Integral non-linearity error measurement through statistical analysis of a multi-tone test signal

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Abstract

Conventionally, the integral non-linearity of an analogue to digital converter (ADC) is measured using a pure sine wave test signal [1]. The measured amplitude probability distribution function (APDF) of the ADC output is compared with the ideal distribution and the differences used to calculate integral non-linearity error (INL). Accuracy of this method is limited by the practical problem of generating a pure sine wave. Others have proposed random noise as an alternative to the sine wave test signal [2]. Unfortunately, a much larger number of samples is required for this method to achieve a given confidence level. This is due mainly to the need to be sure that they fit a Normal distribution with sufficient accuracy. The approach reported in the literature has been to fit a Tchebycshev polynomial to the INL result [3]. However, results measured using noise do not correlate well with those of sine wave tests. A new approach to testing will be described that makes use of a periodic signal derived from pseudo random binary sequences and offers the potential for increased accuracy in measuring INL.

1. Introduction

A multi-tone, multi-level test signal can be generated by low pass filtering a pseudo random binary sequence (PRBS). The APDF and spectral distribution of this signal depends on the generating polynomial used. A PRBS can be implemented using a shift register with exclusive-or feedback from certain taps of the register. Particular selections of taps produce maximal length or m-sequences [4]. These have the property that their spectrum consists of spectral lines of uniform amplitude and frequency spacing. As the sequence length is increased the spectral lines become closer together and the APDF approaches a Normal distribution. The APDF of a long sequence can be calculated using the error-function for a Normal distribution. However, the number of samples required to approximate a Normal distribution is a significant disadvantage. In practice, as the peak amplitude of the test signal is limited, the true distribution is a truncated Normal one.

An alternative approach is adopted for this present paper. Figure 1 depicts the test signal generator. Two relatively short m-sequences are used. Each produces an APDF that is significantly different from Normal.
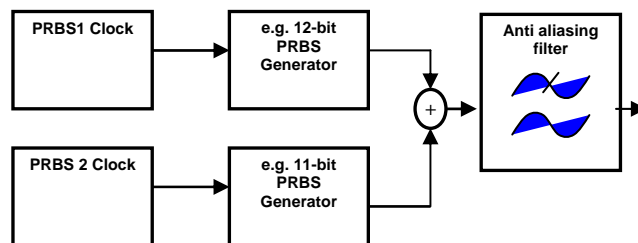

FIGURE 1 Test signal generator

When the clock rate of each shift register is synchronised to a common frequency and the two

multi level signals are added, the combined signal is periodic but the APDF is closer to a Normal distribution. The APDF of this signal was compared to a uniform distribution and the error is depicted in Figure 2. Figure 3 shows the same process applied to a signal with a truncated Normal distribution. The Y axis is the error between a uniform PDF and the PDF of the undistorted test signal in least significant bit amplitude units (LSBs). The X axis is the ADC code value in decimal.
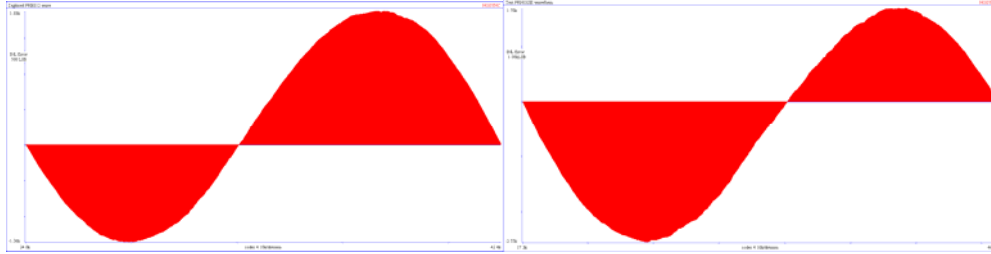

FIGURE 2 APDF 'error' of test signal   FIGURE 3 APDF 'error' of Normal distribution

 It can be seen that both these figures have similar plots. The new test signal can be generated with greater purity than a sine wave and requires a smaller number of samples than required by one long sequence. However, the measurement of linearity error is problematic as the ideal APDF of the undistorted test signal must be known for INL to be calculated using the conventional method. One solution to this is to generate and store the APDF depicted in Figure 2 as a reference. As the storage required increases with the number of bits of resolution of the ADC, this approach becomes impractical for high speed ADCs. A method where the ideal APDF could be calculated from a limited set of basis functions would reduce the need for storage. The error-function can be used to calculate the plot in Figure 3, and as mentioned earlier, this is the INL method proposed by others. A summary of an investigation of a new approach that makes use of improved modelling of the APDF error function is presented next.

2. APDF 'error' of the new test signal

A variety of polynomial and statistical curve fitting methods were used in an investigation of the most accurate mathematical description for the characteristics shown in Figures 2 and 3. This approach was investigated to see if it offered any advantages when compared with the error-function distribution. Reduced calculation time would be an advantage for real time measurement and correction of non-linearity. The truncated Normal distribution used for Figure 3 was quantised to 16 bits so that one LSB is 1 part in 65536 of the full scale amplitude range. The peak to peak Y axis shown in Figure 2 is 4,000 LSB, or one sixteenth of full scale range. In contrast, the error in using a Normal distribution is 18,000 LSB or more than a quarter of full scale range, as shown in Figure 3.

The problem was approached by considering what type of functions would best fit this characteristic. Visual inspection indicates that one cycle of a sinewave of suitable amplitude, phase and offset might be a good starting point. Investigations were undertaken using MATLAB curve-fitting functions. Polynomial fitting produced limited success as the fit near the end points of the plot was poor. The best result was obtained by fitting a Fourier series to the plot. This was a relatively simple solution and required only 5 harmonics. Equation (1) below gave the best fit and the coefficients are shown in Table 1. The residual error after fitting is shown in Figure 4.

$F(x) = a_0 + a_1\cos(x\omega) + b_1\sin(x\omega) + a_2\cos(2x\omega) + b_2\sin(2x\omega)$
$+ a_3\cos(3x\omega) + b_3\sin(3x\omega) + a_4\cos(4x\omega) + b_4\sin(4x\omega) + a_5\cos(5x\omega) + b_5\sin(5x\omega)$ (1)

Coefficient	a_0	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4	a_5	b_5
Result of fit	-162.4	1266	963.8	-574.1	1337	393.8	-540.9	-206.6	179.4	79.31	-47.8
Harmonic Ratios				$-2a_1$	$7b_1$	$-1.25a_2$	$-3b_2$	$-2a_3$	$-3b_3$	$-3a_4$	$-3b_4$

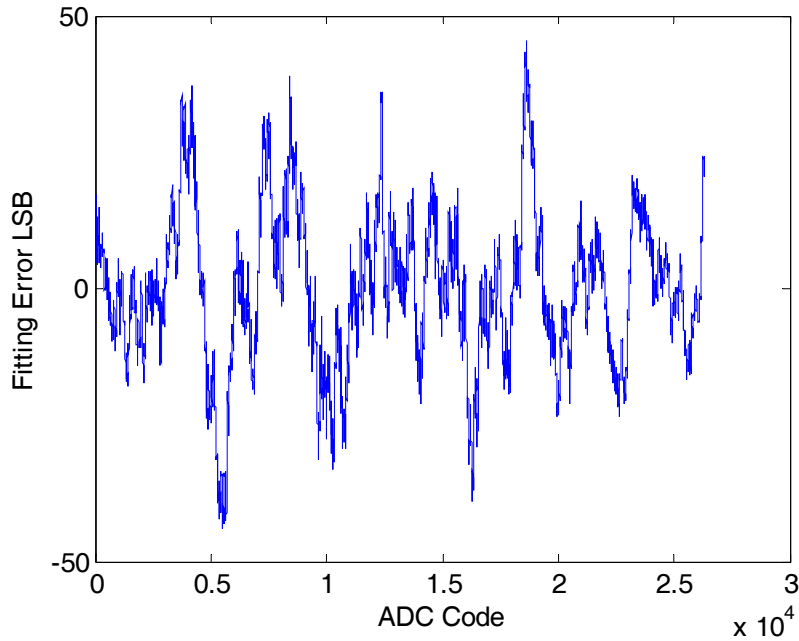TABLE 1   Best fit Fourier Coefficient values

FIGURE 4   Residual error from 11 coefficient Fourier fit

The truncated Normal distribution required only three harmonics for the best fit, and the residual error appeared to be not periodic. Furthermore, the harmonic coefficients had no obvious integer relationship so the method of fitting a Fourier series was not successful for a Normal distribution as it would not speed up calculations. In contrast, Figure 4 shows the APDF of the residual for the new test signal. The plot in Figure 4 does not fit well to any of the standard curve fitting functions.

This result suggests the possibility of tailoring the APDF of the test signal. Ideally, there would then be no residual error in this 11 coefficient fit. This area of research will be investigated in future work as it was beyond the scope of the project.The standard deviation of this residual error was 14.08 LSB with a maximum error of +/- 40 LSB compared to the original deviation from a uniform distribution of +2.4k to – 1.5k LSB. As the simulation used 16 bit quantisation, this error is less than the quantising error of an ideal 13 bit ADC. The Coefficients described in Table 1 are therefore a useful basis for calculating the APDF of the undistorted test signal when testing a 12 bit ADC to an accuracy of better than 0.5 bits. The integer harmonic ratios shown in Table 1 indicate that a fast algorithm could be produced by first fitting a_0, a_1 and b_1 to the data and then calculating the other coefficients before a final fit.

3. Results

The non-linearity of an ADC was simulated by assuming that it could be modelled by a third order polynomial as shown in equation 2 where the values of the coefficients are a = 0, b = 1, c = 0.01, d = 0.01.

$$Y(t) = a + b\ x(t) + c\ x(t)^2 + d\ x(t)^3 \qquad\qquad (2)$$

The test signal was applied to this model and the distortion signal separated from the original test signal by using two synchronised comb filters. As these comb filters remove only the test signal this distortion signal has the same APDF as would have been obtained by subtracting the APDF of the test signal from the distorted test signal. This is equivalent to calculating the integral non-linearity error. Next, the APDF 'error' of this linearity error signal was calculated using a uniform distribution as the reference. When the cubic term was set to zero the APDF shown in Figure 5 was produced. By comparing this with Figure 2 it can be seen that the quadratic term has produced an APDF with even symmetry. If the APDF of the undistorted test signal can be approximated using the results in Section 2 then a comparison of both APDFs could enable the quadratic coefficient 'c' to be determined with a precision close to 13 bits.
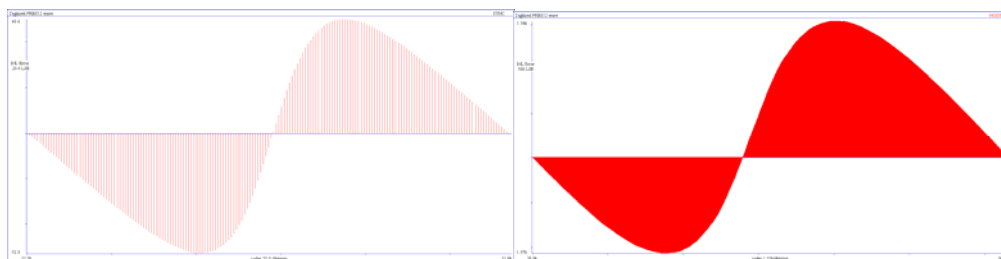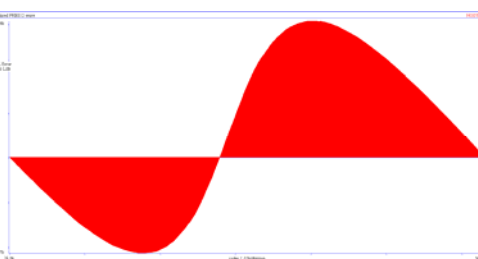

FIGURE 5 Quadratic   FIGURE 6 Cubic

In contrast, when the quadratic term is set to zero the APDF becomes asymmetrical and is depicted in Figure 6. Curve fitting of these error functions is being investigated as a method of obtaining the value of the coefficients of Equation 2.

4. Conclusion

The results reported provide a basis for calculating the integral non-linearity error of an ADC by using a Fourier series to represent the APDF of the test signal. This reduces the storage requirements for the calculation and avoids the need for a large number of test samples. Furthermore, the change in symmetry of the APDF error of the distortion signal could facilitate calculation of the coefficients in a power series model of the non-linearity. EPSRC support under grant EP/D04264X/1 is gratefully acknowledged.

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