THE ROLE OF PUBLIC SECTOR

IN ECONOMIC PERFORMANCE

THEORETICAL ANALYSES OF THE EFFECT OF FISCAL POLICIES ON LONG-RUN GROWTH WITH A SELF-INTERESTED GOVERNMENT

&

EMPIRICAL STUDY OF THE ECONOMIC RELATIONSHIP BETWEEN THE GOVERNMENT AND NONGOVERNMENT SECTORS IN CHINA

by

XIAODONG CHEN

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Abstract

This thesis responds to the development policy debate on whether the popular Beijing Consensus is an alternative, especially for developing countries, to the Washington Consensus of market-friendly policies. The advocates of the Beijing Consensus have overlooked the facts that China's development model has much in common with the Washington Consensus, China has profited from the globalisation, and the reform for establishing a market-oriented economy is the key factor of the rapid development and outstanding achievements in China. The critics have reluctantly acknowledged the successful strategies employed by the policy makers in Beijing, however, they branded China as an authoritarian country, and are wilfully or unintentionally blind to the diversity, inclusiveness and competitiveness in the 'quasi-nonpartisan' Chinese political system. This thesis explain how a farsighted government can increase long-run growth by fiscal policies, why free market and the ambitious government are not contradictory in China, and why so many 'democratic' countries violate the first two prescriptions of the Washington Consensus: fiscal discipline, and public expenditure priority to pro-growth investment. The theoretical analyses show that the strategy of a farsighted government is to sacrifice the first several generations but benefit all future generations through cutting nonproductive public spending and giving expenditure priority to productivity-enhancing expenditure, so that a higher growth rate leads to a 'quasi-Pareto improvement' among generations. Nevertheless, farsighted fiscal policy together with 'hedonistic citizens' has a side effect, the welfare loss for both the government and the citizens. The Barro model is an exception in which a farsighted government has no place to increase the growth rate because welfare maximisation equals growth maximisation. The empirical results reveal the strong substitution relationship between the government and nongovernment capital, and thus the general CES technology rather than the Cobb-Douglas technology is the suitable structure containing the two types of capital in the production function. However, the government capital has become more complementary to the nongovernment capital due to the deepening reforms of SOEs and fiscal policies since 1992.

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Chapter 1: Introduction

Since the end of the Cold War, most politicians and economists have agreed that the Washington Consensus has achieved a glorious and definitive victory. The term Washington Consensus was first conceived in 1989 by John Williamson to refer to a set of ten market-based policy recommendations. Williamson (2002) presents that these commonly shared themes promoted by Washington-based institutions, such as the IMF, World Bank, and U.S. Treasury Department, were necessary for the recovery of Latin American countries from the economic and financial crises in the 1980s. Subsequent to the minting of the term, it has been fairly widely used as a form of neoliberal manifesto which supports the standpoint that market force rather than government planning should shape economic outcomes. Some neoliberal scholars even argue that the market-democratic capitalism is the best and final model of the human society. Nevertheless, since 1990s, the world has experienced multiple financial turmoils, economic stagnation, and even the collapse of several nations' economic systems. The most recent and more severe 2008 financial crisis has further eroded confidence in the Western. Some economists blame these crises on the implementations of the Washington Consensus, and therefore the phrase 'Beijing Consensus' seems a more popular term in debates about economic and political policies. The new conception was coined by Joshua Cooper Ramo (2004) to pose China's development model as an alternative, especially for nations in the Third World, to the Washington Consensus of market-friendly policies.

Before we draw a conclusion on whether China's national economic strategy is disrupting the Washington consensus, we had better review the ten economic policy prescriptions: (1) Fiscal discipline with avoidance of large deficits; (2) Redirection of public expenditure from subsidies, especially subsidies to cover the losses of SOEs, toward broad-based provision of key pro-growth, pro-poor services such as primary education, primary health care and infrastructure development; (3) Reform tax system by broadening the tax base and adopting moderate marginal tax rates; (4) Liberalising interest rates by market forces; (5) Competitive exchange rates; (6) Liberalization of trade; (7) Liberalization of inward foreign direct investment; (8) Privatisation of SOEs; (9) Deregulation; (10) Legal security for property rights.

From China's experiences in economic development, we can find that the orientation of China's reform generally follows the ten policy recommendations:

Currently, China does even better than most capitalist countries in (1), (2), (3), (6) and (7), with much low deficits relative to GDP, high ratio of public productive expenditures to GDP, relative low tax rates, the crown of global trade champion, and the crown of champion in FDI stock (including FDI in Hong Kong).

China tries its best to fulfill (10) but should do more. In 2011 China received 526,412 applications, exceeding those in any other country, according to the World Intellectual Property Organisation, a UN body which follows 125 patent offices, China accounted for 72% of the world's patent-filing growth between 2009 and 2011.

China partially accepts the ideas of (4), (5), (8) and (9) by gradual reform rather than neoliberal economic shock therapy. Most leaders of the Communist Party assume that the free market is powerful and generally efficient but not perfect, so that market force should dominate the economic performance but the state with pragmatic policies is essential to maintaining social stability and high economic growth in the long run, and sometimes protecting free market from the attacks from both domestic and foreign speculators. China favors globalisation, meanwhile, maintains economic sovereignty and pursues its own priorities. For example, China had privatised, restructured and even closed lots of unprofitable SOEs in the 1990s, but now still keeps more than 100 state-owned enterprise giants mainly in energy, transportation, telecommunication and banking industries. These giants not only provide relatively cheap infrastructure and low-interest loan to stimulate the economy, but also keep China's economic independence. Despite regarded as absurd and inefficient monsters by many advocators of Washington Consensus, most of these SOEs have performed better than their competitors outside China since the 2008 economic crisis; moreover, they followed the government's stimulus policies in the crisis and helped China to achieve high growth rates (9.6%, 9.2% and 10.4%) from 2008 to 2010. According to the International Comparison Program (2014), hosted by the World Bank, mainland China's economy based on Purchasing Power Parities (PPPs) is forecasted to be the world's largest in 2014, occupying about 17% of the global economy. Ramo (2004) emphasises that China actively seeks self-determination from outside pressure, as it is imposed by 'hegemonic powers' such as the United States.

In brief, China's economic development model has much in common with the Washington Consensus; Nevertheless, Chinese government dissents in part from the neoliberal ideology or 'market fundamentalism'.

Due to China's high-speed growth in the past three decades, most criticism has been focused on the political system in China. Williamson argues that the system is authoritarianism as opposed to democracy. Other neoliberals also call it as single-party system or one-party state. However, these descriptions are inaccurate. The characteristics of the political system in China can be summarised as follows:

Firstly, there are other political parties which participate in the National People's Congress and the Chinese People's Political Consultative Conference, some ministers and national leaders are also from these parties.

Secondly, there is no appreciable distinction of ideology between different parties, but there are different viewpoints on how to reform China within each party and among ordinary Chinese people. Constant trial and error experimentation is recognized by different political factions. Hence, political pluralism does exist in China.

Thirdly, there is stiff competition among officials. Local performance evaluations are based, to a large extent, on economic development and social stability within their jurisdictions. Their achievements rather than promises and provocative speeches in election campaigns shape their fortunes in political career.

Fourthly, the collective decision-making mechanism in Politburo is different from the system in the United States in which the supreme power is concentrated in the hands of the President.

Fifthly, some Politburo members hold powerful regional positions, thus both central enthusiasm and local initiative are stimulated; some competent and relatively young successors are elected into the Central Committee and Politburo, therefore both long-term and immediate interests are taken into account when the Communist Party makes decisions.

Finally, senior officials in the Politburo Standing Committee are in the same age group and most are below 70 years old. The President and the Primer would not counteract each other's efforts even they were once competitors, because they only have a ten-year tenure and will retire together. They are in the common fate group and do not have a second chance after the end of term, so they must cooperate with each other after the election.

Overall, the current political system in China is much close to a nonpartisan system feathered by fierce competition before election and fairly close cooperation between the top decision makers and their successors and within the Politburo after the election. It is also called as a ‘quasi-nonpartisan political system’ in this thesis.

Why do most people in the world prefer a system with two political monopolies the ‘Apple Party’ and the ‘Banana Party’ to another system with a big party named the ‘Fruits Party’? We can not ignore the diversity and competitiveness in the current Chinese political system.

There is no doubt that China should reform the current political system and establish a more democratic society. Nevertheless, democracy is not necessary equal to a two-party or multiparty system.

Williamson (2002) argues:

> Despite the significant differences in the interpretation of fiscal discipline, I would maintain that there is very broad agreement in Washington that large and sustained fiscal deficits are a primary source of macroeconomic dislocation in the forms of inflation, payments deficits, and capital flight. They result not from any rational calculation of expected economic benefits, but from a lack of the political courage or honesty to match public expenditures and the resources available to finance them. Unless the excess is being used to finance productive infrastructure investment, an operational budget deficit in excess of around 1 to 2 percent of GNP is prima facie evidence of policy failure.

Williamson has not explained why there is ‘a lack of the political courage or honesty’ in a democracy while China, with an ‘authoritarian’ government, can obey the fiscal discipline.

This thesis is initially motivated to explain why China's economic model as an alternative development philosophy exactly has much in common with the currently dominant ideology the Washington Consensus, meanwhile, it is featured by a powerful and pro-development state, selective borrowing of foreign ideas and Deng Xiaoping's pragmatic strategies.

This thesis focuses on the effect of fiscal policies on long-run growth and the economic relationship between the government and nongovernment sectors. The contents are related to the policy prescriptions (1), (2), (3), (8) and (9) in the Washington Consensus. Moreover, this thesis provides the political reasons behind how and why a government can increase long-run growth rate through farsighted fiscal policies and why politicians in 'democratic' countries have a shorter time horizon.

This thesis is organised as follows: Chapter 2 surveys the literature with regard to the endogenous growth models with fiscal policies for the theoretical analyses in Chapter 3, subsequently provides important studies on investment, infrastructure development, reforms of SOEs and fiscal policies in China, and the method of indirect inference, as the literature for the empirical work in Chapter 4. In Chapter 3, five models are employed to illustrate whether it is possible for a farsighted government to increase the long-run growth rate. This chapter contributes to the literature with lots of new results due to the assumption of self-interested (farsighted or shortsighted) government. Chapter 4 investigates the economic relationship between government and nongovernment capital in China by the method of indirect inference, revealing the strong substitution relationship and thus concludes that the CES structure is better than Cobb-Douglas function in describing the economic relationship between the two types of capital in the production function. Chapter 5 summarises the important results in Chapter 3 and Chapter 4, gives a link between the theoretical and empirical results, and finally concludes this thesis.

Chapter 2: Literature Review

2.1 Introduction

The scope for fiscal policy to affect economic growth is dependent on the underlying model of growth. When the Solow-Swan model, which assumes decreasing marginal productivities to each input, dominated the neoclassical growth theorists' view of economic development, fiscal policy was powerless in explaining the long-run growth. The pioneering work of Barro (1990) opened the window to a rich literature on endogenous growth models with fiscal policies. The relevant works are introduced in Section 2.2 as the literature for the theoretical analyses in Chapter 3. Section 2.3 surveys the literature of investment and fiscal policies in China and offers the background of national conditions for the empirical work in Chapter 4 which employs an exogenous growth model to examine the economic relationship between government and nongovernment capital in China. Section 2.4 is also related to Chapter 4, it is devoted to a brief review of the literature of indirect inference which is used as the methodology of testing and estimating economic models using China's data. Section 2.5 summarises this Chapter.

2.2 Growth Models, Fiscal Policies and Ramsey Policy Problem

2.2.1 Exogenous Growth Models

Following the publication of important papers of Solow (1956) and Swan (1956), study of economic growth became one of the central topics of the Economics profession until the early 1970s. The key aspect of the Solow-Swan model is the neoclassical production function which assumes constant returns to scale with positive and decreasing marginal productivities to each input, and a constant-saving-rate rule to form a simple general equilibrium model. The limitation of diminishing returns to capital makes the Solow-Swan model difficult to explain the long-run per capita growth. In the 1960s, some neoclassical growth theorists patched this deficiency up by assuming that technological progress occurred in an exogenous manner. This modification reconciled the theory with a positive and nearly constant per capita growth rate. The exogenous growth model in Arrow and Kurz (1970) assumes that the productive public capital stimulates aggregate productivity.

2.2.2 Endogenous Growth Models

After the mid-1980s, a group of Economic theorists recognized the significance of long-run Economic growth and they wanted to find out endogenous mechanics to explain long-run growth without appealing to exogenous changes in technology and demographic factors. The work of Arrow (1962), Uzawa (1965), and Sidrauski (1967) formed the basis for this research. Romer (1986), Lucas (1988), and Rebelo (1991) omit exogenous technological progress, instead, think that growth goes indefinitely because investment in human capital has spillover effect on economy and escapes the diminishing return to capital accumulation

The endogenous growth theory is further supported by incorporating imperfect competition and R&D activity to the growth model. The important studies in the late 1980s and early 1990s are Romer (1987, 1990), Aghion and Howitt (1992) and Grossman and Helpman (1991).

2.2.3 Endogenous Growth with Fiscal Policies

2.2.3.1 Public Service vs Public Capital

Barro (1990) introduced a model with a flow of productive public service where the growth-maximisation income tax rate coincides with the welfare-maximisation tax rate. The framework, as the first endogenous growth model in which productive government expenditure determines the long-run growth rate, is frequently cited in the literature, and is often modified and extended into different models to explain the effects of fiscal policies on the growth. Following Barro (1990), many early studies in the literature including Turnovsky (1996) also treat the current flow of public spending as the source of contribution to productive capacity. Barro and Sala-i-Martin (1992) summarises three categories of public service as a productive input for private poruducers in the literature: publicly-provided private goods which are rival and excludable; publicly-provided public goods which are non-rival and non-excludable; and publicly-provided goods subject to congestion which are rival but to some extent non-excludable.

However, Barro's assumption that the public spending as a flow takes a role in the macroeconomic production is less plausible from the viewpoint of the work of Arrow and Kurz (1970) which argues that the government expenditure only indirectly influences the production through a stock of public capital. The empirical studies by Aschauer (1989) and Munnell and Cook (1990) defend the assumption of Arrow and Kurz. The importance of pubic capital is also presented in the survey of the empirical literature by Sturm, Kuper and de Haan (1998).

Futagami, Morita and Shibata (1993) (abbreviated to 'FMS') combined the Barro model with Arrow and Kurz's assumption, showing in that the welfare-maximisation tax rate is smaller than the rate that maximises the growth. In addition, they also analysed the transitional dynamics of their model and proved the existence of a unique steady growth equilibrium with private and public capital. Other studies by Baxter and King (1993), Glomm and Ravikumar (1994) and Cassou and Lasing (1998) also suggest that the accumulated stock of public capital rather than the flow of government expenditure is more relevant to production process.

Greiner and Hanusch (1998) extended the FMS model by assuming that the productive public spending is divided into investment in public capital and subsidy for private investment as in Judd (1985). Moreover, the tax revenue is also used to finance lump-sum transfer to the household. Their study concludes that maximising growth is not equivalent to maximising welfare on the balanced growth path.

Ghosh and Roy (2004) combined the Barro model and the FMS model, allowing both public service and public capital to exist in the production function. Their study explores present-versus-future trade-off in the public sector and emphasises the that the effect of the public sector on the economy depends not only on the income tax rate but also on the allocation of tax revenues between the public service and the accumulation of public capital. Their analysis shows that the growth rate, the share of public spending in the output, the proportion of investment in public capital in total public spending, and the proportion of private investment in total private spending all are lower in the equilibrium outcome than in the centralised optimal outcome.

2.2.3.2 Productivity Enhancing vs Utility Enhancing

The public service or public capital is often used to either enhance the private productivity, such as Barro (1990), Futagami, Morita and Shibata (1993) and Ghosh and Roy (2004), or enhance the utility, such as Xie (1997). Turnovsky and Fisher (1995) include both public consumption expenditure and public infrastructure expenditure in the same model.

Different from the previous studies of others, the paper of Chatterjee and Ghosh (2011) examines the effect of fiscal policy on macroeconomic performance and welfare by assuming that the government collects tax revenue to finance the accumulation of public capital which simultaneously provides both productivity and utility enhancing services to the private sector.

Similar to Chatterjee and Ghosh (2011), Misch, Gemmell and Kneller (2013) allow for the possibility that public service or public capital entail mixed effects in both utility and production. Moreover, their model, as in Devarajan, Swaroop and Zou (1996), Baier and Glomm (2001) and Ott and Turnovsky (2006), is based on CES technology rather than Cobb–Douglas technology, allowing greater complementarity between productive public spending and private capital. Their work focuses on the trade-off between growth and welfare maximisation and points out that the growth-maximisation tax rate can lie above, below, or on the welfare-maximisation equivalent. However, even relatively large differences in growth- and welfare-maximisation tax rates translate into relatively small differences in growth rates, and, in some cases, welfare levels.

Agénor (2008) provides a more complex endogenous growth framework with government spending in both health and infrastructure. Health influences individual welfare, in addition to affecting growth through enhancing the productivity of individuals; meanwhile infrastructure services affect the production as well as the provision of health services. The rate of time preference depends negatively on consumption of health services, relative to income.

2.2.3.3 Congestion

As argued by Edwards (1990), Barro and Sala-i-Martin (1992), Turnovsky (1997), Fisher and Turnovsky(1998) and Eicher and Turnovsky (2000), public goods are characterized by some degree of congestion. Edwards (1990) proposes five congestion models. Eicher and Turnovsky (2000) specify two notions of congestion in a growth context: relative congestion and aggregate (absolute) congestion. The former indicates the level of services derived by an individual from the provision of a public good in terms of his share of individual capital stock in the aggregate capital stock; the latter shows how aggregate usage of the service alone influences the services received by an individual.

The commonly cited paper by Barro and Sala-i-Martin (1992) illustrates that income tax, as a user fee for rival but non-excludable public goods, prevents the growth rate from being too high, something that lump-sum taxation can not achieve. Turnovsky (1997) captures congestion effects in a model with public capital and demonstrates that a time-varying income tax in the decentralised economy could replicate the first-best optimum in both short run and long run.

However, the treatment of congestion is often restricted to its effects on either production or utility, depending on whether the public good is used to consumption or investment. Different from the previous studies, the work of Chatterjee and Ghosh (2011) allows that congested public capital plays a dual role and shows that the consumption tax can be distortionary when the public capital is subject to congestion. Optimal fiscal policy involves using both income- and consumption- based tax or subsidy policies as corrective instruments for congestion. The optimal fiscal policy rules indicate greater flexibility in the choice of corrective policy instruments relative to the previous literature where the consumption tax is generally viewed as the least distortionary source of public finance.

Economides, Park, and Philippopoulos (2011) focus on the optimal allocation of tax revenues between productive and nonproductive public goods. The numerical results reveal that when productivity-enhancing public goods are subject to congestion, if the society values more public consumption goods and services, the more growth-promoting policies the government should choose, in other words, the government should give priority to public investment. In a growing economy, the government realises that a large tax base is needed to finance the public consumption goods and services. Hence, it makes its allocation decision to boost economic growth through productive public expenditure and then enlarge the tax base. In other words, a nongrowing society cannot afford the provision of public consumption goods and services. Only when there are “unrealistically” strong preferences over public consumption should the government follow the conventional policy recipe, namely, not only to tax more but also to make public consumption occupy a larger share in public expenditure.

2.2.3.4 Tax Categories

Lump-sum tax only reduces people's available income and it does not interfere with the incentive for consumption or capital accumulation. Hence, it is considered to be Pareto-efficient in the taxation literature. Barro (1990) demonstrates that lump-sum tax could replicate the centralised optimal resource allocation.

However, lump-sum tax is difficult to be implemented in reality. Therefore, distortionary taxes are still the main revenue sources for almost all countries. Following Barro (1990) and Futagami, Morita and Shibata (1993), subsequent studies by Glomm and Ravikumar (1994), Cassou and Lansing (1998), Turnovsky (1997), Aschauer (2000), and Marrero and Novales (2005) all assume the public revenue is from raised from proportional income tax.

Recently the consumption tax has occupied a significant place in tax reform, especially in Japan and the United States. Meanwhile, consumption tax is commonly regarded as a non-distortionary fiscal instrument which does not affect private economic decisions. However, the only condition under which consumption tax is also distortionary in intertemporal models is when the work-leisure choice is endogenously determined, which is illustrated by Milesi-Ferretti and Roubini (1998) and Turnovsky (2000).

Capital tax is also a feasible choice either in the economic theory or in the real world. Judd (1985) and Park and Philippopoulos (2005) use capital tax in their models. The former focuses on redistributive taxation, rather than as a means of funding public goods; while the latter assumes the capital tax is used to finance government consumption and production services in an endogenous growth model.

Chamley (1985) studies optimal wage taxation, exempting capital income from taxation, in an economy with infinite horizon. The extension by including capital-income tax is done by Chamley (1986) who finds that the optimal capital-income tax rate is zero in the long run.

Some studies use at least two tax instruments, for instances, both income tax and consumption tax in Chatterjee and Ghosh (2011), both capital- and wage- based fiscal instruments in Correia (1996), taxation from capital, wage and consumption in Jones, Manuelli and Rossi (1997) and Renström (1997).

2.2.3.5 Transitional Dynamics

Similar to the framework for studying optimal fiscal policies in Barro (1990), the models in Barro and Sali-i-Martin (1995), Alesina and Rodrik (1994) and Devereux and Wen (1998) demonstrate that the optimal income tax rate is time invariant and there are no transitional growth dynamics.

Based on Barro (1990), Futagami, Morita and Shibata (1993) developed a model with transitional dynamics by assuming the public capital rather than the public service is used to enhance the private productivity; while the endogenous growth model in Park and Philippopoulos (2005) still assumes that the flow of public expenditure affects the economic performance, but allows the expenditure to be used to finance both consumption and production as Lee (1992) and Cazzavillan (1996) so that there is transitional dynamics and a sufficient condition for existence and uniqueness of the balanced growth path.

Recently, lots of endogenous growth models with fiscal policies have transitional dynamics, but most of them focus on the properties of long-run general equilibrium, such as Ghosh and Roy (2004), Economides and Philippopoulos (2008), Economides, Park, and Philippopoulos (2011).

2.2.3.6 Time Inconsistency vs Time Consistency

As mentioned by Simaan and Cruz (1973) and Kydland (1977), the open-loop solution of a Stackelberg differential game is generally affected by time inconsistency. Under open-loop, the government, as the leader in the Stackelberg differential game, chooses an optimal policy, constrained by representative private agent as the follower, at the beginning of the game. However, whether the government honor its commitment is questioned. The literature on time inconsistency of optimal fiscal and monetary policies includes Kydland and Prescott (1977), Calvo (1978), Barro and Gordon (1983) and Calvo and Obstfeld (1988).

The study of Cellini and Lambertini (2007), based on the model in Xie (1997), proves that optimal fiscal policy and consumption in Stackelberg game are not only time consistent but also subgame perfect since the optimal policy is stationary and the consumption plan is independent of the state variable from the beginning.

Ortigueira (2006) introduces the optimal time-consistent tax policy in an exogenous growth model with leisure and public consumption in the utility function. Malley, Philippopoulos and Economides (2002) characterise the Markov time-consistent tax policy in an endogenous growth economy where the government raises tax revenues on total income to finance public consumption and production services. However, their model has two limitations: the allocation of government spending between consumption and production services is exogenously given; and they use a logarithmic utility function and physical capital fully depreciates each period. Under these parametric restrictions, the Ramsey policy is not subject to a time consistency problem and it is identical with the Markov perfect solution. Novales, Pérez and Ruiz (2014) overcome the two restrictions by a constant relative risk aversion (CRRA) utility function, incomplete depreciation of capital and an endogenously time-varying split of government spending between public consumption and production services. When comparing the optimal Markov-perfect and Ramsey policies, they find that the income tax rate is higher, the share of public consumption in public expenditure is higher and economic growth rate is slightly lower under the Markov government than their counterparts under the Ramsey government.

2.2.3.7 First-best Solution vs Second-best Solution

Many studies compare the second-best solution under decentralised competitive equilibrium with the first-best solution for the social planner in a centralised economy to check whether the second-best solution is optimal. Barro (1990) finds that the income tax rate of the second-best solution equals that of first-best solution, while both saving rate and growth rate are higher in the centralised model and a lump sum tax can reproduce the first-best allocation. Ghosh and Roy (2004) suggest that saving rate, income tax rate and growth rate in the centralised economy are higher than their counterparts in the decentralised economy and the optimal outcome attained by the social planner can be replicated in a decentralised economy if the public expenditure is financed by a consumption tax or a tax on income from labour. Turnovsky (1997) demonstrates in his model with congestion effects that a time-varying income tax in the decentralised economy could reproduce the first-best optimum in both short run and long run. However, Economides, Park, and Philippopoulos (2011) show that the government in the decentralised economy can not implement the optimal allocation.

2.2.4 Self-Interested Politicians

Most studies in the literature assume the objective of the public sector is to maximise either the growth rate or the utility attained by the representative household.

Barro (1990) considers the alternative that the self-interested government can earn the net revenue (the difference between the tax revenue and the expenditure for the households) to buy goods and then receive utility by consuming these goods. However, this assumption is infeasible in reality, especially in democratic countries where politicians must face electoral constraints.

The work of Ploeg and Klundert (1991) allows the discount factor of politicians to be larger than or equal to that of citizens and differ and shows that the behavior of self-interested politicians raise utility-enhancing public expenditure meanwhile cuts public investment and thus reduces the economic growth rate as well as private agents' welfare. This result is concordant with the empirical evidence given by Tanzi and Lutz (1993). However, their paper uses flow of public expenditure rather than stock of public capital in the production for analytical convenience. It also eliminates the case that the government may be more patient than households. Furthermore, it does not explain why these short-sighted politicians are condoned by the voters repeatedly in representative democracy.

The model in Acemoglu, Golosov and Tsyvinski (2008) also allows the discount factors of households and politicians to differ and uses linear labor and capital taxes to finance public expenditure. The result shows that political economy considerations made by self-interested politicians lead to a new source of distortions in the resource allocation because of the necessity to satisfy the political sustainability constraints; when politicians are less patient than the citizens, tax distortions remain even in the long run . In Acemoglu, Golosov and Tsyvinski (2010), non-linear taxes are used in a dynamic economy subject to a self-interested politician, without any commitment power, who is partly controlled by the citizens. The study of Acemoglu, Golosov and Tsyvinski (2011) deals with dynamic taxation of capital and labour in the Ramsey model and assumes fiscal policies are made by a self-interested politician who can not commit to policies. The conclusion is that if the politician is benevolent, the Chamley–Judd result of zero long-run taxes still holds; while if the politician is less patient than the citizens, the best equilibrium from the viewpoint of the citizens involves long-run capital taxation. Similar to Ploeg and Klundert (1991), Acemoglu, Golosov and Tsyvinski (2008, 2010, 2011) do not address the case of farsighted politicians who are more patient than the citizens.

2.3 Investment and Fiscal policies in China

2.3.1 Investment

After the founding of People's Republic of China (PRC) in 1949, China turned to the Soviet Union as the example of economic development, as well as the chief trading partner and technology source. Chinese government developed a massive socialist industrial complex through public investment. The state poured resources into capital-intensive firms to produce metals, machinery, and chemicals. Naughton (2007) labels this this development strategy as 'Big Push industrialization' since overwhelming priority was given to channeling the maximum available investment into heavy industry.

The economic reform was introduced by Deng Xiaoping in 1978. Maddison (2007) presents that since the end of 1970s, the share of government investment in total investment has experienced a sharp drop and now the private saving (via the banking system) rather than the state is the most important source to finance the investment, meanwhile the economy has been opened to foreign investment.

The annual data from National Bureau of Statistics of China show that the share of investment in GDP in 1952 is less than 23%. It has been increased to around 48% in recent years. Kuijs (2005) investigates the investment and saving in China, suggesting that high investment and saving are key features of China's pattern of growth. The study also reveals that investment has been high since 1990 with household and government investment stable at rates comparable to other countries, while the investment from enterprises, ranging between 27 and 35 percent of GDP, makes China's investment-GDP ratio higher than that in any other large economy.

Zhou (2007) shows that the incentives at the local level are the key factor that leads to large-scale investment projects. Officials in local governments are motivated primarily by political considerations. As the performance evaluations in China are based, to a large extent, on GDP growth within their jurisdictions, they have a strong incentive to promote investment and form favorable business environment for both domestic and foreign investors.

2.3.2 Infrastructure

Infrastructure facilities have played an important role in China's economic development, especially after the 1978 reform. Sahoo, Dash, and Nataraj (2010) investigate the role of infrastructure in promoting growth in China for the period from 1975 to 2007 and conclude that infrastructure development has significant positive contribution to growth than other types of investment. The investment in infrastructure has accounted for a large proportion of GDP, for example, it was about 14% of GDP in 2006. A massive development of infrastructure facilities has been a key factor of China's sustained high growth and increased competitiveness since the 1990s. D ́emurger (2001) gives empirical evidence on the links between infrastructure investment and economic growth in China by using panel data from 24 provinces in the period between 1985 and 1998. The empirical results also indicate that transport facilities are a key differentiating factor in explaining the economic growth gap between different provinces and point to the role of telecommunication in reducing the burden of isolation.

Government investment is most important source of infrastructure development especially before the 1978 reform. Sahoo, Dash, and Nataraj (2010) present that direct budget spending on urban infrastructure includes expenditures from both central and local government. Since urban infrastructure is a local responsibility, a vast majority of infrastructure facilities are invested by local governments.

2.3.3 SOEs in China

After 1949, market forces were replaced by regulatory devices and lots of SOEs were built. Maddison (2007) argues that 'there was a distinct preference for large enterprises which were expected to be more vertically integrated than in a capitalist market economy'. Almost all investment was carried out by the government and SOEs before 1978. At the same time, the SOEs were responsible for providing housing, education and health services to workers and even their families. With the commitment to full employment, SOEs could not dismiss workers who were redundant, lazy or inefficient.

However, with the reform in the state-owned sectors, the SOEs have become more efficient and competitive. Hay, Morris, Liu and Yao (1994) analyse different aspects of behaviours of SOEs in China, including production and costs, employment, profit margins and profitability, finance, investment decisions, and autonomy, and conclude that the reform programme was successful and in many respects in the 1980s the SOEs began to behave like Western firms.

In China, the largest four commercial banks are the direct descendants of the planned-economy bank system. Naughton (2007) investigates the financial system in China and points out that the Big Four state-owned commercial banks account for three quarters of total banking system assets in 2003. According to Relbanks 2014 statistics on largest banks by total assets, Industrial and Commercial Bank of China is the largest bank in the world with assets of 3.2 trillion dollars. All of the four state-owned commercial banks are among the top ten in the world, together with assets of about 10.7 trillion dollars. Since 1978, more and more savings from households and unincorporated businesses flow into the state-owned commercial banks and these banks have become a key channel for household surpluses to be invested by firms.

2.3.4 Tax System

Before 1978 China had adopted a highly centralised fiscal system. SOEs were required to turn most profits to fiscal administrative divisions, and local governments must turn over all their revenues to higher governments and finally to the Ministry of Finance, which would then allocated the expenditure of the SOEs and local governments. Since the beginning of the reform and opening up, the taxation system has gone through several important reforms. State Administration of Taxation of China (2012) shows that the establishment of foreign-related taxation system were made as the breakthrough points of tax reform in the early days of the reform; in 1983 and 1984, the reform established the distribution relations between the State and the enterprises within the taxation system; in 1994, the largest-scale and widest-scope tax reform adopted to meet the requirements of socialist market economy system.

After several taxation reforms, for the present, China has 19 tax categories, i.e. value added tax, consumption tax, business tax, enterprise income tax, individual income tax, resource tax, urban and township land use tax, house property tax, city maintenance and construction tax, tax on the use of arable land, land appreciation tax, vehicle purchase tax, vehicle and vessel tax, stamp tax, deed tax, tobacco leaf tax, customs duty, tonnage dues, and fixed assets investment orientation regulatory tax. Of which, most tax categories are proportional rather than progressive. The only two progressive tax categories are individual income tax and land appreciation tax, together occupying 8.6% of the tax revenue in 2012, as shown in National Bureau of Statistics of China (2013).

2.4 Indirect Inference

Indirect inference as a simulation-based method provides a statistical criterion for rejecting or accepting models. Indirect inference can be used to make inferences about the parameters of the structural model by comparing the performance of the auxiliary model based on the simulated data from the structure model with the performance of the auxiliary model based the actual data. Indirect inference has been well known in the literature, since being introduced by Smith (1993). Other influential names are Gourieroux, Monfort and Renault (1993), Canova (2005) and Le, Meenagh, Minford and Wickens (2011). Gourieroux, Monfort and Renault (1993) name the method of indirect inference as ‘incorrect’ criterion in the sense that it does not directly give a consistent estimator of the parameters of interest. Indirect inference is useful in estimating models for which the likelihood function or other criterion function is difficult to evaluate, especially those with nonlinear dynamic structures, or latent variables, or incomplete data.

Minford, Theodoridis and Meenagh (2009) use a VAR model as the auxiliary model to test a full open economy model of the UK which has been in forecasting use for three decades. The process of testing by indirect inference is to bootstrap the structural residuals and generate a large number of sample replications, based on which, a distribution of the VAR parameters is obtained, finally test whether the VAR parameters from actual data lies within this distribution at some level of confidence.

The method of indirect inference has also been used to estimate models in many studies. Liu and Minford (2014) summarise the basic idea of indirect estimation: to choose a set of parameters for the structural model, so that, when this model is simulated, it generates estimates of the auxiliary model as close as possible to those obtained from actual data. In other words, indirect estimation chooses the structural parameters that can minimise the distance between the two sets of estimated parameters.

Detailed procedures of both testing and estimating models by indirect inference, together with an example code that deals with a simple three-equation model, are given by Le and Meenagh (2013).

2.5 Conclusion

Endogenous growth models contain strong predictions regarding the function of fiscal instruments in economic growth. The theoretical literature on the relationship between fiscal policies and long-run growth has mainly focused on the effects of public service and public capital, productivity-enhancing goods and utility enhancing goods, congestion and non-congestion, distortionary taxation and non-distortionary taxation, time consistency and time inconsistency. However, few studies illustrate the different results with a self-interested government, especially when the government is more farsighted than the private sector.

There is a rich literature about investment and fiscal policies in China. Many studies present that the investment was dominated by the government and SOEs between 1949 and 1978, and the government expenditure was mainly from the state-owned sector. Since 1978, there has been a sharp drop in the government size. Today, investment is mainly financed from private saving via the state-owned commercial banks; market forces play a much bigger role in resource allocation; meanwhile, the SOEs become more efficient and competitive; the public investment is mainly used to finance the infrastructure development rather than subsidise the operation of state-owned sector.

During the last two decades, indirect inference, as a simulation-based method, has become more and more influential in testing and estimating economic models. Many empirical researchers introduce the procedures of how to use indirect inference, and example codes are also offered in the literature.

Chapter 3: How to Increase Long-Run Growth for a Farsighted Government

3.1 Introduction

The pioneering work of Barro (1990) incorporates the flow of public service into private production and thus creates the first endogenous growth model in which long-run economic growth comes from fiscal policy, suggesting that there is an inverted U-shaped-curve relationship between the public productive expenditure and economic growth, as the increasing cost of distortionary taxation which is necessary to finance the government spending overtakes the declining benefit of productivity-enhancing expeniture. Following Barro (1990), Futagami, Morita and Shibata (abbreviated to 'FMS') (1993) modify Barro's model by using stock of public capital in the production function. Xie (1997) designs a model in which income tax revenue finances the public consumption instead of enhancing private productivity. However, these authors do not address the consequences of fiscal policies decided by a farsighted or shortsighted government. This chapter overcomes these shortcomings and gives numerical results of different government time horizons in a model which includes both utility-enhancing and productivity-enhancing public goods. The rest of this chapter is organised as follows.

In Section 3.2, a centralised AK model illustrates the resource allocation arranged by a farsighted government would sacrifice the first several generations but benefit all future generations.

Section 3.3 goes beyond the previous work of Xie (1997) by adding the centralised models and comparing the decentralised and centralised results, incorporating a self-interested government and comparing the solutions under growth maiximisation, benevolent government, farsighted government and shortsighted government. Some important results are: (a) The optimal tax rate set by a self-interested government in the open-loop Stackelberg game is not only time consistent, but part of a subgame perfect equilibrium. (b) With an impatient government, it is possible that the growth rate in centralize model is smaller than the rate in the decentralised model. (c) With a farsighted government, the overall utility, the birth of the first benefited person in generation utility, the instantaneous utility, all living people's generation utility exceed their counterparts with a benevolent government in chronological order. (d) The more farsighted the government is, the faster the overall utility, the generation utility and the instantaneous utility exceed their counterparts with a benevolent government. (e) The inefficiency is not only due to tax distortion, but also from the time-preference difference between the representative and the government.

Section 3.4 revisit the Barro model and conclude that, with Cobb-Douglas production function, the optimal fiscal policy is independent from the government's time preference, which means a farsighted government can not realise a higher growth rate by fiscal instrument.

Section 3.5 extends the FMS model by assuming that the tax rate can be time invariant and the government can be self-interested. Moreover this section compares the decentralised and centralised results, and concludes that, in both the decentralised and the centralised FMS models, a more patient government would tax more to accumulate public capital, leading to a higher ratio of public capital to private capital and then a higher growth rate.

Section 3.6 presents a model with both a flow of public service in utility function and a stock of public capital in production function and illustrates that a farsighted government can increase long-run growth rate by two fiscal channels: reducing the ratio of unproductive public spending in GDP and increasing the ratio of productivity-enhancing public expenditure in GDP.

Section 3.7 discusses the result of this chapter, and gives conclusions, including the reason why why so many 'democratic' counties violate the first two prescriptions of the Washington Consensus: fiscal discipline and public expenditure priority to pro-growth investment; in contrast, the farsighted leaders in Singapore and China have pragmatic concern with serving the long-run benefits of their people.

The appendix shows the basic structure of this chapter and presents important results.

3.2 Increase the Growth in the Simplest Centralised AK Model

The purpose of this section is to illustrate the concept of 'quasi-Pareto improvement among generations' by a farsighted government. This section adopts a simple AK model without fiscal instrument. Capital in the AK model is broadly defined to encompass physical and human capital. Both the centralised and decentralised solutions are presented. The centralised solutions of saving rate and growth rate depend on whether the government is farsighted, benevolent or shortsighted.

3.2.1 The Structure of the Centralised AK Model

In a centralised economy, the government controls the allocation of resources between representative's consumption and investment. The representative is a consumer-cum-producer who produces a single good which is either consumed or invested as capital for later production.

The objective of the government is to maximise the representative's intertemporal utility with the government's own rate of time preference:

$$\int_0^\infty e^{-\rho_G t} u(c)\,dt \quad here \quad \rho_G > 0 \tag{2.1}$$

$$u(c) = \ln c \tag{2.2}$$

where c is the representative's consumption, ρ_G is the government's rate of time preference which is potentially different from the representative's rate of time preference ρ.

A shortsighted (impatient) government is less patient than the citizens, hence

$$\rho_G > \rho \tag{2.3}$$

A longsighted (patient) government is more patient than the citizens, hence

$$\rho_G < \rho \tag{2.4}$$

The representative is a household-producer who produces y with the AK model.

$$y = f\left(k\right) = Ak \tag{2.5}$$

where k is capital, the initial capital $k(0) > 0$ is given, A is a technological scale factor.

With zero depreciation rate, the budget constraint is

$$\dot{k} = y - c = Ak - c \tag{2.6}$$

3.2.2 The Centralised Equlibrium

The government maximises (2.1) the representative's intertemporal utility with the government's own rate of time preference, subject to the budget constraint (2.6). To solve the government's problem, set up the current-value Hamiltonian:

$$H_G \equiv \ln c + \lambda_k \left(y - c\right) \tag{2.7}$$

λ_k is the dynamic multiplier associated with (2.6).

The transversality condition is

$$\lim_{t\to\infty} e^{-\rho_G t}\lambda_k k = 0 \tag{2.8}$$

First order conditions:

$$\begin{aligned} \frac{\partial H_G}{\partial c} &= \frac{1}{c} - \lambda_k = 0 && \left(2.9a\right) \\ \frac{\partial H_G}{\partial k} &= \lambda_k \frac{\partial y}{\partial k} = \rho_G \lambda_k - \dot{\lambda}_k && \left(2.9b\right) \end{aligned} \tag{2.9}$$

(2.9a) and (2.9b) imply

$$\frac{\dot{c}}{c} = \frac{\partial y}{\partial k} - \rho_G \tag{2.10}$$

(2.5) and (2.10) imply

$$\frac{\dot{c}}{c} = A - \rho_G \tag{2.11}$$

Hence the consumption growth rate is a constant. A positive consumption growth rate imply

$$A - \rho_G > 0 \tag{2.12}$$

The condition of bounded utility is

$$\lim_{t \to \infty} \frac{\dot{u}}{u} < \rho_G \tag{2.13}$$

(2.2) imply

$$\frac{\dot{u}}{u} = \frac{\frac{\partial u}{\partial c}\dot{c}}{u} = \frac{1}{u} \cdot \frac{\dot{c}}{c} = \frac{1}{\ln c}\left(A - \rho_G\right) \tag{2.14}$$

(2.13) and (2.14) imply

$$\lim_{t \to \infty} c > \exp\left(\frac{A - \rho_G}{\rho_G}\right) \tag{2.15}$$

This inequality condition is satisfied since (2.11) and (2.12) ensure a constant positive growth rate.

3.2.3 Proof of Balanced Growth Path

(2.6) and (2.9b) imply

$$\frac{\dot{\lambda}_k}{\lambda_k} = \rho_G - A \tag{2.16}$$

Hence

$$\lambda_k = \lambda_k(0)e^{(\rho_G - A)t} \tag{2.17}$$

(2.6) and (2.9a) imply

$$\dot{k} = Ak - \frac{1}{\lambda_k} \tag{2.18}$$

(2.17) and (2.18) imply

$$\dot{k} = Ak - \frac{1}{\lambda_k(0)}e^{(A-\rho_G)t} \tag{2.19}$$

The solution of k is

$$k = \frac{1}{\lambda_k(0)\rho_G}e^{(A-\rho_G)t} + \left[k(0) - \frac{1}{\lambda_k(0)\rho_G}\right]e^{At} \tag{2.20}$$

(2.17) and (2.20) imply

$$\lambda_k k = \frac{1}{\rho_G} + \left[\lambda_k(0)k(0) - \frac{1}{\rho_G}\right]e^{\rho_G t} \tag{2.21}$$

Substitute (2.21) into the transversality condition (2.8), we have

$$\lambda_k(0)k(0) = \frac{1}{\rho_G} \tag{2.22}$$

Substitute (2.22) into (2.20), it can be seen that the second term in (2.20) is zero and hence the equilibrium is on balanced growth path (BGP).

$$\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\dot{y}}{y} = \gamma_C = A - \rho_G \tag{2.23}$$

where γ_C is the growth rate of all the three variables, the subscript ‘C’ denotes ‘*centralised* model’.

(2.9a), (2.22) and (2.23) imply

$$\frac{c}{k} = \rho_G \tag{2.24}$$

Hence

$$\begin{aligned} k &= k(0)e^{(A-\rho_G)t} && (2.25a) \\ c &= \rho_G k(0)e^{(A-\rho_G)t} && (2.25b) \\ y &= Ak(0)e^{(A-\rho_G)t} && (2.25c) \end{aligned} \tag{2.25}$$

The saving ratio is

$$s_C = \frac{\dot{k}}{y} = \frac{\dot{k}}{k} \cdot \frac{k}{y} = \frac{A-\rho_G}{A} \tag{2.26}$$

3.2.4 Comparison

In the decentralised model, the representative rather than the government makes decision on the resource allocation. It is straightforward to derive the growth rate and saving rate in the decentralised model by replacing ρ_G by ρ.

$$\gamma_D = A - \rho \tag{2.27}$$

$$s_D = \frac{A-\rho}{A} \tag{2.28}$$

Assume:

$$\begin{aligned} \rho_G &= \rho_{GP} < \rho && \textit{for a patient government} && (2.29a) \\ \rho_G &= \rho_{GI} > \rho && \textit{for a impatient government} && (2.29b) \\ \rho_G &= \rho_{GB} = \rho && \textit{for a benevolent government} && (2.29c) \end{aligned} \tag{2.29}$$

Hence, we have

$$\begin{aligned} &\gamma_{CI} < \gamma_{CB} = \gamma_D < \gamma_{CP} && (2.30a) \\ &s_{CI} < s_{CB} = s_D < s_{CP} && (2.30b) \end{aligned} \tag{2.30}$$

Proposition 3.2 A:

In the centralised model, the saving rate and growth rate are larger than, or equal to, or smaller than their counterparts in the decentralised model if the government is more patient than, or as patient as, or less patient than the representative.

3.2.5 Experiment

Assume the representative is a family with infinite generations. We set $A = 1.5$, the inter-generational rate of time preference $\rho = 0.5$, and $k(0) = 1$. Table 1 demonstrates the effect of government's rate of time preference ρ_G on consumption c and instantaneous utility u of different generations.

Table 1: Effect of ρ_G on Consumption and Welfare of Different Generations

Generation		1	2	3	4	5	10
Farsighted	c	0.30	1.00	3.31	10.98	36.45	14706.24
$\rho_G = 0.3$	u	-1.20	0.00	1.20	2.40	3.60	9.60
Benevolent	c	0.50	1.36	3.69	10.04	27.30	4051.54
$\rho_G = 0.5$	u	-0.69	0.31	1.31	2.31	3.31	3.61
Shortsighted	c	0.70	1.56	3.47	7.72	17.17	2086.67
$\rho_G = 0.7$	u	-0.36	0.44	1.24	2.04	2.84	7.64

We treat the solutions with benevolent government as the benchmark and conclude that the first several generations are worse off (with respect to consumption and instantaneous utility) if the government is farsighted while all future generations are better off; The first several generations are better off under a shortsighted government, but all future generations are worse off.

In the centalised regime where the government is more patient than the citizens, the resource allocation sacrifices the first several generations but benefits all future generations. It is a 'quasi-Pareto improvement' among generations when compared with the case with a benevolent government.

3.3 Increase the Growth in a Model with Nonproductive Public Expenditure

This section develops a model similar to Xie (1997), but there is no restriction on whether the government is benevolent or self-interested. With a self-interested government, the optimal fiscal policy and consumption are both time consistent and subgame perfect. This conclusion is the same as that under a benevolent government. Moreover, in both the decentralised and centralised models, the less patient a government is, the higher the optimal tax rate is, and the lower the growth rate is. If the government is very impatient, the growth rate in the centralised model is smaller than the rate in the decentralised model.

3.3.1 The Structure of the Xie Model

The objective of the representative is to maximise his intertemporal utility:

$$\int_0^\infty e^{-\rho t} u dt \quad here \quad \rho > 0 \tag{3.1}$$

$$u(c,h) = \ln c + \varphi \ln h \quad here \quad \varphi > 0 \tag{3.2}$$

where c is consumption, h is the flow of public consumption expenditure, ρ is the rate of time preference, φ is the weight given to public consumption service relative to private consumption. (Note: $\varphi = 1$ in Xie (1997).)

The model assumes that the public consumption is provided without user charges and not subject to congestion.

The representative is a household-producer who produces y with the AK model.

$$y = f(k) = Ak \tag{3.3}$$

where k is capital, the initial capital $k(0) > 0$ is given, A is a technological scale factor.

The government's budget constraint is

$$h = \tau y \quad here \quad 0 < \tau < 1 \tag{3.4}$$

where τ is the income tax rate that can be time variant if necessary for optimisation.

With zero depreciation rate, the budget constraint is

$$\dot{k} = (1-\tau) y - c = (1-\tau) Ak - c \tag{3.5}$$

3.3.2 Decentralised Competitive Equilibrium

The representative household-producer maximises his intertemporal utility, given tax rate τ and public consumption service h. To characterise the representative's response to the fiscal policy by a set of first order conditions and a transversality condition, set up the current -value Hamiltonian firstly:

$$H \equiv \ln c + \varphi \ln h + \lambda_k \left[(1-\tau) y - c \right] \tag{3.6}$$

The transversality condition is

$$\lim_{t \to \infty} e^{-\rho t} \lambda_k k = 0 \tag{3.7}$$

First order conditions:

$$\begin{aligned} \frac{\partial H}{\partial c} &= \frac{1}{c} - \lambda_k = 0 && (3.8a) \\ \frac{\partial H}{\partial k} &= \lambda_k (1-\tau) \frac{\partial y}{\partial k} = \rho \lambda_k - \dot{\lambda}_k && (3.8b) \end{aligned} \tag{3.8}$$

(3.8a) and (3.9b) imply

$$(1-\tau) \frac{\partial y}{\partial k} = \rho + \frac{\dot{c}}{c} \tag{3.9}$$

(3.9) is the Euler equation which can be written as

$$\dot{c} = c\left[\left(1-\tau\right)\frac{\partial y}{\partial k} - \rho\right] \quad (3.10)$$

(3.3) and (3.10) imply

$$\dot{c} = c\left[\left(1-\tau\right)A - \rho\right] \quad (3.11)$$

$$\frac{\dot{c}}{c} = \left[\left(1-\tau\right)A - \rho\right] \quad (3.12)$$

(3.11) and (3.5) constitute the decentralised competitive equilibrium (DCE):

$$\begin{aligned} \dot{c} &= c\left[\left(1-\tau\right)A - \rho\right] & (3.13a) \\ \dot{k} &= \left(1-\tau\right)Ak - c & (3.13b) \end{aligned} \quad (3.13)$$

The system (3.13) expresses the motion of consumption c and capital k as functions of government policy instrument τ.

Substitute (3.8a) into (3.13b), we have

$$\lambda_k \dot{k} = \lambda_k \left(1-\tau\right)Ak - 1 \quad (3.14)$$

(3.3) and (3.8b) imply

$$\dot{\lambda}_k = \lambda_k \left[\rho - \left(1-\tau\right)A\right] \quad (3.15)$$

Multiply both sides of (3.15) by k, we have

$$\dot{\lambda}_k k = \lambda_k \left[\rho - \left(1-\tau\right)A\right]k \quad (3.16)$$

The sum of (3.14) and (3.16) gives

$$\frac{d\left[\lambda_k k\right]}{dt} = \dot{\lambda}_k k + \lambda_k \dot{k} = \lambda_k k \rho - 1 \quad (3.17)$$

Hence the solution of $\lambda_k k$ is

$$\lambda_k k = \frac{1}{\rho} - Be^{\rho t} \quad (3.18)$$

It is necessary to set $B = 0$ so that the transversality condition (3.7) can be satisfied. Hence,

$$\lambda_k k = \frac{1}{\rho} \tag{3.19}$$

Substitute (3.8a) into (3.19), we have

$$\frac{c}{k} = \rho \tag{3.20}$$

(3.5) and (3.20) imply

$$\frac{\dot{k}}{k} = (1-\tau)A - \rho \tag{3.21}$$

Hence the CDE (3.13) becomes

$$\begin{aligned} \dot{c} &= c\left[(1-\tau)A-\rho\right] && (3.22a) \\ \dot{k} &= k\left[(1-\tau)A-\rho\right] && (3.22b) \end{aligned} \tag{3.22}$$

The solution is

$$\begin{aligned} k &= k(0)\exp\int_0^t \left\{\left[1-\tau(\varsigma)\right]A-\rho\right\} d\varsigma && (3.23a) \\ c &= \rho k(0)\exp\int_0^t \left\{\left[1-\tau(\varsigma)\right]A-\rho\right\} d\varsigma && (3.23b) \end{aligned} \tag{3.23}$$

If the tax rate τ is a constant, $(1-\tau)A$ can be treated as a technological scale factor for the representative. As in Section 3.2, the whole economy is on the BGP.

Hence, the long-run growth rate is

$$\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\dot{h}}{h} = \frac{\dot{y}}{y} = \gamma_D = (1-\tau)A - \rho \tag{3.24}$$

On BGP, the consumption-capital ratio is

$$\frac{c}{k} = \rho \tag{3.25}$$

On BGP, the saving rate is

$$s_D = \frac{\dot{k}}{y} = \frac{\dot{k}}{k} \cdot \frac{k}{y} = \frac{(1-\tau)A - \rho}{A} = (1-\tau) - \frac{\rho}{A} \tag{3.26}$$

On BGP, the after-tax saving rate is

$$s_D^A = \frac{\dot{k}}{(1-\tau)y} = (1-\tau)^{-1} s_D = \frac{(1-\tau)A - \rho}{(1-\tau)A} = 1 - \frac{\rho}{(1-\tau)A} \tag{3.27}$$

3.3.3 Decentralised Equilibrium with Growth Maximisation

(3.12) implies that if the growth rate reaches the maximum, the government should not impose tax. Hence

$$\tau_{DG} = 0$$

The ratio of public consumption to private consumption is

$$\frac{h_{DG}}{c_{DG}} = 0$$

where the subscript 'DG' denotes '*Decentralised* model with *Growth* maximisation'.

In this model, the government spending is nonproductive and enters the utility function. Hence, the growth is maximised when there is no nonproductive expenditure.

The solution of γ_{DG} is

$$\gamma_{DG} = A - \rho$$

The saving rate under the growth maximisation is

$$s_{DG} = \frac{A - \rho}{A}$$

Since the tax rate is zero, the after-tax saving rate under the growth maximisation is

$$s_{DG}^{A} = s_{DG} = \frac{A-\rho}{A}$$

Summary:

$$\begin{aligned}
&\tau_{DG} = 0 && (3.28a) \\
&\gamma_{DG} = A-\rho && (3.28b) \\
&\frac{c_{DG}}{k_{DG}} = \rho && (3.28c) \\
&\frac{h_{DG}}{c_{DG}} = 0 && (3.28d) \\
&s_{DG} = \frac{A-\rho}{A} && (3.28e) \\
&s_{DG}^{A} = \frac{A-\rho}{A} && (3.28f)
\end{aligned} \tag{3.28}$$

3.3.4 Decentralised General Equilibrium with a Self-Interested Government

For a benevolent government in the decentralised economy, the problem is to run the public sector to maximise its citizens' intertemporal utility; while a self-interested government has its own utility function. The difference between the two intertemporal utility functions is just that the government has a different rate of time preference.

$$\int_0^{\infty} e^{-\rho_G t} u dt \quad here \quad \rho_G > 0 \; and \; \rho_G \neq \rho \tag{3.29}$$

In the Stackelberg play, the government chooses a sequence of tax rates at date zero to maximise its intertemporal utility (3.29), subject to is budget constraints (3.4), the representative's budget constraint (3.5) and the Euler equation (3.11).

The current-value Hamiltonian for the government is

$$\begin{aligned} H^{GS} \equiv \ln c + \varphi \ln(\tau A k) + \mu_k \left[(1-\tau) A k - c \right] \\ + \mu_c c \left[(1-\tau) A - \rho \right] \end{aligned} \tag{3.30}$$

First order conditions:

$$\begin{aligned}
&\frac{\partial H^{GS}}{\partial \tau} = \frac{\varphi}{\tau} - A\mu_k k - A\mu_c c = 0 && (3.31a) \\
&\frac{\partial H^{GS}}{\partial k} = \frac{\varphi}{k} + (1-\tau)A\mu_k = \rho_G \mu_k - \dot{\mu}_k && (3.31b) \qquad (3.31) \\
&\frac{\partial H^{GS}}{\partial c} = \frac{1}{c} - \mu_k + \mu_c\left[(1-\tau)A - \rho\right] = \rho_G \mu_c - \dot{\mu}_c && (3.31c)
\end{aligned}$$

(3.31a)-(3.31c) imply

$$\begin{aligned}
&\tau = \frac{\varphi}{A(\mu_k k + \mu_c c)} && (3.32a) \\
&\dot{\mu}_k = \mu_k\left[\rho_G - (1-\tau)A\right] - \frac{\varphi}{k} && (3.32b) \qquad (3.32) \\
&\dot{\mu}_c = \mu_k - \frac{1}{c} + \mu_c\left[\rho_G + \rho - (1-\tau)A\right] && (3.32c)
\end{aligned}$$

Multiply all terms of (3.22b) by μ_k, we have

$$\mu_k \dot{k} = \mu_k k\left[(1-\tau)A - \rho\right] \tag{3.33}$$

Multiply all terms of (3.32b) by k, we have

$$\dot{\mu}_k k = \mu_k k\left[\rho_G - (1-\tau)A\right] - \varphi \tag{3.34}$$

Sum up (3.33) and (3.34), we have

$$\mu_k \dot{k} + \dot{\mu}_k k = \frac{d(\mu_k k)}{dt} = \mu_k k(\rho_G - \rho) - \varphi \tag{3.35}$$

The solution of $\mu_k k$ is

$$\mu_k k = \frac{\varphi}{\rho_G - \rho} + Q_1 e^{(\rho_G - \rho)t} \tag{3.36}$$

where Q_1 is a constant.

(3.20) and (3.36) imply

$$\mu_k = \left[\frac{\varphi}{\rho_G - \rho} + Q_1 e^{(\rho_G - \rho)t} \right] \frac{\rho}{c} \tag{3.37}$$

Substitute (3.37) into (3.32c), we have

$$\dot{\mu}_c = \left[\frac{\varphi}{\rho_G - \rho} + Q_1 e^{(\rho_G - \rho)t} \right] \frac{\rho}{c} - \frac{1}{c} + \mu_c \left[\rho_G + \rho - (1-\tau) A \right] \tag{3.38}$$

Multiply all terms of (3.38) by c, we have

$$\dot{\mu}_c c = \frac{\varphi\rho}{\rho_G - \rho} - 1 + Q_1 \rho e^{(\rho_G - \rho)t} + \mu_c c \left[\rho_G + \rho - (1-\tau) A \right] \tag{3.39}$$

Multiply all terms of (3.11) by μ_c, we have

$$\mu_c \dot{c} = \mu_c c \left[(1-\tau) A - \rho \right] \tag{3.40}$$

Sum up (3.39) and (3.40), we have

$$\dot{\mu}_c c + \mu_c \dot{c} = \frac{d(\mu_c c)}{dt} = \mu_c c \rho_G + \frac{\varphi\rho}{\rho_G - \rho} - 1 + Q_1 \rho e^{(\rho_G - \rho)t} \tag{3.41}$$

The solution of $\mu_c c$ is

$$\mu_c c = \frac{1}{\rho_G} - \frac{\varphi\rho}{\rho_G (\rho_G - \rho)} - Q_1 e^{(\rho_G - \rho)t} + Q_2 e^{\rho_G t} \tag{3.42}$$

where Q_2 is a constant.

Substitute (3.36) and (3.42) into (3.32a), we have

$$\tau = \frac{\varphi}{A \left(\frac{1+\varphi}{\rho_G} + Q_2 e^{\rho_G t} \right)} \tag{3.43}$$

To satisfy the condition $\tau \geq 0$ for any t including when t tends to infinity, we must set $Q_2 = 0$, and then we have the solution of τ in the decentralised equilibrium with a self-interested government as

$$\tau_{DS}=\frac{\rho_G\varphi}{A(1+\varphi)}$$

The growth rate is

$$\gamma_{DS}=(1-\tau_{DS})A-\rho=A-\frac{\rho_G\varphi}{(1+\varphi)}-\rho$$

The solution of the ratio of public consumption to private consumption is

$$\frac{h_{DS}}{c_{DS}}=\frac{\tau_{DS}Ak}{\rho k}=\frac{\rho_G\varphi}{\rho(1+\varphi)}$$

The saving rate is

$$s_{DS}=\left[1-\frac{\rho_G\varphi}{A(1+\varphi)}\right]-\frac{\rho}{A}$$

The after-tax saving rate is

$$s_{DS}^A=1-\frac{\rho}{\left[1-\frac{\rho_G\varphi}{A(1+\varphi)}\right]A}$$

Summary:

$$\begin{aligned}
\tau_{DS}&=\frac{\rho_G\varphi}{A(1+\varphi)} && (3.44a)\\
\gamma_{DS}&=A-\frac{\rho_G\varphi}{1+\varphi}-\rho && (3.44b)\\
\frac{c_{DS}}{k_{DS}}&=\rho && (3.44c)\\
\frac{h_{DS}}{c_{DS}}&=\frac{\rho_G\varphi}{\rho(1+\varphi)} && (3.44d)\\
s_{DS}&=\left[1-\frac{\rho_G\varphi}{A(1+\varphi)}\right]-\frac{\rho}{A} && (3.44e)\\
s_{DS}^A&=1-\frac{\rho}{\left[1-\frac{\rho_G\varphi}{A(1+\varphi)}\right]A} && (3.44f)
\end{aligned} \qquad (3.44)$$

(3.19) shows that, at any instant, the co-state variable of the representative (follower) λ_k is independent of the tax rate τ_{DS} set by the government (leader), hence the game is time consistent. (See Karp and Lee (2003).)

(3.44a) reveals that the tax rate τ_{DS} is dependent of the capital at any instant, hence the optimal tax rate is subgame perfect.

Propostition 3.3A:

The optimal tax rate set by the self-interested government in the open-loop Stackelberg game is not only time consistent, but part of a subgame perfect equilibrium, which ensures the DCE is on the BGP.

3.3.5 Decentralised General Equilibrium with a Benevolent Government

In the literature, the policy adopt by the benevolent government in a decentralised model is often called as 'second-best optimal policy'. (See, for example, Economides, Park, and Philippopoulos (2011).) The solution with a benevolent government is just a special case of the general solution with a self-interested government when $\rho_G = \rho$. (See Cellini and Lambertini (2007) for details, they gives the solution for $= 1$.)

It is straightforward to give the summary below:

$$\left.\begin{aligned}
&\tau_{DB} = \frac{\rho\varphi}{A(1+\varphi)} && (3.45a)\\
&\gamma_{DB} = A - \frac{\rho\varphi}{1+\varphi} - \rho && (3.45b)\\
&\frac{c_{DB}}{k_{DB}} = \rho && (3.45c)\\
&\frac{h_{DB}}{c_{DB}} = \frac{\varphi}{(1+\varphi)} && (3.45d)\\
&s_{DB} = \left[1 - \frac{\rho\varphi}{A(1+\varphi)}\right] - \frac{\rho}{A} && (3.45e)\\
&s_{DB}^{A} = 1 - \frac{\rho}{\left[1 - \frac{\rho\varphi}{A(1+\varphi)}\right]A} && (3.45f)
\end{aligned}\right\} \qquad (3.45)$$

Hence, (3.44) is a general solution that is suitable for the benevolent government, thus

$$\begin{aligned}
&\frac{d\tau_D}{d\rho_G} > 0 && (3.46a) \\
&\frac{d\gamma_D}{d\rho_G} < 0 && (3.46b) \\
&\frac{d\left(c_D / k_D\right)}{d\rho_G} = 0 && (3.46c) \\
&\frac{d\left(h_D / c_D\right)}{d\rho_G} > 0 && (3.46d) \\
&\frac{ds_D}{d\rho_G} < 0 && (3.46c) \\
&\frac{ds_D^A}{d\rho_G} < 0 && (3.46d)
\end{aligned} \qquad (3.46)$$

Propostition 3.3B:

In the decentralised Xie model, the less patient a government is, the higher the optimal tax rate is set, and the lower the growth rate reaches.

The tax rate under an impatient government is higher than that under a benevolent government, which reduces the disposable income, while the consumption-capital ratio is independent on the government's discount rate, thus less saving is used to accumulate capital, and finally higher tax stunts the economic growth.

A shortsighted government is inclined to promote public consumption and thus reduces the economic growth rate. The reason is that the politicians face uncertainty about election in a democratic society and thus have a shorter time horizon than citizens. In reality, government policies are more complex than the Xie model. With imperfect information, people are difficult to calculate the effects of these policies on the welfare of themselves and their children. Politicians know the shortsighted policies would reduce the welfare of the citizens, but they can dress up their statements to lead the gullible voters up the garden path.

3.3.6 Centralised General Equilibrium with Growth Maximisation

Obviously, maximising the growth rate in the centalised model imply the social planner does not have expenditure, hence the model is the same as that in Section 3.2, or the solution is just the one in 3.3.3 modified by replacing ρ by ρ_G:

$$\left.\begin{array}{ll} \tau_{CG} = 0 & (3.47a) \\ \gamma_{CG} = A - \rho_G & (3.47b) \\ \dfrac{c_{CG}}{k_{CG}} = \rho_G & (3.47c) \\ \dfrac{h_{CG}}{c_{CG}} = 0 & (3.47d) \\ s_{CG} = \dfrac{A-\rho_G}{A} & (3.47e) \\ s_{CG}^{A} = \dfrac{A-\rho_G}{A} & (3.47f) \end{array}\right. \qquad (3.47)$$

3.3.7 Centralised General Equilibrium with a Self-Interested Government

The current-value Hamiltonian for the government is

$$H^{GS} \equiv \ln c + \varphi \ln(\tau Ak) + \mu_k \left[(1-\tau)Ak - c\right] \qquad (3.48)$$

First order conditions:

$$\left.\begin{array}{ll} \dfrac{\partial H^{GS}}{\partial \tau} = \dfrac{\varphi}{\tau} - A\mu_k k = 0 & (3.49a) \\ \dfrac{\partial H^{GS}}{\partial k} = \dfrac{\varphi}{k} + (1-\tau)A\mu_k = \rho_G \mu_k - \dot{\mu}_k & (3.49b) \\ \dfrac{\partial H^{GS}}{\partial c} = \dfrac{1}{c} - \mu_k = 0 & (3.49c) \\ \dfrac{\partial H^{GS}}{\partial \mu_k} = (1-\tau)Ak - c = \dot{k} & (3.49d) \end{array}\right. \qquad (3.49)$$

(3.49a)-(3.49d) imply

$$\begin{aligned}
&\tau = \frac{\varphi}{A\mu_k k} && (3.50a) \\
&\dot{\mu}_k = \mu_k\left[\rho_G - (1-\tau)A\right] - \frac{\varphi}{k} && (3.50b) \\
&\mu_k = \frac{1}{c} && (3.50c) \\
&\dot{k} = (1-\tau)Ak - c && (3.50d)
\end{aligned} \qquad (3.50)$$

Multiply all terms of (3.50d) by μ_k and use (3.50c), we have

$$\mu_k \dot{k} = \mu_k k(1-\tau)A - 1 \qquad (3.51)$$

Multiply all terms of (3.50b) by k, we have

$$\dot{\mu}_k k = \mu_k k\left[\rho_G - (1-\tau)A\right] - \varphi \qquad (3.52)$$

Sum up (3.51) and (3.52), we have

$$\mu_k \dot{k} + \dot{\mu}_k k = \frac{d(\mu_k k)}{dt} = \mu_k k \rho_G - 1 - \varphi \qquad (3.53)$$

The solution of $\mu_k k$ is

$$\mu_k k = \frac{1+\varphi}{\rho_G} + Q_3 e^{\rho_G t} \qquad (3.54)$$

where Q_3 is a constant.

Substitute (3.54) into (3.50a), we have

$$\tau = \frac{\varphi}{A\left(\frac{1+\varphi}{\rho_G} + Q_3 e^{\rho_G t}\right)} \qquad (3.55)$$

To satisfy the condition $\tau \geq 0$ for any t including when t tends to infinity, we must set $Q_3 = 0$, then τ in the centralised equilibrium with a self-interested government is

$$\tau_{CS} = \frac{\rho_G \varphi}{A(1+\varphi)} \qquad (3.56)$$

(3.50a), (3.50c) and (3.56) imply

$$\frac{c_{CS}}{k_{CS}} = \frac{\rho_G}{1+\varphi}$$

Hence we have

$$\frac{h_{CS}}{c_{CS}} = \frac{\tau A k}{\frac{\rho_G}{1+\varphi} k} = \varphi$$

The above constant ratios imply the equilibrium is on BGP, hence we have

$$\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\dot{h}}{h} = \frac{\dot{y}}{y} = \gamma_{CS} = A - \rho_G \tag{3.57}$$

Summary:

$$\begin{aligned}
&\tau_{CS} = \frac{\rho_G \varphi}{A(1+\varphi)} && (3.58a) \\
&\gamma_{CS} = A - \rho_G && (3.58b) \\
&\frac{c_{CS}}{k_{CS}} = \frac{\rho_G}{1+\varphi} && (3.58c) \\
&\frac{h_{CS}}{c_{CS}} = \varphi && (3.58d) \\
&s_{CS} = \frac{A - \rho_G}{A} && (3.58e) \\
&s_{CS}^A = \frac{A - \rho_G}{\left[1 - \frac{\rho_G \varphi}{A(1+\varphi)}\right] A} && (3.58f)
\end{aligned} \tag{3.58}$$

If the government is benevolent, in other words, $\rho_G = \rho$, we replace ρ_G by ρ in (3.58) to obtain the solution.

$$
\begin{aligned}
&\frac{d\tau_C}{d\rho_G} > 0 && (3.59a)\\
&\frac{d\gamma_C}{d\rho_G} < 0 && (3.59b)\\
&\frac{d\left(c_C / k_C\right)}{d\rho_G} = 0 && (3.59c)\\
&\frac{d\left(h_C / c_C\right)}{d\rho_G} > 0 && (3.59d)\\
&\frac{ds_C}{d\rho_G} < 0 && (3.59c)\\
&\frac{ds_C^A}{d\rho_G} < 0 && (3.59d)
\end{aligned}
\tag{3.59}
$$

Propostition 3.3C:

In the centralised Xie model, the less patient a government is, the higher the optimal tax rate is set, and the lower the growth rate is.

3.3.8 Compare Decentralised Outcome with Centralised Outcome when the Government Is Self-Interested

From (3.44) and (3.58), we find the tax rate in the centralised model coincides with that in the decentralised model. If $\rho_G \leq \rho$, the government is farsighted or benevolent, we have

$$
\begin{aligned}
&\tau_{CS} = \tau_{DS} && (3.60a)\\
&\gamma_{CS} \geq \gamma_{DS} && (3.60b)\\
&\frac{c_{CS}}{k_{CS}} \leq \frac{c_{DS}}{k_{DS}} && (3.60c)\\
&\frac{h_{CS}}{c_{CS}} \geq \frac{h_{DS}}{c_{DS}} && (3.60d)\\
&s_{CS} \geq s_{DS} && (3.60e)\\
&s_{CS}^A \geq s_{DS}^A && (3.60f)
\end{aligned}
\tag{3.60}
$$

In the centralised model, the 'dictatorial' government can order the representative to allocate more disposable income in saving (investment) rather than consumption, when compared with the allocation in the decentralised model. There are two reasons: the government, as a social planner, exploits the externality; the government is more patient than the representative (when $\rho_G < \rho$).

However, the growth rate in the centralised model is not always higher than that in the decentralised model, which is different from the conclusions of other researchers in the literature since they assume the government is benevolent. If the government is extremely impatient, it may command the representative to allocate more disposable income in private consumption. The share of consumption in the disposable income is higher than the share in the decentralised model, which lowers the saving rate and further reduces the growth rate.

Compare (3.44) with (3.58), we have

$$\gamma_{CS} < \gamma_{DS} \quad \frac{c_{CS}}{k_{CS}} > \frac{c_{DS}}{k_{DS}} \quad \frac{h_{CS}}{c_{CS}} < \frac{h_{DS}}{c_{DS}} \quad s_{CS} < s_{DS} \quad s_{CS}^A < s_{DS}^A \quad if \quad \rho_G > (1+\varphi)\rho \tag{3.61}$$

Propostition 3.3D:

If $\boldsymbol{\rho_G > (1+\varphi)\rho}$, the growth rate in the centralised model is smaller than the rate in the decentralised model.

3.3.9 Effect of the Government's Time Preference on Welfare in the Decentralised Model

We can regard the representative as a representative family with infinite generations and assume:

(a). There are infinite families in the economy.

(b). Each family has one member at any time.

(c). The dates of birth are randomly distributed among different families, some people are born before date zero.

(d). Each generation survives for l periods and dies when the next generation is born so that each family has one living member.

Define the critical date $\bar{x}$ as the threshold of the instantaneous utility with a farsighted government surpassing that with a benevolent government.

Define the critical date $\ddot{x}$ as the threshold of date of overall utility with a farsighted government surpassing that with a benevolent government. At date $\bar{x}$, the overall utility with a farsighted government has surpassed that with a benevolent government since future instantaneous utility with a farsighted government is higher, hence

$$\ddot{x} < \bar{x} \tag{3.62}$$

Define the critical date $\tilde{x}$ as the date of birth of the first person who will be benefited from the farsighted fiscal policy with respect to 'generation utility' (the overall utility in one generation, see (3.80) or (3.81) below). There are infinite families and people in the economy, hence, the time when the first beneficiary's generation utility with a farsighted government surpasses his generation utility with a benevolent government must be the date of his death $\tilde{x} + l$. (Otherwise at least one person born before $\tilde{x}$ has benefited from the farsighted government in terms of generation utility before $\tilde{x} + l$, but it is impossible.) At date $\tilde{x} + l$, all living people are born after $\tilde{x}$, hence $\tilde{x} + l$ is the first date when all living people are beneficiaries of the farsighted policy.

At date $\bar{x}$, no one has already benefited from the farsighted fiscal policy with respect to 'generation utility' because, before $\bar{x}$, all died and living people have suffered from the farsighted fiscal policy since the instantaneous utility is lower than that with a benevolent government. At date $\bar{x} + l$, all living people have had higher instantaneous utility since they are born. Hence,

$$\bar{x} < \tilde{x} + l < \bar{x} + l$$

$$\tilde{x} < \bar{x} < \tilde{x} + l < \bar{x} + l \tag{3.63}$$

Divided the welfare (overall utility) at date $\ddot{x}$ into two parts

$$U(\ddot{x}) = U^{\ddot{x} \to \ddot{x}+l}(\ddot{x}) + U^{\ddot{x}+l \to \infty}(\ddot{x}) \tag{3.64}$$

The first part is the discounted utility from $\ddot{x}$ to $\ddot{x}+l$, the second part is the discounted utility after $\ddot{x}+l$. At date $\ddot{x}$, the overall utility with a farsighted government is equal to that with a benevolent government. Hence

$$U_{DS}\left(\ddot{x}\right)=U_{DB}\left(\ddot{x}\right) \tag{3.65}$$

Farsighted policy leads to higher growth, hence

$$U_{DS}^{\ddot{x}+l\to\infty}\left(\ddot{x}\right)>U_{DB}^{\ddot{x}+l\to\infty}\left(\ddot{x}\right) \tag{3.66}$$

(3.65) and (3.66) imply

$$U_{DS}^{\ddot{x}\to\ddot{x}+l}\left(\ddot{x}\right)<U_{DB}^{\ddot{x}\to\ddot{x}+l}\left(\ddot{x}\right) \tag{3.67}$$

At date $\ddot{x}$, the new born people's generation utility with a farsighted government will be still less than that with a benevolent government. Hence

$$\ddot{x}<\tilde{x} \tag{3.68}$$

(3.63) and (3.68) imply

$$\ddot{x}<\tilde{x}<\bar{x}<\tilde{x}+l \tag{3.69}$$

Proposition 3.3E:

With a farsighted government, the overall utility, the birth of the first benefited person in generation utility, the instantaneous utility, all living people's generation utility exceed their counterparts with a benevolent government in chronological order.

Since the economy is on BGP, the instantaneous utility for the citizens at date x is

$$u\left(c\left(x\right),h\left(x\right)\right)=\ln\left[c\left(0\right)e^{\gamma x}\right]+\varphi\ln\left[h\left(0\right)e^{\gamma x}\right] \tag{3.70}$$

(3.3), (3.4) and (3.44c) imply that the instantaneous utility is

$$u\left(c\left(x\right),h\left(x\right)\right)=\left(1+\varphi\right)\left[\ln k\left(0\right)+\gamma x\right]+\ln\rho+\varphi\ln\left(\tau A\right) \tag{3.71}$$

(3.44a), (3.44b) and (3.71) imply

$$u\left(c(x),h(x)\right)=(1+\varphi)\left[\ln k(0)+\left(A-\frac{\rho_G\varphi}{1+\varphi}-\rho\right)x\right]+\ln\rho+\varphi\ln\frac{\rho_G\varphi}{1+\varphi}$$

$$\begin{aligned}u\left(c(x),h(x)\right)=(1+\varphi)\left[\ln k(0)+(A-\rho)x\right]+\ln\rho+\varphi\ln\frac{\varphi}{1+\varphi}\\+\varphi\left(\ln\rho_G-\rho_G x\right)\end{aligned}\tag{3.72}$$

$\bar{x}$ is obtained by solving

$$u\left(c_{DS}(x),h_{DS}(x)\right)\geq u\left(c_{DB}(x),h_{DB}(x)\right)\tag{3.73}$$

$$\begin{aligned}&\ln\rho_G-\rho_G x\geq\ln\rho-\rho x\\&x\geq\frac{\ln\rho-\ln\rho_G}{\rho-\rho_G}\quad if\quad \rho_G<\rho, the\ government\ is\ farsighted\end{aligned}$$

Hence

$$\bar{x}=\frac{\ln\rho-\ln\rho_G}{\rho-\rho_G}\tag{3.74}$$

From (3.74), we have

$$\frac{\partial\bar{x}}{\partial\rho_G}>0\quad if\quad \rho_G<\rho\tag{3.75}$$

The BGP welfare of the representative is calculated as the overall utility expressed by a formula including relevant parameters, growth rate, the given value of initial private capital (initial value of the state variable), and sometimes ratios of relevant variables to private capital. (See Barro (1990) and Misch, Gemmell and Kneller (2013).)

The welfare for the citizens at date x is

$$U(x)=\int_0^\infty e^{-\rho t}\left\{\ln\left[c(x)e^{\gamma t}\right]+\varphi\ln\left[h(x)e^{\gamma t}\right]\right\}dt$$

$$U(x)=\int_0^\infty e^{-\rho t}\left\{\ln\left[c(0)e^{\gamma(x+t)}\right]+\varphi\ln\left[h(0)e^{\gamma(x+t)}\right]\right\}dt$$

$$U(x)=\int_0^{\infty} e^{-\rho t}\left\{\ln\left[k(0)\rho e^{\gamma(x+t)}\right]+\varphi\ln\left[k(0)\frac{\rho_G\varphi}{1+\varphi}e^{\gamma(x+t)}\right]\right\}dt$$

$$U(x)=\int_0^{\infty} e^{-\rho t}\left[(1+\varphi)\ln k(0)+\ln\rho+\varphi\ln\frac{\rho_G\varphi}{1+\varphi}+\gamma(1+\varphi)(x+t)\right]dt$$

Substitute (3.44b) into the above equation, we have

$$U(x)=\int_0^{\infty} e^{-\rho t}\left[(1+\varphi)\ln k(0)+\ln\rho+\varphi\ln\frac{\rho_G\varphi}{1+\varphi}+\left(A-\frac{\rho_G\varphi}{1+\varphi}-\rho\right)(1+\varphi)(x+t)\right]dt$$

$$U(x)=\frac{(1+\varphi)\ln k(0)+\ln\rho+\varphi\ln\rho_G+\varphi\ln\frac{\varphi}{1+\varphi}+\left(A-\frac{\rho_G\varphi}{1+\varphi}-\rho\right)(1+\varphi)x}{\rho}$$
$$+\int_0^{\infty} e^{-\rho t}\left[\left(A-\frac{\rho_G\varphi}{1+\varphi}-\rho\right)(1+\varphi)t\right]dt$$

$$U(x)=\frac{(1+\varphi)\ln k(0)+\ln\rho+\varphi\ln\rho_G+\varphi\ln\frac{\varphi}{1+\varphi}+\left(A-\frac{\rho_G\varphi}{1+\varphi}-\rho\right)(1+\varphi)x}{\rho}$$
$$+\frac{\left(A-\frac{\rho_G\varphi}{1+\varphi}-\rho\right)(1+\varphi)}{\rho^2}$$

$$U(x)=\frac{(1+\varphi)\ln k(0)+\ln\rho+\varphi\ln\frac{\varphi}{1+\varphi}+(A-\rho)(1+\varphi)x}{\rho}+\frac{(A-\rho)(1+\varphi)}{\rho^2}$$
$$+\frac{\varphi\ln\rho_G-\rho_G\varphi x}{\rho}-\frac{\rho_G\varphi}{\rho^2}$$

$$U(x)=\frac{(1+\varphi)\ln k(0)+\ln\rho+\varphi\ln\frac{\varphi}{1+\varphi}+(A-\rho)(1+\varphi)x}{\rho}$$
$$+\frac{(A-\rho)(1+\varphi)}{\rho^2}+\frac{\varphi}{\rho}\left(\ln\rho_G-\rho_G x-\frac{\rho_G}{\rho}\right) \tag{3.76}$$

$\ddot{x}$ is obtained by solving

$$U_{DS}(x)\geq U_{DB}(x) \tag{3.77}$$

$$\ln \rho_G - \rho_G x - \frac{\rho_G}{\rho} \geq \ln \rho - \rho x - \frac{\rho}{\rho}$$

$$x \geq \frac{(\ln \rho - \ln \rho_G)}{\rho - \rho_G} - \frac{1}{\rho} \quad if \quad \rho_G < \rho$$

Hence

$$\ddot{x} = \frac{(\ln \rho - \ln \rho_G)}{\rho - \rho_G} - \frac{1}{\rho} \tag{3.78}$$

From (3.78), we have

$$\frac{\partial \ddot{x}}{\partial \rho_G} > 0 \quad if \quad \rho_G < \rho \tag{3.79}$$

The generation utility for the generation born at date x is

$$\ddot{U}(x) = \int_x^{x+l} e^{-\rho(t-x)} \left[\ln c(t) + \varphi \ln h(t) \right] dt \tag{3.80}$$

The generation utility can be written as

$$\ddot{U}(x) = \int_0^l e^{-\rho t} \left\{ \ln\left[c(x) e^{\gamma t} \right] + \varphi \ln\left[h(x) e^{\gamma t} \right] \right\} dt$$

$$\ddot{U}(x) = \int_0^l e^{-\rho t} \left[(1+\varphi) \ln k(0) + \ln \rho + \varphi \ln \frac{\rho_G \varphi}{1+\varphi} + \left(A - \frac{\rho_G \varphi}{1+\varphi} - \rho \right) (1+\varphi)(x+t) \right] dt$$

$$\ddot{U}(x) = \left[(1+\varphi) \ln k(0) + \ln \rho + \varphi \ln \frac{\varphi}{1+\varphi} + (A-\rho)(1+\varphi) x \right] \frac{1-e^{\rho l}}{\rho}$$
$$+ (\ln \rho_G - \rho_G x) \frac{\varphi\left(1-e^{-\rho l}\right)}{\rho} + \int_0^l e^{-\rho t} \left(A - \frac{\rho_G \varphi}{1+\varphi} - \rho \right) (1+\varphi) t dt$$

$$\ddot{U}(x) = \left[(1+\varphi) \ln k(0) + \ln \rho + \varphi \ln \frac{\varphi}{1+\varphi} + (A-\rho)(1+\varphi) x \right] \frac{1-e^{\rho l}}{\rho}$$
$$+ (\ln \rho_G - \rho_G x) \frac{\varphi\left(1-e^{-\rho l}\right)}{\rho} + (A-\rho)(1+\varphi) \left(\frac{1-e^{-\rho l}}{\rho^2} - \frac{l e^{-\rho l}}{\rho} \right)$$
$$- \rho_G \varphi \left(\frac{1-e^{-\rho l}}{\rho^2} - \frac{l e^{-\rho l}}{\rho} \right)$$

$$\ddot{U}(x)=\left[(1+\varphi)\ln k(0)+\ln\rho+\varphi\ln\frac{\varphi}{1+\varphi}+(A-\rho)(1+\varphi)x\right]\frac{1-e^{-\rho l}}{\rho}$$
$$+(A-\rho)(1+\varphi)\left(\frac{1-e^{-\rho l}}{\rho^{2}}-\frac{le^{-\rho l}}{\rho}\right)\tag{3.81}$$
$$+\left(\ln\rho_G-\rho_G x-\frac{\rho_G}{\rho}+\rho_G\frac{le^{-\rho l}}{1-e^{-\rho l}}\right)\frac{\varphi\left(1-e^{-\rho l}\right)}{\rho}$$

$\tilde{x}$ is obtained by solving

$$\ddot{U}_{DS}(x)\geq\ddot{U}_{DB}(x)\tag{3.82}$$

$$\ln\rho_G-\rho_G x-\frac{\rho_G}{\rho}+\rho_G\frac{le^{-\rho l}}{1-e^{-\rho l}}\geq\ln\rho-\rho x-\frac{\rho}{\rho}+\rho\frac{le^{-\rho l}}{1-e^{-\rho l}}$$
$$x\geq\frac{(\ln\rho-\ln\rho_G)}{\rho-\rho_G}-\frac{1}{\rho}+\frac{le^{-\rho l}}{1-e^{-\rho l}}\quad if\quad\rho_G<\rho$$

Hence

$$\tilde{x}=\frac{(\ln\rho-\ln\rho_G)}{\rho-\rho_G}-\frac{1}{\rho}+\frac{le^{-\rho l}}{1-e^{-\rho l}}\tag{3.83}$$

From (3.83), we have

$$\frac{\partial\tilde{x}}{\partial\rho_G}>0\quad if\quad\rho_G<\rho\tag{3.84}$$

Since

$$-\frac{1}{\rho}+\frac{le^{-\rho l}}{1-e^{-\rho l}}<0\tag{3.85}$$

$$-\frac{1}{\rho}+\frac{le^{-\rho l}}{1-e^{-\rho l}}+l>0\tag{3.86}$$

(3.74), (3.78) and (3.83) also prove (3.69).

Prove: $-\frac{1}{\rho}+\frac{le^{-\rho l}}{1-e^{-\rho l}}<0$

Set $q_1 = e^z - 1 - z$

$\because q_1 = 0$ if $z = 0$, $\dfrac{\partial q_1}{\partial z} = 0$ if $z = 0$, $\dfrac{\partial q_1}{\partial z} > 0$ if $z > 0$.

$\therefore q_1 = e^z - 1 - z > 0$ if $z > 0$.

$\therefore e^{\rho l} - 1 - \rho l > 0$ since $\rho l > 0$

$\therefore 1 - e^{-\rho l} - \rho l e^{-\rho l} > 0$

$\therefore 1 - e^{-\rho l} - \rho l e^{-\rho l} > 0$

$\therefore -\dfrac{1}{\rho} + \dfrac{l e^{-\rho l}}{1 - e^{-\rho l}} < 0$

Prove: $-\dfrac{1}{\rho} + \dfrac{l e^{-\rho l}}{1 - e^{-\rho l}} + l > 0$

Set $q_2 = z e^z - e^z + 1$

$\because q_2 = 0$ if $z = 0$, $\dfrac{\partial q_2}{\partial z} = 0$ if $z = 0$, $\dfrac{\partial q_2}{\partial z} > 0$ if $z > 0$.

$\therefore q_2 = z e^z - e^z + 1 > 0$ if $z > 0$.

$\therefore \rho l e^{\rho l} - e^{\rho l} + 1 > 0$ since $\rho l > 0$

$\therefore \dfrac{\rho l}{1 - e^{-\rho l}} > 1$

$\therefore -\dfrac{1}{\rho} + \dfrac{l}{1 - e^{-\rho l}} > 0$

$\therefore -\dfrac{1}{\rho} + \dfrac{l e^{-\rho l}}{1 - e^{-\rho l}} + l > 0$

(3.74), (3.79) and (3.84) imply the below proposition.

Proposition 3.3F:

The more farsighted the government is, the faster the overall utility, the generation utility and the instantaneous utility exceed their counterparts with a benevolent government.

3.3.10 Welfare Loss with Conflicting Time Preferences

From (3.76), we have, at the beginning, the welfare of the representative family is

$$U(0)=\frac{(1+\varphi)\ln k(0)+\ln\rho+\varphi\ln\dfrac{\varphi}{1+\varphi}}{\rho}+\frac{(A-\rho)(1+\varphi)}{\rho^2}+\frac{\varphi}{\rho}\left(\ln\rho_G-\frac{\rho_G}{\rho}\right) \tag{3.87}$$

$$U_{DS}(0)-U_{DB}(0)=\frac{\varphi}{\rho}\left(\ln\rho_G-\ln\rho-\frac{\rho_G-\rho}{\rho}\right)<0 \tag{3.88}$$

(3.88) implies that the representative family has welfare loss when the government is not benevolent.

The government's intertemporal utility is

$$U^G(0)=\frac{(1+\varphi)\ln k(0)+\ln\rho+\varphi\ln\dfrac{\varphi}{1+\varphi}}{\rho_G}+\frac{(A-\rho)(1+\varphi)}{\rho_G^2}+\frac{\varphi}{\rho_G}(\ln\rho_G-1) \tag{3.89}$$

If the representative family is obedient and set its time preference equal to the government's, then the government is 'benevolent' again, and the government's intertemporal utility becomes

$$U_{DB'}^G(0)=\frac{(1+\varphi)\ln k(0)+\ln\rho_G+\varphi\ln\dfrac{\varphi}{1+\varphi}}{\rho_G}+\frac{(A-\rho_G)(1+\varphi)}{\rho_G^2}+\frac{\varphi}{\rho_G}(\ln\rho_G-1) \tag{3.90}$$

We have

$$U_{DS}^{G}\left(0\right)-U_{DB'}^{G}\left(0\right)=\frac{1}{\rho_G}\left[\ln\rho-\ln\rho_G-\frac{\left(\rho-\rho_G\right)\left(1+\varphi\right)}{\rho_G}\right]<0 \tag{3.91}$$

As illustrated before, the Pareto optimal resource allocation can be realised in the centralised models and the Ramsey solutions in the decentralised models are second best. In other words, welfare loss arises due to the tax distortion in the decentralised models. However tax distortion is not the only reason of inefficient allocation, welfare loss is also from different time preferences between the representative and the government. For the representative, he currently prefer a benevolent government; while the patient government hopes the citizens are obedient and adjust their time preference to the same level of the government's.

3.4 No Chance to Increase the Growth in a Model with Productive Public Service

Barro (1990) extended the previous endogenous growth models to incorporate the flow of productive public service in the production function. The elasticity of substitution in Cobb-Douglas function is unity, resulting in that the growth-maximisation income tax rate is also the rate which maximises the intertemporal utility of the representative, therefore, a government can not increase the growth rate even if it is more patient than the representative.

3.4.1 The Structure of the Barro Model

The objective of the representative household-producer is to maximise his intertemporal utility：

$$\int_0^\infty e^{-\rho t} u(c)\,dt \tag{4.1}$$

$$u(c) = \ln c \tag{4.2}$$

where c is consumption, ρ is the rate of time preference.

The household-producer produces y according to a Cobb-Douglas technology.

$$y = f(k, g) = Ak^{1-\alpha} g^{\alpha} \tag{4.3}$$

where k is private capital and g is the flow of public service. $k(0) = k_0 > 0$ is given, A is a technological scale factor, α and $1-\alpha$ are the output elasticities of g and k, respectively.

The government's budget constraint is

$$g = \tau y \quad here \quad 0 < \tau < 1 \tag{4.4}$$

where τ is the income tax rate.

The household-producer's budget constraint is

$$\dot{k} = (1-\tau)\,y - c \tag{4.5}$$

3.4.2 Decentralised Competitive Equilibrium

The representative maximises his intertemporal utility, given τ and g. To solve the representative's problem, set up the current-value Hamiltonian

$$H \equiv \ln c + \lambda_k\left[(1-\tau)\,y - c\right] \tag{4.6}$$

The transversality condition is

$$\lim_{t\to\infty} e^{-\rho t}\lambda_k k = 0 \tag{4.7}$$

The first order conditions are

$$\begin{aligned}
&\frac{\partial H}{\partial c} = \frac{1}{c} - \lambda_k = 0 && (4.8a)\\
&\frac{\partial H}{\partial k} = \lambda_k\,(1-\tau)\frac{\partial y}{\partial k} = \rho\lambda_k - \dot{\lambda}_k && (4.8b)
\end{aligned} \tag{4.8}$$

(4.8) implies

$$(1-\tau)\frac{\partial y}{\partial k} = \rho + \frac{\dot{c}}{c} \tag{4.9}$$

(4.9) is the Euler equation which can be written as

$$\dot{c} = c\left[(1-\tau)\frac{\partial y}{\partial k} - \rho\right] \tag{4.10}$$

The DCE is:

$$\begin{aligned}
&\dot{c} = c\left[(1-\tau)A(1-\alpha)k^{-\alpha}g^{\alpha} - \rho\right] && (4.11a)\\
&\dot{k} = (1-\tau)Ak^{1-\alpha}g^{\alpha} - c && (4.11b)
\end{aligned} \tag{4.11}$$

(4.3) and (4.4) imply

$$\frac{g}{k} = \left(A\tau\right)^{\frac{1}{1-\alpha}} \tag{4.12}$$

Substitute (4.12) into (4.11), the DCE is

$$\begin{aligned} \dot{c} &= c\left[\left(1-\tau\right)\tau^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}\left(1-\alpha\right)-\rho\right] && \left(4.13a\right) \\ \dot{k} &= \left(1-\tau\right)\tau^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}k-c && \left(4.13b\right) \end{aligned} \tag{4.13}$$

3.4.3 Long-Run Growth-Maximisation Policy

(4.13) shows that as long as the tax rate τ is constant, the DCE is on BGP. Hence

$$\frac{\dot{y}}{y} = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{g}}{g} = \gamma \tag{4.14}$$

(4.13a) and (4.14) imply that the long-run growth rate is

$$\gamma = \left(1-\tau\right)\tau^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}\left(1-\alpha\right)-\rho \tag{4.15}$$

To maximise the growth rate γ, we have the solution for tax rate τ

$$\tau_{DG} = \alpha \tag{4.16}$$

Hence the maximum growth rate is

$$\gamma_{DG} = A^{\frac{1}{1-\alpha}}\left(1-\alpha\right)^{2}\alpha^{\frac{\alpha}{1-\alpha}}-\rho \tag{4.17}$$

3.4.4 Decentralised General Equilibrium Solution

The government chooses a sequence of tax rates at date zero to maximise the representative’s intertemporal utility with the government’s own time preference as below. If the government is self-interested, then $\rho_G \neq \rho$; if the government is benevolent, then $\rho_G = \rho$.

$$\int_0^\infty e^{-\rho_G t} u dt \quad here \quad \rho_G > 0 \tag{4.18}$$

The current-value Hamiltonian for the self-interested government is

$$\begin{aligned} H^{GS} \equiv \ln c + \mu_k \left[(1-\tau)\tau^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} k - c \right] \\ + \mu_c c \left[(1-\tau)\tau^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} (1-\alpha) - \rho \right] \end{aligned} \tag{4.19}$$

First order conditions are

$$\begin{aligned}
&\frac{\partial H^{GS}}{\partial \tau} = \left[\mu_k k + \mu_c c(1-\alpha) \right] \tau^{\frac{\alpha}{1-\alpha}-1} \left[-\tau + \frac{\alpha}{1-\alpha}(1-\tau) \right] = 0 && (4.20a) \\
&\frac{\partial H^{GS}}{\partial k} = \rho_G \mu_k - \dot{\mu}_k && (4.20b) \\
&\frac{\partial H^{GS}}{\partial c} = \rho_G \mu_c - \dot{\mu}_c && (4.20c) \quad (4.20) \\
&\frac{\partial H^{GS}}{\partial \mu_k} = \dot{k} && (4.20d) \\
&\frac{\partial H^{GS}}{\partial \mu_c} = \dot{c} && (4.20f)
\end{aligned}$$

(4.20a) implies the solution of the tax rate is

$$\tau_{DS} = \alpha \tag{4.21}$$

ρ_G does not enter the solution of the tax rate even the government is benevolent.

Proposition 3.4A:

With Cobb-Douglas production function in Barro model, the fiscal policy is independent of either the representative's or the government's time preference.

3.4.5 Discussion

The solution $\tau = \alpha$ is of the below characters:

(a) It is dependent on neither the representative's time preference nor the government's.
(b) It is the solution for both growth maximisation and utility maximisation.
(c) It is optimal in both short run and long run since it maximises both one-period and overall utility.
(d) It is both second-best and first-best solutions. In the centralised model, the growth rate is higher, but the optimum solution of tax rate is still the same as that in the decentralised model. (See Barro (1990) for details.)
(e) It is optimal for both household-producer and the government; it is optimal for both benevolent and self-interested government.

Since the Ramsey second-best solution is also the growth-maximisation solution in the Barro model, the government can not increase the growth rate even if the government is more farsighted than the citizens. Furthermore, it is still impossible for a patient government to increase the economic growth by fiscal instrument in the centralised model, which coincides with the conclusion of Devarajan, Swaroop and Zou (1996): 'productive expenditures, when used in excess, could become unproductive'. The key is that the elasticity of substitution in Cobb-Douglas production function is unity and thus maximising the growth rate is efficient. When a general CES production function is used in the Barro model, the tax rate set by the benevolent government will be smaller than the rate that maximises the growth rate if the elasticity of substitution is less than unity. (See Devarajan, Swaroop and Zou (1996) and Misch, Gemmell and Kneller (2013), their models use a general CES technology.) When the growth-maximisation tax rate lies above the welfare-maximisation equivalent, a farsighted government would achieve a higher growth rate than the benevolent government. (The detailed comparison under CES production function with a flow of public productive expenditure is not given here since the purpose of this section is to illustrate that in some special case, the discount rate of the government does not affect the growth, in other words, whether an ambitious government can increase the growth depends on the form of production function.)

3.5 Increase the Growth in a Model with Productive Public Capital

Following Barro (1990), Futagami, Morita and Shibata (FMS) (1993) developed a similar endogenous growth model in 1993. The only difference is that the FMS model assumes that the tax revenue finances the public investment which is accumulated into public capital. In other words, the public capital rather than the public service takes a role in the production. The FMS model concludes that the optimal tax rate in the decentralised model is smaller than the rate that attains the maximum growth rate. For tractability, the FMS model assumes that the tax rate is time invariant. This section examines FMS's conclusion by the typical Ramsey method and makes the assumption that the tax rate can be time variant if it is profitable for the leading player, the government.

3.5.1 The Structure of the FMS Model

The objective of representative household-producer is to maximise his intertemporal utility

$$\int_0^\infty e^{-\rho t} u(c)\, dt \tag{5.1}$$

$$u(c) = \ln c \tag{5.2}$$

where c is consumption, ρ is the rate of time preference.

The household-producer produces y according to a Cobb-Douglas technology as in Barro (1990), Glomm and Ravikumar (1997) and Economides and Philippopoulos (2008).

$$y = f\left(k, k_g\right) = Ak^{1-\alpha} k_g^{\alpha} \tag{5.3}$$

where k is private capital and k_g is public capital. $k(0)$ and $k_g(0)$ are given and both are positive, A is a technological scale factor, α and $1-\alpha$ are the output elasticities of k_g and k, respectively.

The government's budget constraint is

$$\dot{k}_g = \tau y \quad here \quad 0 < \tau < 1 \tag{5.4}$$

where τ is the income tax rate.

The household-producer's budget constraint is

$$\dot{k} = (1-\tau) y - c \tag{5.5}$$

3.5.2 Decentralised Competitive Equilibrium

The representative household-producer maximises his intertemporal utility, given tax rate τ and public capital k_g. To solve the representative's problem, set up the current-value Hamiltonian

$$H \equiv \ln c + \lambda_k \left[(1-\tau) y - c \right] \tag{5.6}$$

The transversality condition is

$$\lim_{t \to \infty} e^{-\rho t} \lambda_k k = 0 \tag{5.7}$$

The first order conditions are

$$\begin{aligned} \frac{\partial H}{\partial c} &= \frac{1}{c} - \lambda_k = 0 && (5.8a) \\ \frac{\partial H}{\partial k} &= \lambda_k (1-\tau) \frac{\partial y}{\partial k} = \rho \lambda_k - \dot{\lambda}_k && (5.8b) \end{aligned} \tag{5.8}$$

(5.8) implies

$$(1-\tau) \frac{\partial y}{\partial k} = \rho + \frac{\dot{c}}{c} \tag{5.9}$$

(5.9) is the Euler equation which can be written as

$$\dot{c} = c\left[(1-\tau)\frac{\partial y}{\partial k} - \rho\right] \tag{5.10}$$

(5.10) and (5.5) constitute the DCE:

$$\begin{aligned} \dot{c} &= c\left[(1-\tau)A(1-\alpha)k^{-\alpha}k_g^{\alpha} - \rho\right] && (5.11a) \\ \dot{k} &= (1-\tau)Ak^{1-\alpha}k_g^{\alpha} - c && (5.11b) \end{aligned} \tag{5.11}$$

3.5.3 Long-Run Growth-Maximisation Policy

Define the auxiliary variables:

$$\begin{aligned} \frac{c}{k} &= \hat{c} && (5.12a) \\ \frac{k_g}{k} &= \hat{k}_g && (5.12b) \end{aligned} \tag{5.12}$$

On BGP in the long run, we have

$$\frac{\dot{y}}{y} = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{k}_g}{k_g} = \gamma \tag{5.13}$$

$$\begin{aligned} \frac{\dot{\tau}}{\tau} &= 0 && (5.14a) \\ \frac{\dot{\hat{c}}}{\hat{c}} &= 0 && (5.14b) \\ \frac{\dot{\hat{k}}_g}{\hat{k}_g} &= 0 && (5.14c) \end{aligned} \tag{5.14}$$

Hence, (5.11a) becomes

$$(1-\tau)A(1-\alpha)\hat{k}_g^{\alpha} - \rho = \gamma \tag{5.15}$$

(5.3) and (5.4) imply on BGP,

$$\begin{aligned}
&\hat{k}_g = \left(A\tau\gamma^{-1}\right)^{\frac{1}{1-\alpha}} && (5.16a) \\
&\tau = A^{-1}\gamma\hat{k}_g^{1-\alpha} && (5.16b) \\
&\gamma = A\tau\hat{k}_g^{\alpha-1} && (5.16c)
\end{aligned} \qquad (5.16)$$

Substitute (5.16a) into (5.15), we have

$$(1-\tau)A(1-\alpha)\left(A\tau\gamma^{-1}\right)^{\frac{\alpha}{1-\alpha}} - \rho = \gamma$$

$$A^{\frac{1}{1-\alpha}}(1-\alpha)(1-\tau)\tau^{\frac{\alpha}{1-\alpha}} = (\gamma+\rho)\gamma^{\frac{\alpha}{1-\alpha}} \qquad (5.17)$$

To maximise the growth rate γ, the left hand side of equation (5.17) should be maximised, then we have the solution for tax rate τ

$$\tau_{DG} = \alpha \qquad (5.18)$$

Where the subscript 'DG' denotes 'the *Decentralised* model with *Growth* maximisation'.

Proposition 3.5 A (FMS Proposition 3):

The long-run growth rate on BGP in the decentralised model attains its maximum when $\boldsymbol{\tau = \alpha}$.

There is no closed form of γ_{DG} the growth rate with the growth-maximisation policies. The numerical solution of γ_{DG} can be obtained by

$$A^{\frac{1}{1-\alpha}}(1-\alpha)^2\alpha^{\frac{\alpha}{1-\alpha}} = (\gamma_{DG}+\rho)\gamma_{DG}^{\frac{\alpha}{1-\alpha}} \qquad (5.19)$$

From (5.3), (5.12b) and (5.13), we obtain the saving rate

$$s = \frac{\dot{k}}{y} = \frac{\dot{k}}{k}\cdot\frac{k}{y} = \gamma A^{-1}\hat{k}_g^{-\alpha} = \gamma A^{-1}\left(A\tau\gamma^{-1}\right)^{\frac{\alpha}{\alpha-1}} = A^{\frac{1}{\alpha-1}}\tau^{\frac{\alpha}{\alpha-1}}\gamma^{\frac{1}{1-\alpha}} \qquad (5.20)$$

The saving rate under the growth maximisation is

$$s_{DG} = \gamma_{DG} A^{-1} \hat{k}_{gDG}^{-\alpha} = A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha}{\alpha-1}} \gamma_{DG}^{\frac{1}{1-\alpha}} \tag{5.21}$$

The after-tax saving rate is

$$s^A = \frac{\dot{k}}{(1-\tau) y} = (1-\tau)^{-1} s = (1-\tau)^{-1} \gamma A^{-1} \hat{k}_g^{-\alpha} = A^{\frac{1}{\alpha-1}} (1-\tau)^{-1} \tau^{\frac{\alpha}{\alpha-1}} \gamma^{\frac{1}{1-\alpha}} \tag{5.22}$$

The after-tax saving rate under the growth maximisation is

$$s_{DG}^A = (1-\alpha)^{-1} \gamma_{DG} A^{-1} \hat{k}_{gDG}^{-\alpha} = A^{\frac{1}{\alpha-1}} (1-\alpha)^{-1} \alpha^{\frac{\alpha}{\alpha-1}} \gamma_{DG}^{\frac{1}{1-\alpha}} \tag{5.23}$$

The ratio of two types of capital under the growth maximisation is

$$\hat{k}_{gDG} = A^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \gamma_{DG}^{\frac{1}{\alpha-1}} \tag{5.24}$$

3.5.4 Decentralised General Equilibrium with a Benevolent Government

In a decentralised economy, the benevolent government's problem is to choose the path of tax rate τ to maximise the representative's intertemporal utility (5.1) subject to the government's budget constraint (5.4), the representative's budget constraint (5.5) and the Euler equation (5.10).

The current-value Hamiltonian for the government is

$$\begin{aligned} H^{GB} \equiv & \ln c + \mu_{kg} \tau y \\ & + \mu_k \left[(1-\tau) y - c \right] \\ & + \mu_c c \left[(1-\tau) \frac{\partial y}{\partial k} - \rho \right] \end{aligned} \tag{5.25}$$

Where μ_{kg}, μ_k and μ_c are dynamic multipliers associated with (5.4), (5.5) and (5.10) respectively.

First order conditions are

$$
\begin{aligned}
&\frac{\partial H^{GB}}{\partial \tau} = \mu_{kg} y - \mu_k y - \mu_c c \frac{\partial y}{\partial k} = 0 && (5.26a) \\
&\frac{\partial H^{GB}}{\partial k_g} = \mu_{kg} \tau \frac{\partial y}{\partial k_g} + \mu_k (1-\tau) \frac{\partial y}{\partial k_g} + \mu_c c (1-\tau) \frac{\partial^2 y}{\partial k \partial k_g} = \rho \mu_{k_G} - \dot{\mu}_{k_G} && (5.26b) \\
&\frac{\partial H^{GB}}{\partial k} = \mu_{kg} \tau \frac{\partial y}{\partial k} + \mu_k (1-\tau) \frac{\partial y}{\partial k} + \mu_c c (1-\tau) \frac{\partial^2 y}{\partial k^2} = \rho \mu_k - \dot{\mu}_k && (5.26c) \\
&\frac{\partial H^{GB}}{\partial c} = \frac{1}{c} - \mu_k + \mu_c \left[(1-\tau) \frac{\partial y}{\partial k} - \rho \right] = \rho \mu_c - \dot{\mu}_c && (5.26d) \quad (5.26) \\
&\frac{\partial H^{GB}}{\partial \mu_k} = (1-\tau) y - c = \dot{k} && (5.26e) \\
&\frac{\partial H^{GB}}{\partial \mu_c} = (1-\tau) \frac{\partial y}{\partial k} - \rho = \dot{c} && (5.26f) \\
&\frac{\partial H^{GB}}{\partial \mu_{kg}} = \tau y = \dot{k}_g && (5.26g)
\end{aligned}
$$

Substitute the production function (5.3) for y into the first order conditions of the Ramsey problem in (5.26), then (5.26a)-(5.26g) constitute a seven-equation system in the paths of $\tau, c, k, k_g, \mu_c, \mu_k, \mu_{kg}$. This is a general equilibrium with Ramsey second-best allocation.

3.5.5 Long-Run Decentralised General Equilibrium with a Benevolent Government

The first step of traditional method of solving the long-run second-best general equilibrium is to make the equilibrium system stationary. (See Economides and Philippopoulos (2008) and Novales, Pérez and Ruiz (2014).)

Define

$$
\begin{aligned}
&\mathrm{M}_{kg} = \mu_{kg} k_g && (5.27a) \\
&\mathrm{M}_k = \mu_k k && (5.27b) \qquad (5.27) \\
&\mathrm{M}_c = \mu_c c && (5.27c)
\end{aligned}
$$

The dynamic general equilibrium system becomes

$$\mathrm{M}_c(1-\alpha)+\mathrm{M}_k=\frac{\mathrm{M}_{kg}}{\hat{k}_g} \qquad (5.28a)$$

$$\dot{\mathrm{M}}_{kg}=\mathrm{M}_{kg}\rho-(1-\tau)A(1-\alpha)\alpha\hat{k}_g^{\alpha}\mathrm{M}_c -(1-\tau)A\alpha\,\hat{k}_g^{\alpha}\mathrm{M}_k+\tau A(1-\alpha)\hat{k}_g^{\alpha-1}\mathrm{M}_{kg} \qquad (5.28b)$$

$$\dot{\mathrm{M}}_k=\mathrm{M}_k(1-\tau)A\alpha\,\hat{k}_g^{\alpha}+\mathrm{M}_k(\rho-\hat{c}) +\mathrm{M}_c(1-\tau)A(1-\alpha)\alpha\hat{k}_g^{\alpha}-\mathrm{M}_{kg}\tau A(1-\alpha)\hat{k}_g^{\alpha-1} \qquad (5.28c)$$

$$\dot{\mathrm{M}}_c=\mathrm{M}_c\rho+\mathrm{M}_k\hat{c}-1 \qquad (5.28d)$$

$$\dot{\hat{k}}_g=\tau A\hat{k}_g^{\alpha}-(1-\tau)A\hat{k}_g^{1+\alpha}+\hat{c}k_g \qquad (5.28e)$$

$$\dot{\hat{c}}=\hat{c}^2-\rho\hat{c}-(1-\tau)A\alpha\,\hat{k}_g^{\alpha}\hat{c} \qquad (5.28f)$$

$$(5.28)$$

Formed by 6 equations in the paths of $\tau, \hat{c}, \hat{k}_g, \mathrm{M}_{kg}, \mathrm{M}_k, \mathrm{M}_c$, the above system shows a stationary general equilibrium with Ramsey second-best policy.

We can obtain the long-run second-best general equilibrium by setting

$$\dot{\mathrm{M}}_{kg}=\dot{\mathrm{M}}_k=\dot{\mathrm{M}}_c=\dot{\hat{k}}_g=\dot{\hat{c}}=0 \qquad (5.29)$$

Substitute (5.29) into the stationary general equilibrium system (5.28), we have

$$\mathrm{M}_c(1-\alpha)+\mathrm{M}_k=\frac{\mathrm{M}_{kg}}{\hat{k}_g} \qquad (5.30a)$$

$$\mathrm{M}_{kg}\rho+\mathrm{M}_k(\rho-\hat{c})=0 \qquad (5.30b)$$

$$0=\mathrm{M}_k(1-\tau)A\alpha\,\hat{k}_g^{\alpha}+\mathrm{M}_k(\rho-\hat{c}) +\mathrm{M}_c(1-\tau)A(1-\alpha)\alpha\hat{k}_g^{\alpha}-\mathrm{M}_{kg}\tau A(1-\alpha)\hat{k}_g^{\alpha-1} \qquad (5.30c)$$

$$\mathrm{M}_c\rho+\mathrm{M}_k\hat{c}=1 \qquad (5.30d)$$

$$(1-\tau)A\hat{k}_g^{\alpha}=\tau A\hat{k}_g^{\alpha-1}+\hat{c} \qquad (5.30e)$$

$$\hat{c}=\rho\hat{c}+(1-\tau)A\alpha\,\hat{k}_g^{\alpha} \qquad (5.30f)$$

$$(5.30)$$

There are 6 variables $\tau, \hat{c}, \hat{k}_g, \mathrm{M}_{kg}, \mathrm{M}_k, \mathrm{M}_c$ and 6 equations in the above system which characterizes the BGP for Ramsey equilibrium. This system can be solved numerically by setting the parameter values.

3.5.6 A Simple Analytical Method of Solving the Benevolent Government's Long-Run Ramsey Policy

The numerical method in the literature as in 3.5.5 is unconvincing to rebut the argument in Barro (1990) that the optimal tax rate is the same as the rate that maiximises the growth. This section deduces Proposition 5 in Futagami, Morita and Shibata (1993) by qualitative comparison following the Ramsey method in 3.5.4.

We can exploit the first order condition (5.26a) which implies

$$\mu_c c \frac{\partial y}{\partial k} = \left(\mu_{kg} - \mu_k\right) y \tag{5.31}$$

Substitute (5.31) in (5.26b), we have

$$\dot{\mu}_{kg} = \rho \mu_{kg} - \mu_{kg} \frac{\partial y}{\partial k_g} \tag{5.32}$$

On BGP in the long run, $\mu_{kg} k_g$ is a constant, hence we have

$$-\frac{\dot{\mu}_{kg}}{\mu_{kg}} = \frac{\dot{k}_g}{k_g} = \frac{\dot{c}}{c} = \gamma \tag{5.33}$$

Substitute (5.33) in (5.32), we have

$$\frac{\partial y}{\partial k_g} = \rho + \gamma \tag{5.34}$$

Equation (5.34) is just equation (7) in Ghosh and Roy (2004).

(5.34) imlies

$$\gamma_{DB} = A\alpha \hat{k}_{gDB}^{\alpha-1} - \rho \tag{5.35}$$

where subscript 'DB' means '*Decentralised* model with a *Benevolent* Government'.

Equations (5.15) and (5.35) imply that

$$A\alpha \hat{k}_{gDB}^{\alpha-1} = \left(1-\tau_{DB}\right) A\left(1-\alpha\right) \hat{k}_{gDB}^{\alpha} \tag{5.36}$$

Hence the ratios of two types of capital is

$$\hat{k}_{gDB} = \frac{\alpha}{(1-\alpha)(1-\tau_{DB})} \tag{5.37}$$

Substitute (5.35) into (5.16b), we have

$$\tau_{DB} = \alpha - \rho A^{-1}\hat{k}_{gDB}^{1-\alpha} \tag{5.38}$$

Substitute (5.37) into (5.38), we have

$$\tau_{DB} = \alpha - \rho A^{-1}(1-\tau_{DB})^{\alpha-1}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \tag{5.39}$$

τ, γ and $\hat{k}_g$ can be solved one by one numerically through (5.39), (5.37) and (5.35).

From (5.39), we have

$$\tau_{DB} < \alpha \tag{5.40}$$

Proposition 3.5B (FMS Proposition 5):

On BGP in the decentralised model with a benevolent government, the second-best tax rate is lower than the rate which attains the maximum growth rate.

(5.17) shows $\frac{\mathrm{d}\gamma}{\mathrm{d}\tau} > 0$ when $\tau < \alpha$. Hence, (5.18) and (5.40) imply

$$\gamma_{DB} < \gamma_{DG} \tag{5.41}$$

(5.15) implies

$$\hat{k}_g = \left[\frac{\gamma+\rho}{(1-\tau)A(1-\alpha)}\right]^{\frac{1}{\alpha}} \tag{5.42}$$

Proposition 3.5B, (5.41) and (5.42) imply

$$\hat{k}_{gDB} < \hat{k}_{gDG} \tag{5.43}$$

With a benevolent government, the saving rate is

$$s_{DB} = \gamma_{DB} A^{-1} \hat{k}_{gDB}^{-\alpha} = A^{\frac{1}{\alpha-1}} \tau_{DB}^{\frac{\alpha}{\alpha-1}} \gamma_{DB}^{\frac{1}{1-\alpha}} \tag{5.44}$$

With a benevolent government, the after-tax saving rate is

$$s_{DB}^{A} = \left(1-\tau_{DB}\right)^{-1} \gamma_{DB} A^{-1} \hat{k}_{gDB}^{-\alpha} = A^{\frac{1}{\alpha-1}} \left(1-\tau_{DB}\right)^{-1} \tau_{DB}^{\frac{\alpha}{\alpha-1}} \gamma_{DB}^{\frac{1}{1-\alpha}} \tag{5.45}$$

3.5.7 Decentralised General Equilibrium with a Self-Interested Government

The government may have its own time preference

$$\rho_G \neq \rho \tag{5.46}$$

The current-value Hamiltonian for the government is

$$H^{GS} \equiv \ln c + \mu_{kg} \tau y + \mu_k \left[\left(1-\tau\right) y - c\right] + \mu_c c \left[\left(1-\tau\right) \frac{\partial y}{\partial k} - \rho\right] \tag{5.47}$$

First order conditions are

$$\begin{aligned}
&\frac{\partial H^{GS}}{\partial \tau} = \mu_{kg} y - \mu_k y - \mu_c c \frac{\partial y}{\partial k} = 0 && (5.48a) \\
&\frac{\partial H^{GS}}{\partial k_g} = \mu_{kg} \tau \frac{\partial y}{\partial k_g} + \mu_k \left(1-\tau\right) \frac{\partial y}{\partial k_g} + \mu_c c \left(1-\tau\right) \frac{\partial^2 y}{\partial k \partial k_g} = \rho_G \mu_{k_G} - \dot{\mu}_{k_G} && (5.48b) \\
&\frac{\partial H^{GS}}{\partial k} = \mu_{kg} \tau \frac{\partial y}{\partial k} + \mu_k \left(1-\tau\right) \frac{\partial y}{\partial k} + \mu_c c \left(1-\tau\right) \frac{\partial^2 y}{\partial k^2} = \rho_G \mu_k - \dot{\mu}_k && (5.48c) \\
&\frac{\partial H^{GS}}{\partial c} = \frac{1}{c} - \mu_k + \mu_c \left[\left(1-\tau\right) \frac{\partial y}{\partial k} - \rho\right] = \rho_G \mu_c - \dot{\mu}_c && (5.48d) \\
&\frac{\partial H^{GS}}{\partial \mu_k} = \left(1-\tau\right) y - c = \dot{k} && (5.48e) \\
&\frac{\partial H^{GS}}{\partial \mu_c} = \left(1-\tau\right) \frac{\partial y}{\partial k} - \rho = \dot{c} && (5.48f) \\
&\frac{\partial H^{GS}}{\partial \mu_{kg}} = \tau y - c = \dot{k}_g && (5.48g)
\end{aligned} \tag{5.48}$$

The system (5.48) is a general equilibrium with a self-interested government. This system includes both ρ and ρ_G.The resource allocation for the representative after the taxation is affected by his time preference; while the government's decision is not only affected by his own time preference, but also responds to the consumption policy function of the household-producer.

3.5.8 Long-Run Decentralised General Equilibrium with a Self-Interested Government

The stationary general equilibrium system is

$$
\begin{aligned}
&\mathrm{M}_c(1-\alpha)+\mathrm{M}_k=\frac{\mathrm{M}_{kg}}{\hat{k}_g} && (5.49a)\\
&\dot{\mathrm{M}}_{kg}=\mathrm{M}_{kg}\rho_G-\mathrm{M}_c(1-\tau)A(1-\alpha)\alpha\hat{k}_g^{\alpha}\\
&\qquad -\mathrm{M}_k(1-\tau)A\alpha\,\hat{k}_g^{\alpha}+\mathrm{M}_{kg}\tau A(1-\alpha)\hat{k}_g^{\alpha-1} && (5.49b)\\
&\dot{\mathrm{M}}_k=\mathrm{M}_k(1-\tau)A\alpha\,\hat{k}_g^{\alpha}+\mathrm{M}_k(\rho_G-\hat{c}) && \qquad (5.49)\\
&\qquad +\mathrm{M}_c(1-\tau)A(1-\alpha)\alpha\hat{k}_g^{\alpha}-\mathrm{M}_{kg}\tau A(1-\alpha)\hat{k}_g^{\alpha-1} && (5.49c)\\
&\dot{\mathrm{M}}_c=\mathrm{M}_c\rho_G+\mathrm{M}_k\hat{c}-1 && (5.49d)\\
&\dot{\hat{k}}_g=\tau A\hat{k}_g^{\alpha}-(1-\tau)A\hat{k}_g^{1+\alpha}+\hat{c}k_g && (5.49e)\\
&\dot{\hat{c}}=\hat{c}^2-\rho\hat{c}-(1-\tau)A\alpha\,\hat{k}_g^{\alpha}\hat{c} && (5.49f)
\end{aligned}
$$

With a self-interested government, the long-run second-best general equilibrium is

$$
\begin{aligned}
&\mathrm{M}_c(1-\alpha)+\mathrm{M}_k=\frac{\mathrm{M}_{kg}}{\hat{k}_g} && (5.50a)\\
&\mathrm{M}_{kg}\rho_G+\mathrm{M}_k(\rho_G-\hat{c})=0 && (5.50b)\\
&0=\mathrm{M}_k(1-\tau)A\alpha\,\hat{k}_g^{\alpha}+\mathrm{M}_k(\rho_G-\hat{c})\\
&\qquad +\mathrm{M}_c(1-\tau)A(1-\alpha)\alpha\hat{k}_g^{\alpha}-\mathrm{M}_{kg}\tau A(1-\alpha)\hat{k}_g^{\alpha-1} && (5.50c) \qquad (5.50)\\
&\mathrm{M}_c\rho_G+\mathrm{M}_k\hat{c}=1 && (5.50d)\\
&(1-\tau)A\hat{k}_g^{\alpha}=\tau A\hat{k}_g^{\alpha-1}+\hat{c} && (5.50e)\\
&\hat{c}=\rho+(1-\tau)A\alpha\,\hat{k}_g^{\alpha} && (5.50f)
\end{aligned}
$$

This Ramsey system can be solved numerically by setting the values of A, α, ρ, ρ_G.

3.5.9 The Analytical Method of Solving the Self-Interested Government's Long-Run Ramsey Policy

(5.48a) and (5.48b) imply

$$\frac{\partial y}{\partial k_g} = \rho_G + \gamma \tag{5.51}$$

$$\gamma_{DS} = A\alpha \hat{k}_{gDS}^{\alpha-1} - \rho_G \tag{5.52}$$

where subscript 'DS' means '*Decentralised* model with a *Self-interested* Government'.

(5.15) and (5.52) imply that

$$A\alpha \hat{k}_{gDS}^{\alpha-1} - \rho_G = (1-\tau_{DS}) A (1-\alpha) \hat{k}_{gDS}^{\alpha} - \rho \tag{5.53}$$

Hence the ratios of two types of capital is

$$\hat{k}_{gDS} = \frac{\alpha}{(1-\alpha)(1-\tau_{DS})} \left[1 + \frac{\rho_G - \rho}{(1-\tau_{DS}) A (1-\alpha) \hat{k}_{gDS}^{\alpha}} \right]^{-1} \tag{5.54}$$

$$\hat{k}_{gDS} < \frac{\alpha}{(1-\alpha)(1-\tau_{DS})} \quad if \quad \rho_G > \rho \tag{5.55}$$

Substitute (5.52) into (5.16b), we have

$$\tau_{DS} = \alpha - \rho_G A^{-1} \hat{k}_{gDS}^{1-\alpha} \tag{5.56}$$

Hence

$$\tau_{DS} < \alpha \tag{5.57}$$

We also have the below conclusion:

$$\tau_{DS} < \tau_{DB} < \alpha \quad if \quad \rho_G > \rho \tag{5.58}$$

Prove: $\tau_{DS} < \tau_{DB} < \alpha \quad if \quad \rho_G > \rho$

$\because$ (5.17) shows $\frac{d\gamma}{d\tau} > 0$ when $\tau < \alpha$,

$\therefore$ If $\tau_{DB} \le \tau_{DS} < \alpha$, we have $\gamma_{DB} \le \gamma_{DS}$ (P1).

$\because$ $\rho_G > \rho$ and $\gamma_{DB} \le \gamma_{DS}$,

$\therefore$ (5.35) and (5.52) imply $\hat{k}_{gDB} > \hat{k}_{gDS}$.

$\because$ $\tau_{DB} \le \tau_{DS}$, $\hat{k}_{gDB} > \hat{k}_{gDS}$ and $\rho_G > \rho$,

$\because$ (5.15) implies $\gamma_{DB} > \gamma_{DS}$ (P2).

(P1) and (P2) are contradictory.

Hence $\tau_{DS} < \tau_{DB} < \alpha \quad if \quad \rho_G > \rho$

Assume $\rho_{GP} < \rho < \rho_{GI}$, where ρ_{GP} is the rate of time preference of a patient government; ρ_{GI} is the rate of time preference of a impatient government. From (5.58), (5.17), (5.37) and (5.54), it is straightforward to obtain

$$\tau_{DSI} < \tau_{DB} < \tau_{DSP} < \alpha, \quad \gamma_{DSI} < \gamma_{DB} < \gamma_{DSP}, \quad \hat{k}_{gDSI} < \hat{k}_{gDB} < \hat{k}_{gDSP} \tag{5.59}$$

Proposition 3.5C:

In the decentralised FMS model, a more patient government will tax more to accumulate public capital, further leads to a higher ratio of public capital to private capital, and finally has a higher growth rate.

We can compare the solution under a self-interested government with the growth-maximisation solution by the same method in 3.5.8, then we have the summary below.

Summary (DSI vs DB vs DSP vs DG):

$$\begin{aligned} &\tau_{DSI} < \tau_{DB} < \tau_{DSP} < \tau_{DG} = \alpha && (5.60a) \\ &\gamma_{DSI} < \gamma_{DB} < \gamma_{DSP} < \gamma_{DG} && (5.60b) \\ &\hat{k}_{gDSI} < \hat{k}_{gDB} < \hat{k}_{gDSP} < \hat{k}_{gDG} && (5.60c) \end{aligned} \tag{5.60}$$

3.5.10 Centralised Equilibrium with Growth Maximisation

Because of the positive externality of 'taxation and public investment', the decentralised choices of consumption-investment are not Pareto optimal.

However, in the centralised model, the resource allocation among consumption, private capital and public capital is not necessary to be optimal if the tax rate is set arbitrarily. If the tax rate is set arbitrarily, then the current-value Hamiltonian for the benevolent social planner's problem is still the same as equation (5.6) and the Euler equation is still as equation (5.10). The government is not necessary to be benevolent and it may have a different time preference, therefore we just replace ρ by ρ_G.

In the decentralised model $\frac{\partial y}{\partial k}$ is the private marginal return on capital before taxation, while in the centralised model $\frac{\partial y}{\partial k}$ is the social marginal return on capital before taxation. The social planner regards public capital is endogenous since it comes from the output.

On BGP, (5.4) imply

$$\dot{k}_g = \gamma k_g = \tau y \tag{5.61}$$

(5.3) and (5.61) imply that the reduced form of production function is

$$y = A^{\frac{1}{1-\alpha}} \gamma^{\frac{-\alpha}{1-\alpha}} \tau^{\frac{\alpha}{1-\alpha}} k \tag{5.62}$$

Hence, the social marginal return on capital before taxation is

$$\frac{\partial y}{\partial k} = A^{\frac{1}{1-\alpha}} \gamma^{\frac{-\alpha}{1-\alpha}} \tau^{\frac{\alpha}{1-\alpha}} \tag{5.63}$$

The Euler equation becomes

$$(1-\tau) A \left(A\tau\gamma^{-1}\right)^{\frac{\alpha}{1-\alpha}} - \rho_G = \frac{\dot{c}}{c} = \gamma \tag{5.64}$$

Hence we have

$$A^{\frac{1}{1-\alpha}}\left(1-\tau\right)\tau^{\frac{\alpha}{1-\alpha}}=\left(\gamma+\rho_G\right)\gamma^{\frac{\alpha}{1-\alpha}} \tag{5.65}$$

When equation (5.17) is compared with equation (5.65), only one difference on the left hand side is the presence of the term, $1-\alpha$, in the former . Hence, we have the below proposition.

Proposition 3.5D:

The long-run growth rate in the centralised model is always larger that in decentralised for all values of tax rate.

To maximise the growth rate γ, the left hand side of (5.65) should be maximised, then we have the solution for tax rate τ

$$\tau_{CG}=\alpha \tag{5.66}$$

where the subscript 'CG' denotes '*Centralised* model with *Growth* maximisation'.

Proposition 3.5E:

The long-run growth rate in the centralised model attains its maximum when $\boldsymbol{\tau=\alpha}$.

The numerical solution of γ_{CG} the growth rate under the growth-maximisation policies in the centralised model can be obtained by

$$A^{\frac{1}{1-\alpha}}\left(1-\alpha\right)\alpha^{\frac{\alpha}{1-\alpha}}=\left(\gamma_{CG}+\rho_G\right)\gamma_{CG}^{\frac{\alpha}{1-\alpha}} \tag{5.67}$$

Compare (5.19) with (5.67), we have

$$\gamma_{CG}>\gamma_{DG} \tag{5.68}$$

The ratio of two types of capital is

$$\hat{k}_{gCG}=A^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}\gamma_{CG}^{\frac{1}{\alpha-1}} \tag{5.69}$$

Proposition 3.5D, (5.24) and (5.71) imply

$$\hat{k}_{gCG} < \hat{k}_{gDG} \tag{5.70}$$

The saving rate under the growth maximisation in the centralised model is

$$s_{CG} = \gamma_{CG} A^{-1} \hat{k}_{gCG}^{-\alpha} = A^{\frac{1}{\alpha-1}} \alpha^{\frac{\alpha}{\alpha-1}} \gamma_{CG}^{\frac{1}{1-\alpha}} \tag{5.71}$$

The after-tax saving rate under the growth maximisation in the centralised model is

$$s_{CG}^{A} = (1-\alpha)^{-1} \gamma_{CG} A^{-1} \hat{k}_{gCG}^{-\alpha} = A^{\frac{1}{\alpha-1}} (1-\alpha)^{-1} \alpha^{\frac{\alpha}{\alpha-1}} \gamma_{CG}^{\frac{1}{1-\alpha}} \tag{5.72}$$

Proposition 3.5D, (5.21) and (5.71) imply

$$s_{CG} > s_{DG} \tag{5.73}$$

Proposition 3.5D, (5.23) and (5.72) imply

$$s_{CG}^{A} > s_{DG}^{A} \tag{5.74}$$

Summary （CG vs DG）:

$$\begin{aligned}
&\tau_{CG} = \tau_{DG} = \alpha && (5.75a) \\
&\gamma_{CG} > \gamma_{DG} && (5.75b) \\
&\hat{k}_{gCG} < \hat{k}_{gDG} && (5.75c) \\
&s_{CG} > s_{DG} && (5.75d) \\
&s_{CG}^{A} > s_{DG}^{A} && (5.75e)
\end{aligned} \tag{5.75}$$

3.5.11 Centralised Equilibrium with a Benevolent government

To achieve the optimal allocation in the centralised model, the social planner's problem is to choose τ, c, k, k_g to maximise the representative's intertemporal utility (5.1) subject to the government's budget constraint (5.4) and the representative's budget constraint (5.5)

The current-value Hamiltonian for the social planner is

$$H^{CB} \equiv \ln c + \mu_{kg} \tau y \\ + \mu_k \left[(1-\tau) y - c \right] \tag{5.76}$$

First order conditions are

$$\begin{aligned}
&\frac{\partial H^{CB}}{\partial \tau} = \mu_{kg} y - \mu_k y = 0 && (5.77a) \\
&\frac{\partial H^{CB}}{\partial k_g} = \mu_{kg} \tau \frac{\partial y}{\partial k_g} + \mu_k (1-\tau) \frac{\partial y}{\partial k_g} = \rho \mu_{kg} - \dot{\mu}_{k_G} && (5.77b) \\
&\frac{\partial H^{CB}}{\partial k} = \mu_{kg} \tau \frac{\partial y}{\partial k} + \mu_k (1-\tau) \frac{\partial y}{\partial k} = \rho \mu_k - \dot{\mu}_k && (5.77c) \\
&\frac{\partial H^{CB}}{\partial c} = \frac{1}{c} - \mu_k = 0 && (5.77d)
\end{aligned} \tag{5.77}$$

(5.77a) implies

$$\mu_{kg} = \mu_k \tag{5.78}$$

On BGP, we have

$$\frac{\partial y}{\partial k_g} = \frac{\partial y}{\partial k} = \rho + \gamma_{CB}$$

$$A\alpha \hat{k}_{gCB}^{\alpha-1} = A(1-\alpha) \hat{k}_{gCB}^{\alpha}$$

$$\hat{k}_{gCB} = \frac{\alpha}{1-\alpha} \tag{5.79}$$

$$\hat{k}_{gCB} < \hat{k}_{gDB} \tag{5.80}$$

$$\gamma_{CB} = A\alpha^{\alpha} (1-\alpha)^{1-\alpha} - \rho \tag{5.81}$$

(5.35) and (5.37) imply

$$\gamma_{DB} = A\alpha \hat{k}_{gDB}^{\alpha-1} - \rho < A\alpha \left(\frac{1-\alpha}{\alpha} \right)^{1-\alpha} - \rho = A\alpha^{\alpha} (1-\alpha)^{1-\alpha} - \rho$$

Hence

$$\gamma_{CB} > \gamma_{DB} \tag{5.82}$$

Substitute (5.79) and (5.81) into (5.16b), we have

$$\tau_{CB} = \alpha - \rho A^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \tag{5.83}$$

Hence

$$\tau_{CB} < \alpha \tag{5.84}$$

Compare (5.39) with (5.83), we have

$$\tau_{CB} > \tau_{DB} \tag{5.85}$$

With a benevolent government in the centralised model, (5.65) becomes

$$A^{\frac{1}{1-\alpha}}\left(1-\tau_{CB}\right)\tau_{CB}^{\frac{\alpha}{1-\alpha}} = \left(\gamma_{CB}+\rho\right)\gamma_{CB}^{\frac{\alpha}{1-\alpha}} \tag{5.86}$$

(5.86) shows $\frac{d\gamma}{d\tau} > 0$ when $\tau < \alpha$. Hence, (5.85) also implies (5.80).

The saving rate is

$$s_{CB} = \gamma_{CB} A^{-1}\hat{k}_{gCB}^{-\alpha} = A^{\frac{1}{\alpha-1}}\tau_{CB}^{\frac{\alpha}{\alpha-1}}\gamma_{CB}^{\frac{1}{1-\alpha}} = 1-\alpha-\rho A^{-1}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \tag{5.87}$$

(5.44), (5.80), (5.82) and (5.87) imply

$$s_{CB} > s_{DB} \tag{5.88}$$

The after-tax saving rate is

$$s_{CB}^{A} = \left(1-\tau_{CB}\right)^{-1}\gamma_{CB} A^{-1}\hat{k}_{gCB}^{-\alpha} = \frac{A^{\frac{1}{\alpha-1}}\tau_{CB}^{\frac{\alpha}{\alpha-1}}\gamma_{CB}^{\frac{1}{1-\alpha}}}{\left(1-\tau_{CB}\right)} = 1-\rho A^{-1}\alpha^{-\alpha}\left(1-\alpha\right)^{\alpha-1} \tag{5.89}$$

(5.45), (5.85), (5.88) and (5.89) imply

$$s_{CB}^{A} > s_{DB}^{A} \tag{5.90}$$

Summary (CB vs DB):

$$\begin{aligned}
&\tau_{CB} > \tau_{DB} && (5.91a)\\
&\gamma_{CB} > \gamma_{DB} && (5.91b)\\
&\hat{k}_{gCB} < \hat{k}_{gDB} && (5.91c)\\
&s_{CB} > s_{DB} && (5.91d)\\
&s_{CB}^{A} > s_{DB}^{A} && (5.91e)
\end{aligned} \tag{5.91}$$

(5.66) and (5.84) imply

$$\tau_{CB} < \tau_{CG} \tag{5.92}$$

(5.67) and (5.86) imply

$$\left(\gamma_{CB}+\rho\right)\gamma_{CB}^{\frac{\alpha}{1-\alpha}} < \left(\gamma_{CB}+\rho\right)^{\frac{1}{1-\alpha}} = \left[A\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\right]^{\frac{1}{1-\alpha}} = \left(\gamma_{CG}+\rho\right)\gamma_{CG}^{\frac{\alpha}{1-\alpha}} \tag{5.93}$$

$$\gamma_{CB} < \gamma_{CG} \tag{5.94}$$

We also have

$$\hat{k}_{gCB} < \hat{k}_{gCG} \tag{5.95}$$

Prove: $\hat{k}_{gCB} < \hat{k}_{gCG}$

(5.16c) implies $\gamma_{CG} = A\tau_{CG}\hat{k}_{gCG}^{\alpha-1} = A\alpha\hat{k}_{gCG}^{\alpha-1}$

If $\hat{k}_{gCG} \le \hat{k}_{gCB} = \dfrac{\alpha}{1-\alpha}$, we have $\gamma_{CG} \ge A\alpha\hat{k}_{gCB}^{\alpha-1} = A\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}$.

Since $\left(\gamma_{CG}+\rho_G\right)\gamma_{CG}^{\frac{\alpha}{1-\alpha}} > \gamma_{CG}^{\frac{1}{1-\alpha}}$, then $\left(\gamma_{CG}+\rho_G\right)\gamma_{CG}^{\frac{\alpha}{1-\alpha}} > A^{\frac{1}{1-\alpha}}\left(1-\alpha\right)\alpha^{\frac{\alpha}{1-\alpha}}$.

Equation (5.67) is violated.

Hence, we have $\hat{k}_{gCB} < \hat{k}_{gCG}$

(5.95) is intuitively reasonable: $\tau_{CG} = \alpha$, if $\hat{k}_{gCG} < \hat{k}_{gCB} = \frac{\alpha}{1-\alpha}$, we have $\dot{k} + \dot{k}_G > y$, budget constraint would be violated.

3.5.12 Centralised Equilibrium with a Self-Interested government

If the social planner has a different time preference, $\rho_G \neq \rho$, the solution in the long run is obtained by replacing ρ by ρ_G in 3.5.11

Hence

$$\hat{k}_{gCS} = \frac{\alpha}{1-\alpha} \tag{5.96}$$

$$\gamma_{CS} = A\alpha\hat{k}_{gCS}^{\alpha-1} - \rho_G = A\alpha^{\alpha}(1-\alpha)^{1-\alpha} - \rho_G \tag{5.97}$$

$$\tau_{CS} = \alpha - \rho_G A^{-1}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} < \alpha \tag{5.98}$$

$$s_{CS} = \gamma_{CS}A^{-1}\hat{k}_{gCS}^{-\alpha} = A^{\frac{1}{\alpha-1}}\tau_{CS}^{\frac{\alpha}{\alpha-1}}\gamma_{CS}^{\frac{1}{1-\alpha}} = 1-\alpha-\rho_G A^{-1}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \tag{5.99}$$

$$s_{CS}^{A} = (1-\tau_{CS})^{-1}\gamma_{CS}A^{-1}\hat{k}_{gCS}^{-\alpha} = \frac{A^{\frac{1}{\alpha-1}}\tau_{CS}^{\frac{\alpha}{\alpha-1}}\gamma_{CS}^{\frac{1}{1-\alpha}}}{(1-\tau_{CS})} = 1-\rho_G A^{-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1} \tag{5.100}$$

(5.54) and (5.96) imply

$$\hat{k}_{gCSI} = \frac{\alpha}{1-\alpha} = \hat{k}_{gCSP} < \hat{k}_{gDSP} \tag{5.101}$$

(5.52), (5.60b), (5.97) and (5.101) imply

$$\gamma_{DSI} < \gamma_{DSP} < \gamma_{CSP} \tag{5.102}$$

(5.16b), (5.59), (5.101) and (5.102) imply

$$\tau_{DSI} < \tau_{DSP} < \tau_{CSP} \tag{5.103}$$

(5.20), (5.101) and (5.102) imply

$$s_{DSI} < s_{DSP} < s_{CSP} \tag{5.104}$$

(5.22), (5.103) and (5.104) imply

$$s_{DSI}^{A} < s_{DSP}^{A} < s_{CSP}^{A} \tag{5.105}$$

Summary (CSI vs DSI vs DSP vs CSP):

$$\begin{aligned}
&\tau_{DSI} < \tau_{DSP} < \tau_{CSP} && (5.106a)\\
&\gamma_{DSI} < \gamma_{DSP} < \gamma_{CSP} && (5.106b)\\
&\hat{k}_{gCSI} = \frac{\alpha}{1-\alpha} = \hat{k}_{gCSP} < \hat{k}_{gDSP} && (5.106c)\\
&s_{DSI} < s_{DSP} < s_{CSP} && (5.106d)\\
&s_{DSI}^{A} < s_{DSP}^{A} < s_{CSP}^{A} && (5.106e)
\end{aligned} \tag{5.106}$$

From the results in 3.5.11 and (5.96)-(5.98), we have the summary below.

Summary (CSI vs CB vs CSP vs CG):

$$\begin{aligned}
&\tau_{CSI} < \tau_{CB} < \tau_{CSP} < \tau_{CG} = \alpha && (5.107a)\\
&\gamma_{CSI} < \gamma_{CB} < \gamma_{CSP} < \gamma_{CG} && (5.107b)\\
&\hat{k}_{gCSI} = \hat{k}_{gCB} = \frac{\alpha}{1-\alpha} = \hat{k}_{gCSP} < \hat{k}_{gCG} && (5.107c)
\end{aligned} \tag{5.107}$$

3.5.13 Comment

In the FMS model, the production depends on the stock of productive public capital rather than on the flow of public spending as in the Barro model, therefore maximising the growth rate is not efficient for any type of government. There exists transitional dynamics in FMS model because it includes two state variables: private and public capital stocks, which leads to the difference in the welfare aspect of fiscal policy to arise between Barro's and FMS's outcomes.

Once there is a gap between growth-maximisation tax rate and welfare-maximisation tax rate, a farsighted government has the chance to realise its inclination and set a higher tax rate than the rate set by a benevolent government, so that more productive public expenditure is used to finance the production and thus the growth rate will be higher.

3.6 Increase the Growth in a Model with both Productive Public Capital and Nonproductive Public Service

3.6.1 The Structure of the Model with Two Types of Expenditures

The objective of representative household-producer is to maximise his intertemporal utility

$$\int_0^\infty e^{-\rho t} u dt \tag{6.1}$$

$$u(c,h) = \ln c + \varphi \ln h \quad here \quad \varphi > 0 \tag{6.2}$$

where c is consumption, h is the flow of public consumption expenditure, ρ is the rate of time preference, φ is the weight given to public consumption service relative to private consumption. The utility function is the same as that in the Xie model (1997) which is discussed in Section 3.3.

The production function, as that in the FMS model (1993), is based on a Cobb-Douglas technology.

$$y = f\left(k, k_g\right) = Ak^{1-\alpha} k_g^{\alpha} \tag{6.3}$$

where k is private capital and k_g is public capital. $k(0)$ and $k_g(0)$ are given and both are positive, A is a technological scale factor, α and $1-\alpha$ are the output elasticities of k_g and k, respectively.

The government's budget constraints are

$$\begin{aligned} h &= \tau_h y \quad here \quad 0 < \tau_h < 1 && (6.4a) \\ \dot{k}_g &= \tau_{kg} y \quad here \quad 0 < \tau_{kg} < 1 && (6.4b) \\ \tau &= \tau_h + \tau_{kg} \quad here \quad 0 < \tau < 1 && (6.4c) \end{aligned} \tag{6.4}$$

where τ is the total income tax rate, τ_h/τ is the fraction of tax revenue used to finance the public consumption spending, τ_{kg}/τ is the fraction that finances the productive public capital. τ_h and τ_{kg} are set independently.

The household-producer's budget constraint is

$$\dot{k} = \left(1 - \tau_h - \tau_{kg}\right) y - c \tag{6.5}$$

3.6.2 Decentralised Competitive Equilibrium

The representative household-producer maximises his intertemporal utility, given τ_h, τ_{kg}, h and k_g. The current-value Hamiltonian for the representative is

$$H \equiv \ln c + \varphi \ln h + \lambda_k \left[\left(1 - \tau_h - \tau_{kg}\right) y - c\right] \tag{6.6}$$

The transversality condition is

$$\lim_{t \to \infty} e^{-\rho t} \lambda_k k = 0 \tag{6.7}$$

The first order conditions are

$$\left.\begin{aligned} \frac{\partial H}{\partial c} &= \frac{1}{c} - \lambda_k = 0 && (6.8a) \\ \frac{\partial H}{\partial k} &= \lambda_k \left(1 - \tau_h - \tau_{kg}\right) \frac{\partial y}{\partial k} = \rho \lambda_k - \dot{\lambda}_k && (6.8b) \end{aligned}\right. \tag{6.8}$$

(6.8) implies the Euler equation is

$$\dot{c} = c\left[\left(1 - \tau_h - \tau_{kg}\right) \frac{\partial y}{\partial k} - \rho\right] \tag{6.9}$$

(6.9) and (6.5) constitute the DCE:

$$\left.\begin{aligned} \dot{c} &= c\left[\left(1 - \tau_h - \tau_{kg}\right) A (1 - \alpha) k^{-\alpha} k_g^{\alpha} - \rho\right] && (6.10a) \\ \dot{k} &= \left(1 - \tau_h - \tau_{kg}\right) A k^{1-\alpha} k_g^{\alpha} - c && (6.10b) \end{aligned}\right. \tag{6.10}$$

3.6.3 Long-Run Growth-Maximisation Policy

On BGP in the long run, we have

$$\frac{\dot{y}}{y}=\frac{\dot{c}}{c}=\frac{\dot{k}}{k}=\frac{\dot{h}}{h}=\frac{\dot{k}_g}{k_g}=\gamma \tag{6.11}$$

$$\begin{aligned}
&\frac{\dot{\tau}_h}{\tau_h}=0 \qquad (6.12a)\\
&\frac{\dot{\tau}_{kg}}{\tau_{kg}}=0 \qquad (6.12b)
\end{aligned} \tag{6.12}$$

Hence, the long-run growth rate is

$$\gamma=\left(1-\tau_h-\tau_{kg}\right)A\left(1-\alpha\right)k^{-\alpha}k_g^{\alpha}-\rho \tag{6.13}$$

(6.3) and (6.4) imply on BGP,

$$\frac{k_g}{k}=\left(A\tau_{kg}\gamma^{-1}\right)^{\frac{1}{1-\alpha}} \tag{6.14}$$

Substitute (6.14) into (6.13), we have

$$\gamma=\left(1-\tau_h-\tau_{kg}\right)A\left(1-\alpha\right)\left(A\tau_{kg}\gamma^{-1}\right)^{\frac{\alpha}{1-\alpha}}-\rho$$

$$A^{\frac{1}{1-\alpha}}\left(1-\alpha\right)\left(1-\tau_h-\tau_{kg}\right)\tau_{kg}^{\frac{\alpha}{1-\alpha}}=\left(\gamma+\rho\right)\gamma^{\frac{\alpha}{1-\alpha}} \tag{6.15}$$

To maximise the growth rate γ, the left hand side of equation (6.15) should be maximised, then we have the solution of growth maximisation:

$$\begin{aligned}
&\tau_{hDG}=0 \qquad (6.16a)\\
&\tau_{kgDG}=\alpha \qquad (6.16b)
\end{aligned} \tag{6.16}$$

Proposition 3.6 A:

The long-run growth rate on BGP in the decentralised model attains its maximum when $\boldsymbol{\tau_h=0}$ and $\boldsymbol{\tau_{kg}=\alpha}$.

The numerical solution of γ_{DG} can be obtained by

$$A^{\frac{1}{1-\alpha}}\left(1-\alpha\right)^{2}\alpha^{\frac{\alpha}{1-\alpha}}=\left(\gamma_{DG}+\rho\right)\gamma_{DG}^{\frac{\alpha}{1-\alpha}} \tag{6.17}$$

The saving rate is

$$s_{DG}=A^{\frac{1}{\alpha-1}}\alpha^{\frac{\alpha}{\alpha-1}}\gamma_{DG}^{\frac{1}{1-\alpha}} \tag{6.18}$$

The after-tax saving rate is

$$s_{DG}^{A}=A^{\frac{1}{\alpha-1}}\left(1-\alpha\right)^{-1}\alpha^{\frac{\alpha}{\alpha-1}}\gamma_{DG}^{\frac{1}{1-\alpha}} \tag{6.19}$$

When the government maximises the BGP growth rate, the model is the same as the FMS model.

3.6.4 Decentralised General Equilibrium

In the decentralised model, the government maximises the representative's intertemporal utility with the government's own time preference. If the government is self-interested, then $\rho_G \neq \rho$; if the government is benevolent, then $\rho_G = \rho$.

The current-value Hamiltonian for the government is

$$\begin{aligned} H^{GS} \equiv{}& \ln c+\varphi\ln\tau_h y+\mu_{kg}\tau_{kg}y \\ &+\mu_k\left[\left(1-\tau_h-\tau_{kg}\right)y-c\right] \\ &+\mu_c c\left[\left(1-\tau_h-\tau_{kg}\right)\frac{\partial y}{\partial k}-\rho\right] \end{aligned} \tag{6.20}$$

First order conditions are

$$\frac{\partial H^{GS}}{\partial \tau_{kg}} = \mu_{kg} y - \mu_k y - \mu_c c \frac{\partial y}{\partial k} = 0 \qquad (6.21a)$$

$$\frac{\partial H^{GS}}{\partial \tau_h} = \frac{\varphi}{\tau_h} - \mu_k y - \mu_c c \frac{\partial y}{\partial k} = 0 \qquad (6.21b)$$

$$\frac{\partial H^{GS}}{\partial k_g} = \frac{\varphi}{y}\frac{\partial y}{\partial k_g} + \mu_{kg}\tau_{kg}\frac{\partial y}{\partial k_g} + \mu_k\left(1-\tau_h-\tau_{kg}\right)\frac{\partial y}{\partial k_g}$$
$$+\mu_c c\left(1-\tau_h-\tau_{kg}\right)\frac{\partial^2 y}{\partial k \partial k_g} = \rho_G \mu_{k_G} - \dot{\mu}_{k_G} \qquad (6.21c)$$

$$\frac{\partial H^{GS}}{\partial k} = \frac{\varphi}{y}\frac{\partial y}{\partial k} + \mu_{kg}\tau_{kg}\frac{\partial y}{\partial k} + \mu_k\left(1-\tau_h-\tau_{kg}\right)\frac{\partial y}{\partial k}$$
$$+\mu_c c\left(1-\tau_h-\tau_{kg}\right)\frac{\partial^2 y}{\partial k^2} = \rho_G \mu_k - \dot{\mu}_k \qquad (6.21d)$$

$$\frac{\partial H^{GS}}{\partial c} = \frac{1}{c} - \mu_k + \mu_c\left[\left(1-\tau_h-\tau_{kg}\right)\frac{\partial y}{\partial k} - \rho\right] = \rho_G \mu_c - \dot{\mu}_c \qquad (6.21e)$$

$$\frac{\partial H^{GS}}{\partial \mu_{kg}} = \tau_{kg} y = \dot{k}_g \qquad (6.21f)$$

$$\frac{\partial H^{GS}}{\partial \mu_k} = \left(1-\tau_h-\tau_{kg}\right)y - c = \dot{k} \qquad (6.21g) \qquad (6.21)$$

$$\frac{\partial H^{GS}}{\partial \mu_c} = \left(1-\tau_h-\tau_{kg}\right)\frac{\partial y}{\partial k} - \rho = \dot{c} \qquad (6.21h)$$

(6.21a) and (6.21b) imply

$$\frac{\varphi}{y} = \mu_{kg}\tau_h \qquad (6.22)$$

(6.21a) implies

$$\mu_c c \frac{\partial y}{\partial k} = \left(\mu_{kg} - \mu_k\right) y \qquad (6.23)$$

Use (6.22) and (6.23) and obtain the simplified dynamic system:

$$\mu_c c \frac{\partial y}{\partial k} = \left(\mu_{kg} - \mu_k\right) y \qquad (6.24a)$$

$$\frac{\varphi}{y} = \mu_{kg}\tau_h \qquad (6.24b)$$

$$\dot{\mu}_{kg} = \rho_G \mu_{kg} - \mu_{kg}\frac{\partial y}{\partial k_g} \qquad (6.24c)$$

$$\dot{\mu}_k = \rho_G \mu_k - \mu_{kg}\left(\tau_h + \tau_{kg}\right)\frac{\partial y}{\partial k} - \mu_k\left(1 - \tau_h - \tau_{kg}\right)\frac{\partial y}{\partial k} + \mu_c c\left(1 - \tau_h - \tau_{kg}\right)\frac{\partial^2 y}{\partial k^2} \qquad (6.24d) \qquad (6.24)$$

$$\dot{\mu}_c = \rho_G \mu_c - \frac{1}{c} + \mu_k - \mu_c\left[\left(1 - \tau_h - \tau_{kg}\right)\frac{\partial y}{\partial k} - \rho\right] \qquad (6.24e)$$

$$\dot{k}_g = \tau_{kg} y \qquad (6.24f)$$

$$\dot{k} = \left(1 - \tau_h - \tau_{kg}\right) y - c \qquad (6.24g)$$

$$\dot{c} = \left(1 - \tau_h - \tau_{kg}\right)\frac{\partial y}{\partial k} - \rho \qquad (6.24h)$$

The first order conditions (6.24a)-(6.24h) constitute an eight-equation system in the paths of $\tau_h, \tau_{kg}, c, k, k_g, \mu_c, \mu_k, \mu_{kg}$. The system is a general equilibrium with Ramsey second-best allocation.

3.6.5 Long-Run Decentralised General Equilibrium

Define

$$\hat{k}_g = \frac{k_g}{k} \qquad (6.25a)$$

$$\hat{c} = \frac{c}{k} \qquad (6.25b)$$

$$\mathrm{M}_{kg} = \mu_{kg} k_g \qquad (6.25c) \qquad (6.25)$$

$$\mathrm{M}_k = \mu_k k \qquad (6.25d)$$

$$\mathrm{M}_c = \mu_c c \qquad (6.25e)$$

The dynamic general equilibrium system becomes

$$
\left.
\begin{array}{ll}
\dfrac{\varphi}{\tau_h} = \mathrm{M}_{kg}\hat{k}_g^{\alpha-1} & (6.26a) \\
\mathrm{M}_c(1-\alpha)+\mathrm{M}_k = \dfrac{\mathrm{M}_{kg}}{\hat{k}_g} & (6.26b) \\
\dot{\mathrm{M}}_{kg} = \mathrm{M}_{kg}\rho_G - \mathrm{M}_{kg}A\alpha\hat{k}_g^{\alpha-1} + \mathrm{M}_{kg}\tau_{kg}A\hat{k}_g^{\alpha-1} & (6.26c) \\
\dot{\mathrm{M}}_k = \mathrm{M}_k(\rho_G-\hat{c}) + \mathrm{M}_{kg}(\alpha-\tau_h-\tau_{kg})A\hat{k}_g^{\alpha-1} & (6.26d) \\
\dot{\mathrm{M}}_c = \mathrm{M}_c\rho_G + \mathrm{M}_k\hat{c} - 1 & (6.26e) \\
\dot{\hat{k}}_g = \tau_{kg}A\hat{k}_g^{\alpha} - (1-\tau_h-\tau_{kg})A\hat{k}_g^{1+\alpha} + \hat{c}k_g & (6.26f) \\
\dot{\hat{c}} = \hat{c}^2 - \rho\hat{c} - (1-\tau_h-\tau_{kg})A\alpha\,\hat{k}_g^{\alpha}\hat{c} & (6.26g)
\end{array}
\right. \qquad (6.26)
$$

There are 7 equations in the paths of 7 variables $\tau_h, \tau_{kg}, \hat{c}, \hat{k}_g, \mathrm{M}_{kg}, \mathrm{M}_k, \mathrm{M}_c$ in the above stationary general equilibrium with Ramsey second-best policy.

We obtain the long-run second-best general equilibrium by setting

$$
\dot{\mathrm{M}}_{kg} = \dot{\mathrm{M}}_k = \dot{\mathrm{M}}_c = \dot{\hat{k}}_g = \dot{\hat{c}} = 0 \qquad (6.27)
$$

Substitute (6.27) into (6.26), we have

$$
\left.
\begin{array}{ll}
\dfrac{\varphi}{\tau_h} = \mathrm{M}_{kg}\hat{k}_g^{\alpha-1} & (6.28a) \\
\mathrm{M}_c(1-\alpha)+\mathrm{M}_k = \dfrac{\mathrm{M}_{kg}}{\hat{k}_g} & (6.28b) \\
0 = \rho_G - A\alpha\hat{k}_g^{\alpha-1} + \tau_{kg}A\hat{k}_g^{\alpha-1} & (6.28c) \\
0 = \mathrm{M}_k(\rho_G-\hat{c}) + \mathrm{M}_{kg}(\alpha-\tau_h-\tau_{kg})A\hat{k}_g^{\alpha-1} & (6.28d) \\
0 = \mathrm{M}_c\rho_G + \mathrm{M}_k\hat{c} - 1 & (6.28e) \\
0 = \tau_{kg}A\hat{k}_g^{\alpha-1} - (1-\tau_h-\tau_{kg})A\hat{k}_g^{\alpha} + \hat{c} & (6.28f) \\
0 = \hat{c} - \rho - (1-\tau_h-\tau_{kg})A\alpha\,\hat{k}_g^{\alpha} & (6.28g)
\end{array}
\right. \qquad (6.28)
$$

There are 7 variables and 7 equations in the above system which characterizes the BGP Ramsey equilibrium.

3.6.6 The Effect of the Government's Time Preference

Table 2 represents that a patient government prefers to reduce the proportion of nonproductive public expenditure in economic output and increase the share of productive public investment.

Table 2: Effect of ρ_G on $\tau_h, \tau_{kg}, \hat{c}, \hat{k}_g, M_{kg}, M_k, M_c$

ρ_G	τ_h	τ_{kg}	$\hat{c}$	$\hat{k}_g$	M_{kg}	M_k	M_c
0.03	0.0149	0.1796	0.0922	0.4915	9.8769	3.7615	21.7782
0.05	0.0238	0.1489	0.0912	0.4029	5.3125	4.9306	11.0069
0.07	0.0297	0.1257	0.0903	0.3386	3.7372	9.8239	1.6174

Notes: $A = 0.25,\ \alpha = 0.25,\ \rho = 0.05,\ \varphi = 0.25$. With $\varphi = 0$ and $\tau_h = 0$, we can have the numerical solution for FMS model in Section 3.5.

The numerical solution of growth rate γ is obtained by (6.15).

The saving rate is calculated by

$$s = \frac{\dot{k}}{y} = \frac{\dot{k}}{k} \cdot \frac{k}{y} = \gamma A^{-1} \hat{k}_g^{-\alpha} \tag{6.29}$$

The after-tax saving rate is calculated by

$$s^A = \frac{\dot{k}}{\left(1 - \tau_h - \tau_{kg}\right) y} = \left(1 - \tau_h - \tau_{kg}\right)^{-1} s \tag{6.30}$$

s^T is defined as the ratio of total investment to output. Total investment includes both private and public investment.

$$s^T = \frac{\dot{k} + \dot{k}_g}{y} = \frac{\dot{k}\left(1 + \hat{k}_g\right)}{k} \cdot \frac{k}{y} = \left(1 + \hat{k}_g\right) s \tag{6.31}$$

Table 3 shows that, based on the assumed parameters, a patient government leads to a higher private saving rate s^A and a higher total saving rate of the economy s^T, and thus a higher growth rate.

Table 3: Effect of ρ_G on γ, s, s^A, s^T

ρ_G	γ (%)	s	s^A	s^T
0.03	7.6472	0.4699	0.5833	0.7009
0.05	7.3590	0.4739	0.5728	0.6648
0.07	7.0802	0.4762	0.5631	0.6367

Notes: $A = 0.25,\ \alpha = 0.25,\ \rho = 0.05,\ \varphi = 0.25.$

The higher growth rate due to a farsighted government sacrifices the welfare of current generation, but benefits the future generations after a certain period. The numerical results show that although the share of private saving in output decreases, but the share of private saving in disposable income increases because the reduced amount of nonproductive expenditure is much more than the increased amount of public investment, thus the representative has more disposable income and also gives a higher proportion to saving. Both private and public sectors are willing to invest more, and finally the growth rate is higher when compared with the situation under a benevolent government.

3.6.7 Comment

The model in this section is designed by combining the Xie model in Section 3.3 and the FMS model in Section 3.4, therefore, as illustrated in Table 2, a farsighted government has two channels to realize a higher growth rate than that achieved by a benevolent government: to lower τ_h the ratio of nonproductive public expenditure in GDP; on the other hand, to increase τ_{kg} the ratio of productive public expenditure in GDP.

3.7 Discussion and Conclusion

3.7.1 Discussion

A farsighted government would have an incentive to sacrifice the current generation by reducing the ratio of public consumption to GDP, giving expenditure priority to productivity-enhancing public expenditure and thus raising the saving rate, so as to increase the long-run economic growth rate and finally rival other countries by a higher welfare level in the future. However, there is a puzzle: future generations will be beneficiaries of the farsighted fiscal policy but they will prefer a benevolent government at their day. Therefore, it is difficult for politicians to adopt farsighted policies in a two-party or multiparty political system.

The political discount rate will be higher than the private rate of time preference if politicians face uncertainty about next election. Therefore in western ‘democratic’ countries, most politicians have a shorter time horizon than households and favor quick boosts to electoral popularity. The fruits of productive investment, especially in human capital, do not occur until some time has passed, and the fruits may be harvested by opponent parties. The government budget is balanced in this Chapter, however, in practice, shortsighted politicians can increase the social welfare expenditure but keep the tax rates relatively low in short run to curry favour with voters, thus budget deficits are necessary, which will increase the tax burden of future generations. This is why most ‘democratic’ countries do not submit to fiscal discipline, the first policy recommendation of the Washington Consensus.

The politicians could be more farsighted than households if different generations of politicians have the same long-run goal—the success of a political and economic system, which may happen in a one-party or nonpartisan system, especially when the government defend its political system in international ideological battles (for examples, in Taiwan, South Korea and Singapore from the 1950s to the 1980s, and currently in mainland China).

The tax distortion is illustrated by comparing the decentralised results with centralised results. A lump sum tax (poll tax), with zero marginal tax rate, can reproduce the

Pareto optimal allocation in the centralised model in which the government is benevolent. In UK, Poll tax was levied by the governments of John of Gaunt in the 14th century, Charles II in the 17th century and Margaret Thatcher in the 20th century. However, it was unpopular, many opponents thought it shifted the tax burden from the rich to the poor. (See Minford (1990).) Lump sum tax, implemented with the name of ‘contract responsibility system’, was a successful practice in China, partially supplanting the egalitarian distribution method, whereby the state assumed all profits and losses. The system was first adopted in agriculture in 1981 and later extended to other sectors of the economy, by which farmers and managers were held responsible for the profits and losses since they were allowed to keep profits after they submitted a fixed quota. In China, another instrument to exploit the positive external effects and avoid tax distortion is to run SOEs. The government, the owner of the SOEs, prefers a higher ratio of investment to output when compared with the private sectors, because the social marginal productivity is higher than the private marginal return on capital, as demonstrated in Barro (1990).

However, with a self-interested government, lump sum tax can not replicate the Pareto optimal allocation in the centralised model, which differs from Barro’s conclusion because both of the government’s and the representative’s rates of time preference enter the solution of lump sum tax rate and thus the time horizon of the government would affect the overall utility of the representative.

3.7.2 Conclusion

The strategy of a farsighted government is to sacrifice the first several generations but benefit all future generations through cutting nonproductive public spending meanwhile promoting productive public expenditure, so that a higher growth rate leads to a ‘quasi-Pareto improvement’ among generations.

Barro model is an exception, because the elasticity of substitution in the Cobb-Douglas function is unity, which makes the maximisation of the welfare correspond to the maximisation of the growth, and therefore the farsighted government has no place to increase the growth rate.

There are many benefits, such as a higher long-run growth rate and higher welfare for future generations, if the government is more patient than the citizens. However, the farsighted fiscal policy together with 'hedonistic citizens' also leads to welfare loss for both the government and the citizens. The inefficiency in resource allocation is from both tax distortion in a decentralised economy and the divergence of time preference between the private and public sectors. A farsighted government had better seek an accommodation on a time horizon with the citizens.

In practice, shortsighted politicians in most 'democratic' countries usually promote government consumption by budget deficits in hopes of wooing voters, otherwise, they cut public investment rather than social welfare spending to maintain a balanced budget. The long-term consequence is even worse if politicians damage public investment but still keep large budget deficits to finance welfare programs. It is the bad political system that has grievously distorted and blighted the market economy. This is why so many counties violate the first two prescriptions of the Washington Consensus: fiscal discipline and public expenditure priority to pro-growth investment. In contrast, the farsighted governments of Singapore and China have down-to-earth pragmatic concern with serving their people in the long run.

Appendix 3: Basic Structure and Important Results

3.1 Introduction

3.2 Increase the Growth in the Simplest Centralised AK Model

3.2.1 The Structure of the Centralised AK Model

3.2.2 The Centralised Equlibrium

3.2.3 Proof of Balanced Growth Path

$$k = k(0)e^{(A-\rho_G)t} \qquad c = \rho_G k(0)e^{(A-\rho_G)t} \qquad y = Ak(0)e^{(A-\rho_G)t}$$

3.2.4 Comparison

$$\gamma_{CI} < \gamma_{CB} = \gamma_D < \gamma_{CP} \qquad s_{CI} < s_{CB} = s_D < s_{CP}$$

Proposition 3.2 A: In the centralised model, the saving rate and growth rate are larger than, or equal to, or smaller than their counterparts in the decentralised model if the government is more patient than, or as patient as, or less patient than the representative.

3.2.5 Experiment

In the centalised regime with a farsighted government, the resource allocation sacrifices the first several generations but benefits all future generations. It is a 'quasi-Pareto improvement'.

3.3 Increase the Growth in a Model with Nonproductive Public Expenditure

3.3.1 The Structure of the Xie Model

3.3.2 Decentralised Competitive Equilibrium

$$\dot{c} = c\left[(1-\tau)A-\rho\right] \qquad \dot{k} = k\left[(1-\tau)A-\rho\right]$$

3.3.3 Decentralised Equilibrium with Growth Maximisation

$$\tau_{DG} = 0 \qquad \gamma_{DG} = A-\rho \qquad \frac{c_{DG}}{k_{DG}} = \rho \qquad \frac{h_{DG}}{c_{DG}} = 0 \qquad s_{DG} = \frac{A-\rho}{A} \qquad s_{DG}^{A} = \frac{A-\rho}{A}$$

3.3.4 Decentralised General Equilibrium with a Self-interested Government

$$\tau_{DS} = \frac{\rho_G\varphi}{A(1+\varphi)} \qquad \gamma_{DS} = A-\frac{\rho_G\varphi}{1+\varphi}-\rho \qquad \frac{c_{DS}}{k_{DS}} = \rho \qquad \frac{h_{DS}}{c_{DS}} = \frac{\rho_G\varphi}{\rho(1+\varphi)}$$

$$s_{DS} = \left[1-\frac{\rho_G\varphi}{A(1+\varphi)}\right]-\frac{\rho}{A} \qquad s_{DS}^{A} = 1-\frac{\rho}{\left[1-\frac{\rho_G\varphi}{A(1+\varphi)}\right]A}$$

Propostition 3.3A: The optimal tax rate set by the self-interested government in the open-loop Stackelberg game is not only time consistent, but part of a subgame perfect equilibrium, which ensures the DCE is on the BGP.

3.3.5 Decentralised General Equilibrium with a Benevolent Government

$$\frac{d\tau_D}{d\rho_G} > 0 \qquad \frac{d\gamma_D}{d\rho_G} < 0 \qquad \frac{d(c_D/k_D)}{d\rho_G} = 0 \qquad \frac{d(h_D/c_D)}{d\rho_G} > 0 \qquad \frac{ds_D}{d\rho_G} < 0 \qquad \frac{ds_D^A}{d\rho_G} < 0$$

Propostition 3.3B: In the decentralised Xie model, the less patient a government is, the higher the optimal tax rate is set, and the lower the growth rate reaches.

3.3.6 Centralised General Equilibrium with Growth Maximisation

$$\tau_{CG}=0 \qquad \gamma_{CG}=A-\rho_G \qquad \frac{c_{CG}}{k_{CG}}=\rho_G \qquad \frac{h_{CG}}{c_{CG}}=0 \qquad s_{CG}=\frac{A-\rho_G}{A} \qquad s_{CG}^A=\frac{A-\rho_G}{A}$$

3.3.7 Centralised General Equilibrium with a Self-Interested Government

$$\tau_{CS}=\frac{\rho_G\varphi}{A(1+\varphi)} \qquad \gamma_{CS}=A-\rho_G \qquad \frac{c_{CS}}{k_{CS}}=\frac{\rho_G}{1+\varphi} \qquad \frac{h_{CS}}{c_{CS}}=\varphi$$

$$s_{CS}=\left[1-\frac{\rho_G\varphi}{A(1+\varphi)}\right]-\frac{\rho_G}{A} \qquad s_{CS}^A=1-\frac{\rho_G}{\left[1-\frac{\rho_G\varphi}{A(1+\varphi)}\right]A}$$

$$\frac{d\tau_C}{d\rho_G}>0 \qquad \frac{d\gamma_C}{d\rho_G}<0 \qquad \frac{d\left(c_C/k_C\right)}{d\rho_G}=0 \qquad \frac{d\left(h_C/c_C\right)}{d\rho_G}>0 \qquad \frac{ds_C}{d\rho_G}<0 \qquad \frac{ds_C^A}{d\rho_G}<0$$

Propostition 3.3C: In the centralised Xie model, the less patient a government is, the higher the optimal tax rate is set, and the lower the growth rate is.

3.3.8 Compare Decentralised Outcome with Centralised Outcome when the Government Is Self-Interested

Propostition 3.3D: If $\rho_G > (1+\varphi)\rho$, the growth rate in the centralised model is smaller than the rate in the decentralised model.

3.3.9 Effect of the Government's Time Preference on Welfare in the Decentralised Model

$$\ddot{x}<\tilde{x}<\bar{x}<\tilde{x}+l$$

Proposition 3.3E: With a farsighted government, the overall utility, the birth of the first benefited person in generation utility, the instantaneous utility, all living people's generation utility exceed their counterparts with a benevolent government in chronological order.

$$\bar{x}=\frac{\ln\rho-\ln\rho_G}{\rho-\rho_G} \qquad \ddot{x}=\frac{\left(\ln\rho-\ln\rho_G\right)}{\rho-\rho_G}-\frac{1}{\rho} \qquad \tilde{x}=\frac{\left(\ln\rho-\ln\rho_G\right)}{\rho-\rho_G}-\frac{1}{\rho}+\frac{le^{-\rho l}}{1-e^{-\rho l}}$$

Proposition 3.3F: The more farsighted the government is, the faster the overall utility, the generation utility and the instantaneous utility exceed their counterparts with a benevolent government.

3.3.10 Welfare Loss with Conflicting Time Preferences

Tax distortion is not the only reason of inefficient allocation, welfare loss is also from different time preferences between the representative and the government.

3.4 No Chance to Increase the Growth in a Model with Productive Public Service

3.4.1 The Structure of the Barro Model

3.4.2 Decentralised Competitive Equilibrium

$$\dot{c}=c\left[\left(1-\tau\right)\tau^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}\left(1-\alpha\right)-\rho\right] \qquad \dot{k}=\left(1-\tau\right)\tau^{\frac{\alpha}{1-\alpha}}A^{\frac{1}{1-\alpha}}k-c$$

3.4.3 Long-Run Growth-Maximisation Policy

$$\gamma_{DG} = A^{\frac{1}{1-\alpha}} \left(1-\alpha\right)^2 \alpha^{\frac{\alpha}{1-\alpha}} - \rho$$

3.4.4 Decentralised General Equilibrium Solution

Proposition 3.4A: With Cobb-Douglas production function in the Barro model, the fiscal policy is independent of either the representative's or the government's time preference.

3.4.5 Discussion

A farsighted government can not increase economic growth by fiscal instrument in either decentralised or centralised models. The key is that the elasticity of substitution in Cobb-Douglas production function is unity and thus maximising the growth rate is efficient.

3.5 Increase the Growth in a Model with Productive Public Capital

3.5.1 The Structure of the FMS Model

3.5.2 Decentralised Competitive Equilibrium

$$\dot{c} = c\left[\left(1-\tau\right) A\left(1-\alpha\right) k^{-\alpha} k_g^{\alpha} - \rho\right] \qquad \dot{k} = \left(1-\tau\right) A k^{1-\alpha} k_g^{\alpha} - c$$

3.5.3 Long-Run Growth-Maximisation Policy

Proposition 3.5 A: The long-run growth rate on BGP in the decentralised model attains its maximum when $\tau = \alpha$.

3.5.4 Decentralised General Equilibrium with a Benevolent Government

3.5.5 Long-Run Decentralised General Equilibrium with a Benevolent Government

3.5.6 A Simple Analytical Method of Solving the Benevolent Government's Long-Run Ramsey Policy

Proposition 3.5B: On BGP in the decentralised model with a benevolent government, the second-best tax rate is lower than the rate which attains the maximum growth rate.

3.5.7 Decentralised General Equilibrium with a Self-Interested Government

3.5.8 Long-Run Decentralised General Equilibrium with a Self-interested Government

3.5.9 The Analytical Method of Solving the Self-Interested Government's Long-Run Ramsey Policy

$$\tau_{DSI} < \tau_{DB} < \tau_{DSP} < \alpha \qquad \gamma_{DSI} < \gamma_{DB} < \gamma_{DSP} \qquad \hat{k}_{gDSI} < \hat{k}_{gDB} < \hat{k}_{gDSP}$$

Proposition 3.5C: In the decentralised the FMS model, a more patient government will tax more to accumulate public capital, further leads to a higher ratio of public capital to private capital, and finally has a higher growth rate.

Summary (DSI vs DB vs DSP vs DG):

$$\tau_{DSI} < \tau_{DB} < \tau_{DSP} < \tau_{DG} = \alpha \qquad \gamma_{DSI} < \gamma_{DB} < \gamma_{DSP} < \gamma_{DG} \qquad \hat{k}_{gDSI} < \hat{k}_{gDB} < \hat{k}_{gDSP} < \hat{k}_{gDG}$$

3.5.10 Centralised Equilibrium with Growth Maximisation

Proposition 3.5D: The long-run growth rate in the centralised model is always larger that in the decentralised model for all values of tax rate.

Proposition 3.5E: The long-run growth rate in the centralised model attains its maximum when $\tau = \alpha$.

Summary（CG vs DG）:

$$\tau_{CG} = \tau_{DG} = \alpha \qquad \gamma_{CG} > \gamma_{DG} \qquad \hat{k}_{gCG} < \hat{k}_{gDG} \qquad s_{CG} > s_{DG} \qquad s_{CG}^{A} > s_{DG}^{A}$$

3.5.11 Centralised Equilibrium with a Benevolent government

Summary (CB vs DB):

$$\tau_{CB} > \tau_{DB} \qquad \gamma_{CB} > \gamma_{DB} \qquad \hat{k}_{gCB} < \hat{k}_{gDB} \qquad s_{CB} > s_{DB} \qquad s_{CB}^{A} > s_{DB}^{A}$$

3.5.12 Centralised Equilibrium with a Self-Interested government

Summary (CSI vs DSI vs DSP vs CSP):

$$\tau_{DSI} < \tau_{DSP} < \tau_{CSP} \qquad \gamma_{DSI} < \gamma_{DSP} < \gamma_{CSP}$$

$$\hat{k}_{gCSI} = \frac{\alpha}{1-\alpha} = \hat{k}_{gCSP} < \hat{k}_{gDSP}$$

$$s_{DSI} < s_{DSP} < s_{CSP} \qquad s_{DSI}^{A} < s_{DSP}^{A} < s_{CSP}^{A}$$

Summary (CSI vs CB vs CSP vs CG):

$$\tau_{CSI} < \tau_{CB} < \tau_{CSP} < \tau_{CG} = \alpha \qquad \gamma_{CSI} < \gamma_{CB} < \gamma_{CSP} < \gamma_{CG}$$

$$\hat{k}_{gCSI} = \hat{k}_{gCB} = \frac{\alpha}{1-\alpha} = \hat{k}_{gCSP} < \hat{k}_{gCG}$$

3.5.13 Comment

3.6 Increase the Growth in a Model with both Productive Public Capital and Nonproductive Public Service

3.6.1 The Structure of the Model with Two Types of Expenditures

3.6.2 Decentralised Competitive Equilibrium

$$\dot{c} = c\left[\left(1-\tau_h-\tau_{kg}\right)A\left(1-\alpha\right)k^{-\alpha}k_g^{\alpha}-\rho\right] \qquad \dot{k} = \left(1-\tau_h-\tau_{kg}\right)Ak^{1-\alpha}k_g^{\alpha}-c$$

3.6.3 Long-Run Growth-Maximisation Policy

Proposition 3.6 A: The long-run growth rate on BGP in the decentralised model attains its maximum when $\tau_h = 0$ and $\tau_{kg} = \alpha$.

3.6.4 Decentralised General Equilibrium

3.6.5 Long-Run Decentralised General Equilibrium

3.6.6 The Effect of the Government's Time Preference

3.6.7 Comment

3.7 Discussion and Conclusion

3.7.1 Discussion

Future generations will be beneficiaries of the farsighted policy but they will prefer a benevolent government at their day.

A lump sum tax can reproduce the Pareto optimal allocation in the centralised model in which the government is benevolent. However, with a self-interested government, lump sum tax can not replicate the Pareto optimal allocation in the centralised model, which differs from Barro's conclusion.

3.7.2 Conclusion

Chapter 4: Economic Relationship between the Government and Nongovernment Sectors in China

4.1 Introduction

Mainland China has been the fastest growing major economy since the founding of the People's Republic in 1949. Mainland China's sustained and rapid economic growth and increased competitiveness in manufacturing and international trade have been underpinned by a massive capital formation. Before the early 1990s, central control over planning and the state ownership of financial system and certain important industries had enabled the government to mobilise whatever surplus was available and greatly increase the proportion of GDP devoted to investment. In 1992, Deng Xiaoping made his famous southern tour of China, using his travels as a method of reasserting his reformist platform after his retirement and criticising his political opponents who were against further reform and opening up. Subsequently the 'Socialist Market Economy' was legitimised. Since 1992, private investment has been widely encouraged and it has made up a growing share in total investment.

The purpose of this research is to examine the elasticity of substitution between government and nongovernment capital (nongovernment capital is mainly in the private sector after the 1978 reform), check the difference on the orientation of public investment between the two periods — before and after 1992, and finally discuss the policy implications.

The rest of this chapter is structured as follows: In Section 4.2 the economic background of the empirical work in this chapter is briefly presented. Section 4.3 demonstrates the theoretical framework and the modified model which can be directly used in Dynare. Section 4.4 provides the sources of raw data and the method of data processing. Section 4.5 illustrates the methodology of indirect inference in testing models and estimation. The test and estimation results with different production functions are shown in Section 4.6 and Section 4.7, respectively. In order to examine the change in the economic relationship between the government and nongovernment sectors, Section 4.8 deals with the estimations by dividing the data into two periods: 1952 -1992 and 1993-2012. Section 4.9 gives some succinct comments on the empirical results and provides relevant policy implications. Finally, the conclusion is drawn in Section 4.10.

4.2 Background

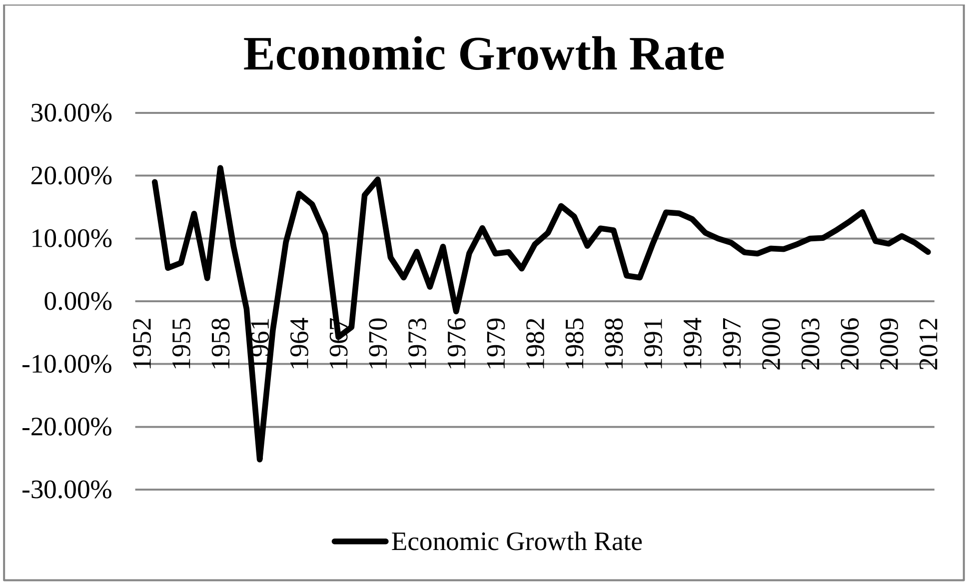
4.2.1 The Economic Performance of Mainland China

Mainland China is the fastest growing major economy in the world, with remarkable growth rates, averaging above 8% after 1949 and around 10% over the past 30 years. According to the International Comparison Program (2014), under the authority of the United Nations and hosted by the World Bank, new GDP assessments based on Purchasing Power Parities (PPPs) put mainland China's economy at 87% of the size of the US in 2011, and mainland China could overtake the United States as the world's largest economy in 2014, occupying about 17% of the global economy. (From now on, 'China' mentioned in this chapter denotes mainland China, excluding Taiwan, Hong Kong and Macau.)

Figure 1 demonstrates the six economic troughs after 1949: four of them happened before 1990 and were all because of political disarrays; the other two were caused by the external factors, the 1997 Asian financial crisis and the 2008 global financial and economic crisis. There were large economic fluctuations before 1978. Since 1978, the government has realised rapid but stable economic growth.

Production grew substantially between 1949 and the beginning of economic reform in 1978. Without the political turmoils, China should have a higher growth rate in that period. However, the economic growth was accompanied by the increase of population. As a result, productive capacity was unable to outdistance essential consumption needs significantly, which offset the efforts of the government to invest more resources in capital goods. The relatively small size of the capital stock per capita led to low productivity per worker, which in turn perpetuated the lack of ability to generate a substantial surplus. In 1979, the family planning policy was introduced to alleviate social, economic, and environmental problems in China, averting more about 300 million births. In the post reform period, the increase in productivity rather than population growth has accounted for China's economic growth. (See Chow and Li (2002).) Since 1949, human capital has increased stably in China. The growth of human capital was mainly from the growing population in the first three decades and chiefly due to the increasing quality and capability of the workforce in the following three decades.

Figure 1: Economic Growth Rate in China



Data Source: National Data, National Bureau of Statistics of China.

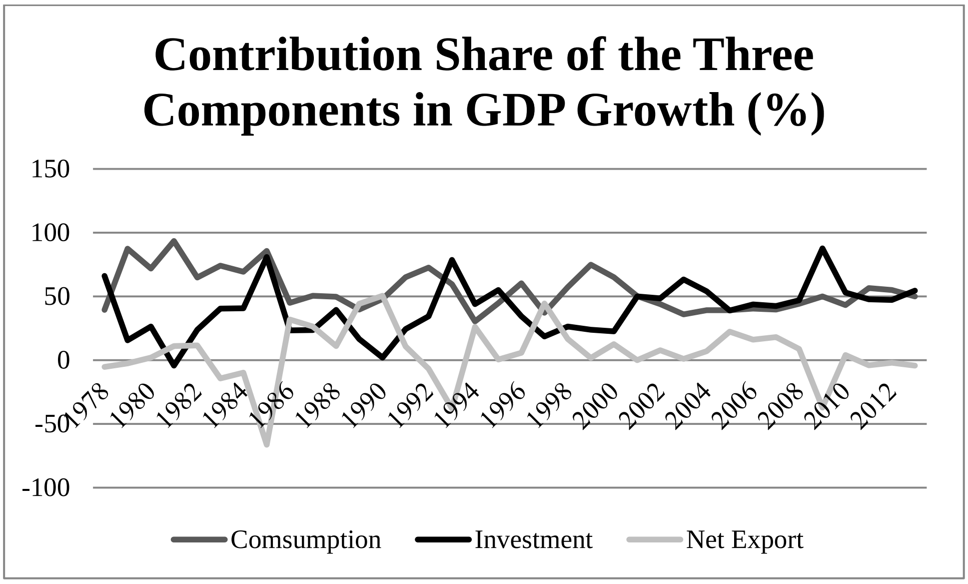
The Six Economic Troughs since 1949:
1960: the third year of the Great Leap Forward; 1959-1961: ‘Three Years of Natural Disasters’ along with policy failures.
1966: the Great Cultural Revolution.
1976: the Tangshan earthquake; the deaths of Zhou Enlai (the Premier), Zhu De (the principal founder of the army) and Mao Zedong (the Chairman); Down with The Gang of Four.
1989: students riot; Zhao Ziyang, the General Secretary of the Communist Party, was ousted from his position; Economic sanctions from Western countries.
1997: the Asian financial crisis.
2008: the global financial crisis.

4.2.2 Investment in China

Investment-led growth is a distinguishing feature of China’s economy. Different from the false impression of many people, Figure 2 reveals that investment contributes much more than net export in China’s economic growth. In 2009 when the closure of the export department resulted in redundancies, investment contributed 87.6% of the total economic growth, protecting China from the concussion of the global economic crisis. As is exhibited in Figure 3, the proportion of investment in GDP has been above 30% since 1970, reaching almost 50% in recent years.

The majority of investment was carried out by the government directly or by state-owned entities before 1978. In the mid-1980s, nongovernment investment began to outpace government investment, as shown in Figure 3. In recent years, more than 80% of the investment has been from the nongovernment sector.

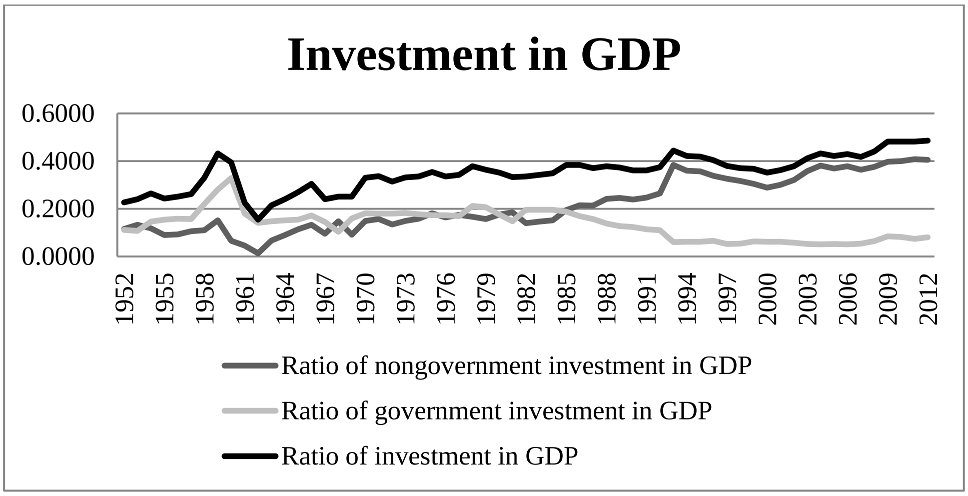
Figure 2: Contribution Shares of the Three Components in GDP Growth



Data Source: National Data, National Bureau of Statistics of China.

Note: Contribution share of the three components in GDP increase refers to the proportion of the increment of each component of GDP to the increment of GDP.

Figure 3: Investment in GDP



Data Source: calculated from the raw data from China Statistical Yearbook (2001, 2007, 2008) and National Data, National Bureau of Statistics of China

4.2.3 Tax System

Before 1978, public expenditure were chiefly from the revenues and profits of the SOEs. Since 1978, however, the planned quotas have been remitted and the profits claimed by the government have been replaced with taxes. In the beginning of the reform, this tax system was adjusted so that differences in the capitalisation and pricing situations among various firms were allowable, but more-uniform tax schedules were introduced in the early 1990s. After 1992, value added tax (Value added tax is proportional to the value realised in the production) has been the largest revenue resource among all tax categories. In 2012, tax revenue accounted for 85.8% in government revenue and most items of tax revenue are proportional rather than progressive. The two progressive tax categories are individual income tax and land appreciation tax, together occupying 8.6% of the total tax revenue in 2012.

4.3 The Macroeconomic Model with Taxation and Public Capital

4.3.1 Model Structure

(a) The utility function:

$$u\left(c_t, h_t\right) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \varphi \frac{h_t^{1-\sigma} - 1}{1-\sigma} \tag{1}$$

The model adopts a transformed 'constant intertemporal elasticity of substitution' (CIES) utility function, where c_t is private consumption (or nongovernment consumption), h_t is public consumption (or government consumption), $\sigma > 0$ and $-\sigma$ is the elasticity of marginal utility with respect to private and public consumption, φ is the weight given to public consumption service relative to private consumption in the utility.

(b) The production function:

$$y_t = f\left(k_t\right) = A_t^{1-\theta}\left[\alpha k_t^r + \left(1-\alpha\right) k_{Gt}^r\right]^{\frac{\theta}{r}} \exp\left(v_t^y\right) \tag{2}$$

where A_t is a deterministic trend which tracks the long-run growth of human capital (including both quantity and quality of the labour force), v_t^y is the logarithm of productivity shock, k_t is nongovernment capital and k_{Gt} is government capital.

The homogenous Constant Elasticity of Substitution (CES) production function was introduced by Arrow, Chenery, Minhas and Solow (1961). The elasticity of substitution measures the percentage change in factor proportions due to a percentage change in the marginal rate of technical substitution.

Equation (2) is an extended CES production function which uses a CES structure to describe the relationship between government and nongovernment capital, but specifies the relationship between the broad capital and A_t as the Cobb–Douglas form. α and $(1-\alpha)$ are the share parameters for nongovernment and government capital respectively, $(1-r)^{-1}$ is the elasticity of substitution between the two types of capital, θ and $(1-\theta)$ are the output elasticities of broad capital and A_t, respectively.

If r approaches zero, in the limit we get the Cobb–Douglas function:

$$y_t = f(k_t) = A_t^{1-\theta} k_t^{\theta\alpha} k_{Gt}^{\theta(1-\alpha)} \exp(v_t^y) \tag{3}$$

(c) The nongovernment capital accumulation:

$$k_{t+1} = i_t + (1-\delta) k_t \tag{4}$$

(d) The household-producer budget constraint:

$$c_t = (1-\tau_t) y_t - i_t \tag{5}$$

(e) The government capital accumulation:

$$k_{Gt+1} = i_{Gt} + (1-\delta_G) k_{Gt} \tag{6}$$

(f) k_{Gt}, τ_t, h_t are exogenous variables, h_t does not enter the solution.

4.3.2 Solution of the Model with CES Production Function

The objective function is

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{7}$$

subject to equations (2), (4) and (5).

Rewrite the problem in the form of Bellman equation.

$$v(k_t, k_{Gt}, \tau_t) = \max_{c_t, k_{t+1}} \left\{ u(c_t) + \beta E_t \left[v(k_{t+1}, k_{Gt+1}, \tau_{t+1}) \right] \right\} \tag{8}$$

subject to:

$$c_t + k_{t+1} - (1-\delta) k_t = (1-\tau_t) A_t^{1-\theta} \left[\alpha k_t^r + (1-\alpha) k_{Gt}^r \right]^{\frac{\theta}{r}} \exp(v_t^y) \tag{9}$$

Derive the Euler equation

$$\frac{\partial v\left(k_t, k_{Gt}, \tau_t\right)}{\partial k_{t+1}} = -c_t^{-\sigma} + \beta E_t\left[\frac{\partial v\left(k_{t+1}, k_{Gt+1}, \tau_{t+1}\right)}{\partial k_{t+1}}\right] = 0 \tag{10}$$

Apply envelope theorem to the first derivative of value function:

$$\frac{\partial v\left(k_t, k_{Gt}, \tau_t\right)}{\partial k_t} = c_t^{-\sigma}\left\{\theta\alpha\left(1-\tau_t\right)A_t^{1-\theta}\left[\alpha k_t^r + \left(1-\alpha\right)k_{Gt}^r\right]^{\frac{\theta}{r}-1} k_t^{r-1}\exp\left(v_t^y\right) + \left(1-\delta\right)\right\} \tag{11}$$

Hence,

$$\begin{aligned} &\frac{\partial v\left(k_{t+1}, k_{Gt+1}, \tau_{t+1}\right)}{\partial k_{t+1}} \\ &= c_{t+1}^{-\sigma}\left\{\theta\alpha\left(1-\tau_{t+1}\right)A_{t+1}^{1-\theta}\left[\alpha k_{t+1}^r + \left(1-\alpha\right)k_{Gt}^r\right]^{\frac{\theta}{r}-1} k_{t+1}^{r-1}\exp\left(v_{t+1}^y\right) + \left(1-\delta\right)\right\} \end{aligned} \tag{12}$$

From equations (10) and (12), obtain:

$$c_t^{-\sigma} = \beta E_t\left\{c_{t+1}^{-\sigma}\left\{\theta\alpha\left(1-\tau_{t+1}\right)A_{t+1}^{1-\theta}\left[\alpha k_{t+1}^r + \left(1-\alpha\right)k_{Gt+1}^r\right]^{\frac{\theta}{r}-1} k_{t+1}^{r-1}\exp\left(v_{t+1}^y\right) + \left(1-\delta\right)\right\}\right\} \tag{13}$$

Thus, we have a system with three endogenous variables

$$\begin{aligned} &y_t = A_t^{1-\theta}\left[\alpha k_t^r + \left(1-\alpha\right)k_{Gt}^r\right]^{\frac{\theta}{r}}\exp\left(v_t^y\right) \\ &c_t^{-\sigma} = \beta E_t\left\{c_{t+1}^{-\sigma}\left\{\theta\alpha\left(1-\tau_{t+1}\right)A_{t+1}^{1-\theta}\left[\alpha k_{t+1}^r + \left(1-\alpha\right)k_{Gt+1}^r\right]^{\frac{\theta}{r}-1} k_{t+1}^{r-1}\exp\left(v_{t+1}^y\right) + \left(1-\delta\right)\right\}\right\} \\ &c_t = \left(1-\tau_t\right)y_t - k_{t+1} + \left(1-\delta\right)k_t \end{aligned} \tag{14}$$

Translate the system (14) into the form with Dynare timing convention

$$\begin{aligned} &y_t = A_t^{1-\theta}\left[\alpha k_{t-1}^r + \left(1-\alpha\right)k_{Gt}^r\right]^{\frac{\theta}{r}}\exp\left(v_t^y\right) \\ &c_t^{-\sigma} = \beta E_t\left\{c_{t+1}^{-\sigma}\left\{\theta\alpha\left(1-\tau_{t+1}\right)A_{t+1}^{1-\theta}\left[\alpha k_t^r + \left(1-\alpha\right)k_{Gt+1}^r\right]^{\frac{\theta}{r}-1} k_t^{r-1}\exp\left(v_{t+1}^y\right) + \left(1-\delta\right)\right\}\right\} \\ &c_t = \left(1-\tau_t\right)y_t - k_t + \left(1-\delta\right)k_{t-1} \end{aligned} \tag{15}$$

Set 1952 as the initial period, so define

$$A_t = A_{1952}\gamma^{(t-1952)} \tag{16}$$

$$\hat{x}_t = \frac{x_t}{A_t} \tag{17}$$

Hence, we have a stationary system:

$$\begin{aligned}
&\hat{y}_t = \left[\alpha \hat{k}_{t-1}^r \gamma^{-r} + (1-\alpha)\hat{k}_{Gt}^r\right]^{\frac{\theta}{r}} \exp\left(v_t^y\right) \\
&\hat{c}_t^{-\sigma} = \beta E_t \left\{\hat{c}_{t+1}^{-\sigma}\gamma^{-\sigma}\left\{\theta\alpha\left(1-\tau_{t+1}\right)\left[\alpha \hat{k}_t^r \gamma^{-r} + (1-\alpha)\hat{k}_{Gt+1}^r\right]^{\frac{\theta}{r}-1} \hat{k}_t^{r-1}\gamma^{1-r} \exp\left(v_{t+1}^y\right) + (1-\delta)\right\}\right\} \\
&\hat{c}_t = (1-\tau_t)\hat{y}_t - \hat{k}_t + (1-\delta)\hat{k}_{t-1}\gamma^{-1}
\end{aligned} \tag{18}$$

where

$$v_t^y = \rho_y v_{t-1}^y + e_t^y \tag{19}$$

$$\tau_t = \tau_E \exp\left(v_t^\tau\right) \quad where \quad v_t^\tau = \rho_\tau v_{t-1}^\tau + e_t^\tau \tag{20}$$

$$\hat{k}_{Gt} = \hat{k}_{GE} \exp\left(v_t^{KG}\right) \quad where \quad v_t^{KG} = \rho_{KG} v_{t-1}^{KG} + e_t^{KG} \tag{21}$$

4.3.3 Solution of the Model with Cobb-Douglas Production Function

If the production function is in the form of Cobb-Douglas as equation (3), the system (15) should become

$$\begin{aligned}
&y_t = A_t^{1-\theta}\left(k_{t-1}^\alpha k_{Gt}^{1-\alpha}\right)^\theta \exp\left(v_t^y\right) \\
&c_t^{-\sigma} = \beta E_t\left\{c_{t+1}^{-\sigma}\left\{\theta\alpha\left(1-\tau_{t+1}\right) A_{t+1}^{1-\theta} k_t^{\theta\alpha-1} k_{Gt+1}^{\theta(1-\alpha)} \exp\left(v_{t+1}^y\right) + (1-\delta)\right\}\right\} \\
&c_t = (1-\tau_t) y_t - k_t + (1-\delta)k_{t-1}
\end{aligned} \tag{22}$$

Finally, the system (18) becomes

$$
\begin{aligned}
&\hat{y}_t = \left(\hat{k}_{t-1}^{\alpha}\gamma^{-\alpha}\hat{k}_{Gt}^{1-\alpha}\right)^{\theta}\exp\left(v_t^y\right) \\
&\hat{c}_t^{-\sigma} = \beta E_t\left\{\hat{c}_{t+1}^{-\sigma}\gamma^{-\sigma}\left\{\theta\alpha\left(1-\tau_{t+1}\right)\hat{k}_t^{\theta\alpha-1}\hat{k}_{Gt+1}^{\theta(1-\alpha)}\gamma^{1-\theta\alpha}\exp\left(v_{t+1}^y\right)+\left(1-\delta\right)\right\}\right\} \\
&\hat{c}_t = \left(1-\tau_t\right)\hat{y}_t - \hat{k}_t + \left(1-\delta\right)\hat{k}_{t-1}\gamma^{-1}
\end{aligned}
\tag{23}
$$

4.4 Data Sources and Data Processing

This chapter employs annual data from 1952 to 2012. The raw data are collected from China Statistical Yearbook (2001, 2007, 2008) and National Data, National Bureau of Statistics of China.

The items of raw data are: (1) Gross Domestic Product (GDP), (2) Household Consumption (private consumption or nongovernment consumption), (3) Gross Capital Formation (total investment), (4) Government Budgetary Revenue, (5) Government Extra-budgetary Revenue, (6) Government Expenditure by Function (1952-2006), (7) Main Items of National Government Expenditure of Central and Local Governments (2007-2012), (8) Extra-budgetary Revenue by Item (1952-2010), (9) Extra-budgetary Expenditure by Item. (Note: since 2011, extra-budgetary revenue and expenditure were included in intra-budgetary revenue and expenditure.)

The National Bureau of Statistics of China does not directly offer the data on government investment. However, the figures can be approximately estimated from corresponding productive expenditure subitems in the items (6)-(9). Therefore nongovernment investment can be calculated by

$$\text{nongovernment investment} = \text{total investment} - \text{government investment}$$

Based on the above nominal data, the real data are obtained by dividing the nominal data by the GDP deflator.

$$\text{real data} = \text{nominal data} / \text{GDP deflator}$$

The base year for calculating GDP deflator is 2000. GDP deflator from 1960 to 2012 is from World Development Indicators (WDI) & Global Development Finance (GDF), The World Bank (issued in July, 2013); GDP deflator from 1952 to 1959 is calculated from the nominal GDP and indices of GDP (1952-1960).

Tax rate is simply estimated by

$$\text{tax rate} = \left(\text{budgetary revenue} + \text{extra-budgetary revenue} \right) / \text{GDP}$$

The approach of calculating capital stock values belongs to the category of the conventional ‘Perpetual Inventory Method’ (PIM). The estimation technique can be expressed as equations (4) and (6). The value of capital stock is estimated by using the annual data of capital formation as the investment.

The data of GDP, nongovernment consumption, nongovernment investment, government investment and tax rates are listed in Appendix 4A.

4.5 Methodology

Indirect inference is a simulation-based method for estimating, or making inferences about, the parameters of economic models. A VAR model can be used as the auxiliary model. The process of indirect inference testing is to bootstrap the structural residuals and generate a large number of sample replications, based on which, a distribution of the VAR parameters is obtained, finally test whether the VAR parameters from actual data lies within this distribution at some level of confidence. The estimation using indirect inference is to choose a set of parameters for the structural model, so that, when this model is simulated, it generates estimates of the auxiliary model as close as possible to the estimates of the auxiliary model using actual data. In other words, indirect estimation chooses the structural parameters that can minimise the distance between the two sets of estimated parameters. There are many famous researchers in the literature, such as Smith (1993), Gourieroux, Monfort and Renault (1993), Canova (2005) and Le, Meenagh, Minford and Wickens (2011).

4.5.1 Indirect Inference Test

Step 1: Calculate the Innovations

Parameters γ and A_{1952} should be estimated so that the deterministic trend A_t is obtained by Equation (16) and then the stationary system (18) can be generated. γ is estimated on the basis of the averaging growth rates of GDP and private consumption. Set the shock in the first equation of system (15) as zero, substitute (16) into the equation, then obtain the value of 'A_{1952}' in each year, and thus estimate the A_{1952} on the basis of the mean value. Parameters τ_E in Equation (20) and $\hat{k}_{GE}$ in Equation (21) are estimated on the basis of the mean values of τ_t and $\hat{k}_{Gt}$, respectively. A_{1952},τ_E and $\hat{k}_{GE}$ could be adjusted around the corresponding mean values. Parameters $\rho_y, \rho_\tau, \rho_{KG}$ in (19)-(21) are estimated by AR(1) models. Finally, innovations are obtained from Equations (19)-(21).

Step 2: Simulate Data

Save the policy functions in Dynare with the DSGE model above, and then obtain N sets of simulated data from the policy functions and N bootstraps of innovations.

Step 3: Estimate the Auxiliary Model

Use a VAR(1) as the auxiliary model, then estimate it with both actual data and the N samples of simulated data. β^a is obtained from the actual data and includes the VAR parameters and the volatilities of the three variables in the VAR model; β^i (i=1,2,…N) is obtained from one sample of simulated data; $\bar{\beta}$ is the average of N sets of β^i. The auxiliary VAR(1) Model is

$$\begin{bmatrix} y_t - y_{ss} \\ k_t - k_{ss} \\ \hat{c}_t - \hat{c}_{ss} \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \\ \beta_{13} & \beta_{23} & \beta_{33} \end{bmatrix} \begin{bmatrix} y_{t-1} - y_{ss} \\ k_{t-1} - k_{ss} \\ \hat{c}_{t-1} - \hat{c}_{ss} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^k \\ \varepsilon_t^c \end{bmatrix} \tag{24}$$

$$\textit{where} \quad y_{ss}, k_{ss}, \hat{c}_{ss} \ \textit{are steady state values}$$

Step 4: Compute Wald Statistics

$$W^x = \left(\beta^x - \bar{\beta}\right)' \Omega^{-1} \left(\beta^x - \bar{\beta}\right) \quad \textit{where} \quad \Omega = \operatorname{cov}\left(\beta^i - \bar{\beta}\right), \quad x = a \ \textit{or} \ i \ (i = 1, 2, \cdots N) \tag{25}$$

Compare W^a with W^{95th} (the 95th percentile of the Wald statistics from the simulated data), if W^a is outside the 95% confidence interval, then the model with the selected parameters does not pass the test, in other words, the actual data can not be well explained. The model can be accepted if

$$WR = \frac{W^a}{W^{95th}} \le 1 \tag{26}$$

4.5.2 Indirect Inference Estimation

As introduced in 4.5.1, indirect inference can be employed to test whether the existing model with a set of parameters can generate the actual data and the Wald statistic measures the distance between the data and the model. However, if this set of structure parameters can not explain the data well then other sets of parameters might be used to explain how the data is generated. If none set of parameters can be found under which the model passes, then the model should be rejected. Even the model has already passed the test, it is still necessary to seek an alternative set of parameters that could reduce the value of WR in (26). The main idea of indirect inference estimation is to seek the set of parameters which minimises the value of WR. The algorithm in Le and Meenagh (2013) is different from that in (26), however minimising the statistic in Equation (3) in Le and Meenagh (2013) is identical to minimising WR here.

4.6 Test Results

4.6.1 Test Result with CES Production Function

The key aspects of PIM are to set the base year capital stock and to estimate the asset depreciation rates. Table 4 and Table 5 show that different values of initial capital stock and depreciation rate have been used in the literature and the choices are controversial.

Any initial value of capital stock (the capital stock in 1952) with moderate depreciation rates makes the ratio of total capital to GDP is around 2 during the period after 1960, which suggests the capital-GDP ratio is about 2 (See Figure 4), similar to those in other countries. The GDP in 1952 was about 274 billion, therefore, the indirect inference test sets the initial capital stock as 600 billion Yuan with the price level in 2000, which is close to Wang and Fan (2000). For the estimations in Section 4.7, the initial capital stock is allowed to be from 500 to 800 billion Yuan.

According to the data of government and nongovernment investment in 1950s, it is reasonable to assume the initial capital was almost equally divided into government and nongovernment sectors. Hence, the initial values of the two types of capital are both set as 300 billion in the test.

Table 4: Literature of Initial Capital Stock (in 2000 Prices, billion Yuan)

Authors	Capital Stock in 1952
Zhang (1991), cited in Zhang and Zhang (2003)	808.1
He (1992), cited in Zhang and Zhang (2003)	186.5
Chow (1993)	707.1
Perkins (1998), cited in Wu (2009)	832.3
Chow and Li (2002)	894.1
Wang and Fan (2000)	646.5
Zhang and Zhang (2003)	323.2

Table 5 privates the literature of depreciation rate in China. The selection of depreciation rates of both types of capital follows Zhang (2008) with 9.6% in the test, and the rates are in an interval from 7% to 12% for estimations.

Table 5: Literature of Depreciation Rate in China (%)

Authors	Depreciation Rate
World Bank (1997)	4.0
Perkins (1998), cited in Wu (2009)	5.0
Chow and Li (2002)	5.4
Wu (2004)	7.0
Maddison (2007)	17.0
Zhang (2008)	9.6

The study of Luo and Zhang (2009) shows that the labour income share in GDP decreases from 52% in 1987 to 40% in 2006. Bai and Qian (2010) present that the labour income share in the industry sector increases from 34.52% to 42.21% in the period between 1978 and 2004. Labour income share is much lower in China than in the US and most of other countries, in other words, capital income share in China is higher than that in most of other countries.

Since there is government capital in the production function, the elasticity of output with respect to total capital should be higher than the capital income share. The average ratio of nongovernment capital to government capital is about 5:4 in the period from 1952 to 2012. Hence, the value of θ is possibly in the interval from 0.55 to 0.75. The value in testing can be set as 2/3=0.667.

Many studies adopt a 3.0 percent or a little larger discount rate according to the real rate of interest on Treasury bonds or other economic figures, therefore, the rate of time preference is fixed as $\beta = 0.97$ for tests with the data in China, a future-oriented society, and it is allowed to be between the lower bound 0.96 and the upper bound 0.99 in estimations.

Table 6 gives the test results with CES Production Function, showing that the model passes the test since the algorithm $\mathrm{WR} < 1$. Moreover, all VAR parameters and variances from actual data are within the 95% lower and upper bounds, in other words, based on the assumed parameters, the model with CES production function captures the reality well.

Table 6: Test Result with CES Production Function

Categories	Actual VAR Coefficients and Volatilities	95% Lower Bound	95% Upper Bound	IN/OUT
β_{11}	0.8206	0.6378	1.0459	IN
β_{21}	0.0573	-0.0888	0.1361	IN
β_{31}	0.1703	-0.4056	0.5985	IN
β_{12}	-0.0978	-0.2880	0.3605	IN
β_{22}	1.0737	0.8136	1.1583	IN
β_{32}	-0.1834	-0.7208	0.8677	IN
β_{13}	0.0102	-0.0671	0.1678	IN
β_{23}	0.0053	-0.0528	0.0781	IN
β_{33}	0.9446	0.5078	1.0135	IN
$\mathrm{var}(\hat{y})$	0.0268	0.0073	0.1456	IN
$\mathrm{var}(\hat{k})$	0.2771	0.0218	1.1913	IN
$\mathrm{var}(\hat{c})$	0.0095	0.0011	0.0285	IN
$W^a = 10.3766$				
$WR = \frac{W^a}{W^{95th}} = 0.3367 < 1$, the model can not be rejected.				

$k_{1951} = 3000$, $k_{G1952} = 3000$, $\delta = 0.096$, $\delta_G = 0.096$, $\beta = 0.97$, $\theta = 0.6667$, $\alpha = 0.55$, $r = 1$, $\sigma = 1.5$, $\gamma = 1.08$. $A_{1952}, \tau_E, \hat{k}_{GE}, \rho_y, \rho_\tau, \rho_{KG}$ are estimated by the methods in 4.5.1. (Note: k_{1951} is written according to the Dynare timing convention.)

The figures of impulse response functions are listed in Appendix 4B.

4.6.2 Test Result with Cobb-Douglas Production Function

The test of the model with Cobb-Douglas production function adopts the same values of $k_{1951}, k_{G1952}, \delta, \delta_G, \beta, \theta, \alpha, \sigma$ as in the test with CES production function, while $A_{1952}, \tau_E, \hat{k}_{GE}, \rho_y, \rho_\tau, \rho_{KG}$ are estimated by the methods in 4.5.1.

Table 7: Test Result with Cobb-Douglas Production Function

Categories	Actual VAR Coefficients and Volatilities	95% Lower Bound	95% Upper Bound	IN/OUT
β_{11}	0.8452	0.7330	1.0752	IN
β_{21}	0.0533	-0.4185	0.2196	IN
β_{31}	0.1856	-0.3629	0.8223	IN
β_{12}	-0.0144	-0.0205	0.2273	IN
β_{22}	1.0554	0.5347	0.8986	OUT
β_{32}	-0.2703	0.1446	0.9134	OUT
β_{13}	0.0195	-0.0700	0.1363	IN
β_{23}	0.0037	-0.1042	0.2141	IN
β_{33}	0.9489	0.4051	1.1459	IN
$\mathrm{var}(\hat{y})$	0.0182	0.0059	0.0645	IN
$\mathrm{var}(\hat{k})$	0.1885	0.0054	0.1130	OUT
$\mathrm{var}(\hat{c})$	0.0065	0.0012	0.0224	IN
$W^a = 1383.2690$				
$WR = \frac{W^a}{W^{95th}} = 46.3973 > 1$, the model is rejected.				

The figures of impulse response functions are listed in Appendix 4C.

Table 7 shows that, with Cobb-Douglas production function, the Wald statistics from the test is very high. Besides, the stationary nongovernment capital cannot be captured well since β_{22}, β_{32} and $\mathrm{var}(\hat{k})$ are far away from the 95% lower and upper bounds. Hence, we can not accept the Cobb-Douglas production function under the assumed parameters.

4.6.3 Compare Test Results with Different Values of r

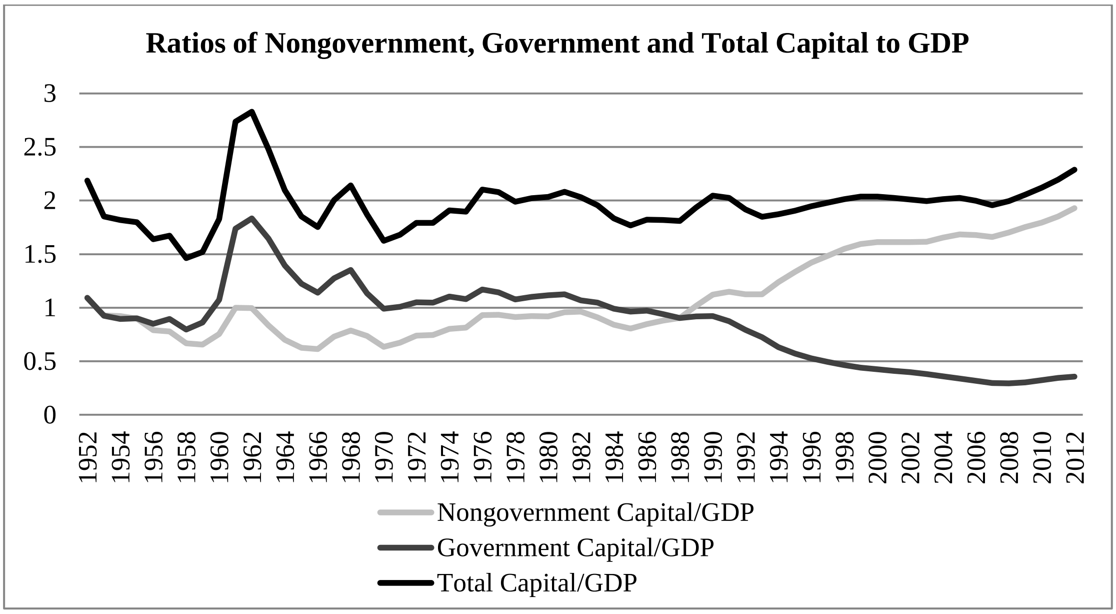
According to the testing results in Table 8, the CES production function is much better than the Cobb-Douglas production function, which is not surprising since CES production function is a general case. However, what is 'odd' is that the results imply the elasticity of substitution between the two types of capital $\frac{1}{1-r}$ is much high since the test result is better if r is closer to unity.

Figure 4 may justify the 'odd' results. The ratio of total capital to GDP is much stable, while the ratio of nongovernment capital to GDP had been doubled in the six decades and the ratio of government capital had decreased to less than half of the initial ratio, suggesting there may be a strong substitution relation between the two types of capital.

Table 8: Test Results with Different Values of r

r value	0	0.2	0.4	0.6	0.8	1
W^a	1383.2690	373.5062	114.4094	39.0750	16.4111	10.3766
WR	46.3973	13.0236	3.9217	1.3285	0.5401	0.3367

Figure 4: Ratios of Private, Public and Total Capital to GDP



Note: The data is estimated by assuming $k_{1951} = 3000$, $k_{G1952} = 3000$, $\delta = 0.096$, $\delta_G = 0.096$.

From Table 8, we can conjecture that the two types of capital are gross substitutes ($0 < r \leq 1$). However, we can not make a conclusion that the model with Cobb-Douglas production function is underperformed without looking at the estimation results. Moreover, the model passes the test when $r = 0.8$, hence, we can not jump to a conclusion that the two types of capital are perfect substitutes ($r = 1$).

4.7 Estimation Results

4.7.1 Estimation with CES Production Function

Table 9 gives the estimation result with CES production function, and again the set of estimated parameters captures the reality well with a low Wald statistics and a low value of WR which is much small than 1.

The parameters obtained from estimation are: $k_{1951} = 2895.7031$, $k_{G1952} = 3999.2188$, $\delta = 0.1192$, $\delta_G = 0.0928$, $\beta = 0.9720$, $\theta = 0.7469$, $\alpha = 0.5012$, $r = 0.9998$, $\sigma = 1.6518$, $\gamma = 1.0702$, $A_{1952} = 4474.7984$, $\tau_E = 0.3475$, $\hat{k}_{GE} = 0.7048$, $\rho_y = 0.7880$, $\rho_\tau = 0.9674$, $\rho_{KG} = 0.9900$. The estimated r is close to 1, implying that the strong substitution between government and nongovernment capital.

Table 9: Estimation Result

Categories	Actual VAR Coefficients and Volatilities	95% Lower Bound	95% Upper Bound	IN/OUT
β_{11}	0.7923	0.5921	1.0131	IN
β_{21}	0.0775	-0.0715	0.1556	IN
β_{31}	0.1818	-0.3850	0.7843	IN
β_{12}	-0.0791	-0.3328	0.4609	IN
β_{22}	1.1108	0.7487	1.1839	IN
β_{32}	-0.3322	-1.1916	1.4248	IN
β_{13}	-0.0060	-0.0519	0.0780	IN
β_{23}	0.0112	-0.0321	0.0426	IN
β_{33}	0.9519	0.7459	1.1394	IN
$\mathrm{var}(\hat{y})$	0.0437	0.0069	0.2048	IN
$\mathrm{var}(\hat{k})$	0.3445	0.0207	1.5947	IN
$\mathrm{var}(\hat{c})$	0.0036	0.0005	0.0226	IN

$W^a = 4.8892$

$WR = \frac{W^a}{W^{95th}} = 0.1502 < 1$, the model can not be rejected.

The figures of impulse response functions are listed in Appendix 4D.

4.7.2 Estimation with Cobb-Douglas Production Function

Table 10 shows that the model with Cobb-Douglas production function does not pass the test because the Wald statistics is very large and the value of WR is larger than 1, which means that any set of estimated parameters, under Cobb-Douglas production function, can not capture the reality well. Hence, the Cobb-Douglas production function is not suitable to describe the relationship between government and nongovernment capital.

The parameters obtained from estimation are: $k_{1951} = 3443.7988$, $k_{G1952} = 3998.9000$, $\delta = 0.1138$, $\delta_G = 0.1192$, $\beta = 0.9895$, $\theta = 0.7498$, $\alpha = 0.7703$, $\sigma = 2.2445$, $\gamma = 1.0780$, $A_{1952} = 4821.4015$, $\tau_E = 0.2621$, $\hat{k}_{GE} = 0.4775$, $\rho_y = 0.8266$, $\rho_\tau = 0.9440$, $\rho_{KG} = 0.9900$.

Table 10: Estimation Result with Cobb-Douglas Production Function

Categories	Actual VAR Coefficients and Volatilities	95% Lower Bound	95% Upper Bound	IN/OUT
β_{11}	0.5924	0.5906	1.0549	OUT
β_{21}	0.1278	-0.2145	0.1744	IN
β_{31}	0.3969	-0.3512	1.2604	IN
β_{12}	0.1180	-0.0143	0.3971	IN
β_{22}	1.0340	0.6248	0.8915	OUT
β_{32}	-0.4426	-0.1252	1.2771	OUT
β_{13}	-0.0685	-0.0405	0.1210	OUT
β_{23}	0.0299	-0.0663	0.0677	IN
β_{33}	1.0409	0.5989	1.1862	IN
$\mathrm{var}(\hat{y})$	0.0122	0.0123	0.1333	OUT
$\mathrm{var}(\hat{k})$	0.1163	0.0196	0.4336	IN
$\mathrm{var}(\hat{c})$	0.0032	0.0015	0.0268	IN
$W^a = 50.9957$				
$WR = \frac{W^a}{W^{95th}} = 1.7445 > 1$, the model is rejected.				

The figures of impulse response functions are listed in Appendix 4E.

4.7.3 Estimation with Different Values of r

The estimation results with different values of r demonstrate that a higher value of r usually generates a lower value of WR, which makes the model more easily pass the test. Table 11 gives the estimation result with $r = 0.25$. Three VAR parameters and the variance of stationary output obtained from the actual data are outside the 95% confidence intervals, however, which does not prevent the model overall fitting the data since WR<1. Therefore we can not reject that there is potentially somewhat complementary relationship between government and nongovernment capital.

The parameters obtained from estimation are: $k_{1951} = 2873.6084$, $k_{G1952} = 3997.8014$, $\delta = 0.1199$, $\delta_G = 0.1199$, $\beta = 0.9899$, $\theta = 0.7499$, $\alpha = 0.7381$, $\sigma = 2.3875$, $\gamma = 1.0705$, $A_{1952} = 5257.6483$, $\tau_E = 0.2665$, $\hat{k}_{GE} = 0.4364$, $\rho_y = 0.8482$ $\rho_\tau = 0.9436$, $\rho_{KG} = 0.9900$.

Table 11: Estimation Result with r = 0.25

Categories	Actual VAR Coefficients and Volatilities	95% Lower Bound	95% Upper Bound	IN/OUT
β_{11}	0.5671	0.5833	1.0730	IN
β_{21}	0.1344	-0.1816	0.1843	IN
β_{31}	0.4496	-0.4230	1.2262	IN
β_{12}	0.0637	-0.0889	0.4321	IN
β_{22}	1.0412	0.6608	0.9480	OUT
β_{32}	-0.3522	-0.3380	1.2807	OUT
β_{13}	-0.0709	-0.0566	0.1203	OUT
β_{23}	0.0303	-0.0600	0.0722	IN
β_{33}	1.0490	0.5875	1.1704	IN
$\mathrm{var}(\hat{y})$	0.0102	0.0104	0.1352	OUT
$\mathrm{var}(\hat{k})$	0.0947	0.0192	0.4601	IN
$\mathrm{var}(\hat{c})$	0.0027	0.0014	0.0248	IN
$W^a = 29.8255$				
$WR = \frac{W^a}{W^{95th}} = 0.9676 < 1$, the model can not be rejected.				

The figures of impulse response functions are listed in Appendix 4F.

4.8 Estimation in Two Periods

4.8.1 Estimation with CES Production Function 1952-1992

The small values of Wald statistics and WR in Table 12 suggest that, based on the data from 1952 to 1992, the model with CES production function can be accepted. All actual VAR Coefficients and Volatilities are within the 95% lower and upper bounds.

The parameters obtained from estimation are: $k_{1951} = 3499.7070$, $k_{G1952} = 3499.5117$, $\delta = 0.1009$, $\delta_G = 0.1048$, $\beta = 0.9700$, $\theta = 0.6063$, $\alpha = 0.5500$, $r = 0.9988$, $\sigma = 1.7406$, $\gamma = 1.0776$, $A_{1952} = 2945.2770$, $\tau_E = 0.4060$, $\hat{k}_{GE} = 0.9672$, $\rho_y = 0.7615$ $\rho_\tau = 0.9147$, $\rho_{KG} = 0.9900$. r is close to 1, implying the strong substitution relationship between the two types of capital from 1952 to 1992.

Table 12: Estimation Result (1952-1992)

Categories	Actual VAR Coefficients and Volatilities	95% Lower Bound	95% Upper Bound	IN/OUT
β_{11}	0.4693	0.1639	0.9018	IN
β_{21}	0.0666	-0.0758	0.2504	IN
β_{31}	0.5367	-0.3910	1.1620	IN
β_{12}	0.0836	-0.2218	0.3566	IN
β_{22}	1.0169	0.8429	1.0809	IN
β_{32}	-0.3870	-0.7997	0.1583	IN
β_{13}	-0.0982	-0.1700	0.0841	IN
β_{23}	0.0124	-0.0225	0.1057	IN
β_{33}	1.0455	0.5985	1.1943	IN
$\mathrm{var}(\hat{y})$	0.0241	0.0105	0.0607	IN
$\mathrm{var}(\hat{k})$	0.0395	0.0165	0.1544	IN
$\mathrm{var}(\hat{c})$	0.0124	0.0047	0.0216	IN

$W^a = 3.9566$

$WR = \frac{W^a}{W^{95th}} = 0.1232 < 1$, the model can not be rejected.

The figures of impulse response functions are listed in Appendix 4G.

4.8.2 Estimation with CES Production Function 1993-2012

Table 13 shows that the values of Wald statistics and WR is small, so that, based on the data from 1993 to 2012, the model with CES production function can be accepted. All actual VAR Coefficients and Volatilities are within the 95% lower and upper bounds.

The parameters obtained from estimation are: $k_{1951} = 3000$, $k_{G1952} = 3000$, $\delta = 0.12$ $\delta_G = 0.07$, $\beta = 0.9760$, $\theta = 0.7400$, $\alpha = 0.5536$, $r = 0.8816$, $\sigma = 1.3114$, $\gamma = 1.0744$, $A_{1993} = 57130.3394$, $\tau_E = 0.1743$, $\hat{k}_{GE} = 0.7051$, $\rho_y = 0.9155$ $\rho_\tau = 0.9950$, $\rho_{KG} = 0.9904$. The elasticity of substitution is $r = 0.8816$, less than that in the period between 1952 and 1992.

Table 13: Estimation Result (1993-2012)

Categories	Actual VAR Coefficients and Volatilities	95% Lower Bound	95% Upper Bound	IN/OUT
β_{11}	0.8895	0.0322	1.0956	IN
β_{21}	0.0594	-0.1105	0.3160	IN
β_{31}	-0.2581	-0.8481	1.3448	IN
β_{12}	0.6383	-0.5769	1.1465	IN
β_{22}	0.8254	0.4165	1.1647	IN
β_{32}	-0.4560	-1.5088	2.4191	IN
β_{13}	0.2006	-0.1966	0.3798	IN
β_{23}	-0.0686	-0.1300	0.0713	IN
β_{33}	1.0200	0.4342	1.2904	IN
$\mathrm{var}(\hat{y})$	0.0260	0.0017	0.0638	IN
$\mathrm{var}(\hat{k})$	0.2067	0.0169	0.6743	IN
$\mathrm{var}(\hat{c})$	0.0004	0.0001	0.0059	IN
$W^a = 10.1558$				
$WR = \frac{W^a}{W^{95th}} = 0.3419 < 1$, the model can not be rejected.				

The figures of impulse response functions are listed in Appendix 4H.

4.9 Comments and Implications

4.9.1 Substitution Between Government and Nongovernment Capital

As illustrated by both test and estimation results, there is strong substitution relationship between government and nongovernment capital. Hence, Cobb-Douglas technological relationship between government and nongovernment capital may not be applicable to describe China's macroeconomy, especially when the economy was based on the dominance of the state-owned sector before 1992. However Table 11 shows that when $r = 0.25$, the estimated set of parameters overall passes the test, hence we can not arbitrarily reject that there is potentially somewhat complementary relationship between government and nongovernment capital.

The substitution relationship between government and nongovernment sectors was in line with the reality in China before 1992. Influenced by the Soviet Union, Chinese government nationalised virtually most private industrial enterprises during the 1950s. Moreover, a large proportion of government productive expenditure was poured into SOEs and then it turned into the capital in nongovernment sector. Under a centrally planned economy, the government 'invaded' the private sector, the difference between the government investment and nongovernment investment was ambiguous. In other words, sometimes the government did what the nongovernment sector should do. After 1978, the government initiated intensive reforms and private investment was encouraged. In 1990s, some SOEs were privatised. Nongovernment capital, especially private capital, then 'fought back' into the field once occupied by the public capital.

4.9.2 Differences Between Two Periods

In the 1980s and 1990s, Chinese government intentionally reduced its share of GDP in order to allow rural and urban households and firms to have more resources and better incentives. Furthermore, the SOEs tended to be more market oriented, they could not receive public investment freely. Meanwhile, public productive investment flowed to infrastructure construction rather than SOEs. The government became a service supplier. Government and nongovernment sectors became more complementary, which is verified by the estimated elasticity of substitution being reduced from close to 1 in the period between 1952 and 1992 to 0.8816 after 1992.

4.9.3 Discussion and Policy Implications

The merits and drawbacks of large-scale public investment thoroughly exhibited in the Maoist era. In that period, consumption, especially in rural areas, was contained by taxes and compulsory delivery quotas, which was imposed in order to finance the public investment and feed the urban population at low prices. Absolute equalitarianism totally destroyed the market forces. Enthusiasm for work was mainly from disciplines and political beliefs rather than economic benefits. Nevertheless, economic performance was a great improvement, as argued by Maddison (2007). Real GDP had expanded more than fivefold; human capital and labour productivity were also enhanced due to better education and doubled average life span; the economic structure was transformed with the industry's share of GDP increasing from one tenth to almost one half. China achieved this despite that self-inflicted wounds hampered the economic growth during the Great Leap Forward (1958-1960) and the Cultural Revolution (1966-1976), and China suffered the political and economic isolation, and hostile diplomatic relations with both the United States and the Soviet Unions. Obviously, without the political instability and the economic sanctions imposed by some Western countries, China would have a better economic performance with the development effort and large-scale public investment. We can not jump rashly to the conclusion that the macroeconomy before the 1978 reform was a complete failure. However, it is safe to draw the conclusion that Deng Xiaoping's path of opening up and economic reform was absolutely superior to the development strategies before the reform, which is supported by higher and more stable economic growth after 1978. Committing to continuing Deng Xiaoping's path, the Chinese government has adopted tactful fiscal policies to deal with the trade-off in the public investment, as simply summarised in three aspects below.

Firstly, the government decentralises the economy, partially privatises the SOEs and transform most of them into joint-stock companies. The state retains ownership of some large SOEs but has little direct control over their operations. Moreover, the government does not pour the public investment into SOEs freely any longer, which lessens the financial burden of the government, making it possible to reduce the tax rate so that households and firms have more resources and better incentives. As a result, even higher saving and investment have been made by a gradual takeover of national saving from government by microeconomic agencies, and then the growth acceleration has been achieved by increased efficiency.

Secondly, the government has changed the structure of public investment. By stopping the free injection of funds into SOEs and cutting the financial support to them, the government has more resources to invest in high-quality and large-quantity infrastructures and improve the investment environment. The government directly or indirectly (through SOEs) builds more expressways, international airports, high-speed railways and telecommunication facilities. The government has transferred from the competitor of the private sector to the service provider of private investors.

Lastly, the government uses public investment, together other policies, as a tool of stabilising the economy when faced with slipping business confidence domestically or externally. Beijing's response to the 2008 economic crisis was swift: development of polysilicon supplies and manufacturing technology of clean cars were declared as national priorities with financial supports and tax advantages. Money poured into manufacturers and overseas acquisition of cheap assets, resources and technology from state-owned companies and banks, local governments expedited approvals for new plants. The leveraged investment from the government and SOEs offset the reduction of private investment, reduced the unemployment, and finally restored confidence and liquidity, achieving rather high growth rates: 9.6%, 9.2% and 10.4% in 2008, 2009 and 2010, respectively. In 2009, 87.6% of the economic growth was contributed by investment.

In brief, the successful experiences of the Chinese government are cutting directly investment in the competitive market, reducing the tax burden, providing better infrastructure facilities, and using public investment to restore macroeconomic stability if necessary and encouraging the economic activities of the SOEs as an intermediary channeling to prop private investment when faced with external crisis.

4.10 Conclusion

This chapter focuses on the elasticity of substitution between government and nongovernment capital through testing and estimating the models with CES and Cobb-Douglas technologies. Both test and estimation results underscore the strong substitution relationship between government and nongovernment capital after the founding of People's Republic of China in 1949, and thus the general CES technology rather than the Cobb-Douglas technology is the suitable structure containing the two types of capittal in the production function.

Furthermore, the estimation results corroborate the difference on the orientation of public investment between the two periods: before and after the legislation of Socialist Market Economy in 1992, revealing that the government capital has become more complementary to the nongovernment capital due to the deepening economic reform.

The empirical results from indirect inference method, along with both failures and successes in the experiences of Chinese government, give meaningful policy implications: macroeconomic intervention in the competitive market through fiscal policies and government industrial monopoly should be constrained in order to stimulate the vitality of the households and firms; a public service-oriented government should be established so that the government and nongovernment sectors have a more complementary and less competitive relationship; when the domestic economy was swept by external crisis, the government should exploit public investment as one of the countermeasures to stabilise the macroeconomy, rescue the confidence of private sector, and then reduce unemployment, especially in the export sector.

Appendix 4

Appendix 4A: Real Data (1952-1982)

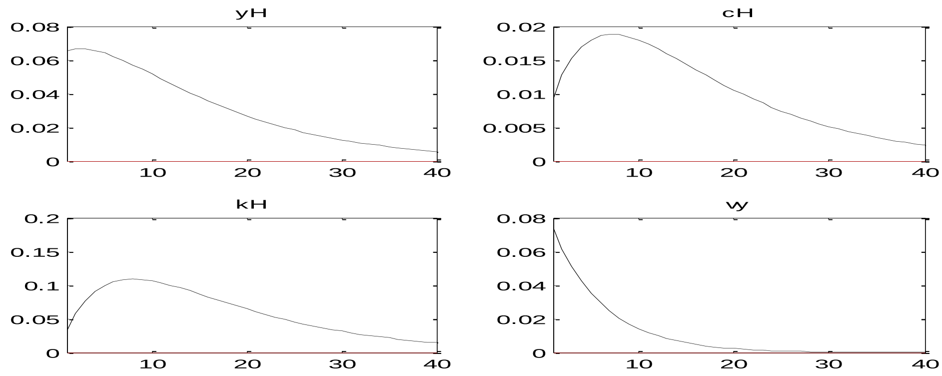
	GDP (100 million Yuan)	Nongovernment Consumption (100 million Yuan)	Nongovernment Investment (100 million Yuan)	Government Investment (100 million Yuan)	Tax Rates
1952	2743.95	1830.64	316.79	304.33	0.28
1953	3265.02	2096.90	433.92	351.82	0.27
1954	3437.42	2200.91	404.76	503.21	0.30
1955	3647.27	2415.21	325.78	561.99	0.29
1956	4155.76	2614.73	384.82	656.55	0.29
1957	4307.32	2769.11	455.51	673.75	0.31
1958	5222.65	2893.04	577.80	1148.43	0.33
1959	5683.32	2729.89	859.56	1595.84	0.41
1960	5614.64	2858.19	371.18	1844.62	0.47
1961	4199.66	2811.36	192.19	753.07	0.34
1962	4004.53	2922.30	54.88	565.67	0.33
1963	4382.73	3000.00	295.91	646.88	0.32
1964	5134.18	3141.24	459.96	776.97	0.32
1965	5927.81	3286.70	678.62	917.58	0.32
1966	6563.60	3587.84	871.37	1130.74	0.34
1967	6189.46	3773.55	589.37	895.98	0.28
1968	5935.58	3708.58	876.18	612.63	0.25
1969	6938.42	4037.59	633.88	1105.83	0.32
1970	8285.03	4438.40	1239.02	1500.59	0.34
1971	8865.18	4610.89	1397.92	1594.41	0.36
1972	9200.22	4874.68	1237.34	1653.06	0.36
1973	9926.67	5226.19	1481.37	1814.87	0.37
1974	10156.17	5340.37	1607.74	1799.98	0.36
1975	11039.78	5629.83	1997.31	1915.39	0.36
1976	10862.36	5861.62	1778.13	1875.37	0.36
1977	11690.03	6016.06	2033.12	1976.00	0.37
1978	13055.95	6300.50	2170.35	2764.83	0.41
1979	14047.65	6955.39	2212.69	2901.07	0.39
1980	15147.02	7768.08	2663.10	2667.46	0.38
1981	15933.42	8559.93	2956.79	2353.31	0.36
1982	17379.53	9477.31	2425.11	3399.90	0.38

Appendix 4A: (continue): Real Data (1983-2012)

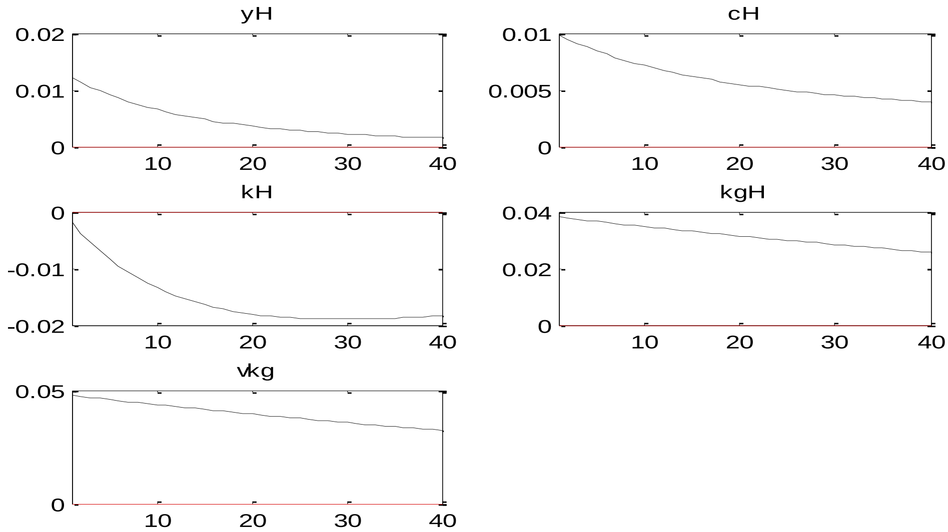
	GDP (100 million Yuan)	Nongovernment Consumption (100 million Yuan)	Nongovernment Investment (100 million Yuan)	Government Investment (100 million Yuan)	Tax Rates
1983	19277.89	10446.49	2812.19	3780.12	0.39
1984	22205.95	11528.03	3381.21	4367.10	0.39
1985	25205.59	13104.28	4916.61	4749.32	0.39
1986	27422.42	14150.25	5888.66	4631.49	0.38
1987	30605.63	15548.48	6518.02	4806.85	0.35
1988	34064.36	17817.26	8212.25	4695.81	0.31
1989	35459.77	18390.23	8715.88	4499.27	0.31
1990	36805.64	18633.48	8779.73	4522.71	0.30
1991	40194.69	19801.81	9937.37	4581.92	0.29
1992	45897.51	22161.78	12154.53	5039.98	0.27
1993	52323.29	24303.42	20128.59	3146.54	0.16
1994	59182.05	26822.45	21295.33	3681.46	0.15
1995	65630.71	30626.90	23446.85	4049.75	0.14
1996	72194.53	34441.53	24405.00	4791.57	0.16
1997	78909.90	36891.99	25798.00	4146.04	0.15
1998	85065.79	39537.69	26950.93	4609.44	0.15
1999	91525.87	42784.65	27914.08	5716.76	0.17
2000	99214.55	45854.60	28668.24	6174.56	0.17
2001	107452.40	48442.78	32285.00	6685.51	0.19
2002	117226.20	51686.87	37634.00	6754.70	0.19
2003	128949.74	54732.56	46312.63	6818.48	0.19
2004	141975.26	57915.35	54122.97	7300.00	0.19
2005	158012.11	62336.56	58264.80	8256.75	0.20
2006	178080.54	67980.12	67335.95	9188.36	0.21
2007	203374.38	73705.05	73950.23	10933.67	0.22
2008	222901.15	79260.70	83867.19	14312.45	0.22
2009	243397.69	88236.91	96675.93	20747.47	0.22
2010	268714.23	94203.35	107513.19	22057.09	0.22
2011	293707.51	104889.89	120040.23	21718.07	0.22
2012	316711.44	116195.95	128529.96	25581.27	0.23

Appendix 4B: IRFs (Test, r=1, 1952-2012)

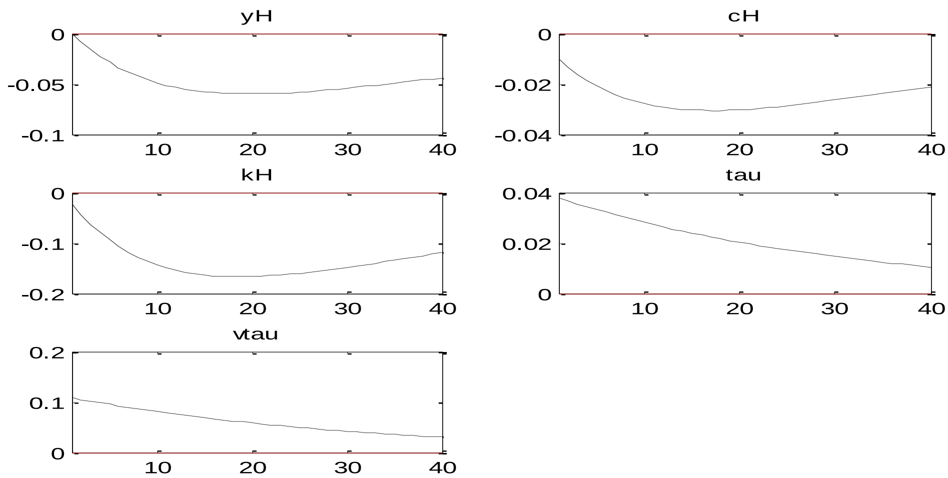
To e^{y} :



To e^{KG}:

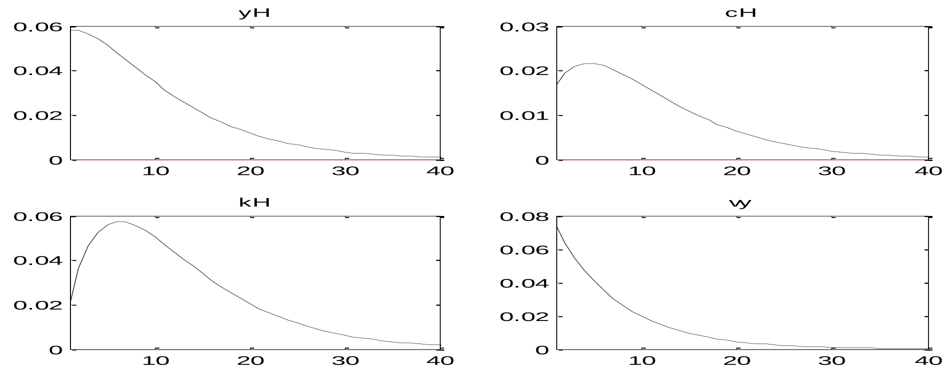


To e^{τ}:

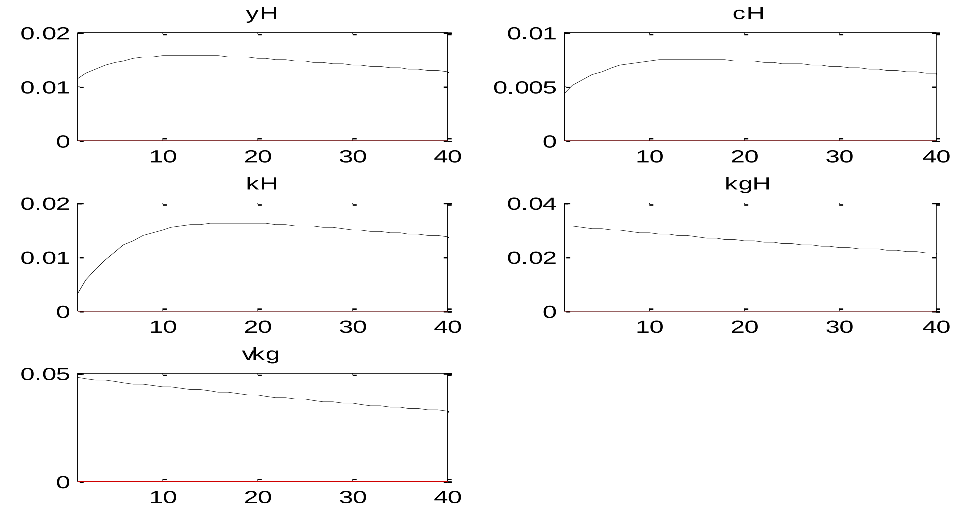


Appendix 4C: IRFs (Test, r=0, 1952-2012)

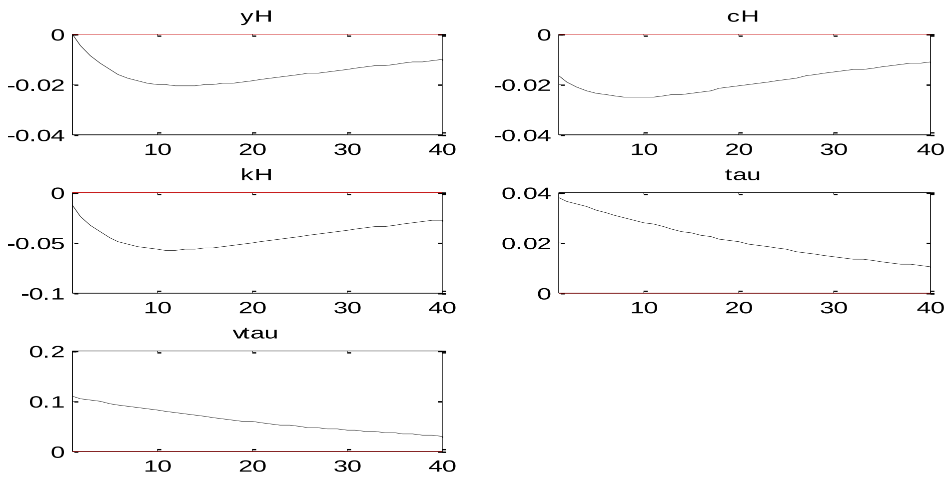
To e^{y} :



To e^{KG}:

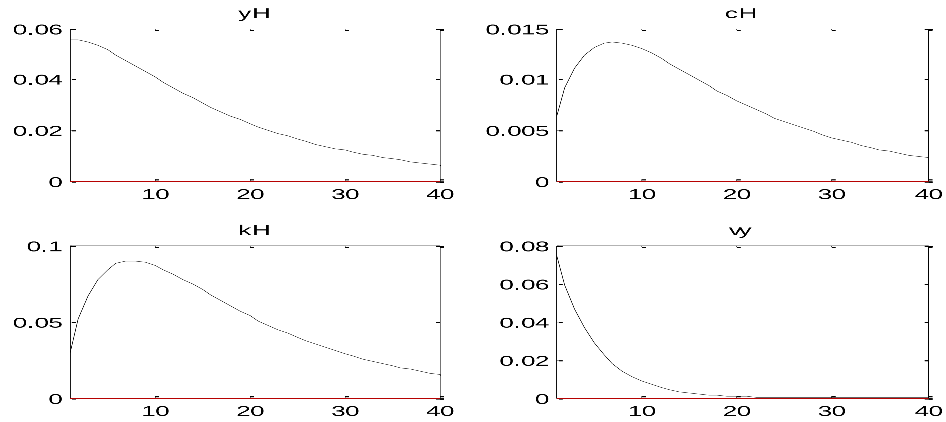


To e^{τ}:

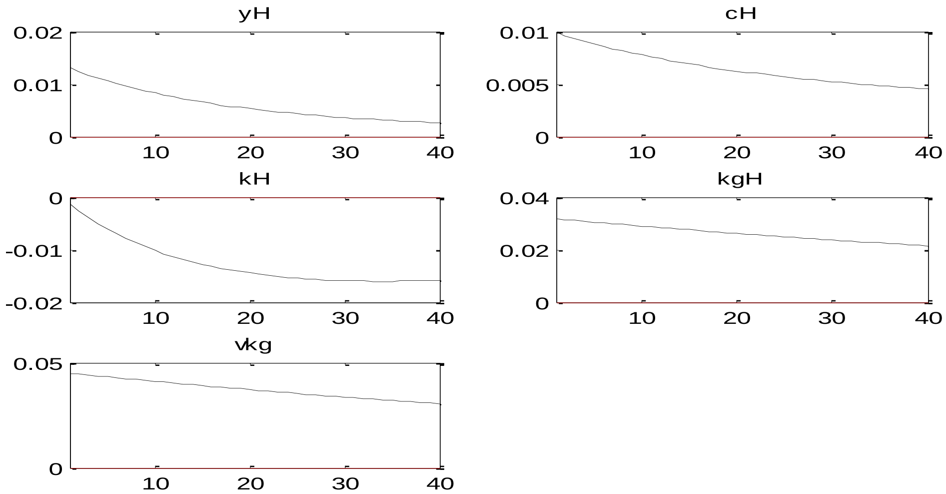


Appendix 4D: IRFs (Estimation, 1952-2012)

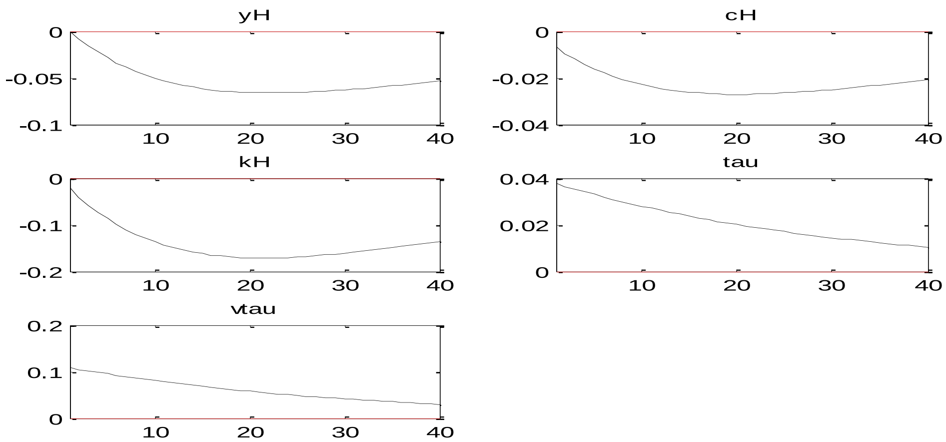
To e^{y} :



To e^{KG}:

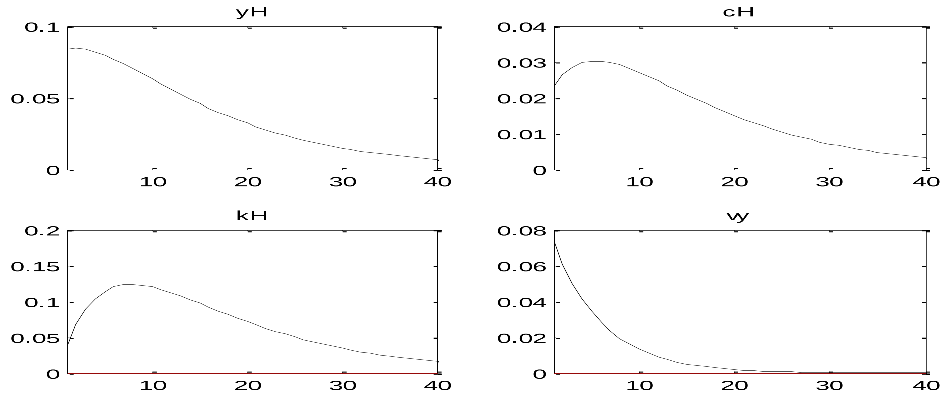


To e^{τ}:

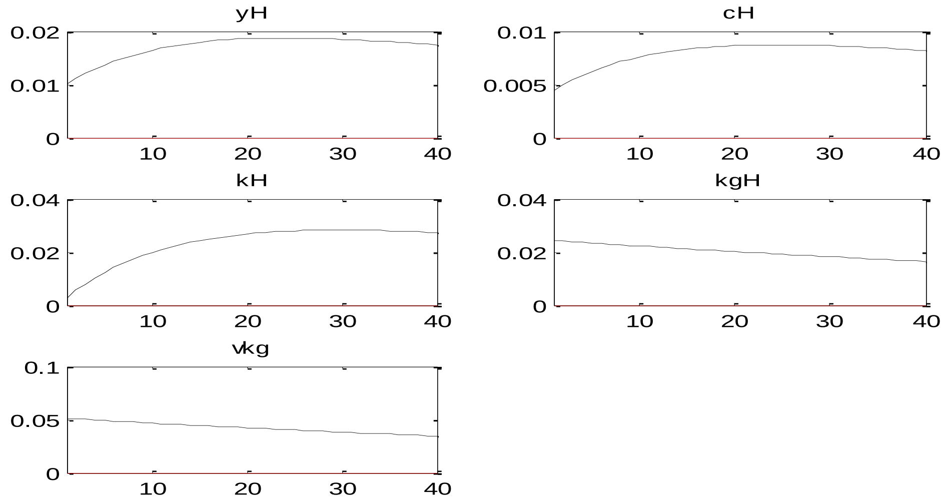


Appendix 4E: IRFs (Estimation, r=0, 1952-2012)

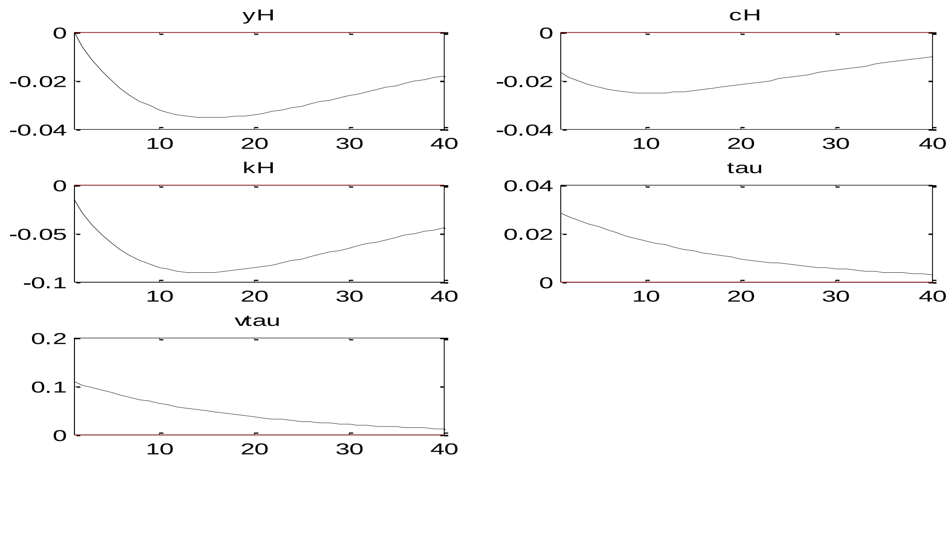
To e^{y} :



To e^{KG}:

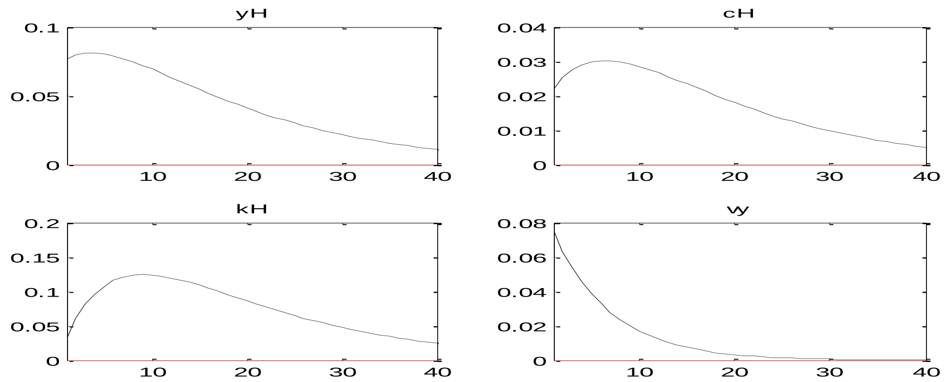


To e^{τ}:

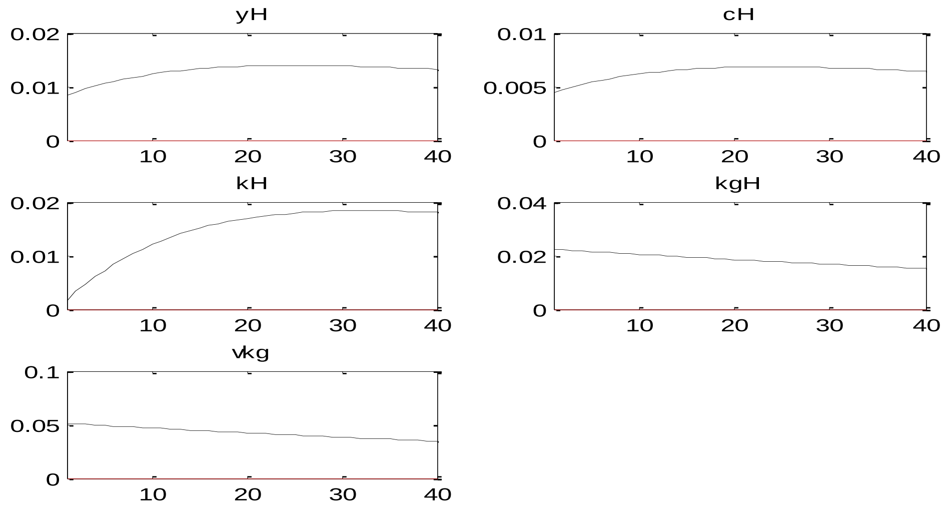


Appendix 4F: IRFs (Estimation, r=0.25, 1952-2012)

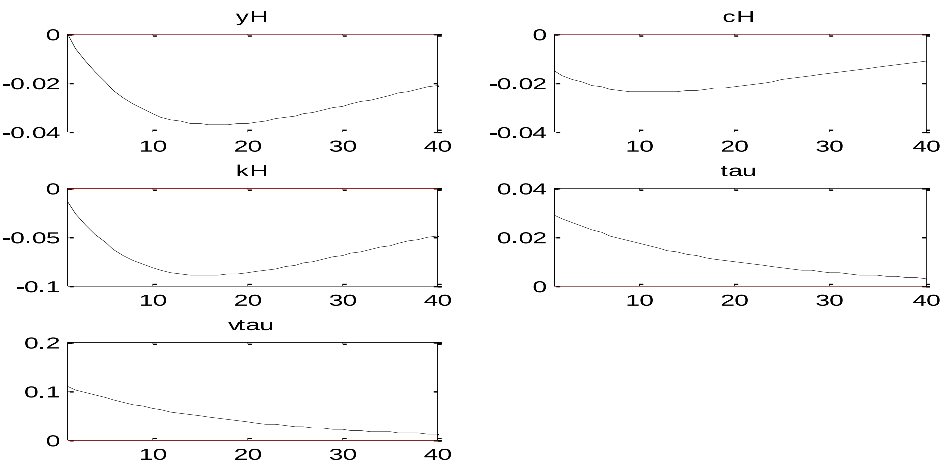
To e^y :



To e^{KG}:

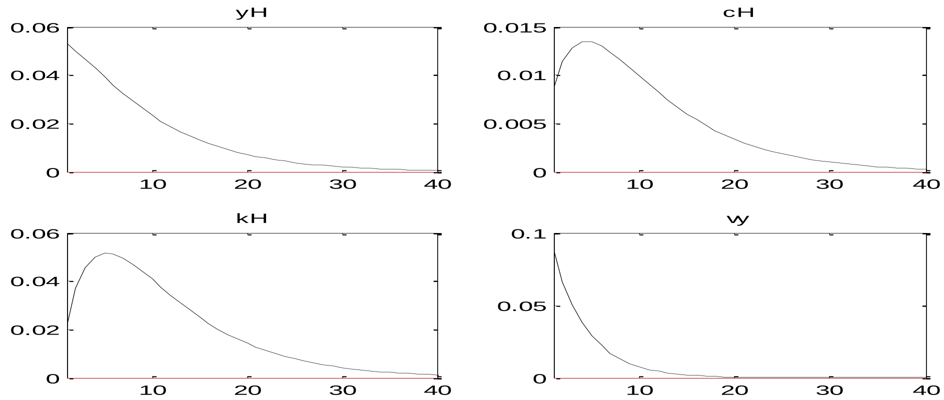


To e^{τ}:

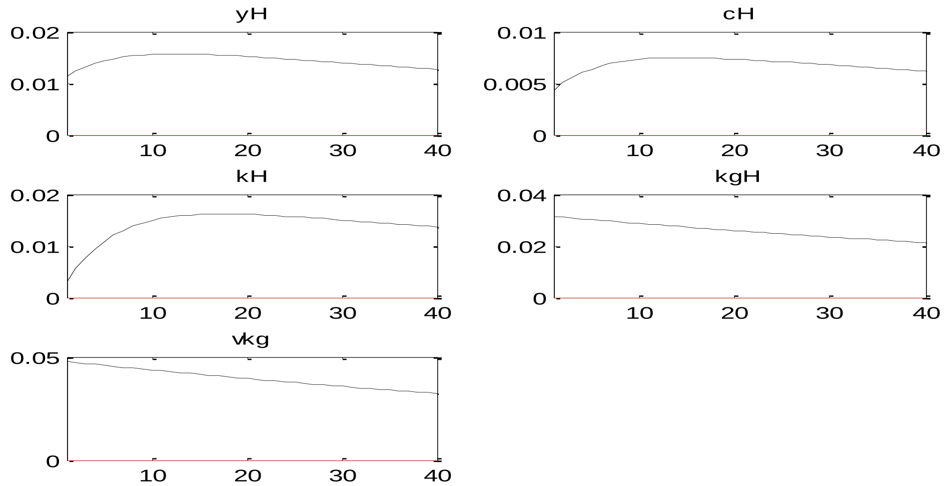


Appendix 4G: IRFs (Estimation, 1952-1992)

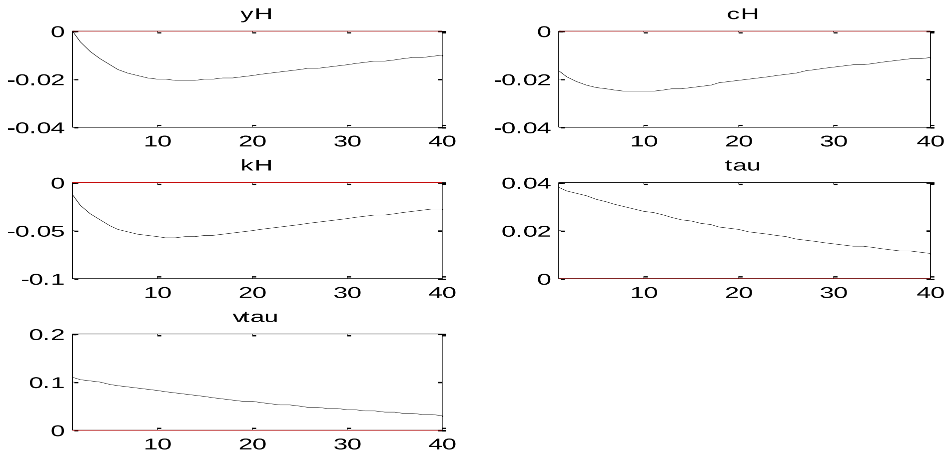
To e^{y} :



To e^{KG}:

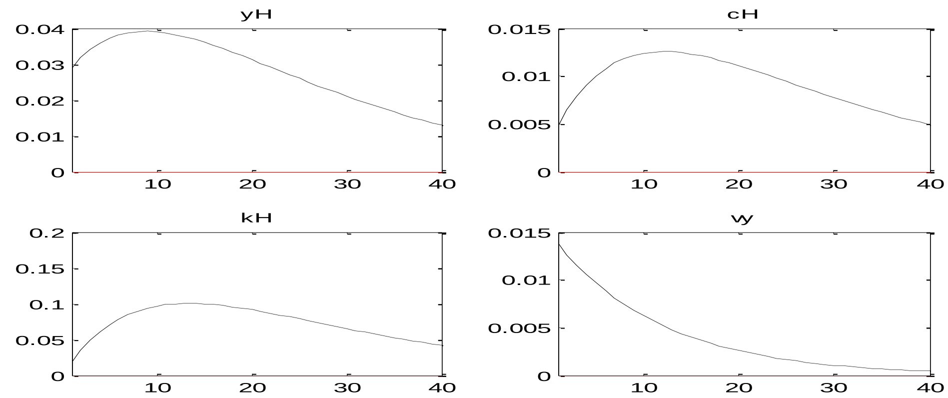


To e^{τ}:

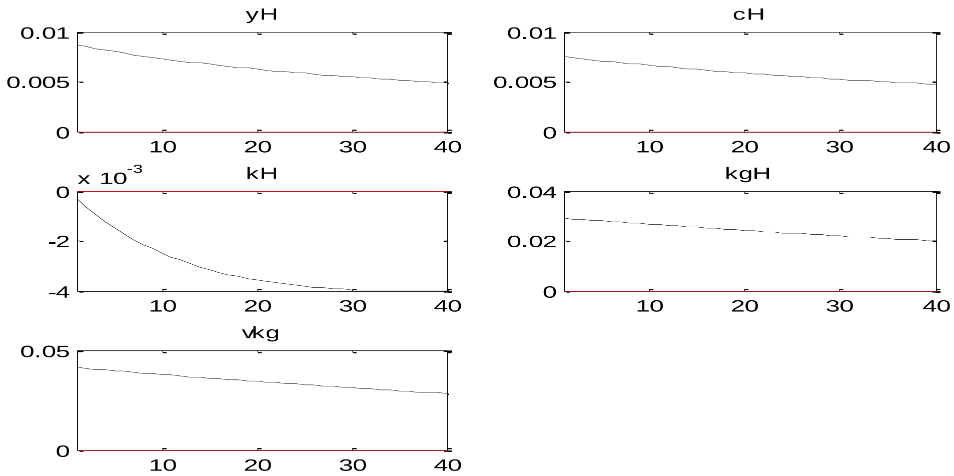


Appendix 4H: IRFs (Estimation, 1993-2012)

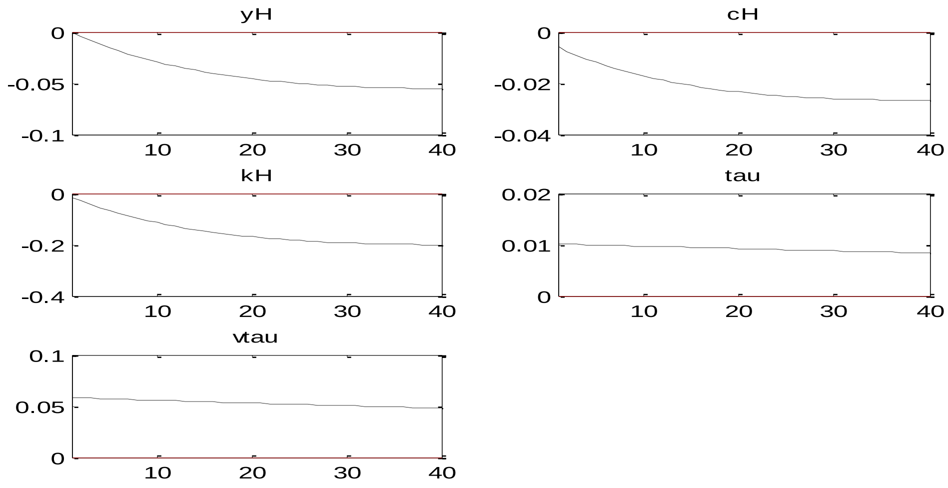
To e^{y} :



To e^{KG}:



To e^{τ}:



Chapter 5: Conclusion

5.1 Review the Main Results in Chapter 3

The strategy of a farsighted government is to sacrifice the first several generations but benefit all future generations through cutting nonproductive public spending and giving expenditure priority to productivity-enhancing expenditure, so that a higher growth rate leads to a 'quasi-Pareto improvement' among generations. The Barro model is an exception in which a farsighted government has no place to increase the growth rate because the Cobb-Douglas production function makes maximisation of welfare correspond to the maximisation of the growth.

Nevertheless, farsighted fiscal policy together with 'hedonistic citizens' has a side effect, the welfare loss for both the government and the citizens. In a decentralised economy, the inefficient resource allocation is caused by both tax distortion and the divergence of time preference between the private and public sectors. A farsighted government had better seek an accommodation on time horizon with the citizens.

If the government is benevolent, a lump sum tax can reproduce the Pareto optimal allocation in the centralised model. However, with a self-interested government, lump sum tax can not realise the Pareto optimality, which differs from Barro's conclusion.

An instrument to exploit the positive external effects and reduce tax distortion is to run SOEs if they can operate efficiently. The government, the owner of the SOEs, prefers a higher ratio of investment to output than the private sector since the social marginal productivity is higher than the private marginal return on capital.

The political discount rate will be higher than the private rate of time preference if politicians face uncertainty about next election. Therefore in western 'democratic' countries, most politicians have a shorter time horizon than households and favor quick boosts to electoral popularity. It is cheaper from an electoral viewpoint to give priority to government consumption rather than public investment. The sly politicians even use large budget deficits to finance welfare programs in the hope of wooing voters. Contrarily, the farsighted leaders in Singapore and China, the two 'quasi-nonpartisan' countries, have down-to-earth pragmatic concern with the long-term interests of their people.

5.2 Review the Main Results in Chapter 4

Chapter 4 examines the elasticity of substitution between government and nongovernment capital through testing and estimating the models with CES and Cobb-Douglas technologies. Both test and estimation results reveal the strong substitution relationship between the two types of capital, and thus the general CES technology rather than the Cobb-Douglas technology is the suitable structure containing the two types of capital in the production function.

Furthermore, the estimation results verify the difference on the orientation of public investment between the two periods (pre and post 1992), showing that the government capital has become more complementary to the nongovernment capital due to the deepening reforms of SOEs and fiscal policies.

The empirical results from indirect inference method, together with both failures and successes in China's experiences during the past six decades, give meaningful policy implications: market forces rather than government intervention and planning should dominate the economy; the government should be service-oriented and market-friendly so that the government and nongovernment sectors have a more complementary and less competitive relationship; nevertheless, when the domestic economy suffers from external crisis, the government can temporarily employ Keynesian stimulation to stabilise the macroeconomy, recover the confidence of private sector, and then reduce unemployment.

5.3 The Link Between Chapter 3 and Chapter 4

As illustrated in Chapter 3, there is a trade-off in the effects of government investment: On the one hand, economic growth is negatively affected by the rising cost of distortionary taxation necessary to finance the productive expenditure. On the other hand, the government could increase the growth rate by raising the investment ratio in output for three reasons. Firstly, the development of public infrastructures induces higher future returns to private investment and hence encourages private investment. Secondly, if the government is more farsighted than the citizens, it will allocate a high proportion of output in SOEs to reinvestment since the operations of SOEs are more or less controlled by the government. Thirdly, the government is enthusiastic over investment because it considers the external effect, as demonstrated by the centralised models in Chapter 3.

If government and nongovernment capital are perfect substitutions, the conclusions derived from the endogenous growth models in Chapter 3 would be somewhat different and both shortcomings and advantages of public investment are strengthened. In such a situation, the government investment only distorts the decisions of the nongovernment sector via the distortionary tax, but does not increase the returns to private production because there is no complementary relationship between public and private capital. This is the logically compelling argument of privatisation. However, the government can act more swiftly and more decisively to stabilise the economy through increasing public investment in the recessions and cutting investment in the economic booms, because the public investment could turn into output without being accompanied by the resources in private sector.

The part of public capital as substitutes of nongovernment capital once dominated complementary public capital in China, however there still existed infrastructure investment before the reform although its share in total public investment was rather lower than later. Moreover, empirical results do not reject somewhat complementary relation between the two types of capital. Hence all conclusions based on 'complementary public spending' in Chapter 3 are tenable even when we research China's macroeconomy.

5.4 Suggestion to Further Research

Due to the problems in availability and accuracy of data, Chapter 4 employs a simple exogenous growth model to investigate the elasticity of substitution between government and nongovernment capital. If relevant data are available, further extension should decompose the public capital into two parts as substitute and complement of nongovernment capital, and also use a CES utility function to allow for a degree of substitutability between government and nongovernment consumption. On the basis of the empirical results of Chapter 4, using a general CES production technology and a linear production function as a special case for the models in Chapter 3 should be considered. Many studies in the literature assume that public goods are subject to congestion. If congestion effects are allowed in the production function and utility function, there may be more interesting implications.

5.5 General Conclusion

When public faith in the efficacy of markets and the competence of politicians is shaken in the West, the policy makers in Beijing still believe in the power of market, however they are convinced that the market economy and an ambitious government can coexist. The current political system in the capitalist West rather than the market machanism is the key factor that seriously distorts and blights the economy.

The leaders of the Communist Party are still studying the outstanding civilised achievements in both economic and political fields from Western countries, however, they do not have blind faith in the market fundamentalism, not mention to the political models in the West. They accept a more general proposition that strategies need to be tailored to the specific circumstances of individual nations, nevertheless, they do not reject the proposals of reform under the pretext of national conditions. This is the primary hallmark of the Beijing Consensus if there really exists the Beijing Consensus as an alternative development philosophy to the currently dominant ideology.

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