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Rough set and PSO-based ANFIS approaches to modeling customer satisfaction for affective product design

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ABSTRACT

Facing fierce competition in marketplaces, companies try to determine the optimal settings of design attribute of new products from which the best customer satisfaction can be obtained. To determine the settings, customer satisfaction models relating affective responses of customers to design attributes have to be first developed. Adaptive neuro-fuzzy inference systems (ANFIS) was attempted in previous research and shown to be an effective approach to address the fuzziness of survey data and nonlinearity in modeling customer satisfaction for affective design. However, ANFIS is incapable of modeling the relationships that involve a number of inputs which may cause the failure of the training process of ANFIS and lead to the 'out of memory' error. To overcome the limitation, in this paper, rough set (RS) and particle swarm optimization (PSO) based-ANFIS approaches are proposed to model customer satisfaction for affective design and further improve the modeling accuracy. In the approaches, the RS theory is adopted to extract significant design attributes as the inputs of ANFIS and PSO is employed to determine the parameter settings of an ANFIS from which explicit customer satisfaction models with better modeling accuracy can be generated. A case study of affective design of mobile phones is used to illustrate the proposed approaches. The modeling results based on the proposed approaches are compared with those based on ANFIS, fuzzy least-squares regression (FLSR), fuzzy regression (FR), and genetic programming-based fuzzy regression (GP-FR). Results of the training and validation tests show that the proposed approaches perform better than the others in terms of training and validation errors.

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1. Introduction

Affective design has been shown to excite psychological feelings of customers and can help improve the emotional aspects of customer satisfaction. It is an important design strategy to enhance customer satisfaction of new products in customer-driven product development. Design attributes, such as shape and color, evoke the affective responses of customers to products. Products with good affective design can help attract customers and influence their choices and preferences, such as loyalty and joy of use [1,2]. The process of affective design includes identifying, measuring, analyzing, and understanding the relationship between the affective needs of the customer domain and the perceptual design attributes in the design domain [3]. One of the major processes of affective design is to determine the design attributes settings of new products such that high, or even optimal, customer affective satisfaction

of the new products can be obtained. To determine the design attribute settings, customer satisfaction models that relate affective responses of customers to design attributes have to be developed first. However, the modeling process is quite complex as the relationships to be modeled can be highly nonlinear and fuzzy. Modeling customer satisfaction for affective product design has been applied in the industry for various product designs, such as the design of vehicle interior [4], office chairs [5], mobile phones [6], and digital camera [7].

A handful of studies previously attempted to model the relationships between affective responses and design attributes using statistical and artificial intelligence methods. Artificial neural network (ANN) was proposed to model the affective relationship in product design [8,9]. An interactive evolutionary system based on neural networks was proposed to analyze the aesthetic perceptions of customers and approximate their aesthetic intentions [10]. Chen et al. developed a prototype system for affective design in which Kohonen's self-organizing map neural network was employed to consolidate the relationships between design

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attributes and affective dimensions [11]. The main advantage of the ANN is the development of models through learning from data without requiring prior knowledge. Although a trained ANN can possibly provide an accurate prediction or classification, it is known as a 'black box' model from which no explicit knowledge of the relationships can be obtained [12].

Multiple linear regression has been used to model affective relationships [13]. The approach is easy to apply, but it assumes that the design attributes in the regression are linear, and the effect of an independent design attribute is the same throughout the entire range of the affective response. A decision support system has been proposed to provide guidelines for optimizing affective satisfaction based on principal component analysis and multiple regression [14]. Petiot and Grognet [15] proposed an explicit modeling method based on a vector field to model affective relationships. You et al. [16] developed the customer satisfaction models for automotive interior material using quantification I analysis. Based on the models, the significance of the design attributes can be identified. Han et al. [17] attempted to evaluate product usability based on statistical regression models that relate usability dimensions and design attributes. However, the above statistical approaches are unable to address the fuzziness involved in the affective responses of customers.

To address the fuzziness of affective modeling, Park and Han proposed a fuzzy rule-based approach to examine customer satisfaction towards office chair designs [18]. They reported that the fuzzy rule-based approach outperformed the multiple linear regression approaches in terms of the number of design attributes to be considered in modeling. A fuzzy expert system with gradient descent optimization was proposed to develop models that relate affective responses to design attributes in fashion product development [19]. Shimizu and Jindo [4] applied a fuzzy regression method to model the relationship between design attributes and affective responses to address the fuzziness of human sensations towards vehicle interior design. Tanaka's fuzzy regression approach was proposed to model customer satisfaction for improving the design of driver seat [20]. However, the fuzzy regression approach is unable to capture nonlinearity of the modeling. Chan et al. introduced genetic programming into fuzzy regression for modeling affective relationships [6]. An evolutionary algorithm was used to construct branches of a tree representing the structures of a model where the nonlinearity of the model could be addressed and the fuzzy regression was then used to determine the fuzzy coefficients of the model. The limitation of this approach is that the size of the search space increases exponentially with the number of nodes and the tree depth.

The hybrid approaches of fuzzy logic and ANN combine the capability of fuzzy logic in the linguistic representation of knowledge and the adaptive learning capability of ANN for automatic generation and optimization of a fuzzy inference system. Fuzzy neural networks have been introduced to establish the relationships between design attributes and consumer affections [21]. Fuzzy neural networks utilize a series of output nodes of the ANN to emulate a fuzzy membership grade of affection intensity and then determine the aggregate value of customer affection through defuzzification. Hsiao and Tsai [22] proposed a method that enables an automatic product form or product image evaluation by means of a neural network-based fuzzy reasoning and genetic algorithm, which was applied to establish relationships between the design attributes of a new product and the customers' affective image. An adaptive neuro-fuzzy inference system (ANFIS) was examined by Kwong and Wong [23] to generate explicit customer satisfaction models which can capture the nonlinearity and fuzziness existing in the modeling. Compared with ANN, a set of fuzzy if-then rules with appropriate membership functions and the internal models can be generated based on ANFIS to

stipulate input-output pairs explicitly. However, the conventional learning algorithms for ANFIS are gradient descent, in which the calculation of gradients in each step is difficult and the use of chain rules may cause a local minimum. These issues have been shown to affect modeling accuracy. On the other hand, ANFIS is not suitable for the modeling problems that involve a number of inputs. If the number of inputs is large, the number of generated fuzzy rules increases exponentially. These increases would cause long computational time and even execution errors. To overcome the limitation and further improve modeling accuracy of ANFIS, in this paper, rough set (RS) and particle swarm optimization (PSO)-based ANFIS approaches are proposed to modeling customer satisfaction for affective design.

The organization of this paper is as follows: Section 2 describes how the proposed approaches are used to model customer satisfaction for affective design. In Section 3, a case study of mobile phone design is described to illustrate the proposed approaches. The validation of the proposed approaches is shown in Section 4. Finally, conclusions are given in Section 5.

2. Modeling customer satisfaction using RS and PSO-based ANFIS approaches

To address the deficiency of ANFIS for modeling affective relationships, RS and PSO-based ANFIS approaches are proposed in this research. Since ANFIS is incapable for application in those modeling problems that involve a number of attributes, in the proposed approaches, RS theory is introduced to reduce the number of inputs and determine indispensable design attributes for generating customer satisfaction models. The PSO-based ANFIS approach is introduced to develop nonlinear customer satisfaction models, in which PSO is used to determine the optimal values of antecedent parameters in membership functions, such that the errors between the predictive customer satisfaction values and the actual customer satisfaction values can be minimized. Fig. 1 shows a flowchart of the proposed approaches to modeling customer satisfaction for affective design.

2.1. ANFIS structure

ANFIS is a multilayer feed-forward network in which the neural network is regarded as a learning algorithm and fuzzy reasoning is used to map inputs into an output [24]. It is a fuzzy inference

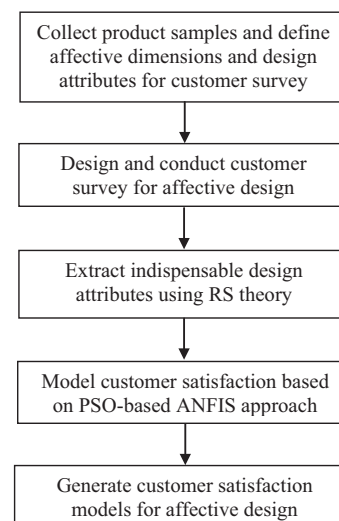


Fig. 1. The flowchart of the proposed approaches.

system implemented in the framework of adaptive neural networks. Fig. 2 shows the architecture of a typical ANFIS with two inputs and one output. To facilitate an illustration of the mathematical aspect of ANFIS, each input of ANFIS is assumed to have two linguistic descriptions. In fact, if more linguistic descriptions are involved, the process of ANFIS is still the same but the ANFIS structure would be more complex as the numbers of nodes in the layers 1 to 3 increase correspondingly.

If both inputs, x_1 and x_2 , have two linguistic descriptions (e.g., low and high), a membership function is used to represent each description. Hence, $\mu_i(x_1)$ denotes the membership function for the i th linguistic description of x_1 , and $\lambda_j(x_2)$ denotes the membership function of the j th linguistic description of x_2 , where $i = 1, 2$ and $j = 1, 2$. Thus, four membership functions are available for all inputs as defined by the four nodes in Layer 1 (L1). Different types of membership functions such as triangular, trapezoidal, Gaussian, bell-shaped, sigmoidal and polynomial based membership function with symmetrical shape and equal spread were compared in previous studies and the results indicated that triangular membership function could perform more effectively and provided better accuracy than the other membership functions in a fuzzy system [25]. Triangular-shaped membership functions have consistency property and are easier to perform fuzzy arithmetic [26]. Therefore, in this research, triangular-shaped membership functions are adopted and defined below.

$$\mu_i(x_1) = \begin{cases} \frac{x_1 - a_i}{b_i - a_i} & a_i \leq x_1 \leq b_i \\ \frac{c_i - x_1}{c_i - b_i} & b_i \leq x_1 \leq c_i \\ 0 & \text{Otherwise} \end{cases} \quad \text{and} \quad \lambda_j(x_2) = \begin{cases} \frac{x_2 - s_j}{t_j - s_j} & s_j \leq x_2 \leq t_j \\ \frac{u_j - x_2}{u_j - t_j} & t_j \leq x_2 \leq u_j \\ 0 & \text{Otherwise} \end{cases} \quad (1)$$

where (a_i, b_i, c_i) and (s_j, t_j, u_j) are triangular fuzzy numbers. The parameters in this layer are referred to as antecedent parameters.

At L2, one rule is used to denote the outcome for each combination of x_1 and x_2 . Hence, the total number of rules is $2 \times 2 = 4$. The fuzzy rules can be generally expressed as follows:

$$R_{ij} : \text{IF } x_1 \text{ is } \mu_i \text{ AND } x_2 \text{ is } \lambda_j, \text{ THEN } f_{ij} = p_{ij}x_1 + q_{ij}x_2 + r_{ij} \quad (2)$$

where p_{ij} , q_{ij} , and r_{ij} are the parameters of the internal models f_{ij} of the fuzzy rules R_{ij} and they are consequent parameters. The outputs of this layer are described as follows:

$$w_{ij} = \mu_i(x_1) \cdot \lambda_j(x_2) \quad (\forall i = 1, 2, j = 1, 2) \quad (3)$$

where w_{ij} represents the firing strength of each fuzzy rule. The firing strength indicates the degree to which R_{ij} is satisfied. The connection weight between L2 and L3 is \bar{w}_{ij} as defined by (4), which is

the normalized firing strength. The larger the value of \bar{w}_{ij} implies that R_{ij} is more significant.

$$\bar{w}_{ij} = \frac{w_{ij}}{W} \quad \text{where } W = \sum_i \sum_j w_{ij} \quad (\forall i = 1, 2, j = 1, 2) \quad (4)$$

At L3, the internal model of R_{ij} is a first-order Takagi–Sugeno fuzzy model [27] as defined by (5).

$$f_{ij} = p_{ij}x_1 + q_{ij}x_2 + r_{ij} \quad (\forall i = 1, 2, j = 1, 2) \quad (5)$$

At L4, a single node is used to compute the overall output as the summation of all incoming signals. The mathematical formulation of the node is defined by (6).

$$y = \sum_{i=1}^2 \sum_{j=1}^2 \bar{w}_{ij} \cdot f_{ij} = \sum_{i=1}^2 \sum_{j=1}^2 \bar{w}_{ij} \cdot (p_{ij}x_1 + q_{ij}x_2 + r_{ij}) \quad (6)$$

From (6), explicit models can be generated by combining of all the normalized firing strengths and the corresponding internal models of all the fuzzy rules. The learning algorithm of an ANFIS is to determine the parameters (a_i, b_i, c_i) , (s_j, t_j, u_j) , and (p_{ij}, q_{ij}, r_{ij}) , such that the error between the ANFIS output and the training data can be minimized.

2.2. Determination of inputs for ANFIS using RS theory

Attribute reduction is a process of finding an optimal subset of all attributes following certain criteria so that the attribute subset is sufficient to represent the classification relation of data. A proper choice of attribute subsets can reduce the input number of ANFIS, thus simplify its structure, and shorten computational time. The RS theory was proposed by Pawlak [28], which is based on equivalence relations or indiscernibility in the classification of objects. The approximation space of a RS is the classification of the domain of interest into disjoint categories [29]. RS theory handles inconsistent information using two approximations, the upper and lower approximations. The upper and lower approximations represent the indiscernible object classifications that possess sharp descriptions on concepts but with no sharp boundaries.

A design table with 4-tuple can be expressed as $S = (U, Q, V, \rho)$, where U is the universe that is a finite and non-empty set of object; Q is a finite set of attributes; $V = \cup_{q \in Q} V_q$, where V_q is a domain of the attribute q ; The information function is $\rho : U \times Q \rightarrow V$, such that $\rho(s, q) \in V_q$ for every $q \in Q, s \in U$, and $\exists(q, v)$, where $q \in Q$ and $v \in V_q$ are descriptions of S .

Assuming a subset of the set of attributes, $R \subseteq Q$, two objects, $x, y \in U$, are indiscernible with respect to R if and only if $\rho(x, r) = \rho(y, r)$ for $\exists r \in R$. The indiscernibility relation, which is the equivalence relation defined on set U , is written as $\text{ind}(R)$. $\text{ind}(R)$ partitions the universe U into disjoint subsets, and $U/\text{ind}(R)$ is used to denote these partitions of U . The lower and upper approximation of a set $Y \subseteq U$ can be defined as follows:

$$\underline{R}Y = \cup \{X : X \in U/\text{ind}(R), X \subseteq Y\} \quad (7)$$

$$\overline{R}Y = \cup \{X : X \in U/\text{ind}(R), X \cap Y \neq \emptyset\} \quad (8)$$

where $\underline{R}Y$ consists of all objects in U that certainly belong to Y , and $\overline{R}Y$ consists of all objects in U that possibly belong to Y under the equivalent relation R .

Elements belonging only to the upper approximation compose the boundary region (BN) or the doubtful area. It represents the area which cannot be certainly classified into Y or to its complement. Mathematically, a boundary region can be expressed as follows:

$$BN(Y) = \overline{R}Y - \underline{R}Y \quad (9)$$

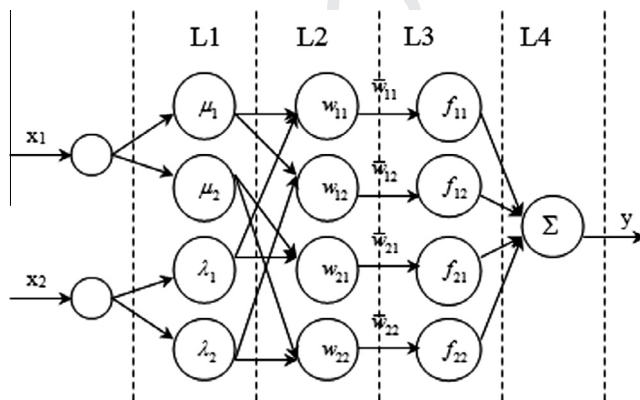


Fig. 2. An ANFIS with four layers and two inputs.

The positive region $Pos_R(Y)$ and the negative region $Neg_R(Y)$ of Y on R are defined by (10) and (11), respectively.

$$Pos_R(Y) = \underline{RY} \quad (10)$$

$$Neg_R(Y) = U - Pos_R(Y) \quad (11)$$

Based on the above definitions, attribute reduction is defined as follows:

If R is a set of equivalent relation, $r \in R$, and $Pos_R(Y) \neq Pos_{R-\{r\}}(Y)$, namely, $ind(R) \neq ind(R - \{r\})$, R is the independent attribute and r is the indispensable attribute in R , otherwise r is dispensable.

If R is independent, $R \subseteq P$ and $ind(R) = ind(P)$, R is a reduction of P , $R \in RED(P)$. $RED(P)$ represents the set of all the attribute reductions of P . The intersection of $RED(P)$ is the core of P , which is expressed as $Core(P)$.

The number of each design attribute appearing in the attribute reductions reflects the importance of each design attribute. A larger number implies that the corresponding design attribute is more important. Based on the numbers, ranking of the design attributes can be performed and the top ranking attributes are selected as the inputs of the PSO-based ANFIS.

2.3. Determination of parameters for ANFIS using PSO and LSE

The learning algorithm of an ANFIS aims to determine the antecedent and consequent parameters such that the error between the ANFIS output and the actual output can be minimized. Jiang proposed a hybrid learning algorithm which is composed of a forward pass and a backward pass to complete training and updating in an adaptive network [30]. Referring to the ANFIS structure (Fig. 2), given the values of antecedent parameters, the overall output can be expressed as a linear combination of the consequent parameters as follows:

$$\begin{aligned} y &= \sum_{i=1}^2 \sum_{j=1}^2 \bar{w}_{ij} (p_{ij}x_1 + q_{ij}x_2 + r_{ij}) \\ &= \bar{w}_{11}(p_{11}x_1 + q_{11}x_2 + r_{11}) + \bar{w}_{12}(p_{12}x_1 + q_{12}x_2 + r_{12}) \\ &\quad + \bar{w}_{21}(p_{21}x_1 + q_{21}x_2 + r_{21}) + \bar{w}_{22}(p_{22}x_1 + q_{22}x_2 + r_{22}) \\ &= (\bar{w}_{11}x_1)p_{11} + (\bar{w}_{11}x_2)q_{11} + (\bar{w}_{11})r_{11} + (\bar{w}_{12}x_1)p_{12} + (\bar{w}_{12}x_2)q_{12} + (\bar{w}_{12})r_{12} \\ &\quad + (\bar{w}_{21}x_1)p_{21} + (\bar{w}_{21}x_2)q_{21} + (\bar{w}_{21})r_{21} + (\bar{w}_{22}x_1)p_{22} \\ &\quad + (\bar{w}_{22}x_2)q_{22} + (\bar{w}_{22})r_{22} = A\theta \end{aligned} \quad (12)$$

where $A = (\bar{w}_{11}x_1, \bar{w}_{11}x_2, \bar{w}_{11}, \bar{w}_{12}x_1, \bar{w}_{12}x_2, \bar{w}_{12}, \bar{w}_{21}x_1, \bar{w}_{21}x_2, \bar{w}_{21}, \bar{w}_{22}x_1, \bar{w}_{22}x_2, \bar{w}_{22})$ and θ is a vector of the consequent parameters $(p_{11}, q_{11}, r_{11}, p_{12}, q_{12}, r_{12}, p_{21}, q_{21}, r_{21}, p_{22}, q_{22}, r_{22})$. The number of the consequent parameters of (12) is 12. If there are t training data sets, the dimensions of A , θ , and y are $t \times 12$, 12×1 , and $t \times 1$, respectively.

In the forward pass, the antecedent parameters are fixed, and the input signals go forward to calculate each node output until matrix A in (12) is obtained. The consequent parameters are then determined using the least square estimation (LSE) method. An LSE value of θ , $\hat{\theta}$, aims at minimizing the squared error $\|A\theta - y\|^2$, which is calculated based on the following formulations.

$$\hat{\theta}_{i+1} = \hat{\theta}_i + \frac{S_{i+1}a_{i+1}(b_{i+1}^T - a_{i+1}^T\hat{\theta}_i)}{1 + a_{i+1}^T S_{i+1} a_{i+1}} \quad (13)$$

$$S_{i+1} = S_i - \frac{S_i a_{i+1} a_{i+1}^T S_i}{1 + a_{i+1}^T S_i a_{i+1}} \quad (14)$$

where a_i^T is the i th row vector of matrix A , b_i^T is the i th element of y , $i = 1, \dots, t$; S_i is the covariance matrix and $S_0 = \gamma I$; γ is a positive large number; and I is the identity matrix with a 12×12 dimension.

The predictive output \hat{y} of ANFIS is obtained based on the identified value of $\hat{\theta}$.

$$\hat{y} = A\hat{\theta} \quad (15)$$

In the backward pass, the error rates propagate backward, and the antecedent parameters are updated. The conventional algorithm for updating the antecedent parameters is the gradient descent method. However, it is very difficult to determine the best learning rate in the gradient descent method, and the convergence of antecedent parameters based on the method is slow. In this study, a PSO algorithm is introduced to determine and update the antecedent parameters. PSO has a high degree of stability and has been demonstrated to have fast convergence. It does not rely on the derivative nature of objective function and can achieve global optimization by comparing objective function values time after time.

PSO is a popular search algorithm based on the social behavior of a bird flock [31]. In PSO, every potential solution of the optimization problem can be imagined as being a point in a D -dimensional search space. This point is called a 'particle'. Particles fly in search space with a certain speed, which is dynamically adjusted according to its own and its companions' flight experience. Every particle has a fitness value determined by the objective function and knows its current position and its own current best position, p_{best} . The p_{best} can be seen as the particle's own flying experience. In addition, every particle also knows the global best position g_{best} , which has the best value in p_{best} . The g_{best} can be seen as its companions' flying experience for the particle. Every particle uses the following information to change their current location: (1) the current location; (2) the current speed; (3) the distance between the current location and its own best location; and (4) the distance between the current location and the global best location. The optimization search is achieved by the iteration of a particle swarm which is formed by a group of random initialized particles.

A swarm is composed of m particles flying in the D -dimension in a certain speed. Every particle changes its position based on considering its own historical best position and other particles' historical best position. The position for the i th particle is $x_i = (x_{i1}, x_{i2}, \dots, x_{id})$, where $1 \leq i \leq m$ and $1 \leq d \leq D$. D is the dimension of the search space as well as the number of antecedent parameters. The speed for the i th particle is $v_i = (v_{i1}, v_{i2}, \dots, v_{id})$. The historical best position of the i th particle, which has the minimum fitness value, is $p_i = (p_{i1}, p_{i2}, \dots, p_{id})$. The best position g_{best} for the whole swarm is $p_g = (p_{g1}, p_{g2}, \dots, p_{gd})$, $j \in \{1, 2, \dots, m\}$. The final result of p_g denotes the optimal values of the antecedent parameters. The process of updating the speed and the position of the particle based on the idea of inertia weight [32] is expressed as follows:

$$v_{id}^{k+1} = \omega v_{id}^k + c_1 r_1 (p_{id}^k - x_{id}^k) + c_2 r_2 (p_{gd}^k - x_{id}^k) \quad (16)$$

$$x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \quad (17)$$

where v_{id}^k and x_{id}^k are the speed vector and the position vector of the i th particle at the k th iteration, respectively; k is the number of iterations; ω is the inertia weight, the value of which decides the quantity inherited from the current speed of the particle; c_1 and c_2 are learning factors and are usually set as 2; The values of r_1 and r_2 are randomly chosen from the range $[0, 1]$.

2.4. Proposed RS and PSO-based ANFIS approaches to modeling customer satisfaction for affective design

The processes of modeling customer satisfaction for affective design based on the proposed approaches are shown as follows:

Step 1: A customer survey is designed and conducted to obtain affective responses of customers on products.

Step 2: Once survey data is obtained, the values of the affective responses are discretized and used as the outputs. Based on the survey data, RS theory is introduced to identify redundant attributes and generate a list of attribute reductions.

Step 3: Based on the list of the attribute reductions, the number of each design attribute appearing in the list is calculated. The ranking for all the design attributes is obtained based on the number and the important design attributes are then selected as the inputs of PSO-based ANFIS.

Step 4: Using the extracted design attributes as the inputs, the ANFIS is trained by the hybrid learning algorithm of PSO and LSE. The initialization for a particle swarm is first conducted, including iteration number, swarm size, dimension of search space, search range and learning factors. The speed and position of each particle are initialized randomly.

Step 5: In the first iteration, the initial position of every particle is used as the initial individual best position p_b , and the position vector of each particle is used as the antecedent parameters of ANFIS in sequence. The initial iteration is followed by calculating the values of membership functions μ_i and λ_j , the firing strength w_{ij} , and the normalized firing strength \bar{w}_{ij} using (1), (3), and (4), respectively. Based on the input data sets and the initial values of the consequent parameters, the values of the fuzzy rule f_{ij} are determined based on (5). Therefore, the outputs of all nodes reach L4. The final output y is then obtained using (6). LSE is used to identify the consequent parameters $\hat{\theta}$ using (13) and (14). The identified $\hat{\theta}$ and the matrix A in (12) are then used to compute for the value of the predictive output \hat{y} based on (15). Next, the mean absolute percentage error (ME) between the model output \hat{y} and the actual value for the i th particle is calculated, which is also the fitness value ME_i^1 of the i th particle in the first iteration. ME_i^1 is recorded as the initial individual best fitness value p_{best} . The particle which has the smallest value in ME_i^1 is selected as the best particle. The particle's position vector is defined as the initial global best position p_g , and its fitness value is defined as the initial global best fitness value g_{best} .

Step 6: The iteration is continued by $n + 1 \rightarrow n$. In each iteration, the speed vector v_{id}^{n+1} and the position vector x_{id}^{n+1} for each particle are updated based on (16) and (17), respectively. Then, the ME_i^n of the i th particle in the n th iteration is calculated based on the updated position of particles. The current fitness value ME_i^n is compared with p_{best} for each particle. If the value of ME_i^n is smaller than p_{best} , the individual best fitness value p_{best} is set as the value of ME_i^n , and the particle's individual optimal position along with its new position $p_b = x_{id}^n$ are updated.

Step 7: The iteration stops when the pre-defined number of iterations is satisfied. The global best fitness value g_{best} is updated by selecting the smallest value in p_{best} and the number of the best particle is then recorded. The global best position p_g is decided as the position of the selected best particle. The values of p_g are the identified antecedent parameters and the values of $\hat{\theta}$ are the identified consequent parameters.

Step 8: Based on the antecedent and consequent parameters, the customer satisfaction models can be obtained using (1), (3), (4), and (6). The fuzzy rules are generated based on (2).

3. Case study

A case study of mobile phone design is used in this study to illustrate the proposed approaches to model the relationships

between affective responses and design attributes. A total of 32 mobile phones of various brands were selected. Morphological analysis was used to study the representative attributes of mobile phones as numerical data sets. Table 1 shows the nine representative design attributes: top shape, bottom shape, side shape, function button shape, number buttons style, screen size, thickness, layout, and weight, which are denoted as $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$, and x_9 , respectively. Design attributes have different numbers of form which range from 3 to 6. Four affective dimensions were used to evaluate the affective design of the mobile phones. They are simple–complex (S–C), unique–general (U–G), high-tech–classic (H–C), and handy–bulky (H–B), which are denoted as y_1, y_2, y_3 and y_4 , respectively. A survey was conducted using a questionnaire, in which a five-point scale was used to assess the mobile phone appearance corresponding to the four affective dimensions. Design profiles of the samples and the means of the affective responses of respondents to the S–C, U–G, H–C, and H–B of the samples are shown in Table 2.

3.1. Determination of inputs for PSO-based ANFIS

With the survey data, Rosetta software was employed to extract important design attributes. Rosetta is a toolkit for analyzing tabular data within the framework of RS theory and can be used to support the overall data mining and knowledge discovery process including initial browsing and preprocessing of the data, computation of minimal attribute sets, generation of descriptive patterns, and validation [33]. The previous research has shown that genetic algorithm based RS approach can obtain reducts effectively with high classification accuracy and derive larger number of reducts [34]. Therefore, in this study, the genetic reducer in Rosetta is used to conduct attributes reduction. The set of attribute reductions for S–C obtained from the software is shown in Table 3. The numbers for design attributes $x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$, and x_9 are 14, 11, 18, 13, 9, 19, 14, 10, and 17, respectively. Based on the numbers, the ranking of importance of the design attributes is $x_6 > x_3 > x_9 > x_7 = x_1 > x_4 > x_2 > x_8 > x_5$. Similarly, the ranking results of the nine design attributes for U–G, H–C, and H–B are $x_1 > x_7 > x_5 = x_4 = x_3 > x_6 = x_2 > x_9 = x_8, x_1 > x_3 > x_7 = x_5 > x_9 = x_4 > x_8 = x_6 = x_2$, and $x_9 > x_7 > x_4 > x_5 = x_1 > x_2 > x_6 = x_3 > x_8$, respectively.

In order to determine the number of inputs, the first two, three and four design attributes in the ranking were selected as inputs to model customer satisfaction. Using S–C as an example, if the number of inputs is two, the input attributes are x_3 and x_6 . The general form of w_{ij} and \bar{w}_{ij} can be expressed by (1), (3), and (4), as follows:







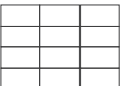
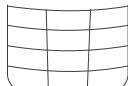


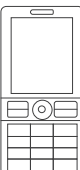
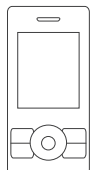

$$w_{ij} = a_{ij}x_3x_6 + b_{ij}x_3 + c_{ij}x_6 + d_{ij} \quad (18)$$

$$\bar{w}_{ij} = \frac{a_{ij}x_3x_6 + b_{ij}x_3 + c_{ij}x_6 + d_{ij}}{W = \sum_{i=1}^3 \sum_{j=1}^3 (a_{ij})x_3x_6 + \sum_{i=1}^3 \sum_{j=1}^3 (b_{ij})x_3 + \sum_{i=1}^3 \sum_{j=1}^3 (c_{ij})x_6 + \sum_{i=1}^3 \sum_{j=1}^3 (d_{ij})} \quad (19)$$

where

$$a_{ij} = \begin{cases} \frac{1}{(b_i - a_i)(t_j - s_j)} & a_i \leq x_3 \leq b_i \text{ and } s_j \leq x_6 \leq t_j \\ \frac{-1}{(b_i - a_i)(u_j - t_j)} & a_i \leq x_3 \leq b_i \text{ and } t_j \leq x_6 \leq u_j \\ \frac{-1}{(c_i - b_i)(t_j - s_j)} & b_i \leq x_3 \leq c_i \text{ and } s_j \leq x_6 \leq t_j \\ \frac{1}{(c_i - b_i)(u_j - t_j)} & b_i \leq x_3 \leq c_i \text{ and } t_j \leq x_6 \leq u_j \\ 0 & \text{otherwise} \end{cases}$$

Table 1
Morphological analysis on the 32 mobile phone samples.

Design attributes	Alt. 1	Alt. 2	Alt. 3	Alt. 4	Alt. 5	Alt. 6
Top shape (x_1)	Line and no fillet	Arc and no fillet	Line and small fillet	Arc and small fillet	Irregular	Curve
Bottom shape (x_2)	Line and no fillet	Arc and no fillet	Line and small fillet	Arc and small fillet	Irregular	Curve
Side shape (x_3)						
Function button shape (x_4)	Round	Square and round inner	Small squares	Large square	Wide large	No number buttons
Number buttons style (x_5)					Other style	No number buttons
Screen size (x_6)	≤ 2.2 in	2.4–2.8 in	≥ 3 in	One piece		
Thickness (x_7)	≤ 10 mm	11–14 mm	15–18 mm	≥ 19 mm		
Layout (x_8)				Other layout		
Weight (x_9)	≤ 80 g	83–100 g	101–120 g	125–140 g	141–149 g	≥ 150 g

Alt. – Alternative.

$$b_{ij} = \begin{cases} \frac{-s_j}{(b_i - a_i)(t_j - s_j)} & a_i \leq x_3 \leq b_i \text{ and } s_j \leq x_6 \leq t_j \\ \frac{u_j}{(b_i - a_i)(u_j - t_j)} & a_i \leq x_3 \leq b_i \text{ and } t_j \leq x_6 \leq u_j \\ \frac{s_j}{(c_i - b_i)(t_j - s_j)} & b_i \leq x_3 \leq c_i \text{ and } s_j \leq x_6 \leq t_j \\ \frac{-u_j}{(c_i - b_i)(u_j - t_j)} & b_i \leq x_3 \leq c_i \text{ and } t_j \leq x_6 \leq u_j \\ 0 & \text{otherwise} \end{cases}$$

$$c_{ij} = \begin{cases} \frac{-a_i}{(b_i - a_i)(t_j - s_j)} & a_i \leq x_3 \leq b_i \text{ and } s_j \leq x_6 \leq t_j \\ \frac{a_i}{(b_i - a_i)(u_j - t_j)} & a_i \leq x_3 \leq b_i \text{ and } t_j \leq x_6 \leq u_j \\ \frac{c_i}{(c_i - b_i)(t_j - s_j)} & b_i \leq x_3 \leq c_i \text{ and } s_j \leq x_6 \leq t_j \\ \frac{-c_i}{(c_i - b_i)(u_j - t_j)} & b_i \leq x_3 \leq c_i \text{ and } t_j \leq x_6 \leq u_j \\ 0 & \text{otherwise} \end{cases}$$

$$d_{ij} = \begin{cases} \frac{a_i s_j}{(b_i - a_i)(t_j - s_j)} & a_i \leq x_3 \leq b_i \text{ and } s_j \leq x_6 \leq t_j \\ \frac{-a_i u_j}{(b_i - a_i)(u_j - t_j)} & a_i \leq x_3 \leq b_i \text{ and } t_j \leq x_6 \leq u_j \\ \frac{-c_i s_j}{(c_i - b_i)(t_j - s_j)} & b_i \leq x_3 \leq c_i \text{ and } s_j \leq x_6 \leq t_j \\ \frac{c_i u_j}{(c_i - b_i)(u_j - t_j)} & b_i \leq x_3 \leq c_i \text{ and } t_j \leq x_6 \leq u_j \\ 0 & \text{otherwise} \end{cases}$$

Given that

$$O_{ij} = \bar{w}_{ij} \cdot f_{ij} = \frac{(a_{ij}x_3x_6 + b_{ij}x_3 + c_{ij}x_6 + d_{ij})(p_{ij}x_3 + q_{ij}x_6 + r_{ij})}{W}$$

$$= \frac{a_{ij}p_{ij}(x_3)^2x_6 + a_{ij}q_{ij}x_3(x_6)^2 + b_{ij}p_{ij}(x_3)^2 + c_{ij}q_{ij}(x_6)^2 + (c_{ij}p_{ij} + b_{ij}q_{ij} + a_{ij}r_{ij})x_3x_6 + (d_{ij}p_{ij} + b_{ij}r_{ij})x_3 + (d_{ij}q_{ij} + c_{ij}r_{ij})x_6 + d_{ij}r_{ij}}{W} \quad (20)$$

552 The customer satisfaction model for S–C can be formulated by
551 (5), as follows:

In this study, triangular-shaped membership functions are
used. Both inputs have three linguistic descriptions: small, med-

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$$y = \sum_{i=1}^3 \sum_{j=1}^3 O_{ij} = \sum_{i=1}^3 \sum_{j=1}^3 \bar{w}_{ij} \cdot f_{ij} = \frac{AP(x_3)^2 x_6 + AQx_3(x_6)^2 + BP(x_3)^2 + CQ(x_6)^2 + (CP + BQ + AR)x_3x_6 + (DP + BR)x_3 + (DQ + CR)x_6 + DR}{Ax_3x_6 + Bx_3 + Cx_6 + D} \quad (21)$$

552 where
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$$AP = \sum_{i=1}^3 \sum_{j=1}^3 a_{ij}p_{ij}, \quad AQ = \sum_{i=1}^3 \sum_{j=1}^3 a_{ij}q_{ij}, \quad BP = \sum_{i=1}^3 \sum_{j=1}^3 b_{ij}p_{ij},$$

$$555 \quad CQ = \sum_{i=1}^3 \sum_{j=1}^3 c_{ij}q_{ij}, \quad CP = \sum_{i=1}^3 \sum_{j=1}^3 c_{ij}p_{ij},$$

$$556 \quad BQ = \sum_{i=1}^3 \sum_{j=1}^3 b_{ij}q_{ij}, \quad AR = \sum_{i=1}^3 \sum_{j=1}^3 a_{ij}r_{ij}, \quad DP = \sum_{i=1}^3 \sum_{j=1}^3 d_{ij}p_{ij},$$

$$559 \quad BR = \sum_{i=1}^3 \sum_{j=1}^3 b_{ij}r_{ij}, \quad DQ = \sum_{i=1}^3 \sum_{j=1}^3 d_{ij}q_{ij},$$

$$560 \quad CR = \sum_{i=1}^3 \sum_{j=1}^3 c_{ij}r_{ij}, \quad DR = \sum_{i=1}^3 \sum_{j=1}^3 d_{ij}r_{ij}, \quad A = \sum_{i=1}^3 \sum_{j=1}^3 a_{ij},$$

$$562 \quad B = \sum_{i=1}^3 \sum_{j=1}^3 b_{ij}, \quad C = \sum_{i=1}^3 \sum_{j=1}^3 c_{ij}, \quad D = \sum_{i=1}^3 \sum_{j=1}^3 d_{ij}$$

ium, and large. The parameter settings of the proposed approaches
for two inputs, three inputs and four inputs are shown in Table 4.
Using the two inputs as an example, six sets of the antecedent
parameters (a_i, b_i, c_i) are available and the number of antecedent
parameters to be identified is $6 \times 3 = 18$. The number of fuzzy rules
is $3 \times 3 = 9$, and the number of consequent parameters to be
trained is $9 \times 3 = 27$. The size of the particle swarm was set as
30. The number of dimensions of the search space for PSO is 18,
which is equal to the number of the antecedent parameters. The
iteration number is directly related to the search time which was
determined as 200 through the repeated operations to make sure
that the least number of iterations and the proper search range
can be obtained. The upper and lower values of the inertia weight
 w are 0.9 and 0.1, respectively. The learning factors c_1 and c_2
were set as 2. The proposed approaches were implemented using a
Matlab software package to generate models that relate affective
responses and the design attributes. Assuming that the values of
the inputs belong to the left range of the membership function,
the generated S–C models with two inputs, three inputs and four
inputs are obtained as shown in (22)–(24), respectively.

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Table 2
Design matrix of 32 mobile phone samples.

Phone No.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	S–C	U–G	H–C	H–B
1	3	3	1	3	2	2	3	1	2	1.85	3.62	2.97	2.56
2	3	3	2	2	1	1	2	1	2	2.59	3.44	3.15	2.79
3	6	6	1	1	5	1	4	1	4	2.88	2.76	3.21	3.32
4	4	4	3	1	6	1	2	2	2	2.41	2.65	2.88	2.59
5	3	4	3	4	6	1	2	2	3	2.06	2.85	2.53	2.47
6	3	3	1	5	6	2	3	2	4	2.71	2.41	2.15	3.18
7	1	1	2	4	6	2	4	2	4	3.26	2.53	2.47	3.18
8	1	1	1	2	6	2	2	2	2	2.79	2.74	2.50	2.71
9	3	4	6	1	6	1	3	2	2	2.91	2.65	2.85	3.12
10	4	4	3	6	4	1	2	1	2	2.65	2.82	3.00	2.15
11	2	2	6	5	6	2	4	2	3	2.76	2.62	2.47	3.18
12	2	2	6	3	6	2	3	2	4	2.71	2.56	2.41	3.38
13	6	6	6	4	6	1	3	2	2	2.09	2.76	2.85	2.71
14	4	4	2	6	6	3	2	3	2	2.21	2.09	2.09	1.94
15	4	3	6	1	6	2	3	2	4	2.44	2.82	2.71	3.09
16	3	3	6	5	6	3	3	2	5	2.62	2.15	2.35	2.94
17	3	3	2	6	6	3	2	3	3	2.12	2.53	2.35	3.03
18	2	4	6	5	2	1	1	1	2	2.50	3.38	2.97	2.59
19	3	3	1	4	5	2	3	1	3	2.41	3.00	3.00	3.03
20	4	4	6	5	1	1	2	1	3	2.68	3.68	3.53	3.06
21	4	4	1	1	2	1	2	1	2	2.88	3.35	3.29	3.12
22	6	4	3	1	4	2	2	1	3	2.88	2.94	2.97	2.97
23	3	3	6	2	3	1	3	1	3	3.12	3.38	3.15	3.56
24	5	5	1	4	3	1	2	1	1	2.50	2.85	3.24	2.62
25	4	4	6	1	6	1	3	2	2	2.44	3.21	3.06	3.09
26	3	6	5	1	6	2	3	2	3	2.68	2.97	2.85	3.32
27	1	1	5	1	6	1	2	2	3	2.65	2.79	2.79	2.91
28	3	3	4	1	6	3	2	3	4	2.00	1.91	1.91	2.53
29	4	4	2	1	6	2	2	2	3	2.41	2.47	2.21	2.56
30	4	4	4	5	2	2	3	1	2	3.26	3.15	2.82	3.03
31	3	3	1	6	6	2	3	4	3	3.38	2.79	2.76	3.18
32	3	3	1	1	6	2	3	2	6	2.32	2.62	2.56	3.50

Table 3
Attribute reduction sheet for S–C.

No.	Reduct	Support	Length
1	{ x_1, x_3, x_9 }	100	3
2	{ x_3, x_4, x_6 }	100	3
3	{ x_4, x_6, x_9 }	100	3
4	{ x_2, x_6, x_9 }	100	3
5	{ x_2, x_3, x_6 }	100	3
6	{ x_2, x_6, x_7 }	100	3
7	{ x_4, x_5, x_6 }	100	3
8	{ x_2, x_3, x_5, x_9 }	100	4
9	{ x_1, x_3, x_6, x_8 }	100	4
10	{ x_2, x_3, x_4, x_9 }	100	4
11	{ x_1, x_5, x_7, x_9 }	100	4
12	{ x_1, x_6, x_8, x_9 }	100	4
13	{ x_1, x_2, x_3, x_4 }	100	4
14	{ x_1, x_5, x_6, x_9 }	100	4
15	{ x_1, x_6, x_7, x_9 }	100	4
16	{ x_1, x_3, x_5, x_6 }	100	4
17	{ x_2, x_3, x_4, x_8 }	100	4
18	{ x_3, x_6, x_7, x_9 }	100	4
19	{ x_3, x_5, x_6, x_9 }	100	4
20	{ x_3, x_5, x_6, x_7 }	100	4
21	{ x_1, x_4, x_7, x_9 }	100	4
22	{ x_2, x_3, x_4, x_5 }	100	4
23	{ x_1, x_2, x_3, x_8 }	100	4
24	{ x_3, x_4, x_7, x_9 }	100	4
25	{ x_2, x_4, x_7, x_9 }	100	4
26	{ x_3, x_7, x_8, x_9 }	100	4
27	{ x_2, x_3, x_8, x_9 }	100	4
28	{ x_1, x_4, x_8, x_9 }	100	4
29	{ x_1, x_5, x_6, x_7 }	100	4
30	{ x_1, x_4, x_6, x_7 }	100	4
31	{ x_3, x_6, x_7, x_8 }	100	4
32	{ x_1, x_6, x_7, x_8 }	100	4
33	{ x_4, x_6, x_7, x_8 }	100	4

Table 4
Parameter settings of the proposed approaches for different inputs.

Parameters	Two inputs (x_6 and x_3)	Three inputs (x_6, x_3 and x_9)	Four inputs (x_6, x_3, x_9 and x_7)
Number of antecedent parameters	18	27	36
Number of consequent parameters	27	108	405
Number of fuzzy rules	9	27	81
Dimensions of the search space	18	27	36
The size of particle swarm		30	
Iteration number		200	
Inertia weight		[0.1, 0.9]	
Learning factors		2	

Table 5
Comparison of modeling results for two inputs, three inputs and four inputs.

Training results	Two inputs (x_6 and x_3)	Three inputs (x_6, x_3 and x_9)	Four inputs (x_6, x_3, x_9 and x_7)
Structure (number of terms)	12	28	64
ME (%)	4.1071×10^{-2}	6.7214×10^{-2}	4.0846×10^{-2}
VoE (%)	3.3555×10^{-2}	7.9524×10^{-2}	3.2517×10^{-2}

$$\begin{aligned}
 &0.0019x_3(x_6)^2x_7x_9 + 0.0007(x_3)^2x_6x_7x_9 - 0.0020x_3x_6x_7(x_9)^2 \\
 &+ 0.0021x_3x_6(x_7)^2x_9 + 0.0232(x_6)^2x_7x_9 + 0.0299x_3(x_6)^2x_7 \\
 &+ 0.0021x_3(x_6)^2x_9 - 0.0039(x_3)^2x_7x_9 + 0.0660(x_3)^2x_6x_7 \\
 &- 0.0016(x_3)^2x_6x_9 - 0.0019x_3x_7(x_9)^2 + 0.0177x_6x_7(x_9)^2 \\
 &- 0.0108x_3x_6(x_9)^2 - 0.0009x_3(x_7)^2x_9 + 0.0276x_6(x_7)^2x_9 \\
 &+ 0.0271x_3x_6(x_7)^2 + 0.0198x_3x_6x_7x_9 + 0.3469(x_6)^2x_7 + 0.0304(x_6)^2x_9 \\
 &+ 0.0524x_3(x_6)^2 - 0.0111(x_3)^2x_7 - 0.0049(x_3)^2x_9 + 0.0819(x_3)^2x_6 \\
 &+ 0.0142x_7(x_9)^2 - 0.0010x_3(x_9)^2 - 0.0730x_6(x_9)^2 - 0.0055(x_7)^2x_9 \\
 &- 0.0074x_3(x_7)^2 + 0.2745x_6(x_7)^2 + 0.3810x_6x_7x_9 + 0.1925x_3x_6x_7 \\
 &- 0.0525x_3x_6x_9 - 0.0443x_3x_7x_9 + 0.4586x_6x_7 - 0.7456x_6x_9 + 0.3896x_3x_6 \\
 &- 0.3440x_3x_7 - 0.0459x_3x_9 - 0.0566x_7x_9 + 0.6697(x_6)^2 - 0.0124(x_3)^2 \\
 &+ 0.0418(x_9)^2 - 0.0896(x_7)^2 - 0.0962x_6 - 0.5378x_3 + 0.2785x_9 \\
 &y_1 = \frac{-0.2842x_7 - 0.1167}{0.0656x_3x_6x_7x_9 - 0.0581x_3x_7x_9 - 0.0635x_6x_7x_9 - 0.0714x_3x_6x_7} \\
 &- 0.0699x_3x_6x_9 + 0.0562x_7x_9 + 0.0633x_3x_7 + 0.0692x_6x_7 + 0.0619x_3x_9 \\
 &+ 0.0677x_6x_9 + 0.0762x_3x_6 - 0.0612x_7 - 0.0600x_9 - 0.0675x_3 \\
 &- 0.0738x_6 + 0.0653
 \end{aligned} \quad (24)$$

To compare the modeling results based on the two inputs, three inputs and four inputs, ME and variance of errors (VoE) were adopted, as defined by (25) and (26), respectively.

$$ME = \frac{1}{t} \sum_{k=1}^t \frac{|\hat{y}_k - y_k|}{y_k} \cdot 100 \quad (25)$$

$$VoE = \frac{1}{t-1} \sum_{k=1}^t \left(\frac{|\hat{y}_k - y_k|}{y_k} \cdot 100 - ME \right)^2 \quad (26)$$

where t is the number of data sets. \hat{y}_k is the k th predictive output based on the identified model and y_k is the k th actual output based on the survey data.

$$y_1 = \frac{0.1170(x_3)^2x_6 - 0.1421x_3(x_6)^2 - 0.1561(x_3)^2 + 1.6626(x_6)^2 + 1.0187x_3x_6 - 0.8097x_3 + 0.4499x_6 + 0.0269}{0.3900x_3x_6 - 0.3870x_3 - 0.4065x_6 + 0.4035} \quad (22)$$

$$\begin{aligned}
 &0.0228x_3(x_6)^2x_9 - 0.0171(x_3)^2x_6x_9 + 0.0058x_3x_6(x_9)^2 + 0.0541(x_6)^2x_9 \\
 &+ 0.3443x_3(x_6)^2 + 0.0005(x_3)^2x_9 + 0.0098(x_3)^2x_6 + 0.0216x_6(x_9)^2 \\
 &- 0.0040x_3(x_9)^2 + 0.1790x_3x_6x_9 + 0.8213(x_6)^2 + 0.0028(x_3)^2 + 0.0009(x_9)^2 \\
 &+ 0.3974x_6x_9 - 0.9325x_3x_6 - 0.2384x_3x_9 - 0.1120x_6 + 0.5569x_3 \\
 &- 0.1383x_9 - 0.0755 \\
 &y_1 = \frac{0.0742x_3x_6x_9 - 0.0829x_3x_9 - 0.0941x_6x_9 - 0.0740x_3x_6 + 0.1051x_9}{+ 0.0828x_3 + 0.0939x_6 - 0.1049}
 \end{aligned} \quad (23)$$

The training errors and structure of the generated models are compared in Table 5. From the table, it can be seen that the values of ME and VoE for two inputs, three inputs and four inputs all are very small and have the same order of magnitude. However, the number of terms of the generated models based on the four inputs and three inputs are five times and two times more than that with two inputs, respectively. On the other hand, the number of fuzzy rules generated for the four inputs and three inputs are nine times

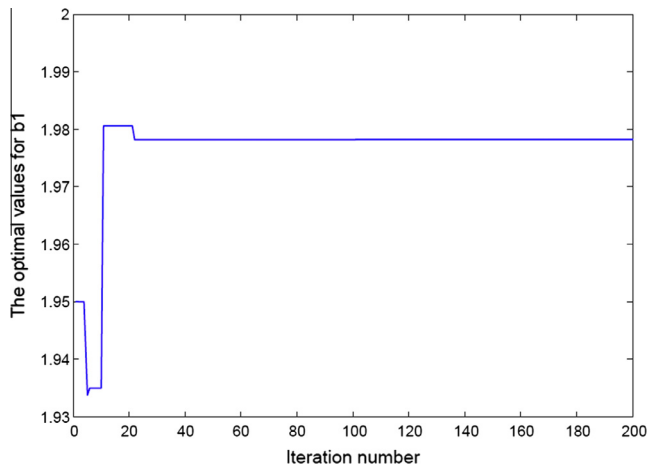


Fig. 3. Results of the iteration process of PSO for b_1 .

and three times more than that for the two inputs, respectively. Therefore, having more inputs could substantially increase the complexity of the models and cause long computational time. In this research, models with two inputs were selected because of their simpler structures and good training accuracy.

Using S-C as an example, the inputs were selected as x_6 and x_3 , namely 'screen size' and 'side shape'. Though intuitively the attributes 'layout' and 'number buttons style' are relevant to the S-C dimension, when we look at the morphological analysis of the 32 mobile phones and also their product images, it can be noted that the larger screen size of mobile phones is, the less number of buttons and larger screen layout are. The effect of 'screen size' is more dominated compared with the other two attributes in the survey data. Thus, the 'screen size' and 'side shape' were picked up by the algorithms.

3.2. Evaluation of the proposed approaches

To evaluate the effectiveness of the proposed approaches, the modeling results based on the proposed approaches are compared

Table 6
Developed models and their training results.

Affective responses	Methods	Generated models	Training error (%)	
			ME	VoE
S-C	FLSR	$y_1 = (2.1624, 2.4883) + (0.1424, 0.4980)x_1 + (-0.1442, 0.4677)x_2 + (0.0350, 0)x_3 + (0.0423, 0.3680)x_4 + (-0.0047, 0)x_5 + (-0.1014, 1.2000)x_6 + (0.0323, 0.8275)x_7 + (0.0487, 0)x_8 + (0.0832, 0)x_9$	11.9358	86.7696
	FR	$y_1 = (2.2926, 0.0398) + (0.0794, 0.8812)x_1 + (-0.0532, 1.5028)x_2 + (0.0268, 0.4970)x_3 + (0.0080, 0.4426)x_4 + (-0.0299, 11.4561)x_5 + (-0.3197, 0.1294)x_6 + (0.1200, 0.1075)x_7 + (0.0898, 0.0815)x_8 + (0.1048, 0.0391)x_9$	10.5370	99.3563
	GP-FR	$y_1 = (3.0111, 0)x_1x_8 + (-0.0655, 1.9000)$	8.6447	63.2346
	RS-PSO-ANFIS	$y_1 = \frac{0.1170(x_3)^2x_6 - 0.1421x_3(x_6)^2 - 0.1561(x_3)^2 + 1.6626(x_6)^2 + 1.0187x_3x_6 - 0.8097x_3 + 0.4499x_6 + 0.0269}{0.3900x_3x_6 - 0.3870x_3 - 0.4065x_6 + 0.4035}$	4.1071×10^{-2}	3.3555×10^{-2}
U-G	FLSR	$y_2 = (3.0714, 1.2023) + (-0.0208, 0.1323)x_1 + (0.0311, 0.0706)x_2 + (0.0183, 0.0205)x_3 + (0.0197, 0.2203)x_4 + (-0.1360, 0.0752)x_5 + (-0.0692, 0.7018)x_6 + (0.2407, 0.4483)x_7 + (-0.0305, 0.2812)x_8 + (-0.0704, 0.0680)x_9$	8.7704	22.1099
	FR	$y_2 = (3.7652, 0) + (-0.0265, 0.0115)x_1 + (-0.0181, 0.0431)x_2 + (0.0164, 0)x_3 + (-0.0172, 0.0310)x_4 + (-0.1966, 0.0625)x_5 + (-0.1601, 0)x_6 + (0.2352, 0)x_7 + (0.0234, 0)x_8 + (-0.0913, 0)x_9$	6.9932	16.8974
	GP-FR	$y_2 = (3.7400, 0)x_5 + (-0.1025, 0)(x_1 + x_5x_6) + (-0.0322, 0.4066)$	5.9050	22.7076
	RS-PSO-ANFIS	$y_2 = \frac{-0.0295(x_1)^2x_7 + 0.1912x_1(x_7)^2 - 0.0334(x_1)^2 - 0.1516(x_7)^2 + 0.0155x_1x_7 + 0.0122x_1 + 0.0346x_7 + 1.8739}{0.2141x_1x_7 - 0.2943x_1 - 0.9145x_7 + 1.2570}$	4.1701×10^{-2}	2.2427×10^{-2}
H-C	FLSR	$y_3 = (2.8018, 1.1861) + (0.0294, 0.1507)x_1 + (0.0521, 0.0930)x_2 + (0.0370, 0)x_3 + (-0.0125, 0.2105)x_4 + (-0.0822, 0.1407)x_5 + (-0.1852, 0.6564)x_6 + (0.1857, 0.2869)x_7 + (-0.0602, 0.1308)x_8 + (-0.0227, 0.2046)x_9$	6.8264	23.0816
	FR	$y_3 = (3.4891, 0) + (0.0407, 0)x_1 + (-0.0052, 0)x_2 + (0.0236, 0)x_3 + (0.0124, 0.0509)x_4 + (-0.0544, 0.0790)x_5 + (-0.3748, 0)x_6 + (0.0953, 0)x_7 + (-0.1003, 0)x_8 + (-0.0440, 0)x_9$	6.2405	21.2462
	GP-FR	$y_3 = (4.3690, 0.0826)x_8 + (-0.9904, 0.1261)x_8^2 + (0.1857, 2.3901)x_6 + (-0.3144, 0.2013)$	4.8831	20.9459
	RS-PSO-ANFIS	$y_3 = \frac{0.3778(x_1)^2x_3 - 0.2061x_1(x_3)^2 - 0.0577(x_1)^2 + 1.7976(x_3)^2 + 0.6227x_1x_3 - 0.2026x_1 - 6.3250x_3 + 5.8372}{0.0394x_1x_3 - 0.0595x_1 - 0.0613x_3 + 0.0925}$	1.3829×10^{-2}	1.7254×10^{-3}
H-B	FLSR	$y_4 = (1.4996, 0.6132) + (0.0259, 0.2954)x_1 + (0.0518, 0.2442)x_2 + (0.0404, 0)x_3 + (-0.0528, 0.0018)x_4 + (-0.0157, 0.0446)x_5 + (-0.0023, 0.4259)x_6 + (0.3466, 0.0134)x_7 + (0.0112, 0.3099)x_8 + (0.1391, 0.0515)x_9$	9.0608	42.9147
	FR	$y_4 = (1.7395, 0) + (0.0352, 0.0117)x_1 + (0.0493, 0.0167)x_2 + (0.0215, 0)x_3 + (-0.0456, 0.0043)x_4 + (-0.1063, 0.1479)x_5 + (-0.2709, 0)x_6 + (0.2904, 0)x_7 + (0.3296, 0)x_8 + (0.2166, 0)x_9$	8.6941	39.4641
	GP-FR	$y_4 = (2.6600, 0)x_9x_7 + (0.0711, 0.0171)x_5x_2x_6 + (-0.0110, 0.3819)$	7.3000	48.9273
	RS-PSO-ANFIS	$y_4 = \frac{-0.0689(x_7)^2x_9 + 0.0595x_7(x_9)^2 + 0.4928(x_7)^2 - 0.0146(x_9)^2 + 0.1840x_7x_9 + 0.5159x_7 - 0.2617x_9 + 0.4097}{0.1614x_7x_9 - 0.1698x_7 - 0.2114x_9 + 0.2224}$	1.2640×10^{-2}	3.6467×10^{-3}

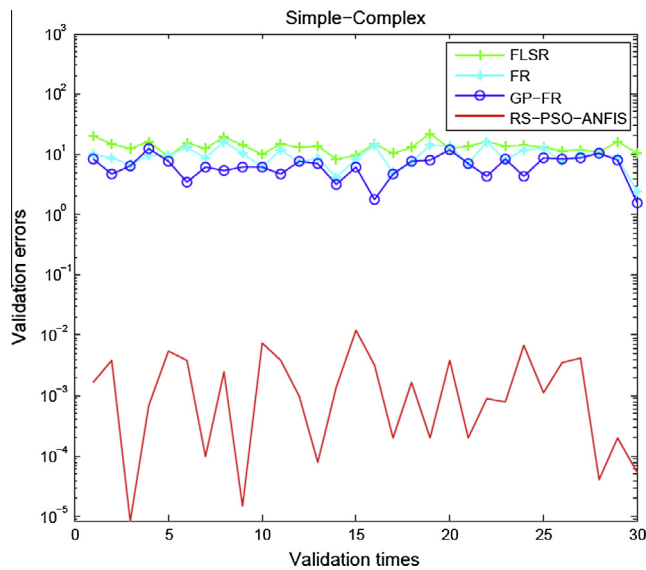


Fig. 4. Validation results of the models for S-C.

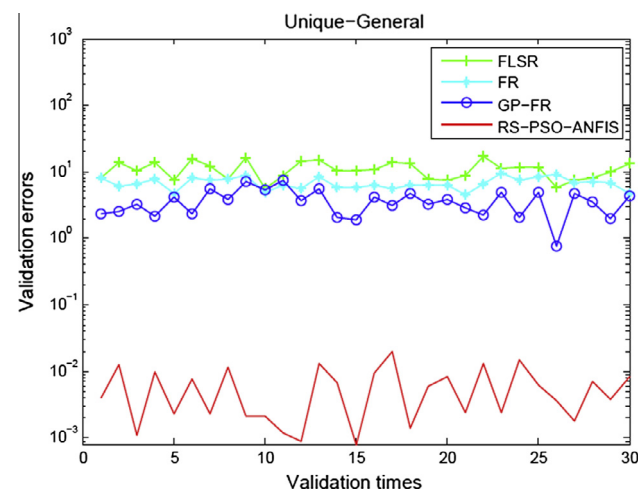


Fig. 5. Validation results of the models for U-G.

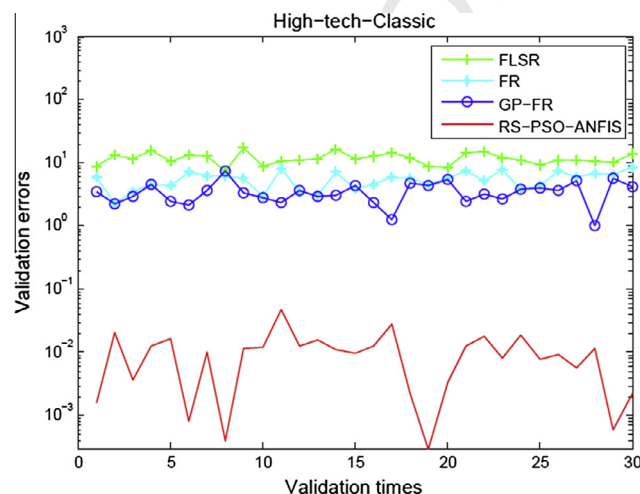


Fig. 6. Validation results of the models for H-C.

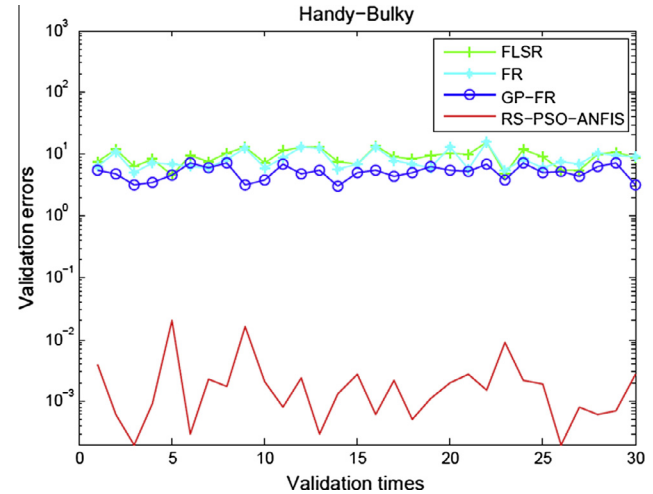


Fig. 7. Validation results of the models for H-B.

Table 7

Means and variances of the validation errors for the four affective dimensions.

Affective responses	Validation error	FLSR	FR	GP-FR	RS-PSO-ANFIS
S-C	ME (%)	13.5352	9.5727	6.7991	0.0024
	VoE (%)	74.5989	83.2105	35.8331	4.7686×10^{-5}
U-G	ME (%)	10.8509	6.7610	3.7295	0.0062
	VoE (%)	72.7121	14.6969	12.9238	1.6056×10^{-4}
H-C	ME (%)	11.9449	5.4254	3.6154	0.0110
	VoE (%)	40.1570	16.5305	12.1622	6.2361×10^{-4}
H-B	ME (%)	9.2836	8.4071	5.6903	0.0028
	VoE (%)	44.1524	37.2721	25.9304	1.2190×10^{-4}

with those based on ANFIS, fuzzy least-squares regression (FLSR), fuzzy regression (FR) and genetic programming based fuzzy regression (GP-FR). However, the ANFIS models could not be developed since the training process of ANFIS was a failure and an 'out of memory' error occurred, because its structure was too complex. Considering that the PSO-based ANFIS is a stochastic method, 30 runs on the proposed approaches were conducted, and the mean of the 30 runs was calculated. The generated fuzzy rules (R_{ij}) for the S-C, U-G, H-C, and H-B are shown in Appendix A, where $i = 1, 2, 3, j = 1, 2, 3$. The optimal value setting of the antecedent parameters is determined through the iteration of PSO. Fig. 3 shows the results of the iteration process of PSO for the center of the first membership function b_1 for S-C.

The same survey data was also used to develop the models based on the proposed approaches, FLSR, FR and GP-FR approaches for the four affective dimensions. Table 6 shows the developed models, training errors, and the variance of training errors. From the table, it can be seen that all the developed models can capture the fuzziness of the modeling. However, only the models developed based on the proposed approaches and the GP-FR models can address the nonlinearity of the modeling. The table also shows that the values of ME and VoE based on the proposed approaches are the smallest compared with those based on the other three approaches.

4. Validation of the proposed approaches

A total of 30 validation tests were conducted to further evaluate the effectiveness of the proposed methodology. In each validation test, five data sets were randomly selected from the 32 data sets

as the testing data sets, and the remaining 27 data sets were used to develop the customer satisfaction models. The validation tests primarily aim to compare the validation errors of the generated customer satisfaction models based on the proposed approaches with those based on FLSR, FR, and GP-FR.

FLSR is developed based on the definition of weighted fuzzy arithmetic and the least squares fitting criterion [35]. Different values for h ($0 \leq h < 1$) were selected to examine how h affects the results of FLSR [36]. It was found that the changes of h value do not affect the center value of each fuzzy coefficient but influence the value of spread. Also, when a larger value of h is chosen, the prediction capability of the models would increase. Thus, in this study, the h value of FLSR was set as 0.9 for obtaining good prediction capability. After a number of trials using different h values within a range of $[0, 1]$, the h values of FR were set as 0.9 for S-C and 0.5 for U-G, H-C, and H-B, as these settings led to the smallest modeling errors. For GP-FR, the population size and the number of iteration were set as 40 and 200, respectively. The generation gap, crossover probability, and mutation probability were set as 0.8, 0.7, and 0.3, respectively. The maximum depth of tree was set as 5. The parameter settings of the generated models based on the proposed approaches are shown in Section 3.1. The validation errors and VoE were obtained using (25) and (26), respectively. The 30 validation results for the S-C, U-G, H-C, and H-B models based on the four methods are shown in Figs. 4–7, respectively. The lines with '+', '*', 'O', and the solid line '-' denote the validation results of the FLSR, FR, GP-FR, and the proposed approaches, respectively. Table 7 shows the mean validation errors and the mean VoE for the four affective dimensions S-C, U-G, H-C, and H-B based on the four approaches. From the figures and the table, it can be seen that the proposed approaches outperform the other three approaches in modeling customer satisfaction for affective design in terms of prediction errors, mean validation errors and mean VoE for all the affective dimensions.

5. Conclusion

ANFIS was shown to be an effective approach to generate explicit customer satisfaction models for affective design, and can address both fuzziness and nonlinearity of the modeling. However, it is incapable of modeling the problems that involve a number of inputs. Additionally, the conventional learning algorithm of ANFIS is based on the gradient descent method, which leads to slow convergence of the parameters. In this paper, RS and PSO-based ANFIS approaches to modeling customer satisfaction for affective design are proposed to overcome the limitation and further improve the modeling accuracy. In the proposed approaches, RS theory is introduced to reduce the number of inputs and determine the indispensable attributes as the inputs of PSO-based ANFIS. PSO is employed to determine the parameter settings of the ANFIS which can provide better modeling accuracy. A case study of affective product design of mobile phones was conducted to illustrate and validate the proposed approaches. The four affective dimensions, namely, S-C, U-G, H-C, and H-B, were considered. A total of 30 validation tests were conducted to evaluate the effectiveness of the proposed approaches. At the beginning, we included all the nine design attributes as the inputs of an ANFIS, but 'out of memory' error occurred and the training process of ANFIS failed due to highly complex structure of the ANFIS. With the proposed approaches, explicit customer satisfaction models can be generated which can address both the nonlinearity and fuzziness of the modeling. Compared with the FLSR, FR, and GP-FR approaches in modeling customer satisfaction for affective design, the proposed approaches perform better than all these

approaches in terms of training errors and validation errors. Future work could involve a study of determining optimal settings of design attributes for affective product design based on the generated customer satisfaction models. On the other hand, some techniques could be explored to simplify the structures of the generated customer satisfaction models.

Acknowledgement

The work described in this paper was fully supported by a grant from The Hong Kong Polytechnic University (Project No. G-YK81).

Appendix A

Fuzzy rules for S-C are shown as follows:

- R_{11} : IF x_6 is μ_1 AND x_3 is λ_1 , THEN $f_{11} = 1.5117x_6 - 0.3394x_3 + 1.3808$
- R_{12} : IF x_6 is μ_1 AND x_3 is λ_2 , THEN $f_{12} = 0.7060x_6 + 1.4597x_3 + 0.3531$
- R_{13} : IF x_6 is μ_1 AND x_3 is λ_3 , THEN $f_{13} = 0.4722x_6 + 0.1991x_3 + 0.1180$
- R_{21} : IF x_6 is μ_2 AND x_3 is λ_1 , THEN $f_{21} = 2.3674x_6 - 0.2254x_3 + 3.9512$
- R_{22} : IF x_6 is μ_2 AND x_3 is λ_2 , THEN $f_{22} = -8.5420x_6 + 1.4312x_3 - 4.2708$
- R_{23} : IF x_6 is μ_2 AND x_3 is λ_3 , THEN $f_{23} = 0.0369x_6 - 0.0121x_3 + 0.0092$
- R_{31} : IF x_6 is μ_3 AND x_3 is λ_1 , THEN $f_{31} = 0.0035x_6 + 0.0817x_3 + 0.0026$
- R_{32} : IF x_6 is μ_3 AND x_3 is λ_2 , THEN $f_{32} = 0.0051x_6 + 0.0823x_3 + 0.0026$
- R_{33} : IF x_6 is μ_3 AND x_3 is λ_3 , THEN $f_{33} = 0.0101x_6 + 0.0805x_3 + 0.0025$

Fuzzy rules for U-G are shown as follows:

- R_{11} : IF x_1 is μ_1 AND x_7 is λ_1 , THEN $f_{11} = -0.4917x_1 - 0.8716x_7 + 4.3940$
- R_{12} : IF x_1 is μ_1 AND x_7 is λ_2 , THEN $f_{12} = -0.0379x_1 + 0.5080x_7 - 0.8927$
- R_{13} : IF x_1 is μ_1 AND x_7 is λ_3 , THEN $f_{13} = 0.1869x_1 + 0.3145x_7 + 0.0393$
- R_{21} : IF x_1 is μ_2 AND x_7 is λ_1 , THEN $f_{21} = -0.4399x_1 + 5.3350x_7 + 3.5210$
- R_{22} : IF x_1 is μ_2 AND x_7 is λ_2 , THEN $f_{22} = -0.9409x_1 + 2.5969x_7 + 0.9266$
- R_{23} : IF x_1 is μ_2 AND x_7 is λ_3 , THEN $f_{23} = 0.0400x_1 + 0.1038x_7 + 0.0130$
- R_{31} : IF x_1 is μ_3 AND x_7 is λ_1 , THEN $f_{31} = 0.0941x_1 + 0.0065x_7 + 0.0029$
- R_{32} : IF x_1 is μ_3 AND x_7 is λ_2 , THEN $f_{32} = 0.0835x_1 + 0.0086x_7 + 0.0026$
- R_{33} : IF x_1 is μ_3 AND x_7 is λ_3 , THEN $f_{33} = 0.0811x_1 + 0.0203x_7 + 0.0025$

Fuzzy rules for H-C are shown as follows:

- R_{11} : IF x_1 is μ_1 AND x_3 is λ_1 , THEN $f_{11} = -0.2365x_1 + 9.5598x_3 + 4.8719$
- R_{12} : IF x_1 is μ_1 AND x_3 is λ_2 , THEN $f_{12} = 45.9082x_1 + 12.3290x_3 - 245.3522$
- R_{13} : IF x_1 is μ_1 AND x_3 is λ_3 , THEN $f_{13} = 0.0496x_1 + 0.0775x_3 + 0.0024$
- R_{21} : IF x_1 is μ_2 AND x_3 is λ_1 , THEN $f_{21} = -0.0533x_1 - 36.4828x_3 - 0.3111$
- R_{22} : IF x_1 is μ_2 AND x_3 is λ_2 , THEN $f_{22} = 41.5233x_1 - 46.2008x_3 + 20.1252$
- R_{23} : IF x_1 is μ_2 AND x_3 is λ_3 , THEN $f_{23} = 0.0042x_1 + 0.0993x_3 + 0.0031$
- R_{31} : IF x_1 is μ_3 AND x_3 is λ_1 , THEN $f_{31} = 0.1029x_1 + 0.0204x_3 + 0.0032$
- R_{32} : IF x_1 is μ_3 AND x_3 is λ_2 , THEN $f_{32} = 0.0462x_1 + 0.0063x_3 + 0.0014$
- R_{33} : IF x_1 is μ_3 AND x_3 is λ_3 , THEN $f_{33} = 0.0445x_1 + 0.0445x_3 + 0.0014$

Fuzzy rules for H-B are shown as follows:

- R_{11} : IF x_7 is μ_1 AND x_9 is λ_1 , THEN $f_{11} = 0.9047x_7 + 0.5873x_9 + 0.6901$
- R_{12} : IF x_7 is μ_1 AND x_9 is λ_2 , THEN $f_{12} = -4.4788x_7 + 0.6989x_9 - 2.2507$
- R_{13} : IF x_7 is μ_1 AND x_9 is λ_3 , THEN $f_{13} = 0.0026x_7 + 0.0204x_9 + 0.0006$
- R_{21} : IF x_7 is μ_2 AND x_9 is λ_1 , THEN $f_{21} = 0.2146x_7 + 0.9237x_9 + 0.0952$
- R_{22} : IF x_7 is μ_2 AND x_9 is λ_2 , THEN $f_{22} = -1.4136x_7 + 0.6612x_9 - 0.8071$
- R_{23} : IF x_7 is μ_2 AND x_9 is λ_3 , THEN $f_{23} = 0.0153x_7 + 0.1222x_9 + 0.0038$
- R_{31} : IF x_7 is μ_3 AND x_9 is λ_1 , THEN $f_{31} = 0.3298x_7 + 0.2137x_9 + 0.0412$
- R_{32} : IF x_7 is μ_3 AND x_9 is λ_2 , THEN $f_{32} = 0.1251x_7 + 0.1017x_9 + 0.0156$
- R_{33} : IF x_7 is μ_3 AND x_9 is λ_3 , THEN $f_{33} = 0$

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