The final publication is available at Springer via http://dx.doi.org/10.1007/s11242-014-0395-3

Revised September 2014

Title: Analytical solutions for ground temperature profiles and stored energy using meteorological data

Authors: Peter John Cleall[1], José Javier Muñoz-Criollo[2], Stephen William Rees[3]

Affiliations:

[1] Corresponding Author and Senior Lecturer, Cardiff School of Engineering, Cardiff University, Cardiff, CF24 3AA, Wales, UK. E-mail: Cleall@cardiff.ac.uk

[2] Post Graduate Research Student, Cardiff School of Engineering, Cardiff University, Cardiff, CF24 3AA, Wales, UK

[3] Lecturer, Cardiff School of Engineering, Cardiff University, Cardiff, CF24 3AA, Wales, UK. E-mail: ReesS@cardiff.ac.uk

Corresponding Author: Dr Peter Cleall
Cardiff School of Engineering
Cardiff University
Cardiff, CF24 3AA
Wales, UK
E-mail: cleall@cardiff.ac.uk
Tel. 029 20875795
Fax. 029 20874004

Abstract

Analytical solutions to estimate temperature with depth and stored energy within a soil column based upon readily available meteorological data are presented in this paper which are of particular relevance in the field of ground heat extraction and storage. The transient one-dimensional heat diffusion equation is solved with second kind (Neumann) boundary conditions at the base and third kind (Robin) boundary conditions, based on a heat balance, at the soil surface. In order to describe the soil-atmosphere interactions, mathematical expressions describing the daily and annual variation of solar radiation and air temperature are proposed. The presented analytical solutions are verified against a numerical solution and applied to investigate a case study problem based upon results of a field experiment. It is shown that the proposed analytical approach can offer a reasonable estimate of the thermal behaviour of the soil requiring no information from the soil other than its thermal properties Comparisons of predicted and measured soil temperature profiles and stored energy transients demonstrate there is reasonable overall agreement. The research contributes a practical approach that can provide surface boundary data that is vital in the thermal analysis of many engineering problems. Applications include; inter-seasonal heat transfer, energy piles and other more established ground source heat utilization methods.

Keywords: Soil, stored energy, thermal, analytical, heat transfer.

1. Introduction

The estimation of ground temperature profiles is important for several engineering applications that use the soil as a reservoir or source of thermal energy. Examples of these applications are the minimisation of thermal losses and passive heating and cooling of buildings (e.g. Rees et al. 2000; Zoras 2009), ground source heating (e.g. Florides and Kalogirou 2007); shallow energy piles (e.g. Wood et al 2010) and inter seasonal thermal energy storage (Bobes-Jesus et al. 2013; Pinel et al. 2011). These applications are highly dependent on the amount of energy present in the near-surface region of the soil and its temporal variation. Subsequently one of the first steps in the process of evaluation of their implementation is related with the assessment of ground temperature profiles and overall ground energy storage. To provide sufficient details such assessments are usually performed with the aid of theoretical models solved by numerical methods (e.g. Qin et al, 2002; Yumrutaş et al. 2005; Laloui et al 2006). These have the advantage of being able to include a high range of complexities within the domain of interest for example, different physical processes, materials, geometries, boundary conditions, etc. However, if the problem is relatively simple, it can be approached analytically. An analytical solution is usually simpler, easier to implement computationally and offers detailed insight about the underlying physical processes. Also, analytical solutions can be helpful in establishing reasonable initial conditions for more comprehensive numerical simulations when no other information is available.

Analytical solutions have been applied to solve the diffusion equation and the diffusion-convection equation in soil in various different fields. For example, heat diffusion has been studied in relation to the interaction between buildings and soil (Hagentoft 1996a; Hagentoft 1996b; Jacovides et al. 1996, Hollmuller and Lachal, 2014) and the diffusion of contaminants in porous media composed of two or more layers layers (Li and Cleall 2010; Chen et al. 2009). Convection and diffusion have been analysed together in relation with water infiltration (Gao et al. 2003; Wang et al. 2012) and general solute transport in porous media under various boundary conditions (Li and Cleall 2011). Water infiltration in unsaturated soils have also been studied using Richard's equation (Huang and Wu 2012). Approximate analytical solutions have been used to study heat and moisture transfer including phase change (thawing) in soils (Kurylyk, 2014). In each of these approaches three main types of boundary conditions are considered. These are: *first type* (also known as Dirichlet type), which specify the value of the variable at the boundary; *second type* boundary conditions (also known as Neumann type) which specify the value of the derivative of a variable at the boundary; and *third type* boundary conditions (also known as Robin type), these specify both (as a linear combination) the value of the variable and its derivative at the boundary.

The limitations of analytical solutions typically result from the simplification of certain aspects of the problem. Some of the first analytical approaches to estimate the temperature of the ground (Michopoulos et al. 2010; Mihalakakou et al. 1997) and coupled heat diffusion and water infiltration (Shao et al. 1998) relied on the assumption of fixed boundary conditions (constant or periodic). These approaches individually achieved objectives of including more than one physical process, more complex geometries (Chuangchid and Krarti 2001) or the actual operation of a heat exchanger used for heating a building (Yumrutaş et al. 2005). In recent years the inclusion of time dependent boundary conditions of the *second type* (Adam and Markiewicz, 2002; Wang 2012; Wang and Bou-Zeid 2012) and of the *third type* (Cleall and Li 2011) has gained more attention to describe in more detail the energy and mass transfer interactions at the soil surface. With regard to the boundary condition at the bottom of the domain it is common to either fix it at an estimated average temperature or assume an insulated (no heat flux) boundary condition. The implication of this last assumption is to neglect any geothermal heat flux. This is typically the case in consideration of the near soil surface (Davies 2013), however, where this assumption cannot be made, the inclusion of a constant heat flux at the bottom that takes into account this term is not difficult.

This paper presents a new analytical solution to the transient one dimensional heat diffusion equation using a flux boundary condition equal to zero at the bottom of the domain and a *third kind* (Robin) boundary condition at its surface. This enables surface heat fluxes directly related to meteorological conditions to be realistically represented. To achieve this, two mathematical expressions for meteorological variables are proposed and compared against daily and hourly experimental meteorological data. These expressions and the proposed analytical solution are then used to consider a field-scale case-study with the results obtained from the analytical solution compared against hourly experimental recordings of soil temperature profiles and estimates of stored energy.

2. Mathematical formulation

2.1. General Solution

The general form for the one dimensional homogeneous transient heat diffusion equation defined in a finite domain of length L is

$$\frac{d^2T}{dz^2} = \frac{1}{\alpha}\frac{dT}{dt} \quad \text{in} \quad \begin{matrix} 0 \le z \le L \\ t > 0 \end{matrix} \tag{1}$$

where T is the temperature of the soil and α is the thermal diffusivity. The solution of this equation can obtained following the approach given in (Özişik 2002) for various boundary conditions using the integral transform technique. The boundary conditions and initial condition considered here are defined as:

$$f_1(t) = -k\frac{dT}{dz} + h_1 T \quad \text{at} \quad \mathrm{z} = 0,\ t > 0 \tag{2}$$

$$f_2(t) = k\frac{dT}{dz} + h_2 T \quad \text{at} \quad \mathrm{z} = L,\ t > 0 \tag{3}$$

$$T = F(\mathrm{z}) \quad \text{in} \quad 0 \le z \le L,\ t = 0 \tag{4}$$

where h_1 and h_2 are the heat transfer coefficient at z=0 (soil surface) and z=L respectively, and k is the soil thermal conductivity. In the case where a Robin boundary condition $f_1(t)$ is applied at $z=0$, a zero heat flux boundary condition is applied at $z=L$ and a constant initial condition F_i is used, the solution has the form:

$$T(\mathrm{z},t) = \sum_{m=1}^{\infty} 2\left(\frac{\beta_m^2 + H_1^2}{L(\beta_m^2 + H_1^2) + H_1}\right) e^{-\alpha\beta_m^2 t} \cos\beta_m (L - z)\left[\frac{F_i \sin(\beta_m L)}{\beta_m} + \frac{\alpha\cos(\beta_m L)}{k_1}\int_{t'=0}^{t} e^{\alpha\beta_m^2 t'} f_1(t')dt'\right] \tag{5}$$

where $H_1=h_1/k$ and the eigenvalues β_m are the positive roots of:

$$\beta \tan\beta = H_1 \tag{6}$$

2.2 Energy stored in the soil

The description of the soil's temperature profile with depth given by equation (5) allows the calculation of the energy stored (J/m^2) in a column of soil of depth L with reference to the energy present in the soil at an arbitrary reference time as:

$$Q(\mathrm{z},t) = \rho c_p \int_0^L \left[T(\mathrm{z},t) - T(\mathrm{z},t_{ref})\right]dz \tag{7}$$

where ρ and c_p are the density and specific heat capacity of the soil, $T(z,t,)$ is the temperature profile at time t and $T(z,t_{ref})$ is the temperature profile at a reference time t_{ref}.

2.3. Boundary condition at the soil surface

The boundary condition at the soil surface (z=0) is based on consideration of the heat energy balance at the surface of the soil and can be defined by:

$$-k\frac{dT}{dz}=(1-\alpha_S)R+4\sigma T_{0,\mathrm{K}}^3\varepsilon_G\varepsilon_{sky}^{0.25}T_{a,\mathrm{K}}-4\sigma T_{0,K}^3\varepsilon_G T_K+h_E(q_a-q_G)+h_C T_a-h_C T \quad (8)$$

where α_s is the soil albedo (Garratt 1994), R (W/m^2) is solar radiation, σ (W/m^{2}K^4) is the Steffan-Boltzmann constant, T_a and $T_{a,K}$ is air temperature in (˚C) and (K) respectively, (variables and constants used to calculate the terms in equation (8) are summarized in Table 1). $T_{0,K}$ (K) is an average temperature that arises from the linearization of the infrared heat transfer equation (Duffie and Beckman 2006) and is defined as:

$$T_{0,K}=\left[0.25\left(\varepsilon_{sky}^{0.5}T_{a,K}^2+T_{G,K}^2\right)\left(\varepsilon_{sky}^{0.25}T_{a,K}+T_{G,K}\right)\right]^{1/3} \quad (9)$$

$T_{G,K}$is the temperature of the soil surface in (K), ε_G is the emissivity of the soil surface (Garratt 1994), ε_{sky} is the sky emissivity (Edinger and Brady 1974; Herb et al. 2008) defined as:

$$\varepsilon_{sky}=n+0.67(1-n)\left(q_a/100\right)^{0.08} \quad (10)$$

where n is a cloud factor with a non-dimensional value from 0 to 1. q_G (Pa) and q_a (Pa) are the vapour pressure for the soil surface and air respectively and are defined as:

$$q_G=\exp\left(\frac{\psi M_{H_2O}g}{RT_{G,K}}\right)611\exp\left(\frac{L_v M_{H_2O}}{R}\left(\frac{1}{273.15K}-\frac{1}{T_{G,K}}\right)\right) \quad (11)$$

$$q_a=\left(\frac{H_r}{100}\right)611\exp\left(\frac{L_v M_w}{R}\left(\frac{1}{273.15K}-\frac{1}{T_{a,K}}\right)\right) \quad (12)$$

where ψ is the surface water pressure in (m) (the average value of saturation and wilting point for clay provided in (Garratt 1994) is used), M_w is the molecular weight of water (kg/mol), g (m/s^2) is the acceleration of gravity, R (J/molK) is the gas constant, L_v (J/kg) is the latent heat of vaporization of water and H_r (%) is the relative humidity. An expression for the saturation vapour pressure can be found in (North and Erukhimova 2009), while the term for the relative humidity of the soil is defined in (Philip and de Vries 1957).

The heat transfer coefficients for evaporative (h_E) and convective (h_C) heat flux can be defined following the approach given by (Jansson et al. 2006). This approach assumes a turbulent heat transfer process in the surface of the soil and has the advantage of using relatively simple heat transfer coefficients:

$$h_E = \frac{\rho_a L_V}{r_a} \tag{13}$$

$$h_C = \frac{\rho_a c_p}{r_a \eta} \tag{14}$$

where ρ_a (kg/m^3) is the air density, c_p (J/kgK) is air specific heat capacity, η (Pa/K) is the psychrometric constant and r_a (s/m) is the aerodynamic resistance defined (for neutral conditions (Garratt 1994)) as:

$$r_a = \frac{\log\left(\frac{z_{ref}}{z_{mr}}\right)\log\left(\frac{z_{ref}}{z_{hr}}\right)}{k_{vk}^2 u} \tag{15}$$

where u (m/s) is the wind velocity, k_{vk} is the Von Karman constant, z_{ref} (m) is the height at which wind speed and air temperature measurements were made, z_{mr} and z_{hr} (m) are the relative roughness for momentum and heat respectively of the soil surface in its interaction with the atmospheric boundary and their values are taken from (Garratt 1994) and (Kotani and Sugita 2005) respectively. The psychrometric constant is defined as:

$$\eta = \frac{c_{p,a} P M_a}{\mathrm{LM}_w} \tag{16}$$

where P is the atmospheric pressure (Pa) and M_a is the molecular weight of air (kg/mol). Others (Edinger and Brady 1974; Herb et al. 2008) use different approaches to define these heat transfer coefficients which are useful for cases were non turbulent processes can be assumed (low wind speeds) that take into account forced and natural convection, however these coefficients are, relatively more complex and not readily amenable for inclusion in the form of analytical solution presented here.

Equation (8) can be rewritten in the form of equation (2), to subsequently be used in the solution of equation (5). For this, average values for air temperature, wind speed and relative humidity are required to calculate some of these coefficients (namely ε_{sky}, $T_{0,K}$ and q_a) that otherwise would be unsuitable to include in an analytical approach. Also, the evaporative term q_G is dependent on the temperature of the surface of the soil. An average temperature for the soil surface can be estimated by integrating equation (8) over a full yearly cycle so as to consider a quasi-equilibrium scenario (i.e. zero net heat flux) after expressions for solar radiation and air temperature have been defined.

2.4. Mathematical expressions for meteorological variables

In order to solve equation (5) using equation (8) as a boundary condition it is necessary to formulate expressions for the meteorological variables required. Mathematical expressions for solar radiation are available in the literature (Duffie and Beckman 2006). In general these expressions are functions of geographical parameters and provide the amount of radiation between sunrise and sunset, however, they are not suitable for use here because for a continuous analytical solution a function that is applicable during night time is required. In this paper we offer two simplified mathematical expressions for idealised daily and annual variations of solar radiation and air temperature that can be constructed using widely available averaged meteorological data.

The expression for solar radiation builds upon another expression for daily variations given in the literature (Lumb 1964). Here this expression is expanded to include annual variation. An equation for variation in solar radiation is proposed here as:

$$R(t) = \frac{\pi}{2}\left(\cos^2(\gamma t) - \cos(\gamma t) + \frac{4-\pi}{2\pi}\right)\left(\mathrm{R}_1\cos(\varphi t) + \mathrm{R}_2\right) \tag{17}$$

where t is given in seconds taking the origin at midyear (July 1st), φ is the annual period defined as $2\pi/31557600$ s (2π divided by 365.25 days in seconds), and γ is the daily period defined as $2\pi/86400$ s (2π divided by 24 hours in seconds). R_1 and R_2 are coefficients, that can be determined from the meteorological conditions for summer and winter (the summer and winter periods can be arbitrarily defined based on localised conditions). These coefficients are defined as:

$$R_1 = 0.5(A - B) \tag{18}$$

$$R_2 = 0.5(A + B) \tag{19}$$

where A and B are the summer and winter daily average solar radiation respectively.

A similar sinusoidal expression is proposed to represent the diurnal air temperature variation as in general air temperature variations correlate to insolation. For simplicity a sinusoidal daily variation with its maximum at midday and the minimum at midnight is assumed. The annual variation is mainly sinusoidal with maximums and minimums at summer and winter respectively but incorporates an additional sine term to take into account typically observed slightly higher values in spring and slightly lower values in autumn. The proposed expression is:

$$T_a(t) = T_1\left[\cos(\varphi t) + 0.5\sin(\varphi t)\right] + T_2 - \left\{T_3\left[\cos(\varphi t) + 0.5\sin(\varphi t)\right] + T_4\right\}\cos(\gamma t) \quad (20)$$

where t is given in seconds taking the origin at midyear (1st July). T_1, T_2, T_3 and T_4 are coefficients determined from the meteorological conditions for mid-summer and mid-winter periods. They are calculated as:

$$T_1 = 0.5(C - D) \quad (21)$$

$$T_2 = 0.5(C + D) \quad (22)$$

$$T_3 = (E - F) \quad (23)$$

$$T_4 = 0.5(E + F) \quad (24)$$

where coefficients *C*, *D*, *E*, *F* are defined as the mid-summer daily average, mid-winter daily average, mid-summer average amplitude, and mid-winter average amplitude respectively.

The average value for solar radiation and air temperature defined by these mathematical expressions can be calculated by averaging equations (17) and (20) over a suitable period of time (e.g. four years). It can be found that the average value for solar radiation and air temperature is given by R_2 and T_2 respectively.

Due to the relatively random nature of variations in relative humidity and wind speed across an annual time span, mathematical expressions for these variables have not been developed and instead it is proposed that annual averages based on values from meteorological data sets are used.

3. Verification

The analytical solution proposed here is verified via consideration of a hypothetical problem. The results obtained from the analytical solutions are compared with those from a numerical solution using the finite-element method (Cleall et al. 2007; Seetharam et al. 2007). A number of analyses have been undertaken with varying values of material parameter and system coefficients to investigate the uniqueness of the solutions. Results of a typical analysis follow.

Problem statement: A 20 m deep layer of soil is defined with an initially uniform temperature of 14 °C. Hypothetical soil material parameters (k= 1 W/mK, c_p= 800 J/kgK, ρ= 2000 kg/m^3), values for the coefficients of equations (17) and (20) (A= 250 W/m^2; B= 20 W/m^2; C= 16 ˚C; D= 3.6 ˚C; E= 2.5 ˚C; F= 5 ˚C), an average value for soil surface temperature of 8.7 ˚C (calculated, as explained before, by integrating equation (8) over a full yearly cycle), a cloud factor of 0 and annual averages of relative humidity (80.6 %) and wind speed (1.14 m/s) are assumed. The finite element analysis discretised the domain with 512 2-noded equally sized elements and used a constant time step of 1800 seconds, full details of the numerical approach used can be found in Seetharam et al (2007). Comparison of the temperature profiles and energy stored obtained from both the proposed solution and the alternative numerical solution are presented in figures 1 and 2 for the 1st, 40th, and 80th year of analysis.

Figure 1 compares analytical and numerical temperature profiles for 4 sampling dates for 3 different years. The year is taken to comprise 365.25 days and the sampling points have been homogeneously distributed in each year and approximately correspond to calendar dates of 1st January (t1), 1st April (t2), 1st July (t3) and 1st October (t4). It can be seen that the analytical and numerical results are in excellent agreement and that the temperature profiles for the 40th and 80th years are identical implying that a stationary state has been reached.

Figure 2 shows the comparison of stored energy, for year 40th, calculated analytically using equation (7) and numerically using:

$$Q_N(z_i, t_j) = \rho C_p \sum_{i=0}^{m} \left[T_N(z_i, t_j) - T(z_i, t_{ref}) \right] \Delta z_i \qquad (25)$$

where Δz_i is the length of cell i. In both cases, analytical and numerical, a constant reference temperature of 8.7 ˚C (the temperature at the bottom of the domain at year 40th) has been used. The maximum relative error between numerical and analytical is less than 0.1%. Again it can be seen that the analytical and numerical results are in excellent agreement.

4. Application to a case-study

A two year long demonstration project commissioned by the British Highways Agency in order to assess the feasibility of use of inter-seasonal heat storage systems to provide thermal maintenance to highways and heating for buildings was reported by the Transport Research Laboratory (TRL) (Carder et al. 2007). The project was carried out between July 2005 and May 2007 at Toddington, UK. Boreholes up to 12.875 m deep were drilled and temperature sensor arrays placed inside. Two of these boreholes were located far from the location of the storage system, and served as control boreholes, the remaining boreholes were distributed on a highway section and recorded the ground temperature evolution through time while the inter-seasonal heat storage system was active. The specific data used for this work corresponds to one of the control boreholes, and as such the storage system need not be considered further. No details regarding regular surface maintenance above this borehole (e.g. grass cutting) are provided in (Carder et al. 2007). However site visits by the authors indicate it is reasonable to assume that the surface was subject to a natural cycle of plant growth (mainly grass).

TRL set up a meteorological station and performed recordings of solar radiation, air temperature, wind speed, relative humidity and precipitation every 15 minutes from July 2005 to May 2007 (Carder et al. 2007). Hourly average values from this station are used in this work to compare against results obtained from the mathematical expressions proposed to describe the meteorological conditions. This approach offers the advantage of testing the ability of the proposed expressions, fitted to readily available long term meteorological data, to represent localised short term measured data.

The proposed mathematical expressions for solar radiation and air temperature have been fitted to meteorological data recordings reported by the British Atmospheric Data Centre (UK Meteorological Office 2012) and the Met Office (UK Meteorological Office) for the period from 1985 to 2004 to investigate their appropriateness and ability to represent realistically the diurnal and seasonal variations. For the purpose of this work, a monitoring station located in Hertfordshire, UK (coordinates 51.8062 latitude, -0.3585 longitude) was selected as it offers suitable daily and hourly meteorological data and is also relatively near (17 km) to the site of the experimental project for which localised meteorological data and soil temperature profiles were also recorded. The variables obtained to allow calculation of the coefficients used in the mathematical expressions for solar radiation (17) and air temperature (20) are summarized in Tables 2 and 3. These variables represent average values for mid-summer and mid-winter periods which in this study are defined respectively as from 25th June to 5th July and from 25th December to 5th January. These periods were chosen since they are expected to contain the maximum and minimum values of the variables. Due to data availability, cloud cover information was obtained from a monitoring station located at Bedford (coordinates 52.2265 latitude, -0.46376 longitude, approx. 31 km from the experimental site). The station has reported hourly cloud cover data from November 2008 allowing the determination of an average cloud factor value of 0.59 for the five year period (2009-2013). It is assumed that this value is representative of the amount of cloud cover present in any other year.

Annual averages of relative humidity (80.6 %) and wind speed (1.14 m/s) based on values recorded during the two-year long (2005-2006) demonstration project are used in the subsequent application of the proposed analytical solution to consider a 20 m deep soil column. The proposed solution also requires a set of material parameters to describe the soil thermal properties these have been based on those reported in (Carder et al. 2007) for the soil at this site and are summarised in Table 4.

5. Results

Figure 3 and 4 present comparisons of daily average values generated with the proposed mathematical expressions for solar radiation (equation (17)) and air temperature (equation (20)) with equivalent measured data for the period 1985-2004. In both cases it can observed that the predicted data are constrained by the well-defined maximums and minimums. These values, as discussed before, are based on the average values for summer and winter. As would be expected the data with higher daily average values for solar radiation correspond to summer months while those with lower values correspond to winter months. It can also be seen that in each month the experimental data tend to have a wider range of lower values this is because the mathematical expression for the predicted data is idealized and in no way takes into account the effect of cloud cover which will decrease the amount of solar radiation that reaches the soil surface. These effects result in the spread of data points displayed in figure 3 having a trapezoidal like shape. As before, the data with the higher average values of daily temperature shown in figure 4 correspond to summer months while those with lower values correspond to winter months. It can be seen that the predicted data for air temperature offer a better comparison with the ideal line included in the figure and that it offers a better correlation factor than the case for solar radiation. This is probably due to the fact that air temperature is not as highly impacted by the presence of cloud cover. It is noted that if the average value for maximum daily summer temperatures and the average value for minimum daily winter temperatures are used an improved linear fit in figure 4 could be obtained. However daily averages for summer and winter have been used to retain homogeneity with the definition of coefficients for solar radiation. Implementation of averaged values in the proposed solution is trivial (i.e. simply by revising the definition of the coefficients of equation (20)) and either approach can be adopted to achieve the best fit with measured data.

Figures 5 and 6 present comparisons of experimental and predicted daily average values for solar radiation and air temperature respectively for 2005-2006. This permits testing of the proposed expressions for solar radiation and air temperature with an independent subset of data. The experimental values shown are taken from (UK Meteorological Office 2012; UK Meteorological Office). It can be seen that the correlation values are in general similar to those obtained for the period 1985-2004 which was used to establish the coefficients in the expressions.

Fig. 7 and Fig. 8 present comparisons of hourly values of solar radiation and air temperature from the proposed expressions with equivalent data recorded on site by TRL (Carder et al. 2007) from September 2005 to August 2006. In Fig. 7 a pattern of stratification of the data points can be observed with data points forming horizontal bands. These 'bands' are mostly composed for points belonging to summer months. They arise because as equation (17) approaches its maximum in mid-summer it tends to flatten and predict similar values for corresponding hours from mid-May to mid-August while the experimental values are affected by the relatively random presence of clouds.

Fig. 8 shows experimental and predicted hourly air temperature values. A general trend of underestimation of the predicted temperatures can be observed. It is worth noting that the period considered was warmer (on average by 0.5 °C) than for the previous 20 years. In particular, the average air temperature for the last 20 years was 9.7 °C while the average air temperature for 2005-2006 calculated using TRL data was 10.2 °C. These differences are more marked if they are considered at a monthly level, where the average for July and January for the last 30 years was 16.2 °C and 4.1 °C respectively and 20 °C and 3.4 °C for July 2006 and January 2006 respectively. This in part explains the general under prediction of temperatures seen in Fig. 8. It can also be observed in figure 7 that a limited number of small negative night time values are given by equation (17) due to its sinusoidal and continuous nature, this is illustrated more clearly in Fig. 9. These unavoidable limitations are acknowledged but it is noted that the overall daily solar radiation is still realistic as seen in Fig. 3 where the negative values are absent as it is presenting averaged daily values. Fig. 9 also illustrates the effect of clouds as well as the effect of variation of day length.

Fig. 10 and Fig. 11 show the comparison of soil temperatures obtained by applying equations (17) and (20) in equation (5) (using the material data provided in Table 4 and a domain depth of 20 m) against experimental data from a control borehole of TRL for three different depths.

An average cloud factor of 0.59 has been used in equation (10) to take into account the effect of clouds in the infrared terms in equation (8) Fig. 10 shows the comparison for the temperature sensor at 0.025 m. Although the correlation factor tends to be low due to the random nature of the experimental data caused in part by the random nature of the daily meteorological data, it can be seen that the analytical solution offers a reasonable description of the thermal behaviour of the soil.

Fig. 11 shows the comparison for the temperature sensors located at 1.025 m and 12.875 m. These results indicate that as the depth increases the correlation factor tend to increase. However, for deeper sections of the soil this trend no longer holds, this is due to the fact that the temperature variations in the ground are very small. At depth of 12.875 m, where it would be expected that the soil would maintain at a relatively constant value the analytical solution proposed in this work reasonably predicts the experimental value with a maximum error of 1.3 °C. It is worth noting that the proposed model assumes a homogeneous free heat flux boundary condition at the bottom of the soil column which is at a depth of 20 m. The advantage of this approach over one that considers a first type (Dirichlet) boundary condition at the base is that no assumption of soil temperature at depth is required.

Transient variations in stored energy can be obtained via use of equation (7) and consideration of measured temperature profiles. As the experimental temperatures are discrete data, linear interpolation is used to approximate continuous profiles. Fig. 12 shows comparisons of the calculated and estimated measured stored energy in a column of soil 12.875 m deep. It can be observed that the proposed model is able to offer realistic estimates in the relative change in seasonal energy storage. It is noted that there is a trend of a slight underestimation of energy stored. This is related to the fact that the period compared, as mentioned previously, was slightly warmer than the longer term average of the period used to calibrate equations that represent the surface weather condition.

6. Conclusions

Analytical solutions to estimate the soil temperature with depth and stored energy were presented in this paper. The boundary conditions used are of the *second kind* (Neumann) at the bottom and of the *third kind* (Robin) based on a heat balance at the soil surface. In order to describe the soil-atmosphere interactions, mathematical expressions describing the daily and annual variation of solar radiation and air temperature have been proposed. The analytical solutions were shown to correlate well with numerical solutions from a finite-element analysis.

The presented analytical solutions were used to investigate a case study problem base upon results of a field experiment reported by others. Predicted soil temperature profiles and stored energy transients have been compared against experimental recordings for over one year. Also the predicted meteorological data has been compared against widely available public records and against data recorded on site. The main differences found between the predicted and experimental data are due to the random nature of certain meteorological variables (e.g. clouds) and the inevitable variability in average data for a particular year in comparison to averages from a longer term data set. The results show that the analytical approach proposed can offer a reasonable estimate of the thermal behaviour of the soil requiring no information from the soil other than its thermal properties. This work provides a useful tool in applications requiring estimations of the soil temperature profiles, for example in the field of ground heat extraction and storage, or in numerical problems where a reasonable initial state can minimise the computational time to reach a convergent steady state.

Acknowledgments

The authors gratefully acknowledge the support given to the second author whose PhD studies were funded by CONACYT (the Mexican National Council of Science and Technology) and SEP (Mexican Secretariat of Public Education). Also the supply by TRL/HA of the source data published in (Carder et al. 2007) is gratefully acknowledged.

References

Adam D, Markiewicz R. Nutzung der geothermischen energie mittels erdberuhrter

bauwerke. Osterr Ing Archit;147, (2002)

Bobes-Jesus, V., Pascual-Muñoz, P., Castro-Fresno, D., Rodriguez-Hernandez, J.: Asphalt solar collectors: A literature review. Appl. Energy. 102, 962–970 (2013).

Carder, D.R., Barker, K.J., Hewitt, M.G., Ritter, D., Kiff, A.: Performance of an interseasonal heat transfer facility for collection, storage and re-use of solar heat from the road surface. (2007).

Chen, Y., Xie, H., Ke, H., Chen, R. An analytical solution for one-dimensional contaminant diffusion through multi-layered system and its applications Environmental Geology, 58(5), 1083-1094, (2009) Chuangchid, P., Krarti, M.: Foundation heat loss from heated concrete slab-on-grade floors. Build. Environ. 36, 637–655 (2001).

Cleall, P.J., Li, Y.-C.: Analytical Solution for Diffusion of VOCs through Composite Landfill Liners. J. Geotech. Geoenvironmental Eng. 137, 850–854 (2011).

Cleall, P.J., Seetharam, S.C., Thomas, H.R.: Inclusion of some aspects of chemical behavior of unsaturated soil in thermo/hydro/chemical/mechanical models. I: Model development. J. Eng. Mech.-Asce. 133, 338–347 (2007).

Davies, J.H.: Global map of solid Earth surface heat flow. Geochem. Geophys. Geosystems. 14, 4608–4622 (2013).

Duffie, J.A., Beckman, W.A.: Solar Engineering of Thermal Processes. Wiley (2006).

Edinger, J.E., Brady, D.K.: Heat Exchange and Transport in the Environment. John Hopkins University (1974).

Florides, G., Kalogirou, S.: Ground heat exchangers—A review of systems, models and applications. Renew. Energy. 32, 2461–2478 (2007).

Gao, Z., Fan, X., Bian, L.: An analytical solution to one-dimensional thermal conduction-convection in soil. Soil Sci. 168, 99–107 (2003).

Garratt, J.R.: The Atmospheric Boundary Layer. Cambridge University Press (1994).

Hagentoft, C.E.: Heat losses and temperature in the ground under a building with and without ground water flow .1. Infinite ground water flow rate. Build. Environ. 31, 3–11 (1996)(a).

Hagentoft, C.E.: Heat losses and temperature in the ground under a building with and without ground water flow .2. Finite ground water flow rate. Build. Environ. 31, 13–19 (1996)(b).

Herb, W.R., Janke, B., Mohseni, O., Stefan, H.G.: Ground surface temperature simulation for different land covers. J. Hydrol. 356, 327–343 (2008).

Hollmuller P. and Lachal B.: Air–soil heat exchangers for heating and cooling of buildings: Design guidelines, potentials and constraints, system integration and global energy balance. Applied Energy. 119, 476–487, (2014).

Huang, R.Q., Wu, L.Z.: Analytical solutions to 1-D horizontal and vertical water infiltration in saturated/unsaturated soils considering time-varying rainfall. Comput. Geotech. 39, 66–72 (2012).

Jacovides, C.P., Mihalakakou, G., Santamouris, M., Lewis, J.O.: On the ground temperature profile for passive cooling applications in buildings. Sol. Energy. 57, 167–175 (1996).

Jansson, C., Almkvist, E., Jansson, P.: Heat balance of an asphalt surface: observations and physically-based simulations. Meteorol. Appl. 13, 203–212 (2006).

Kotani, A., Sugita, M.: Seasonal variation of surface fluxes and scalar roughness of suburban land covers. Agric. For. Meteorol. 135, 1–21 (2005).

Kurylyk, B. L., McKenzie, J. M., MacQuarrie, K. T. B., Voss, C. I.: Analytical solutions for benchmarking cold regions subsurface water flow and energy transport models: One-dimensional soil thaw with conduction and advection. Advances in Water Resources. 70, 172–184 (2014).

Laloui, L., Nuth, M., Vulliet, L. Experimental and numerical investigations of the behaviour of a heat exchanger pile. International Journal for Numerical and Analytical Methods in Geomechanics, 30 (8), 763-781 (2006)

Li, Y.-C., Cleall, P.J.: Analytical Solutions for Contaminant Diffusion in Double-Layered Porous Media. J. Geotech. Geoenvironmental Eng. 136, 1542–1554 (2010).

Li, Y.-C., Cleall, P.J.: Analytical solutions for advective–dispersive solute transport in double-layered finite porous media. Int. J. Numer. Anal. Methods Geomech. 35, 438–460 (2011).

Lumb, F.E.: The influence of cloud on hourly amounts of total solar radiation at the sea surface. Q. J. R. Meteorol. Soc. 90, 43–56 (1964).

Michopoulos, A., Papakostas, K.T., Mavrommatis, T., Kyriakis, N.: Comparative assessment of eight models predicting the ground temperature. JP J. Heat Mass Transf. 4, 119–135 (2010).

Mihalakakou, G., Santamouris, M., Lewis, J.O., Asimakopoulos, D.N.: On the application of the energy balance equation to predict ground temperature profiles. Sol. Energy. 60, 181–190 (1997).

North, G.R., Erukhimova, T.L.: Atmospheric Thermodynamics: Elementary Physics and Chemistry. Cambridge University Press (2009).

Özişik, N.: Boundary Value Problems of Heat Conduction. Dover Publications, Incorporated (2002).

Philip, J.R., de Vries, D.A.: Moisture movement in porous materials under temperature gradients. Trans. Am. Geophys. Union. 38, 222–232 (1957).

Pinel, P., Cruickshank, C.A., Beausoleil-Morrison, I., Wills, A.: A review of available methods for seasonal storage of solar thermal energy in residential applications. Renew. Sustain. Energy Rev. 15, 3341–3359 (2011).

Qin , Z., Berliner, P., Karnieli, A.: Numerical solution of a complete surface energy balance model for simulation of heat fluxes and surface temperature under bare soil environment. Applied Mathematics and Computation. 130 (1), 171–200 (2002).

Rees, S.W., Adjali, M.H., Zhou, Z., Davies, M., Thomas, H.R.: Ground heat transfer effects on the thermal performance of earth-contact structures. Renew. Sustain. Energy Rev. 4, 213–265 (2000).

Seetharam, S.C., Thomas, H.R., Cleall, P.J.: Coupled thermo/hydro/chemical/mechanical model for unsaturated soils—Numerical algorithm. Int. J. Numer. Methods Eng. 70, 1480–1511 (2007).

Shao, M., Horton, R., Jaynes, D.B.: Analytical solution for one-dimensional heat conduction-convection equation. Soil Sci. Soc. Am. J. 62, 123–128 (1998).

UK Meteorological Office: Met Office Integrated Data Archive System (MIDAS) Land and Marine Surface Stations Data (1853-current), http://badc.nerc.ac.uk/view/badc.nerc.ac.uk__ATOM__dataent_ukmo-midas.

UK Meteorological Office: Met Office: UK climate summaries, http://www.metoffice.gov.uk/climate/uk/index.html.

Wang, L., Gao, Z., Horton, R., Lenschow, D.H., Meng, K., Jaynes, D.B.: An Analytical Solution to the One-Dimensional Heat Conduction–Convection Equation in Soil. Soil Sci. Soc. Am. J. 76, 1978–1986 (2012).

Wang, Z.-H.: Reconstruction of soil thermal field from a single depth measurement. J. Hydrol. 464–465, 541–549 (2012).

Wang, Z.-H., Bou-Zeid, E.: A novel approach for the estimation of soil ground heat flux. Agric. For. Meteorol. 154–155, 214–221 (2012).

Wood,C.J., Liu,H., Riffat, S.B. An investigation of the heat pump performance and ground temperature of a piled foundation heat exchanger system for a residential building, Energy, 35(12), 4932-4940 (2010)

Yumrutaş, R., Kanoğlu, M., Bolatturk, A., Bedir, M.Ş.: Computational model for a ground coupled space cooling system with an underground energy storage tank. Energy Build. 37, 353–360 (2005).

Zoras, S.: A Review of Building Earth-Contact Heat Transfer. Adv. Build. Energy Res. 3, 289–314 (2009).

Table 1 - Summary of variables and constants used to calculate parameters in equation (11)

ρ_a (kg/m^3)	1.2041	L_v (J/kg)	2.45E6	z_{mr} (m)	1E-3
$c_{p,a}$ (J/kgK)	1012	z_{ref} (m)	3	z_{hr} (m)	1E-3
k_{vk}	0.41	P (Pa)	101325	M_w (kg/mol)	0.0180153
M_a (kg/mol)	0.02897	ψ (m)	-75.2025	R (J/molK)	8.3144621
g(m/s^2)	9.8	α_s	.15	σ (W/m^{2}K^4)	5.67E-8
ε_G	0.97				

Table 2: Summary of values used to calculate coefficients for the mathematical expression for solar radiation equation (17). Based on data from (UK Meteorological Office 2012)

A	Mid-summer daily average	204.2 W/m^2
B	Mid-winter daily average	21.3 W/m^2

Table 3: Summary of values used to calculate coefficients for the mathematical expression for air temperature equation (20). Based on data from (UK Meteorological Office 2012).

C	Mid-summer daily average	15.4 ˚C
D	Mid-winter daily average	3.6 ˚C
E	Mid-summer average amplitude	2.7 ˚C
F	Mid-winter average amplitude	4.2 ˚C

Table 4: Soil material parameters (Carder et al. 2007) and domain depth.

k	Soil thermal conductivity	1.2 W/mk
ρ	Soil density	1960 kg/m^3
c_p	Soil specific capacity	840 J/kgK
α	Soil thermal diffusivity ($=k/\rho c_p$)	
L	Depth of the domain	20 m

Figure Captions

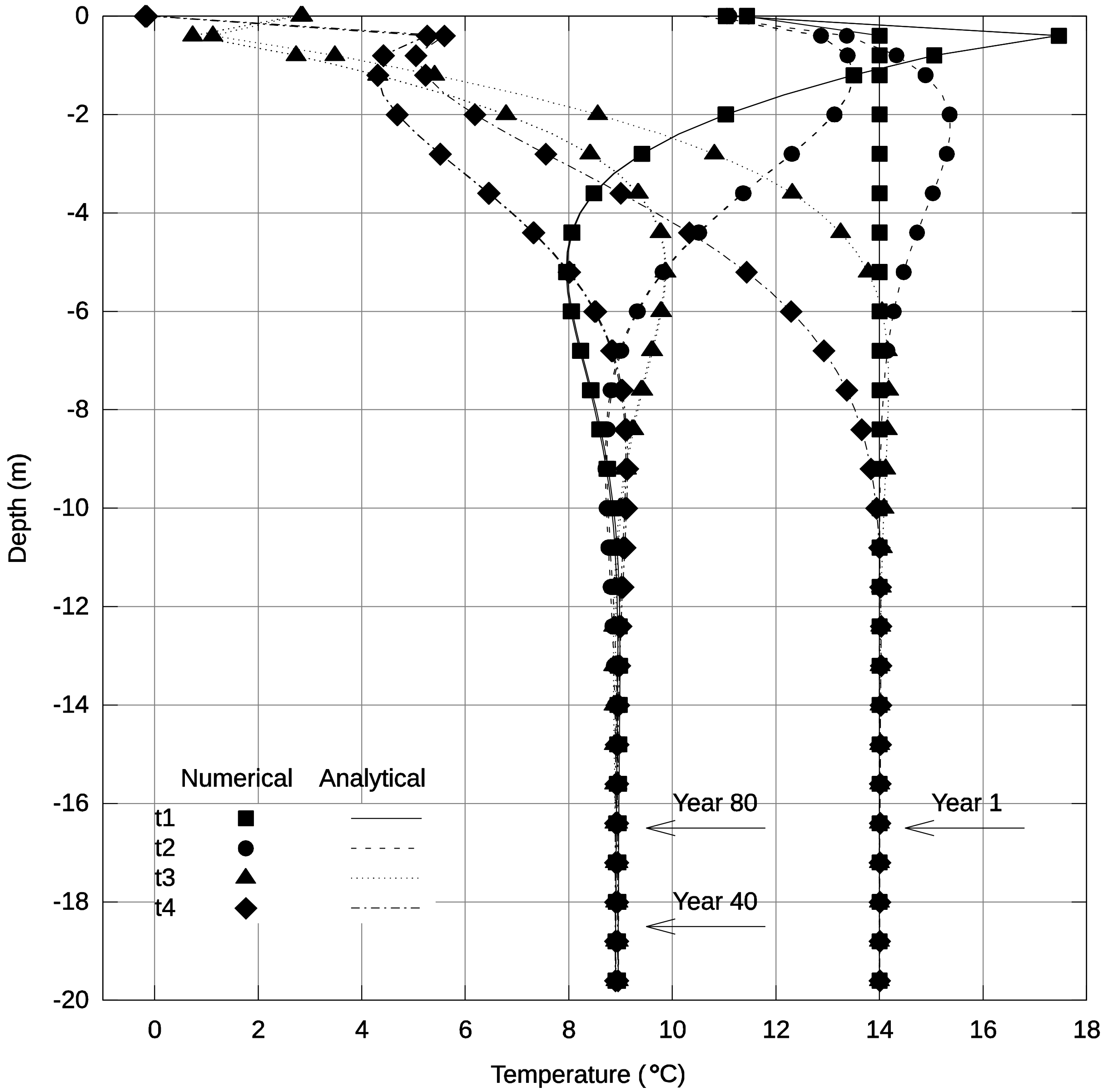

Fig. 1 Comparison of analytical and numerical results for 4 dates for 3 different years (1st, 40th and 80th). 1st January (t1), 1st April (t2), 1st July (t3) and 1st October (t4) of each year

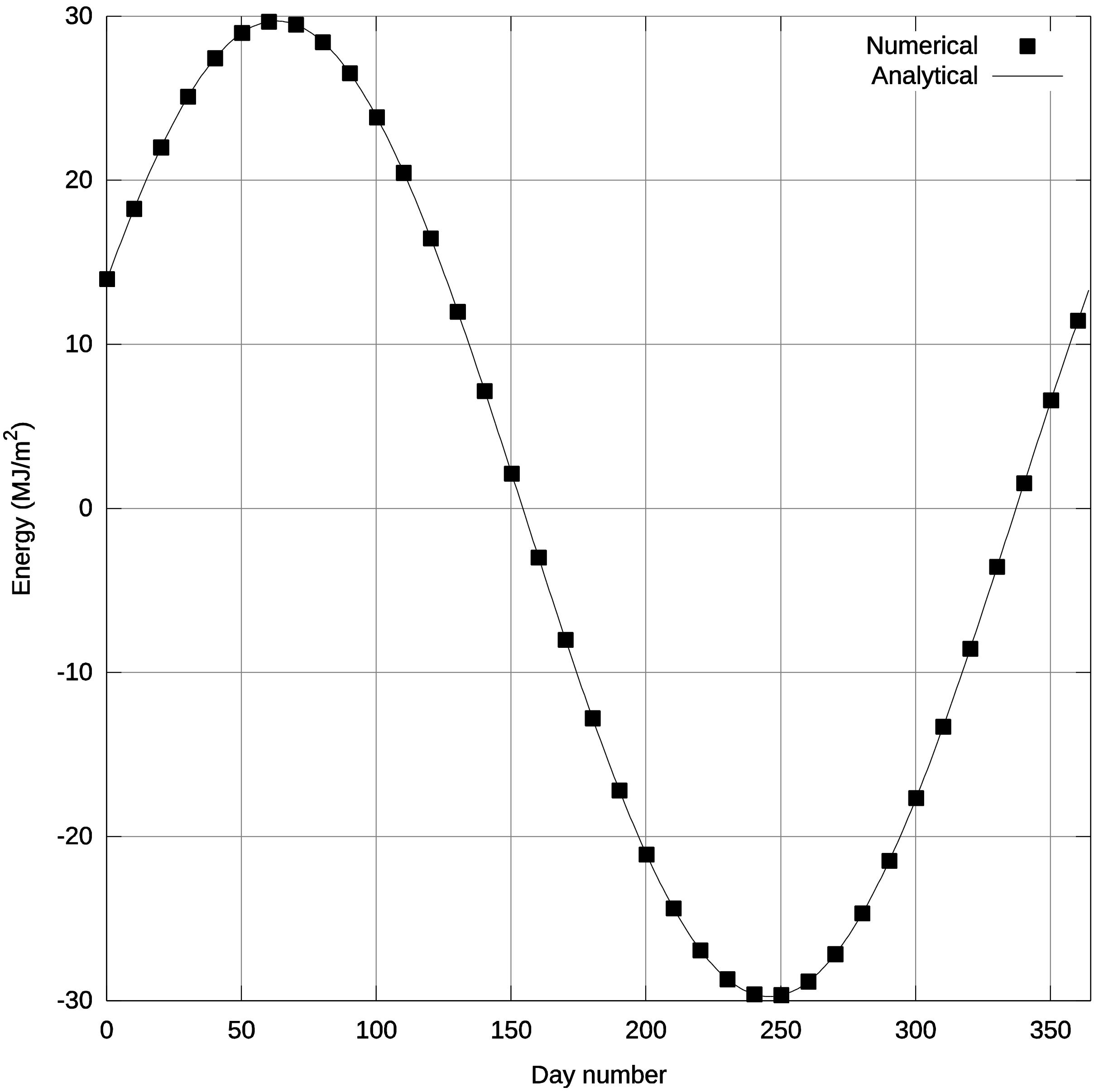

Fig. 2 Comparison of stored energy calculated analytically using equation (7) and numerically using equation (25) in a column of soil of 20 m for year 40^{th}

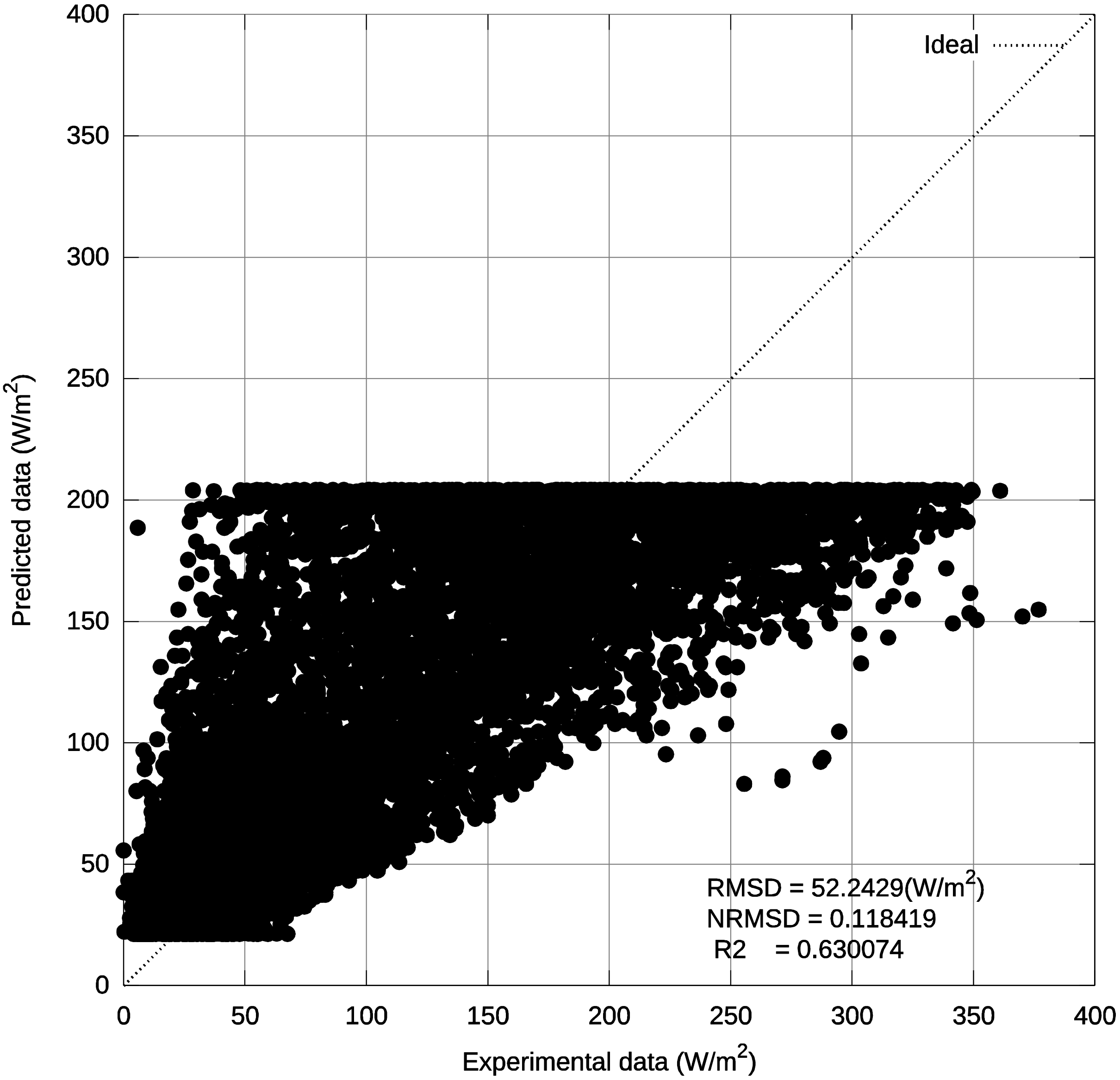

Fig. 3 Comparison of daily average values for solar radiation predicted with equation (17) with data from (UK Meteorological Office 2012) for 1985-2004

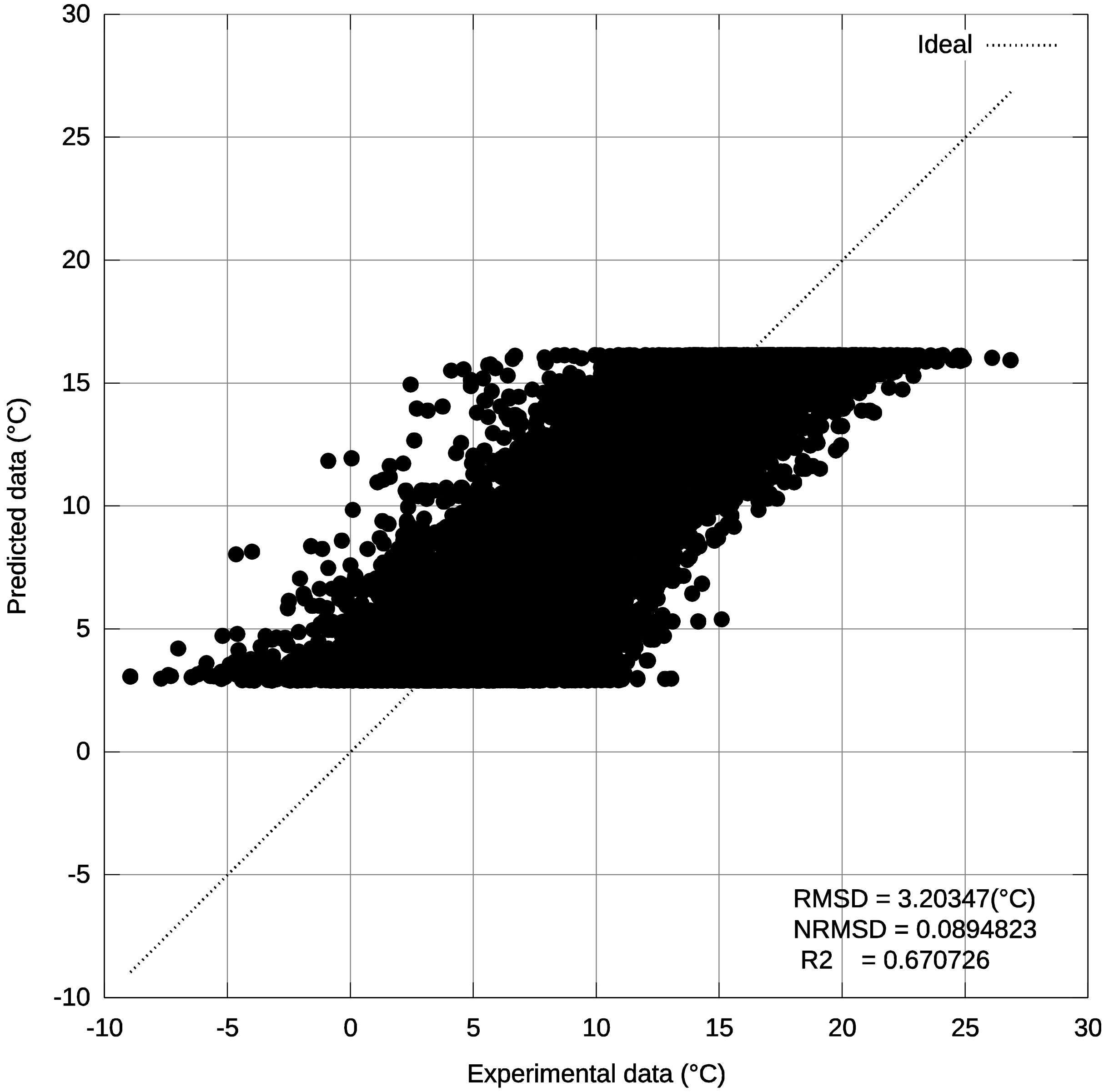

Fig. 4 Comparison of daily average values for air temperature predicted with equation (20) with data from (UK Meteorological Office 2012) for 1985-2004

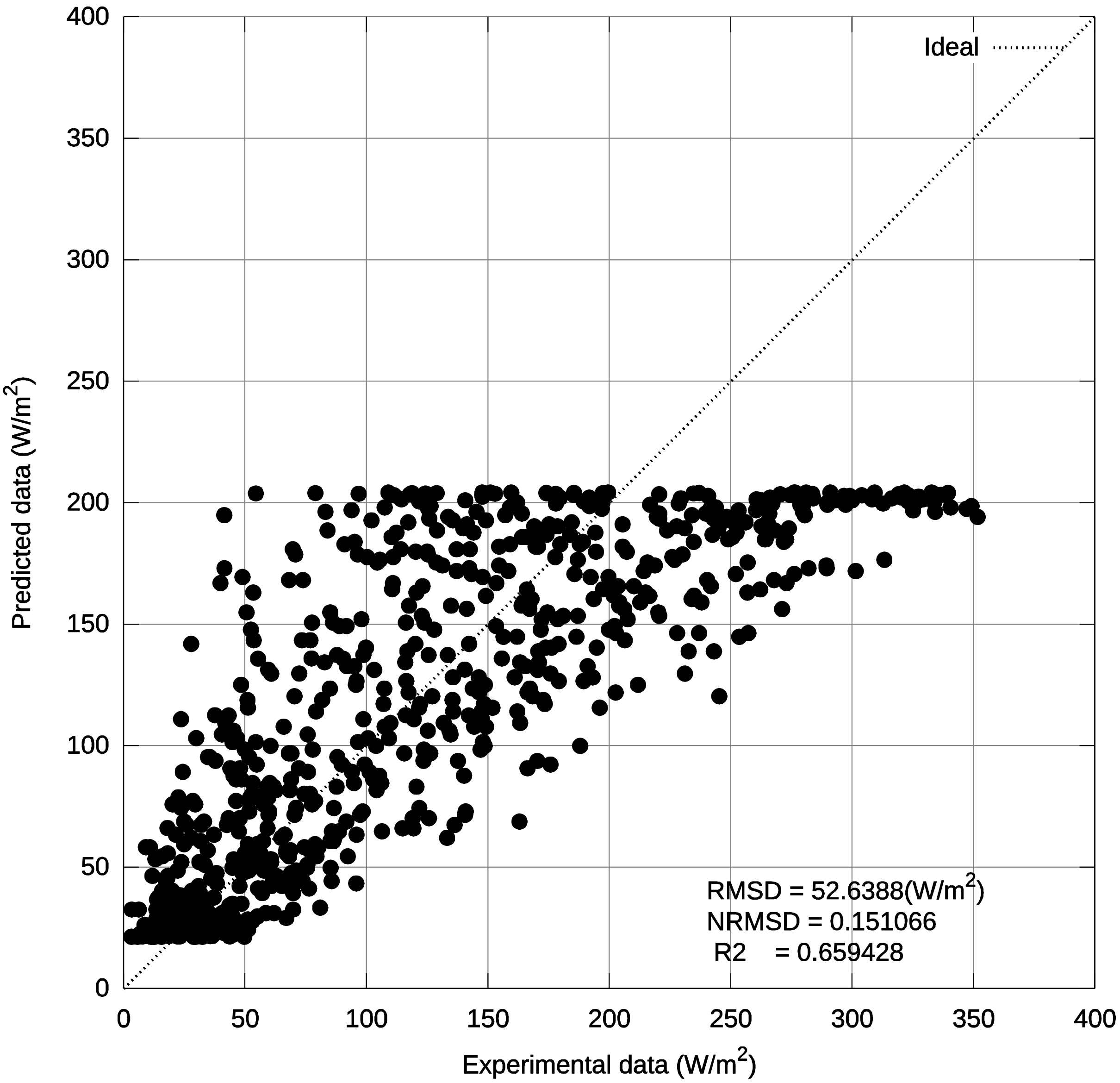

Fig. 5 Comparison of daily average values for solar radiation predicted with equation (17) with data from (UK Meteorological Office 2012) for 2005-2006

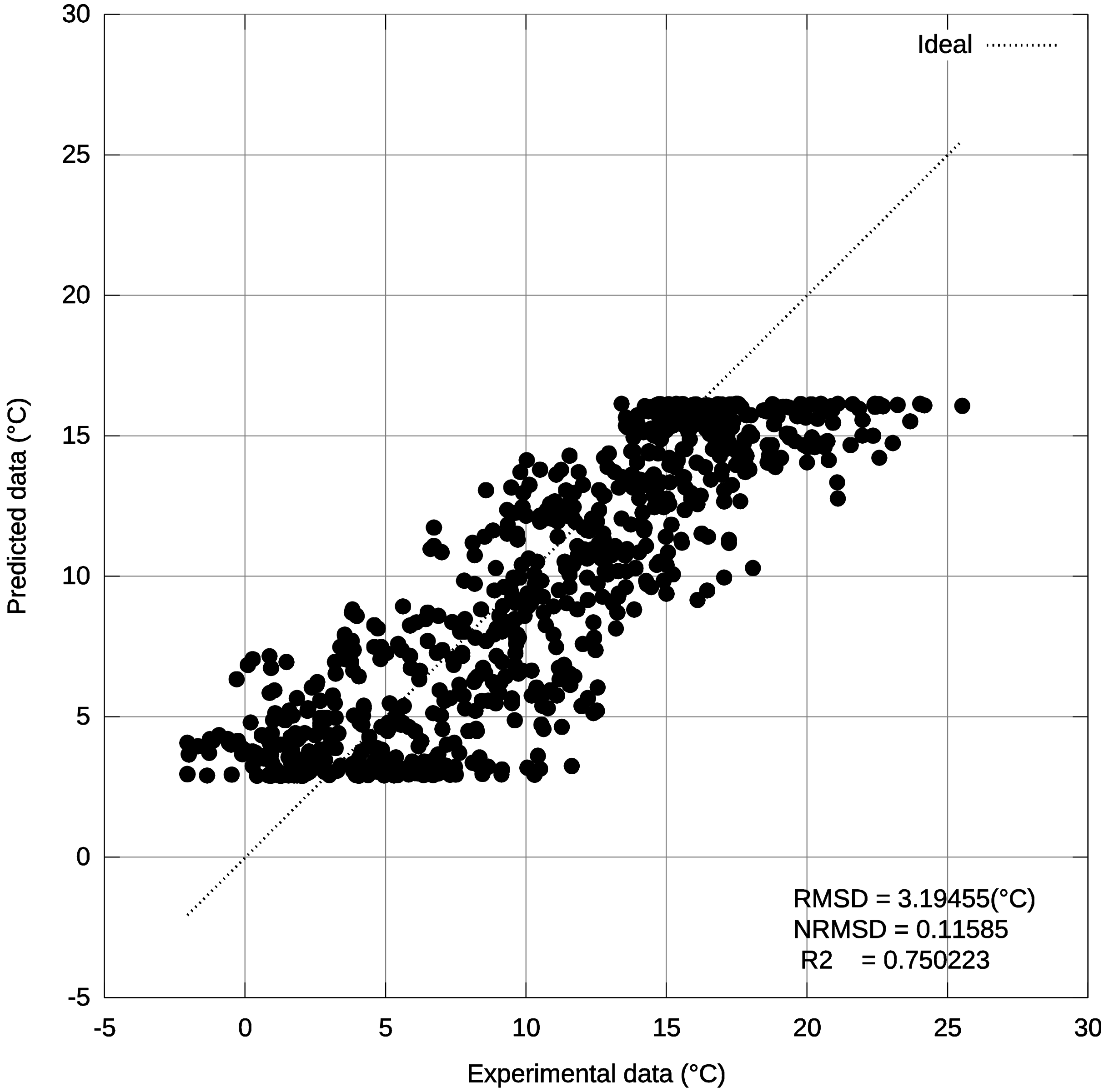

Fig. 6 Comparison of daily average values for air temperature predicted with equation (20) with data from (UK Meteorological Office 2012) for 2005-2006

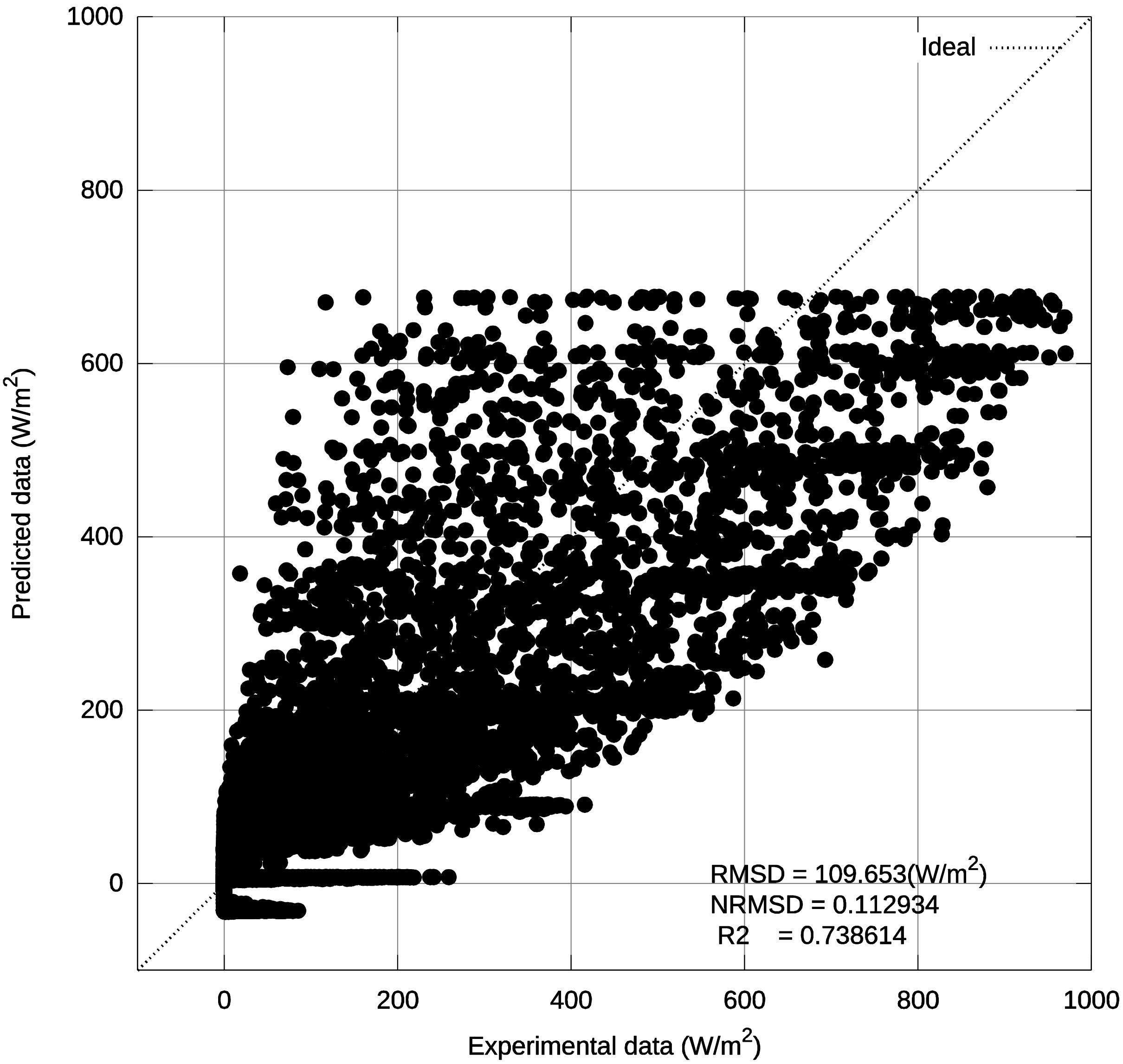

Fig. 7 Comparison of hourly average values for solar radiation predicted with equation (17) with data measured on site provided by (Carder et al. 2007) from September 2005 to August 2006

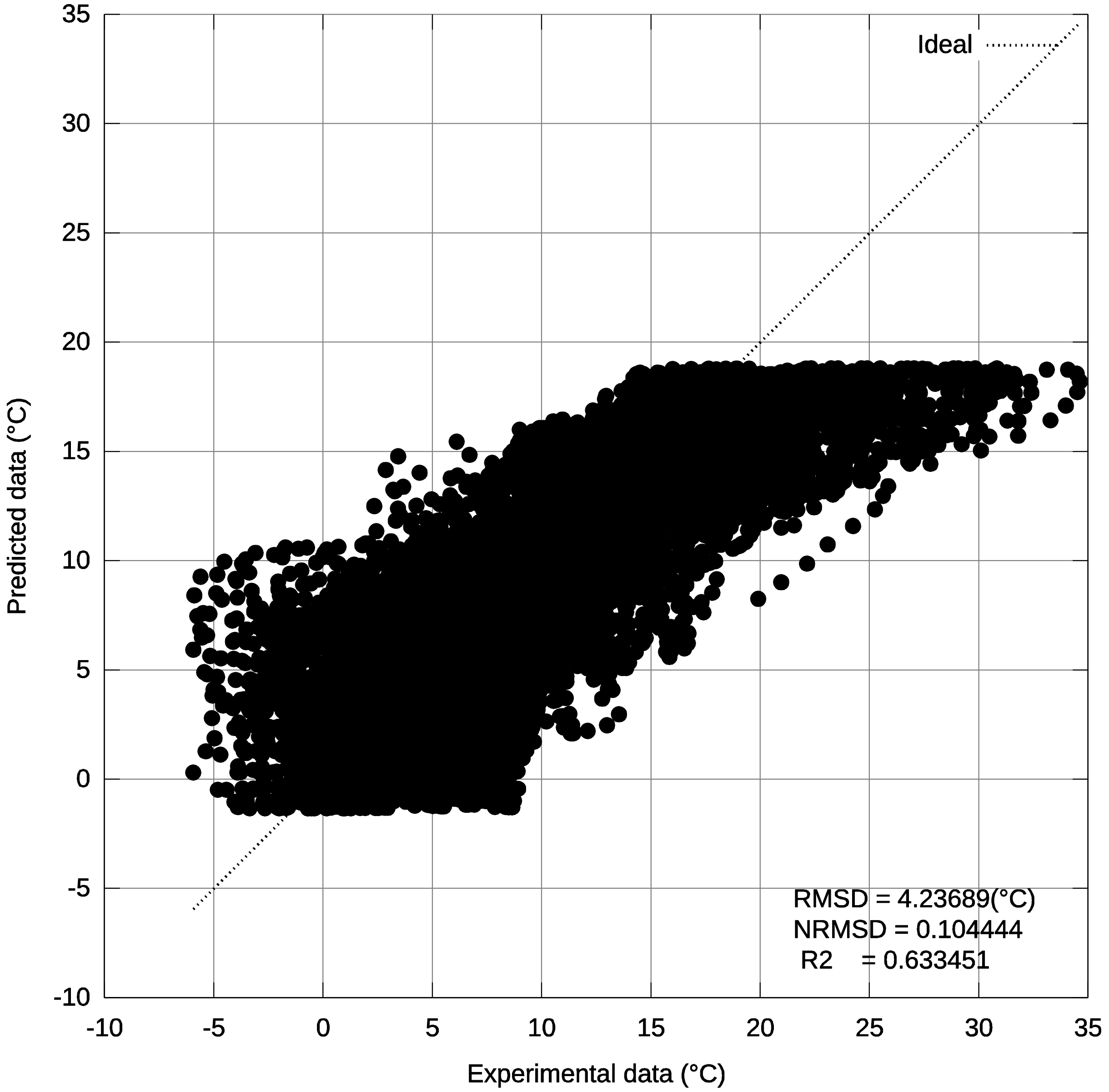

Fig. 8 Comparison of hourly average values for air temperature predicted with equation (20) with data measured on site provided by (Carder et al. 2007) from September 2005 to August 2006

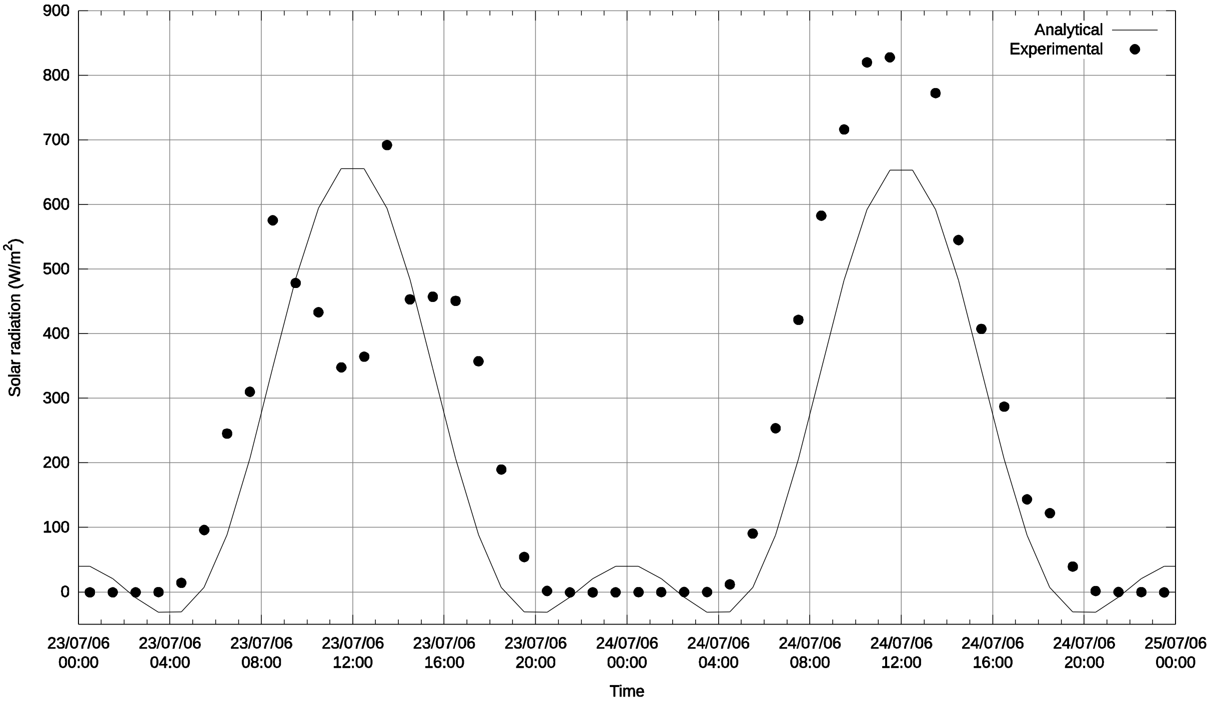

Fig. 9 Comparison of solar radiation values predicted by equation (17) and measured on site by (Carder et al. 2007) for 2 days during summer 2006

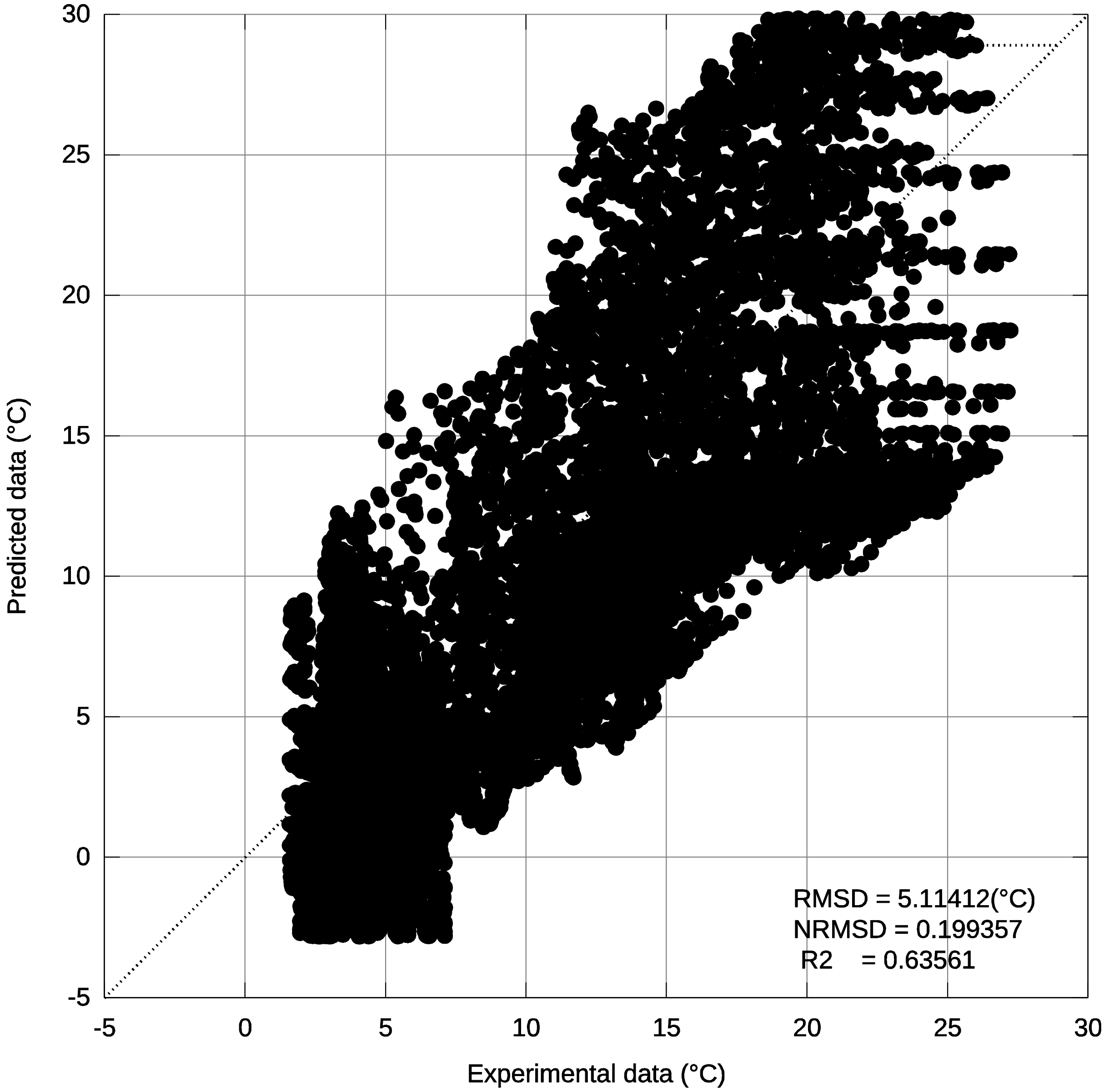

Fig. 10 Comparison of predicted vs. experimental soil temperatures at 0.025 m depth for the period September 2005 to August 2006

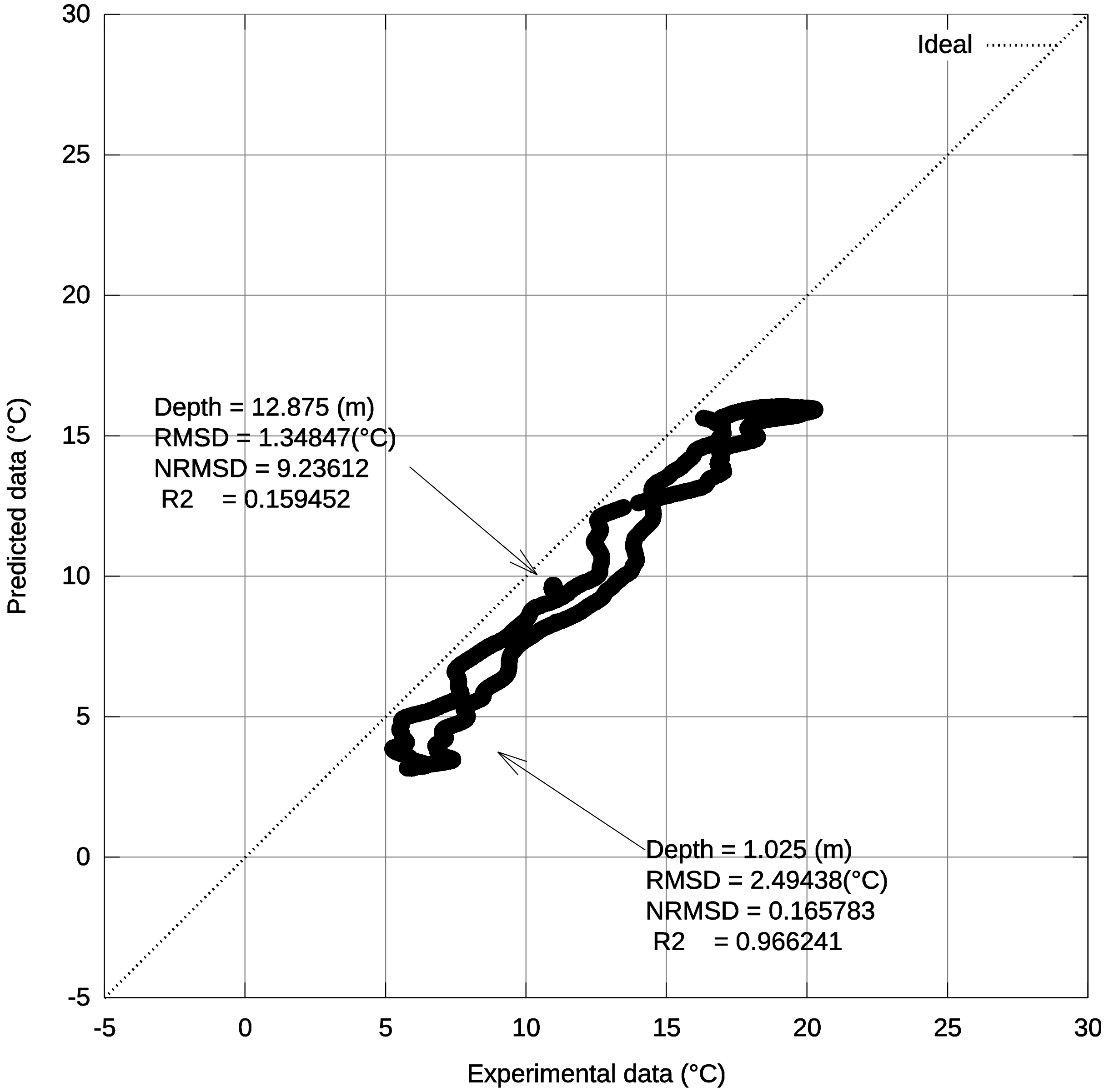

Fig. 11 Comparison of predicted vs. experimental soil temperatures at 1.025 m and 12.875 m depth for the period September 2005 to August 2006

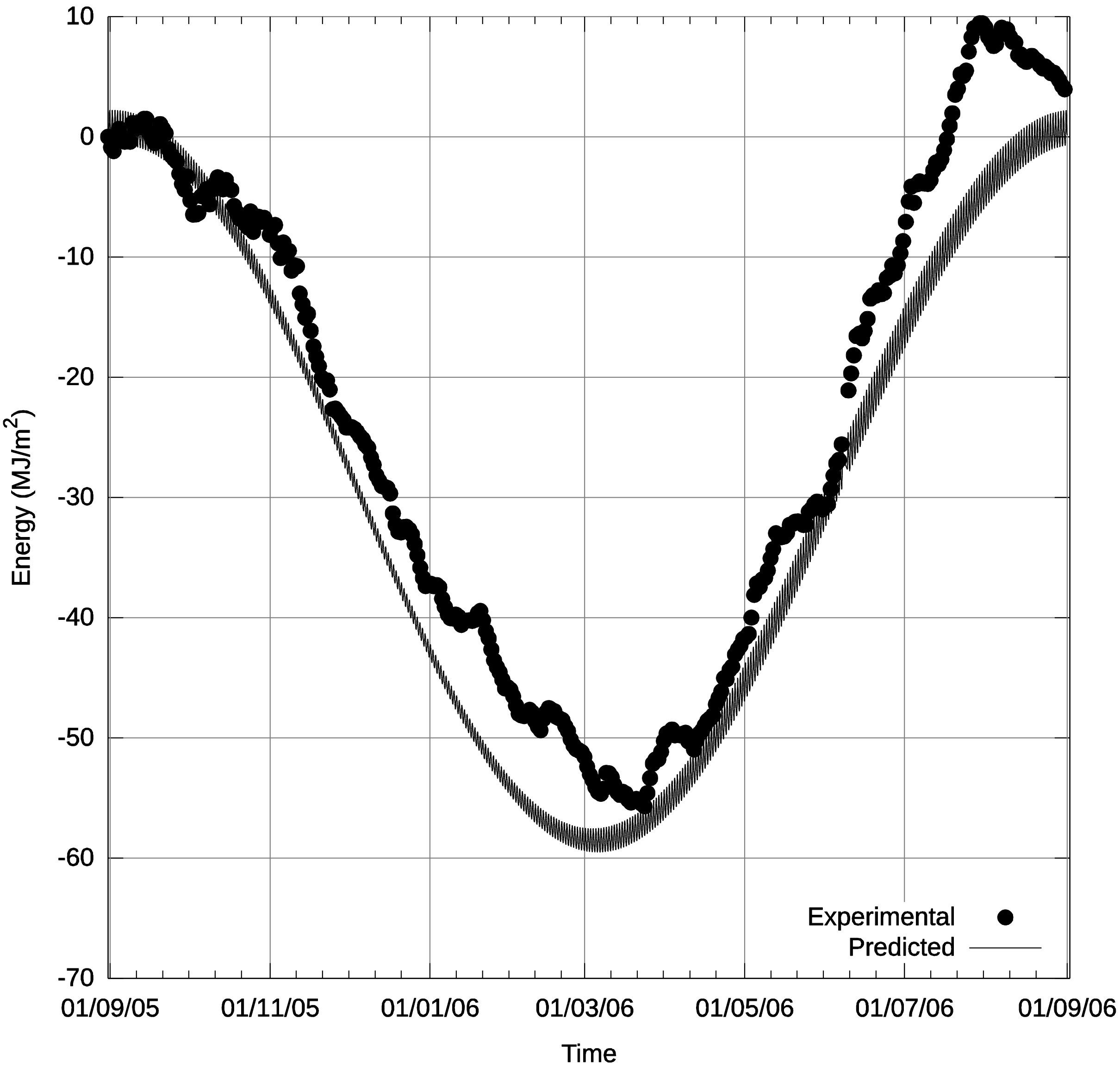

Fig. 12 Transient variation of stored energy in a column of soil 12.875 m depth for the period September 2005 to August 2006