$$\left(\frac{P_t}{P_{t-1}} \right)^{1-\phi_p} = \left[(1-\alpha) \left(\frac{P_t^* \widetilde{P}_t}{P_{t-1}} \right)^{(1-\phi_p)} + \alpha \left(\frac{\widetilde{P}_t}{\widetilde{P}_{t-1}} \right)^{1-\phi_p} \right] \qquad (1.25)$$

Similarly, the aggregate wage (1.12) can be written as

$$\left(\frac{W_t}{W_{t-1}} \right)^{1-\phi_w} = (1-\omega) \left(\frac{W_t^* \widetilde{W}_t}{W_{t-1}} \right)^{1-\phi_w} + \omega \left(\frac{\widetilde{W}_t}{\widetilde{W}_{t-1}} \right)^{1-\phi_w} \qquad (1.26)$$

The aggregate output is given by:

$$Y_t = \frac{Z_t K_{t-1}^{\nu} N_t^{1-\nu}}{DP_t}, \qquad (1.27)$$

where $N_t = \int_0^1 N_t(f)\partial f$ is aggregate employment; $K_{t-1} = \int_0^1 K_{t-1}(h)\partial h = \int_0^1 K_{t-1}(f)\partial f$ is the aggregate capital stock; and $DP_t = \int_0^1 \left(\frac{P_t}{P_t(f)\widetilde{P}_t} \right)^{-\phi_p} df$ is a measure of price dispersion across firms and it is

$$DP_t = (1-\alpha) \left(\frac{P_t}{P_t^* \widetilde{P}_t} \right)^{\phi_p} + \alpha \left(\frac{P_t}{P_{t-1}} \right)^{\phi_p} \left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}} \right)^{-\phi_p} DP_{t-1} \qquad (1.28)$$

1.2.4 Monetary and fiscal policy

The monetary authority anchors the nominal side of the economy using an simple reaction function given by empirical specifications (Canzoneri et al. (2004) used the data over the Volcker and Greenspan years 1979:3 - 2003:2):

$$i_t = 0.222 + 0.82 i_{t-1} + 0.35552 \pi_t + 0.032384 \, (output \ gap)_t + \varepsilon_{i,t}, \qquad (1.29)$$

where $\pi_t = \log \left(\frac{P_t}{P_{t-1}} \right)$ and $\varepsilon_{i,t}$ is the interest rate shock. However, it is not optimal in the normative sense, because it is not the one that derived from maximizing the expected utility of

the representative household. Rotemberg and Woodford (1998) showed that a simple interest-rate feedback rule, having the response to the inflation and output does almost as well as a more complicated rule which is optimal in their maximising expected utility framework (this model has only price stickiness, however, we still can base our rule specification on this argument). Canzoneri et al. (2004) say that the interest rule and its estimation does not show clearly how the rule should be interpreted in this NNS model. The reason is that the model cannot provide estimates of potential output. Therefore, the benchmark output gap is defined as the difference between actual output and the steady state output.

For simplicity we omit the fiscal shock:

$$\log G_t = \varepsilon_{g,t}, \tag{1.30}$$

since Canzoneri et al.(2004) found that government spending shocks, at least as modelled, did not have much effect in the model.

1.3 Solving for optimal indexation

Since the considered model is complex and nonlinear, its solution can be only derived using the numerical method. However, one may want to understand the insights of how such model works, this can be done by loglinearising all the equations. Still solving the loglinearisation of all original assumptions can be very difficult, in order to make the task possible, some simplifications are necessarily made. Everything is the same, the capital is assumed to be exogenous and made a non-tradeable endowment resource, while the labour market is assumed to be competitive.

1.3.1 A simplified model with exogenous capital and a competitive labour market

(1) Each firm is a price-setter, which forms the expectation of its price for period t based on micro information of period t and macro information of period $(t - 1)$. Each firm f minimises its total cost subject to its production function:

$$\min \ TC_t(f) = W_t L_t(f)$$

$$s.t \ Y_t(f) = Z_t \bar{K}^\nu L_t(f)^{1-\nu}$$

where $\bar{K}$ is exogenous and $\log Z_t = \rho_1 \log Z_{t-1} + z_t$. The labour demand function is derived from the above production function $L_t(f) = \left(\frac{Y_t(f)}{Z_t \bar{K}^\nu}\right)^{\frac{1}{1-\nu}}$. The firm's cost minimising problem implies that

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the nominal marginal cost is

$$MC_t = \frac{\partial TC_t(f)}{\partial \bar{Y}_t(f)} = \frac{\partial TC_t(f)}{\partial L_t(f)} \frac{\partial L_t(f)}{\partial \bar{Y}_t(f)} = \frac{1}{1-\nu} W_t L_t(f)^\nu \frac{1}{Z_t \bar{K}^\nu}$$

and thus, loglinearised real marginal cost is

$$\log mc_t = \log W_t - \log P_t + \nu \log L_t(f) - \log Z_t \qquad (1.31)$$

In regards to households, each of them maximises the life-time expected welfare subject to his budget constraint and labour demand, but without the capital accumulation constraint in this simple set up. However, besides the assumption of fully complete contingent claims that make the households homogeneous in their consumption decisions, the competitive labour market means they are homogeneous in labour supply also. The welfare maximisation implies every household supplies N_t^χ units of labour: $N_t^\chi = \frac{W_t}{P_t C_t}$ and its log-linear form is given by

$$\chi \log N_t = \log W_t - \log P_t - \log C_t \qquad (1.32)$$

The competitive labour market also means that in equilibrium the supply of and demand for labour must be equal so that the equation (1.31) becomes

$$\log mc_t = \log W_t - \log P_t + \nu \log N_t - \log Z_t$$

(2) The production function is given as:

$$\log Y_t = \log Z_t + (1 - \nu) \log N_t \qquad (1.33)$$

(3) Ignoring government spending, the market clearing condition gives

$$\log Y_t = \log C_t \qquad (1.34)$$

(4) We use a simple Taylor Rule, without lags and with the real interest rate assumed to be set in response to inflation and the output gap, with a monetary shock:

$$r_t = \tau \pi_t + \sigma(\log Y_t - \log Z_t) + \log M_t \qquad (1.35)$$

Adding in the Euler equation $\log C_t = \log C_{t+1} - r_t$ and allowing for market clearing gives us an

Aggregate Demand curve:

$$\log Y_t = \frac{1}{1 - \sigma^* B^{-1}} \left\{ -\tau \sigma^* \pi_t + \sigma \sigma^* \log Z_t - \sigma^* \log M_t \right\}$$

where $\log Z_t = \rho_1 \log Z_{t-1} + z_t$; $\log M_t = \rho_2 \log M_{t-1} + \mu_t$; z_t and μ_t are $i.i.d.$; B^{-1} is the forward operator instructing one to lead the variable but keeping the expectations data-set constant; and $\sigma^* = \frac{1}{1+\sigma}$.

(5) The reset price level loglinearised around its equilibrium (equation 1.62 in Appendix 1 section 1.3.5) is:

$$\log P_t^* = \frac{(1 - \alpha\beta) E_t \left(\log mc_t + \log P_t - \log \tilde{P}_t \right)}{1 - \alpha\beta B^{-1}} \tag{1.36}$$

This is rewritten using equations $(1.31), (1.32), (1.33)$ and (1.34) (equation 1.63 in Appendix 1 section 1.3.5) :

$$\log P_t^* = \frac{(1 - \alpha\beta)}{1 - \alpha\beta B^{-1}} E_t \left(\frac{1 + \chi}{1 - \nu} (\log Y_t - \log Z_t) + \log P_t - \log \tilde{P}_t \right) \tag{1.37}$$

Notice that though the firm at t knows its own marginal cost and its own price, in order to forecast the future paths of variables required to set its price it needs to know Y_t, P_t and $\tilde{P}_t$ which are macro variables; of these it only knows $\tilde{P}_t$. It does of course know Z_t, its own productivity level.

(6) Appendix 1 section 1.3.5 shows the log-linearised form of the aggregate price equation as

$$\log P_t - \log \tilde{P}_t = \alpha \left(\log P_{t-1} - \log \tilde{P}_{t-1} \right) + (1 - \alpha) \log P_t^* \tag{1.38}$$

(7) The loglinearised price dispersion, $\log DP_t$, is derived using a conventional second order Taylor expansion around $P_{t-i}^* = 1$ (Appendix 1 section 1.3.5):

$$\log DP_t = \frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1 - \alpha) \left[1 - \alpha^i (1 - \alpha) \right] \left(\log P_{t-i}^* \right)^2 - \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p \alpha^{i+j} (1 - \alpha)^2 \log P_{t-i}^* \log P_{t-j}^* \tag{1.39}$$

Solving the model under rational indexation

We now solve the model in turn under rational (this section) and lagged indexation (next section), so that we may compare the two welfare expressions. Our notation is as follows: $E^{t-1} x_t = E(x_t | \Phi_{t-1})$; $E_t x_t = E(x_t | \Phi_{t-1}, \phi_t)$; $x_t^{UE} = x_t - E^{t-1} x_t$, where Φ_{t-1} is the full information set from period $t - 1$ and ϕ_t is the limited information set available (to the agent forming expectations) for period t.

Assume under rational indexation that

$$\log \tilde{P}_t = E \left(\log P_t | \Phi_{t-1} \right) \equiv E^{t-1} \log P_t$$

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Applying this assumption to equation (1.38), we get:

$$E^{t-1} \log P_t^* = -\frac{\alpha}{1-\alpha} \left(\log P_{t-1} - E^{t-2} \log P_{t-1} \right) \tag{1.40}$$

and

$$E^{t-1} \log P_{t+i}^* = E^{t-1} E^t \log P_{t+i}^* = 0; \ \forall i \geqslant 1$$

Using the rational expectation assumption and equation (1.40), the reset price is therefore:

$$\log P_t^* = E^{t-1} \log P_t^* + \log P_t^{*UE} = \log P_t^{*UE} - \frac{\alpha}{1-\alpha} \log P_{t-1}^{UE}, \tag{1.41}$$

where from equation (1.38) (Appendix 1 section 1.3.5)

$$log P_t^{UE} = (1-\alpha) \, log P_t^{*UE} \tag{1.42}$$

and from equation (1.37) and the assumption that firms have knowledge of their own micro information (productivity, prices and costs) in period t as well as the macro information of period $(t-1)$:

$$
\begin{aligned}
\log P_t^{*UE} &= \log P_t^* - E^{t-1} \log P_t^* \\
&= (1-\alpha\beta) \, \Sigma_{i=0} \, (\alpha\beta)^i \left(\frac{1+\chi}{1-\nu} \right) \left[E^{t-1} \log Y_{t+i} - E^t \log Z_{t+i} \right] \\
&\quad - (1-\alpha\beta) \, \Sigma_{i=0} \, (\alpha\beta)^i \left(\frac{1+\chi}{1-\nu} \right) \left[E^{t-1} \log Y_{t+i} - E^{t-1} \log Z_{t+i} \right] \\
&= (1-\alpha\beta) \left(\frac{1+\chi}{1-\nu} \right) \frac{1}{1-\alpha\beta\rho_1} \, (-z_t)
\end{aligned}
\tag{1.43}
$$

Hence, given equations (1.41), (1.42) and (1.43) the reset price is rewritten as

$$\log P_t^* = \log P_t^{*UE} - \alpha \log P_{t-1}^{*UE} = \chi' \frac{(1-\alpha L)}{(1-\alpha)} \, (-z_t), \tag{1.44}$$

where $\chi' = (1-\alpha)(1-\alpha\beta) \left(\frac{1+\chi}{1-\nu} \right) \frac{1}{1-\alpha\beta\rho_1}$.

This in turn implies two things:

(a) $E^{t-1} \log P_{t+j}^* = (1-\alpha\beta) \, \Sigma_{i=0} \, (\alpha\beta)^i \left(\frac{1+\chi}{1-\nu} \right) \left[E^{t-1} \log Y_{t+i} - E^{t-1} \log Z_{t+i} \right]_{+j} = 0 \ \forall j \geqslant 1$

$\Leftrightarrow E^{t-1} \log Y_{t+i} = E^{t-1} \log Z_{t+i}, \ \forall i \geqslant 1$

(b) $\quad E^{t-1} \log P_t^* = (1 - \alpha\beta) \left(\dfrac{1 + \chi}{1 - \nu} \right) \left[E^{t-1} \log Y_t - E^{t-1} \log Z_t \right] = -\dfrac{\alpha}{1 - \alpha} \log P_{t-1}^{UE}$

Now we look at the output side of the model, where under rational expectations output consists of the expected output and the surprise change in output. First, using the Aggregate Demand curve we obtain for surprise output:

$$\log Y_t^{UE} = -\tau\sigma^* \log P_t^{UE} + \sigma\sigma^* z_t - \sigma^* \mu_t \tag{1.45}$$

and from the restrictions on expected reset prices above:

$$E^{t-1} \log Y_t = E^{t-1} \log Z_t - \left(\frac{\alpha}{1 - \alpha} \right) \frac{1}{(1 - \alpha\beta)} \left(\frac{1 - \nu}{1 + \chi} \right) \log P_{t-1}^{UE}.$$

The latter equation is in turn written as follows

$$E^{t-1} \log Y_t = E^{t-1} \log Z_t + v' z_{t-1}, \tag{1.46}$$

where $v' = \left(\frac{\alpha}{1-\alpha} \right) \frac{1}{(1-\alpha\beta)} \left(\frac{1-\nu}{1+\chi} \right)$ and we have used the expressions above for surprise prices and reset prices.

Using equations (1.42) and (1.43), the unexpected component of output can be written

$$\log Y_t^{UE} = \sigma^*(\sigma + \tau\chi') z_t - \sigma^* \mu_t \tag{1.47}$$

The aggregate output therefore is just a sum of its expected and unexpected components- equations (1.46) and (1.47)

$$\log Y_t = \rho_1 \log Z_{t-1} + v' z_{t-1} + \sigma^*(\sigma + \tau\chi') z_t - \sigma^* \mu_t \tag{1.48}$$

and

$$\log Y_t - \log Z_t = v' z_{t-1} + \sigma^*(\tau\chi' - 1) z_t - \sigma^* \mu_t \tag{1.49}$$

Welfare under rational indexation Under the flexible price and wage assumption, the welfare level would be

$$u_t^{FLEX} = \log Z_t \tag{1.50}$$

However, in the economy of price rigidity and competitive labour market, the welfare function is

$$u_t = \log C_t - \bar{N}^{\chi+1} \log N_t \tag{1.51}$$

where $\log C_t = \log Y_t = \log Z_t + (1 - \nu) \log N_t - \log DP_t$ and $\log N_t = \frac{1}{1-\nu} (\log Y_t - \log Z_t)$.

We evaluate expected welfare in terms of its deviation from the flex-price optimum:

$$E\left(u_t - u_t^{FLEX}\right) = E\left[\left(\frac{1 - \nu - \bar{N}^{\chi+1}}{1 - \nu}\right) \left\{ \begin{array}{c} v'z_{t-1} + \sigma^*(\tau\chi' - 1)z_t \\ \\ -\sigma^*\mu_t \end{array} \right\} \right] - E \log DP_t \qquad (1.52)$$

Notice that the unconditional mean of the first element in this expected welfare term is 0. So effectively we only consider the second term, where it is known respectively from equations (1.39) and (1.44) that

$$\begin{aligned} E\left(u_t - u_t^{FLEX}\right) &= -E \log DP_t \\ &= -\frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i (1 - \alpha)\left[1 - \alpha^i (1 - \alpha)\right] var\left(\log P_{t-i}^*\right) \\ &+ \Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j} (1 - \alpha)^2 Cov\left(\log P_{t-i}^*, \log P_{t-j}^*\right) \end{aligned}$$

and

$$\log P_t^* = -\chi'' z_t + \alpha\chi'' z_{t-1},$$

where $\chi'' = \frac{\chi'}{1-\alpha}$. With variances and covariances of the renewed price derived in Appendix 1 section 1.3.5, the expected welfare is now

$$E\left(u_t - u_t^{FLEX}\right) = -\phi_p\alpha\chi''^2 var(z) \qquad (1.53)$$

Solving the model with lagged indexation

Turning to the solution under lagged indexation, we write the index as

$$\log \tilde{P}_t = E^{t-1} \log P_t + (k \log P_{t-1} - E^{t-1} \log P_t) \qquad (1.54)$$

where if $k = 1$ then there is full lagged indexation $\log \tilde{P}_t = \log P_{t-1}$, and if $k = 0$ then there is a no indexation and $\log \tilde{P}_t = 0$. We will focus here exclusively on the case of full lagged indexation, $k = 1$.

Using this assumption and equation (1.38), the general price is

$$\log P_t = E^{t-1} \log P_t + v_{t-1} + \frac{(1 - \alpha) \log P_t^*}{1 - \alpha L},$$

where $v_{t-1} = \log P_{t-1} - E^{t-1} \log P_t = -E^{t-1}\pi_t$. This equation can also be written as

$$\left(\log P_t - E^{t-1} \log P_t\right) - v_{t-1} + \alpha v_{t-2} - \alpha \log P_{t-1}^{UE} = (1 - \alpha) \log P_t^*$$

and taking the expectation E^{t-1} throughout and manipulating this equation, we get expected reset price

$$E^{t-1}\log P_t^* = \frac{-v_{t-1} + \alpha v_{t-2} - \alpha \log P_{t-1}^{UE}}{1-\alpha} \tag{1.55}$$

What we see is that the expected reset price now contains the last two expected inflation rates; these terms are the bias in indexation away from its rational value.

We use equation (1.37) and its expectation

$$E^{t-1}\log P_t^* = \frac{(1-\alpha\beta)}{1-\alpha\beta B^{-1}} E^{t-1} E_t \left(\frac{1+\chi}{1-\nu} (\log Y_t - \log Z_t) + \log P_t^{UE} - v_{t-1} \right)$$

to obtain unanticipated reset price

$$\log P_t^{*UE} = (1-\alpha\beta) \left(\frac{1+\chi}{1-\nu} \right) \frac{1}{1-\alpha\beta\rho_1} (-z_t) \tag{1.56}$$

Solving for $\log P_t$ and $\log P_t^*$ under lagged indexation Using the equations for inflation together with the Aggregate Demand curve above yields (Appendix 1 section 1.3.5):

$$\pi_t = \frac{1-\alpha}{1-\alpha L} \frac{1-\alpha\beta}{1-\alpha\beta B^{-1}} E_t \left[\chi^* \left(\frac{1}{1-\sigma^* B^{-1}} \right) \left\{ -\tau\sigma^*\pi_t - \sigma^*(1 - B^{-1})\log Z_t - \sigma^*\log M_t \right\} + \pi_t \right] \tag{1.57}$$

where $\chi^* = \frac{1+\chi}{1-\nu}$. The solution for the general inflation rate has the Wold decomposition form $\pi_t = \Sigma_{i=0}\zeta_i\mu_{t-i} + \Sigma_{i=0}\xi_i z_{t-i}$; use undetermined coefficients to solve for π_t. Thus the solution for ν_t is by implication:

$$\nu_t = -E^t\pi_{t+1} = -\Sigma_{i=0}\zeta_{i+1}\mu_{t-i} - \Sigma_{i=0}\xi_{i+1}z_{t-i} \tag{1.58}$$

We once again find the solution for output's deviation from its flexprice value which now becomes:

$$\log Y_t - \log Z_t = \left(\log Y_t^{UE} - z_t \right) + E^{t-1}(\log Y_t - \log Z_t)$$

$$= (\sigma^*(\tau\chi' - 1)z_t - \sigma^*\mu_t) +$$

$$\left\{ \frac{1}{(1-\alpha\beta)\chi^*} \right\} \left(\begin{array}{c} \alpha\chi' z_{t-1} - \frac{\alpha(1+\beta)}{1-\alpha}v_{t-1} + \\ \frac{\alpha}{1+\alpha}v_{t-2} + \frac{\alpha\beta}{1-\alpha}E^{t-1}v_t \end{array} \right) \tag{1.59}$$

Welfare under lagged indexation Expected welfare is

$$E \left(u_t - u_t^{FLEX} \right) = E \left[\frac{1-\nu-\bar{N}^{\chi+1}}{1-\nu} \left(\begin{array}{c} (\sigma^*(\tau\chi' - 1)z_t - \sigma^*\mu_t) + \\ \left\{ \frac{1}{(1-\alpha\beta)\chi^*} \right\} \left(\begin{array}{c} \alpha\chi' z_{t-1} - \frac{\alpha(1+\beta)}{1-\alpha}v_{t-1} + \\ \frac{\alpha}{1+\alpha}v_{t-2} + \frac{\alpha\beta}{1-\alpha}E^{t-1}v_t \end{array} \right) \end{array} \right) \right] - E \log DP_t \tag{1.60}$$

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But again, we only have to consider the second term in this welfare expression, where

$$-E \log DP_t = -\frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1 - \alpha) \left[1 - \alpha^i (1 - \alpha) \right] var \left(\log P_{t-i}^* \right)$$

$$+ \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p \alpha^{i+j} (1 - \alpha)^2 \, Cov \left(\log P_{t-i}^*, \log P_{t-j}^* \right)$$

with

$$\log P_t^* = \frac{\chi'}{1 - \alpha} (-z_t + \alpha z_{t-1}) - \left(\frac{\nu_{t-1} - \alpha \nu_{t-2}}{1 - \alpha} \right) \quad \text{(Appendix 1 section 1.3.5)} \tag{1.61}$$

We have the following expressions: $\nu_{t-1} = \log P_{t-1} - E^{t-1} \log P_t = -E^{t-1} \pi_t$ is correlated with z_{t-1}; but $\nu_{t-2} = \log P_{t-2} - E^{t-2} \log P_{t-1}$ is uncorrelated with z_t and z_{t-1}. Assume that $-\frac{\nu_{t-1} - \alpha \nu_{t-2}}{1-\alpha} = \psi_0 z_{t-1} + q_{t-1}$, where $\psi_0 z_{t-1}$ combines all terms in z_{t-1}, and q_{t-1} is uncorrelated with z_{t-1} and z_t. Comparing this with the new reset price under rational indexation, this lagged indexation's renewed price function has the extra term $-\frac{\nu_{t-1} - \alpha \nu_{t-2}}{1-\alpha}$. For the task below, we temporarily take the expected welfare under rational expectation as a benchmark. To compare the expected welfare under lagged indexation to the benchmark, we need to investigate whether this extra term in renewed price improves or worsens the expected welfare level in respect to the benchmark.

It can be seen that there are two elements in this term. The first, $\psi_0 z_{t-1}$, is potentially helpfully correlated with the lagged term in the rational indexation solution for $\log P_t^*$; hence it could potentially reduce $E \log DP_t$. The second, q_{t-1}, adds noise to the solution and hence must increase $E \log DP_t$. We can find a closed form expression for the latter but the former requires numerical calculation. In Appendix 2 section 1.3.6 we look at whether this latter term can be signed unambiguously.

What we find is that for productivity shocks the effect of rational compared with lagged indexation is ambiguous; extra noise is introduced by the lagged index but some of it is correlated with the lagged productivity in a potentially helpful way. For monetary shocks rational indexation is unambiguously superior because in this case these shocks have no effect on the reset price and therefore on welfare; under lagged indexation monetary shocks at $t - 1$ and before all enter the current reset price setting.

Conclusions from the simplified model

What we find is that it is optimal within this simplified version of the model to index reset prices to the rational expectation of the price level in the face of monetary shocks; in the face of productivity shocks it is ambiguous. We thus find, in line with earlier work, that the type of indexation will depend importantly on the monetary regime. Thus, to put it rather crudely, provided monetary shocks are large enough, rational indexation will be optimal.

We find that should rational indexation be chosen, expected welfare is invariant to monetary policy; and the economy's Phillips Curve defaults to a 'New Classical' one where output depends on current and lagged real shocks but otherwise only on monetary surprises as in Lucas (1972); there is an echo here of Sargent and Wallace's (1975) famous irrelevance result three decades ago[5]. The intuition behind this result is that rational indexation builds into prices the effect of any shocks known at time $t-1$. Whatever has happened at $t-1$ is, in the case of the productivity shock, built into the expected real reset price for next period; this fixes expected real marginal cost and hence expected real output. The expected price index is then calculated as the necessary price increase that will accommodate this and the expected level of interest rates. Unexpected monetary shocks have no effect on prices because they have been pre-set in this way. Thus only lagged money shocks affect prices while only unexpected money shocks affect output under rational indexation in this model.

We wish to compare indexation by rationally expected ('rational') and lagged inflation. Rational indexation builds into price-setting the effect of any influences known at time $t-1$. Thus expected money supply and output will be translated directly into the indexation element. Unanticipated money supply shocks will have no effect on prices as in Sargent and Wallace (1975). Only current productivity shocks (perceived by the representative agent at the micro level) will cause an unexpected movement in prices; under Calvo pricing only those able to react to changing marginal cost will have been able to change their prices and so relative prices will have been distorted, causing a welfare loss. In the following period this effect is perceived at the macro level (and leads to a movement in the indexation element for that period), which in turn causes those firms that are now able to move their relative prices to do so in reaction to this previously unknown change in prices and costs. Hence one obtains an effect of the productivity shock current and lagged, on the reset price level (which determines relative prices); but no effect of monetary shocks.

Under lagged indexation, the effect of an unexpected money supply shock will again have no effect on the current reset price and therefore on actual prices because it is not observed; again the productivity shock will have an effect because perceived at the micro level. Next period lagged indexation will add to all prices this effect of the lagged productivity shock. In addition those who are price-resetters will change their prices in response to the expected effects of the indexation as well as those of both lagged shocks. This process plays out over time as the lagged indexation continues to move the general price level and those able to change prices each period react to this.

Welfare depends on relative price dispersion. Under rational indexation this depends solely on

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[5] Notice too that in a corollary of this point, again echoing Sargent and Wallace (ibid.), price level determinacy cannot be produced by an interest-rate rule targeting inflation unless the lagged price level is given; yet the model cannot generate such a lagged price level under such a regime unless again the twice-lagged price level was fixed and so on ad infinitum. It is necessary when using such a rule to specify the lagged price level exogenously via an initial condition, presumably indicating that at some previous point a different rule was being pursued.

the variance of the productivity shock; under lagged indexation it depends on the variance of both shocks. Hence if monetary shocks are significant this will mean that rational indexation will dominate lagged. If there are purely productivity shocks the comparison is ambiguous: This is because it is in principle possible for lagged indexation to produce lower relative price dispersion if it generates a smoother profile of price-resetting due to its dynamics. What we are seeing here is that under Calvo pricing indexation cannot remove the price-dispersing effect of productivity shocks and the size of these over time can be modified by different mechanical indexation mechanisms. However rational indexation can remove entirely the price-dispersing effects of monetary shocks- and this is its welfare contribution.

When we relax the assumption of restricted current information and allow agents to observe all current shocks then this strong result for monetary policy and shocks no longer holds. Not only do monetary shocks influence expected welfare but so do changes in the interest rate response to shocks; the reason lies in the immediacy with which monetary actions impact on expectations. Thus the invariance of welfare to monetary shocks and policy disappears with the broadening of the information assumption. This is illustrated below, in Table 1.1, for two nominal interest rate rules, one targeting inflation and the other targeting the price level.

Inflation target			Price target		
Productivity	Monetary	Total	Productivity	Monetary	Total
−0.001363	−0.001026	−0.002389	−0.00087	−0.000498	−0.00137
The interest rate rules here are for nominal interest rates, not as above for the real interest rate					

Table 1.1: Expected welfare for rational indexation under interest rule with inflation and price level targets, assuming full current information (standard deviation of 0.01 for monetary and productivity shock)

1.3.2   Simulation of the full model

We now use numerical methods to investigate the full model; we use Dynare (Juillard, 2003). We compute the impulse response functions of the model's variables under the different indexing processes in the face of temporary productivity, unexpected and expected monetary shocks. Throughout the simulation, we use a discount factor β of 0.99. The Cobb-Douglas capital share parameter, $\nu = 0.25$, implies that the output-labour elasticity is 0.75. The wage and price markup rates are $\mu_w = \mu_p = 1.167$. The constant probability determining the degree of price stickiness is $\alpha = 0.67$; this implies that an average price contract duration is 3 periods, while the probability of wage resetting is assumed to be $1 - \omega = 0.25$ in every period, implying an average contract length of 4 periods. The Frisch elasticity of labour supply is 0.33. These are the calibration parameters used by Canzoneri et al. (2004) in their benchmark specification. We also use the default assumption about information used by them and others who use this approach: namely that agents know all current
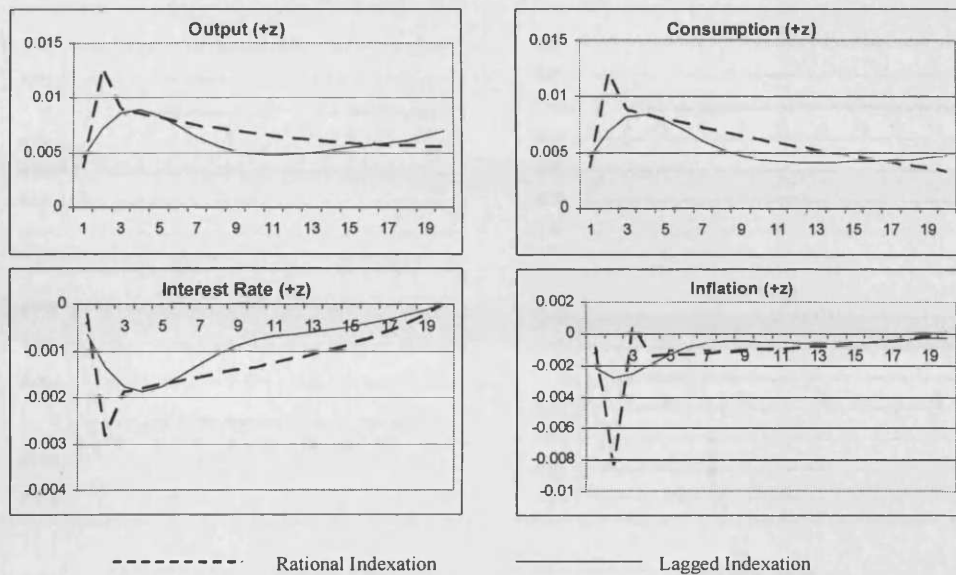
Figure 1.1: Dynamic paths after an unexpected 0.01 rise in productivity

information, macro and micro, when they form their expectations at t; the indexation formula still uses the lag information because of lags in application. Hence our initial simulations use the model exactly as in this literature except for the addition of the indexation formula, so that we can gauge the effect solely of adding in this one element. We later investigate alternative assumptions to check the robustness of the results- within this we check for the stricter information assumption made above in the analytic section, that only micro information is known at t. It does in fact turn out that our initial results are highly robust.

Our simulations' results are presented graphically. The impulse response functions show the mean value at each date after the initial shock for each variable.

Firstly, Figure 1.1 shows the impulse response functions produced by a positive temporary productivity shock that occurs in period 1. In both cases, the rise in productivity initially causes a boom. However, rational expectation indexing brings higher average output and consumption than the other indexation model.

Secondly, we assume an unanticipated negative monetary shock: the interest rate increases in period 1. In all cases, this contractionary policy causes recession, which eventually disappears. From Figure 1.2 we can see that under rational indexation output and consumption return very quickly to their steady states, while under the other case they exhibit some degree of inertia. The response of inflation shows that the shock's effect under rational indexation dies out quickly, whereas the other case shows more gradual responses.
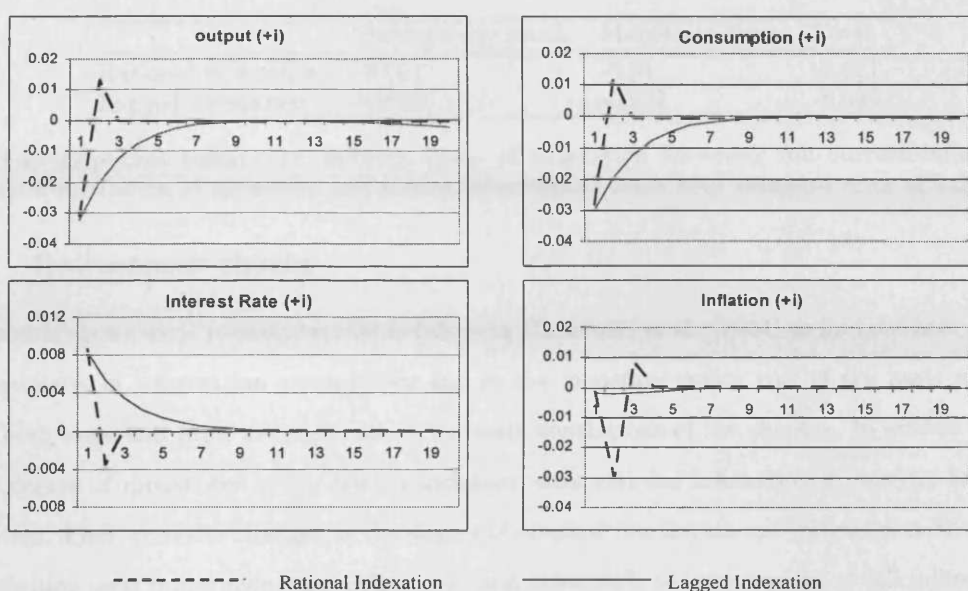
Figure 1.2: Dynamic paths after an unexpected contractionary monetary policy

In the more general cases of wage rigidity and full information, it is harder to disentangle the effects analytically and we resort to numerical simulation. Under full information money supply shocks now have a current effect under rational and lagged indexation. While with wage rigidity wage indexation enters and acts similarly on wages to price indexation on prices. What rational indexation does in these more complex set-ups is to allow wage/price-resetting agents to react to the current monetary shock knowing that all agents will add its expected effect into wages/prices next period. With these moving to absorb the demand stimulus rapidly excess demand should be limited and so relative wages/prices should be little disturbed. Lagged indexation implies that the money supply shock will lead to a slower corresponding change in wages and prices as the lags work through; hence excess demand and relative wages/prices would be more disturbed. This is what the simulations show. For productivity shocks the results, as for the simplified set-up above, are less clear cut– the reason again being that indexation cannot avoid the wage/price dispersing effect of productivity shocks.

Finally we show the model's computations under stochastic simulation of both monetary and productivity shocks for expected welfare in terms of deviations from the flex-optimum (the numbers are expressed as a flow per quarter — i.e. 'expected permanent welfare' in units of log consumption, that is fractions of permanent consumption). Table 1.2 shows that, in line with our analytical version, rational indexation dominates lagged for monetary shocks. For productivity shocks it slightly dominates but the two regimes are close, indicating the presence of the trade-off we found in the analytical model.

	Productivity shock	Monetary shock	Total
Rational indexation	-0.001	-0.01	-0.011
Lagged indexation	-0.002	-0.021	-0.023

Table 1.2: Expected welfare for different types of indexation assuming full current information (stochastic simulation of monetary and productivity shocks, each with standard error of 0.01)

1.3.3 Robustness checks

This section shows some robustness checks following Canzoneri et al. (2004) to find out how changes in parameters, in information assumptions and in the monetary policy rule of the basic model in which both wage and price are rigid, affect the main conclusions of the chapter. In general we find a high degree of robustness in our basic conclusion, that rational indexation is superior to lagged indexation. First, consider changes in the degree of nominal inertia; second variations in the output gap definition used in the monetary policy rule; and third with the assumption of full information.

In Table 1.4 presents the expected welfare costs of different degrees of nominal rigidity (from the benchmark of $(\alpha, \omega) = (0.67, 0.75)$); also with a variation in the Frisch elasticity from its benchmark of 3 to a value of 7. The first row shows the expected loss in the case of $(\alpha, \omega) = (0.67, 0.75)$ and the second row gives the expected loss in welfare in $(\alpha, \omega) = (0.5, 0.5)$. In both cases, the rational indexation's expected welfare losses are about half of what they are under the lagged indexation. The last row also shows that when Calvo rigidity is as low as $(\alpha, \omega) = (0.2, 0.2)$, rational indexation still dominates lagged indexation, though the gap narrows.

(α, ω)	Indexation	$\chi = 3$			$\chi = 7$		
		Productivity	Monetary	Total	Productivity	Monetary	Total
$(0.67, 0.75)$	Rational	-0.0012	-0.00967	-0.01087	-0.00173	-0.02132	-0.02305
	Lagged	-0.00191	-0.02066	-0.02257	-0.00303	-0.04417	-0.04720
$(0.5, 0.5)$	Rational	-0.00116	-0.01107	-0.01223	-0.00153	-0.02385	-0.02538
	Lagged	-0.00138	-0.02000	-0.02138	-0.00184	-0.04263	-0.04447
$(0.2, 0.2)$	Rational	-0.00090	-0.01365	-0.01455	-0.00091	-0.02900	-0.02991
	Lagged	-0.00097	-0.01580	-0.01677	-0.00110	-0.03433	-0.03543

Table 1.3: Expected welfare for different types of indexation with alternative degrees of price and wage inertia assuming full information set

Based on Rotemberg and Woodford (1997), Canzoneri et al. (2004) switch from the benchmark monetary rule, where the output gap was defined as the gap between the actual output and steady state output, to the 'good' policy that defines the output gap as the difference between the actual and flex-price output and show how this change alters the measure of welfare costs. The results for this experiment are reported in Table 1.4.

It is clear that there are differences between the two rules. The 'good' policy delivers higher expected welfare under both rational and lagged indexation (monetary shocks are unaffected because they do not alter the distance between the standard and the true output gap) but the ranking of the two indexation methods remains the same.

Policy	Indexation	$\chi = 3$			$\chi = 7$		
		Productivity	Monetary	Total	Productivity	Monetary	Total
Bad	Rational	-0.0012	-0.00967	-0.01087	-0.00173	-0.02132	-0.02305
	Lagged	-0.00191	-0.02066	-0.02257	-0.00303	-0.04417	-0.04720
Good	Rational	-0.00074	-0.00967	-0.01041	-0.00076	-0.02132	-0.02208
	Lagged	-0.00151	-0.02066	-0.02217	-0.00209	-0.04417	-0.04626

Table 1.4: Expected welfare for different types of indexation with alternative policies assuming full current information

We also look at the expected welfare under both lagged and rational indexation when only micro information is assumed to be known at period t. Table 1.5 is the same as Table 1.3 - and Table 1.6 as Table 1.4 - apart from this information assumption. Welfare is slightly worse since agents are assumed to have less current information but the ranking between rational and lagged indexation is again unaffected.

(α, ω)	Indexation	$\chi = 3$			$\chi = 7$		
		Productivity	Monetary	Total	Productivity	Monetary	Total
(0.67, 0.75)	Rational	-0.00381	-0.00098	-0.0048	-0.00627	-0.00232	-0.00859
	Lagged	-0.01459	-0.0121	-0.02669	-0.02528	-0.02563	-0.0509
(0.5, 0.5)	Rational	-0.0042	-0.00098	-0.00518	-0.00778	-0.00232	-0.0101
	Lagged	-0.01213	-0.0184	-0.03054	-0.02217	-0.03998	-0.06216

Table 1.5: Expected welfare for different types of indexation with alternative degrees of price and wage inertia assuming micro current information

(α, ω)	Indexation	$\chi = 3$			$\chi = 7$		
		Productivity	Monetary	Total	Productivity	Monetary	Total
Bad	Rational	-0.00381	-0.00098	-0.0048	-0.00627	-0.0232	-0.00859
	Lagged	-0.01459	-0.0121	-0.03669	-0.02528	-0.02563	-0.0509
Good	Rational	-0.00371	-0.00098	-0.00469	-0.00604	-0.0232	-0.00836
	Lagged	-0.01147	-0.0121	-0.02358	-0.01928	-0.02562	-0.0449

Table 1.6: Expected welfare for different types of indexation with alternative policies assuming micro current information

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1.3.4 Conclusion

We conclude that on theoretical grounds the Calvo contract should be adjusted for rationally expected rather than lagged inflation in the presence of monetary noise. The implications of the resulting model for monetary policy are radical if agents only observe local current information and there is price-setting only: neither monetary shocks nor monetary policy have any effect on welfare in this case, in an echo of Sargent and Wallace's (1975) famous policy irrelevance result. This strong result however no longer holds if all current shocks are observed as is usually assumed in this literature or if there is wage-setting. Further research would be worthwhile to discover the effects of different monetary regimes on the degree and nature of indexation and other nominal protection within the Calvo model. But what we have found here is that the Calvo contract does not provide a model of prolonged nominal rigidity so much as one of prolonged relative price rigidity combined with only brief (one period) nominal rigidity. As such it does not appear to be a helpful underpinning for New Keynesian models; nevertheless it may well be a fit and proper candidate for full empirical examination.

1.3.5 Appendix 1: Loglinearised model

Equation (1.36)

The reset price is given as

$$P_j^*(f) = \mu_p \frac{E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \frac{MC_j}{P_j} Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p+1} \tilde{P}_j}{E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \tilde{P}_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p}}$$

The loglinearised reset price level around its equilibrium is derived, using $\log X_t = \log E_t \Sigma_{i=0}^{\infty} \alpha^i Z_{t+i} = \Sigma_{i=0}^{\infty} (1-\alpha)\alpha^i \log Z_{t+i}$:

$$\log P_t^* = \log \mu_p + \log \left[\frac{E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \frac{MC_j}{P_j} Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p+1} \tilde{P}_j}{E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \lambda_j \tilde{P}_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p}} \right]$$

$$= \log \mu_p + \log \left(E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \Lambda_j mc_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p} \right) - \log \left(E_t \sum_{j=t}^{\infty} (\alpha\beta)^{j-t} \Lambda_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p-1} \right)$$

$$= \log \mu_p + E_t \left[\begin{array}{c} \Sigma_{j=t}^{\infty}(1-\alpha\beta)(\alpha\beta)^j \log \left(\Lambda_j mc_j \left(\frac{P_j}{\tilde{P}_j}\right)^{\phi} Y_j \right) \\ -\Sigma_{j=t}^{\infty}(1-\alpha\beta)(\alpha\beta)^j \log \left(\Lambda_j Y_j \left(\frac{P_t}{\tilde{P}_j}\right)^{\phi_p-1} \right) \end{array} \right]$$

$$= \log \mu_p + E_t \left[\Sigma_{j=t}^{\infty}(1-\alpha\beta)(\alpha\beta)^j \left(\log \Lambda_j + \log mc_j + \phi_p \left(\log P_j - \log \tilde{P}_j \right) + \log Y_j \right) \right]$$

$$- E_t \left[\Sigma_{j=t}^{\infty}(1-\alpha\beta)(\alpha\beta)^j \left(\log \Lambda_j + \log Y_j + (\phi_p - 1)(\log P_j - \log \tilde{P}_j) \right) \right]$$

$$= \log \mu_p + E_t \Sigma_{j=t}^{\infty}(1-\alpha\beta)(\alpha\beta)^j \left(\log mc_j + \log P_j - \log \tilde{P}_j \right) \tag{1.62}$$

Equation (1.37)

$$\log P_t^* = \frac{(1-\alpha\beta) E_t \left(\log mc_t + \log P_t - \log \tilde{P}_t \right)}{1 - \alpha\beta B^{-1}} =$$

$$= (1.31) \frac{(1-\alpha\beta)}{1 - \alpha\beta B^{-1}} E_t \left(\log W_t - \log P_t + \nu \log N_t - \log Z_t + \log P_t - \log \tilde{P}_t \right)$$

$$= (1.32) \frac{1-\alpha\beta}{1 - \alpha\beta B^{-1}} E_t \left(\chi \log N_t + \log C_t + \nu \log N_t - \log Z_t + \log P_t - \log \tilde{P}_t \right)$$

$$= (1.34)(1.33) \frac{1-\alpha\beta}{1 - \alpha\beta B^{-1}} E_t \left(\begin{array}{c} \frac{\log Y_t - \log Z_t}{1-\nu} \chi + (\log Y_t - \log Z_t) + \\ \nu \frac{\log Y_t - \log Z_t}{1-\nu} + \log P_t - \log \tilde{P}_t \end{array} \right)$$

$$= \frac{(1-\alpha\beta)}{1 - \alpha\beta B^{-1}} E_t \left(\frac{1+\chi}{1-\nu} (\log Y_t - \log Z_t) + \log P_t - \log \tilde{P}_t \right) \tag{1.63}$$

Equation (1.38)

The first order differential of the general price level in equation (1.25) is given by:

$$\left(1 - \phi_p\right) P_0^{-\phi_p} \partial P_t = (1-\alpha)\left(1 - \phi_p\right) P_t^{*(-\phi_p)} \tilde{P}_t^{1-\phi_p} \partial P_t^* + (1-\alpha)\left(1 - \phi_p\right) P_t^{*(1-\phi_p)} \tilde{P}_t^{-\phi_p} \partial \tilde{P}_t +$$

$$+ \alpha \left(1 - \phi_p\right) \left(\frac{P_{t-1}}{\tilde{P}_{t-1}}\right)^{1-\phi_p} \tilde{P}_t^{-\phi_p} \partial \tilde{P}_t +$$

$$\alpha \left(1 - \phi_p\right) \tilde{P}_t^{1-\phi_p} \left[\frac{1}{\tilde{P}_{t-1}^{1-\phi_p}} P_{t-1}^{-\phi_p} \partial P_{t-1} + P_{t-1}^{1-\phi_p} \tilde{P}_{t-1}^{\phi_p-2} \partial \tilde{P}_{t-1} \right]$$

Divide both sides of the equation by $(1 - \phi_p)$ and also, assume that prices at equilibrium equal to 1 and $\frac{\partial \log X_t}{\partial X_t} = \frac{1}{X_t}$ to get:

$$\partial \log P_t = (1-\alpha) \, \partial \log P_t^* + (1-\alpha) \partial \log \tilde{P}_t + \alpha \partial \log \tilde{P}_t + \alpha \left[\partial \log P_{t-1} - \partial \log \tilde{P}_{t-1} \right]$$

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Take the integral to find the price:

$$\log P_t - \log \tilde{P}_t = \frac{(1-\alpha)\ln P_t^*}{1-\alpha L} \qquad (1.64)$$

Equation (1.39)

Use a conventional second order Taylor expansion of $\log DP_t$, where DP_t is

$$DP_t = \left[(1-\alpha)\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}\right]^{\frac{\phi_p}{\gamma}} \left[(1-\alpha)\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*(-\phi_p)}\right]$$

letting $\gamma = 1 - \phi_p$. We assume that $DP_t = x_t * y_t$, therefore $\ln DP_t = \log x_t + \log y_t$ and $\partial \log DP_t = \partial \log x_t + \partial \log y_t$, where $x_t = \left[(1-\alpha)\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}\right]^{\frac{\phi_p}{\gamma}}$ and $y_t = \left[(1-\alpha)\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*(-\phi_p)}\right]$. Using $\partial \log x = \frac{\partial x}{x}$ gives therefore,

$$\partial \log x_t = \partial \log \left[(1-\alpha)\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}\right]^{\frac{\phi_p}{\gamma}}$$

$$= \Sigma_{i=0}^\infty \frac{\frac{\phi_p}{\gamma}\left[(1-\alpha)\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}\right]^{\frac{\phi_p}{\gamma}-1}\gamma(1-\alpha)\alpha^i P_{t-i}^{*(\gamma-1)}}{\left[(1-\alpha)\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}\right]^{\frac{\phi_p}{\gamma}}}\partial P_{t-i}^*$$

$$+ \frac{1}{2}\Sigma_{i=0}^\infty \left[\frac{\phi_p\alpha^i(\gamma-1)P_{t-i}^{*(\gamma-2)}\left[\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}\right] - \phi_p\alpha^i P_{t-i}^{*(\gamma-1)}\alpha^i\gamma P_{t-i}^{*(\gamma-1)}}{\left[\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}\right]^2}\right]\left(\partial P_{t-i}^*\right)^2 +$$

$$+ \frac{1}{2}2\Sigma_{j=0}^\infty\Sigma_{i=0}^\infty \left[-\frac{\phi_p\alpha^i P_{t-i}^{*(\gamma-1)}\gamma\alpha^j P_{t-j}^{*(\gamma-1)}}{\left[\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}\right]^2}\right]\partial P_{t-i}^*\partial P_{t-j}^*$$

$$= \Sigma_{i=0}^\infty \frac{\phi_p\alpha^i P_{t-i}^{*(\gamma-1)}}{\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}}\partial P_{t-i}^* + \frac{1}{2}\Sigma_{i=0}^\infty \left[\frac{\phi_p\alpha^i P_{t-i}^{*(\gamma-2)}}{\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}}\left(\gamma - 1 - \frac{\gamma\alpha^i P_{t-i}^{*\gamma}}{\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}}\right)\right]\left(\partial P_{t-i}^*\right)^2$$

$$- \Sigma_{j=0}^\infty\Sigma_{i=0}^\infty \left[\frac{\phi_p\alpha^i P_{t-i}^{*(\gamma-1)}\gamma\alpha^j P_{t-j}^{*(\gamma-1)}}{\left[\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}\right]^2}\right]\partial P_{t-i}^*\partial P_{t-j}^*$$

$$= \Sigma_{i=0}^\infty \frac{\phi_p\alpha^i P_{t-i}^{*\gamma}}{\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}}\partial \ln P_{t-i}^* + \frac{1}{2}\Sigma_{i=0}^\infty \left[\frac{\phi_p\alpha^i P_{t-i}^{*\gamma}}{\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}}\left(\gamma - 1 - \frac{\gamma\alpha^i P_{t-i}^{*\gamma}}{\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}}\right)\right]\left(\partial \ln P_{t-i}^*\right)^2$$

$$- \Sigma_{j=0}^\infty\Sigma_{i=0}^\infty \left[\frac{\phi_p\alpha^i P_{t-i}^{*\gamma}\gamma\alpha^j P_{t-j}^{*\gamma}}{\left[\Sigma_{i=0}^\infty \alpha^i P_{t-i}^{*\gamma}\right]^2}\right]\partial \ln P_{t-i}^*\partial \ln P_{t-j}^* \qquad (1.65)$$

and

$$\partial \ln y_t = \partial \ln \left[(1-\alpha) \Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]$$

$$= \Sigma_{i=0}^{\infty} \frac{1}{\left[(1-\alpha) \Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]} (1-\alpha) \alpha^i \left(-\phi_p\right) P_{t-i}^{*\left(-\phi_p-1\right)} \partial P_{t-i}^{*}$$

$$+ \frac{1}{2} \Sigma_{i=0}^{\infty} \left[\frac{\left(-\phi_p \alpha^i \left(-\phi_p-1\right) P_{t-i}^{*\left(-\phi_p-2\right)} \left(\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right) - \right)}{\left[\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]^2} \right] \left(\partial P_{t-i}^{*} \right)^2$$

$$+ \frac{1}{2} 2 \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \left[-\frac{-\alpha^i \phi_p P_{t-i}^{*\left(-\phi_p-1\right)} \alpha^j \left(-\phi_p\right) P_{t-j}^{*\left(-\phi_p-1\right)}}{\left[\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]^2} \right] \partial P_{t-i}^{*} \partial P_{t-j}^{*}$$

$$= \Sigma_{i=0}^{\infty} \frac{-\alpha^i \phi_p P_{t-i}^{*\left(-\phi_p-1\right)}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \partial P_{t-i}^{*} + \frac{1}{2} \Sigma_{i=0}^{\infty} \left[\frac{\phi_p \alpha^i P_{t-i}^{*\left(-\phi_p-2\right)}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \left(\begin{array}{c} \phi_p + 1- \\ \frac{\alpha^i \phi_p P_{t-i}^{*-\phi_p}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \end{array} \right) \right] \left(\partial P_{t-i}^{*} \right)^2$$

$$- \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \left[\frac{\alpha^i \phi_p P_{t-i}^{*\left(-\phi_p-1\right)} \alpha^j \phi_p P_{t-j}^{*\left(-\phi_p-1\right)}}{\left[\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]^2} \right] \partial P_{t-i}^{*} \partial P_{t-j}^{*}$$

$$= -\Sigma_{i=0}^{\infty} \frac{\alpha^i \phi_p P_{t-i}^{*-\phi_p}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \partial \log P_{t-i}^{*} + \frac{1}{2} \Sigma_{i=0}^{\infty} \left[\frac{\phi_p \alpha^i P_{t-i}^{*-\phi_p}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \left(\begin{array}{c} \phi_p + 1- \\ \frac{\alpha^i \phi_p P_{t-i}^{*-\phi_p}}{\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)}} \end{array} \right) \right] \left(\partial \log P_{t-i}^{*} \right)^2$$

$$- \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \left[\frac{\alpha^i \phi_p P_{t-i}^{*-\phi_p} \alpha^j \phi_p P_{t-j}^{*-\phi_p}}{\left[\Sigma_{i=0}^{\infty} \alpha^i P_{t-i}^{*\left(-\phi_p\right)} \right]^2} \right] \partial \log P_{t-i}^{*} \partial \log P_{t-j}^{*} \tag{1.66}$$

Evaluating this at $P_{t-i}^{*} = 1$ gives

$$\partial \ln x_t = \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1-\alpha) \partial \ln P_{t-i}^{*} + \frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1-\alpha) \left[(1-\phi_p) \left(1 - \alpha^i (1-\alpha) \right) - 1 \right] \left(\partial \ln P_{t-i}^{*} \right)^2$$

$$- \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p (1-\phi_p) \alpha^{i+j} (1-\alpha)^2 \partial \ln P_{t-i}^{*} \partial \ln P_{t-j}^{*} \tag{1.67}$$

and

$$\partial \ln y_t = -\Sigma_{i=0}^{\infty} \phi_p \alpha^i (1-\alpha) \partial \ln P_{t-i}^{*} + \frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1-\alpha) \left[\phi_p + 1 - \phi_p \alpha^i (1-\alpha) \right] \left(\partial \ln P_{t-i}^{*} \right)^2$$

$$- \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p^2 \alpha^{i+j} (1-\alpha)^2 \partial \ln P_{t-i}^{*} \partial \ln P_{t-j}^{*} \tag{1.68}$$

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Equation (1.42)

$$\ln P_t - \ln \tilde{P}_t = \alpha \left(\ln P_{t-1} - \ln \tilde{P}_{t-1} \right) + (1 - \alpha) \ln P_t^* \iff$$

$$\log P_t^{UE} = \alpha \left(\ln P_{t-1} - \ln \tilde{P}_{t-1} \right) + (1 - \alpha) \left(\log P_t^{*UE} - \frac{\alpha}{1 - \alpha} \log P_{t-1}^{UE} \right)$$

$$= \alpha \log P_{t-1}^{UE} + (1 - \alpha) \left(\log P_t^{*UE} - \frac{\alpha}{1 - \alpha} \log P_{t-1}^{UE} \right) = (1 - \alpha) \log P_t^{*UE} \qquad (1.69)$$

Equation (1.53)

$$E \left(u_t - u_t^{FLEX} \right) = -E \ln DP_t$$

$$= -\frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1 - \alpha) \left[1 - \alpha^i (1 - \alpha) \right] var \left(\log P_{t-i}^* \right)$$

$$+ \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p \alpha^{i+j} (1 - \alpha)^2 Cov \left(\log P_{t-i}^*, \log P_{t-j}^* \right)$$

$$= (1) + (2)$$

and

$$\log P_t^* = -\chi'' z_t + \alpha \chi'' z_{t-1}$$

Therefore, $var \left(\log P_{t-i}^* \right) = \left(1 + \alpha^2 \right) \chi''^2 var(z)$ and (1) is

$$(1) = -\frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1 - \alpha) \left[1 - \alpha^i (1 - \alpha) \right] \left(1 + \alpha^2 \right) \chi''^2 var(z)$$

$$= -\frac{1}{2} \phi_p (1 - \alpha) \left(1 + \alpha^2 \right) \chi''^2 var(z) \Sigma_{i=0}^{\infty} \alpha^i \left[1 - \alpha^i (1 - \alpha) \right]$$

$$= -\frac{1}{2} \phi_p (1 - \alpha) \left(1 + \alpha^2 \right) \chi''^2 var(z) \frac{2\alpha}{1 - \alpha^2}$$

$$= -\phi_p \chi''^2 \frac{\alpha \left(1 + \alpha^2 \right)}{1 + \alpha} var(z)$$

and (2) is

$$(2) = \phi_p (1 - \alpha)^2 \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \alpha^{i+j} E \left(\log P_{t-i}^* \log P_{t-j}^* \right)$$

$$= \phi_p (1 - \alpha)^2 \begin{bmatrix} i = 0; \ j = 1, 2... \\ \alpha \left(-\alpha \chi''^2 \right) var(z) \\ i = 1; \ j = 2, 3... \\ \alpha^3 \left(-\alpha \chi''^2 \right) var(z) \\ \vdots \end{bmatrix}$$

$$= -\alpha \chi''^2 \phi_p (1 - \alpha)^2 \frac{\alpha}{1 - \alpha^2} var(z)$$

Thus,

$$E\left(u_t - u_t^{FLEX}\right) = -\phi_p \alpha \chi''^2 var(z)$$

Equation (1.57)

Under lagged indexation we start by deriving the equation (1.57) as in the text. Manipulate the equations for inflation and the Aggregate demand curve respectively:

$$\pi_t = \log P_t - \log \tilde{P}_t = \frac{(1-\alpha)\log P_t^*}{1 - \alpha L}$$

and

$$\log Y_t = \frac{1}{1 - \sigma^* B^{-1}} \left\{-\tau\sigma^*\pi_t + \sigma\sigma^* \log Z_t - \sigma^* \log M_t\right\}$$

to get

$$(1 - \alpha L)\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{1 - \alpha\beta B^{-1}} E^t \left(\chi^* (\log Y_t - \log Z_t) + \pi_t\right)$$

and then

$$\pi_t = \frac{1-\alpha}{1-\alpha L}\frac{1-\alpha\beta}{1-\alpha\beta B^{-1}} E_t \left[\chi^* \left(\frac{1}{1 - \sigma^* B^{-1}}\right)\left\{-\tau\sigma^*\pi_t - \sigma^*(1 - B^{-1})\log Z_t - \sigma^* \log M_t\right\} + \pi_t\right]$$

Due to the assumption that in period t producers know both macro information from period $(t-1)$ and micro information in period t, the above equation is rewritten as:

$$(1 - \alpha L)\pi_t = (1-\alpha)(1-\alpha\beta)\chi^*\sigma^*\frac{E_t}{(1 - \alpha\beta B^{-1})(1 - \sigma^* B^{-1})}(1 - B^{-1})(-\log Z_t)$$
$$- (1-\alpha)(1-\alpha\beta)\chi^*\sigma^*\frac{E^{t-1}\log M_t}{(1 - \alpha\beta B^{-1})(1 - \sigma^* B^{-1})}$$
$$- \frac{(1-\alpha)(1-\alpha\beta)(\chi^*\tau\sigma^*)}{(1 - \alpha\beta B^{-1})(1 - \sigma^* B^{-1})}E^{t-1}\pi_t + \frac{(1-\alpha)(1-\alpha\beta)}{(1 - \alpha\beta B^{-1})}E^{t-1}\pi_t \qquad (1.70)$$

Equation (1.70) can be written as:

$$(1 - \alpha L)\pi_t - \frac{(1-\alpha)(1-\alpha\beta)}{(1 - \sigma^* B^{-1})}E^{t-1}\pi_t + \frac{(1-\alpha)(1-\alpha\beta)(\chi^*\tau\sigma^*)}{(1 - \alpha\beta B^{-1})(1 - \sigma^* B^{-1})}E^{t-1}\pi_t$$
$$= (1-\alpha)(1-\alpha\beta)\chi^*\sigma^*\frac{E_t(1 - B^{-1})(-\log Z_t)}{(1 - \alpha\beta B^{-1})(1 - \sigma^* B^{-1})} - (1-\alpha)(1-\alpha\beta)\chi^*\sigma^*\frac{E^{t-1}\log M_t}{(1 - \alpha\beta B^{-1})(1 - \sigma^* B^{-1})}$$

The LHS of this equation is rearranged into:

$$
\begin{bmatrix}
(1 - \alpha L)\left(1 - \alpha\beta E^{t-1}B^{-1}\right) \\
(1 - \sigma^* E^{t-1}B^{-1})\pi_t \\
-(1-\alpha)(1-\alpha\beta)(1-\sigma^* E^{t-1}B^{-1})E^{t-1}\pi_t \\
+(1-\alpha)(1-\alpha\beta)\chi^*\tau\sigma^* E^{t-1}\pi_t
\end{bmatrix}
=
\begin{bmatrix}
1 - \alpha L - \begin{pmatrix} \sigma^* + \alpha\beta - \\ (1-\alpha)(1-\alpha\beta)\sigma^* \end{pmatrix} E^{t-1}B^{-1} \\
+\alpha\beta\sigma^* E^{t-1}B^{-2} + \alpha\left(\sigma^* + \alpha\beta\right)E^{t-2} \\
-\alpha^2\beta\sigma^* E^{t-2}B^{-1} \\
+(1-\alpha)(1-\alpha\beta)\chi^*\tau\sigma^* E^{t-1}
\end{bmatrix}\pi_t
$$

$$
=
\begin{pmatrix}
1 - 0.67L + 1.42 E^{t-1}B^{-1} \\
+0.56 E^{t-1}B^{-2} + 1.01 E^{t-2} \\
-0.38 E^{t-2}B^{-1} + 0.88 E^{t-1}
\end{pmatrix}\pi_t
\qquad (1.71)
$$

while the RHS is reduced to :

$$
(1-\alpha)(1-\alpha\beta)\chi^*\sigma^*
\begin{pmatrix}
\frac{E_t(1-B^{-1})(-\log Z_t)}{(1-\alpha\beta E^t B^{-1})(1-\sigma^* E^t B^{-1})}\left(1 - \alpha\beta E^{t-1}B^{-1}\right)\left(1 - \sigma^* E^{t-1}B^{-1}\right) \\
-E^{t-1}\log M_t
\end{pmatrix}
$$

$$
= (1-\alpha)(1-\alpha\beta)\chi^*\sigma^*
\begin{pmatrix}
\frac{(\rho_1 - 1)}{(1-\alpha\beta\rho_1)(1-\sigma^*\rho_1)}\left(1 - \alpha\beta E^{t-1}B^{-1}\right)\left(1 - \sigma^* E^{t-1}B^{-1}\right)\log Z_t \\
-\rho_2 \log M_{t-1}
\end{pmatrix}
$$

$$
= \frac{(1-\alpha)(1-\alpha\beta)\chi^*\sigma^*}{(1-\alpha\beta\rho_1)(1-\sigma^*\rho_1)}
\begin{bmatrix}
(\rho_1 - 1)\begin{pmatrix} 1 - \alpha\beta E^{t-1}B^{-1} - \\ \sigma^* E^{t-1}B^{-1} + \alpha\beta E^{t-1}B^{-2} \end{pmatrix}\log Z_t - \\
-\rho_2\left(1 - \alpha\beta\rho_1\right)\left(1-\sigma^*\rho_1\right)\log M_{t-1}
\end{bmatrix}
$$

$$
= \chi^{**}
\begin{bmatrix}
(\rho_1 - 1)\left(1 - \alpha\beta E^{t-1}B^{-1} - \sigma^* E^{t-1}B^{-1} + \alpha\beta E^{t-1}B^{-2}\right)\log Z_t - \\
-\rho_2\left(1 - \alpha\beta\rho_1\right)\left(1-\sigma^*\rho_1\right)\log M_{t-1}
\end{bmatrix}
\qquad (1.72)
$$

We are only interested in z_{t-i}, therefore the RHS can be simplified to:

$$
\chi^{**}(\rho_1 - 1)\left(1 - \alpha\beta E^{t-1}B^{-1} - \sigma^* E^{t-1}B^{-1} + \alpha\beta E^{t-1}B^{-2}\right)\log Z_t
$$

$$
= \chi^{**}(\rho_1 - 1)\left(\log Z_t - (\sigma^* + \alpha\beta)\rho_1^2 \log Z_{t-1} + \alpha\beta\rho_1^3 \log Z_{t-1}\right)
$$

$$
= \chi^{**}(\rho_1 - 1)\left(\frac{z_t}{1 - \rho_1 L} - \rho_1^2\left(\sigma^* + \alpha\beta - \alpha\beta\rho_1\right)\frac{z_{t-1}}{1 - \rho_1 L}\right)
$$

Multiply both RHS and LHS by $(1 - \rho_1 L)$,

$$\pi_t - (\rho_1 + \alpha)\,\pi_{t-1} + (1-\alpha)(1-\alpha\beta)\left(\tau\sigma^*\chi^* - 1\right)\left(E^{t-1}\pi_t - \rho_1 E^{t-2}\pi_{t-1}\right)$$

$$+\left((1-\alpha)(1-\alpha\beta)\,\sigma^* - (\sigma^* + \alpha\beta)\right)\left(E^{t-1}\pi_{t+1} - \rho_1 E^{t-2}\pi_t\right) - \alpha^2\beta\sigma^*\left(E^{t-2}\pi_{t+1} - \rho_1 E^{t-3}\pi_t\right)$$

$$+\alpha\left(\sigma^* + \alpha\beta\right)\left(E^{t-2}\pi_t - \rho_1 E^{t-3}\pi_{t-1}\right) + \rho_1\alpha\pi_{t-2} + \alpha\beta\sigma^*\left(E^{t-1}\pi_{t+2} - \alpha E^{t-2}\pi_{t+1}\right)$$

$$= \chi^{**}\left(\rho_1 - 1\right)\left(z_t - \rho_1^2\left(\sigma^* + \alpha\beta - \alpha\beta\sigma^*\rho_1\right)z_{t-1}\right) \tag{1.73}$$

Given the assumption that $\pi_t = \Sigma_{i=0}^{\infty}\xi^i z_{t-i}$, we collect terms:

(z_t)

$$\xi_0 = \chi^{**}\left(\rho_1 - 1\right) \tag{1.74}$$

(z_{t-1})

$$\xi_1\left((1-\alpha)(1-\alpha\beta)\left(\tau\sigma^*\chi^* - 1\right) + 1\right) - (\rho_1 + \alpha)\,\xi_0 + \alpha\beta\sigma_3^*\xi + \left((1-\alpha)(1-\alpha\beta)\,\sigma^* - (\sigma^* + \alpha\beta)\right)\xi_2$$

$$= -\chi^{**}\left(\rho_1 - 1\right)\rho_1^2\left(\sigma^* + \alpha\beta - \alpha\beta\sigma^*\rho_1\right) \tag{1.75}$$

(z_{t-2})

$$0 = \xi_2\left(1 + (1-\alpha)(1-\alpha\beta)\left(\tau\sigma^*\chi^* - 1\right)\left(1 - \rho_1\right) + \alpha\left(\sigma^* + \alpha\beta\right)\right) - \left(\begin{array}{c}(\rho_1 + \alpha) + \rho_1(1-\alpha)\\ (1-\alpha\beta)\left(\tau\sigma^*\chi^* - 1\right)\end{array}\right)\xi_1$$

$$+ \xi_3\left((1-\alpha)(1-\alpha\beta)\,\sigma^* - (\sigma^* + \alpha\beta) - \alpha\beta\sigma^*\left(\alpha + \rho_1\right)\right) + \alpha\beta\sigma_4^*\xi_4 + \rho_1\alpha\xi_0 \tag{1.76}$$

$(z_{t-i}, \ i \geq 3)$

$$0 = \rho_1\alpha\xi_1 - \left(\begin{array}{c}(\rho_1 + \alpha) + \rho_1(1-\alpha)\\ (1-\alpha\beta)\left(\tau\sigma^*\chi^* - 1\right)\end{array}\right)\xi_2 + \alpha\beta\sigma^*\xi_5 + \left(\begin{array}{c}1 + (1-\alpha)(1-\alpha\beta)\left(\tau\sigma^*\chi^* - 1\right) - \\ \rho_1\left((1-\alpha)(1-\alpha\beta)\,\sigma^* - (\sigma^* + \alpha\beta)\right)\\ +\alpha\left(\sigma^* + \alpha\beta\right) + \rho_1\alpha^2\beta\sigma^*\end{array}\right)\xi_3$$

$$+ \left((1-\alpha)(1-\alpha\beta)\,\sigma^* - (\sigma^* + \alpha\beta) - \alpha\beta\sigma^*\left(\alpha + \rho_1\right) - \rho_1\alpha\left(\sigma^* + \alpha\beta\right)\right)\xi_4 \tag{1.77}$$

The last term can be generalised as a 4th order difference equation:

$$\xi_{i+2} - 0.763\xi_{i+1} + 3.43\xi_i - 4.32\xi_{i-1} + 1.1125\xi_{i-2} = 0 \quad \text{for } i \geq 3, \tag{1.78}$$

which has characteristic roots of $[x = -0.226 \pm 1.904i; x = 0.86093 \text{ and } x = 0.35153]$. Given the

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first two roots are very small, the equation can be reduced to

$$(1 - 0.351513L)(1 - 0.86093L)\,\xi_i = 0 \tag{1.79}$$

$$\text{or } \xi_i - 1.2125\xi_{i-1} + 0.3026\xi_{i-2} = 0 \ \text{ for } i \geq 3 \tag{1.80}$$

Therefore, from equation(1.74), (1.75), (1.76) and (1.79), we can build the system of equations and solve for ξ_0 and ξ_1.

$$-0.464 = \xi_0$$

$$0.393 = 1.88\xi_1 - 1.6\xi_0 + 0.564\xi_3 + 1.42\xi_2$$

$$0 = 1.5713\xi_2 - 2.34\xi_1 - 0.518\xi_3 + 0.564\xi_4 + 0.623\xi_0$$

$$0 = \xi_3 - 1.2125\xi_2 + 0.3\xi_1$$

$$0 = \xi_4 - 1.2125\xi_3 + 0.3\xi_2$$

The solution is $\xi_0 = -0.464$ and $\xi_1 = -0.151$.

Equation (1.61)

Substituting equation (1.42)

$$\log P_t^{UE} = (1 - \alpha) \log P_t^{*UE}$$

into equation (1.55)

$$E^{t-1} \log P_t^* = \frac{-v_{t-1} + \alpha v_{t-2} - \alpha \log P_{t-1}^{UE}}{1 - \alpha}$$

we get

$$E^{t-1} \log P_t^* = \frac{-v_{t-1} + \alpha v_{t-2} - \alpha (1 - \alpha) \log P_{t-1}^{*UE}}{1 - \alpha} \tag{1.81}$$

Under the rational expectation and equation (1.56), the new reset price is

$$\log P_t^* = E^{t-1} \log P_t^* + \log P_t^{*UE} =$$

$$= \frac{-v_{t-1} + \alpha v_{t-2}}{1 - \alpha} - \alpha \log P_{t-1}^{*UE} + \log P_t^{*UE}$$

$$= \frac{-v_{t-1} + \alpha v_{t-2}}{1 - \alpha} - \alpha \left(\frac{1 - \alpha\beta}{1 - \alpha\beta\rho_1} \right) \left(\frac{1 + \chi}{1 - \nu} \right) (-z_{t-1}) + \left(\frac{1 - \alpha\beta}{1 - \alpha\beta\rho_1} \right) \left(\frac{1 + \chi}{1 - \nu} \right) (-z_t)$$

$$= \frac{-v_{t-1} + \alpha v_{t-2}}{1 - \alpha} + \frac{\chi'}{1 - \alpha} (-z_t + \alpha z_{t-1}) \tag{1.82}$$

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1.3.6 Appendix 2: Welfare

The first one involves all the elements with q_{t-1} in the expression for $E \log DP_t$.

$$E \ln DP_t (q_{t-1}) = \frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1 - \alpha) \left[1 - \alpha^i (1 - \alpha)\right] var \left(\log P_{t-i}^*\right)$$

$$- \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p \alpha^{i+j} (1 - \alpha)^2 Cov \left(\log P_{t-i}^*, \log P_{t-j}^*\right)$$

$$= A(q_{t-1}) + B(q_{t-1})$$

Here, we only include in the variance of $\log P_t^*$ the terms in q_{t-1} where q_t follows some autocorrelation process $q_{t-i} = \rho_{i-j} q_{t-j}$, so that $var (\log P_t^* [q_{t-1}]) = var(q)$. Therefore

$$A (q_{t-1}) = \phi_p \frac{\alpha}{1 + \alpha} var(q) \tag{1.83}$$

Consider $B (q_{t-1})$ now. First we assume that $\rho_{i-j} = 1$; then

$$B (q_{t-1}) = -\phi_p (1 - \alpha)^2 \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \alpha^{i+j} \rho_{i-j} var (q) \begin{bmatrix} i = 0; \ j = 1, 2... \\ \alpha + \alpha^2 + ... \\ i = 1; \ j = 2, 3... \\ \alpha^3 + \alpha^4 + ... \\ \vdots \end{bmatrix} = -\frac{\alpha \phi_p var (q)}{1 + \alpha} \tag{1.84}$$

Summing up equations (1.83) and (1.84) gives

$$E \ln DP_t(q_{t-1}) = 0 \tag{1.85}$$

However, if $\rho_{i-j} < 1$ for any $i - j$, then this term must be negative. Thus for example suppose that $\rho_{i-j} = \rho^{i-j}$ so that q is a first-order autocorrelation process, then

$$B (q_{t-1}) = -\phi_p (1 - \alpha)^2 \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \alpha^{i+j} \rho^{i-j} var (q) \begin{bmatrix} i = 0; \ j = 1, 2... \\ \alpha \rho + \alpha^2 \rho^2 + ... \\ i = 1; \ j = 2, 3... \\ \alpha^3 \rho + \alpha^4 \rho^2 + ... \\ \vdots \end{bmatrix} = -\frac{\alpha \rho (1 - \alpha) \phi_p var (q)}{(1 + \alpha) (1 - \alpha \rho)}$$

$$\tag{1.86}$$

As a result of equations (1.83) and (1.86), the expected price dispersion is

$$E \ln DP_t \left(q_{t-1} \right) = \phi_p \frac{\alpha}{1+\alpha} \left(var(q) \right) - \frac{\alpha \rho \left(1 - \alpha \right)}{\left(1 + \alpha \right) \left(1 - \alpha \rho \right)} \phi_p var \left(q \right)$$

$$= \phi_p \frac{\alpha}{1+\alpha} \left(\frac{1-\rho}{1-\alpha\rho} \right) var \left(q \right) \tag{1.87}$$

So comparing equations (1.85) and (1.87), we find that this term q_{t-1} must raise $E \log DP_t$, this in turn reduces the expected welfare. Note that q contains all terms in money shocks and all terms in productivity shocks from $(t-2)$ backwards.

The second part involves all other terms that are not q_{t-1}, that is the term $\psi_0 z_{t-1}$. Thus it analyses the effect of this term $\psi_0 z_{t-1}$ on expected welfare. So looking at this part of $\log P_t^*$:

$$\log P_t^* (\psi_0 z_{t-1}) = \frac{\chi'}{1-\alpha} \left(\alpha z_{t-1} - z_t \right) + \psi_0 z_{t-1},$$

we find

$$E \ln DP_t (\psi_0 z_{t-1}) = \frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i \left(1 - \alpha \right) \left[1 - \alpha^i \left(1 - \alpha \right) \right] var \left(\log P_{t-i}^* \right)$$

$$- \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p \alpha^{i+j} \left(1 - \alpha \right)^2 Cov \left(\log P_{t-i}^*, \log P_{t-j}^* \right)$$

$$= A(\psi_0 z_{t-1}) + B(\psi_0 z_{t-1}).$$

Given

$$A(\psi_0 z_{t-1}) = \phi_p \frac{\alpha}{1+\alpha} \left(\chi''^2 + \left(\psi_0 + \alpha \chi'' \right)^2 \right) var(z)$$

and

$$B(\psi_0 z_{t-1}) = \phi_p \frac{\alpha \left(1 - \alpha \right)}{1+\alpha} \chi'' \left(\psi_0 + \alpha \chi'' \right) var(z)$$

this part of expected price dispersion is

$$E \ln DP_t \left(\psi_0 z_{t-1} \right) = \phi_p \frac{\alpha}{1+\alpha} \left(\chi''^2 + \left(\psi_0 + \alpha \chi'' \right)^2 + \left(1 - \alpha \right) \chi'' \left(\psi_0 + \alpha \chi'' \right) \right) var(z) \tag{1.88}$$

Since equation (1.88) is derived using the part of equation (1.61), which partly consists equation for $\log P_t^*$ under the rational expectation indexing, we compare equation (1.88) and the expected price dispersion under rational indexing (equation (1.53)) :

$$E \ln DP_t = \phi_p \frac{\alpha}{1+\alpha} \left(\chi''^2 + \left(\alpha \chi'' \right)^2 + \left(1 - \alpha \right) \alpha \chi''^2 \right) var(z) \tag{1.89}$$

It follows that the difference between $E \log DP_t$ under lagged and rational indexation due to the term in $\psi_0 z_{t-1}$ is: $\phi_p \frac{\alpha}{1+\alpha} \left(\left(\psi_0 \right)^2 \left[1 + \frac{(1+\alpha)\chi''}{\psi_0} \right] \right) var(z)$ from which it can be seen that for this term

to worsen welfare under lagged indexation requires that $\frac{(1+\alpha)\chi''}{\psi_0}$ should be positive (or if negative should be greater than -1 which can effectively be ruled out). Now by construction $\psi_0 = \frac{-1}{1-\alpha}(-\xi_1)$ so the (relevant sufficient) condition for the term to worsen welfare is that $\xi_1 > 0$.

The coefficient ξ_1 comes from the path of prices in the period after the shock; this cannot be solved out analytically as it involves solving for all the ξ_i in the equation from Appendix 1.3.5 (involving among other things finding the two stable roots of a fourth order difference equation). Here we find it numerically using the calibration of Canzoneri et al. (2004); the value of ξ_1 turns out to be -0.151, which implies that the term in $\psi_0 z_{t-1}$ improves welfare.

The reason for this is that rational indexation causes reset prices to follow a first order moving average in which the lagged shock in prices when there is a productivity shock is corrected. The pattern is jagged and therefore costly in the first two quarters. Under lagged indexation the reset price follows a moving average in the path of inflation which responds smoothly to a productivity shock; the path is therefore smoother in the first two quarters.

What we have found overall therefore is that for productivity shocks the effect of rational compared with lagged indexation is ambiguous; extra noise is introduced by the lagged index but some of it is correlated with the lagged productivity in a potentially helpful way. For monetary shocks rational indexation is unambiguously superior because in this case these shocks have no effect on the reset price and therefore on welfare; under lagged indexation monetary shocks at $t-1$ and before all enter the current reset price setting.

Chapter 2

Optimising indexation arragements and their implications for monetary policy

I now introduce a further strand of this thesis: optimal monetary policy analysis and in particular whether an improvement in policy can be brought about by moving from this highly successful policy of inflation targets to price-level targets. It turns out that there is a link between optimising arrangements for indexation and optimising monetary policy. The monetary policy framework influences optimal indexation, and the indexation scheme influences optimal monetary policy. Hence it is useful to consider what is involved in the choice of monetary policy, and in particular the choice between inflation and price-level targets.

This chapter as well as the previous chapter are a mixture of 'normative' and 'positive' economics as follows. We want to find the indexation that gives the highest welfare to agents under a given standard monetary rule. This could be regarded as 'normative' in the sense that it finds what is the best for general welfare; but in fact it is 'positive' in the sense that it is assumed they will act in this way that is best for them — how they do so is discussed in the introduction (e.g. there might be some coordinating groups such as unions of governments; or there might be some evolutionary process). Thus this part of the chapter is positive in the sense that it is discovering (on this assumption) what agents will do. The other part of this chapter is entirely normative, about what is the better monetary policy rule, one based on inflation target or one based on a price-level target. It should be noticed that this is not an attempt to find 'optimal monetary policy', since this would be a very ambitious task that is beyond the scope of this chapter. The idea is only to check one important dimension in the area of simple rules.

In this chapter, first there is a general review of the literature on these questions. We go on in the rest section to considering the issue under NNS models with optimal choice of indexation regimes. Finally, we conclude with some observations on the merits of different monetary rules in the context of optimal indexation.

2.1 Literature review

There has been a long tradition in studying optimal indexation going back to the papers by Gray (1976), Fischer (1977) and Barro (1977) which is described in chapter 1. Beside this literature, there has also been intensive research on the optimal target of the monetary rule. However, there had been no attention paid to the role of the contracts structure in evaluating the alternative monetary policy until the works by Minford, Nowell and Webb (2003) and Minford and Nowell (2003) using the overlapping wage contract model. They find that the choice of wage contract's structure depends on the monetary policy shock. That is, if the shocks in the economy are persistent, more wage indexation is required. Next, they find that there is interplay of endogenous indexation and optimal monetary policy and that price-level rules improved welfare compared with inflation targeting rules, and in so doing reduced the degree of indexation dramatically. As it was expressed in chapter 1: in the NNS models, prices and wages must be indexed to general ongoing inflation to allow them to catch up with the fundamental. Therefore, it is interesting how the private sector's choice of indexation and central bank's choice of monetary policy target interact in this framework. Based on the same NNS model the more general question of optimal indexation are asked when the indexation choice includes the possibility of partial indexation and of varying weights on rational and lagged indexation. It also attempts to establish how this optimising choice would respond to the nature of monetary policy, which is restricted in the New Keynesian manner to an interest rate rule and a choice between inflation and price-level targeting. Finally, it explores the implications for the choice of monetary target.

2.1.1 Debate about inflation vs. price-level targeting

The monetary policy rule is accepted to be the one that sets the nominal interest rate and with the long-term goal of price stability. Wicksell defined the price stability in his 1935 Lectures on Political Economy as "so soon as money becomes a general measure of value, the avoidance of all violent and unexpected fluctuations in its value is of the utmost importance". However, there is less of consensus as to what price stability means in reality. The debate has been whether monetary authorities should choose paths for the price level or for the inflation rate. This question was raised by King (1999), Svensson (1999b) and Parkin (2000). It seems to be a strange matter given that,

because there seemed to be a strong professional consensus on the superiority of inflation targeting over price-level targeting and also the success in the OECD brought by inflation targeting to ensure low inflation and stable world economy. Some economists (e.g. Clarida et al.,1999) have declared victory to use interest rate rules to achieve both low inflation and output stability.

Minford (2004) argues that it is desirable to go back to the world with long-term price level stationarity such as the Gold Standard period for various reasons that would be discussed during this chapter. However, unlike the period of gold control when the price level could depart from its long-run level for a long time, here the concern is about the stabilisation of the price level close to its long-run level over both the short and long terms. Price-level targeting does not mean the price must stay at unchanged level, it is possible to design it so that this deterministic price level can rise steadily along some path, for example, 2% a year as with today's inflation targeting (Minford, 2004). The difference between the price-level and inflation targeting regimes is that under the latter unanticipated shocks to the price level are treated as bygones and never offset, as a result forecasts of price level at long horizons have a large variance. Under the price-level targeting, however, the past missed target is incorporated into the current target, so that whatever happens to the prices in one period is rolled over into the next as the new base for next period's inflation. The expected future level of prices and the variances of the price level do not increase over time since the overshoots or undershoots of the target are required to be fully made up. However, under inflation targeting, the expected level of future inflation and the variance of inflation do not increase over time, but the mean and variance of future prices do, since unanticipated shocks to the price level are treated as bygones and never be offset. Technically, inflation targeting ensures stationary inflation, but leaves the price level to be $I(1)$, while under the alternative the price level behaves like an $I(0)$ time series. Hence if we think of optimal monetary policy as the science of using monetary policy to optimise welfare which in turn depends on the variance of consumption and leisure, then one can divide this decision into two main choices (a) of nominal target and (b) of size of responses to shocks (or their observable counterparts) together with the instrument of response (interest rates and money supply). Most of the work on monetary policy has concentrated on (b) as if the nominal target of inflation was the obvious choice for (a). However, choosing price-level targets changes the time series property of nominal variables from $I(1)$ (nonstationary) to $I(0)$ (stationary) and this may have important implications for welfare.

Inflation targeting is attractive, because if sticky prices are assumed in a model, variable inflation causes misallocation of resources between firms that can adjust prices and firms that cannot (Carlstrom and Fuerst, 2002). Thus inflation targeting reduces the inefficiencies associated with sticky prices. However, theoretically, Carlstrom and Fuerst (2002) suggest that price-level targeting rules may perform better than inflation targeting rules because they build in a backward-looking element

to avoid the possibility of sunspot events that can happen under inflation targeting. Central banks base their policy changes on inflation projections. Then money supply can be adjusted passively by the monetary authority- supplied at whatever level is necessary to achieve the target. Changes in the money supply can be self-fulfilling because public decisions depend on what the public is expected to do, and the public, in turn, bases its behaviour on monetary actions, and there is nothing to pin down either. To avoid sunspot events, options include: a constant money growth rule; aggressive changes in interest rates in response to inflation; or basing the bulk of the response on past inflation, as in price level targeting. Also under an inflation target, past inflation misses do not affect future policy actions: there is base drift. But with a price-level target, past misses must affect future policy actions because the monetary policy must get the price back to the path, so there is no base drift.

It is also strange to think that agents prefer the rule that targets the inflation rate to the one that stabilises the price level since the price level is the exchange value of money for goods, which in turn come into peoples utility. The conventional argument to support the price-level targeting rule is that it facilitates long-term planning and nominal contracting (Black, Coletti and Monnier, 1998, Duguay, 1994, Feldstein, 1997) because the price level is expected to return to some stable value. This means all goods exchanged over time can be priced in money terms with the certainty that this prices them intertemporally. In particular, a policy of targeting a fixed price level is particularly appealing as it eliminates all the uncertainty surrounding the future price level. In principle, to avoid the uncertainty around the future price, indexation can be applied to allow people to deal with real variables directly when they issue nominal contracts, but in practice, indexation is imperfect, both in timing and in exactness (Minford, 2006).

Barnett and Engineer (2000) state that the literature supports price-level targeting over inflation targeting, showing that it is better for the central bank to gradually wrestle the price level, rather than the inflation rate, back to the target path. Price-level targeting provides a firm nominal anchor for expectations, and thereby conditions private sector expectations in a way that reduces inflation variability and improves welfare. This result contradicts the conventional view that price-level targeting is a "bad idea".

The conventional consensus against price-level targeting is, for example, expressed in Fischer (1994) that the benefits from more stable long-term nominal contracting are not substantial given that other means (e.g. indexed bonds, contingent contracts) exist to ameliorate long-run price uncertainty. Also, from the practical point, McCallum (1999) argues that the net reduction in price uncertainty in the United States under price-level targeting would be small. But the main argument is about the costs occurring due to adopting price-level targeting. Price-level targeting induces both higher short-run inflation and output variability than does inflation targeting (Fischer, 1994, Lebow, Roberts, and Stockton, 1992). Shocks that move the price level above (below) the target path lead the

monetary authority to disinflate (inflate) with lower (higher) than average inflation to move towards the target path. This movement of inflation above and then below trend induces short-run inflation variability relative to inflation targeting, since the inflation targeting allows price-level drift and aims for only the target inflation rate. Greater inflation variability induces greater output variability along the short-run Phillips curve. Also, when nominal rigidities are present, the deflationary policy might induce recession. That is the deflationary monetary policy can be used to decrease aggregate demand and inflation, but since prices are sticky, they will still be high thus demand would fall further than otherwise, and given that demand determines output, output will decrease more than in the case of flexible prices. Fischer (1994) writes, "Price level targeting is thus a bad idea, one that would add unnecessary short-term fluctuations to the economy. It is also true... that there is more variability and uncertainty about short-term inflation rates with a price-level target than with a target inflation rate". Gaspar and Smets (2002) point out that this consensus was very strong as a result of the disinflation experience of the 1980s. Therefore, disinflating the economy in response to a one-time shock to prices did not appear to be a good idea. This view was then backed up by simulation studies by Lebow, Roberts and Stockton (1992), and Haldane and Salmon (1995). They showed that, in models dominated by backward-looking expectations, there is a trade-off between low-frequency variability in the price level and high-frequency variability in inflation and output. Black et al (1998), Smets (2000), Dittmar, Gavin and Kydland (1999) and Vestin (1999) find that the cost of inflation targeting against price-level targeting results in increased and even increasing variability in price, but it leads to lower inflation and output volatility than price-level targeting in the short run.

Recently, interest has revived in avoiding base drift. There are two practical reasons for this. First, the stochastic properties of the inflation process cannot be taken as given when considering alternative monetary policy regimes. Focusing on the post-war period it would seem that persistence in inflation has changed significantly when comparing the period up to 1960, where inflation seems to be roughly stationary, with the subsequent period up to 1990, where it exhibits a unit root (Christiano and Fitzgerald, 2003). In the 1990s, after inflation was kept low , both the degree of persistence and the variability of inflation seem to have been reduced (Gaspar and Smets, 2000). These changes in the nature of inflation make it interesting to investigate the costs and benefits of price-level targeting using the model with forward-looking agents. Svensson (1999a) shows that once the effect of price level targeting on the formation of inflation expectations is taken into account, such a regime does not necessarily lead to increased inflation and output variability and may even reduce it.

The second reason is related to the issue of a zero lower bound on nominal interest rates in a low inflation regime. The Japanese experience with a zero nominal interest rate policy provides a real

world illustration (Bernanke, 1999, Goodfriend, 2000 and Svensson 2001). When inflation is low, central banks worry that a serious recession could require large interest rate cuts, but the interest rate cannot go below zero at which the demand for money is indeterminate and monetary policy cannot help the economy to recover. The problem is worse if prices are falling, since then at the zero bound real interest rates remain positive. This concern has led policy-makers to set inflation targets away from zero to create room for the interest rate to fall if it needs to (Minford, 2006). There is another suggestion to avoid the possibility of a liquidity trap, for example McCallum (2005) in the open economy framework, offering manipulation of the exchange rate when monetary policy cannot work with the nominal interest rate. The alternative in this situation could have been to use credible price-level targeting. It appears to be superior to an inflation targeting rule because it creates an automatic expectation of a future increase in prices, an i.e. inflation expectation, which reduces the ex ante real interest rate and therefore it helps to alleviate the zero bound.

From the theoretical point of view, the challenge to the conventional consensus against the price-level targeting rule comes from a series of papers that show that price-level targeting is welfare-improving. There are two strands to this literature, corresponding to whether or not the central bank can commit to future policy. Given the commitment to future policy, there is the strongest case for price-level targeting when the Phillips curve is New Keynesian. Clarida et al. (1999) and Svensson and Woodford (1999) point out that in a simple New Keynesian model with inflation targeting and the loss function defined on inflation and output, the optimal policy under commitment results in a stationary price level process. Intuitively, the explanation is that given the commitment to limit or even avoid price level drift, the price-level target forms a firm nominal anchor for expectations. Agents know that shocks that move the price level below trend will eventually be countered by measures by the monetary authority to move the price level back to the target path. Because they know this involves inflation above trend in the medium term, agents lower their expectations of inflation, thus reducing inflation and inflation variability and increasing welfare. So it stabilises the effects on inflation expectations and ensures a more stable economy. Black, Macklem and Rose (1998) find that inflation and output variability often decrease when the price level is weighted in the policy rule, allowing expectations to adjust to take account of price-level control. Also, Chadha and Nolan (2003) too compare inflation and price-level targeting in New Keynesian models without indexation. They show that price-level targeting dominates inflation targeting under credible commitment and with sufficient inflation aversion, the inflation-targeting central bank can produce quantitatively similar results to one targeting the price level.

The reason for such a conclusion given the New Keynesian Phillips Curve which is derived from Calvo contracts is that in this set-up the chance for a firm to change its price is limited and it may never get to change its price at all, so any movement in the general price level implies that relative

prices between producers are being pushed away from their flex-price equilibrium. In these models the output affects prices. Prices when changed by those with the chance to reflect their expectations of future marginal cost, proxied with rising cost curves by future capacity utilisation. Then if policy-makers can fix the output level to prevent prices from moving at all, it will stop the disequilibrium from mis-set relative producer prices. Thus in these models, if there are no other distortions, the optimal monetary policy is to stabilise the price level perfectly. Goodfriend and King (2001) find that in a simple NNS model the average mark-up acts like a tax on working effort in RBC models where a constant tax rate is optimal to maximise the expected utility of the representative agent because it would make the real economy respond to shocks as if all prices were perfectly flexible. Therefore, the optimal monetary policy should stabilise the price level.

However, Smets (2000) uses a model with a Calvo-type Phillips Curve to examine the optimal horizon for bringing inflation or the price-level back to their targets and finds that the optimal length gets shorter as the price expectations become more forward-looking and the Phillips Curve becomes steeper. The reason may be that too much forward looking creates too much uncertainty, and the optimal policy brings the nominal variables to their target faster can provide a good anchor. Williams (1999) evaluates different rules in the FRB large-scale model of the U.S in which there is a forward/backward-looking Phillips Curve. He finds that multiperiod inflation targeting ranks highly and that price-level targeting only causes minor output. Commitment to stabilise inflation is good, and replacing its by the price-level targeting does bring more inflation movement and thus output instability but this is very small variation. These models show that long-run price-level can be chosen as an extra long-term target but it would not improve a lot. Commitment in these set-ups removes the benefits of price-level targeting.

The second strand considers a central bank that cannot commit to future policy. Assigned the social loss function, the bank is unable to condition private sector expectations in a desirable way, because the social loss function has an inflation-targeting objective, so that the bank's dynamic program ignores the history prior to the current period. The central bank does not persist in battling past inflation; rather, it looks forward and engages in inflation target.

Svensson (1999d) shows that assigning a central bank a loss function with a price-level-targeting objective yields a price-level target policy that may reduce inflation variability without affecting output variability. This "free-lunch" depends on endogenous output persistence in the New Classical Phillips curve. This result is strong because it is possible to improve both outputs disinflation variability by assigning a loss function with a different weight than the original set. Dittmar and Gavin (2000) and Vestin (2000) extend this analysis to the case where expectations are forward-looking in a New-Keynesian Philips curve. They show that the free-lunch argument applies without the need for a persistence terms in the Phillips Curve. Thus, given a price target objective appears to

improve welfare if expectations are forward-looking or if there is substantial endogenous persistence. Dittmar et al. (1999a) used a simple Neoclassical Phillips curve model and evidence about the persistence in output gaps to show that a price-level targeting regime would likely result in a better inflation-output variability trade-off than an inflation targeting regime. Kiley (1998) argues that the Neoclassical specification is inconsistent with U.S. data, showing that Svensson's set-up is one of policy ineffectiveness on output so there is no trade-off with output stability. He believes there is historical evidence that anticipated monetary policy has had real effects. He concludes that, compared to the case with inflation targeting, price-level targeting would have been found to result in a worse inflation-output variability trade-off if Svensson (1999) had started with a New Keynesian version of the Phillips curve. He derives the expectation for the mean of output in a New Keynesian model, shows that the expectation depends on the lagged price level, and infers from this that trying to stabilise the price level would actually raise the variability of output. However, he does not derive the inflation-output variability trade-off implied by the model and does not experiment with alternative policy rules. Extending on the earlier studies, Dittmar et al. (1999) consider a model with the New-Keynesian Phillips curve recommended by Kiley. They examine the inflation-output variability tradeoffs implied by optimal inflation and price level rules. Also, they assume that lagged output enters the aggregate supply function, to be consistent with the theoretical model of Taylor (1980) and empirical work of Roberts (1995) that there is serial correlation in the error terms of the estimated Phillips Curve. They find even stronger support for price level targeting with the New Keynesian Phillips Curve than with the Neoclassical Phillips Curve because in the world of a sticky price where prices are costly to adjust, a policy that reduces price fluctuations is appropriate. With the Neoclassical Phillips curve, it was found that if the output gap was not too persistent and if lagged output was not present in the Phillips Curve, then inflation targets were better to use. However, with a New Keynesian Phillips Curve, the amount of persistence in the output gap can be very small, the price level targeting regime still produces a more favourable trade-off between output and inflation variability.

2.1.2   Models used to discuss the cases of inflation and price-level targeting

The standard approach

This approach is to find the optimal monetary policy by using some sort of Phillips Curve with a quadratic social objective function in terms of output and inflation. This indeed was introduced by Taylor (1979) using a rational expectations model with staggered wage contracts, he explained that there is no long run trade-off between levels of output and inflation, but policymakers can choose

alternative points along an inflation/output frontier by varying the relative weight on inflation versus output stability. This approach is a standard one to be used in the literature nowadays, though the alternative would be macroeconomic models with optimising foundations that allows an explicit evaluation of outcomes in term of individual welfare. The latter approach avoids the arbitrariness on the choice of definition of price stability and full employment. Woodford (2006) also argues that this approach naturally integrates the theory of optimal monetary policy with the theory of optimal taxation. Nevertheless, the social objective function set out in terms of inflation and output (Taylor, 1979) is correct. Rottemberg and Woodford (1997) showed that this ad-hoc objective function by Taylor (1979) can be derived by a second-order Taylor series approximation around a standard representative agent's utility function in term of consumption and leisure, so that the alternative monetary policies can be ranked according to how well they stabilise inflation on the one hand and how well they stabilise the output gap on the other. It validates the old approach by giving it a microeconomic foundation.

The social objective function is an approximation to the welfare of the representative agent, when expectations of it are taken in forming the best intertemporal plan for monetary policy, it implies a trade-off between variances of inflation and output. Also since monetary policy cannot raise the expected level of output and there is commitment, the expected rate of inflation is equal to the inflation target and this trade-off is the focus of policy (Minford and Peel, 2003). The simple example is in Svensson (1997). The set-up is with the Phillips Curve with some persistence of

$$y_t = \rho y_{t-1} + \alpha \left(\pi_t - \pi_t^e \right) + e_t \qquad (2.1)$$

and the utility under commitment of

$$V \left(y_{t-1} \right) = MaxE_{t-1} \left\{ -0.5 \left(\pi_t - \pi^* \right)^2 - 0.5\lambda \left(y_t - y^* \right)^2 + \beta V \left(y_t \right) \right\} \qquad (2.2)$$

The inflation rate therefore is

$$\pi_t = \pi^* - \frac{\alpha \lambda}{1 + \alpha^2 \lambda - \beta \rho^2} e_t \qquad (2.3)$$

Alternatively, Minford and Nowell. (2003) and Minford and Peel (2003) explain this standard approach using the IS-LM and AD-AS framework with the assumption that the central bank observes current shocks. This representation successfully derives the optimal real interest rate and optimal money supply target. But the problem is the interpretation of this set-up depends on the information assumption and also the assumption about contracts, so under different assumptions, the optimal real interest rates rule always has to be reinterpreted. This leads to the consideration of a different

approach to assess the optimal monetary policy.

New Keynesian models

Here, we only focus on a standard NNS model where the Phillips Curve is derived from Calvo contracts where the nominal rigidity is extremely persistent since firms get limited chances or not at all to change their prices if there is no indexation to a general inflation rates. In this type of model, the causation runs from output to prices (Minford, 2004), so that if the producers expect their future marginal cost of production to increase due to the higher future capacity utilisation since they expect the output going to be higher than its natural rate, and they also have a chance to change their price, they will set their prices at a higher level. However, others who are not given a chance to alter their price will have to keep theirs constant. This dispersion in prices is the source of inefficiency in the economy. If there is no other significant source of distortion (Minford 2004, Collard and Dellas, 2003), the optimal monetary policy only needs to stabilise the price level perfectly- price-level targeting, ensuring the dispersion in prices is zero.

However, it is difficult to have a rule that stabilises price perfectly in the fully specified NNS set-up since there is a potential time-inconsistency problem. Minford and Peel (2003) gives an example, the history of price shocks has generated a highly skewed relative price distortion or the capital shock is depressed, there is an incentive to change price so it can offset this history. However, such policy can be carried out with a mistake. If prices rise for some reasons (e.g. a control failure by central bank), then committing to reduce prices in a subsequent period means that those whose relative prices are already out of equilibrium may not necessary be brought back by the price reversal, since they may not be given the chance to change their prices in this period; and those whose relative prices did not change may be driven out of the equilibrium too if it is their turn to change prices. As a result, once the mistake has already been made- the price has already changed, then the best policy is to stop the prices changing further from whatever they are. That is zero inflation not a price-level fixing rule. Therefore, the commitment to price level stability is weak and maybe even incredible. This implies that the Calvo contract model calls for zero-inflation targeting rules rather than price level ones.

In Goodfriend and King (1997) a return to a fixed-price-level path is undesirable, i.e. price level targeting, they recommend to use inflation target rule, because price-level targeting requires variations in the average mark-up. In the framework of a simple NNS model, they find that optimal monetary policy is to stabilise the average mark-up, which in turn is a function of the inflation at least in the long run (King and Wolman, 1996), so that to achieve a constant path of average up mark up, the desirable inflation targeted must be constant. However, it was found that the smallest value of the average mark-up occurs at a positive inflation rate which is not very different from zero.

On the other hand, the variation in the inflation rate to achieve the desirable price level would move the mark-up around which acts like tax and produce substitution and wealth effects on the economy. In other words, optimal monetary policy is committed to stabilise the price level at its existing level if there are no other distortions than those associated with shocks to prices and hence also output relative to its flex price equilibrium. This implies no further distortions due to price changes: existing distortions, due to past inflation, cannot be affected since each period those changing price are chosen randomly hence one is as likely to add a new distortion by changing prices as one is to offset a previous price changes. This implies inflation targeting rather than price-level targeting.

The other piece of research studying inflation targeting in the NNS model was performed by Gali (2001). He explains the nature and work with inflation target in the model with price rigidity. Using the aggregate price equation

$$\pi_t = (1 - \theta)\left(p_t^* - p_{t-1}\right)$$

changes in inflation arise if and only if firms adjusting their prices in the current period choose prices that are, on average, different from the average level of prices that prevailed in the previous period. The optimal reset price is a mark-up over a weighted average of current and expected future nominal marginal cost, where the mark-up is the optimal frictionless mark-up. The firm changes the relative price over the past price level because it wants to keep the expected relative price unchanged and to avoid any anticipated gap between expected and desired mark-ups:

$$p_t^* - p_{t-1} = \sum_{k=0}^{\infty} (\beta\theta)^k E_t \pi_{t+k} - (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \hat{\mu}_{t+k}$$

These two equations combined give the following expression for inflation

$$\pi_t = \beta E_t \pi_{t+1} - \lambda \hat{\mu}_t$$

which means to stabilise the inflation requires the mark-ups be stabilised. A zero inflation target can be achieved by maintaining mark-ups constant at their frictionless level, so that all firms will be maximising profits at current prices and no one will have an incentive to adjust its price. Attaining a positive inflation target requires holding the average mark-up below its frictionless level. Only in that case will firms adjusting prices in any given period choose to set a price above the average price in the past period. This result is independent of the structure of the economy. However, the optimal monetary rule targeting inflation does depend on the structure of the economy and its implementation can be a problem. Therefore, Gali (2001) assumed using a Taylor rule instead of this optimal one.

The theoretical problem with the Calvo set-up is the idea that some price- and wage-setters never

completely catch up with the fundamentals since they may never be given a chance to change their prices or wages. Minford and Peel (2003) argue that Calvo contracts must be indexed to ongoing inflation. However, the commonly used lagged inflation and core inflation indexing turn out to make the Phillips Curve exploitable in the sense that the monetary policy can affect the output in the long run. The only type of indexation that was considered can avoid this problem is the rational indexation and it converts the Phillips Curve into the original Sargent and Wallace (1975) 'surprise Phillips Curve'. This would entirely stop the misallocation and mispricing involved, except for the period of an unanticipated shock hitting the economy; next period given that the shock is one-off everyone works out how to reset their price appropriately because the movement of those who can reset relative prices is anticipated by those who cannot, and this anticipation is built into expected inflation and thereby effectively frustrated. The simple social objective function is expressed in term of the squared inflation surprise and squared output deviation, but then whether monetary policy targets inflation does not matter in computing the welfare since only its predictability matters. The idea here is that inflation cannot cause relative price distortion for any length of time, people will index to eliminate the effect of inflation on their welfare levels. However, without indexation, these models create big costs from nominal rigidity.

Minford and Peel (2003) argue that neither a price-level target nor inflation target can be used in the NNS models, the Calvo contract model is useless in assessing the costs of different monetary policies. This model in its standard form and modified form fail for many reasons (Minford, 2004). However, it is impossible to think that unconditional variability in either inflation or prices do not matter, since people do not like a variation in prices because it affects their consumption and thus, welfare. The way to avoid this problem is to assume an explicit function for the welfare, where inflation and price level do actually matter, which allows for the analysis of alternative monetary policy. Minford and Peel (2003) want to look at the claim made by Chadha and Nolan (2003) that macro outcomes are not too different if a price level rule is substituted for a low inflation rule. They find that the contract structure is varied endogenously in both a representative model and a long-used forecasting model based on IS/LM presentation and it is a factor making price level targeting a significant macro factor.

Though Minford and Peel (2003) criticise and show the failures of the NNS model, some parts of the argument were based on the partial equilibrium of the NNS model, therefore it is interesting to re-examine the implications of the NNS model on the choice of monetary policy in the dynamic general equilibrium framework, assuming there is the possibility of rational indexation. The fully specified NNS version will provide an explicit welfare function and it assumes that interest rates reacting to the current output gap, the lagged interest rate and either to the inflation deviation from target or price-level deviation from target. Therefore, it is possible to assess the alternative

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monetary policy rule under the modified NNS model. In this analysis, there is no commitment assumption, based on the argument by Minford and Peel (2003) that policymakers acting within a general consensus about optimal rules of monetary behaviour can put an end to welfare-reducing behaviour. Minford (1995) says that the principal in all such policy frameworks is the electorate itself; it can be assumed that sooner or later appropriate institutions for achieving its best interests, which include commitment, will be found. There is also no concern about zero-bound issue and it assumes no sunspots. Indexation operates with a one period lag. The purpose is to extend the standard New Keynesian approach to deal with the best feasible indexation.

2.2 The implications of the NNS model with a full choice of indexation regimes

This chapter investigates the more general question of optimal indexation when the indexation choice includes the possibility of partial indexation and of varying weights on rational and lagged indexation. It attempts to establish how this optimising choice would respond to the nature of monetary policy, which is restricted in the New Keynesian manner to an interest rate rule and a choice between inflation and price-level targeting. Finally it explores the implications for the choice of monetary target. The chapter is based on the model's set-up in Chapter 1.

It is organised as follows. Firstly, consider a world in which the current information available to agents who are setting prices or wages is solely about their own situation ('micro' information); this boils down to them observing their own productivity shock if they are a price-setter (if they are a wage setter they would observe only their own current preference shocks but these are suppressed in this model). Thus in this world there is a lag before agents see the current macro outcome. This includes the consideration of the implications of this world in a linearised model with a flexible labour market, a fixed capital stock and price-setting and then in a full nonlinear model with investment and wage-setting added in.

Secondly this information assumption is relaxed and agents are allowed to observe all current information while they are setting prices or wages; this might occur if statistics are released rapidly or there is very efficient signal extraction from global indicators like interest rates, especially in the context of quarterly data which is our prime frame of reference. Finally, both linearised and full models are revisited under this relaxed information assumption.

2.2.1 Agents only Observe Micro Information

The linearised model with price setters only and fixed capital

The model in Chapter 2 is loglinearised and organised in following orders:

Real marginal cost

$$\log mc_t = \log MC_t - \log P_t = \log W_t - \log P_t + \nu \log N_t - \log Z_t \tag{2.4}$$

The production function

$$\log Y_t = \log Z_t + (1 - \nu) \log N_t \tag{2.5}$$

Ignoring government spending, the market clearing condition gives

$$\log Y_t = \log C_t \tag{2.6}$$

An interest rate rule, without lags and with the real interest rate assumed to be set in response to inflation and the output gap, with a monetary shock:

$$r_t = \tau \pi_t + \sigma(\log Y_t - \log Z_t) + \log M_t \tag{2.7}$$

or with the real interest rate, reacting to price level and output gap, with a monetary shock

$$r_t = \tau \log P_t + \sigma(\log Y_t - \log Z_t) + \log M_t \tag{2.8}$$

Note that these can identically be written as rules for the nominal interest rate using $i_t = r_t + E_t \pi_{t+1}$. Note also that we treat the monetary authorities as having effective full current information; this is the standard assumption made in New Keynesian models, presumably on the grounds that the authorities in practice have good access to information about the current state of the economy, even if the isolated private agent does not as we initially assume here.

Adding in the Euler equation $\log C_t = \log C_{t+1} - r_t$ and allowing for market clearing gives us an Aggregate Demand curve:

$$\log Y_t = \frac{1}{1 - \sigma^* B^{-1}} \left\{ -\tau \sigma^* \pi_t + \sigma \sigma^* \log Z_t - \sigma^* \log M_t \right\} \tag{2.9}$$

or

$$\log Y_t = \frac{1}{1 - \sigma^* B^{-1}} \left\{ -\tau \sigma^* \log P_t + \sigma \sigma^* \log Z_t - \sigma^* \log M_t \right\} \tag{2.10}$$

where $\log Z_t = \rho_1 \log Z_{t-1} + z_t$; $\log M_t = \rho_2 \log M_{t-1} + \mu_t$; z_t and μ_t are $i.i.d.$; B^{-1} is the forward operator instructing one to lead the variable but keeping the expectations data-set constant; $\sigma^* = \frac{1}{1+\sigma}$.

The new reset price

$$\log P_t^* = \frac{1 - \alpha\beta}{1 - \alpha\beta B^{-1}} E_t \left(\frac{1+\chi}{1-\nu} \left(\log Y_t - \log Z_t \right) + \log P_t - \log \tilde{P}_t \right) \tag{2.11}$$

The general price level

$$\log P_t - \log \tilde{P}_t = \alpha \left(\log P_{t-1} - \log \tilde{P}_{t-1} \right) + (1 - \alpha) \log P_t^* \tag{2.12}$$

and the price indexation formula is $\log \tilde{P}_t = k_0 \left(E^{t-1} \log P_t + k_1 \left(\log P_{t-1} - E^{t-1} \log P_t \right) \right)$, where $k_0 \in [0,1]$ and $k_1 \in [0,1]$. If $k_0 = 1$, then prices are fully indexed; and if $k_0 = 0$, then price is not indexed. If $k_1 = 1$, then prices are indexed to the lagged price level; and if $k_1 = 0$, then they are indexed to rationally expected general price level.

Price dispersion is

$$\log DP_t = \frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1 - \alpha) \left[1 - \alpha^i (1 - \alpha) \right] \left(\log P_{t-i}^* \right)^2 \tag{2.13}$$
$$- \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p \alpha^{i+j} (1 - \alpha)^2 \log P_{t-i}^* \log P_{t-j}^*$$

In order to discuss policy within the context of this model, we evaluate expected welfare in terms of its deviation from the flex-price optimum

$$E \left(u_t - u_t^{FLEX} \right) = -E \log DP_t = -\frac{1}{2} \Sigma_{i=0}^{\infty} \phi_p \alpha^i (1 - \alpha) \left[1 - \alpha^i (1 - \alpha) \right] var \left(\log P_{t-i}^* \right) \tag{2.14}$$
$$+ \Sigma_{j=0}^{\infty} \Sigma_{i=0}^{\infty} \phi_p \alpha^{i+j} (1 - \alpha)^2 Cov \left(\log P_{t-i}^*, \log P_{t-j}^* \right)$$

where $u_t^{FLEX} = \log Z_t$ under the flexible price and wage assumption.

Solving the model with restricted private information

Our notation is as follows: $E^{t-1} x_t = E \left(x_t \mid \Phi_{t-1} \right)$; $E_t x_t = E \left(x_t \mid \Phi_{t-1}, \phi_t \right)$; $x_t^{UE} = x_t - E^{t-1} x_t$, where Φ_{t-1} is the full information set from period $t-1$ and ϕ_t is the limited information set available for period t. The plan of this section for solving the model under both price-level targeting and inflation targeting is (a) Solve for the price level $\log P_t$ using the Wold decomposition; (b) Solve

for the reset price level $\log P_t^*$ using unknown parameters that were determined in (a); (c) Using (b) to find the covariances and variances of reset prices, which in turn contribute to the calculation of the expected price dispersion; (d) Expected welfare is the negative of expected price dispersion.

Inflation targeting rule Here we consider the case of an interest rule that targets the inflation rate (at zero for convenience). To begin, we use equations (2.9), (2.12), (2.11), indexation formula and the assumption that firms have knowledge of their own micro information (productivity, prices and costs) in period t as well as the macro information of period $(t-1)$ to express all price related variables in terms of the future expected monetary and real shocks (for more details, see Appendix 2.2.4):

$$
\begin{aligned}
(1-\alpha)\log P_t^* &= \log P_t - k_0\left[E^{t-1}\log P_t + k_1\left(\log P_{t-1} - E^{t-1}\log P_t\right)\right] - \\
&\quad -\alpha\left\{\log P_{t-1} - k_0\left[E^{t-2}\log P_{t-1} + k_1\left(\log P_{t-2} - E^{t-2}\log P_{t-1}\right)\right]\right\} \\
&= \frac{(1-\alpha)(1-\alpha\beta)}{1-\alpha\beta B^{-1}}E_t\left\{
\begin{array}{l}
\frac{1+\chi}{1-\nu}\left[\frac{1}{1-\sigma^*B^{-1}}\left(\begin{array}{c}-\tau\sigma^*\log P_t + \tau\sigma^*\log P_{t-1}\\ +\sigma\sigma^*\log Z_t - \sigma^*\log M_t\end{array}\right) - \log Z_t\right] \\
+ \log P_t - k_0\left[E^{t-1}\log P_t + k_1\left(\log P_{t-1} - E^{t-1}\log P_t\right)\right]
\end{array}\right\}
\end{aligned}
\tag{2.15}
$$

Collecting terms and converting the operators back into leads and lags, we obtain

$$
\begin{aligned}
&\log P_t - (2.5847 + 1.9028k_0k_1)\log P_{t-1} + \left(\begin{array}{c}1.8945 - 0.88889k_0\\ +2.69k_0k_1\end{array}\right)E^{t-1}\log P_t - \\
&- (1.75 + 3.1528k_0k_1 - 1.4915k_0)E^{t-2}\log P_{t-1} + (1.534 + 2.43k_0k_1)\log P_{t-2} \\
&-0.62k_0k_1\log P_{t-3} - 0.62k_0(1-k_1)E^{t-3}\log P_{t-2} + 0.936k_0(1-k_1)E^{t-3}\log P_{t-1} \\
&+0.564(1-k_0+k_0k_1)E^{t-1}\log P_{t+2} - (1.8 - 1.42k_0 + 1.98k_0k_1)E^{t-1}\log P_{t+1} \\
&+(1.6604 - 2.32k_0 + 2.842k_0k_1)E^{t-2}\log P_t \\
&-0.351k_0(1-k_1)E^{t-3}\log P_t + (-0.521 + 0.901k_0 - 0.901k_0k_1)E^{t-2}\log P_{t+1} \\
&= \chi^{**}\left[(\rho_1-1)\left(z_t - \rho_1^2(\alpha\beta + \sigma^* - \alpha\beta\rho_1)z_{t-1}\right) - \rho_2(1-\alpha\beta\rho_1)(1-\sigma^*\rho_1)\mu_{t-1}\right]
\end{aligned}
\tag{2.16}
$$

Given the assumption that $\log P_t = \Sigma_{i=0}^{\infty}\varepsilon_i z_{t-i} + \Sigma_{i=0}^{\infty}\varsigma_i\mu_{t-i}$, equation (2.16) can hold if and only if the sum of the LHS's and the RHS's coefficients on z_t, on z_{t-1}, on z_{t-2}..., are equal respectively (and the same argument for μ_{t-i}). For example these coefficients on the z_{t-i} must satisfy (Appendix 2.2.4 shows the equivalent for the μ_{t-i}:)

(z_t)

$$\varepsilon_0 = \chi^{**}\left(\rho_1 - 1\right) \tag{2.17}$$

(z_{t-1})

$$0.564\left(1 - k_0 + k_0 k_1\right)\varepsilon_3 + \left(-1.8 + 1.42k_0 - 1.98k_0 k_1\right)\varepsilon_2 +$$

$$\left(2.8945 - 0.88889k_0 + 2.69k_0 k_1\right)\varepsilon_1 - \left(2.5847 + 1.9028k_0 k_1\right)\varepsilon_0 \tag{2.18}$$

$$= -\chi^{**}\left(\rho_1 - 1\right)\rho_1^2\left(\alpha\beta + \sigma^* - \alpha\beta\rho_1\right)$$

(z_{t-2})

$$0.564\left(1 - k_0 + k_0 k_1\right)\varepsilon_4 - \left(2.321 - 2.322k_0 + 2.881k_0 k_1\right)\varepsilon_3$$

$$+ \left(4.5549 - 3.2089k_0 + 5.532k_0 k_1\right)\varepsilon_2 \tag{2.19}$$

$$+ \left(-4.3347 + 1.4915k_0 - 5.0556k_0 k_1\right)\varepsilon_1 + \left(1.534 + 2.43k_0 k_1\right)\varepsilon_0 = 0$$

$(z_{t-i}, i \geq 3)$

$$0.564\left(1 - k_0 + k_0 k_1\right)\varepsilon_{i+2} - \left(2.321 - 2.322k_0 + 2.881k_0 k_1\right)\varepsilon_{i+1} +$$

$$\left(4.5549 - 3.56k_0 + 5.883k_0 k_1\right)\varepsilon_i - \left(4.3347 - 2.4275k_0 + 6k_0 k_1\right)\varepsilon_{i-1} + \tag{2.20}$$

$$\left(1.534 - 0.62k_0 + 3.05k_0 k_1\right)\varepsilon_{i-2} - 0.62k_0 k_1 \varepsilon_{i-3} = 0$$

The last equation is a 5th order difference equation under the real shocks and it is identical to the equation (2.40) in Appendix 2.2.4, illustrating the case of monetary shocks. Thus both equations (2.20) and (2.40) are characterised by the 5th order polynomial equation

$$
\begin{aligned}
0 \;=\; & 0.564\left(1 - k_0 + k_0 k_1\right)x^5 - \left(2.321 - 2.322k_0 + 2.881k_0 k_1\right)x^4 + \\
& \left(4.5549 - 3.56k_0 + 5.883k_0 k_1\right)x^3 + \left(-4.3347 + 2.4275k_0 - 6k_0 k_1\right)x^2 \\
& + \left(1.534 - 0.62k_0 + 3.05k_0 k_1\right)x - 0.62k_0 k_1
\end{aligned} \tag{2.21}
$$

We consider all possible combinations of k_0 and k_1 and find stable roots under each combination[1]. In this 5th order difference equation, the number of forward roots is two; therefore the condition for a unique saddle path is two unstable roots (looking backwards). The problem, however, is that many (k_0, k_1) yield the wrong number of unstable roots. Many yield three which implies an overdetermined solution; some only have one which implies nonuniqueness. So we only report the cases that deliver the unique solution. In general, these cases will give three stable roots, called x_1, x_2, and x_3; thus,

---

[1] $k_0 = 0 : 0.2 : 1$ and $k_1 = 0 : 0.2 : 1$

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the equation (2.21) can be reduced to

$$(1 - x_1 L)(1 - x_2 L)(1 - x_3 L)\varepsilon_i = 0$$

$$\text{and } (1 - x_1 L)(1 - x_2 L)(1 - x_3 L)\varsigma_i = 0 \text{ for } i \geq 3 \tag{2.22}$$

for real and monetary shocks respectively. That is

$$\varepsilon_i - (x_1 + x_2 + x_3)\varepsilon_{i-1} + (x_1 x_2 + x_1 x_3 + x_2 x_3)\varepsilon_{i-2} + x_1 x_2 x_3 \varepsilon_{t-3} = 0$$

$$\text{and } \varsigma_i - (x_1 + x_2 + x_3)\varsigma_{i-1} + (x_1 x_2 + x_1 x_3 + x_2 x_3)\varsigma_{i-2} + x_1 x_2 x_3 \varsigma_{i-3} = 0 \text{ for } i \geq 3 \tag{2.23}$$

The values of x_1, x_2, and x_3 under each combination of (k_0, k_1) are derived numerically. Table (2.1) reports the values of x_1 and values of $\sqrt{x_2 \times x_3}$ for all combinations of k_0 and k_1 that give a stable solution path (except the case of $(1,0)$, which will be described separately- Appendix 2.2.4)

(k_0, k_1) combination	x_1	damping factor ($\sqrt{x_2 \times x_3}$)
(0; all)	0	0.932623
(1, 0.2)	0.10047	0.947849
(1, 0.4)	0.16347	0.931673
(1.0.6)	0.20892	0.925109
(1, 0.8)	0.24552	0.953922
(1, 1)	0.2744	0.945944

Table 2.1: Stable roots under inflation targeting

From equations (2.17), (2.18), (2.19), (2.23) and results in Table (1.2), we can build the system of equations and solve for ε_i under different combinations of (k_0, k_1) to get the solution for the real shocks related part of the price level $\log P_t = \Sigma_{i=0}^{\infty}\varepsilon_i z_{t-i}$. While from the equations in Appendix 2.2.4, (2.23) and results in Table (2.1), we find ς_i that gives the part of price which relates to monetary shocks $\log P_t = \Sigma_{i=0}^{\infty}\varsigma_i \mu_{t-i}$. However, in order to find the expected welfare, $\log P_t^*$ must be written in terms of these unknown coefficients (the working is shown in Appendix 2.2.4):

$$\log P_t^*(z_{t-i}) = \frac{-\chi' z_t + \alpha\chi' z_{t-1}}{1 - \alpha} + \frac{-k_0 k_1 (\varepsilon_0 - \varepsilon_1) + (1 - k_0)\varepsilon_1}{1 - \alpha} z_{t-1} +$$
$$\frac{\varepsilon_2 (k_0 k_1 + (1 - k_0)) - \varepsilon_1 (k_0 k_1 (1 + \alpha) + \alpha(1 - k_0)) + \alpha k_0 k_1 \varepsilon_0}{1 - \alpha} z_{t-2} +$$
$$\frac{\varepsilon_3 (k_0 k_1 + (1 - k_0)) - \varepsilon_2 (k_0 k_1 (1 + \alpha) + \alpha(1 - k_0)) + \alpha k_0 k_1 \varepsilon_1}{1 - \alpha} z_{t-3} + . \tag{2.24}$$

and

$$\log P_t^* \left(\mu_{t-i}\right) = \frac{-k_0 k_1 \left(\varsigma_0 - \varsigma_1\right) + \left(1 - k_0\right)\varsigma_1}{1 - \alpha}\mu_{t-1} +$$

$$\frac{\varsigma_2 \left(k_0 k_1 + \left(1 - k_0\right)\right) - \varsigma_1 \left(k_0 k_1 \left(1 + \alpha\right) + \alpha \left(1 - k_0\right)\right) + \alpha k_0 k_1 \varsigma_0}{1 - \alpha}\mu_{t-2} +$$

$$\frac{\varsigma_3 \left(k_0 k_1 + \left(1 - k_0\right)\right) - \varsigma_2 \left(k_0 k_1 \left(1 + \alpha\right) + \alpha \left(1 - k_0\right)\right) + \alpha k_0 k_1 \varsigma_1}{1 - \alpha}\mu_{t-3} + . \quad (2.25)$$

To calculate the expected price dispersion

$$E \log DP_t = \frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i \left(1 - \alpha\right)\left[1 - \alpha^i \left(1 - \alpha\right)\right] Var\left(\log P_{t-i}^*\right)$$

$$-\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j} \left(1 - \alpha\right)^2 Cov(\log P_{t-i}^*, \log P_{t-j}^*),$$

we need to find two components. One is the covariance which arises because of lagged terms being cross-multiplied; these terms are then the variance of the *i.i.d.* error times the various cross-terms. The other is the variance. We only demonstrate the calculation of these two terms under productivity shocks. The analysis under monetary shocks is analogous.

Covariance First, we write equation (2.24) as

$$\log P_t^* \left(z_{t-i}\right) = a_0 z_t + a_1 z_{t-1} + a_2 z_{t-2} + a_3 z_{t-3} + a_4 z_{t-4} + \ldots \quad (2.26)$$

where

$$a_0 = \frac{-\chi'}{1 - \alpha} = -\chi''$$

$$a_1 = \frac{\alpha\chi'}{1 - \alpha} + \frac{-k_0 k_1 \left(\varepsilon_0 - \varepsilon_1\right) + \left(1 - k_0\right)\varepsilon_1}{1 - \alpha}$$

$$a_2 = \frac{\varepsilon_2 \left(k_0 k_1 + \left(1 - k_0\right)\right) - \varepsilon_1 \left(k_0 k_1 \left(1 + \alpha\right) + \alpha \left(1 - k_0\right)\right) + \alpha k_0 k_1 \varepsilon_0}{1 - \alpha}; etc.$$

Second, the covariances under productivity shocks will be, for example

$$Cov(\log P_t^*, \log P_{t-1}^*) = var(z)\left[a_0 a_1 + a_1 a_2 + a_2 a_3 + a_3 a_4 + a_4 a_5 + a_5 a_6 + \ldots\right]$$

$$Cov(\log P_t^*, \log P_{t-2}^*) = var(z)\left[a_0 a_2 + a_1 a_3 + a_2 a_4 + a_3 a_5 + a_4 a_6 + a_5 a_7 + \ldots\right]$$

$$Cov(\log P_t^*, \log P_{t-3}^*) = var(z)\left[a_0 a_3 + a_1 a_4 + a_2 a_5 + a_3 a_6 + a_4 a_7 + a_5 a_8 + \ldots\right], etc. \quad (2.27)$$

The first few cross-terms of these covariances can be calculated explicitly, however, a_i would decay after a certain point at the rate of the dominant root, called ρ, of the three stable roots. This

allows us to write the equations in (2.27) as

$$Cov(\log P_t^*, \log P_{t-1}^*) = var(z)\left[a_0a_1 + a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 + \rho a_5^2 + ...\right]$$

$$Cov(\log P_t^*, \log P_{t-2}^*) = var(z)\left[a_0a_2 + a_1a_3 + a_2a_4 + a_3a_5 + \rho a_4a_5 + \rho^2 a_5^2 + ...\right]$$

$$Cov(\log P_t^*, \log P_{t-3}^*) = var(z)\left[a_0a_3 + a_1a_4 + a_2a_5 + \rho a_3a_5 + \rho^2 a_4a_5 + \rho^3 a_5^2 + \rho^3 a_6^2 + ...\right]$$

$$etc. \tag{2.28}$$

Therefore,

$$-\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}Cov(\log P_t^*, \log P_{t-i}^*) = \frac{-\alpha\phi_p var(z)}{1+\alpha}\left[\begin{array}{c} \frac{\rho}{1-\rho}\left(a_5^2 + a_6^2 + ...\right) + a_0\left(a_1 + a_2 + ... + \frac{a_5}{1-\rho}\right) \\ +a_1\left(a_2 + ... + \frac{a_5}{1-\rho}\right) + a_2\left(a_3 + ... + \frac{a_5}{1-\rho}\right) + \\ a_3\left(a_4 + \frac{a_5}{1-\rho}\right) + a_4\frac{a_5}{1-\rho} \end{array}\right] \tag{2.29}$$

Variance   The second term in the expected price dispersion formula- variance- is calculated as follows:

$$Var(\log P_t^*) = var(z)(a_0^2 + a_1^2 + a_2^2 + a_3^2 + a_4^2 + ...) \tag{2.30}$$

and

$$\frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i\left(1-\alpha\right)\left[1 - \alpha^i\left(1-\alpha\right)\right]Var\left(\log P_{t-i}^*\right) = \frac{\alpha\phi_p var(z)}{1+\alpha}(a_0^2 + a_1^2 + a_2^2 + a_3^2 + a_4^2 + ...)$$

From this point onwards, all calculation for both variance and covariance components is done numerically.

Table (2.2) shows the results of expected welfare under different combinations of (k_0, k_1) :

k_0	k_1	E(Welfare)
0	0	−0.01147
1	0	−0.01006
1	0.2	−0.01235
1	0.4	−0.01222
1	0.6	−0.01172
1	0.8	−0.01199
1	1	−0.01163

Table 2.2: Expected welfare under interest rate rule targeting zero rate inflation

Price targeting rule   Here the analysis is analogous to the one used above for the inflation target regime.Using equations (2.10), (2.11), (2.12), the indexation formula and the assumption that firms

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have knowledge of their own micro information (productivity, prices and costs) in period t as well as the macro information of period $(t-1)$, we write all the terms related to the price level as a function of real and monetary shocks. The results are identical to those for inflation targets under full rational indexation which is again the optimum. Again, monetary policy is impotent in this case.

We obtain the equivalent results to those of inflation targeting in the two following Tables 2.3 and 2.4:

(k_0, k_1) combination	x_1	damping factor $(\sqrt{x_2 \times x_3})$
$(0; \text{all})$	0	0.49174
$(1, 0.2)$	0.88303	0.29339
$(1, 0.4)$	0.85953	0.36410
$(1.0.6)$	0.93064	0.40279
$(1, 0.8)$	0.90543	0.43374
$(1, 1)$	0.88844	0.45787

Table 2.3: Stable roots under price-level targeting

k_0	k_1	E(Welfare)
0	0	-0.01127
1	0	-0.01006
1	0.2	-0.01320
1	0.4	-0.01356
1	0.6	-0.01381
1	0.8	-0.01406
1	1	-0.01426

Table 2.4: Expected welfare under interest rate rule targeting price level

Conclusion from the analytic with restricted private information

Under both type of monetary regime: interest rate rule with inflation target and price level target, the analytic model shows that overall the fully rational indexation for prices is always optimal. This is a confirmation of the result that was derived in our previous chapter for the more restricted comparison of full rational with full lagged indexation. If agents index prices and wages rationally, expected welfare is independent of the choice of monetary policy target under the assumption of micro current information. The reason is that by allowing for rational indexation the New Keynesian Phillips Curve defaults to a New Classical one, where real output only depends on current and lagged real shocks and the monetary surprise, but not on the systematic part of the monetary rule. This is an echo of Sargent and Wallace's (1975) famous irrelevance result. The intuition is that rational indexation builds into prices the effect of any shocks known at time $t-1$. Whatever has happened at $t-1$ is, in the case of the productivity shock, built into the expected real reset price for the next period t; this fixes expected real marginal cost and hence expected real output. The expected price

index is then calculated as the necessary price increase that will accommodate this and the expected level of interest rates. Unexpected monetary shocks have no effect on prices because they have been pre-set in this way. Thus only lagged money shocks affect prices while only unexpected monetary shocks affect output under rational indexation in this model.

Hence with this model the monetary rule has no impact on welfare when current information is solely micro. We now turn to the full model under the same information assumption.

Stochastic simulation results on the full model under micro private information

We proceed to consider the stochastic simulations for expected welfare in terms of deviations from the flex-optimum under both lagged and rational indexation when only micro information is assumed to be known at period t. The aim is to relate these results to those from the analytic model above. Table (2.5) shows that expected welfare is maximised by rational indexation, just as in the analytic model.

	Type of shock	k_0	kw_0	k_1	kw_1	Welfare
Inflation target	Mixed	0	0	1	1	-0.0048
		0	0	.8	.8	-0.01311
		.2	.2	1	1	-0.01626
		1	1	1	1	-0.02669
Price target	Mixed	0	0	1	1	-0.0048
		0	0	.8	.8	-0.01032
		.2	.2	1	1	-0.01120
		1	1	1	1	-0.01994

Table 2.5: Expected welfare with micro current information (standard errors of 0.01 for monetary and productitivty shocks)

We now find that monetary policy does, strictly speaking, have an effect on expected welfare under rational indexation. The reason for this is the introduction of wage-setting. Though prices are only affected by the productivity shock (because it alone is currently observed), the interest rate rule reacts to both inflation (or prices) and to the output gap while monetary shocks also affect the latter. This reaction alters output and so employment; with wages fixed this drives agents away from their flex-price leisure choice, affecting their welfare. We also show in Table (2.6) how monetary policy choices affect expected welfare. The choice of whether to target inflation or prices is irrelevant since it is only the current price shock reaction that matters. Thus what matters in the interest rate rule is the size of the reactions to inflation or prices and to the output gap. Higher inflation or price coefficients worsen the effect of productivity shocks because they dampen price changes which means that real wages do not change as much as they should to match productivity change. Higher reactions to the output gap dampen movements in it and employment which move workers away from their flex-price choices. What we notice is that while there are effects here, they are not at all big, because the utility function does not have much curvature in leisure. In the Calvo model the

big losses arise because of price and wage dispersion. Hence we can say that effectively the results are the same as in the analytic model: full rational indexation is optimal and in this case monetary policy is (effectively) impotent.

Full Rational indexing	Productivity	Monetary	Total
Inflation targeting	−0.00381	−0.00098	−0.0048
Price targeting	−0.00381	−0.00098	−0.0048
Stricter Inflation targeting	−0.00426	−0.00098	−0.00524
Stricter Price targeting	−0.00426	−0.00098	−0.00524
Inflation targeting (higher weight on output gap)	−0.00376	−0.00092	−0.00468
Price targeting (higher weight on output gap)	−0.00376	−0.00092	−0.00468

Table 2.6: Expected welfare with micro current information under different rules (standard errors of 0.01 for monetary and productitivity shocks)

Conclusions on case of micro current information only

What we have found in this case is that full rational indexation is the dominant strategy for private agents. This has strong implications for monetary policy. First, it is irrelevant whether the interest rate rule targets inflation or prices since only the shock to prices or inflation matters and it is the same under both rules. Second, the coefficients of the rule make no difference at all to expected welfare in the analytic model (because current prices respond to current productivity shocks only) and in the full model they make virtually no difference (since they only enter through the effect on employment whose effect on welfare is minor).

2.2.2 Agents Observe Full Current Information

We now turn to the case where full current information is available to private agents. This is the default assumption made in New Keynesian models. As we noted earlier the justification presumably lies in the overlap between the length of time in which prices and wages are not changed at all- a quarter- and the production of current macro information by statistics offices and the private sector itself. In the course of three months price and wage setters may well be fairly well informed about what is going on in that quarter so that the assumption of full knowledge may be a close approximation. At any rate we now explore the implications of this assumption within the full model. Under full information the analytic model becomes too complex to solve under indexation schemes which vary the weights on lagged and rational indexation; we confine ourselves to some insights from the analytic model as far as we can take it.

We check the stochastic simulation under the assumption of full information being available in period t (Table 2.7). The pair $(k0, kw0)$ show the weights on lagged and rational indexation in indexation formulas for prices and wages respectively, while $(k1, kw1)$ shows whether prices and wages are partially or fully indexed. Our stochastic simulations are done for 100 sets of 40 overlapping

shocks- with both productivity and monetary shocks. Similarly to Minford and Nowell (2003), we treat each period outcome as a stochastic experiment of equal likelihood. We ignore the discount rate in calculation of the expected welfare. Firstly, in each set, in the first period, it runs for the first shock and records the welfare of this period. The first period values are then used as the base values for the next period simulation and so on. Then.we have 4000 observations which we average to get expected welfare. This process repeats for each $(k0, kw0, k1, kw1)$, where values of these parameters all belong to the interval of $[0, 1]$ and they move with a step of 0.2. Finally, we compare all the expected welfare values to find the weighting scheme that gives the maximum expected welfare.

Table (2.7) reports the results of stochastic simulations on the full model, showing the optimal indexation scheme under our two shocks to productivity and money.

	k0	kw0	k1	kw1	Best expected welfare
Inflation targeting	0	0	1	1	−0.00572
Price targeting	0	0	0.8	1	−0.00096
Stricter price target	0	0	0.8	1	−0.00034

Shocks assumed to be both monetary and productivity each with standard error of 0.01.

Table 2.7: Optimal index under inflation and price-level targeting with full current information

We note that:

(1) lagged indexation does not have any weight in the optimal indexation scheme.

(2) monetary policy is effective on welfare; as we move from inflation to price targeting and then to stricter price targeting expected welfare improves.

(3) the extent of price indexation also responds endogenously to this change in monetary policy: it drops somewhat. While full rational indexation is best under inflation targeting, price indexation drops to only 80% (though still on the rational index) as price level targeting is introduced.

Let us consider these points in turn.

(1) To understand why lagged indexation does not enter the optimal indexation scheme, we refer back to the chapter 1 where we showed that lagged indexation created an additional correlation between lagged price surprises and lagged prices: this tends to raise the variability of accumulated reset prices on balance. Hence the optimal indexation scheme only has rational indexation in it. The explanation can be briefly described as follows: the reset price under lagged indexation is given as the reset price under rational indexation plus an extra term

$$\log P_t^* \text{ (lagged)} = \log P_t^* \text{ (rational)} - \left(\frac{\nu_{t-1} - \alpha\nu_{t-2}}{1 - \alpha} \right) \tag{2.31}$$

where $v_{t-1} = \log P_{t-1} - E_{t-1} \log P_t$. The reset price under the rational expectation is only a function of the current and lagged productivity shocks. The term $-\left(\frac{\nu_{t-1} - \alpha\nu_{t-2}}{1-\alpha} \right)$ in the reset price under lagged indexation contains all past productivity and monetary shocks. The difference between

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expected welfare under rational indexation and lagged indexation is divided into two parts. The first part consists all the terms (q) that are not related to lagged productivity shock (z_{t-1}), such as all the productivity shocks that occur before or at period $(t-2)$ and all monetary shocks. These shocks are the ones that do not enter the reset price expression under rational indexation. The expected welfare of this part is $-\phi_p \frac{\alpha}{1+\alpha} \left(\frac{1-\rho}{1-\alpha\rho}\right) var\ (q)$ [2] which must worsen the expected welfare under lagged indexation compared with rational indexation. The second part of the expected welfare consists only of terms in z_{t-1} which are hence correlated with welfare under rational indexation. This part is $-\phi_p \frac{\alpha}{1+\alpha} \left(\chi''^2 + (\psi_0 + \alpha\chi'')^2 + (1-\alpha)\chi''(\psi_0 + \alpha\chi'')\right) var(z)$, which turns out for the calibrated values of the model to improve expected welfare compared with rational indexation. In aggregate the comparison between the resulted expected welfare levels under lagged and rational indexation depends on the magnitudes of the two parts above. The simulation results suggest that the first part dominates the second, so that lagged indexation lowers expected welfare by introducing additional lagged shocks into the reset price.

(2) Monetary policy is now effective on expected welfare because full information causes current monetary and productivity shocks to affect both reset prices and wages and the interest rate rule modifies these effects through its reaction coefficients. We can demonstrate this conclusion in the analytic model with its assumptions of no labour market, no capital accumulation, and only price setting. This model's solution for the reset price under full rational indexation is:

$$\log P_t^* = 1.5134 z_t - 1.0141 z_{t-1} - 1.4095 \mu_t + 0.7659 \mu_{t-1} \qquad (2.32)$$

Here the monetary shock and its lagged value join the productivity shock and its lagged value in affecting the reset price and so expected welfare.

Moving from an inflation to a price-level target has the effect of increasing the response of real interest rates to price shocks and so dampening these. The reason is that price-level targeting effectively raises the real interest rate, r_t, response to a price shock because $r_t = R_t - E_t \pi_{t+1}$, which is $r_t = rule - (E_t P_{t+1} - P_t)$. Under inflation targeting, the last term in the real interest rate equation is small or zero so the real interest, r_t, is just the rule; but under price-level targeting, $E_t P_{t+1}$ is small or zero, so the rule becomes $rule + P_t$. Therefore, a price-level target stops prices moving as much as they do under an inflation target. This reduces the variance of the reset price and also that of both of the real wage and of employment which additionally enter the welfare function in the full model. We can see the effect of the reduced reset price variance from simulations of the analytic model under full current information and full rational indexation - Table (2.8) below

(3) We turn last to why indexation is sensitive to monetary policy. We can understand this in terms of the alteration less than full rational indexation creates in the reset price, p_t^*. Using the reset

---

[2] $q_{t-i} = \rho^{i-j} q_{t-j}$

Inflation target			Price target		
Productivity	Monetary	Total	Productivity	Monetary	Total
−0.001363	−0.001026	−0.002389	−0.00087	−0.000498	−0.00137

Table 2.8: Expected welfare for rational indexation with full current information (stochastic simulation of monetary and productitivty shocks, each with standard error of 0.01)

price equation which under full rational indexation can be written as function of unexpected current and lagged price changes:

$$(1 - \alpha) \log P_t^* = \log P_t^{UE} - \alpha \log P_{t-1}^{UE}, \tag{2.33}$$

If we now deviate from full rational indexation by reducing the indexation to $(1 - k)$ this gives us instead

$$(1 - \alpha) \log P_t^* = \log P_t^{UE} + k E_{t-1} \log P_t - \alpha \log P_{t-1}^{UE} - k\alpha E_{t-2} \log P_{t-1}, \tag{2.34}$$

We can see that this creates a potential correlation between $E_{t-1} \log P_t$ and $\log P_{t-1}^{UE}$. Suppose there is a shock to the price level, then under price-level targeting there is a commitment to remove some or all of this shock from next period's price level; thus write $E_{t-1} \log P_t = \rho(1 - \beta) \log P_{t-1}^{UE}$ where ρ is the model-generated persistence in prices and β is the extent of its removal by the price-targeting rule. It follows that the variance of $\log P_t^*$ which enters expected welfare will equal $(\frac{1}{1-\alpha})^2 [Var \log P^{UE}](1 + [k\alpha\rho(1 - \beta)]^2 + \{\alpha^2 + [k\rho(1 - \beta)]^2 - 2k\alpha\rho(1 - \beta)\})$. The difference of this from the variance at $k = 0$ is $(\frac{1}{1-\alpha})^2 [Var \log P^{UE}]([k\alpha\rho(1 - \beta)]^2 + [k\rho(1 - \beta)]^2 - 2k\alpha\rho(1 - \beta))$. For this difference to be negative for positive $\{k, \rho(1 - \beta)\}$ we require that $2\alpha > \rho(1 - \beta)(1 + k\alpha^2)$. Price-level targeting generates a value of β close to unity, hence reducing the right hand side to close to zero- the stricter the closer to zero. However, inflation-targeting tends to induce a positive serial correlation, ρ_π, between rates of inflation; thus the serial correlation between price levels is $1 + \rho_\pi = \rho(1 - \beta)$ here.

So what we find is that Calvo persistence produces a reason to bias indexation away from full in order to induce a helpful correlation offsetting the persistence. Nevertheless it remains optimal to use rational indexation in preference to lagged, essentially because the latter introduces unnecessary extra correlations which tend to raise the variance of $\log P_t^*$.

What are the impulse responses to a monetary shock under the three monetary regimes we have identified? The charts show them in turn for inflation targeting, price targeting and stricter price targeting, the latter two with endogenously slightly lower than full (rational) indexation. What we see is that they have none of the supposed hallmark properties of New Keynesian models: there is little persistence, no 'hump shape' in either inflation or output, but rather there is a brief moving shock oscillation followed by virtually no residual effect at all. Price level targeting increases stability,

the stricter the greater the increase. As for the productivity shock (figure 2.2) we see a rather similar effect to that of a monetary shock superimposed on the steady declining effect of declining productivity on output and consumption. There is plainly no nominal rigidity to speak of in these effects; there is solely an effect of the Calvo mechanism causing relative prices to move in response to both shocks because only a minority of price and wage setters are able to change their relative price currently in response to a current shock.
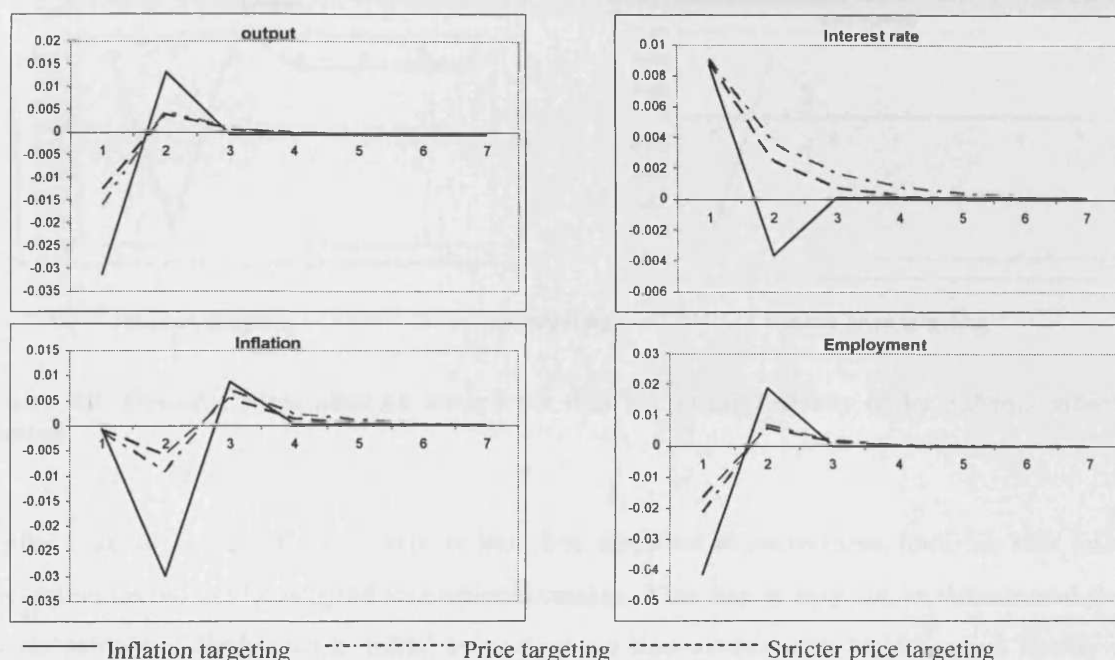
Figure 2.1: Dynamic paths after an unexpected 0.01 rise in interest rate under different monetary regimes

2.2.3 Conclusion

We conclude that the Calvo contract adjusted for rationally expected indexation under both monetary regimes - inflation and price level targeting- delivers the highest expected welfare. This holds under both information assumptions though under full information price-level targeting tends to induce slightly less than full indexation because price shocks are less persistent. Rational indexation eliminates the effectiveness of monetary policy on welfare when there is only price-setting under only current micro information. However in the broader context of wage-setting and full current information, both of which are standard in New Keynesian models, monetary policy regains effectiveness; and the class of interest rate rules that delivers the highest benefits are those which target the price level as strictly as possible, the reason being that this policy minimises the size of shocks to prices and hence of wage and price dispersion. However rational indexation even when less than full ensures that there is very little nominal rigidity in this adapted world of Calvo contracts. It may be

output

interest rate

Inflation

Employment

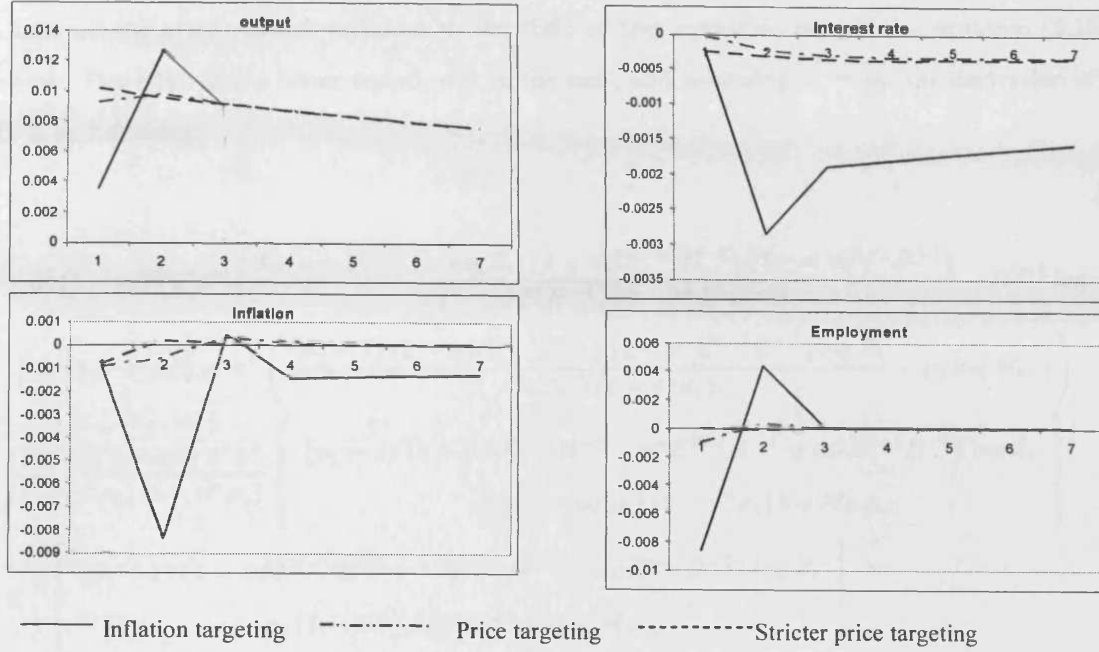——— Inflation targeting   —·—·—· Price targeting   ----------Stricter price targeting

Figure 2.2: Dynamic paths after an unexpected 0.01 rise in productivity under different monetary regimes

replied that in this case Calvo contracts lose their empirical attractiveness, implying that rational indexation should not be adopted for empirical reasons. This may or may not be the case and should be the subject of further work- initial indications are that models with little nominal rigidity may perform rather better empirically than has been realised. The key points we are trying to make in this chapter are first that there is a strong theoretical case for such indexing; second that its effect is to deprive New Keynesian models of their central supposed features; and third that it strengthens the case for price level targeting.

2.2.4 Appendix

Inflation targeting rule under restricted information assumption

Equation (2.15)

$$
\begin{aligned}
&\log P_t - k_0 \left(1 - k_1\right) E^{t-1} \log P_t - \left(k_0 k_1 + \alpha\right) \log P_{t-1} + \alpha k_0 \left(1 - k_1\right) E^{t-2} \log P_{t-1} + \alpha k_0 k_1 \log P_{t-2} \\
&= \frac{(1-\alpha)(1-\alpha\beta)\,\sigma^*\chi^*\left(B^{-1}-1\right)\log Z_t}{\left(1-\alpha\beta E^t B^{-1}\right)\left(1-\sigma^* E^t B^{-1}\right)} - \frac{(1-\alpha)(1-\alpha\beta)\,\sigma^*\chi^* E^{t-1}\log M_t}{\left(1-\alpha\beta E^{t-1} B^{-1}\right)\left(1-\sigma^* E^{t-1} B^{-1}\right)} - \\
&\quad - \frac{(1-\alpha)(1-\alpha\beta)\left[\tau\sigma^*\chi^* - \left(1-\sigma^* B^{-1}\right) + k_0\left(1-k_1\right)\left(1-\sigma^* E^{t-1} B^{-1}\right)\right] E^{t-1}\log P_t}{\left(1-\alpha\beta E^{t-1} B^{-1}\right)\left(1-\sigma^* E^{t-1} B^{-1}\right)} \\
&\quad + \frac{(1-\alpha)(1-\alpha\beta)\left[\tau\sigma^*\chi^* - k_0 k_1\left(1-\sigma^* E^{t-1} B^{-1}\right)\right]\log P_{t-1}}{\left(1-\alpha\beta E^{t-1} B^{-1}\right)\left(1-\sigma^* E^{t-1} B^{-1}\right)}
\end{aligned}
\tag{2.35}
$$

where $\frac{1+\chi}{1-\nu} = \chi^*$.

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Take all the price related variables to the RHS of this equation, we get the equation (2.16) in the text. The RHS of the latter equation is in the text, and assuming $\rho_1 = \rho_2$, the derivation of the LHS is as following:

$$(1-\alpha)(1-\alpha\beta)\chi^*\sigma^* \left(\frac{E_t(1-B^{-1})(-\log Z_t)\left(1-\alpha\beta E^{t-1}B^{-1}\right)(1-\sigma^* E^{t-1}B^{-1})}{(1-\alpha\beta E^t B^{-1})(1-\sigma^* E^t B^{-1})} - E^{t-1}\log M_t \right)$$

$$= (1-\alpha)(1-\alpha\beta)\chi^*\sigma^* \left(\frac{(\rho_1-1)\left(1-\alpha\beta E^{t-1}B^{-1}\right)(1-\sigma^* E^{t-1}B^{-1})\log Z_t}{(1-\alpha\beta\rho_1)(1-\sigma^*\rho_1)} - \rho_2\log M_{t-1} \right)$$

$$= \frac{(1-\alpha)(1-\alpha\beta)\chi^*\sigma^*}{(1-\alpha\beta\rho_1)(1-\sigma^*\rho_1)} \left[\begin{array}{c} (\rho_1-1)\left(1-\alpha\beta E^{t-1}B^{-1}-\sigma^* E^{t-1}B^{-1}+\alpha\beta E^{t-1}B^{-2}\right)\log Z_t \\ -\rho_2(1-\alpha\beta\rho_1)(1-\sigma^*\rho_1)\log M_{t-1} \end{array} \right]$$

$$= \chi^{**} \left[\begin{array}{c} (\rho_1-1)\left(1-\alpha\beta E^{t-1}B^{-1}-\sigma^* E^{t-1}B^{-1}+\alpha\beta E^{t-1}B^{-2}\right)\log Z_t \\ -\rho_2(1-\alpha\beta\rho_1)(1-\sigma^*\rho_1)\log M_{t-1} \end{array} \right]$$

$$= \chi^{**} \left[\begin{array}{c} (\rho_1-1)\left(\log Z_t - (\alpha\beta+\sigma^*)\rho_1^2\log Z_{t-1}+\alpha\beta\rho_1^3\log Z_{t-1}\right) \\ -\rho_2(1-\alpha\beta\rho_1)(1-\sigma^*\rho_1)\log M_{t-1} \end{array} \right]$$

$$= \chi^{**} \left[\begin{array}{c} (\rho_1-1)\left(\frac{z_t}{1-\rho_1 L} - \rho_1^2(\alpha\beta+\sigma^*-\alpha\beta\rho_1)\frac{z_{t-1}}{1-\rho_1 L}\right) \\ -\rho_2(1-\alpha\beta\rho_1)(1-\sigma^*\rho_1)\frac{\mu_{t-1}}{1-\rho_1 L} \end{array} \right] \tag{2.36}$$

Collect unknown coefficients Multiplying both RHS and LHS by $(1-\rho_1 L)$, we get equation (2.16) in the text. Given that $\log P_t = \Sigma_{i=0}^{\infty}\varepsilon_i z_{t-i} + \Sigma_{i=0}^{\infty}\varsigma_i\mu_{t-i}$, we determine the unknown coefficients on μ_{t-i} as follows

(μ_t)

$$\varsigma_0 = 0 \tag{2.37}$$

(μ_{t-1})

$$0.564(1-k_0+k_0 k_1)\varsigma_3 + (-1.8+1.42k_0-1.98k_0 k_1)\varsigma_2 +$$
$$(2.8945-0.8889k_0+2.69k_0 k_1)\varsigma_1 - (2.5847+1.9028k_0 k_1)\varsigma_0 = -0.0933 \tag{2.38}$$

(μ_{t-2})

$$0.564(1-k_0+k_0 k_1)\varsigma_4 - (2.321-2.322k_0+2.881k_0 k_1)\varsigma_3$$
$$+(4.5549-3.2089k_0+5.532k_0 k_1)\varsigma_2 \tag{2.39}$$
$$+(-4.3347+1.4915k_0-5.0556k_0 k_1)\varsigma_1 + (1.534+2.43k_0 k_1)\varsigma_0 = 0$$

$$\left(\mu_{t-i}, i \geq 3 \right)$$

$$0.564 \left(1 - k_0 + k_0 k_1\right) \varsigma_{i+2} - \left(2.321 - 2.322 k_0 + 2.881 k_0 k_1\right) \varsigma_{i+1}$$

$$+ \left(4.5549 - 3.56 k_0 + 5.883 k_0 k_1\right) \varsigma_i - \left(4.3347 - 2.4275 k_0 + 6 k_0 k_1\right) \varsigma_{i-1} \qquad (2.40)$$

$$+ \left(1.534 - 0.62 k_0 + 3.05 k_0 k_1\right) \varsigma_{i-2} - 0.62 k_0 k_1 \varsigma_{i-3} = 0$$

Solution for $\log P_t^*$ under Rational indexation

It is a brief description of solution for $\log P_t^*$. From the equations (2.12) and the assumption of rational expectation, the reset price is

$$\log P_t^* = E^{t-1} \log P_t^* + \log P_t^{*UE} = \log P_t^{*UE} - \frac{\alpha}{1 - \alpha} \log P_{t-1}^{UE} \qquad (2.41)$$

where

$$log P_t^{UE} = \left(1 - \alpha\right) log P_t^{*UE} \qquad (2.42)$$

and from equation of the general price level and the assumption that firms have knowledge of their own micro information in period t as well as the macro information of period $(t - 1)$:

$$\log P_t^{*UE} = \log P_t^* - E^{t-1} \log P_t^*$$

$$= \left(1 - \alpha\beta\right) \Sigma_{i=0} \left(\alpha\beta\right)^i \left(\frac{1 + \chi}{1 - \nu}\right) \left[E^{t-1} \log Y_{t+i} - E^t \log Z_{t+i}\right]$$

$$- \left(1 - \alpha\beta\right) \Sigma_{i=0} \left(\alpha\beta\right)^i \left(\frac{1 + \chi}{1 - \nu}\right) \left[E^{t-1} \log Y_{t+i} - E^{t-1} \log Z_{t+i}\right]$$

$$= \left(1 - \alpha\beta\right) \left(\frac{1 + \chi}{1 - \nu}\right) \frac{1}{1 - \alpha\beta\rho_1} \left(-z_t\right) \qquad (2.43)$$

Hence the renewed price is rewritten as

$$\log P_t^* = \log P_t^{*UE} - \alpha \log P_{t-1}^{*UE} = \chi' \frac{\left(1 - \alpha L\right)}{\left(1 - \alpha\right)} \left(-z_t\right), \qquad (2.44)$$

And the expected welfare under the rational indexation for prices is

$$E(u_t - u_t^{FLEX}) = -\phi_p \alpha \chi''^2 var(z) \qquad (2.45)$$

This solution is identical under two monetary policy rules, that targets inflation and price level.

General Solution for $\log P_t^*$

From equation (2.12)

$$\log P_t^* = \frac{1}{1-\alpha}\left[\log P_t - \log \tilde{P}_t - \alpha\left(\log P_{t-1} - \log \tilde{P}_{t-1}\right)\right]$$

$$= \frac{1}{1-\alpha}\left\{\begin{array}{c}\log P_t - k_0\left(E^{t-1}\log P_t + k_1\left(\log P_{t-1} - E^{t-1}\log P_t\right)\right)\\ -\alpha\left[\log P_{t-1} - k_0\left(E^{t-2}\log P_{t-1} + k_1\left(\log P_{t-2} - E^{t-2}\log P_{t-1}\right)\right)\right]\end{array}\right\}$$

$$= \frac{1}{1-\alpha}\left\{\begin{array}{c}\log P_t - k_0\left(E^{t-1}\log P_t + k_1 v_{t-1}\right)\\ -\alpha\left[\log P_{t-1} - k_0\left(E^{t-2}\log P_{t-1} + k_1 v_{t-2}\right)\right]\end{array}\right\}$$

$$= \frac{1}{1-\alpha}\left\{\begin{array}{c}E^{t-1}\log P_t + \log P_t^{UE} - \alpha\left(E^{t-2}\log P_{t-1} + \log P_{t-1}^{UE}\right)\\ -k_0 E^{t-1}\log P_t - k_0 k_1 v_{t-1} + \alpha k_0 E^{t-2}\log P_{t-1} + \alpha k_0 k_1 v_{t-2}\end{array}\right\} \qquad (2.46)$$

where $\log P_t^{UE} = (1-\alpha)(1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\frac{1}{1-\alpha\beta\rho_1}(-z_t) = -\chi' z_t$. and it does not depend on any monetary shocks.

Therefore, consider only productivity shocks, equation (2.46) can be rewritten as

$$\log P_t^*(z_{t-i}) = \frac{1}{1-\alpha}\left\{\begin{array}{c}\left[-\chi' z_t + \alpha\chi' z_{t-1}\right] + (1-k_0)E^{t-1}\log P_t - \alpha(1-k_0)E^{t-2}\log P_{t-1}\\ -k_0 k_1 v_{t-1} + +\alpha k_0 k_1 v_{t-2}\end{array}\right\}$$

$$(2.47)$$

given that the term $\frac{1}{1-\alpha}\left[-\chi' z_t + \alpha\chi' z_{t-1}\right]$ is equal to $\log P_t^*$ under rational indexation. This equation in term can be written using Wold decomposition:

$$\log P_t^*(z_{t-i}) = \frac{1}{1-\alpha}\left\{\begin{array}{c}\left[-\chi' z_t + \alpha\chi' z_{t-1}\right] + (1-k_0)E^{t-1}\log P_t - \alpha(1-k_0)E^{t-2}\log P_{t-1}\\ -k_0 k_1\left(\log P_{t-1} - E^{t-1}\log P_t\right) + \alpha k_0 k_1\left(\log P_{t-2} - E^{t-2}\log P_{t-1}\right)\end{array}\right\}$$

$$= \frac{1}{1-\alpha}\left\{\begin{array}{c}\left[-\chi' z_t + \alpha\chi' z_{t-1}\right] + (1-k_0)\Sigma_{i=1}^{\infty}\varepsilon_i z_{t-i} - \alpha(1-k_0)_{i=2}^{\infty}\varepsilon_{i-1} z_{t-i}\\ -k_0 k_1\left(\Sigma_{i=1}^{\infty}\varepsilon_{i-1} z_{t-i} - \Sigma_{i=1}^{\infty}\varepsilon_i z_{t-i}\right) + \alpha k_0 k_1\left(\Sigma_{i=2}^{\infty}\varepsilon_{i-2} z_{t-i} - \Sigma_{i=2}^{\infty}\varepsilon_{i-1} z_{t-i}\right)\end{array}\right\}$$

$$= \frac{-\chi' z_t + \alpha\chi' z_{t-1}}{1-\alpha} + \frac{-k_0 k_1\left(\varepsilon_0 - \varepsilon_1\right) + (1-k_0)\varepsilon_1}{1-\alpha}z_{t-1} +$$

$$\frac{\varepsilon_2\left(k_0 k_1 + (1-k_0)\right) - \varepsilon_1\left(k_0 k_1\left(1+\alpha\right) + \alpha(1-k_0)\right) + \alpha k_0 k_1\varepsilon_0}{1-\alpha}z_{t-2} +$$

$$\frac{\varepsilon_3\left(k_0 k_1 + (1-k_0)\right) - \varepsilon_2\left(k_0 k_1\left(1+\alpha\right) + \alpha(1-k_0)\right) + \alpha k_0 k_1\varepsilon_1}{1-\alpha}z_{t-3} + \ldots \qquad (2.48)$$

Assume that there are only monetary shocks μ_{t-i}, then equation (2.46) is

$$\log P_t^* \left(\mu_{t-i} \right) = \frac{1}{1-\alpha} \left\{ \begin{array}{c} \left(1 - k_0\right) E^{t-1} \log P_t - \alpha \left(1 - k_0\right) E^{t-2} \log P_{t-1} \\ -k_0 k_1 \left(\log P_{t-1} - E^{t-1} \log P_t\right) + \alpha k_0 k_1 \left(\log P_{t-2} - E^{t-2} \log P_{t-1}\right) \end{array} \right\}$$

$$= \frac{1}{1-\alpha} \left\{ \begin{array}{c} \left(1 - k_0\right) \Sigma_{i=1}^{\infty} \varsigma_i z_{t-i} - k_0 k_1 \left(\Sigma_{i=1}^{\infty} \varsigma_{i-1} z_{t-i} - \Sigma_{i=1}^{\infty} \varsigma_i z_{t-i}\right) \\ +\alpha k_0 k_1 \left(\Sigma_{i=2}^{\infty} \varsigma_{i-2} z_{t-i} - \Sigma_{i=2}^{\infty} \varsigma_{i-1} z_{t-i}\right) - \alpha \left(1 - k_0\right) \Sigma_{i=1}^{\infty} \varsigma_i z_{t-i} \end{array} \right\}$$

$$= \frac{-k_0 k_1 \left(\varsigma_0 - \varsigma_1\right) + \left(1 - k_0\right) \varsigma_1}{1-\alpha} z_{t-1} +$$
$$\frac{\varsigma_2 \left(k_0 k_1 + \left(1 - k_0\right)\right) - \varsigma_1 \left(k_0 k_1 \left(1 + \alpha\right) + \alpha \left(1 - k_0\right)\right) + \alpha k_0 k_1 \varsigma_0}{1-\alpha} z_{t-2} +$$
$$\frac{\varsigma_3 \left(k_0 k_1 + \left(1 - k_0\right)\right) - \varsigma_2 \left(k_0 k_1 \left(1 + \alpha\right) + \alpha \left(1 - k_0\right)\right) + \alpha k_0 k_1 \varsigma_1}{1-\alpha} z_{t-3} + \ldots \quad (2.49)$$

Chapter 3

Testing NNS models with varying indexation assumptions against a benchmark New Classical model

New Keynesian dynamic stochastic general equilibrium (NK DSGE) models have become popular in macroeconomic research and practice. They have been utilised for policy analysis in institutions such as the IMF, the Federal Reserve, the Bank of England and the European Central Bank. Besides nominal rigidities they often include other frictions such as habit formation, capital adjustment costs and variable capacity utilisation. In modelling nominal rigidities they have usually adopted Calvo's model of contracts (Calvo, 1983). In Calvo's model it is assumed that people can only change their prices stochastically due to menu costs of price change.

However, it is usually assumed that such costs would not apply to an additional general indexation scheme, where price-setters all agree that their prices will go up automatically by some formula in addition to whatever discretionary change they may make. There are many ways in which this could happen, and there must be an optimal arrangement (Le and Minford, 2007), but it is not a focus of this paper. We argue that we do not know exactly what obstructs price flexibility in practice and also, we do not know exactly what might obstruct different indexation arrangements in practice. So even if we discover optimal indexation arrangements, they might not be possible in practice, just as perhaps flexible prices may not be. We have to use empirical evidence to judge the performance of different indexation models against the data. Economists have been willing to embrace Calvo contracts with lagged indexation because they believe these models fit the data well.

Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003) examine the empirical fit of NK DSGE models with lagged indexation. The former study demonstrates that such a model

is able to fit the dynamic responses in the U.S. data, observed inertia in inflation and persistence in output, as well as hump-shaped responses to monetary policy shocks. While Smets and Wouters (2003), using Bayesian estimation, show that the model is able to fit the unconditional moments in the Euro area data as well as predicting comparably to conventional atheoretical VARs, conditional on there being a sufficient number of structural shocks incorporated in the model. Though their testing methods differ the models are rather similar. Both as we have seen assume lagged indexation though Christiano et al. (2005) assume full indexation, while Smets and Wouters (2003) only assume partial indexation.

The indexation assumption seems to be the key to ensure the models fit the data. Collard and Dellas (2006) show that in the absence of backward price indexation, the price and/or wage rigidities themselves cannot generate inertial behaviour for inflation, irrespective of the type of real rigidities in the model. This assumption is not regarded as strict rationality, because the agents are looking backwards. Nevertheless NK DGSE models are growing in popularity due to their empirical success. Recent work, on the other hand, has found that some impulse responses in the data can also be generated by models with very little nominal rigidity, such as a New Classical model or even a classical model (see for example recent work on UK data reported by Minford (2006) and Minford et al. (2007)). Moreover, Le and Minford (2007) have argued on theoretical grounds that indexation should be rationally expected rather than lagged indexation, in the sense that this is likely to optimise the welfare of the representative agent in a general equilibrium model of the New Keynesian type. But it might be argued that this argument can be rejected on empirical grounds because with lagged indexation and other specialised features the NK DSGE models can generate the hump-shaped impulse response functions (IRFs) to monetary shocks found in the data (Christiano et al., 2005). This paper is an attempt to shed some light on this issue.

This paper evaluates the empirical performance of four models based on the NK model by Canzoneri, Cumby and Diba (2004), where the economy is characterised by optimising representative agents, nominal rigidities in prices and wages, capital accumulation and an estimated backward looking monetary policy. The original model has no indexation. But we create two further versions of the model: one assuming full lagged indexation of both prices and wages and one assuming rationally expected indexation instead. Then as a benchmark for all these models of varying degrees of rigidity we set up a flex-price version of the model in which there is a one period information lag; in effect a 'New Classical' version. We can think of these models as ranged along a spectrum of price rigidity from non-indexation at one end to the New Classical at the other. The purpose of the analysis is to discover how far nominal rigidity is required to fit the data.

One of the ways to check the properties of the model, as shown by Christiano et al. (2005), is to use the impulse response functions from a VAR representation of the US data. They find hump-

shaped responses of output and inflation to both monetary and productivity shocks using such a VAR and the IRF bands support the model they propose. The problem with this methodology, as pointed out by Minford (2006), is that a VAR is a set of equations where, with the variables depending solely on each other's lagged values, each variable has its own current error whose behaviour is determined empirically. Theories may be used to restrict a VAR in order to identify an error as a counterpart of a particular shock to the economy, but these theories need to be consistent with the model being tested. Christiano et al (2005) impose the restrictions on the VAR that the error in the interest rate equation is the monetary policy shock and is uncorrelated with the other shocks so that the interest rate has no contemporaneous effect on other variables and vice versa. These restrictions could well appear to have been driven by the desire to find hump-shaped responses which indeed do emerge then from the VAR; but they are not in general implied by the New Keynesian models under consideration. Their model was however then made to fit these restrictions[1]. Minford (2006) argues by contrast that in the NK DSGE model for each structural shock the counterpart in the VAR is a combination of errors - for example if one shocks interest rates there is an immediate effect on most endogenous variables in the VAR under the NK DSGE model's normal specification (as in Canzoneri et al., on which we base this work). Therefore the identification scheme in Christiano et al. (2005) is only right under very special restrictions, whose justification is obscure and apparently ad hoc. The real question one should ask is whether all four of these models can generate IRFs in line with the data as represented by a VAR that is identified by the models themselves. In other words the models themselves, having been specified on theoretical grounds (including such matters as precise information lags), should, as the null hypothesis being tested, supply the restrictions on the VAR. Thus the test (of IRFs etc.) is carried out under the null hypothesis of this model itself.

In this paper, we use the bootstrap method for testing macroeconomic models set out by Minford, Theodoridis and Meenagh (2007). It assumes that the model under consideration is correct and taken as a null hypothesis, and then the model is tested against the data. In the next step, the question asked for the purpose of the test is: if the given model is strictly true, what does the VAR say and can it reject the model, and its dynamic time series representation (the VAR)? The first step

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[1]Christiano et al. (2005) describe the details of their VAR as follows. First, the monetary policy is defined as $R_t = f(\Omega_t) + \varepsilon_t$, where R_t is the Federal Funds rate, f is a linear function, Ω_t is an information set, and ε_t is the monetary policy shock. The monetary policy shock is assumed to be orthogonal to the elements in Ω_t. Then they partition the vector of variables, Y_t, in the VAR as follows:

$$Y_t = \begin{bmatrix} Y_{1t} \\ R_t \\ Y_{2t} \end{bmatrix}$$

The vector, Y_{1t}, is composed of the variables (real GDP, real consumption, the GDP deflator, real investment, the real wage, and labour productivity) whose time t elements are contained in Ω_t, and do not respond contemporaneously to a monetary policy shock, ε_t. The vector Y_{2t} consists of the time t values of all the other variables in Ω_t (real profits, the growth rate of M2, and the S&P500 index scaled by the consumer price index). There are two assumptions. First, the variables in Y_{1t} are assumed to not respond contemporaneously to a monetary shock. Second, the time t information set of the monetary authority consists of current and lagged values of the variables in Y_{1t} and only past values of the variables in Y_{2t}.

is to discover the single equation errors of the model using the data. Then resample the shocks randomly (bootstrapping) to get a set of pseudo-samples, which are used to discover the sampling distribution of the dynamic time series model. The last step is to test whether the parameters of the time series model estimated by the actual data lie within the 95% confidence interval of this sampling distribution, conducted by a Wald-type statistic (or 'M[odel]-metric'). This statistic is constructed as follows, firstly calculate the distance from the average of the bootstrapped parameters for each set of bootstrap parameters; and then calculate the distance of the actual parameters from the bootstrap mean. If the distance of the actual parameter is in a percentile higher than 95th, it means the model is rejected at the 95% level of significance.

The paper is organised as follows. In Section 1, we briefly describe the models under consideration. Section 2 presents the methods used in analysis and testing. In Section 3, we report the results and compare them across the models. Section 4 concludes.

3.1   The Models to be tested

The models are briefly summarised as follows (see Appendix 3.5.1 for details). All three versions of the NK DSGE model have the common assumption that they are characterised by optimising agents, capital accumulation, monopolistic competition in both goods and labour markets, and nominal rigidities in prices and wages. Also, they share an estimated central bank interest rate policy. The interest rate rule has lagged interest rates, inflation and the output gap, which is defined as the difference between actual output and the estimate of potential output. The difference between the models is the assumption of how agents index their prices and wages against the movements of aggregate price and wage levels. In the rational indexation version of the model (RI), all agents raise their prices and wages in line with expected general inflation, formed using all available information at $t-1$. In addition, those who are given a chance to change their price and wage under Calvo contracts also increase their relative price and wage. In the lagged indexation (LI) model, all agents allow their price and wage to rise at the level of the observed lagged inflation, and also Calvo contracts give some agents a chance to change their relative price and wage. However, in the non indexation (NI) model, agents do not increase their wage or price in line with general ongoing inflation, they only increase their relative price and wage if Calvo contracts permit them to.

The last modified version of the model in Canzoneri et al. (2004) we consider here is the New Classical model with flexible price and wage, but with lagged information. Given the above equations, this version is achieved by setting all the Calvo contracts' probabilities to 0, so that $\alpha = 0$ and $\omega = 0$, and all the expectations are formed using the information available in period $t-1$.

3.2 The Testing Procedure

Minford (2006) argues that a model can be tested by estimation to see whether the estimated parameters individually are statistically different from zero. These tests are the necessary condition for the model to be taken seriously as a representation of the data. However, given many models may pass these tests; it is difficult to choose the unique best one. Also, even if the model has passed such a test, it does not necessarily mean that it is correct, because it may not be able to replicate the dynamic behaviour of the economy (autocorrelation and autocovariance). These models must be tested to see whether they produce impulse response functions like those in the data, generate simulated moments like those in the data, generate VAR and other representations like those estimated on the data. Issues arise with all these tests. The identification of the impulse response functions in the data must come from the model to avoid the inconsistency between the two. The data moments approach is theory free; one can choose many different filters, which can produce different results. Finally, there are many different possible time series representation of the data that also faces the problem of filtering.

On the other hand, some economists (beginning with Kydland and Prescott, 1982) avoiding estimation difficulties, such as the unavailability of data on some omitted variables, calibrate the model according to economic theory, under the assumption that the theory is correct, then test it against the dynamic behaviour of the economy. The method used in this analysis is based on a similar idea to test the model's dynamic behaviour against that of the economy. It involves the bootstrap and consists of the following stages (for details, see Minford, Theodoridis and Meenagh, 2007).

The general idea of this analysis is as follows (Minford, Theodoridis and Meenagh, 2007). Assume the structural model, a true model, is the null hypothesis; one can derive the model's predictions about the impulse responses, moments and time series properties of the data, and then test whether these predictions statistically fit with those of the data. The structural model gives the structural errors based on the actual data, so testing against the data under the structural model's identification means these errors are used in generating the structural model's simulated performance. The bootstrap is used to perform the simulation rather than asymptotic distributions, which can be misleading in small samples.

The steps taken in this analysis are organised as follows. First, the structural models are solved and the errors implied by the model conditional on actual data are calculated. Since the model is assumed to be true, these implied errors are true under the null hypothesis. The errors enter the equations of monetary policy, the production fuction (total factor productivity), wage setting, the Euler equation (preferences), investment (marginal capital productivity) and marginal labour productivity.

Second, the null hypothesis assumes that the errors are omitted variables, modelled by autoregressive processes of identically and independently distributed shocks- the residuals from their autoregressive processes. Their empirical distribution is assumed to be given by the actual sample of the residuals, its variance and covariance matrix is therefore the actual one (Minford, Theodoridis and Meenagh, 2007). The random elements in the errors (random shocks) are repeatedly resampled to generate model pseudo-samples, using the vector of autoregressive parameters. Minford (2006) explains the idea of this step that if the model is correct then the estimated shocks will also be the true shocks hitting the model, thus, one may ask what samples could have been produced in the period of estimation had the shocks been different but nevertheless drawn randomly from the same distribution. Given the actual sample shocks; we repeatedly resample them to obtain new samples from the same distribution (they are resampled as a vector in order to preserve their cross-correlation structure). These samples generate new sample data, the pseudo-samples that together indicate a complete range of samples that could have occurred. In total we have 1000 pseudo-samples from the bootstrapping exercise.

Third, we ask what these pseudo-samples (the world according to the model) imply for the possible range of correlations and in general 'dynamic relationships' in the data. We are interested in the relationship of 3 main macroeconomics variables: output, inflation and the interest rate. These relationships are represented by a VAR, which is the parsimonious representation of the data in time-series form. We estimate the specified VAR on all the pseudo-samples to see what the sampling variability implies for their possible estimated values. Also, we take the same VAR and estimate it on the actual sample. HP filtered data is used. The statistical test asks whether the values estimated on the actual data lie within the 95% limits from the range of estimates that the models imply should be found. However, the VAR representation has many parameters, a test that relies on the single-parameter confidence intervals cannot be used as a test of all parameters together. Meenagh, Minford, Theodoridis and Wickens (2007) propose a Wald statistic, based on the principles of indirect inference:

$$\left(\hat{\gamma} - \gamma^0\right)' \Sigma_\gamma^{-1} \left(\hat{\gamma} - \gamma^0\right) \tag{3.1}$$

where $\hat{\gamma}$ is the vector of the VAR coefficient estimates, γ^0 is the vector of the estimates' means and Σ_γ is the Quasi Maximum Likelihood Covariance Matrix of $\hat{\gamma}$. This statistic's distribution can be derived from the bootstrapped sample estimates of the VAR coefficients. If a 95% confidence interval is chosen, then the actual data should generate a parameter combination whose Wald statistic is within the 95th percentile for the model not to be rejected.

We also test the impulse responses implied by the structural model against those of the data. Following our discussion above of the methods adopted by Christiano et al. (1999), we compare

the IRFs of the model with those of the VAR, identifying the shocks from the model itself. Under this procedure we first calculate the effect of a given structural shock on the currrent values of the variables in the VAR- a mapping from the model's structural shock to the VAR shocks. The VAR IRF to this combination of shocks is then calculated; this is the implied IRF to the structural shock in question, under the null hypothesis of the structural model. The 95% confidence bounds for this IRF are found from the VAR bootstraps.

3.3 Results

Under the method of bootstrapped simulations proposed here, we obtain a large number of measures of the models' performance. We treat the Wald statistic as an overall summary of performance. However the other measures can help to indicate where a model can be improved.

We begin by considering the models over the three decades from the 1970s to the 1990s. For this we have assumed the interest rate setting rule estimated by Canzoneri et al from 1979 onwards. The defence of this would be Orphanides' argument that using real-time data the Federal Reserve followed a similar rule in the earlier years. However, even if the Fed did so, the problem remains of the relationship between what they thought the data was and what it actually was. Effectively we are assuming that whatever misperception they had would be a constant, unresponsive to changing shocks; thus our bootstrapped samples would measure accurately the effects of shocks in changing the 'base case'. Needless to say, this assumption can be questioned. Accordingly we also look solely at the sample for which Canzoneri et al estimated their monetary policy function, and we redo our analysis for the sample period 1979-2003. Nevertheless nothing essential changes in the assessment- see Appendix 3.5.4- and so we concentrate our exposition on the three-decade sample period, 1970-1999.

The first point to note is that in terms of the overall Wald statistic evaluation all the models fail, whether at the default 95% we use throughout or the more tolerant 99% level of confidence. This can be seen in the table 3.1 that sets out the key results for the VAR(1) equations; again we focus our discussion on the VAR(1) representation of the data, since higher order VARs do not change the picture but introduce more complexity into the data description which makes it harder to see the main features of the comparison betwen data and models.

Looking at table **3.1**, we can see that out of the nine VAR coefficients four or even five are rejected in every case. This makes it extremely probable that the models will be rejected overall, since we should expect only 5% or so of the parameters to be rejected under overall acceptance. However as we would expect it is a generally different set of coefficients that are rejected in each case and thereby hangs the tale of each model's vulnerability.

Let us consider the models in turn along a spectrum of price flexibility- from non-indexed (NI) as the least flexible, through the lagged indexation model (LI), to rationally indexed (RI), to finally the flex-price version with a short information lag (New Classical, NC). We can consider them on various aspects of the data which are summarised in the VAR coefficients. These are the partial effects in statistical terms. Thus the diagonal terms measure the partial autocorrelations or 'own persistence' effects- the extent to which if a shock occurred to eg inflation alone it would persist in next period's inflation. The off-diagonal terms measure the partial cross-effects; thus for example the cross-effect of lagged interest rates on output tells us about the transmission effect of an interest rate shock on next period's output. While it is usual to compare data and model outcomes in terms of total correlations, this alternative approach unpacks the statistical relationships into their partial components. These partial components are in turn the elements that are assembled with varying weights to create the Impulse Response Functions implied by the VAR under varying combinations of shocks to the variables in the VAR.

Inflation persistence

We may begin with 'inflation persistence' as defined here by the partial autocorrelation of inflation in the VAR. This is a natural place to begin in evaluating these models since much of their motivation has been to explain data persistence in inflation as well as output. In the data this inflation persistence coefficient is fairly low at 0.27, indicating that statistically were other than inflation shocks to be absent inflation would settle down quite quickly. Not surprisingly, this is not what the LI model generates; the lagged indexation term produces, by construction, a high degree of persistence in the inflation process independently of non-inflation shocks to the model. This can be seen from Figure 3.1 which shows the impulse response function of the model to a monetary policy shock. The 95% range for this VAR coefficient from the bootstrap samples generated by the LI model is the highest of all the models at 0.81-0.97, massively above the 0.27 of the data-generated VAR.

More surprisingly perhaps the RI model generates nearly as much inflation persistence with a 95% range from 0.70-0.94. However RI is not proof against an inflation surprise by construction; furthermore in the period after the surprise the model produces a further inflation rise as agents anticipate the further price changes by those not permitted by the Calvo mechanism to change prices at once- see Figure 3.1 for the model's IRF to a monetary policy shock. Thus RI too generates excess inflation persistence.

This shared discrepancy with the data is so large that on its own it destroys the capacity of these two models to pass the Wald Statistic test. The difference of the data-generated coefficient from the bootstrap mean, on which the Wald Statistic operates, is no less than ten times its standard error.

By contrast the other two models get close to the inflation persistence data-based coefficient, as one might expect from their IRFs to a monetary policy shock shown in Figure 3.1. The NI model just

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encompasses it from above, as its 95% range is 0.21-0.57. This model has limited inflation persistence because its Phillips Curve is purely forward-looking so that when an inflation shock occurs those who can change prices do so at once; those who do so next period will not react unless the shock is repeated or persists for some independent reason. Thus any persistence of the inflation shock comes from the built in autocorrelation of the shock itself.

Finally the NC model just fails to encompass the inflation persistence coefficient from below, with a 95% range of -0.17-0.26. Plainly it generates little inflation persistence since while the shocks causing inflation are persistent themselves the monetary policy reactions are designed to offset them rapidly.

So one VAR coeffiecient alone turns out to be of extreme importance in evaluating these models' performance. In brief two of these models fail because they produce massively excessive inflation persistence.

Parameters	Actual	RI			NC		
		Lower	Upper	State	Lower	Upper	State
y_y	0.9587	0.5063	0.8746	OUT	0.6287	0.9158	OUT
y_r	−0.6779	−0.8136	0.1215	IN	−0.0753	0.0896	OUT
y_pi	−0.2746	−0.4322	−0.1243	IN	−0.0503	0.0356	OUT
r_y	0.0694	−0.2921	−0.1099	OUT	−0.1361	0.3224	IN
r_r	0.7751	0.0126	0.5252	OUT	0.6454	0.9202	IN
r_pi	−0.0713	−0.1908	−0.0419	IN	−0.0721	0.0751	IN
pi_y	0.0867	−0.1810	0.0883	IN	−1.0523	0.3070	IN
pi_r	0.3217	−0.6228	0.1028	OUT	−0.2555	0.5047	IN
pi_pi	0.2677	0.7033	0.9441	OUT	−0.1730	0.2552	OUT
		M-metric=100%			M-metric=100%		
Parameters	Actual	LI			NI		
		Lower	Upper	State	Lower	Upper	State
y_y	0.9587	0.6160	0.9403	OUT	0.5131	0.8476	OUT
y_r	−0.6779	−0.0959	0.2149	OUT	−0.382	0.4870	OUT
y_pi	−0.2746	−0.4946	−0.1950	IN	−0.8134	−0.0678	IN
r_y	0.0694	−0.1515	0.1792	IN	−0.0379	0.0849	IN
r_r	0.7751	0.6060	0.9374	IN	0.6263	0.9194	IN
r_pi	−0.0713	0.1179	0.4516	OUT	−0.0094	0.2543	OUT
pi_y	0.0867	−0.1197	0.1218	IN	−0.0630	0.0927	IN
pi_r	0.3217	−0.2645	−0.0028	OUT	−0.3012	0.1167	OUT
pi_pi	0.2677	0.8105	0.9712	OUT	0.2150	0.5669	IN
		M-metric=100%			M-metric=100%		

Table 3.1: VAR parameters and M-metric for different models

Other features of the VAR

We now turn to other features of the data summarised in this VAR. The most problematic for these models is the transmission of interest rate shocks to output. The data-based VAR sets this cross-effect at -0.68, a powerfully negative effect from well-known data regularities. This cannot be captured by the NC model whose mean cross-effect on VARs based on model-generated data is close to zero and its 95% range -0.08-0.09. In a flex-price model output remains close to equilibrium supply; real interest rate movements will affect demand but not equilibrium supply and so not output.
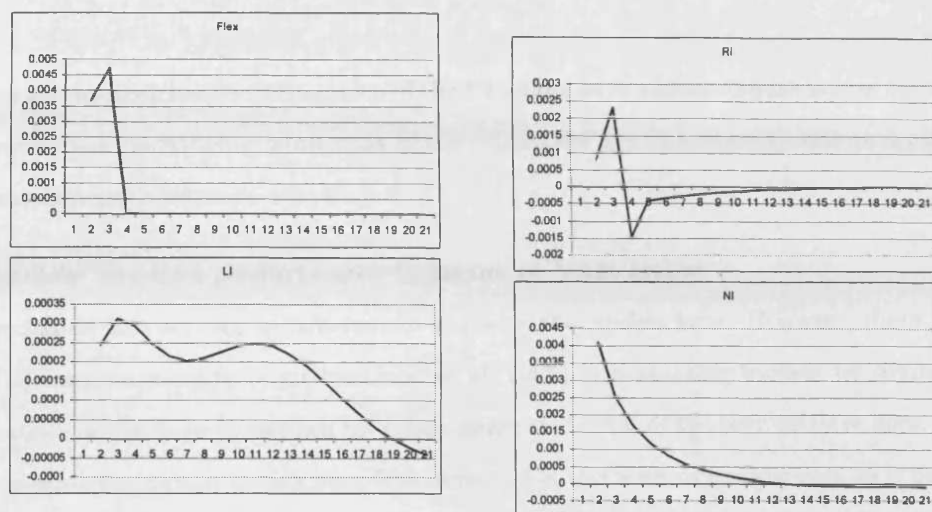
Figure 3.1: Inflation IRFs to monetary policy for different models

The discrepancy of this cross-effect from the bootstrap mean is about 15 standard errors, a massive t-value that puts it out of contention.

The NI model also fails on this cross-effect, though not so badly. Its difficulty is the standard conflict between productivity and demand shocks. A positive demand shock raises interest rates and raises demand and output; a positive productivity shock lowers inflation and interest rates and raises future output.

Let us finally focus on the NI model which remains the one so far not eliminated by a massive discrepancy. Its problems lie in having several moderate discrepancies. The one on the cross-effect of interest rates on output we have just seen. It also fails to capture the positive partial association between a rise in interest rates and a rise in future inflation- coming in the data from the Fisher effect as interest rates anticipate future inflation. In the NI model a shock productivity rise causes a fall in inflation and in interest rates (as discussed above) which gives a positive correlation, however a monetary policy shock generates a fall in interest rates accompanied by a rise in inflation, and so a negative correlation. Against a data-based coefficient of 0.32 the NI model-generated VAR range is -0.30-0.12.

Then we can look at the cross-effect of inflation on interest rates which in the data is slightly negative at -0.07. The NI model works in response to an inflation shock by raising interest rates for some time until the shock disappears- a positive effect; its model-generated 95% range is -0.01-0.25.

Finally we can examine output persistence, on the data set at a high 0.96. The NI model determines output via demand; when shocks occur they decay at rates of around 0.9 and produce interest rate reactions designed to push demand back to normal. Hence persistence in output is dampened by monetary policy, giving a model-generated range of 0.51-0.85, materially below the

data.

Thus we can summarise the perfomance of the NI model as moderate failure across several aspects of the transmission mechanism, while that of the NC model lies in a massive failure on the interest rate transmission mechanism to output.

The models' implied performance in terms of VAR IRFs:

The discussion just set out is sufficient to evaluate the models here. However, there has been a great deal of interest recently (e.g Christiano et al, 2005) in evaluating models by comparing their IRFs with those of the data as implied by a data-generated VAR of the sort we have here. So we now consider the same evaluation in this way. The difficulty of this form of comparison, as is obvious from brief reflection, is that one cannot easily isolate the source of the discrepancies observed between these IRFs.

The VAR coefficients summarise the transmission process in the data in response to some set of initial shocks to the variables. Thus one can determine the Impulse Response Function implied by the VAR to any combination of such shocks. However in order to determine what the IRF will be to a structural shock such as TFP or monetary policy one needs a way of mapping the structural shock into such a combination; this can only be supplied by a theory implying the mapping- ie. a model. Under the null of a particular model this can be done by using that model. This means that we can test whether the model implies the same IRF as the data. We do this first by imposing the model's mapping of shocks on the data-generated VAR. Then we compare this with the same mapping of shocks onto the model-generated VARs, specifically their 95% bounds. The data-generated VAR will use the VAR coefficients appropriate to the implied shock combination, which supplies the weights on the various coefficients. The 95% bounds on the model-generated VARs will be weighted averages of the VAR coefficients that trigger the 95% bounds at each horizon..

It might be asked why not compare the model's deterministic IRF directly with the data-generated VAR IRF? The reason is that under the null of the model the VAR is not necessarily a transformation of the model and therefore an exact reproduction of it; rather it is a data description, an approximation of the model within the bounds of the VAR form, which will have a distribution determined by the model. If it was an exact reproduction, then the distribution of its IRF would be around the model's deterministic IRF; but as it is in general not (indeed we cannot easily determine what the exact transformation is, and certainly we do not attempt to do so here), we must determine its distribution from the bootstrap samples and ask how a VAR of the chosen form would vary if the model was generating the samples on which it is estimated.

For this comparison of models with the data we focus for each model on the IRFs for the relevant shocks that determine its performance. Firstly, we identify which shocks contribute the most in explaining the variable variations. We note that the real business cycle focused primarily on

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productivity shocks. In the RBC view, productivity shocks drive fluctuations in real variables; a New Keynesian adds monopolistic competition and nominal inertia to the RBC model to create a model in which both productivity shocks and demand shocks play a role in the cyclical movements of interest rates and inflation. The variance decomposition (Table 3.2) shows that the dominant shocks for NI are monetary policy and investment; for LI monetary policy and TFP; for RI and NC investment and TFP. Therefore, what we see is that the importance of demand shocks increases with the degree of nominal rigidity. For RI and NC the dominant TFP and investment shocks are both from productivity, respectively total and capital. For LI and NI the addition of monetary policy as dominant is of course from the demand side. This is in line with what we would expect from the nature of these models.

Them we look at the IRFs to these main disturbances. As we have argued the failings of each model in terms of its VAR coefficients will emerge in the IRFs in a way that cannot easily be disentangled because a given shock will first be mapped by the model into some combination of variable shocks and second this combination will engage the VAR coefficients in a particular weighting; hence the overall weighting implied by a particular shock will be the result of the blending of these two factors in a way that is highly complex. However we can say in a general way that the failings will show up in a sprinkling of data-generated IRFs lying outside the model-generated 95% bounds; we might expect a failing model to produce IRFs that lies outside for more than 5% of the observations.

RI					
	Monetary policy	Euler	Investment	Relative factor productivity	Wage-setting	Total factor productivity
Y	18.78	20.10	35.09	2.39	3.89	19.76
R	23.84	12.13	19.13	6.12	5, 50	33.29
PI	9.50	9.65	15.78	7.53	8.46	49.08
LI					
	Monetary policy	Euler	Investment	Relative factor productivity	Wage-setting	Total factor productivity
Y	20.80	19.53	48.78	0.89	2.74	7.27
R	64.12	1.34	3.76	5.52	5.58	19.68
PI	1.62	1.30	4.27	13.73	11.78	67.30
NI					
	Monetary policy	Euler	Investment	Relative factor productivity	Wage setting	Total factor productivity
Y	78.60	0.77	16.68	0.08	1.95	1.92
R	31.06	0.42	48.29	2.30	8.27	9.66
PI	65.11	0.36	8.62	2.30	7.61	15.99
NC					
	Monetary policy	Euler	Investment	Relative factor productivity	Labour supply	Total factor productivity
Y	0.43	2.45	30.24	0.27	7.74	58.87
R	1.72	35.14	28.79	0.26	4.12	29.97
PI	3.71	13.20	26.28	0.33	6.73	49.75

Table 3.2: Variance Decomposition for different models

Total factor productivity shock

Total factor productivity shock
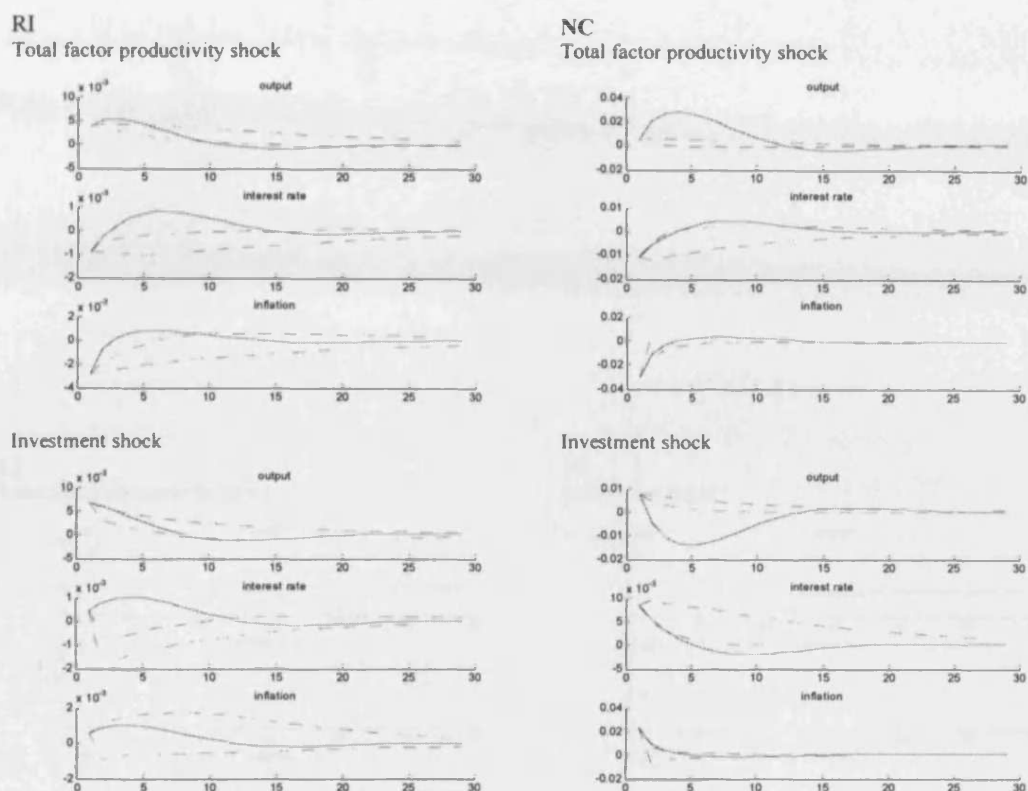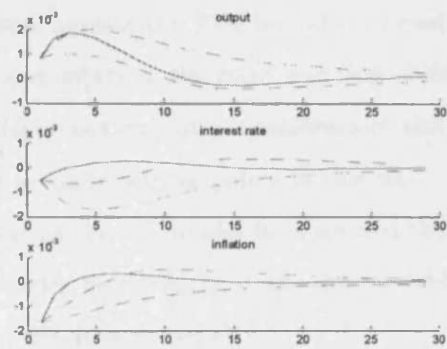

Investment shock

Investment shock

Figure 3.2: RI's and NC's IRFs to dominant shocks

Christiano et al (2005) emphasised that 'hump-shaped' IRFs were implied by the data for inflation and output. So it is of interest that under the nulls of the models here several of the data-generated IRFs exhibit no such humps either for inflation or for output or for both. Of course this may be because the models are rejected by the data and so provide poor identifying restrictions on the VAR. Christiano et al (2005) identified the VAR for the monetary policy shock by restrictions extraneous to the model they used; this was that the shock was independent of contemporaneous output and inflation. They then imposed this restriction on their model as well. Here we have simply used the models as specified; this restriction is hard to justify within these models.

Turning to the IRFs and their distributions we can see that indeed there are a large percentage of observations that lie outside the bounds for each model for the relevant shocks.

The most striking regularity is in the rejection of the output IRFs for the NC model; its data-generated VAR does imply a classic hump shape for output and the model-generated VARs simply cannot pick up these humps by a wide margin. The next most flexible model, RI, does not imply output or inflation humps for its data-generated VAR; it fails widely to pick up the IRFs but particularly badly for interest rates. LI again produces humps in output in the data-generated VAR; but the monetary policy IRF badly fails to pick up this hump. Even though it captures its interest
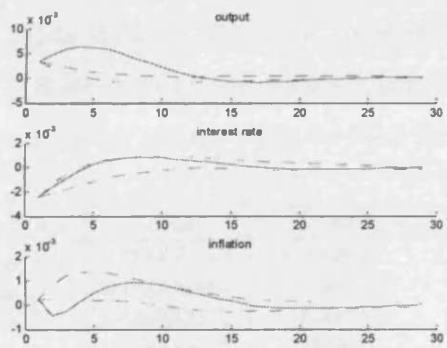
LI
Total factor productivity shock

NI
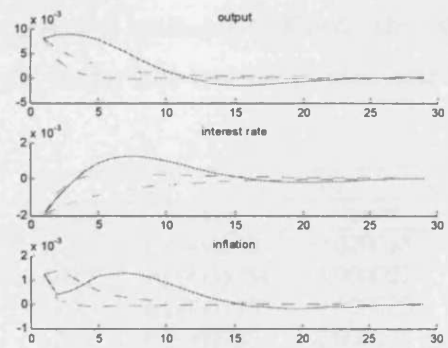Investment shock
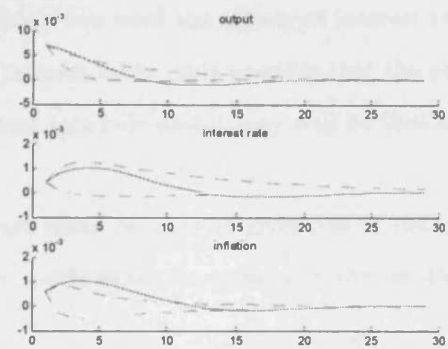
Monetary shock

Monetary shock

Figure 3.3: LI's and NI's IRFs to dominants shocks

rate profile, it fails otherwise on all IRFs. Finally the least flexible model, NI, fails to capture any of the IRFs at all, though none of the failures are dramatic.

Commenting on strengths and weaknesses of each model

In our discussion above we have identified two main reasons why these models have failed. The first was on inflation persistence where the two models with indexation greatly exaggerate inflation persistence. The second was in the interest rate transmission mechanism of the other two models.

We found that the first problem did not occur when the indexation schemes were dropped. This suggests therefore that during this US sample period no indexation was used.

The second problem was particularly widespread in the NI model, for which three out of the four coefficients outside the 95% bounds involved interest rates; two were the effects of interest rates on output and inflation, the third was that of inflation on interest rates. It is possible that the problem lies in the monetary policy specification; this is an interest rate rule and it may well be that the Fed was not actually setting policy in this way.

As far as the NC model is concerned the interest rate effect on output gives one of the massive discrepancies between the data- and model-based VAR coefficients. So again the interest rate rule may well be at fault here too.

In addition for the NC model there is a massive discrepancy for the coefficient of inflation on output, while for both NI and NC models output persistence is inadequate. Finally the variances of the VAR shocks for both models largely lie outside the model 95% bounds; in particular for NC inflation is far too volatile while for NI it is too stable.

Parameters	Actual	RI			NC		
		Lower	Upper	State	Lower	Upper	State
σ_{yy}	0.00005455	0.000020	0.000052	OUT	0.0000248	0.000048	OUT
σ_{yr}	0.00000485	−0.000015	−0.000004	OUT	−0.0000034	0.000027	IN
$\sigma_{y\pi}$	0.00000615	−0.000010	0.000008	IN	0.0000129	0.000083	OUT
σ_{rr}	0.00000628	0.000004	0.000008	IN	0.0000735	0.000139	OUT
$\sigma_{r\pi}$	0.00000318	0.000001	0.000008	IN	−0.0003260	−0.000165	OUT
$\sigma_{\pi\pi}$	0.00001406	0.000015	0.000033	OUT	0.0006093	0.001093	OUT

Parameters	Actual	LI			NI		
		Lower	Upper	State	Lower	Upper	State
σ_{yy}	0.00005455	0.0000034	0.0000072	OUT	0.000025	0.000050	OUT
σ_{yr}	0.00000485	−0.0000062	−0.0000022	OUT	−0.000013	−0.000005	OUT
$\sigma_{y\pi}$	0.00000615	−0.0000002	0.0000018	OUT	−0.000001	0.000007	IN
σ_{rr}	0.00000628	0.0000038	0.0000092	IN	0.000003	0.000007	IN
$\sigma_{r\pi}$	0.00000318	−0.0000017	0.0000004	OUT	−0.000003	0.000001	OUT
$\sigma_{\pi\pi}$	0.00001406	0.0000017	0.0000040	OUT	0.000006	0.000011	OUT

Table 3.3: Variances and Covariances of residuals of different models

This suggests some other features of the model may also be at fault. Hence further work is needed to establish possible improvements in both these models.

However we may make a negative comment based on these results: that there is no evidence that a high degree of price inflexibility is required in the ultimate model specification.

3.4 Conclusion

We use a newly developed bootstrap method for testing macroeconomic models according to their dynamic performance by Minford et al. (2007). We apply this method to three versions of the New Keynesian model and a further flexprice (New Classical) version with lagged information. On the main test criterion, the Wald statistic based on the VAR parameters, we have found that given the set of data here all models, NK and NC models, are rejected by the data massively. It does seem on the evidence here that indexation of any sort was unlikely given the substantially excessive inflation persistence it generates. However the flexible price model does no worse than the simple Calvo contract model, suggesting that when the model is improved sufficiently to pass this test, price inflexibility will not necessarily feature in the specification.

Going through this analysis we hoped the empirical evidence would shed some light on the validity of our theoretical finding that the rational indexation is superior to lagged indexation in Calvo contracts (Le and Minford, 2007), but what we seem to have found is rather different: that both forms of indexation were unlikely in the sample period, and that, even though we cannot gauge the extent of price flexibility, there is no evidence in favour of substantial price rigidity.

3.5 Appendices

3.5.1 Listing of models

A. NNS models with different indexation regimes

Indexation formulae are:

(a) RI: Rational indexation model is

$$\tilde{P}_t = E_{t-1}P_t \tag{3.2}$$

$$\widetilde{W}_t = E_{t-1}W_t \tag{3.3}$$

(b) LI: Lagged indexation model is

$$\tilde{P}_t = P_{t-1} \tag{3.4}$$

$$\widetilde{W}_t = W_{t-1} \tag{3.5}$$

(c) NI: Non indexation model is

$$\tilde{P}_t = 1 \tag{3.6}$$

$$\widetilde{W}_t = 1 \tag{3.7}$$

Consumption

$$C_t = \frac{1}{P_t \lambda_t} \tag{3.8}$$

Capital constraint

$$K_t = (1 - \delta)K_{t-1} + I_t - \frac{1}{2}\psi \left[\frac{I_t}{K_{t-1}} - \delta \right]^2 K_{t-1} \tag{3.9}$$

$$\frac{R_t}{W_t} = \frac{\nu}{1 - \nu} \frac{N_t(f)}{K_{t-1}(f)} \tag{3.10}$$

Marginal cost

$$MC_t = \frac{1}{\nu^\nu (1 - \nu)^{(1-\nu)}} \frac{R_t^\nu W_t^{1-\nu}}{Z_t} \tag{3.11}$$

$$\lambda_t P_t = \xi_t - \xi_t \psi \left[\frac{I_t}{K_{t-1}} - \delta \right] \tag{3.12}$$

$$\xi_t = \beta E_t \left\{ \lambda_{t+1} R_{t+1} + \xi_{t+1} \left([1 - \delta] - \frac{1}{2} \psi \left[\frac{I_{t+1}}{K_t} - \delta \right]^2 + \psi \left[\frac{I_{t+1}}{K_t} - \delta \right] \frac{I_{t+1}}{K_t} \right) \right\}, \tag{3.13}$$

Price setting behaviour:

$$P_t^* = \mu_p \frac{PB_t}{PA_t}, \tag{3.14}$$

$$PB_t = \alpha \beta E_t PB_{t+1} + \lambda_j \frac{MC_t}{P_t} \left(\frac{P_t}{\widetilde{P}_t} \right)^{\phi_p + 1} \widetilde{P}_t Y_t \tag{3.15}$$

$$PA_t = \alpha \beta E_t PA_{t+1} + \lambda_t \left(\frac{P_t}{\widetilde{P}_t} \right)^{\phi_p} \widetilde{P}_t Y_t \tag{3.16}$$

Aggregate price

$$\left(\frac{P_t}{P_{t-1}} \right)^{1 - \phi_p} = (1 - \alpha) \left(\frac{P_t^* \widetilde{P}_t}{P_{t-1}} \right)^{(1 - \phi_p)} + \alpha \left(\frac{\widetilde{P}_t}{\widetilde{P}_{t-1}} \right)^{1 - \phi_p} \tag{3.17}$$

Aggregate output

$$Y_t = \frac{Z_t K_{t-1}^{\nu} N_t^{1-\nu}}{DP_t} \tag{3.18}$$

Price dispersion

$$DP_t = (1 - \alpha) \left(\frac{P_t}{P_t^*(f) \widetilde{P}_t} \right)^{\phi_p} + \alpha \left(\frac{P_t}{P_{t-1}} \right)^{\phi_p} \left(\frac{\widetilde{P}_t}{\widetilde{P}_{t-1}} \right)^{-\phi_p} DP_{t-1} \tag{3.19}$$

Market clearing condition

$$Y_t = C_t + I_t + G_t \tag{3.20}$$

Wage setting behaviour

$$W_t^{*(1 + \phi_w \chi)} = \mu_w \frac{WB_t}{WA_t} \tag{3.21}$$

$$WB_t = \omega \beta E_t WB_{t+1} + N_t^{1+\chi} \left(\frac{W_t}{\widetilde{W}_t} \right)^{\phi_w (1+\chi)} \tag{3.22}$$

$$WA_t = \omega \beta E_t WA_{t+1} + \lambda_t \widetilde{P}_t N_t \left(\frac{W_t}{\widetilde{W}_t} \right)^{\phi_w} \tag{3.23}$$

Aggregate wage

$$\left(\frac{W_t}{\widetilde{W}_t} \right)^{1 - \phi_w} = (1 - \omega) W_t^{*(1 - \phi_w)} + \omega \left(\frac{W_{t-1}}{\widetilde{W}_{t-1}} \right)^{1 - \phi_w} \tag{3.24}$$

Welfare

$$U_t = E_t \sum_{\tau=t}^{\infty} \beta^{\tau - t} \left[\log C_t - \frac{1}{(1 + \chi)} AL_\tau \right], \tag{3.25}$$

Average disutility of work

$$AL_t = N_t^{1+\chi} DW_t \tag{3.26}$$

Wage dispersion

$$DW_t = (1 - \omega) \left(\frac{W_t^*(h)\widetilde{W_t}}{W_t} \right)^{-\phi_w(1+\lambda)} + \omega \left(\frac{W_{t-1}}{W_t} \right)^{-\phi_w(1+\lambda)} \left(\frac{\widetilde{W_t}}{\widetilde{W}_{t-1}} \right)^{-\phi_w(1+\lambda)} DW_{t-1} \qquad (3.27)$$

Euler equation

$$E_t \frac{\lambda_{t+1}}{\lambda_t} = E_t \Delta_{t,t+1} = \frac{1}{1+i} \qquad (3.28)$$

Monetary policy

$$i_t = 0.222 + 0.82 i_{t-1} + 0.35552 \pi_t + 0.032384 \, (output \; gap)_t + \varepsilon_{i,t}, \qquad (3.29)$$

B. New Classical Model

All expectations using $t-1$ information; $\alpha = \omega = 0$.

3.5.2 IRFs of the models

Figure 3.4: IRFs for Rational Indexation Model to different structural shocks
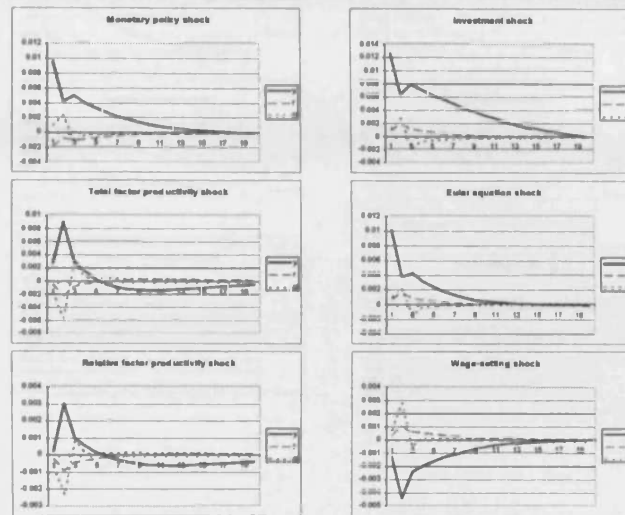
Figure 3.5: IRFs for Lagged Indexation Model to different structural shocks
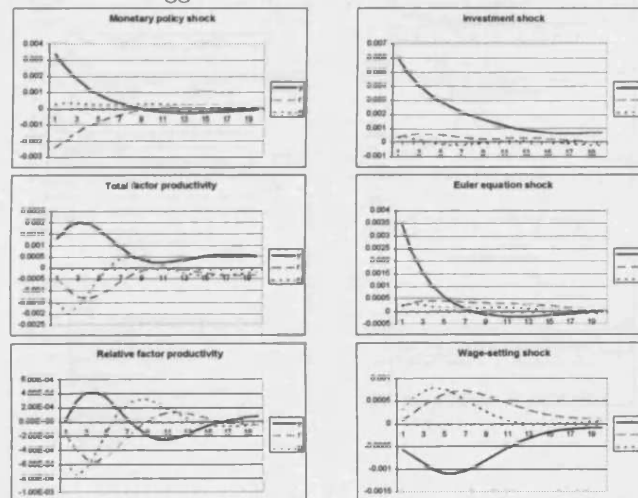
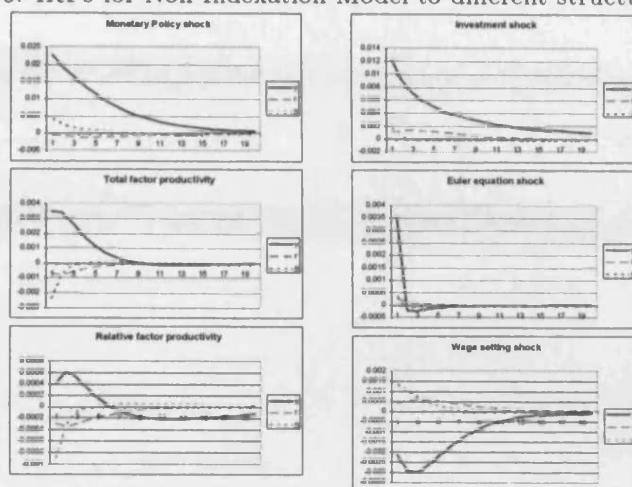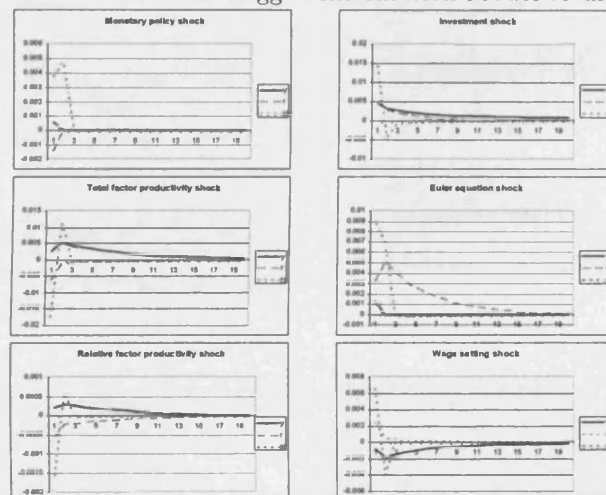Figure 3.6: IRFs for Non-Indexation Model to different structural shocks

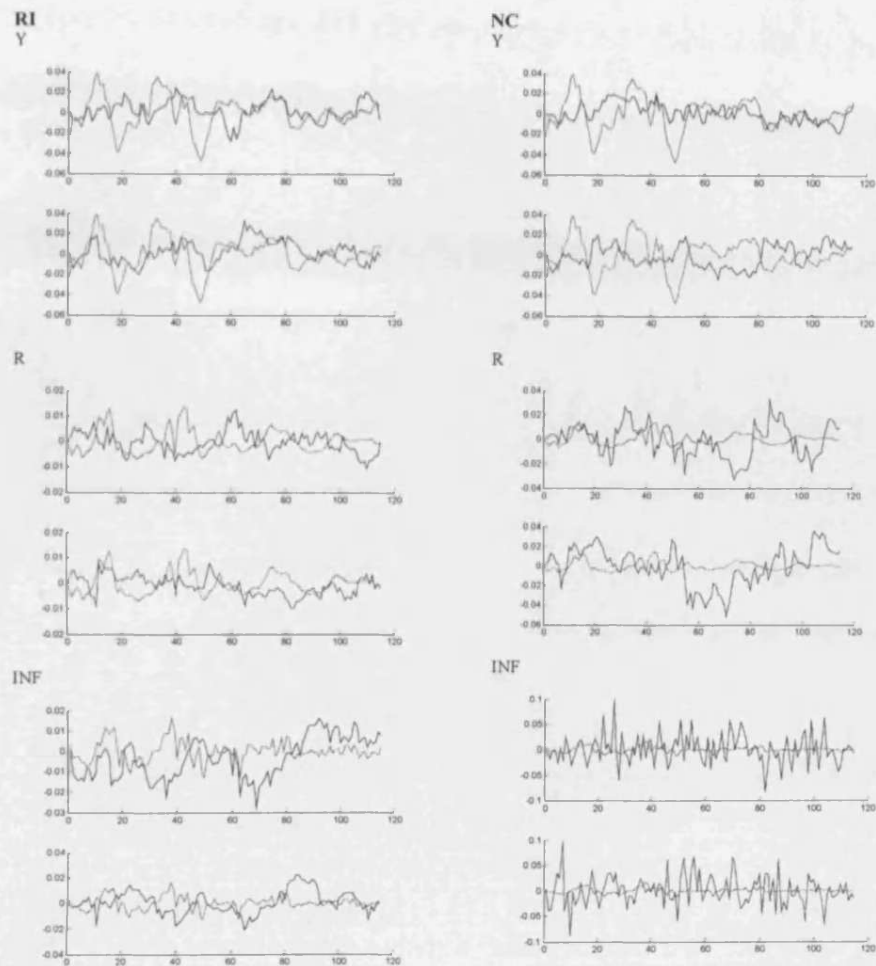Figure 3.7: IRFs for Flexible Price with Lagged Information Model to different structural shocks
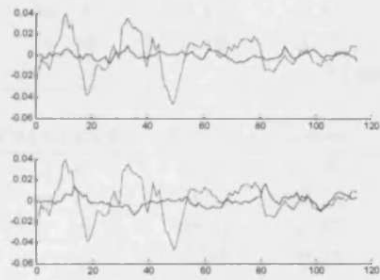
Figure 3.8: Actual vs. bootstrap samples

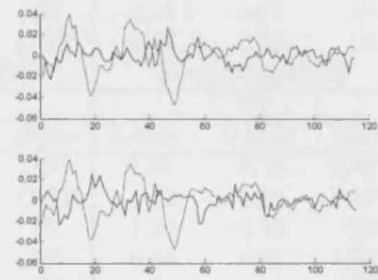### 3.5.3 Actual (red) vs. bootstrap (blue) data of the sample 1970-1999

Figure 3.9: Actual vs. bootstrap samples
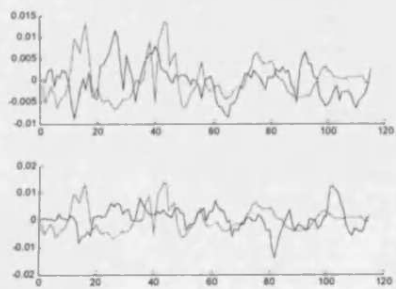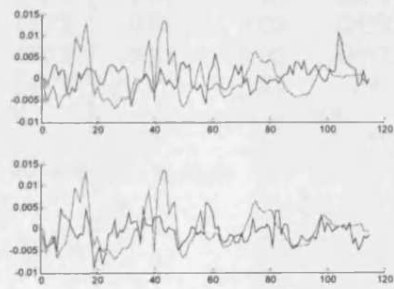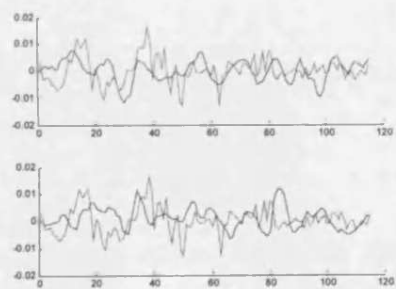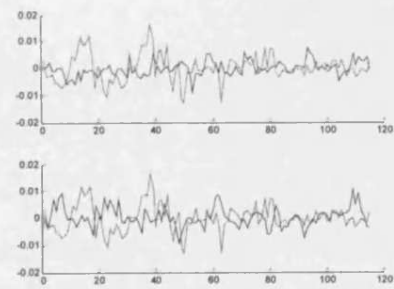
3.5.4 VAR(1) parameters for the sample of 1979.02-2003.02

Parameters	Actual	RI			NC		
		Lower	Upper	State	Lower	Upper	State
y_y	0.99	0.80	1.14	IN	0.51	0.87	OUT
y_r	-0.53	-0.30	0.89	OUT	-0.09	0.12	OUT
y_pi	-0.31	-0.29	0.04	OUT	-0.07	0.04	OUT
r_y	0.07	-0.30	-0.13	OUT	-0.19	0.37	IN
r_r	0.72	0.02	0.51	OUT	0.59	0.93	IN
r_pi	-0.10	-0.15	-0.03	IN	-0.09	0.09	OUT
pi_y	0.11	-0.14	0.23	IN	-1.55	0.02	OUT
pi_r	0.12	-0.80	0.44	IN	-0.16	0.72	IN
pi_pi	0.16	0.77	1.02	OUT	-0.17	0.32	IN
		M-metric=100%			M-metric=100%		
Parameters	Actual	LI			NI		
		Lower	Upper	State	Lower	Upper	State
y_y	0.99	0.58	1.07	IN	0.38	0.78	OUT
y_r	-0.53	-0.14	0.43	OUT	-1.12	0.39	IN
y_pi	-0.31	-0.67	-0.15	IN	-1.24	0.29	IN
r_y	0.07	-0.23	0.13	IN	-0.03	0.05	OUT
r_r	0.72	0.49	0.90	IN	0.59	0.86	IN
r_pi	-0.10	0.13	0.47	OUT	-0.01	0.28	OUT
pi_y	0.11	-0.39	-0.06	OUT	-0.08	0.03	OUT
pi_r	0.12	-0.57	-0.13	OUT	-0.34	0.09	OUT
pi_pi	0.16	0.74	1.01	OUT	0.12	0.57	IN
		M-metric=100%			M-metric=100%		

Table 3.4: VAR parameters for different models

Chapter 4

Overall Conclusions

In this thesis we look at Calvo contracts, which are the basis of the current generation of New Keynesian models, widely include indexation to general inflation. We argue that the indexing formula should be expected inflation rather than lagged inflation. This is likely to optimise the welfare of the representative agent in a general equilibrium model of the New Keynesian type. This is shown analytically for a simplified model and by numerical simulation for a full model with price and wage contracts as well as capital. The consequence of such indexation is that monetary policy no longer has any effect on welfare.This strong result however does not hold if all current shocks are observed or if there is wage-setting, but in this case, the Calvo contact does not provide a model of prolonged nominal rigidity so much as one of prolonged relative price rigidity combined with brief nominal rigidity.

The thesis also discusses the wider choice of optimal indexation in the New Keynesian model with Calvo contracts, where the indexation choice includes the possibility of partial indexation and of varying weights on rational and lagged indexation. It also investigates how the optimising choice would respond to the nature of monetary policy and what are its implications on the choice of monetary policy. It still finds that the Calvo contract adjusted for rationally expected indexation under both inflation and price targeting rules delivers the highest expected welfare under both restricted and full information about current shocks. Rational indexation eliminates the effectiveness of monetary policy on welfare when there is only price-level targeting under restricted current information. If we include both wage setting and full current information, monetary policy is effective, and a price-level targeting rule delivers the highest benefits because it minimises the size of shocks to prices. However, even less than full rational indexation ensures that there is very little nominal rigidity in the adapted world of Calvo contracts.

We use a new bootstrap method, developed by Minford et. al (2007), to test our models according to their dynamic performance. We found that the Calvo set up in the New Keynesian framework

is comprehensively rejected and so is the flexprice New Classical model. I hoped that the empirical evidience would support our theoretical conclusion of the superiority of the rational indexation over lagged indexation in Calvo contracts, but what we seem to have found is rather different: both forms of indexation were unlikely in the chosen sample period, and there is no evidence for the need to include substantial price rigidity in macroeconomic models.

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