Product Differentiation, the Volume of Trade and Profits under Cournot and Bertrand Duopoly*

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Abstract

This article analyses how product differentiation affects the volume of trade under duopoly using Shubik-Levitan demand functions rather than the Bowley demand functions used by Bernhofen (2001). The drawback of Bowley demand functions is that an increase in product differentiation increases the size of the market so the increase in the volume of trade may be the result of the increase in the size of the market rather than the increase in product differentiation *per se*. The Shubik-Levitan demand functions have the advantage that an increase in product differentiation does not increase the size of the market, but consumers still have a 'love of variety'. It is shown that the volume of trade in terms of quantities falls with increasing product differentiation when the trade cost is relatively low, but rises with increasing product differentiation when the trade cost is relatively high. Among the results, it is shown that the trade liberalisation is more likely to be profitable under Cournot duopoly than under Bertrand duopoly for differentiated products with a positive trade cost.

Keywords: Product Differentiation, Cournot Oligopoly, Bertrand Oligopoly.

JEL Classification: F12, F13, L13.

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1. Introduction

An important aspect of the positive theory of international trade is to explain the volume of trade between countries and its determinants. The 'new trade theory' explains intra-industry trade using monopolistic competition and oligopoly models that stress the role of imperfect competition, economies of scale and product differentiation. Conventional wisdom suggests that the volume of intra-industry trade is increasing in the degree of product differentiation, and this view is supported by the results of Bernhofen (2001). This paper will re-examine how product differentiation affects the volume of trade under oligopoly.

The most widely used measure of intra-industry trade is the Grubel-Lloyd index [see Grubel and Lloyd (1975)] that measures the proportion of international trade in an industry that is intra-industry trade. This has been extensively used in empirical work to demonstrate the extent of intra-industry trade, and to explain inter-industry variations in intra-industry trade in terms of variables such as scale economies, product differentiation, and market structure.[1] Generally, the econometric results suggest that product differentiation and scale economies have a positive effect on intra-industry trade as measured by the Grubel-Lloyd index or similar indices. However, this is not consistent with the theory since Ethier (1982), using a monopolistic competition model, showed that the Grubel-Lloyd index is invariant to changes in the degree of product differentiation and the extent of scale economies. For this reason, Harrigan (1994) argued that the implications of the monopolistic competition model should be tested by looking at the volume of trade rather than the Grubel-Lloyd index. He looked at the contribution of scale economies to the volume of trade and found some evidence that the volume of trade was higher in industries with large economies of scale. Schmitt and Yu (2001) provide a theoretical explanation for this result in a monopolistic competition model with traded and non-traded goods. Therefore, when looking at the effect of product differentiation on intra-industry trade, it seems prudent to analyse the effect on the volume of intra-industry trade rather than the effect on the Grubel-Lloyd index.[2]

Intra-industry trade under oligopoly was first analysed by Brander (1981) in a homogeneous product Cournot duopoly with segmented markets using linear demand functions. This analysis was extended to the case of general demand functions, but still with with homogeneous products, by Brander and Krugman (1983). Clarke and Collie (2003) considered the case of Bertrand duopoly with differentiated products using Bowley linear demand functions. Bernhofen (2001) analysed the effect of product differentiation on the volume of trade in a symmetric oligopoly model using Bowley linear demand functions and assuming that the trade cost is zero. He showed that the volume of trade (in terms of quantities and values) was increasing in the degree of product differentiation under Cournot oligopoly and under Bertrand oligopoly.[3] The drawback of Bowley demand functions is that an increase in product differentiation increases the size of the market so the increase in the volume of trade may be the result of the increase in the size of the market rather than the increase in product differentiation *per se*.

This article will re-examine how product differentiation affects the volume of trade under duopoly using Shubik-Levitan demand functions rather than the Bowley demand functions used by Bernhofen (2001), and assuming that the trade cost is positive rather than zero. The Shubik-Levitan demand functions have the advantage that an increase in product differentiation does not increase the size of the market as happens with the Bowley demand functions, but consumers still have a 'love of variety'. Without this market expansion effect from product differentiation, it is shown that the volume of trade in terms of quantities falls with increasing product differentiation when the trade cost is relatively low, but rises with increasing product differentiation when the trade cost is relatively high. When the trade cost is zero, the volume of trade in terms of values is decreasing in the degree of product differentiation if the marginal cost of the firms is sufficiently high. An alternative measure of the volume of trade is the market share of imports and this is increasing in the degree of product differentiation if the trade cost is positive with both Shubik-Levitan and Bowley demand functions. Qualitatively similar results are obtained under both Cournot duopoly and Bertrand duopoly, although it is shown that the volume of trade is higher under Bertrand duopoly than under Cournot duopoly except when the trade cost is relatively high.

Also, the paper extends the analysis of Anderson, Donsimoni, and Gabszewicz (1989) and Bernhofen (2001) about the profitability of trade liberalisation by considering the case of differentiated products under both Cournot and Bertrand duopoly with a positive trade cost. It is shown that trade liberalisation is more likely to be profitable under Cournot duopoly than under Bertrand duopoly.

2. The Volume of Trade under Cournot Duopoly

The model is similar to the symmetric two-country model considered by Bernhofen (2001) except for a different specification of the demand functions and the addition of a trade cost.[4] There are two identical countries (labelled *A* and *B*) with one firm located in each country. Firm one is located in country *A* and firm two is located in country *B*. The firms produce differentiated products and compete as Cournot duopolists in each market. The firms are identical and both have constant marginal cost c. Markets are assumed to be segmented and the firms incur a trade cost k when products are traded between the two countries. The output of firm one sold in its home market (country *A*) is x_{1A} and its exports to country *B* are x_{1B}, while the output of firm two sold in its home market (country *B*) is x_{2B} and its exports to country *A* are x_{2A}. The price of the product of firm one is p_{1A} in country *A* and p_{1B} in country B, while the price of the product of firm two is p_{2A} in country A and p_{2B} in country B. Since marginal cost is constant and markets are segmented, each market can be analysed independently.

Rather than using the Bowley specification of demand functions employed by Bernhofen (2001), the demand functions will be assumed to be the Shubik-Levitan specification of demand functions, although expressed in a manner that makes it easier to compare them with the Bowley demand functions. Preferences in country A can be represented by a quadratic quasi-linear utility function:

$$U = \alpha\left(x_{1A} + x_{2A}\right) - \frac{\beta}{2(1+\varphi)}\left(x_{1A}^2 + x_{2A}^2 + 2\varphi x_{1A}x_{2A}\right) + z \tag{1}$$

where z is a numeraire good produced by a competitive industry using constant returns to scale technology. Note that this utility function exhibits a 'love of variety' just like the Bowley demand functions used by Bernhofen (2001).[5] Utility maximisation yields the linear inverse demands facing the two oligopolistic firms in country A:

$$p_{1A} = \alpha - \frac{\beta}{1+\varphi}\left(x_{1A} + \varphi x_{2A}\right), \qquad p_{2A} = \alpha - \frac{\beta}{1+\varphi}\left(x_{2A} + \varphi x_{1A}\right) \tag{2}$$

The parameter α is the maximum willingness to pay of the consumers, β is inversely related to the size of the market ($1/\beta$ is proportional to the size of the market), and φ is the degree of product substitutability that ranges from $\varphi = 0$ when the products are independent to $\varphi = 1$ when the products are perfect substitutes. Note that the slope of the Shubik-Levitan inverse demand function, $\beta/(1+\varphi)$, depends upon the degree of product substitutability whereas the slope of the Bowley inverse demand function does not depend upon the degree of product substitutability. Since the model is symmetric, demand functions are the same for country *B*, but with a subscript *B* rather than a subscript *A*.

An increase in the degree of product differentiation does not affect the total size of the market with Shubik-Levitan demand functions, whereas an increase in product differentiation increases the size of the market with Bowley demand functions. With Shubik-Levitan demand functions, if the price of both products is $p_{1A} = p_{2A} = \bar{p}$ then total consumption is: $x_{1A} + x_{2A} = 2(\alpha - \bar{p})/\beta$, which is independent of the degree of product substitutability, φ. With Bowley demand functions, where the inverse demand facing firm one in country *A* is: $p_{1A} = \alpha - \beta(x_{1A} + \varphi x_{2A})$ and the demand facing firm two in country *A* is: $p_{2A} = \alpha - \beta(x_{2A} + \varphi x_{1A})$, if the price of both products is $p_{1A} = p_{2A} = \bar{p}$ then total consumption is: $x_{1A} + x_{2A} = 2(\alpha - \bar{p})/\beta(1+\varphi)$, which is decreasing in the degree of product substitutability so an increase in product differentiation increases the size of the market.[6]

Under international trade, in country *A*, the profits of the domestic firm (firm one) are $\pi_{1A} = (p_{1A} - c)x_{1A}$, and the profits of the foreign firm (firm two) are $\pi_{2A} = (p_{2A} - c - k)x_{2A}$. Using the demand functions in (2), it is straightforward to solve for the Cournot equilibrium outputs of the two firms in the two countries:

$$
\begin{aligned}
x_{1A}^C = x_{2B}^C &= \begin{cases} \dfrac{(1+\varphi)\left[k\varphi + (\alpha - c)(2-\varphi)\right]}{\beta\left(4-\varphi^2\right)} & \text{for } 0 \le k < \bar{k} \equiv \dfrac{2-\varphi}{2}(\alpha - c) \\ \dfrac{(\alpha - c)(1+\varphi)}{2\beta} & \text{for } k \ge \bar{k} \end{cases} \\
x_{2A}^C = x_{1B}^C &= \begin{cases} \dfrac{(1+\varphi)\left[(\alpha - c)(2-\varphi) - 2k\right]}{\beta\left(4-\varphi^2\right)} & \text{for } 0 \le k < \bar{k} \\ 0 & \text{for } k \ge \bar{k} \end{cases}
\end{aligned}
\tag{3}
$$

There will be no trade between the two countries if the trade cost is higher than: $k \geq \overline{k}$, where $\overline{k}$ is the prohibitive trade cost. Clearly, the prohibitive trade cost is decreasing in the degree of product substitutability so intra-industry trade is more likely to occur the higher is the degree of product differentiation.[7]

Substituting (3) into (2) and solving for the Cournot equilibrium prices of the two firms in the two countries yields:

$$p_{1A}^C = p_{2B}^C = \begin{cases} c + \dfrac{(2-\varphi)(\alpha - c) + k\varphi}{4-\varphi^2} & \text{for } 0 \leq k < \overline{k} \\ \dfrac{(\alpha + c)}{2} & \text{for } k \geq \overline{k} \end{cases}$$
$$p_{2A}^C = p_{1B}^C = \begin{cases} c + k + \dfrac{(2-\varphi)(\alpha - c) - 2k}{4-\varphi^2} & \text{for } 0 \leq k < \overline{k} \\ \dfrac{(2-\varphi)\alpha + \varphi c}{2} & \text{for } k \geq \overline{k} \end{cases} \tag{4}$$

The total volume of intra-industry trade between the two countries measured in terms of physical quantities is given by: $V_Q^C = x_{2A}^C + x_{1B}^C$. Assuming that the trade cost is below the prohibitive level, $k < \overline{k}$, the effect of a change in the degree of product substitutability on the volume of trade can be seen by differentiating V_Q^C with respect to φ using (3), which yields:

$$\frac{\partial V_Q^C}{\partial \varphi} = \frac{\partial x_{2A}^C}{\partial \varphi} + \frac{\partial x_{1B}^C}{\partial \varphi} = 2\frac{(\alpha - c)(2-\varphi)^2 - 2k\left(4 + 2\varphi + \varphi^2\right)}{\beta\left(4-\varphi^2\right)^2} \tag{5}$$

This is positive (negative) if the relative trade cost is less (greater) than the critical value $\kappa_Q^C \equiv k_Q^C/(\alpha - c) \equiv (2-\varphi)^2/2\left(4 + 2\varphi + \varphi^2\right)$, which is shown in figure one as a function of the degree of product substitutability, φ. Therefore, in contrast to Bernhofen (2001), the volume of trade decreases with the degree of product differentiation when the trade cost is sufficiently low (in the region below the critical trade cost in figure one). The explanation is that an increase in product differentiation lessens competition between the two firms with the result that both firms reduce output thereby reducing the volume of trade. Whereas, in Bernhofen (2001), the effect of lessening competition was outweighed by the increase in market size due to the increase in product differentiation with the result that the outputs of both firms increased. When the trade cost is sufficiently high (in the region above the critical trade cost in figure one), the protective effect of the trade cost afforded to the domestic firm is reduced by an increase in product differentiation with the result that the output of the foreign firm increases thereby increasing the volume of trade.

Alternatively, the volume of trade could also be measured in terms of value rather than physical quantities, which is given by: $V_V^C \equiv p_{2A}^C x_{2A}^C + p_{1B}^C x_{1B}^C$. For simplicity, as in Bernhofen (2001), consider only the case when the trade cost is zero. Then, to see how the volume of trade in terms of value varies with the degree of product substitutability, differentiate V_V^C with respect to φ using (3) and (4), which yields:

$$\begin{aligned}\frac{\partial V_V^C}{\partial \varphi} &= p_{2A}^C \frac{\partial x_{2A}^C}{\partial \varphi} + x_{2A}^C \frac{\partial p_{2A}^C}{\partial \varphi} + p_{1B}^C \frac{\partial x_{1B}^C}{\partial \varphi} + x_{1B}^C \frac{\partial p_{1B}^C}{\partial \varphi} \\ &= \frac{2(\alpha - c)}{\beta(2+\varphi)^3}\left[2c(1+\varphi) - \alpha\varphi\right]\end{aligned} \tag{6}$$

This will be positive (negative) if the marginal cost is greater (lower) than the critical value $c_V^C \equiv \alpha\varphi/2(1+\varphi)$. Since $\varphi \in [0,1]$, a sufficient condition for the derivative (6) to be positive for all values of the degree of product substitutability is that $c > \alpha/4$.[8] Again, in contrast to Bernhofen (2001), the volume of intra-industry trade in terms of value is decreasing in the degree of product differentiation. The explanation is that the effect of product differentiation on the volume of trade in terms of value consists of a quantity effect, which was shown to be negative when $k = 0$, and a price effect, which is positive since $\partial p_{2A}^{C} / \partial \varphi = \partial p_{1B}^{C} / \partial \varphi > 0$, so the overall effect is ambiguous. However, since the effect of product differentiation on price is solely due to the effect on the price-cost margin, see (4), if the marginal cost is sufficiently high then the proportional change in the price will be small and the quantity effect will outweigh the price effect.

A different measure of the extent of international trade that takes account of market size is import penetration or the market share of imports, which measures the volume of trade relative to the size of the market. The market share of imports in terms of physical quantities is given by imports as a proportion of total consumption:

$$M_Q^C \equiv \frac{x_{2A}^C}{x_{1A}^C + x_{2A}^C} = \frac{(2-\varphi)(\alpha - c) - 2k}{(2-\varphi)\left[2(\alpha - c) - k\right]} \tag{7}$$

To see how the market share of imports depends upon the degree of product substitutability, differentiate (7) with respect to φ using (3), which yields:

$$\frac{\partial M_Q^C}{\partial \varphi} = -\frac{2k}{\left[2(\alpha - c) - k\right](2-\varphi)^2} < 0 \tag{8}$$

The market share of imports is always decreasing in the degree of product substitutability if the trade cost is positive so it is increasing in the degree of product differentiation. When the trade cost is zero, the market share of imports is one-half regardless of the degree of product substitutability. It can be shown that the market share of imports is the same with Bowley demand functions as with Shubik-Levitan demand functions. Therefore, the market share of imports provides a robust measure that does not depend upon the specification of demand.

These results are summarised in the following proposition:

***Proposition 1**: In a symmetric Cournot duopoly with Shubik-Levitan demand functions: (i) the volume of trade in terms of quantities is increasing in the degree of product substitutability if the trade cost $k < k_Q^C$; (ii) when the trade cost is zero, $k = 0$, the volume of trade in terms of value is everywhere increasing in the degree of product substitutability if $c > \alpha/4$; (iii) when the trade cost is positive, $k > 0$, the market share of imports is decreasing in the degree of product substitutability.*

With Shubik-Levitan demand functions, the volume of trade in terms of quantities and values may decrease with greater product differentiation whereas it increases with Bowley demand functions. When the trade cost is relatively high, conventional wisdom is restored since the volume of trade in terms of quantities is increasing in the degree of product differentiation. The market share of imports is also increasing in the degree of product differentiation with Shubik-Levitan demand functions and with Bowley demand functions.

3. The Volume of Trade under Bertrand Duopoly

As concern is often expressed about the robustness of results in models of trade under oligopoly, and to allow a comparison between Cournot duopoly and Bertrand duopoly, this section will analyse trade volume under Bertrand duopoly rather than Cournot duopoly with Shubik-Levitan demand functions. The results of Bernhofen (2001) about the volume of trade with Bowley demand functions held under both Cournot and Bertrand duopoly. The model is the same as Clarke and Collie (2003) except for the specification of the demand function.[9] Inverting the indirect demand functions in (2) yields the direct demand functions in country *A* facing firm one and firm two:

$$x_{1A} = \frac{1}{\beta(1-\varphi)}\left[\alpha(1-\varphi) - p_{1A} + \varphi p_{2A}\right], \qquad x_{2A} = \frac{1}{\beta(1-\varphi)}\left[\alpha(1-\varphi) + \varphi p_{1A} - p_{2A}\right] \quad (9)$$

Following Clarke and Collie (2003) and allowing for the possibility of corner solutions, it is straightforward to solve for the Bertrand equilibrium prices of the two firms in the two countries:

$$p_{1A}^B = p_{2B}^B = \begin{cases} c + \dfrac{k\varphi + (\alpha - c)(2 - \varphi^2 - \varphi)}{(4 - \varphi^2)} & \text{for } 0 \le k < \underline{k} \equiv \dfrac{(\alpha - c)(2 - \varphi - \varphi^2)}{2 - \varphi^2} \\ c + \dfrac{1}{\varphi}\left[k - (1 - \varphi)(\alpha - c)\right] & \text{for } \underline{k} < k < \overline{k} \equiv \dfrac{2 - \varphi}{2}(\alpha - c) \\ \dfrac{(\alpha + c)}{2} & \text{for } k \ge \overline{k} \end{cases} \tag{10}$$

$$p_{2A}^B = p_{1B}^B = \begin{cases} c + k + \dfrac{(\alpha - c)(2 - \varphi^2 - \varphi) - (2 - \varphi^2)k}{(4 - \varphi^2)} & \text{for } 0 \le k < \underline{k} \\ c + k & \text{for } k \ge \underline{k} \end{cases}$$

Substituting these prices into the direct demand functions (9) yields the sales of the two firms in the two countries:

$$x_{1A}^B = x_{2B}^B = \begin{cases} \dfrac{k\varphi + (\alpha - c)(2 - \varphi^2 - \varphi)}{\beta(1 - \varphi)(4 - \varphi^2)} & \text{for } 0 \le k < \overline{k} \\ \dfrac{(1 + \varphi)(\alpha - c - k)}{\beta\varphi} & \text{for } \underline{k} < k < \overline{k} \\ \dfrac{(\alpha - c)(1 + \varphi)}{2\beta} & \text{for } k \ge \overline{k} \end{cases} \tag{11}$$

$$x_{2A}^B = x_{1B}^B = \begin{cases} \dfrac{(\alpha - c)(2 - \varphi^2 - \varphi) - k(2 - \varphi^2)}{\beta(1 - \varphi)(4 - \varphi^2)} & \text{for } 0 \le k < \underline{k} \\ 0 & \text{for } k \ge \underline{k} \end{cases}$$

The domestic firm in each country can set the monopoly price if the trade cost $k > \overline{k}$, but there will be no imports if the trade cost $k > \underline{k}$. For $\underline{k} < k < \overline{k}$, there are no imports but the presence of the foreign firm has a pro-competitive effect on the price set by the domestic firm. The prohibitive trade cost, $\underline{k}$, is decreasing in the degree of product substitutability so, as in the case of Cournot duopoly, intra-industry trade is more likely to occur the higher is the degree of product differentiation.

As in the case of Cournot duopoly, the volume of intra-industry trade measured in terms of physical quantities is given by: $V_Q^B = x_{2A}^B + x_{1B}^B$. To see how the volume of trade varies with the degree of product substitutability, differentiate V_Q^B with respect to φ using (11), which yields:

$$\frac{\partial V_Q^B}{\partial \varphi} = \frac{\partial x_{2A}^B}{\partial \varphi} + \frac{\partial x_{1B}^B}{\partial \varphi} = 2\frac{(\alpha - c)(2-\varphi-\varphi^2) - k(8-4\varphi-2\varphi^2+\varphi^4)}{\beta(1-\varphi)^2(4-\varphi^2)^2} \tag{12}$$

This is positive (negative) if the relative trade cost is less (greater) than the critical value $\kappa_Q^B \equiv k_Q^B/(\alpha - c) \equiv (2-\varphi-\varphi^2)/(8-4\varphi-2\varphi^2+\varphi^4)$, which is shown in figure two as a function of the degree of product substitutability, φ. Therefore, in contrast to Bernhofen (2001), the volume of trade decreases with the degree of product differentiation when the trade cost is sufficiently low (in the region below the critical trade cost in figure two).

Alternatively, the volume of trade could also be measured in terms of value rather than physical quantities, which is given by: $V_V^B \equiv p_{2A}^B x_{2A}^B + p_{1B}^B x_{1B}^B$. For simplicity, as in Bernhofen (2001), consider only the case when the trade cost is zero. Then, to see how the volume of trade in terms of value varies with the degree of product substitutability, differentiate V_V^B with respect to φ using (10) and (11), which yields:

$$\begin{aligned}\frac{\partial V_V^B}{\partial \varphi} &= p_{2A}^B \frac{\partial x_{2A}^B}{\partial \varphi} + x_{2A}^B \frac{\partial p_{2A}^B}{\partial \varphi} + p_{1B}^B \frac{\partial x_{1B}^B}{\partial \varphi} + x_{1B}^B \frac{\partial p_{1B}^B}{\partial \varphi} \\ &= \frac{2(\alpha - c)}{\beta(2-\varphi)^3}[2c - \alpha\varphi]\end{aligned} \tag{13}$$

This will be positive (negative) if the marginal cost is greater (lower) than the critical value $c_V^B \equiv \alpha\varphi/2$. Since $\varphi \in [0,1]$, a sufficient condition for the derivative (13) to be positive for all values of the degree of product substitutability is that $c > \alpha/2$. Then, in contrast to Bernhofen (2001), the volume of intra-industry trade in terms of value is decreasing in the degree of product differentiation.

The market share of imports in terms of physical quantities under Bertrand duopoly, when $k < \underline{k}$, is given by:

$$M_Q^B \equiv \frac{x_{2A}^B}{x_{1A}^B + x_{2A}^B} = \frac{\left(2-\varphi-\varphi^2\right)\left(\alpha-c\right)-k\left(2-\varphi^2\right)}{\left(2-\varphi-\varphi^2\right)\left[2\left(\alpha-c\right)-k\right]} \tag{14}$$

When $k \geq \underline{k}$, imports are zero under Bertrand duopoly so the market share of imports is zero.

To see how the market share of imports depends upon the degree of product substitutability, differentiate (7) with respect to φ using (3) yields:

$$\frac{\partial M_Q^B}{\partial \varphi} = -\frac{k\left(2+\varphi^2\right)}{\left(2-\varphi-\varphi^2\right)^2\left[2\left(\alpha-c\right)-k\right]} < 0 \tag{15}$$

The market share of imports is always decreasing in the degree of product substitutability if the trade cost is positive so it is increasing in the degree of product differentiation.

These results are summarised in the following proposition:

Proposition 2: *In a symmetric Bertrand duopoly with Shubik-Levitan demand functions: (i) the volume of trade in terms of quantities is increasing in the degree of product substitutability if the trade cost* $k < k_Q^B$*; (ii) when the trade cost is zero,* $k = 0$*, the volume of trade in terms of value is everywhere increasing in the degree of product substitutability if*

$c > \alpha/2$; *(iii) when the trade cost is positive,* $k > 0$*, the market share of imports is decreasing in the degree of product substitutability.*

The results under Bertrand duopoly are qualitatively similar to the results under Cournot duopoly so the results are robust with regards to market structure. The difference between the results presented here and those of Bernhofen (2001) is the specification of the demand functions.

To see how the volume of trade in terms of physical quantities depends upon market structure, subtract the volume of trade under Cournot duopoly, V_Q^C, from the volume of trade under Bertrand duopoly, V_Q^B, which yields:

$$V_Q^B - V_Q^C = \frac{2\varphi^2}{\beta(1-\varphi)(4-\varphi^2)}\left[(1-\varphi)(\alpha-c)-k\right] \tag{16}$$

Clearly, this is positive (negative) if the relative trade cost is less (greater) than the critical value $\kappa_Q^\Delta \equiv k_Q^\Delta/(\alpha-c) \equiv (1-\varphi)$, which is shown in figure three as a function of the degree of product substitutability, φ.[10] The volume of trade is larger (smaller) under Bertrand duopoly than under Cournot duopoly in the region below (above) the critical relative trade cost.

Subtracting the market share of imports under Bertrand duopoly (14) from the market share of imports under Cournot duopoly (7), when $k < \underline{k}$, yields:

$$M_Q^C - M_Q^B = \frac{\varphi^3 k}{(2-\varphi)(2-\varphi-\varphi^2)\left[2(\alpha-c)-k\right]} > 0 \tag{17}$$

If $\underline{k} \le k < \overline{k}$ then the market share of imports is zero under Bertrand duopoly but positive under Cournot duopoly so $M_Q^C > M_Q^B$. Hence, if the trade cost is positive then the market share of imports is higher under Cournot duopoly than under Bertrand duopoly. These results lead to the following proposition:

Proposition 3: *(i) The volume of trade in terms of physical quantities is larger (smaller) under Bertrand duopoly than under Cournot duopoly,* $V_Q^B > (<) V_Q^C$*, if the trade cost is less (greater) than* k_Q^{Δ}*. (ii) The market share of imports is higher under Cournot duopoly than under Bertrand duopoly if the relative trade cost is positive.*

The Bertrand duopoly model yields qualitatively similar results to the Cournot duopoly model, although there are quantitative differences as spelled out in Proposition 3, so the results can be considered to be fairly robust.

4. Trade Liberalisation and Profits

Another issue addressed by Bernhofen (2001) was the effect of trade liberalisation on the profitability of firms when products are differentiated, which extended the analysis of Anderson, Donsimoni, and Gabszewicz (1989) who considered primarily the case of homogeneous products. The analysis will be extended further in this section by considering the role of trade costs together with product differentiation on the profitability of trade liberalisation, and by comparing the profitability of trade liberalisation under Cournot duopoly and Bertrand duopoly.

The total profits of the firms are the sum of profits from the domestic market plus the profits from the export market so the total profits of firm one are: $\Pi_1 = \pi_{1A} + \pi_{1B}$, and for firm two are: $\Pi_2 = \pi_{2A} + \pi_{2B}$. From (3) and (4), the total profits of the two firms under Cournot duopoly are: $\Pi_1^C = \Pi_2^C =$

$$\begin{cases} \dfrac{(1+\varphi)}{\beta(4-\varphi^2)^2}\left[\{(2-\varphi)(\alpha-c)+k\varphi\}^2+\{(2-\varphi)(\alpha-c)-2k\}^2\right] & \text{for } 0\le k<\bar{k} \\ \dfrac{(\alpha-c)^2(1+\varphi)}{4\beta} & \text{for } k\ge\bar{k} \end{cases} \tag{18}$$

The case when the trade cost is prohibitive, $k\ge\bar{k}$, is equivalent to autarky as there is no trade and the domestic firm in each market can set the monopoly price. Intra-industry trade will occur between the two countries if the trade cost is less than the prohibitive level. Comparing total profits under international trade, $0\le k<\bar{k}$, with total profits under autarky, $k\ge\bar{k}$, it can be shown that profits are higher under international trade if the relative trade cost is less than the critical value:

$$\kappa_{\Pi}^{C}\equiv\frac{k_{\Pi}^{C}}{\alpha-c}\equiv\frac{(2-\varphi)}{2(4+\varphi^2)}\left(4-4\varphi-\varphi^2\right) \tag{19}$$

The critical value is shown in figure four as a function of the degree of product substitutability together with the prohibitive trade cost. It is decreasing in the degree of product substitutability and it is positive if $\varphi<\varphi_{\Pi}^{C}=2\left(\sqrt{2}-1\right)\approx 0\cdot 83$. Therefore, trade liberalisation is profitable if the trade cost is relatively low and the products are sufficiently differentiated, and it is always unprofitable if the products are sufficiently close substitutes, $\varphi>\varphi_{\Pi}^{C}$.

From (10) and (11), the total profits of the two firms under Bertrand duopoly are: $\Pi_1^B=\Pi_2^B=$

$$\begin{cases} \dfrac{\left[\left(2-\varphi-\varphi^2\right)\left(\alpha-c\right)+k\varphi\right]^2+\left[\left(2-\varphi-\varphi^2\right)\left(\alpha-c\right)-k\left(2-\varphi^2\right)\right]^2}{\beta\left(4-\varphi^2\right)^2\left(1-\varphi\right)} & \text{for } 0\le k<\underline{k} \\ \dfrac{\left(1+\varphi\right)\left[k-\left(1-\varphi\right)\left(\alpha-c\right)\right]\left(\alpha-c-k\right)}{\beta\varphi^2} & \text{for } \underline{k}\le k<\overline{k} \\ \dfrac{\left(\alpha-c\right)^2\left(1+\varphi\right)}{4\beta} & \text{for } k\ge\overline{k} \end{cases} \quad (20)$$

Again, the case when the trade cost is prohibitive, $k \ge \overline{k}$, is equivalent to autarky as there is no trade and the domestic firm in each market can set the monopoly price. When $\underline{k} \le k < \overline{k}$, there is no trade but the presence of the foreign firm has a pro-competitive effect on the price set by the domestic firm. As a result, the profits of the domestic firm are reduced and, since it makes zero profits from exports to the foreign market, trade liberalisation has reduced the total profits of both firms. When $0 \le k < \underline{k}$, intra-industry trade will occur between the two countries. Comparing total profits under international trade, $0 \le k < \underline{k}$, with total profits under autarky, $k \ge \overline{k}$, it can be shown that profits are higher under international trade if the trade cost is less than the critical value:

$$\kappa_\Pi^B \equiv \frac{k_\Pi^B}{\left(\alpha-c\right)} \equiv \frac{2\left(2-\varphi-\varphi^2\right)^2-\varphi\left(4-\varphi^2\right)\sqrt{\left(1-\varphi^4\right)}}{2\left(4-3\varphi^2+\varphi^4\right)} \quad (21)$$

The critical trade cost is shown in figure four as a function of the degree of product substitutability. It is decreasing in the degree of product substitutability and it is positive if $\varphi < \varphi_\Pi^B = 0\cdot 61$. Comparing the critical values of the trade cost in figure four, it can be seen that trade liberalisation is profitable for a larger parameter set in the case of Cournot duopoly than in the case of Bertrand duopoly. In the region labelled ***S*** in figure four, trade liberalisation is profitable under Cournot duopoly whereas it is unprofitable under Bertrand duopoly. This may not seem surprising given the result of Singh and Vives (1984) that industry profits are higher under Cournot duopoly than under Bertrand duopoly. However, recently Zanchettin (2006) showed that industry profits may be higher under Bertrand duopoly than under Cournot duopoly when there is a cost asymmetry as in the corner solution considered by Clarke and Collie (2003), i.e. in the region where $\underline{k} \le k < \overline{k}$.[11] Comparing international trade profits when $\underline{k} \le k < \overline{k}$, using (18) and (20), it can be shown that international trade profits will be higher under Bertrand duopoly than under Cournot duopoly if the relative trade cost is greater than the critical value:

$$\kappa_Z \equiv \frac{k_Z}{\alpha - c} \equiv \frac{(2-\varphi)}{8 - 2\varphi^2 + \varphi^4}\left(4 - \varphi^2 - \varphi^3\right) \tag{22}$$

This critical value is shown in figure four as a function of the degree of product substitutability, and international trade profits are higher under Bertrand duopoly than under Cournot duopoly in the region labelled ***Z***. However, international trade is never more profitable than autarky in this region so it does not change the conclusion that trade liberalisation is more likely to be profitable under Cournot duopoly than under Bertrand duopoly.

These results are summarised in the following proposition:

Proposition 4: *Total profits of the two firms are higher under international trade than under autarky if the trade cost* $k < k_\Pi^C$ *under Cournot duopoly and if the trade cost* $k < k_\Pi^B$ *under Bertrand duopoly. International trade is more likely to be profitable under Cournot duopoly than under Bertrand duopoly since* $k_\Pi^B < k_\Pi^C$.

These results were derived using the Shubik-Levitan demand functions, but the results would be unchanged with Bowley demand functions as these results are obtained by comparing profits under international trade with those under autarky for a given degree of product differentiation.

5. Conclusions

This article has analysed how product differentiation affects the volume of trade under duopoly using Shubik-Levitan demand functions rather than the Bowley demand functions used by Bernhofen (2001). The drawback of Bowley demand functions is that an increase in product differentiation increases the size of the market so the increase in the volume of trade may be the result of the increase in the size of the market rather than the increase in product differentiation *per se*. The Shubik-Levitan demand functions have the advantage that an increase in product differentiation does not increase the size of the market, but consumers still have a 'love of variety'. Without this market expansion effect from product differentiation, it was shown that the volume of trade in terms of quantities falls with increasing product differentiation when the trade cost is relatively low, but rises with increasing product differentiation when the trade cost was relatively high. When the trade cost is zero, the volume of trade in terms of values is decreasing in the degree of product differentiation if the marginal cost of the firms is sufficiently high. An alternative measure of the volume of trade is the market share of imports and this is increasing in the degree of product differentiation if the trade cost is positive with both Shubik-Levitan and Bowley demand functions. Qualitatively similar results are obtained both under Cournot duopoly and under Bertrand duopoly, although there are quantitative differences, so the results can be considered to be fairly robust. Also, international trade was shown to be more likely to be profitable under Cournot duopoly than under Bertrand duopoly.

References

Anderson, Simon, Marie-Paule Donsimoni, and Jean J. Gabszewicz. 1989. "Is International Trade Profitable to Oligopolistic Industries?" *International Economic Review* 30 (4):725-733.

Bernhofen, Daniel M. 2001. "Product Differentiation, Competition and International Trade." *Canadian Journal of Economics* 34 (4):1010-1023.

Brander, J., and P. Krugman. 1983. "A Reciprocal Dumping Model of International-Trade." *Journal of International Economics* 15 (3-4):313-321.

Brander, James A. 1981. "Intra-Industry Trade in Identical Commodities." *Journal of International Economics* 11 (1):1-14.

Clarke, Roger, and David R. Collie. 2003. "Product differentiation and the gains from trade under Bertrand duopoly." *Canadian Journal of Economics* 36 (3):658-673.

Ethier, Wilfred J. 1982. "National and International Returns to Scale in the Modern Theory of International Trade." *The American Economic Review* 72 (3):389-405.

Greenaway, David, and Chris Milner. 1986. *The Economics of Intra-Industry Trade*. Oxford: Basil Blackwell.

Grubel, Herbert G., and Peter J. Lloyd. 1975. *Intra-Industry Trade: The Theory and Measurement of International Trade in Differentiated Products*. London: Macmillan.

Harrigan, James. 1994. "Scale Economies and the Volume of Trade." *The Review of Economics and Statistics* 76 (2):321-328.

Leahy, Dermot, and J. Peter Neary. 2011. "Oligopoly and Trade." In *Palgrave Handbook of International Trade*, edited by Daniel M. Bernhofen, Rodney Falvey, David Greenaway and Udo Kreickemeier. Basingstoke: Palgrave Macmillan.

Martin, Stephen. 2002. *Advanced Industrial Economics*. Second ed. Oxford: Blackwell.

Schmitt, Nicolas, and Zhihao Yu. 2001. "Economies of scale and the volume of intra-industry trade." *Economics Letters* 74 (1):127-132. doi: 10.1016/s0165-1765(01)00519-5.

Singh, N., and X. Vives. 1984. "Price and Quantity Competition in a Differentiated Duopoly." *Rand Journal of Economics* 15 (4):546-554.

Zanchettin, Piercarlo. 2006. "Differentiated Duopoly with Asymmetric Costs." *Journal of Economics & Management Strategy* 15 (4):999-1015. doi: 10.1111/j.1530-9134.2006.00125.x.

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[1] See Greenaway and Milner (1986) for a detailed explanation of the Grubel-Lloyd index and a survey of early empirical work on inter-industry variations in intra-industry trade.

[2] In any symmetric monopolistic or oligopolistic competition model, the Grubel-Lloyd index will be equal to one and obviously will not vary with the degree of product differentiation.

[3] For a recent survey of the literature on oligopoly and trade that considers Cournot duopoly and Bertrand duopoly with differentiated products in a unified framework see Leahy and Neary (2011).

[4] Basically, apart from the specification of the demand functions, the model is the same as Brander (1981) or Brander and Krugman (1983).

[5] If the consumer consumes two units of one variety and none of the other variety then utility is: $U(2,0)=U(0,2)=2\alpha-2\beta/(1+\varphi)+z$ whereas if the consumer consumes one unit of each variety then utility is: $U(1,1)=2\alpha-\beta+z$ so the utility function exhibits a 'love of variety' since $U(1,1)-U(2,0)=\beta(1-\varphi)/(1+\varphi)>0$ if the products are differentiated, $0\leq\varphi<1$.

[6] For a discussion of the relationship between the Bowley and Shubik-Levitan specifications of demand functions see chapter three of Martin (2002).

[7] This may explain the positive relationship between product differentiation and the Grubel-Lloyd index of intra-industry trade found in empirical studies. Intra-industry trade is more likely to occur when products are highly differentiated as the prohibitive trade cost is high.

[8] This is an example of a situation where the common practice of normalising the marginal cost of the firms at zero is not an innocuous assumption.

[9] Clarke and Collie (2003) use Bowley demand functions rather than Shubik-Levitan demand functions, but their results about the gains from trade would still hold with Shubik-Levitan demand functions as they compare welfare under free trade with welfare under autarky for a given degree of product substitutability.

[10] This follows from the result in lemma 2 of Zanchettin (2006) about the inefficient firm in a closed economy since the foreign firm is analogous to the inefficient firm due to the trade cost.

[11] Although Zanchettin (2006) considers a closed economy, industry profits in the closed economy are equal to free trade profits of a firm in this symmetric model. The relevant result is Proposition 2, where it should be noted that the parameter *a* is equal to the relative trade cost in this model.

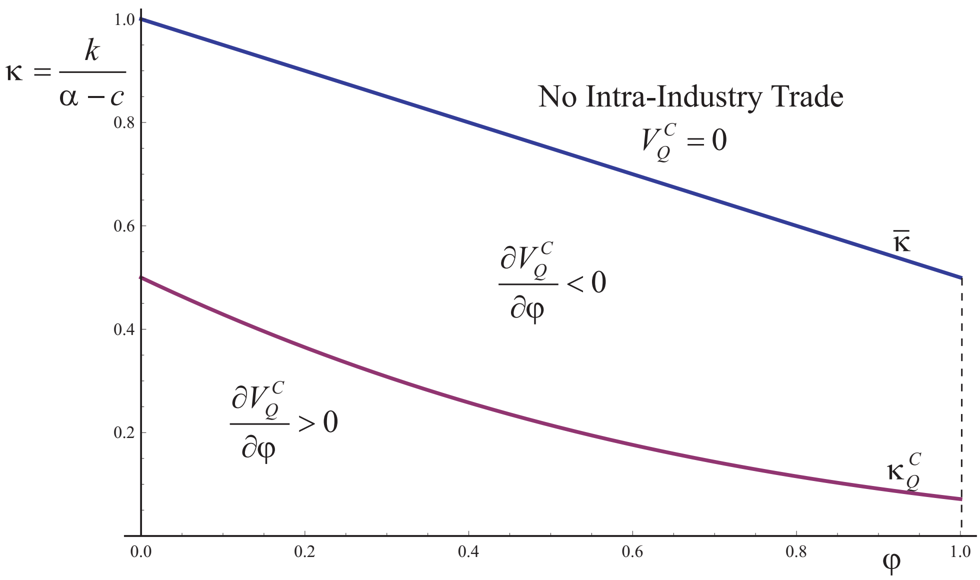

Figure 1: Critical Trade Cost under Cournot Duopoly

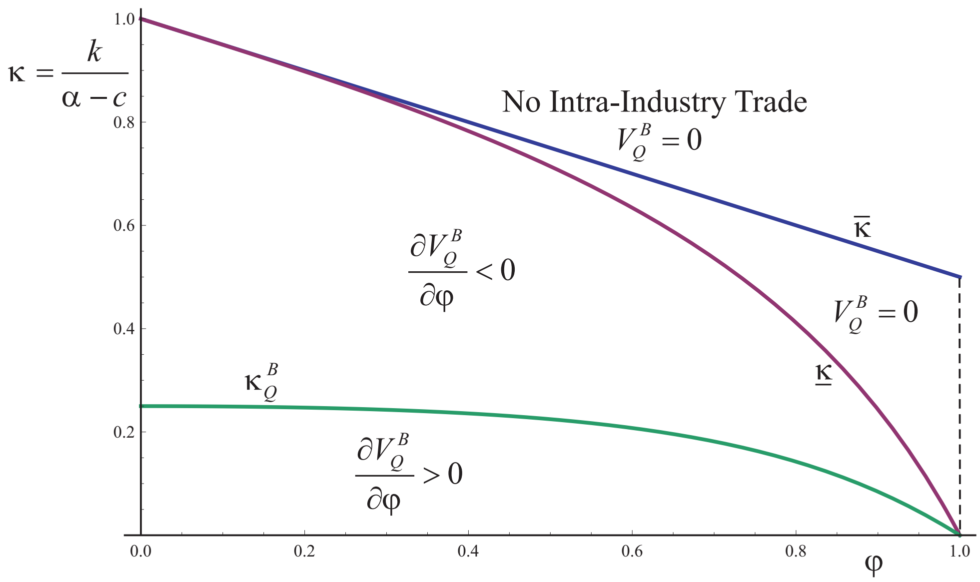

Figure 2: Critical Trade Cost under Bertrand Duopoly

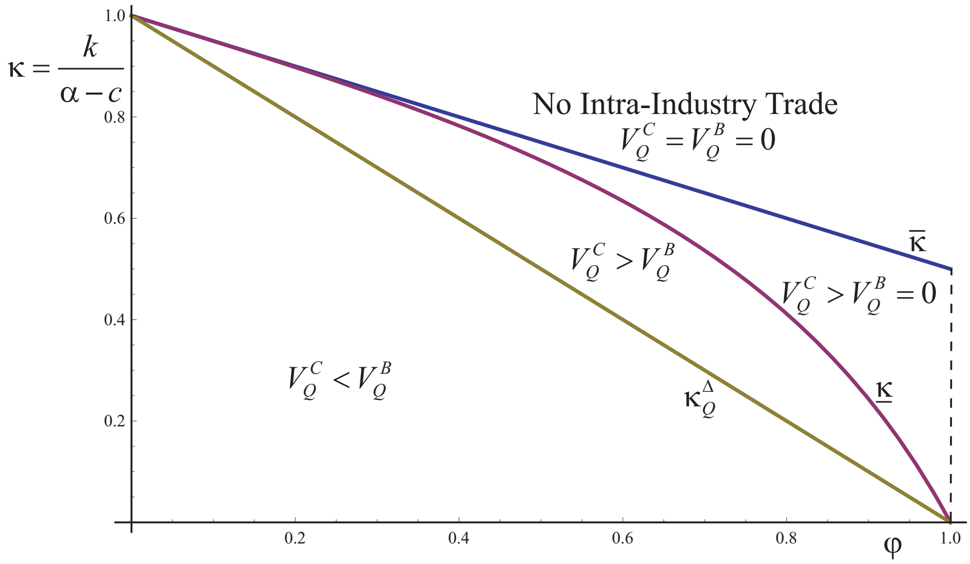

Figure 3: Comparison of the Volume of Trade under Cournot and Bertrand Duopoly

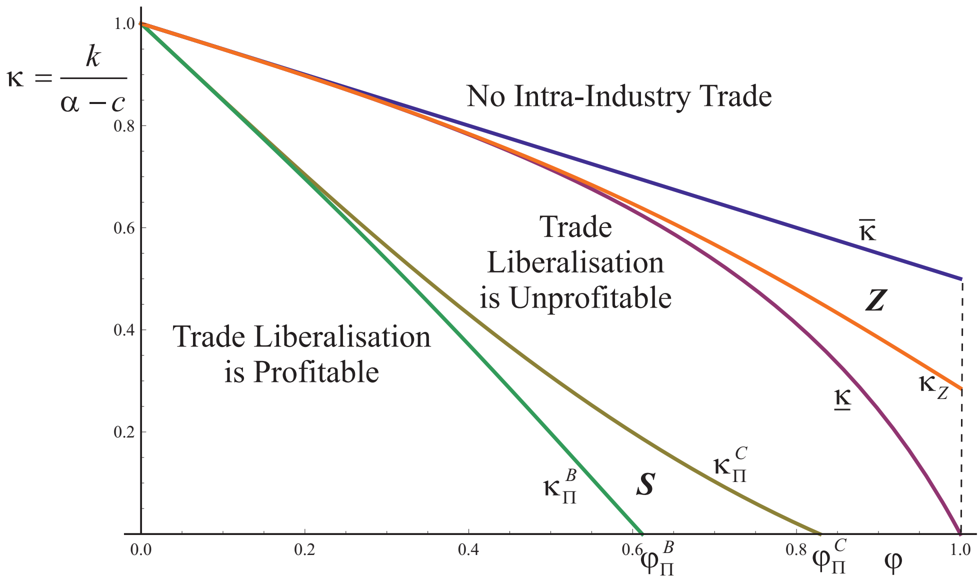

Figure 4: Profitability of Trade Liberalisation under Cournot and Bertrand Duopoly