

Appendix for “R&D and Aggregate Fluctuations”

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Appendix 1: Equilibrium

The equilibrium in this economy is described by constraints (1) and (4), the accumulation equations for the stock of R&D, (3), and capital, (7), and the following optimality conditions:

$$1 = \beta E_t \left\{ \frac{1}{x_{Ct+1}} \left[\frac{1 - \delta_K}{x_{Zt+1}} + \alpha_2 Z_t \frac{Y_{Ct+1}}{K_{Ct+1}} \right] \right\}, \quad (\text{A1})$$

$$\frac{C_t}{1 - H_{Ct}} = \frac{1 - \alpha_1 - \alpha_2}{\varphi_C} \frac{Y_{Ct}}{H_{Ct}}, \quad (\text{A2})$$

$$\frac{C_t}{1 - H_{Rt}} = \frac{1 - \lambda}{\varphi_R} \frac{Y_{Rt}}{H_{Rt}} \kappa \Xi_t, \quad (\text{A3})$$

$$\alpha_2 \frac{Y_{Ct}}{K_{Ct}} = \lambda \frac{Y_{Rt}}{K_{Rt}}, \quad (\text{A4})$$

$$1 = \beta E_t \frac{1}{x_{Ct+1}} \left\{ \frac{\alpha_1}{\kappa \Xi_t} \frac{Y_{Ct+1}}{R_{t+1}} + (1 - \delta_R) x_{\Xi t+1} \right\}, \quad (\text{A5})$$

where $x_{Ct} = C_t/C_{t-1}$ and $x_{\Xi t} = \Xi_t/\Xi_{t-1}$. Condition (A1), is the optimal condition for next period capital stock. Conditions (A2) and (A3) correspond to the optimal choice for work effort in the consumption-good and the R&D sector, respectively. Condition (A4) determines the optimal allocation of capital across sectors while condition (A5) determines the optimal choice for next period stock of R&D.

Appendix 2: VAR Estimation

We define \mathbf{y}_t as $[\Delta \ln(P_{Kt}/P_{GDPT}), \Delta \ln(Y_{Rt}/H_{Rt}), \Delta \ln(Y_{Ct}/H_{Ct}), \ln H_{Rt}, \ln H_{Ct}, \mathbf{\Lambda}_t]'$ where $\Delta \equiv 1 - L$ with L being the lag operator, P_{Kt} is the nominal price of capital investment and P_{GDPT} is the GDP price index. The nominal price of capital investment is deflated by the GDP deflator as in Altig et al. (2011). Vector $\mathbf{\Lambda}_t$, consists of the inflation rate and the nominal interest rate. Let $\varepsilon_t = [\varepsilon_{1t}, \varepsilon_{2t}]'$ where $\varepsilon_{1t} = [\varepsilon_{Zt}, \varepsilon_{Jt}, \varepsilon_{At}]'$ and $\varepsilon_{2t} = [\varepsilon_{Rt}, \varepsilon_{Ct}, \varepsilon_{It}, \varepsilon_{INt}]'$. The estimation strategy follows the methodology of Fisher (2006) which borrows from Shapiro and Watson (1988). Each regression row of (11) is estimated sequentially. For instance, the first equation of (11) is

$$\begin{aligned} \Delta \ln \left(\frac{P_K}{P_{GDP}} \right)_t &= \Phi_P + \Phi_{PP}(L) \Delta \ln \left(\frac{P_K}{P_{GDP}} \right)_{t-1} + \Phi_{PR}(L) \Delta \ln \left(\frac{Y_R}{H_R} \right)_t + \\ &\Phi_{PRH} \ln(H_{Rt}) + \Phi_{PCH} \ln(H_{Ct}) + \Phi_{PC}(L) \Delta \ln \left(\frac{Y_C}{H_C} \right)_t + \Phi_{P\Lambda}(L) \mathbf{\Lambda}_t + \varepsilon_{Zt}. \end{aligned} \quad (\text{A6})$$

As indicated by Fisher (2006), restriction 1 is equivalent to imposing a unit root in each of the lag polynomials associated with $\Delta \ln(Y_{Rt}/H_{Rt})$, $\Delta \ln(Y_{Ct}/H_{Ct})$, $\ln(H_{Rt})$, $\ln(H_{Ct})$ and $\mathbf{\Lambda}_t$. Doing so, the coefficients of (A6) become $\Phi_{Pi}(L) = \tilde{\Phi}_{Pi}(L)(1-L)$ and the regression is rewritten as

$$\begin{aligned} \Delta \ln \left(\frac{P_K}{P_{GDP}} \right)_t &= \Phi_P + \Phi_{PP}(L) \Delta \ln \left(\frac{P_K}{P_{GDP}} \right)_{t-1} + \tilde{\Phi}_{PR}(L) \Delta^2 \ln \left(\frac{Y_R}{H_R} \right)_t + \\ &\tilde{\Phi}_{PRH} \Delta \ln(H_{Rt}) + \tilde{\Phi}_{PCH} \Delta \ln(H_{Ct}) + \tilde{\Phi}_{PC}(L) \Delta^2 \ln \left(\frac{Y_C}{H_C} \right)_t + \\ &\tilde{\Phi}_{P\Lambda}(L) \Delta \mathbf{\Lambda}_t + \varepsilon_{Zt}. \end{aligned} \quad (\text{A7})$$

Since investment-specific shocks are not orthogonal to the variables on the right hand side, ordinary least squares will give inconsistent estimates. According to our economic model the

exogenous shock ε_{Zt} is uncorrelated with variables at $t-1$. Consequently, N lags of variables $\Delta^2 \ln(Y_{Rt}/H_{Rt})$, $\Delta^2 \ln(Y_{Ct}/H_{Ct})$, $\Delta \ln(H_{Rt})$, $\Delta \ln(H_{Ct})$ and $\Delta \mathbf{\Lambda}_t$ are used as instruments.

To identify the other two shocks, restrictions 2 and 3 are imposed on the second and third equation of (11) following the same methodology. Since investment specific shocks have an impact on labor productivity in the R&D sector in the long-run, the estimate of ε_{Zt} is used as an instrument in the second regression to ensure that $\widehat{\varepsilon}_{Jt}$ will be orthogonal to $\widehat{\varepsilon}_{Zt}$. Likewise, estimates $\widehat{\varepsilon}_{Zt}$ and $\widehat{\varepsilon}_{Jt}$ are used as instruments in the third regression to ensure orthogonality with $\widehat{\varepsilon}_{At}$

Note that system (11) can be written as

$$\begin{pmatrix} \mathbf{C}^{11} & \mathbf{C}^{12} \\ 3 \times 3 & 3 \times 4 \end{pmatrix} \begin{pmatrix} \mathbf{y}_{1t} \\ 3 \times 1 \end{pmatrix} = \begin{pmatrix} \Psi^{11}(L) & \Psi^{12}(L) \\ 3 \times 3 & 3 \times 4 \end{pmatrix} \begin{pmatrix} \mathbf{y}_{1t-1} \\ 3 \times 1 \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ 3 \times 1 \\ \mathbf{C}^{21} & \mathbf{C}^{22} \\ 4 \times 3 & 4 \times 4 \end{pmatrix} \begin{pmatrix} \mathbf{y}_{2t} \\ 4 \times 1 \end{pmatrix} = \begin{pmatrix} \Psi^{21}(L) & \Psi^{22}(L) \\ 4 \times 3 & 4 \times 4 \end{pmatrix} \begin{pmatrix} \mathbf{y}_{2t-1} \\ 4 \times 1 \end{pmatrix} + \begin{pmatrix} \varepsilon_{2t} \\ 4 \times 1 \end{pmatrix}, \quad (\text{A10})$$

where $\mathbf{y}_{1t} = [\Delta \ln(P_{Kt}/P_{GDPt}), \Delta \ln(Y_{Rt}/H_{Rt}), \Delta \ln(Y_{Ct}/H_{Ct})]'$ and $\mathbf{y}_{2t} = [\ln H_{Rt}, \ln H_{Ct}, \mathbf{\Lambda}_t]'$. Notice that the coefficients \mathbf{C}^{11} , \mathbf{C}^{12} , $\Psi^{11}(L)$ and $\Psi^{12}(L)$ are derived by unravelling the estimates from (A7), (A8) and (A9). Therefore, the first three equations of the system are exactly identified. On the contrary, the last four equations of (A10) cannot be identified because the structural error ε_{2t} cannot be identified separately from the reduced-form error $(\mathbf{C}^{22})^{-1} \varepsilon_{2t}$. Nevertheless, the shocks in ε_{2t} can be identified up to a particular transformation. It can be shown that there is a family of observational equivalent parameterizations of the structural form where the responses of \mathbf{y}_{2t} to the shocks in ε_{1t} are invariant. To see this,

let Θ be the following orthonormal matrix:

$$\Theta = \begin{pmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 4} \\ \mathbf{0}_{4 \times 3} & \theta_{4 \times 4} \end{pmatrix},$$

where \mathbf{I} denotes the identity matrix and θ is an orthonormal matrix. Premultiplying both sides of (A10) by Θ , the last four equations can be written in reduced form as

$$\mathbf{y}_{2t} = (\mathbf{C}^{22})^{-1} \Psi^{21}(L) \mathbf{y}_{1t-1} + (\mathbf{C}^{22})^{-1} \Psi^{22}(L) \mathbf{y}_{2t-1} - (\mathbf{C}^{22})^{-1} \mathbf{C}^{21} \mathbf{y}_{1t} + \Gamma \epsilon_{2t}, \quad (\text{A11})$$

where $\Gamma = (\theta \mathbf{C}^{22})^{-1}$ and $\epsilon_{2t} = \theta \varepsilon_{2t}$. Let $\widehat{\mathbf{C}}^{22}$ be an estimate of \mathbf{C}^{22} and $\widehat{\varepsilon}_{2t}$ be the corresponding fitted disturbances. An alternative estimate of \mathbf{C}^{22} is $\widetilde{\mathbf{C}}^{22} = \theta \widehat{\mathbf{C}}^{22}$ with corresponding disturbances $\widetilde{\varepsilon}_{2t} = \theta \widehat{\varepsilon}_{2t}$. The estimates $\widehat{\mathbf{C}}^{22}$ and $\widetilde{\mathbf{C}}^{22}$ fit the data equally well. If $(\widehat{\mathbf{C}}^{22})^{-1}$ is lower triangular then the last two equations in (21) can be estimated sequentially using the residuals of the previously estimated equations. Suppose that $(\widehat{\mathbf{C}}^{22})^{-1}$ is not lower triangular. Since $\widehat{\mathbf{C}}^{22}$ is nonsingular, there exist an orthonormal matrix θ and a lower triangular matrix \mathbf{R} such that $\widehat{\mathbf{C}}^{22} = \theta' \mathbf{R}$. It follows that $\theta \widehat{\mathbf{C}}^{22} = \mathbf{R}$ is lower triangular, which implies that $\widehat{\Gamma} = (\widehat{\mathbf{C}}^{22})^{-1} \theta'$ is lower triangular. Consequently, the fourth equation in (A10) is estimated using $\widehat{\varepsilon}_{Zt}$, $\widehat{\varepsilon}_{Jt}$ and $\widehat{\varepsilon}_{At}$ as regressors to ensure orthogonality with $\widehat{\varepsilon}_{Rt}$ and the fifth equation is estimated using $\widehat{\varepsilon}_{Zt}$, $\widehat{\varepsilon}_{Jt}$, $\widehat{\varepsilon}_{At}$ and $\widehat{\varepsilon}_{Rt}$ as regressors to ensure orthogonality with $\widehat{\varepsilon}_{Ct}$. The sixth and the seventh equations are estimated in a similar way. All four equations are estimated by IV, using N lags of \mathbf{y}_t as instruments.

Appendix 3: Data

The nominal value of R&D is the sum of the costs of the R&D activity of both private and government organizations.¹ Private organizations consist of businesses such as private universities and colleges, private hospitals, charitable foundations, other nonprofit institutions serving households and most Federally Funded Research and Development Centers (FFRDC). Government organizations consist of the Federal Government, state and local governments (excluding universities and colleges), public universities and colleges, and FFRDC administered by state and local governments (primarily public universities and colleges). The BEA first compiles nominal R&D investment data from NSF surveys and then adjusts them accordingly so that the final series are statistically and conceptually consistent with the definitions in the NIPA tables.

Real R&D investment is derived by deflating detailed current-dollar expenditures by appropriate price indexes. Two price indexes are constructed and utilized in the satellite account: an input price index and an aggregate output-based price index. The former is based on an aggregation of detailed price indexes for the inputs used to create R&D output while the latter is a weighted average of the output prices of other products produced by 14 R&D-intensive industries with weights corresponding to each industry's share of annual business R&D investment. The output-based price index is the best price measure available in capturing productivity growth in R&D-intensive industries and thus, it is used throughout our analysis to deflate nominal R&D investment.²

¹Expenditures on R&D by government and nonprofit institutions are included in consumption expenditures. For more information refer to Mataloni and Moylan (2007).

²For a detailed discussion about the price indices refer to Okubo et al. (2006) and Lee and Schmidt (2010)

Employment data in the R&D sector: The NSF reports data on domestic employment by R&D performing companies which does not include universities and government. Although there are various statistics for employment from NSF surveys, there are difficulties in constructing an aggregate measure of R&D employment series. First, there are no complete data for all years of our sample and second, it is unclear which of the participants in the surveys are actually involved in performing R&D activities. Given those issues and since R&D investment by universities and government constitutes, on average, only 20 percent of total R&D investment we approximate aggregate employment for R&D by the domestic employment of R&D performing companies.

References

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